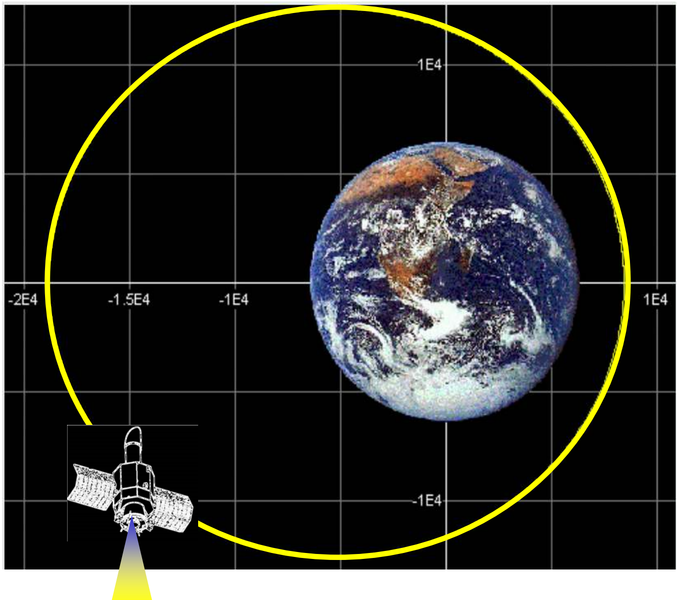
Lecture 13 – Programmed Control

- Optimal control: programmed, synthesis
- Rocket ascend
- Grade change in process control
  - Example
- QP optimization
- Flexible dynamics: input shaping, input trajectory
  - example
- Robotics path planning

Open-loop (programmed) control

- Control $u(t)$ found by solving an optimization problem. Constraints on control and state variables.
- Used in space, missiles, aircraft FMS
  - Mission planning
  - Complemented by feedback corrections
- Sophisticated mathematical methods were developed in the 60s-70s to overcome computing limitations.



$$\dot{x} = f(x,u,t)$$

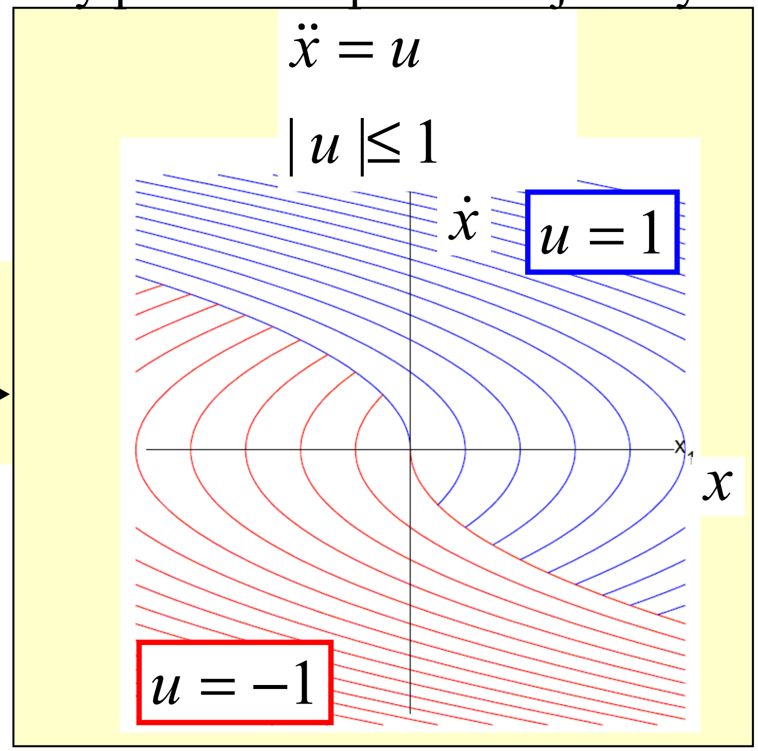
$$J(x,u,t) \to \min$$

$$x \in \mathbf{X}, u \in \mathbf{U}$$

Optimal control: $u = u_*(t)$

Optimal Control Synthesis

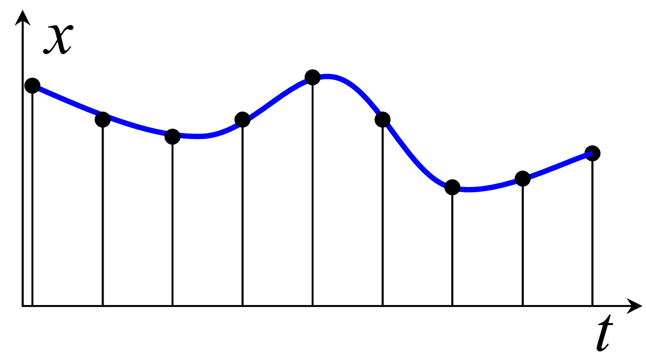
- Find optimal control program for *any* initial conditions
- At any point in time apply control that is optimal for the current state. This is *feedback* control!
  - Optimality principle: an optimal trajectory part is an optimal trajectory
- Example: LQ control
  - Linear system; Quadratic index
  - Analytically solvable problem
- Example: Time optimal control
  - 2^{nd} order system →



- Toy problems, building blocks
- MPC - will discuss in Lecture 14

Optimal Control

- Formulate performance index and constraints
- Programmed control
  - compute optimal control as a time function for particular initial (and final) conditions
- Modern practical approach
  - Discretize trajectory and control
  - Solve using a Nonlinear Programming (NLP) software package
  - Use sparse matrix arithmetics



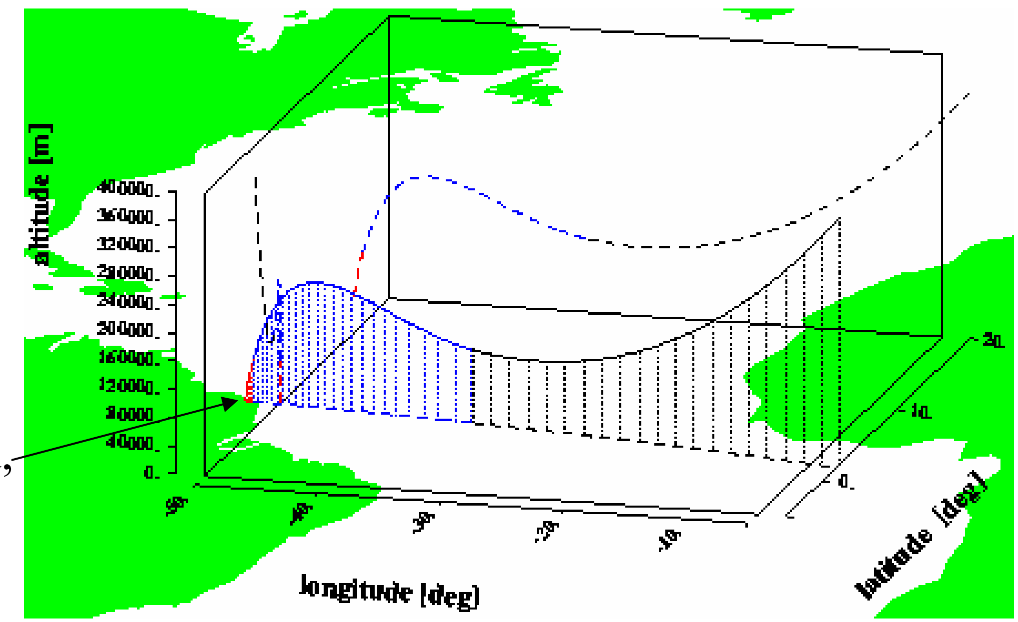
NLP: Minimize: $J(X) \to \min$

subject to: $h(X) = 0$

$g(X) \le 0$

Ascend trajectory optimization

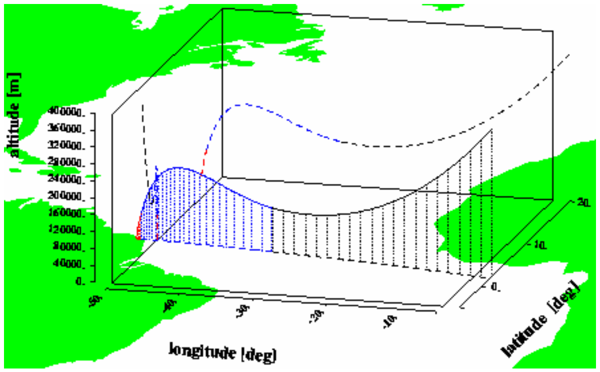
- Rocket launch vehicles
  - fuel (payload) optimality
  - orbital insertion constraint
  - flight envelope constraints
  - booster drop constraint



Kourou, French Guiana

Trajectory optimization

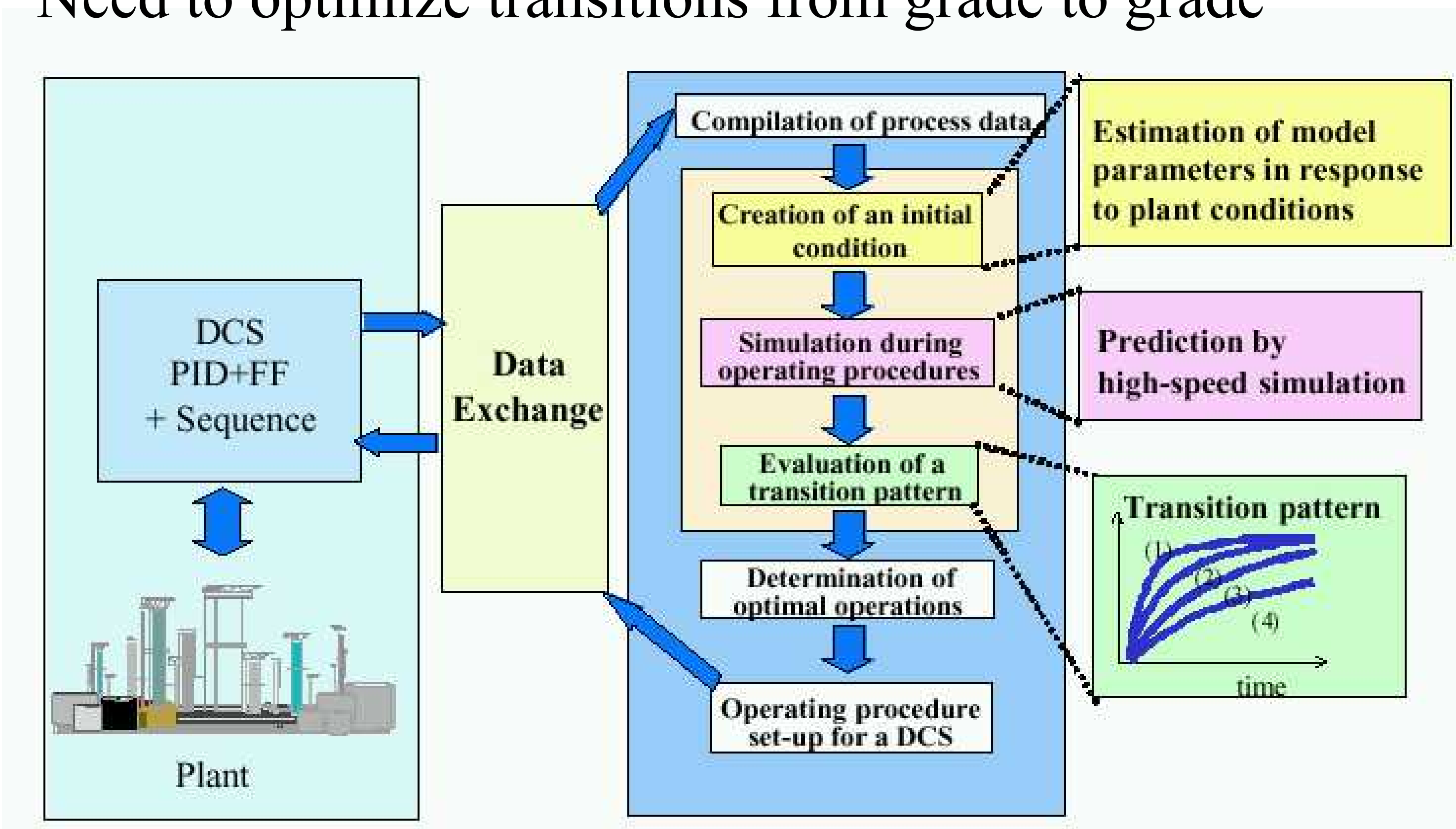
- Nonlinear constrained optimization problem
- Nonlinear Programming (NLP): not QP, not LP
  - Iterative optimization methods: Gradient, Newton, Levenberg-Marquardt, SQP, SSQP
  - Need supervision by a human



- Recall: QP, LP are guaranteed to always produce a solution if the problem is feasible
  - Can be possibly used on-line, inside a control loop

Optimization of Process Transitions

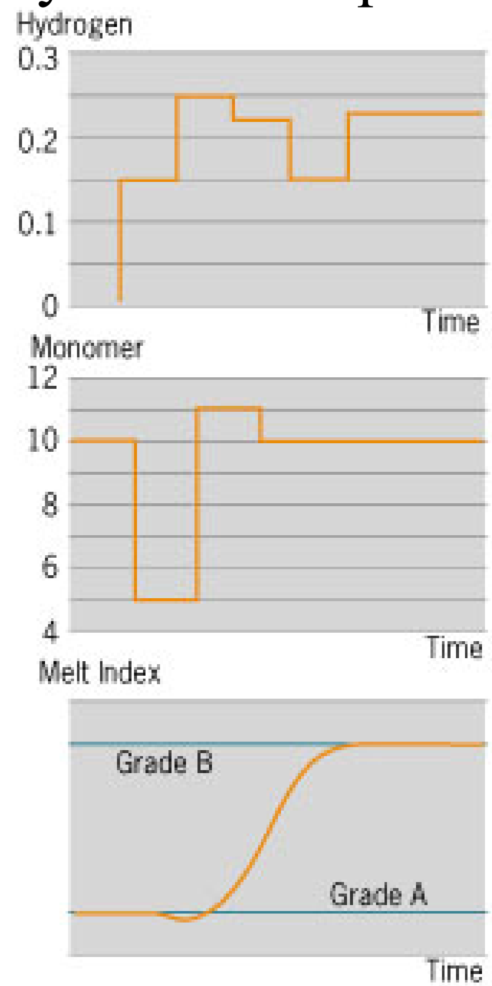
- Process plants manufacture different product varieties (grades)
- Need to optimize transitions from grade to grade



Product Grade Change

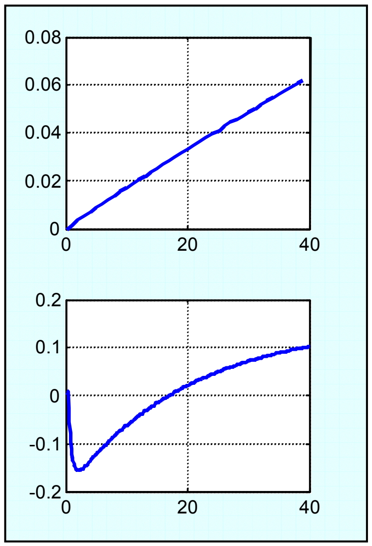
- The requirement: to change from manufacturing grade A to grade B with minimum off-spec production.
- The implementation: using detailed models of process and operating procedures.
- The results: optimum setpoint trajectories for key process controls during the changeover, resulting in minimum lost revenue.

Polymerization process



Grade change control example

- Example process model: chemical reactor



$u \rightarrow$ (process) $\rightarrow y_1$ internal temperature $\quad P_1 = \dfrac{0.001746}{z - 0.995}$

$\rightarrow y_2$ quality variable $\quad P_2 = P_1 - z^{-5}\dfrac{0.03625}{z - 0.8187}$

- The process is in an initial steady state: $u = 0$; $y_1 = y_2 = 0$
- Need to transition, as quickly as possible, to other steady state: $u =$ const; $y_1 =$ const; $y_2 = y_d$

Grade change control example

- Linear system model in the convolution form

$$y = h * u$$

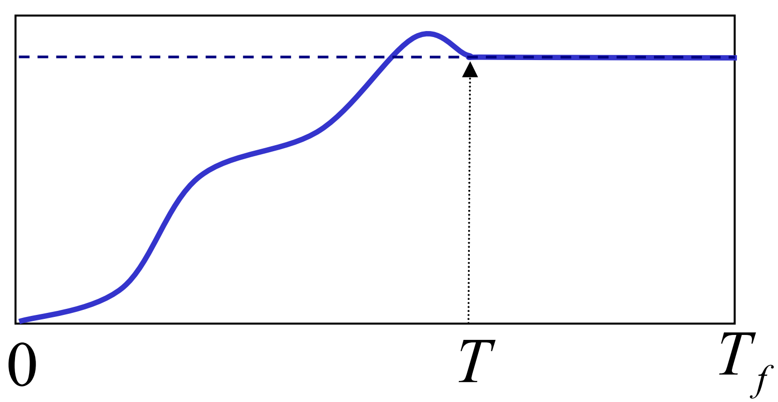
- Quadratic-optimal control

$$\sum_t \left|y_2(t) - y_d\right|^2 + r\left|u(t) - u(t-1)\right|^2 \to \min$$

- Equality constraint (process transitioning to the new grade)

$$\dot{y}_1(t) \equiv 0, y_2(t) \equiv y_d, \text{ for } T \le t \le T + T_f$$

- Inequality constraints
  - Control $\left|u(t)\right| \le u_*$
  - Temperature $\left|y_2(t)\right| \le T_*$



Grade change control example

- Sampled time: $t = 1, \ldots, N$
- Vector-matrix formulation of the problem

$$U = \begin{bmatrix} u(1) \\ \vdots \\ u(N) \end{bmatrix}, Y_1 = \begin{bmatrix} y_1(1) \\ \vdots \\ y_1(N) \end{bmatrix}, Y_2 = \begin{bmatrix} y_2(1) \\ \vdots \\ y_2(N) \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} H_1 U \\ H_2 U \end{bmatrix} = HU$$

$$H_1 U = h_1 * u$$
$$H_2 U = h_2 * u$$

- H is a block-Toeplitz matrix

$$H_1 = \begin{bmatrix} h_1(1) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ h_1(N-1) & \cdots & h_1(1) & 0 \\ h_1(N) & \cdots & h_1(2) & h_1(1) \end{bmatrix}$$

Grade change control example

- Dynamics as an equality constraint:

$$HU - Y = 0$$

- Quadratic-optimal control

$$J = (Y_2 - Y_d)^T (Y_2 - Y_d) + rU^T D^T DU \rightarrow \min$$

$$J = rU^T D^T DU + {Y_2}^T Y_2 - 2{Y_d}^T Y_2 + \ldots \rightarrow \min$$

- Inequality constraints
  - Control $-u_* \le U \le u_*$
  - Temperature $0 \le Y_1 \le T_*$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$Y_d = y_d \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Notations

Terminal constraint

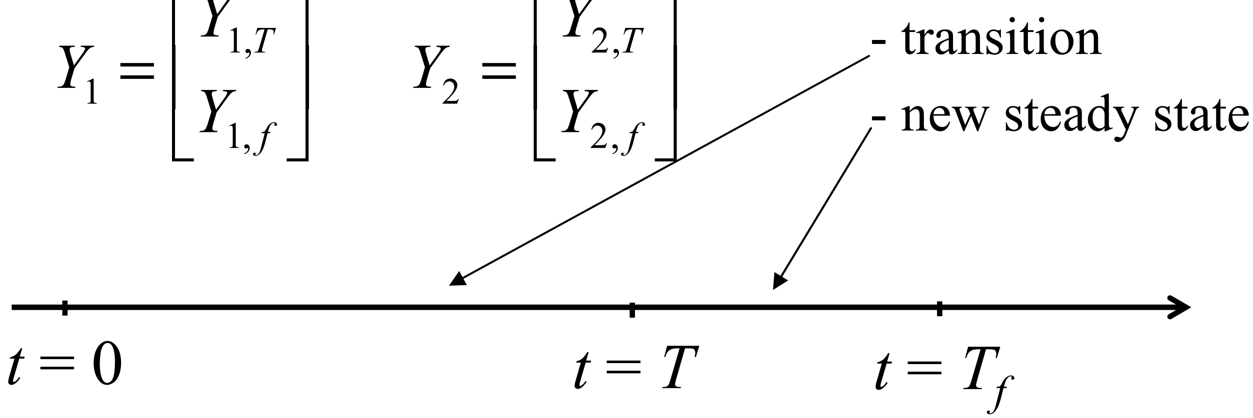
- Equality constraints (new grade steady state)

$$DY_{1,f} = 0$$ - steady in the end

$$Y_{2,f} = Y_{d,f}$$ - at target in the end

$$D = \begin{bmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -1 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} Y_{1,T} \\ Y_{1,f} \end{bmatrix} \qquad Y_2 = \begin{bmatrix} Y_{2,T} \\ Y_{2,f} \end{bmatrix}$$



Quadratic Programming

- QP Problem:

$$Ax \le b$$

$$A_{eq}x = b_{eq}$$

$$J = \frac{1}{2}x^T Qx + f^T x \to \min$$

- Matlab Optimization Toolbox: **`QUADPROG`**

$$x = \begin{bmatrix} U \\ Y_1 \\ Y_2 \end{bmatrix}$$

$$A = \ldots \qquad Q = \ldots$$

$$A_{eq} = \ldots \qquad f = \ldots$$

$$b_{eq} = \ldots$$

See previous slides

Sim

QP Program for the grade change, no terminal constraint; slow T=40

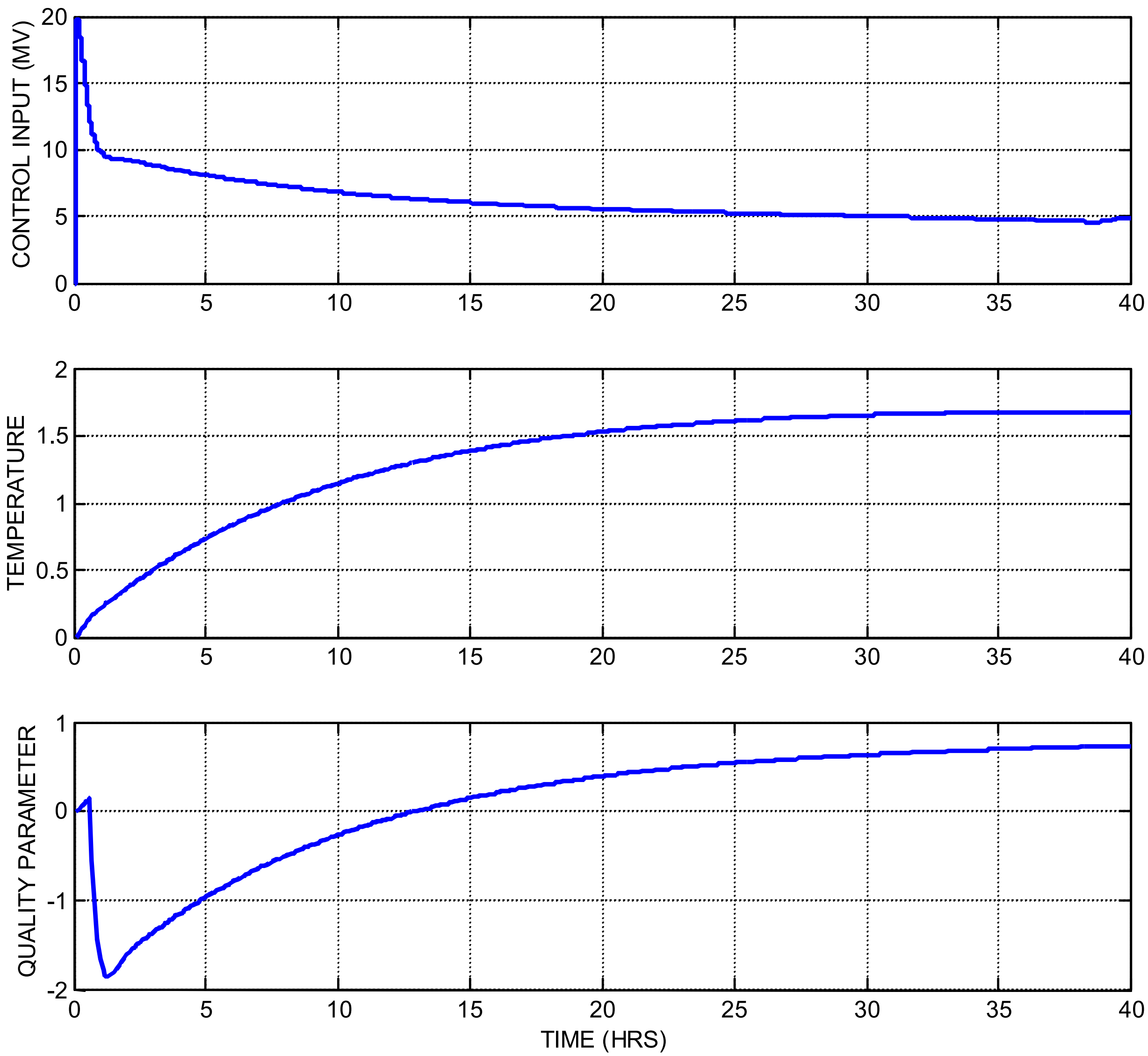
$y_d = 0.75$

$\tau = 0.1$

$r = 0.05$

$T_* = 2$

$u_* = 20$



Sim

QP Program for the grade change with a terminal constraint at $T = 8$

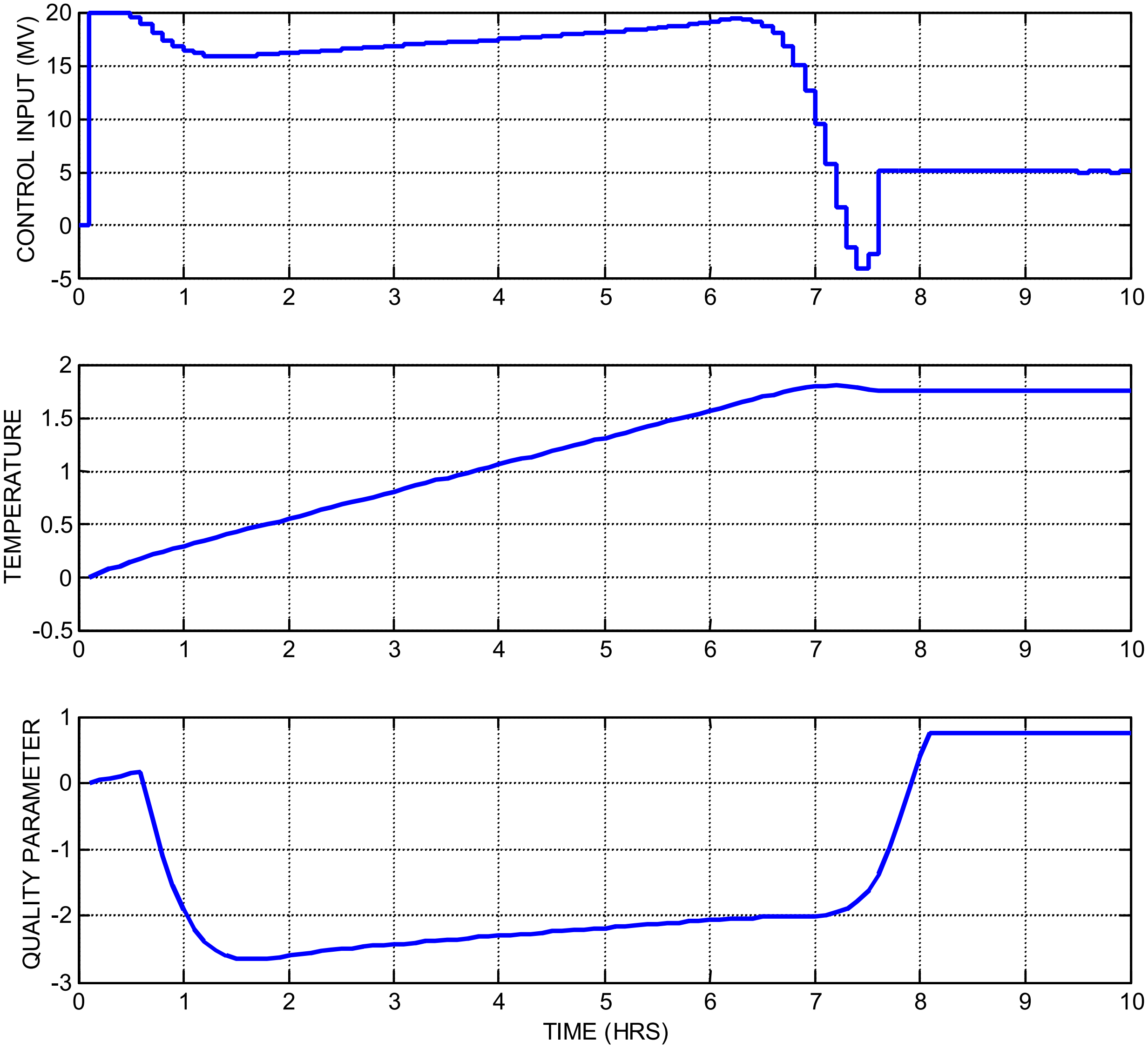
$y_d = 0.75$

$\tau = 0.1$

$r = 0.05$

$T_* = 2$

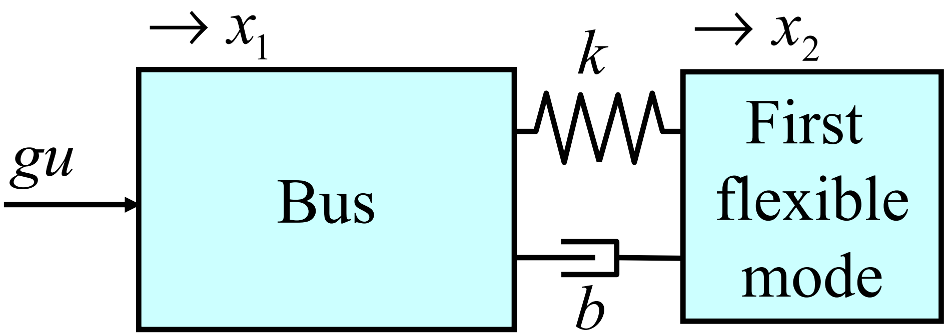
$u_* = 20$

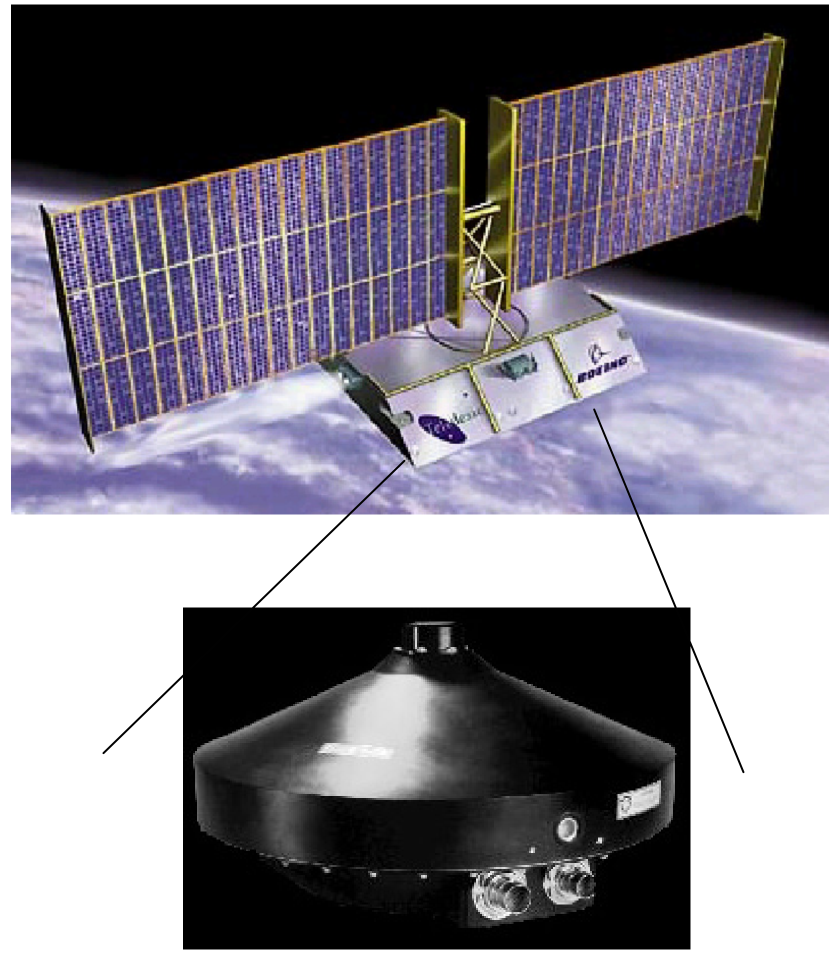


Flexible Satellite Slew Control

- Single flexible mode model
- Franklin, Section 9.2

$$J_1\ddot{x}_1 = -k(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) + gu$$
$$J_2\ddot{x}_2 = k(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2)$$





reaction wheel

Flexible Satellite Slew Control

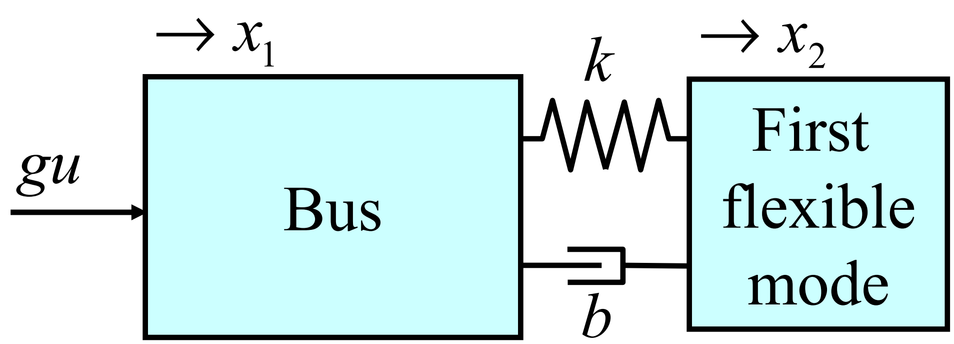
- Linear system model

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/J_1 & -b/J_1 & k/J_1 & b/J_1 \\ 0 & 0 & 0 & 1 \\ k/J_2 & b/J_2 & -k/J_2 & -b/J_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ g/J_1 \\ 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} x_1 \\ x_2 - x_1 \\ \dot{x}_1 \end{bmatrix}$$

- slew angle
- deformation
- slew rate



Flexible Satellite Slew Control

- Linear system model in the convolution form

$$y = h * u$$

- Quadratic-optimal control

$$\sum_t |u(t)|^2 \rightarrow \min$$

- Equality constraint (system coming to the target slew angle)

$$y(t) \equiv y_d, \text{ for } T \le t \le T + T_f$$

- Inequality constraints
  - Control $|u(t)| \le 1$
  - Deformation $|y_2(t)| \le d_*$
  - Slew rate $|y_3(t)| \le v_*$

Flexible Satellite Slew Control

- Sampled time: $t = 1, \ldots, N$
- Y is a $3N$ vector; H is a block-Toeplitz matrix

$$Y = HU$$

$$y = \begin{bmatrix} x_1 \\ x_2 - x_1 \\ \dot{x}_1 \end{bmatrix}$$

- Quadratic-optimal control

$$U^T U \to \min$$

- Equality constraint (system coming to the target slew angle)

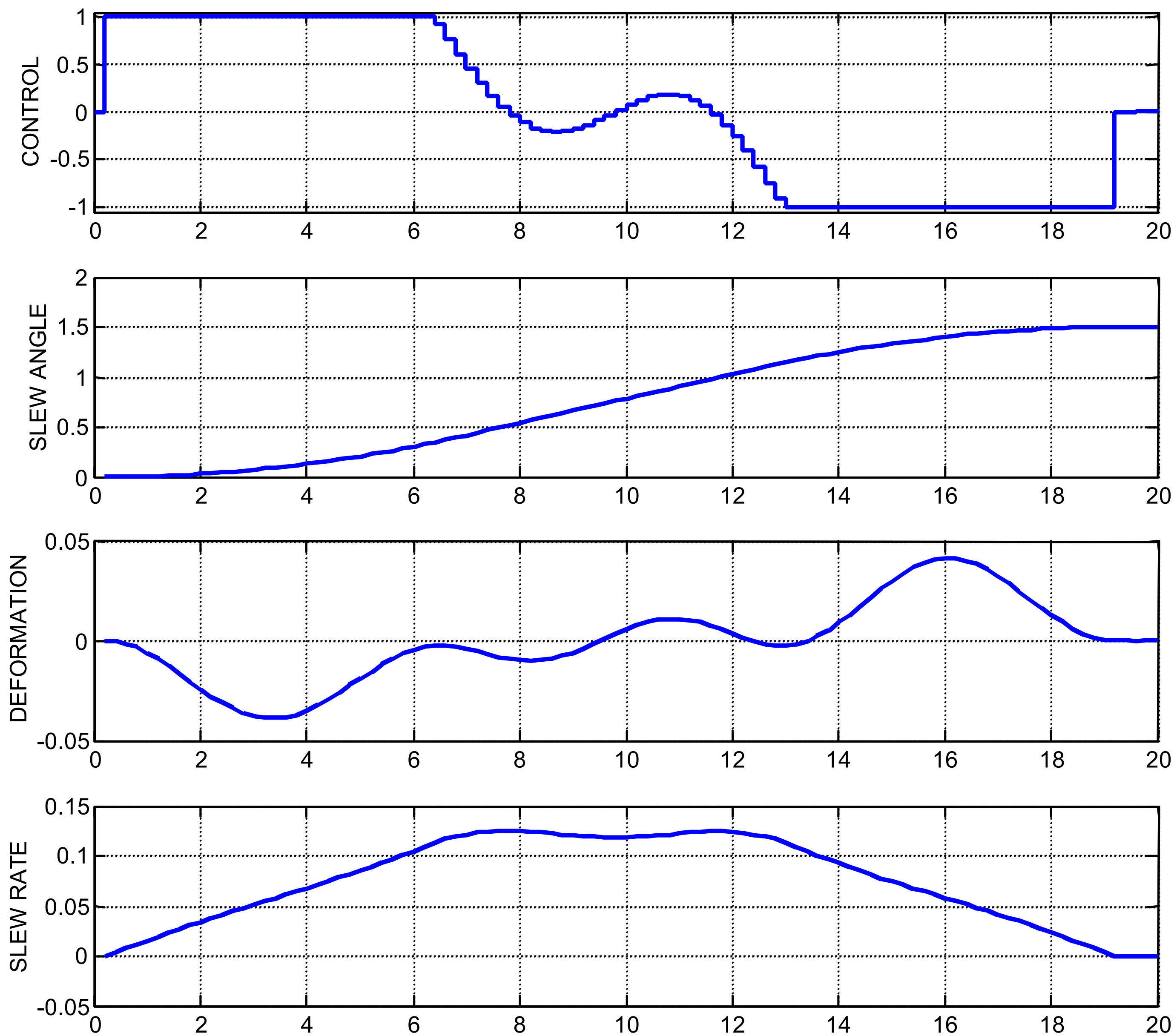
$$S_1 Y = Y_d$$

- Inequality constraints
  - Control $\quad -1 \le U \le 1$
  - Deformation $\quad d_* \le S_2 Y \le d_*$
  - Slew rate $\quad v_* \le S_3 Y \le v_*$

This is a QP problem

Sim

QP Program for the flexible satellite slew



$g = 0.02$

$J_1 = 1$

$J_2 = 0.1$

$k = 0.091$

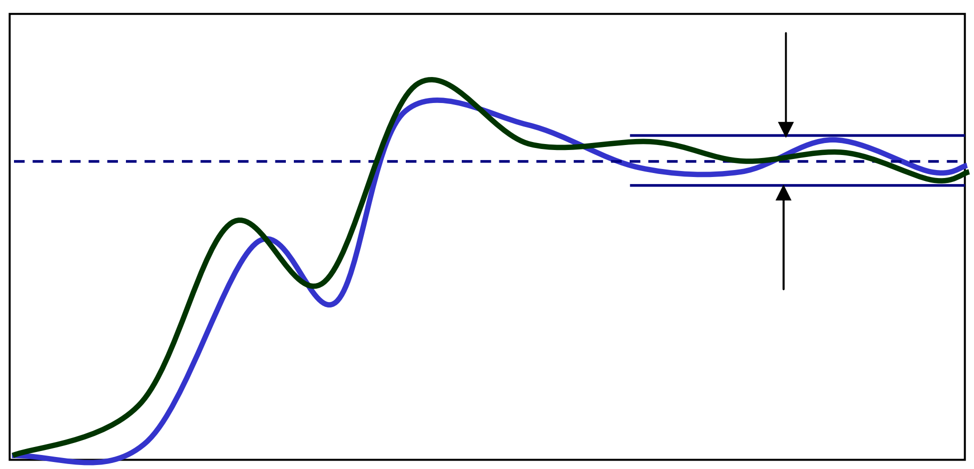
$b = 0.0036$

$d_* = 0.02$

$v_* = 0.2$

Robust design approach

- Replace exact terminal constraint by a given residual error
- Consider the system for several different values of parameters and group the results together
- As an optimality index, consider an average performance index or the worst residual error



Mobile Robot Path Planning

$$F(\xi(\cdot), \eta(\cdot), t_f) \;\to\; \min$$

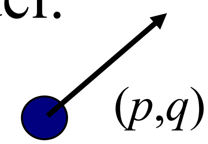
$$\ddot{\xi} = p, \;\; \|p\| \le p_{max},$$

$$\ddot{\eta} = q, \;\; \|q\| \le q_{max},$$

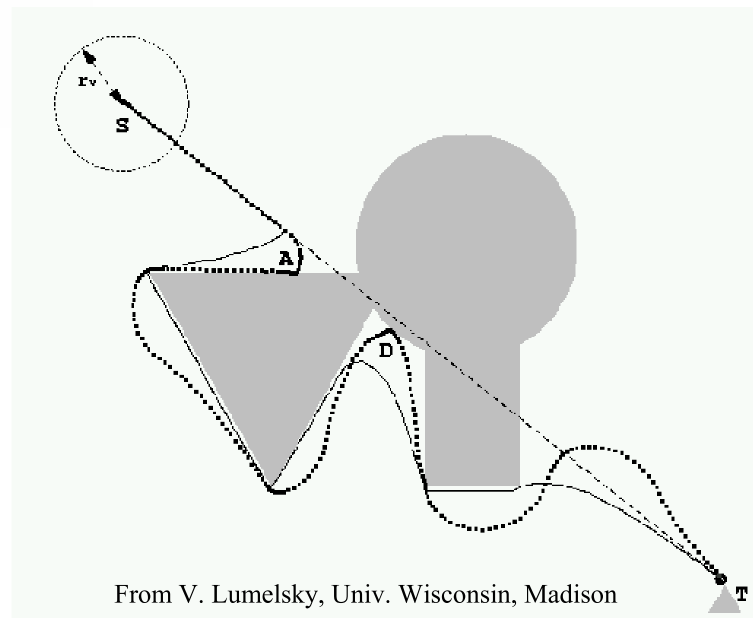
$$\xi(0) = \xi_0, \;\; \eta(0) = \eta_0, \;\; \dot{\xi}(0) = \dot{\xi}_0, \;\; \dot{\eta}(0) = \dot{\eta}_0,$$

$$\eta(t_f) = \eta(t_f) = \dot{\xi}(t_f) = \dot{\eta}(t_f) = 0$$

Point mass model:



Constrained optimization problem of finding an optimal path



From V. Lumelsky, Univ. Wisconsin, Madison

Future Combat Systems (FCS)

- Technology-based transformation of military force
- Ground and air robotics vehicles
- Application of robotics research
- Path planning and optimization are important