Homework Assignment #1 for

EE392m - Control Engineering in Industry, Spring 2005

1. TCP Reno congestion control

Consider the TCP/IP flow control model (Slides 28-29, Lecture 1).
The equations of the system dynamics are

$$\dot{x}(t) = \frac{1-q}{\tau^2} - \frac{1}{2} q x^2(t)$$

where the variables are as follows

- x - transmission rate
- τ - round trip time
- q - packet loss probability

(a) Find the steady state transmission rate
(b) Write equations of the system linearized around the steady state
(c) Find the linearized model settling time for the loss probability q=1%, and the round trip time τ=200ms

2. Missile range control

Consider the ballistic missile range control problem (Slide 22, Lecture 1).
Consider a missile fired to achieve the range of 500 km. The nominal active trajectory ends at the height H=20km and the distance L=10km from the launch site towards the target. At the end of the nominal active trajectory (terminal point), the missile velocity is aimed at the 45 degree angle. The engine cutoff time is calculated by monitoring the following value becoming zero

$$\delta r(t) = f_1 \Delta V_x(t) + f_2 \Delta V_y(t) + f_3 \Delta X(t) + f_4 \Delta Y(t)$$

This is a weighted combination of the deviations from the terminal values for the velocity components and coordinates.

(a) Compute f_1, f_2, f_3, f_4 assuming that the coordinates are in meters and velocity componnts in m/s. Neglect air drag, Earth curvature, and gravity non-uniformity

Hint

Make a function $r = F(V_x, V_y, L, H)$ that computes the range from the terminal velocities and terminal point coordinates. Use it first to find the nominal value of the terminal velocity (given the angle), then to compute f_1, f_2, f_3, f_4 in the linearized map.

3. Watt's governor

Consider the Watt's governor (Slide 14, Lecture 1). The equations of the system dynamics are

$$J\dot{\omega}_E = k\cos\phi - T_L$$

$$\omega_G = n\omega_E$$

$$ml\ddot{\phi} = l\left(m\omega_G^2 l \sin\phi\cos\phi - mg\sin\phi - b\dot{\phi}\right)$$

The parameters values are as follows

System parameter	**Notation**	**Value**
Engine moment of intertia	J	100 kg m^2
Engine torque gain for bob deflection angle	k	141.4 N m^2
Load torque	T_L	100 N m^2
Governor gear ratio	N	1

Flyball mass	*M*	1 kg
Bob length	*L*	1 m
Viscous damping coefficient	*b*	2 kg /s

(a) Derive the nonlinear model of the system in the state space form $\dot{x} = f(x)$

(b) Find the steady state and derive the system linearized dynamics around the steady state in the form $\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = 0$.

(c) Is the linearized system stable or unstable, judging by the obtained a_1, a_2, a_3?

(d) Simulate system with the same initial conditions for the linear and nonlinear models. As the initial conditions choose the steady state engine speed and the bob angle, then decrease the initial bob position by 0.5 rad compared to the steady state.

(e) [BONUS] Assuming that all other parameters are as in the table, find value of b providing the best response of the bob angle $\phi(t)$ with the same initial conditions as in (d). Brute force computation is Ok.

Hints

You do not need to do all the work analytically, though this would be Ok too. A Matlab script that defines all the parameter values in the beginning and generates answers to the questions would be just fine. Linearization can be done numerically, not necessarily analytically. Use Matlab help on **`LTIMODELS`** to find how to change from one system representation to another. The function **`poly`** might be useful too.

4. Impulse response model

MATLAB data file **`SidewaysStep.mat`** on the course web site contains step response data obtained by fine grid simulation of the sideways heat PDE model (Lecture 2, Slides 23 and 24). The arrays `y` and `tm` in the file contain the sampled step response data and the sampling times.

(a) Estimate an impulse response for the system

(b) Find the dominant time constant of the impulse response

(c) Find and plot the heat outflow if temperature outside is always 0. The temperature inside is 60 for $t \leq 0$, then is ramping up to t=75 linearly over the time from 0 to 5. It then ramps down back to 20 also linearly over the time 5 to 20. What is the maximal heat flux achieved?

Hint

You might find MATLAB function `PRONY` useful