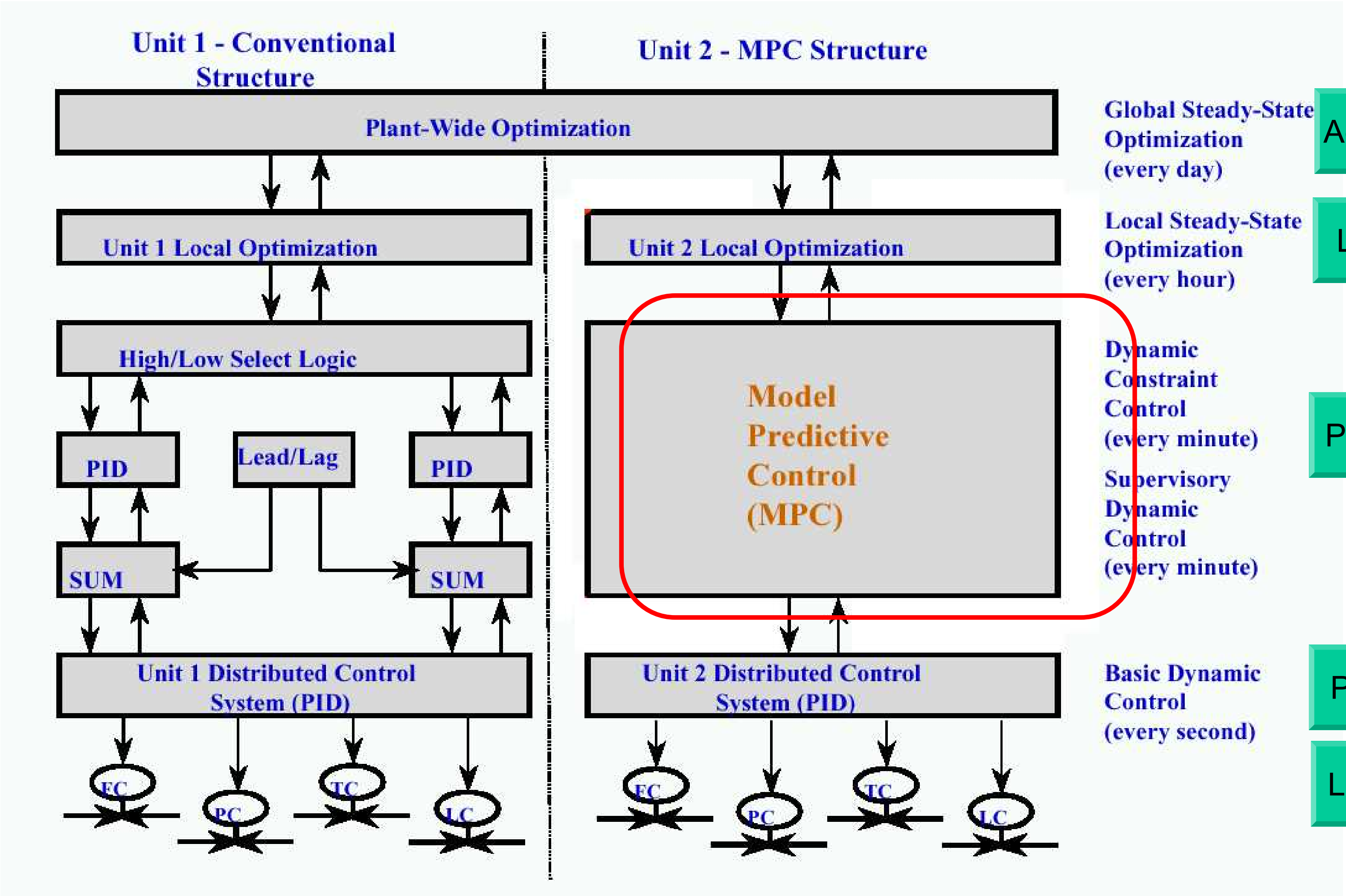
Lecture 15 - Model Predictive Control
Part 2: Industrial MPC

- Prediction model
- Control optimization
- Receding horizon update
- Disturbance estimator - feedback
- IMC representation of MPC

Cascade loops

Control Hierarchy

V&V sim



Unit 1 - Conventional Structure

Unit 2 - MPC Structure

Plant-Wide Optimization

Unit 1 Local Optimization

Unit 2 Local Optimization

High/Low Select Logic

PID

Lead/Lag

PID

SUM

SUM

Model Predictive Control (MPC)

Unit 1 Distributed Control System (PID)

Unit 2 Distributed Control System (PID)

FC PC TC LC

FC PC TC LC

Global Steady-State Optimization (every day)

Local Steady-State Optimization (every hour)

Dynamic Constraint Control (every minute)

Supervisory Dynamic Control (every minute)

Basic Dynamic Control (every second)

Across optimiz

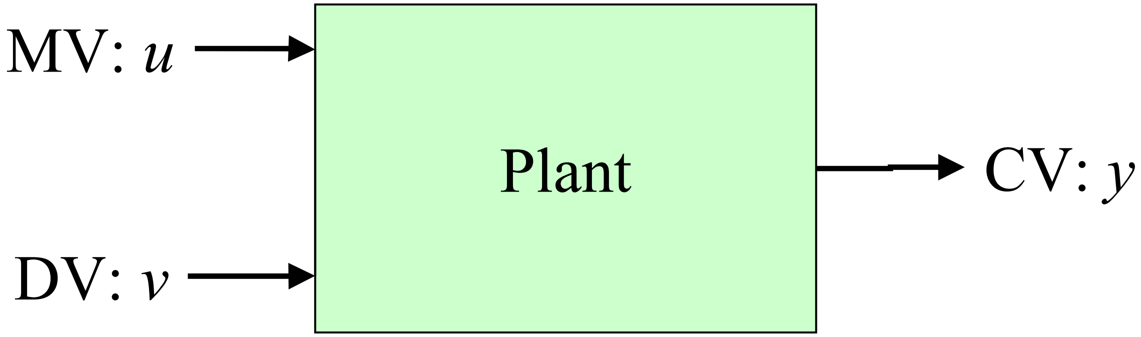
Local optimiz

Path optimiz

PID

Loop shaping

MPC Setup



Plant structure:

- CV - controlled variables – y
  - plant outputs, output errors
- MV - manipulated variables – u
  - control inputs
- DV - disturbance variables - v
  - disturbances and setpoints

Models for MPC

- FIR (Finite Impulse Response) model
  - is used in some formulation
- FSR (Finite Step Response) model
  - Broadly used in process control

$$y(t) = \sum_{k=1}^{n} S^U(k)\Delta u(t-k) + \sum_{k=1}^{n} S^D(k)\Delta v(t-k) + d$$

- Compact notation

$$y(t) = (s^U * \Delta u)(t) + (s^D * \Delta v)(t) + d$$

$$\Delta = 1 - z^{-1}$$

$$h^U = \Delta s^U;$$

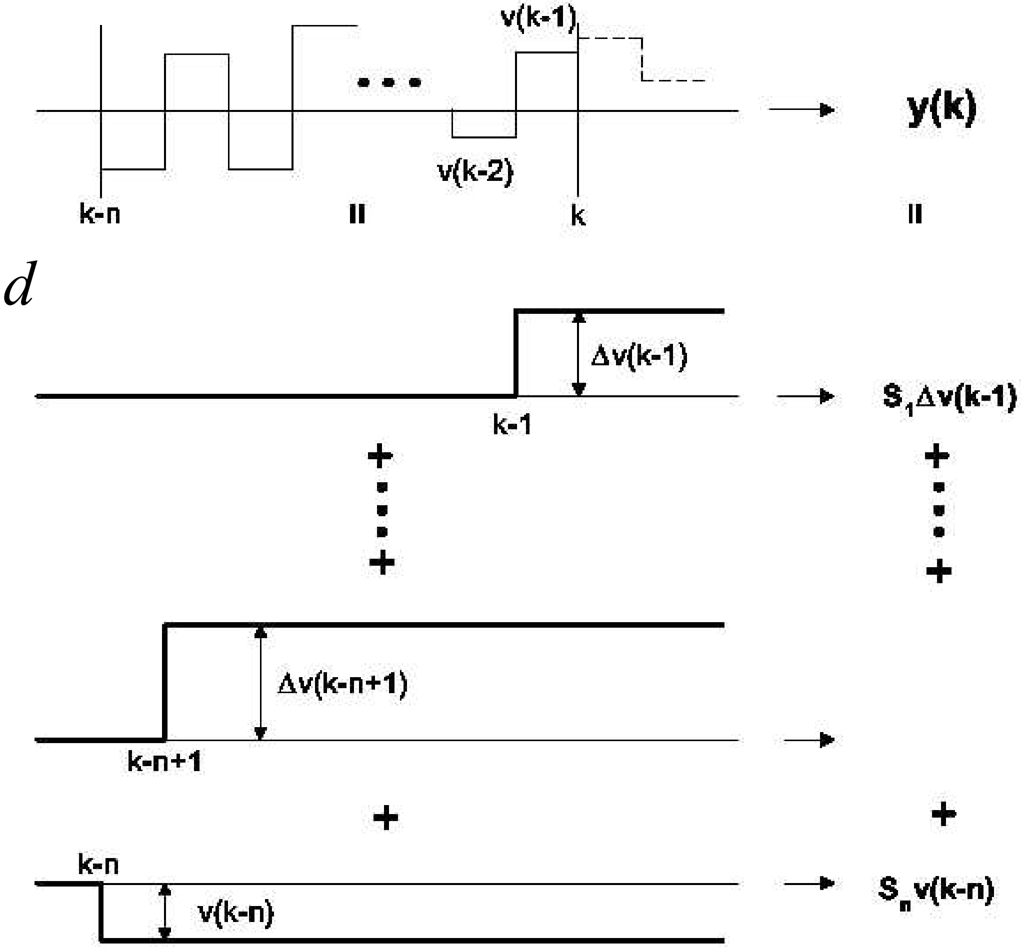
$$s^U * \Delta u = \Delta s^U * u = h^U * u$$

Finite Step Response Model

FSR model

$$y(t)=\sum_{k=1}^{n}S(k)\Delta v(t-k)+d$$

- Ignores anything that happened more than *n* steps in the past
- This is attributed to a constant disturbance *d*



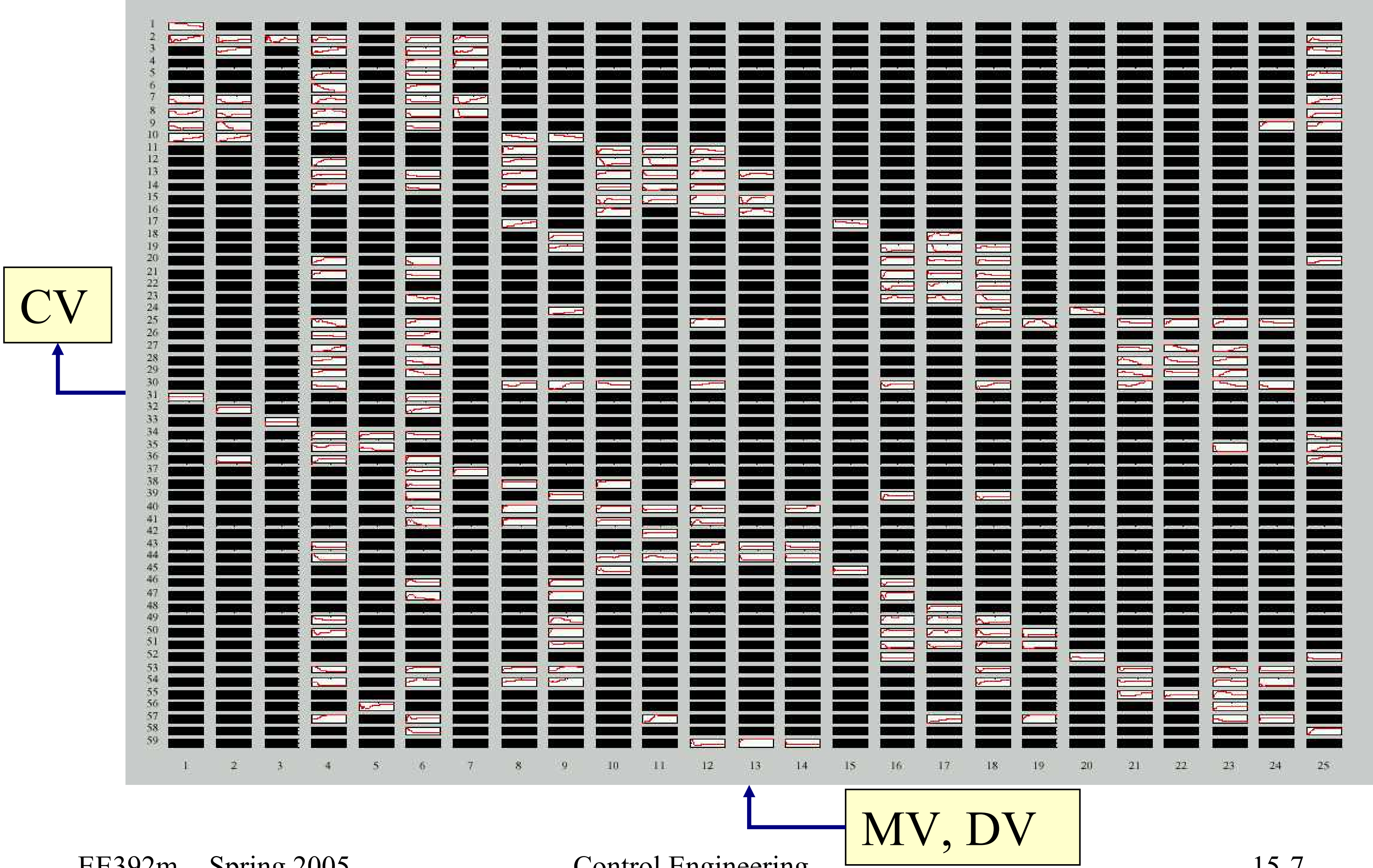
MPC Model Identification

- Identification is a part of most industrial MPC packages
- Step (bump test) or PRBS

FIR model ID

- Step responses are directly used as models

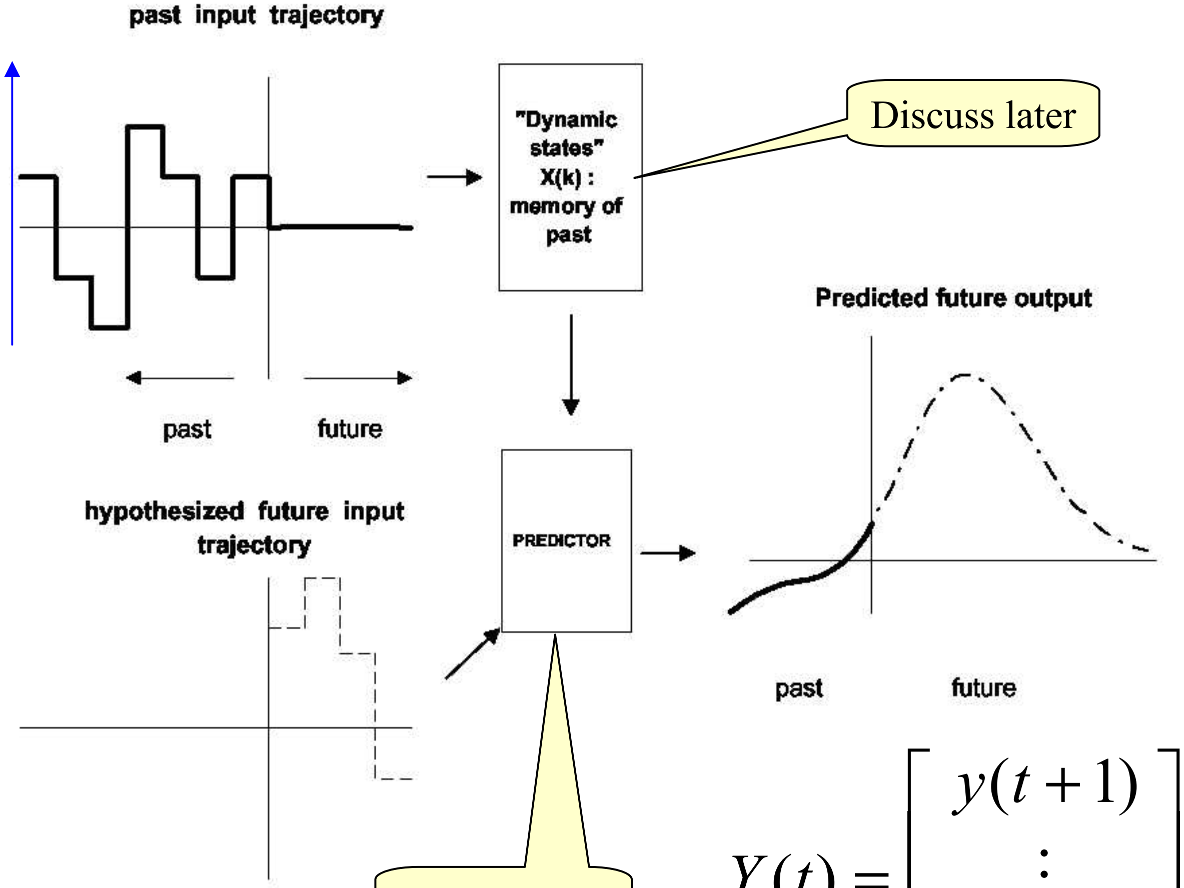
MPC Process Model Example



Prediction Model

$$\underline{U}(t) = \begin{bmatrix} \Delta u(t) \\ \vdots \\ \Delta u(t-n+1) \end{bmatrix}$$

n - memory depth



Discuss later

FSR model

$$U(t) = \begin{bmatrix} \Delta u(t+1) \\ \vdots \\ \Delta u(t+N) \end{bmatrix}$$

$$Y(t) = \begin{bmatrix} y(t+1) \\ \vdots \\ y(t+N) \end{bmatrix}$$

N – prediction horizon

System State for FSR Model

- MPC Concept

$$x(t) \rightarrow [\text{MPC Problem Solver}] \rightarrow u(t)$$

- FSR model state

$$x(t) = \begin{bmatrix} \underline{U}(t) \\ \underline{V}(t) \end{bmatrix}$$

- FSR model in state-space form

$$\begin{aligned} x(t+1) &= Ax(t) + B^U \Delta u(t) + B^D \Delta v(t) \\ y(t) &= Cx(t) + d \end{aligned}$$

Past control moves

$$\underline{U}(t) = \begin{bmatrix} \Delta u(t) \\ \vdots \\ \Delta u(t-n+1) \end{bmatrix}$$

Past disturbances

$$\underline{V}(t) = \begin{bmatrix} \Delta v(t) \\ \vdots \\ \Delta v(t-n+1) \end{bmatrix}$$

Exercise: what are the matrices A, B^U, B^D, C?

Prediction Model

$$y(t) = (s^U * \Delta u)(t) + (s^D * \Delta v)(t) + d$$

CV Prediction

Toeplitz matrix

Future MV

Hankel matrix

Past MV

Hankel matrix

Past DV

Unmeasured disturbance

$$Y = \Psi U + \Phi \cdot \underline{U}(t) + \Phi^D \underline{V}(t) + Dd$$

$$\Psi = \begin{bmatrix} S^U(1) & 0 & \cdots & 0 \\ S^U(2) & S^U(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S^U(N) & S^U(N-1) & \cdots & 0 \end{bmatrix} \quad \Phi^D = \begin{bmatrix} S^D(2) & S^D(1) & \cdots & S^D(n+1) \\ S^D(3) & S^D(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S^D(n+1) & 0 & \cdots & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Toeplitz matrix

Hankel matrix

Future impact of the disturbance

Optimization of Future Inputs

$$Y(t) = \Psi U(t) + \underbrace{\Phi \cdot \underline{U}(t) + \Phi^D \underline{V}(t) + Dd}_{Y^*(t)}$$

$Y^*(t)$ - free response of the system

- MPC optimization problem

$$J = Y^T(t) Q^Y Y(t) + U^T(t) R U(t) \rightarrow \min$$

$$Q^Y = \begin{bmatrix} Q^y & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Q^y \end{bmatrix}, R = \begin{bmatrix} R^u & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R^u \end{bmatrix}$$

- Caveat: the problem might be ill-conditioned, then there is a need for regularization (an additional quadratic term in *J*)

Optimization Constraints

- MV constraints

$$-\Delta u_{\max} \le \Delta u(t) \le \Delta u_{\max}$$

$$u_{\min} \le u(t) \le u_{\max}$$

$$\Longrightarrow$$

$$-\Delta u_{\max} \le U \le \Delta u_{\max}$$

$$u_{\min} \le \Sigma U - C \le u_{\max}$$

$$u(t+k) = u(t) + \sum_{j=2}^{k} \Delta u(t)$$

- CV constraints

$$y_{\min}(t) \le y(t) \le y_{\max}(t) \quad \Longrightarrow \quad Y_{\min} \le Y \le Y_{\max}$$

- Terminal constraint:

$$y(t+k) = 0; \qquad \Delta u(t+k) = 0 \ \text{ for } \ k \ge p$$

QP Solution

- QP Problem:

$$Aq \le b$$

$$A_{eq} q = b_{eq}$$

$$J = \frac{1}{2} q^T Q q + f^T q \to \min$$

$$q = \begin{bmatrix} U \\ Y \end{bmatrix}$$

Decision vector:
predicted MVs, CVs

- Standard QP codes can be used

Control Update

- System dynamics as an equality constraint in optimization

$$Y = \underbrace{\Psi U}_{\text{Forced response}} + \underbrace{Y^{*}(t)}_{\text{Free response}}$$

$$Y^{*} = \begin{bmatrix} \Phi & \Phi^{D} \end{bmatrix} \cdot x(t) + d(t)$$

- Update of the system state

$$x(t+1) = Ax(t) + B^{U}\Delta u(t) + B^{D}\Delta v(t)$$

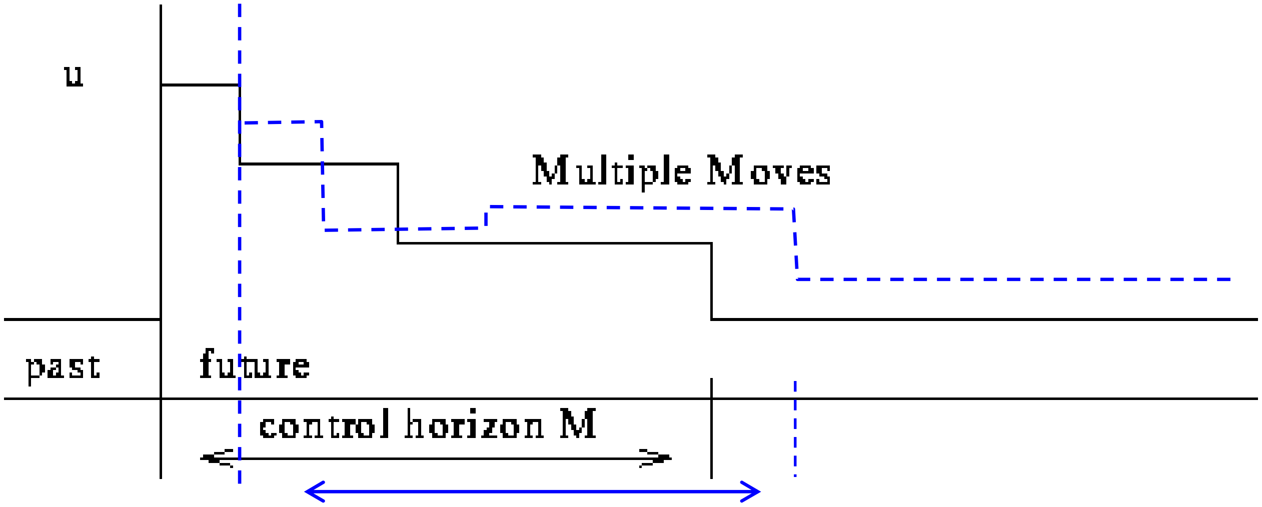
- Optimization problem solution at step t :

$$\{U_{OPT}, Y_{OPT}\} = \arg\min\left[J(U,Y)\right]$$

Receding Horizon Control

- Use the first computed control value only
- Repeat at each t

$$\Delta u(t+1) = [1 \quad 0 \quad \ldots \quad 0] \cdot U_{OPT}(t)$$



State Update and Estimation

- FSR model state update – delay line

$$x(t) = \begin{bmatrix}\underline{U}(t)\\ \underline{V}(t)\end{bmatrix} \qquad \underline{U}(t) = \begin{array}{c} \downarrow\!\!\longleftarrow \Delta u(t+1) \\ \begin{bmatrix}\Delta u(t)\\ \vdots \\ \cancel{\Delta u(t-n+1)}\end{bmatrix}\searrow \end{array} \qquad \underline{V}(t) = \begin{array}{c} \downarrow\!\!\longleftarrow \Delta v(t+1) \\ \begin{bmatrix}\Delta v(t)\\ \vdots \\ \cancel{\Delta v(t-n+1)}\end{bmatrix}\searrow \end{array}$$

- Disturbance estimator

$$d(t+1) = d(t) + k_I\left(y_m(t) - y(t)\right)$$

Unmeasured disturbance

Actually observed CV

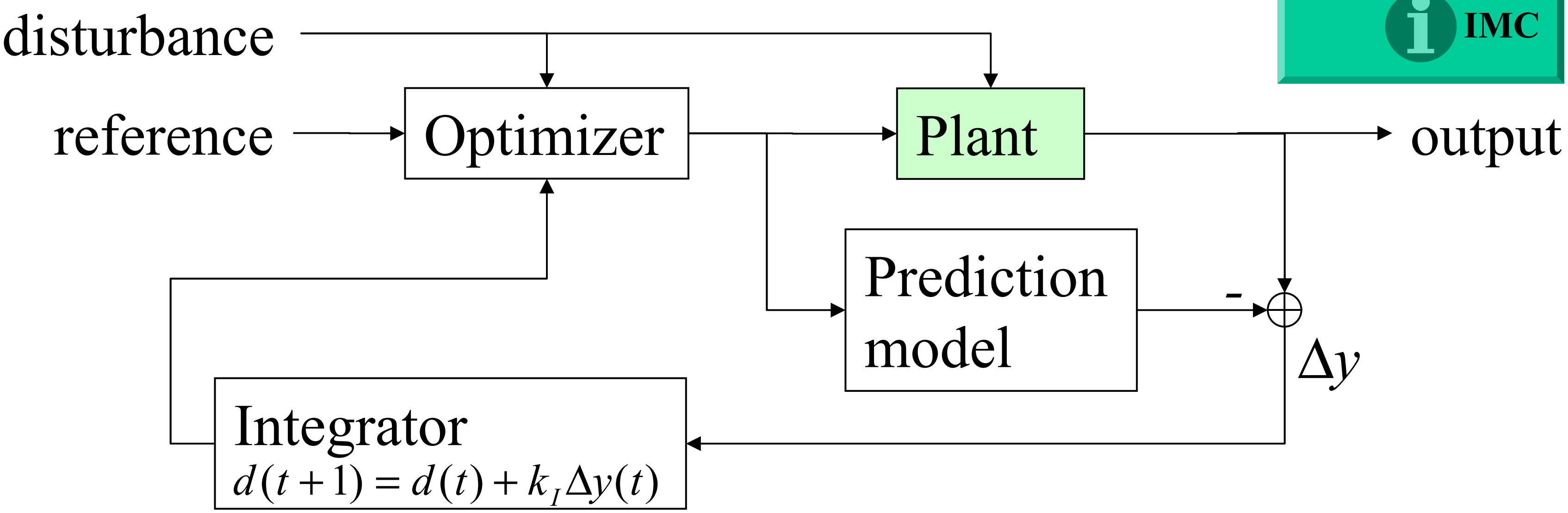
CV Prediction based on the current state $x(t)$. Computed at the previous step

- Disturbance estimator = Integrator feedback

MPC as IMC

- MPC with disturbance estimator is a special case of IMC

IMC

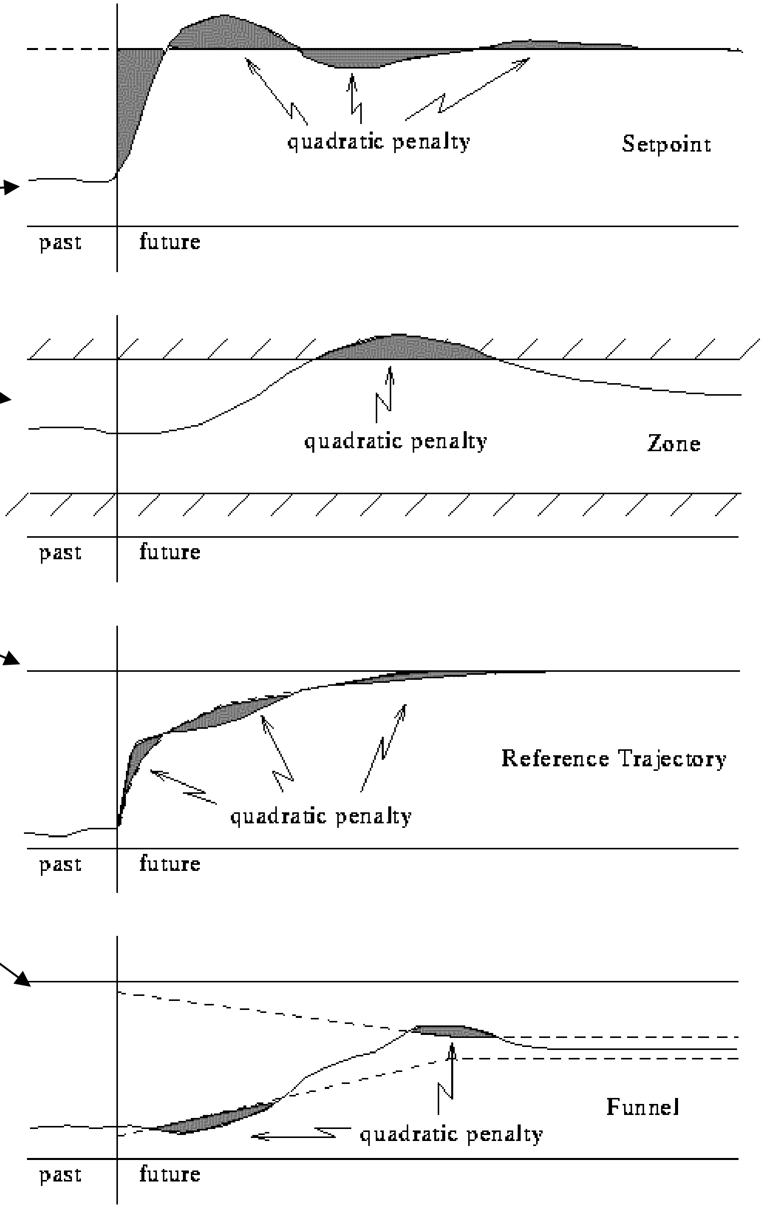


- Closed-loop dynamics (filter dynamics)
  - integrator in the disturbance estimator
  - $2n$ poles $z = 0$ in the FSR model update

zeros poles

Optimization detail

- Setpoint
- Zone
- Trajectory
- Funnel



- Soft constraints (quadratic penalties) and hard constraints for MV, CV
- Regularization
  - penalties
  - singular value thresholding

Soft Constraints

- Consider a QP Problem

$$J = \frac{1}{2} x^T Q x + f^T x \to \min$$

subject to: $Ax \le b$

$A_s x \le b_s$ soft constraint, might be infeasible

- Slack variable formulation: $w = 0$ yields the original problem

Large penalty for constraint violation

$$J = \frac{1}{2} x^T Q x + \frac{1}{2} w^T S w + f^T x \to \min$$

$\begin{bmatrix} x \\ w \end{bmatrix}$ new augmented decision vector

subject to:

$Ax \le b$

$A_s x - b_s \le w$

slack

$0 \le w$

Industrial MPC Features

- Industrial strength products that can be used for a broad range of applications
- Flexibility to plant size, automated setup
- Based on step response/impulse response model
- On the fly reconfiguration if plant is changing
  - MV, CV, DV channels taken off control or returned into MPC
  - measurement problems (sensor loss), actuator failures
- Systematic handling of multi-rate measurements and missed measurement points
  - do not update the disturbance estimate d if the data is missing

Technical detail

- Tuning of MPC feedback control performance is an issue.
  - Works in practice, without formal analysis
  - Theory requires
    - Large (infinite) prediction horizon or
    - Terminal constraint
- Additional tricks for
  - a separate static optimization step
  - integrating and unstable dynamics
  - active constraints
  - regularization
  - shape functions for control
  - different control horizon and prediction horizon
  - ...