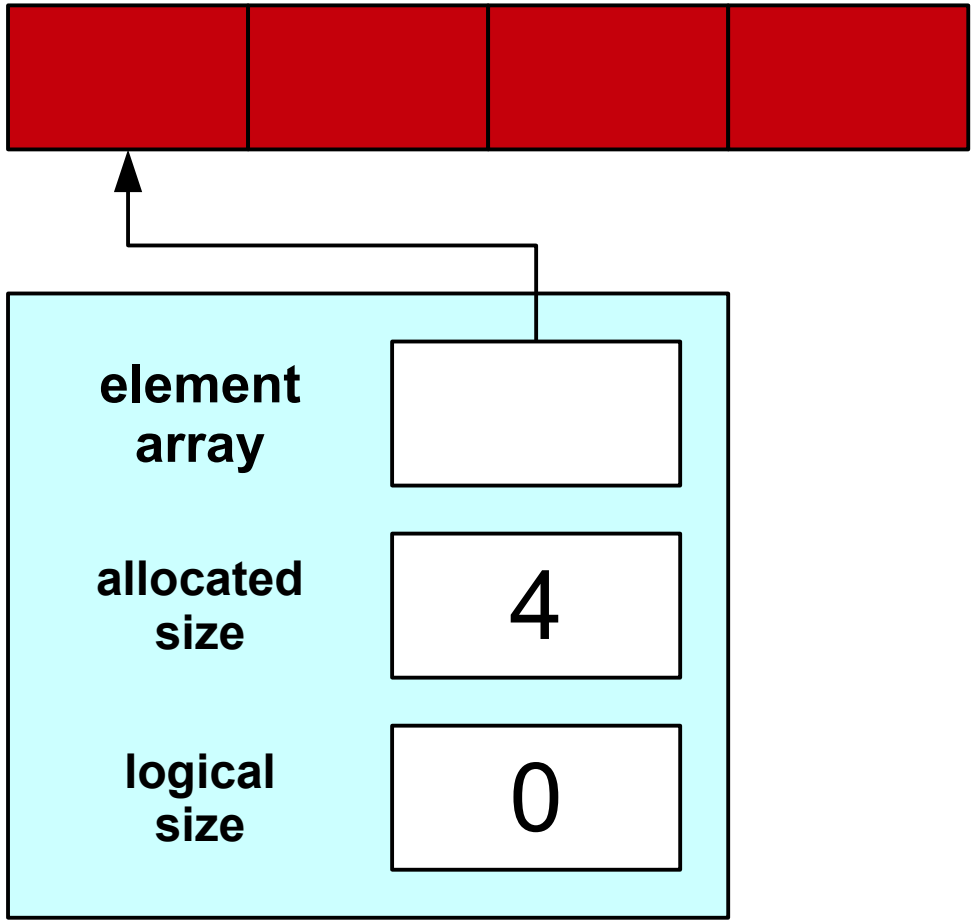
Implementing Abstractions

Part Two

Previously, on CS106B...

A Bounded Stack



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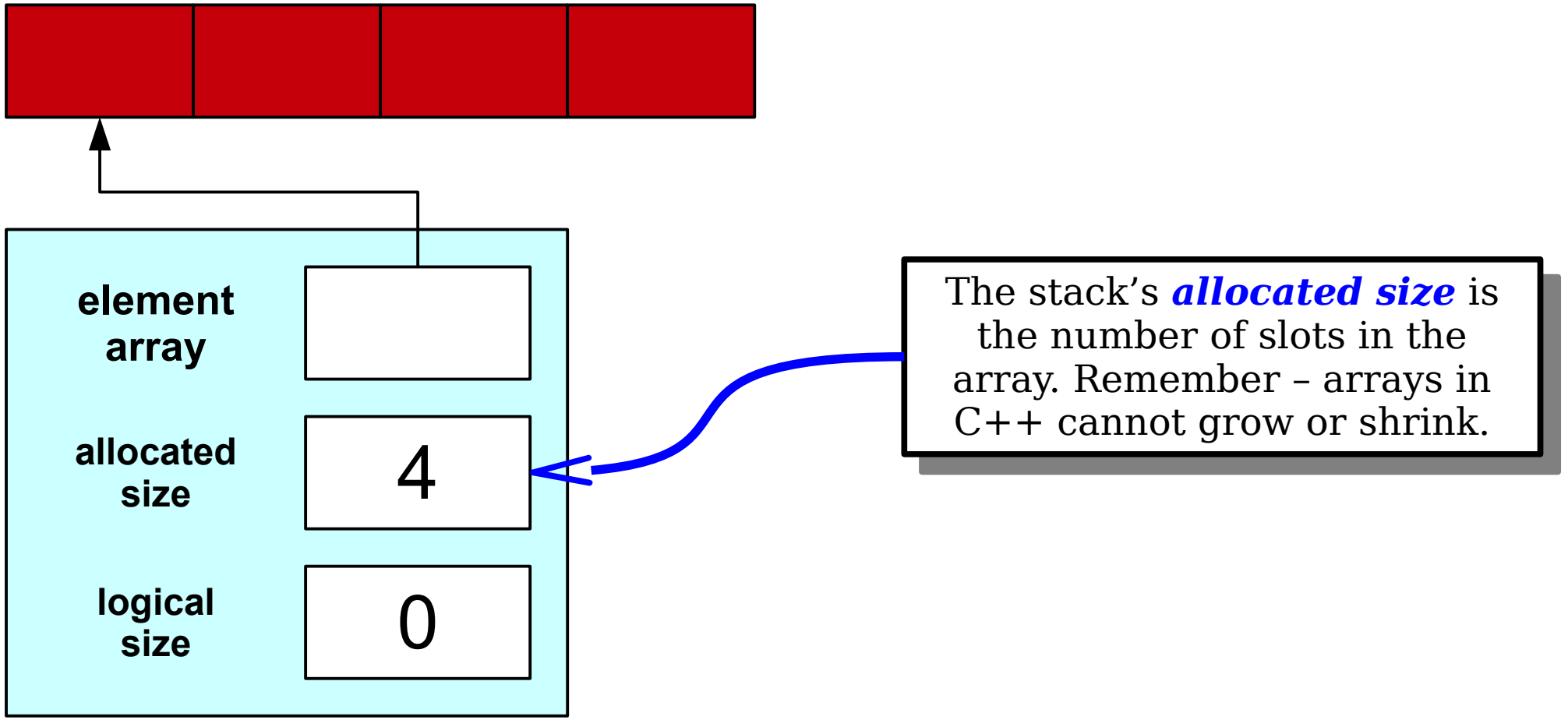

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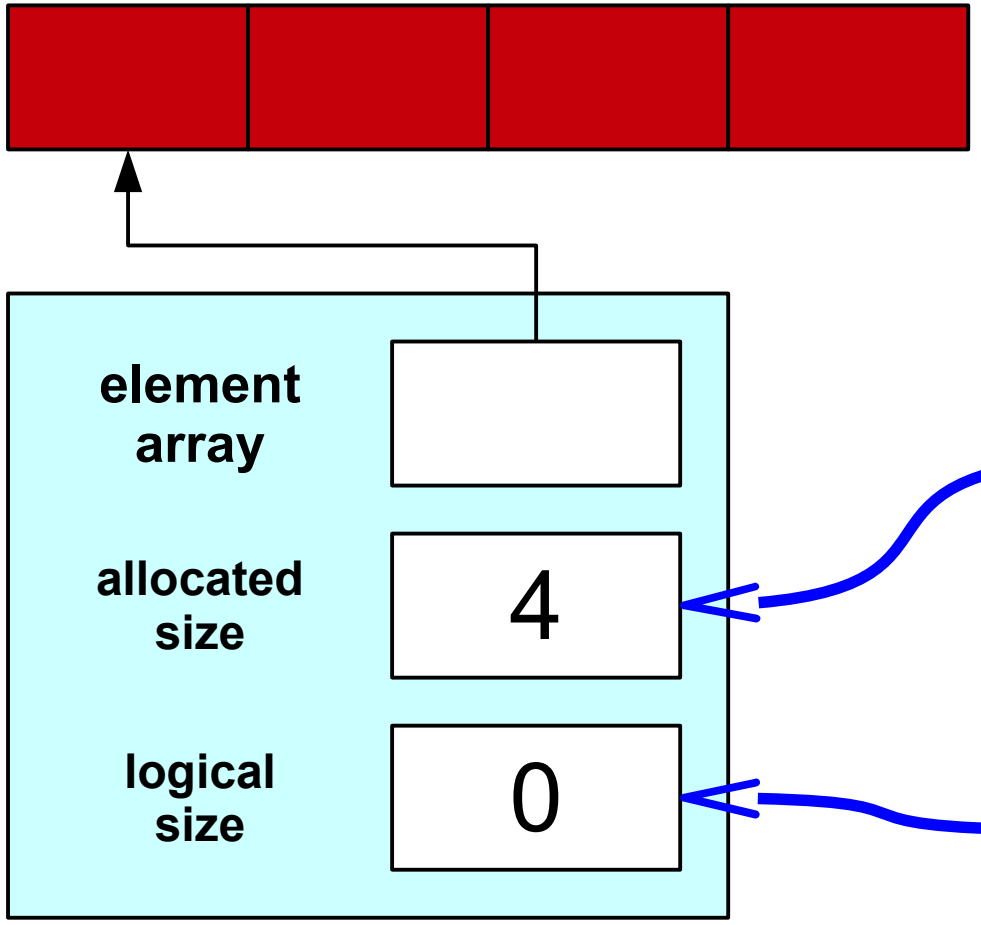
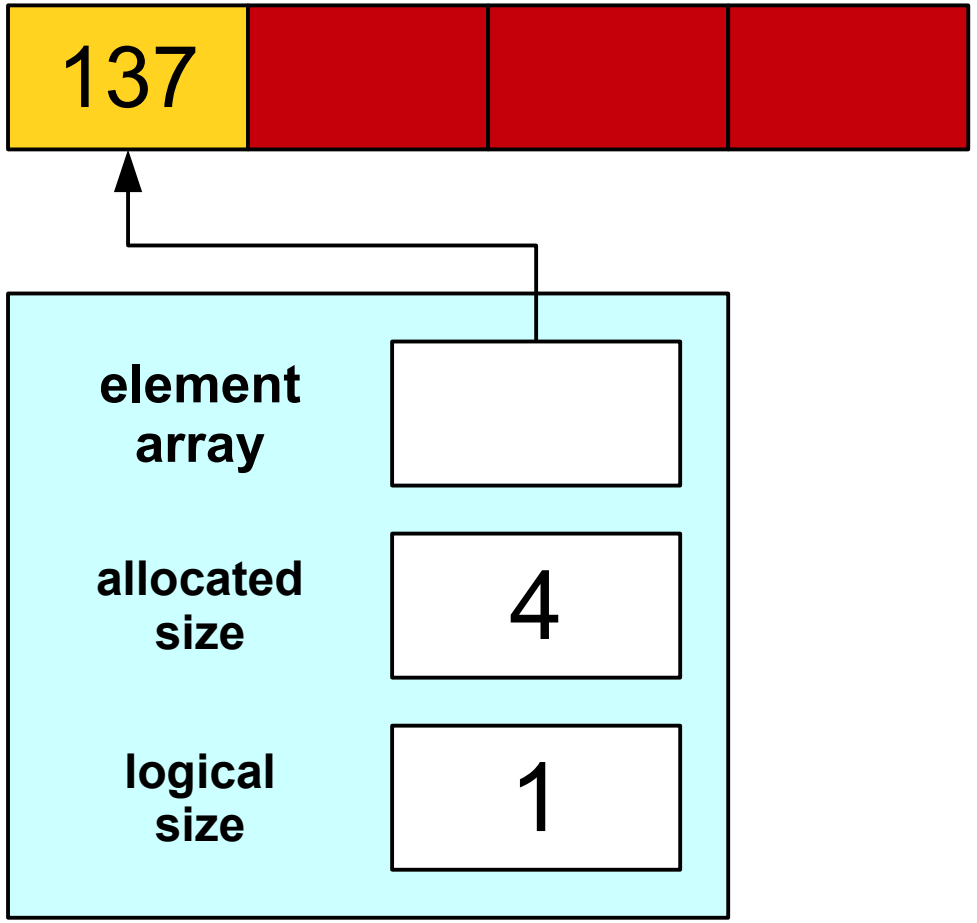


The stack's ***allocated size*** is the number of slots in the array. Remember – arrays in C++ cannot grow or shrink.

The stack's ***logical size*** is the number of elements actually stored in the stack. This lets us track how much space we're actually using.

A Bounded Stack



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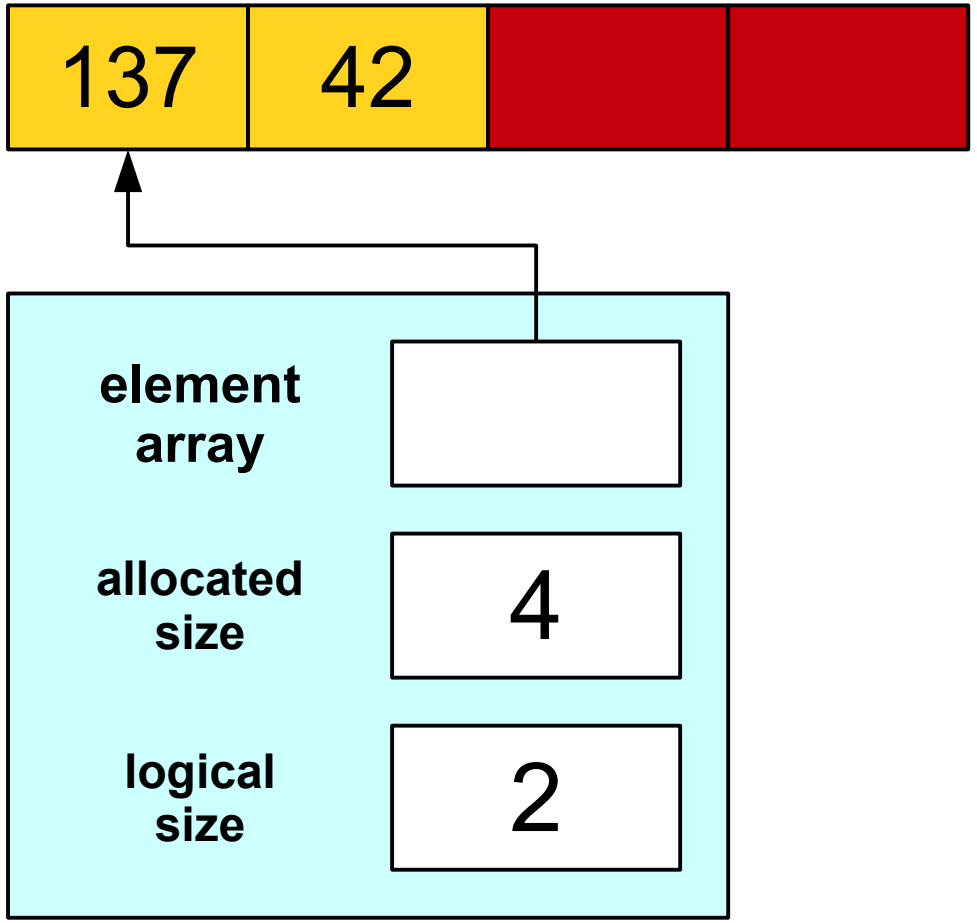

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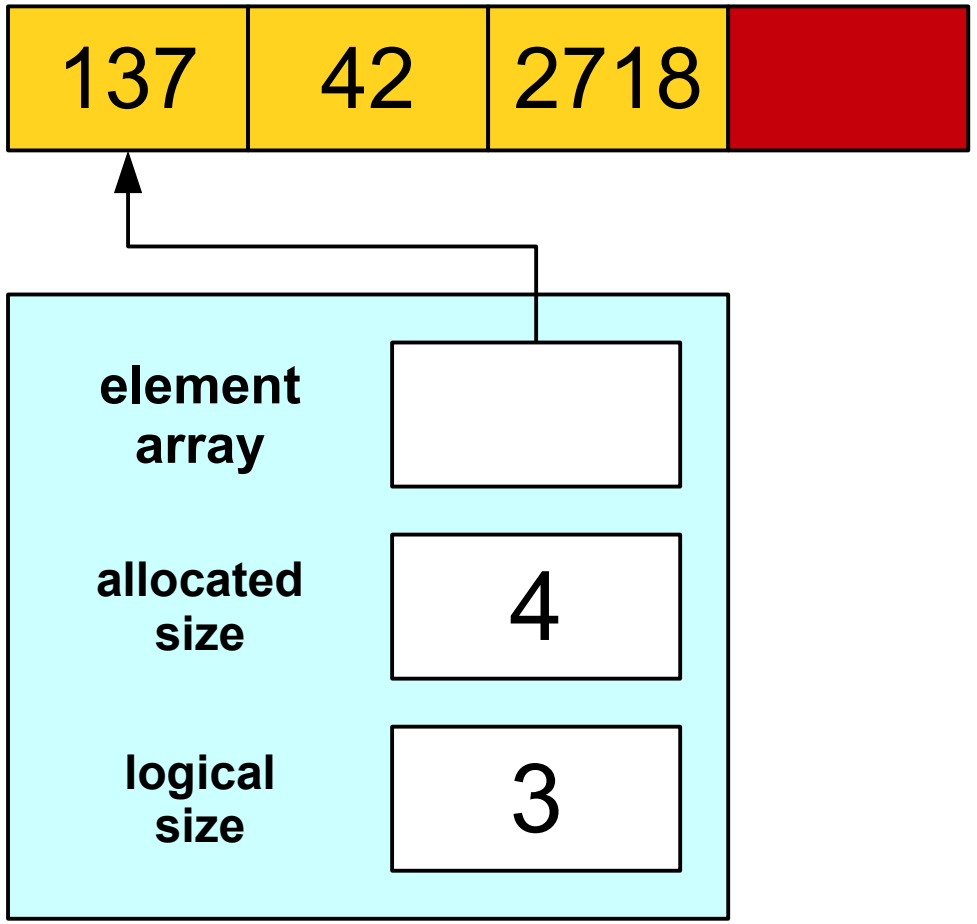

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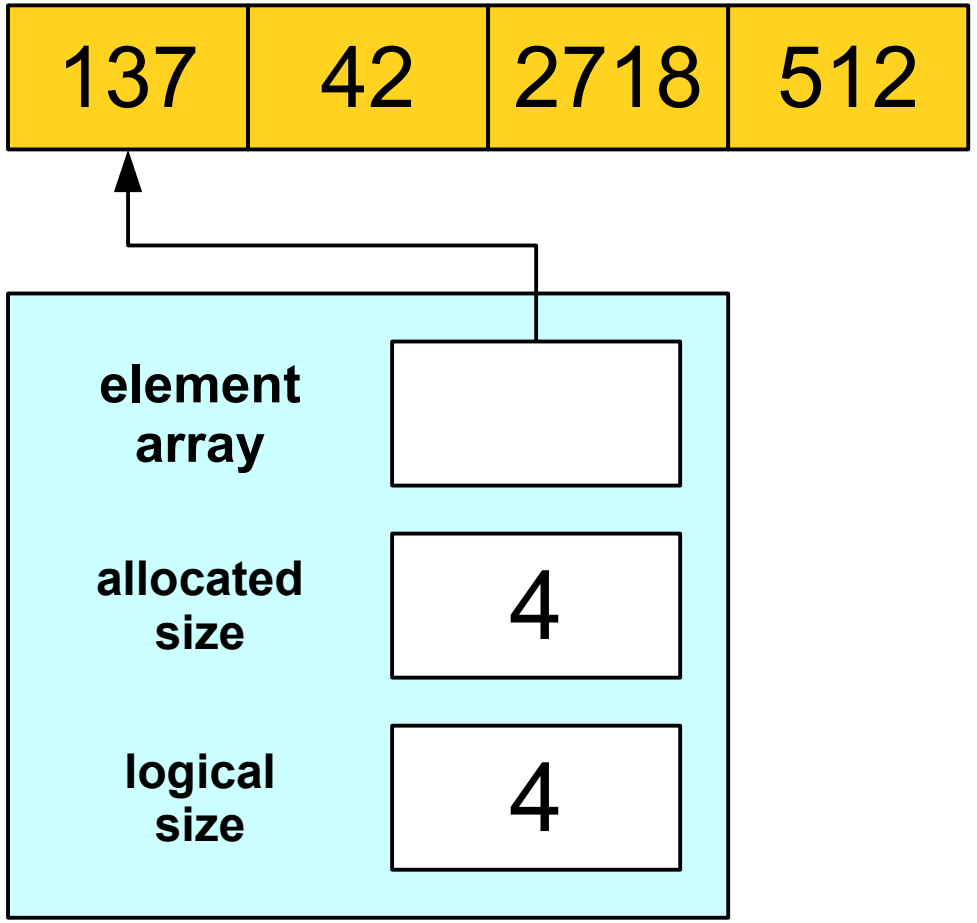

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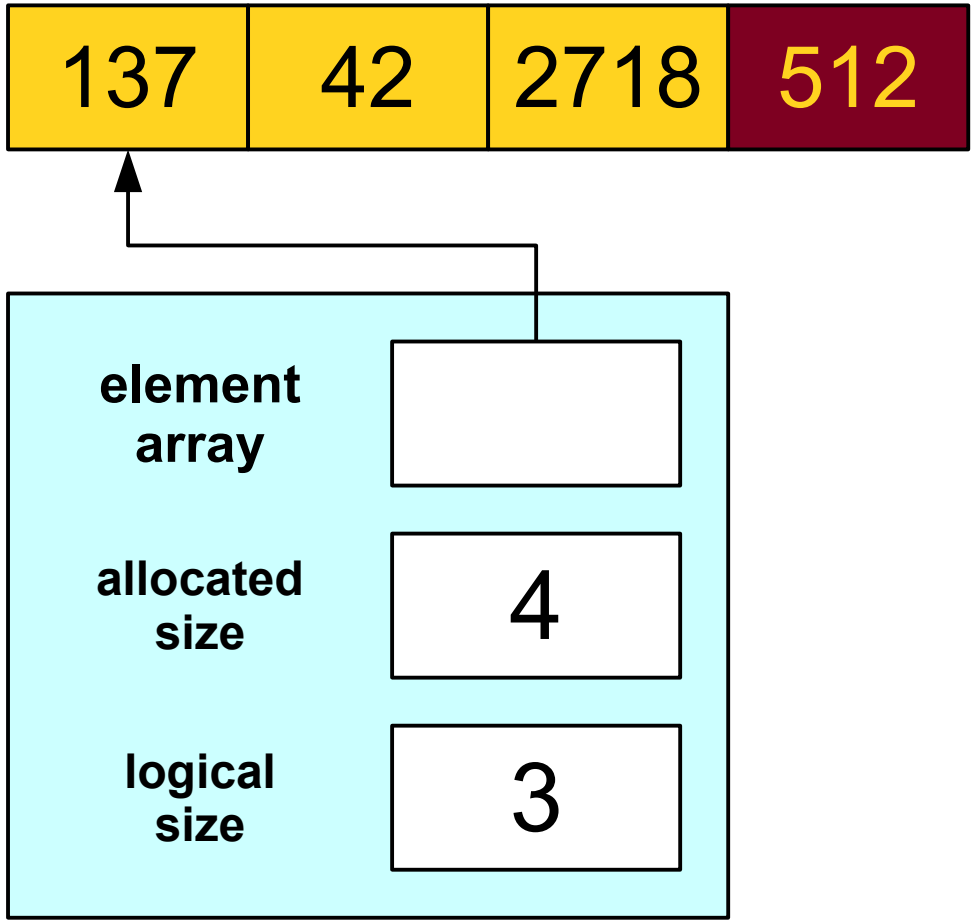

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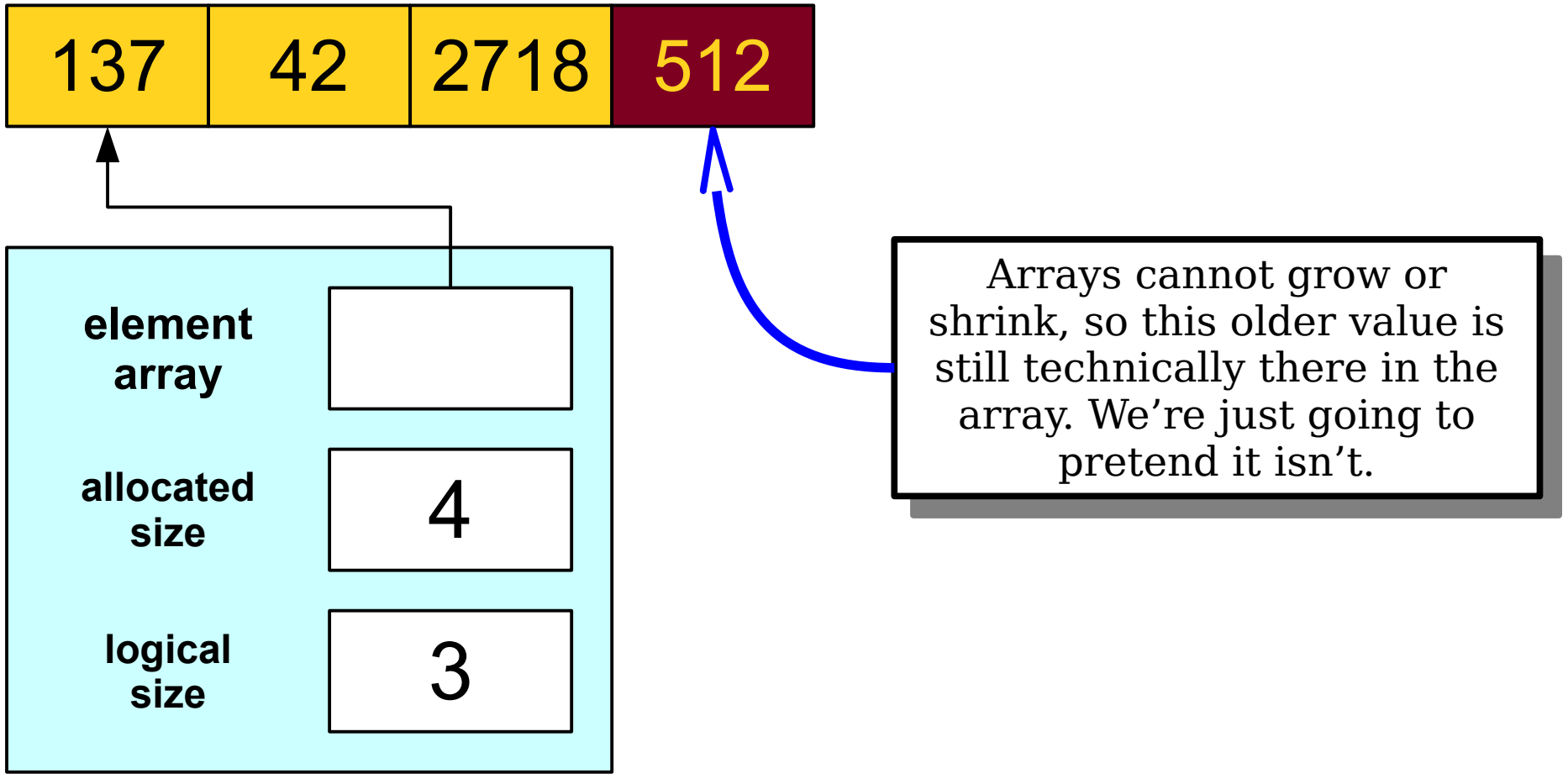

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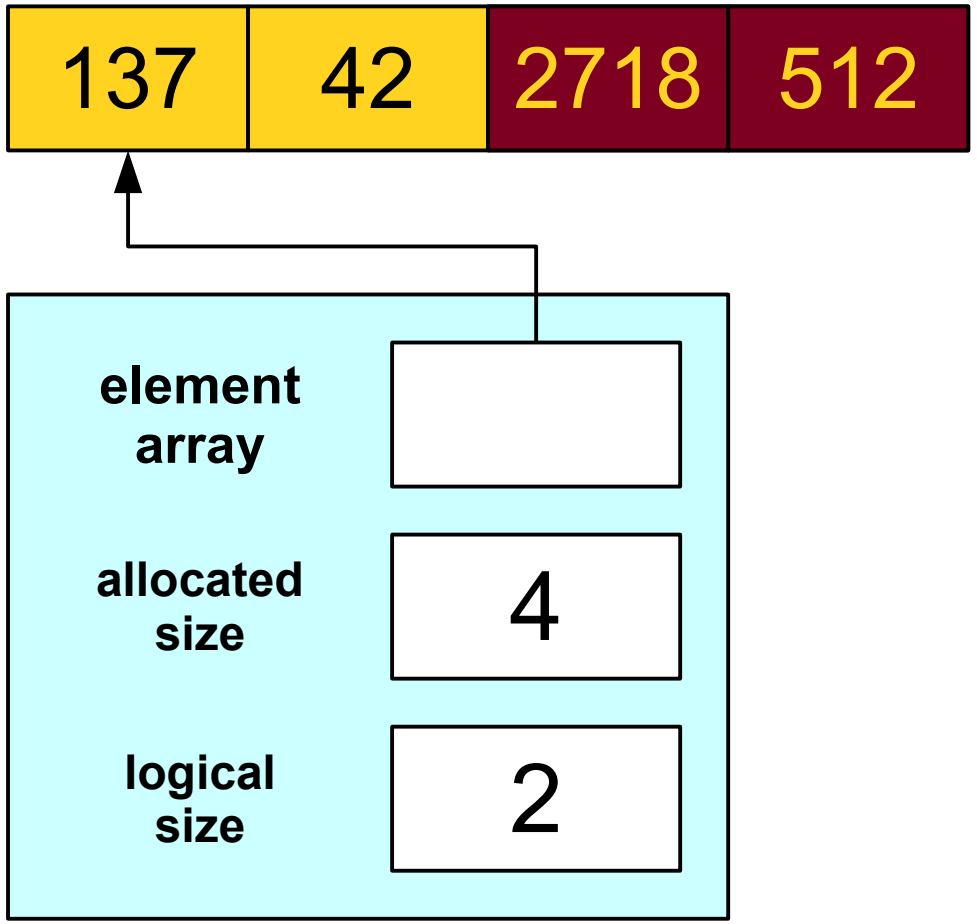

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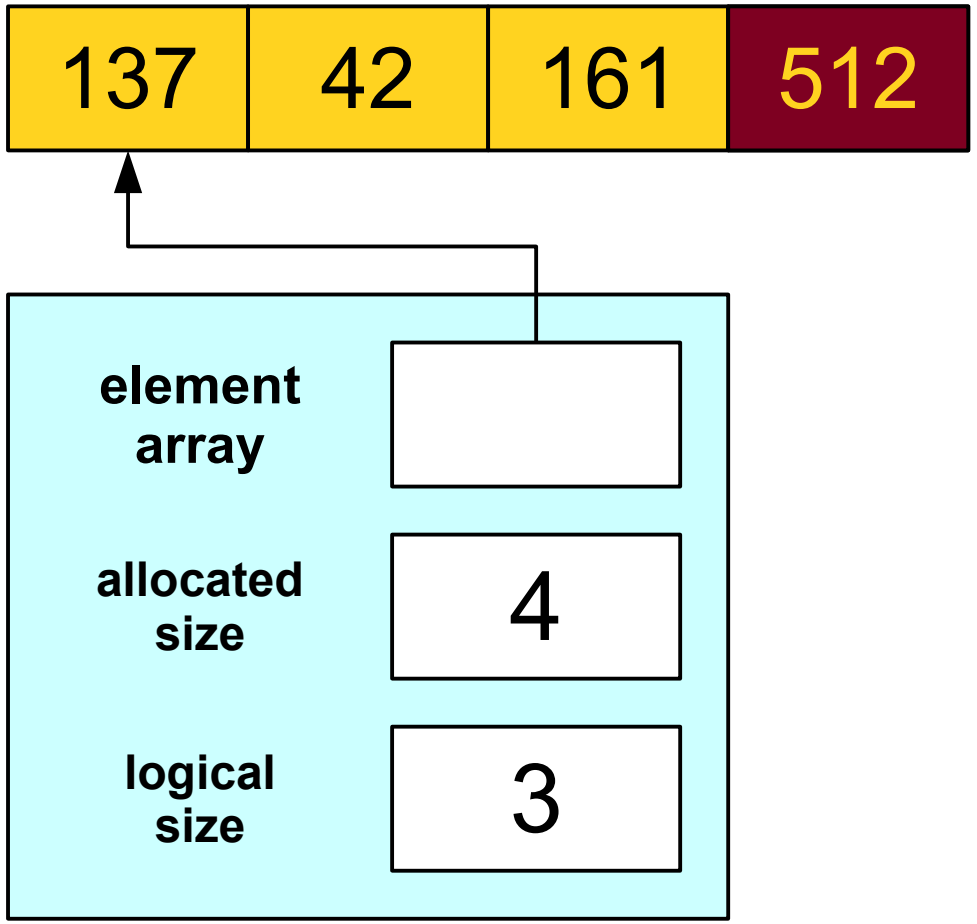

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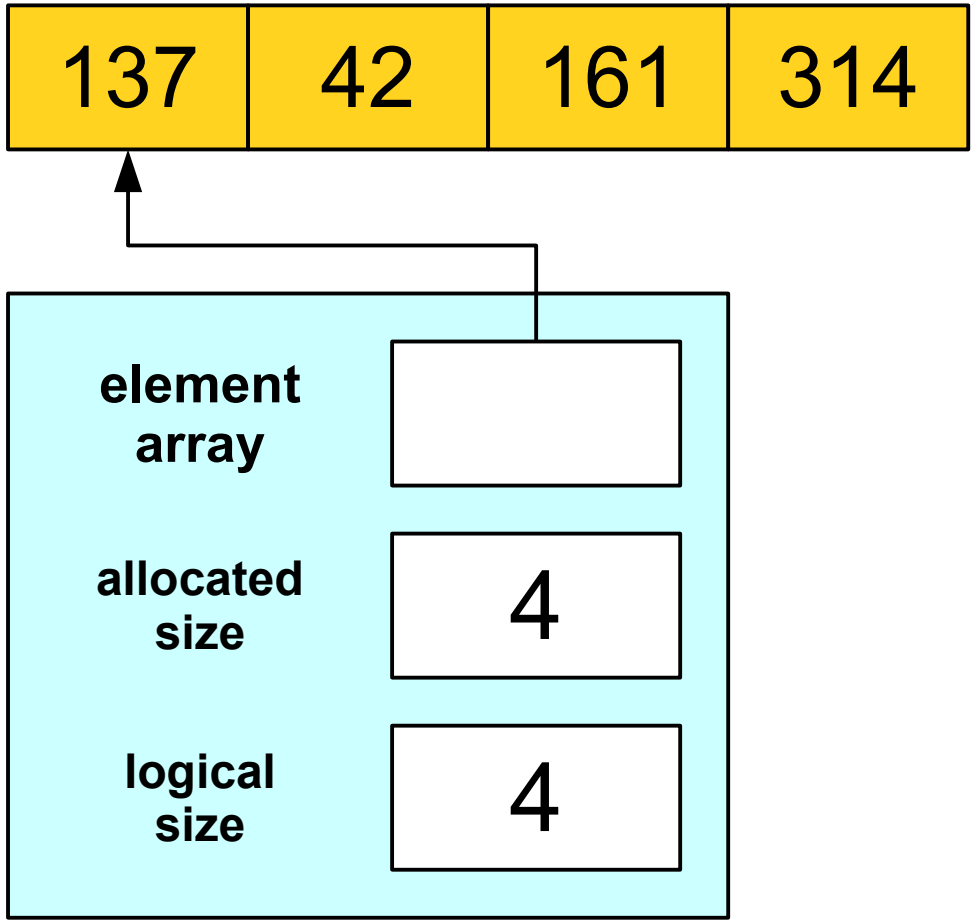
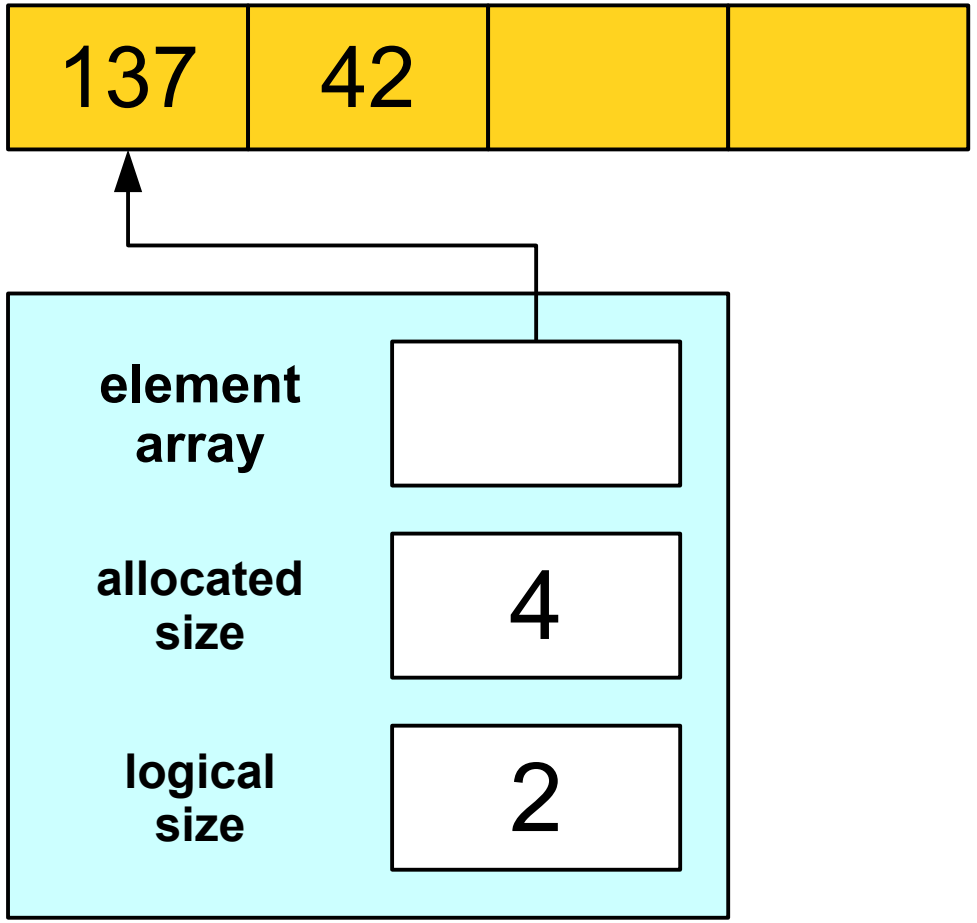


New Stuff!

Running out of Space

- Our current implementation very quickly runs out of space to store elements.
- What should we do when this happens?

An Initial Idea



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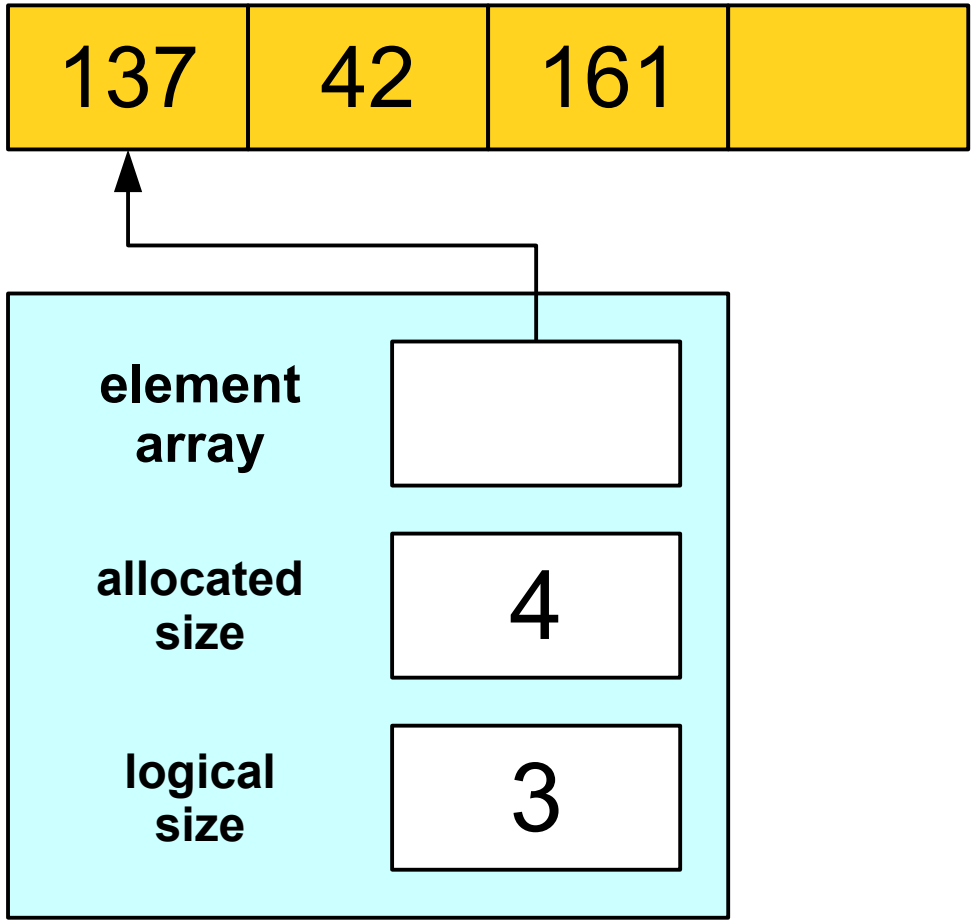

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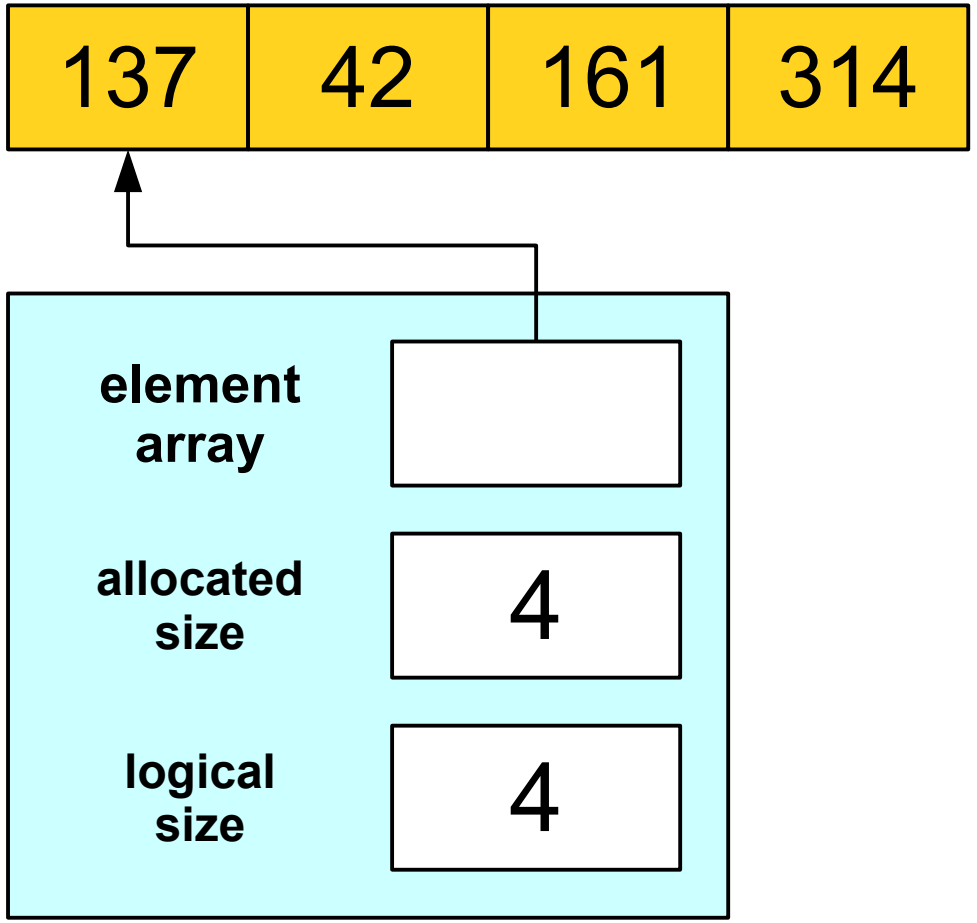

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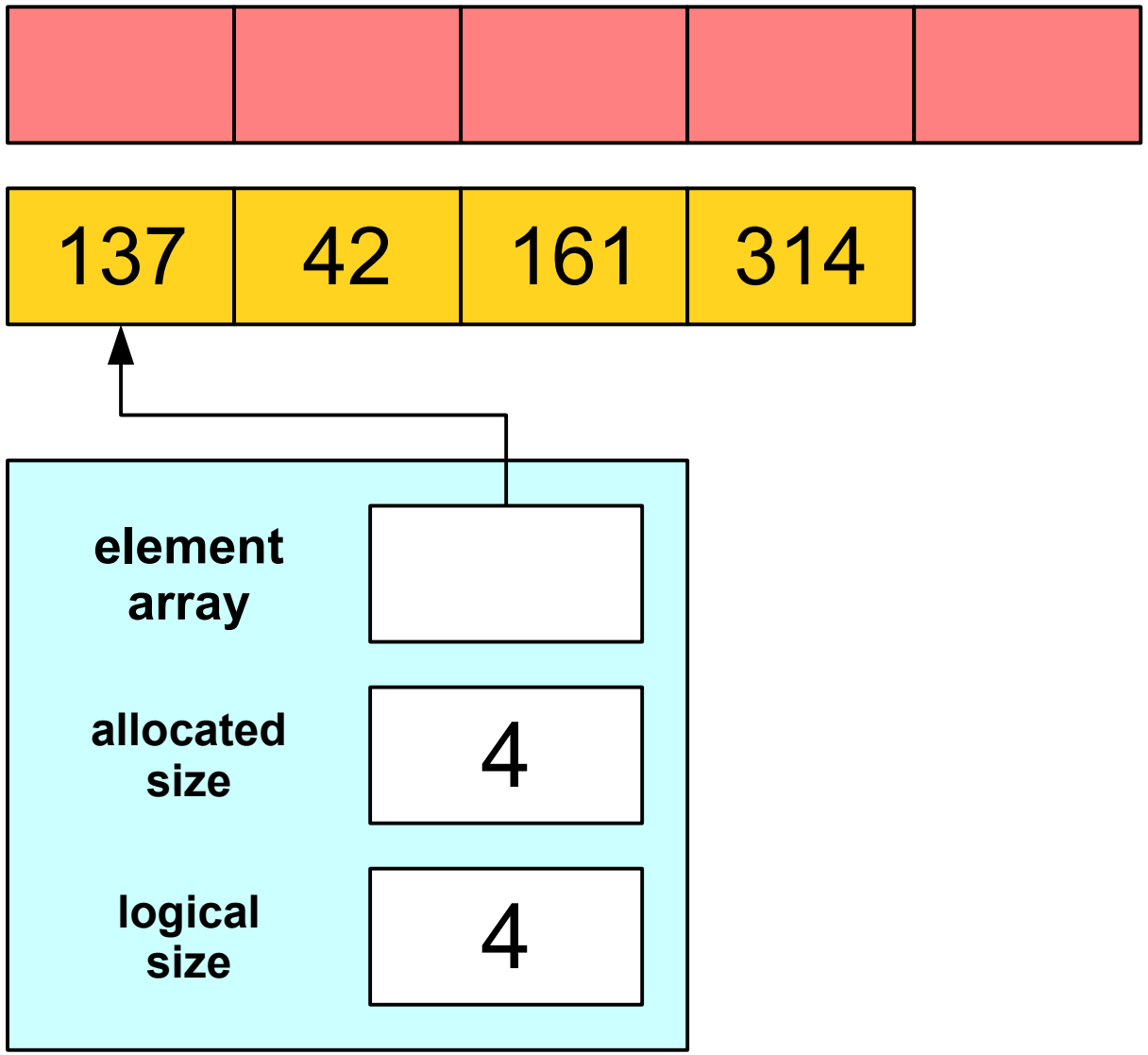

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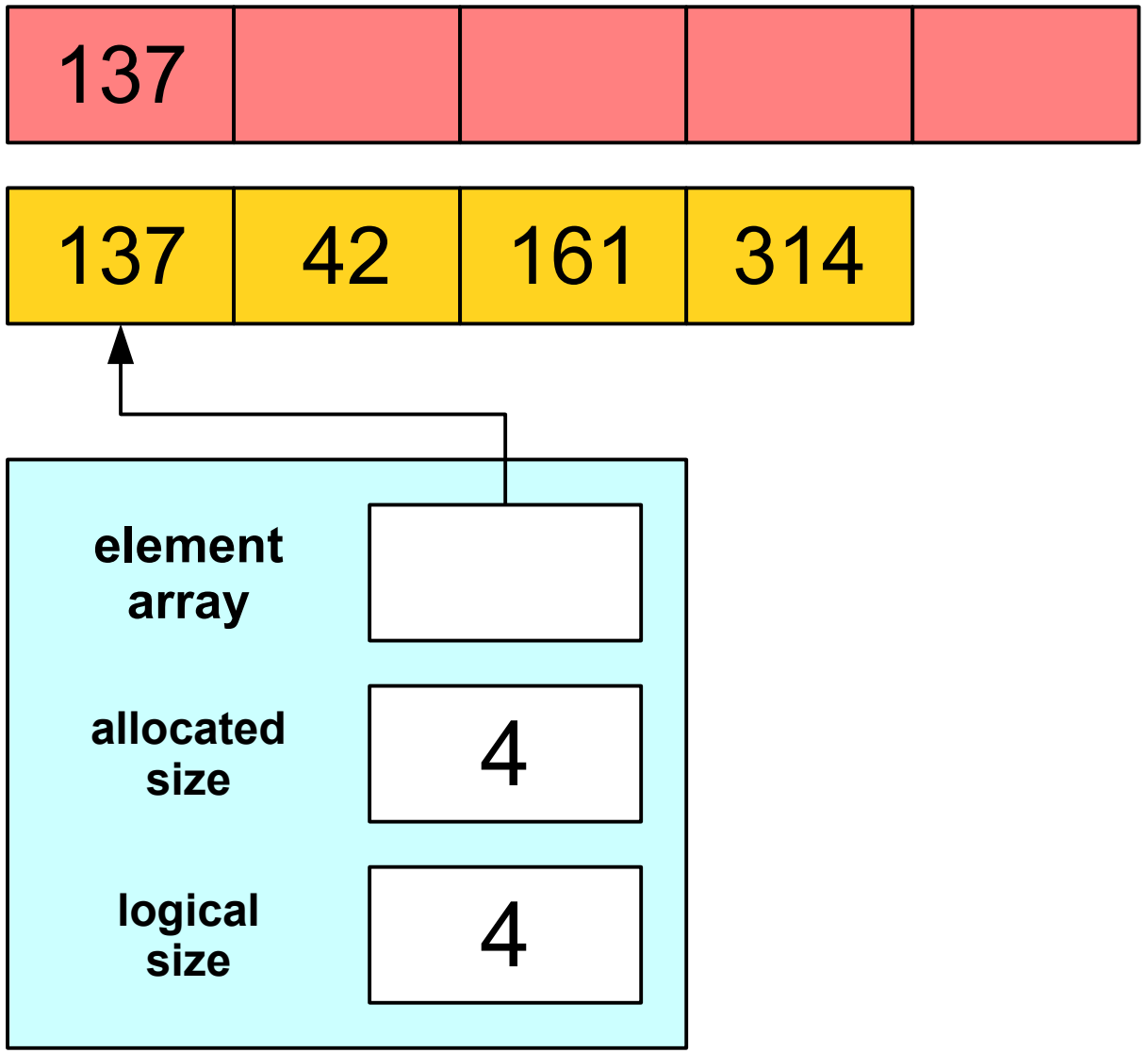

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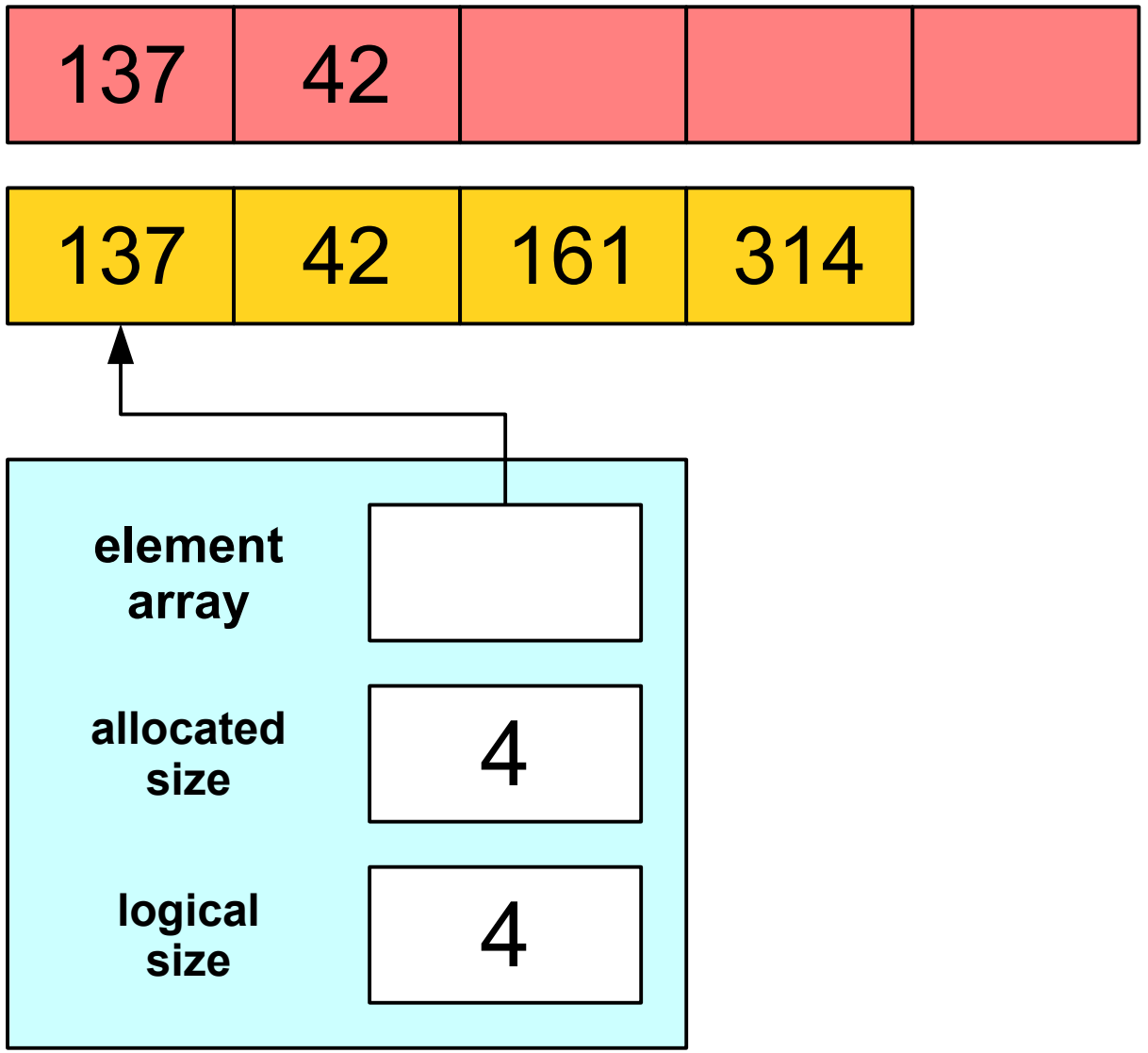

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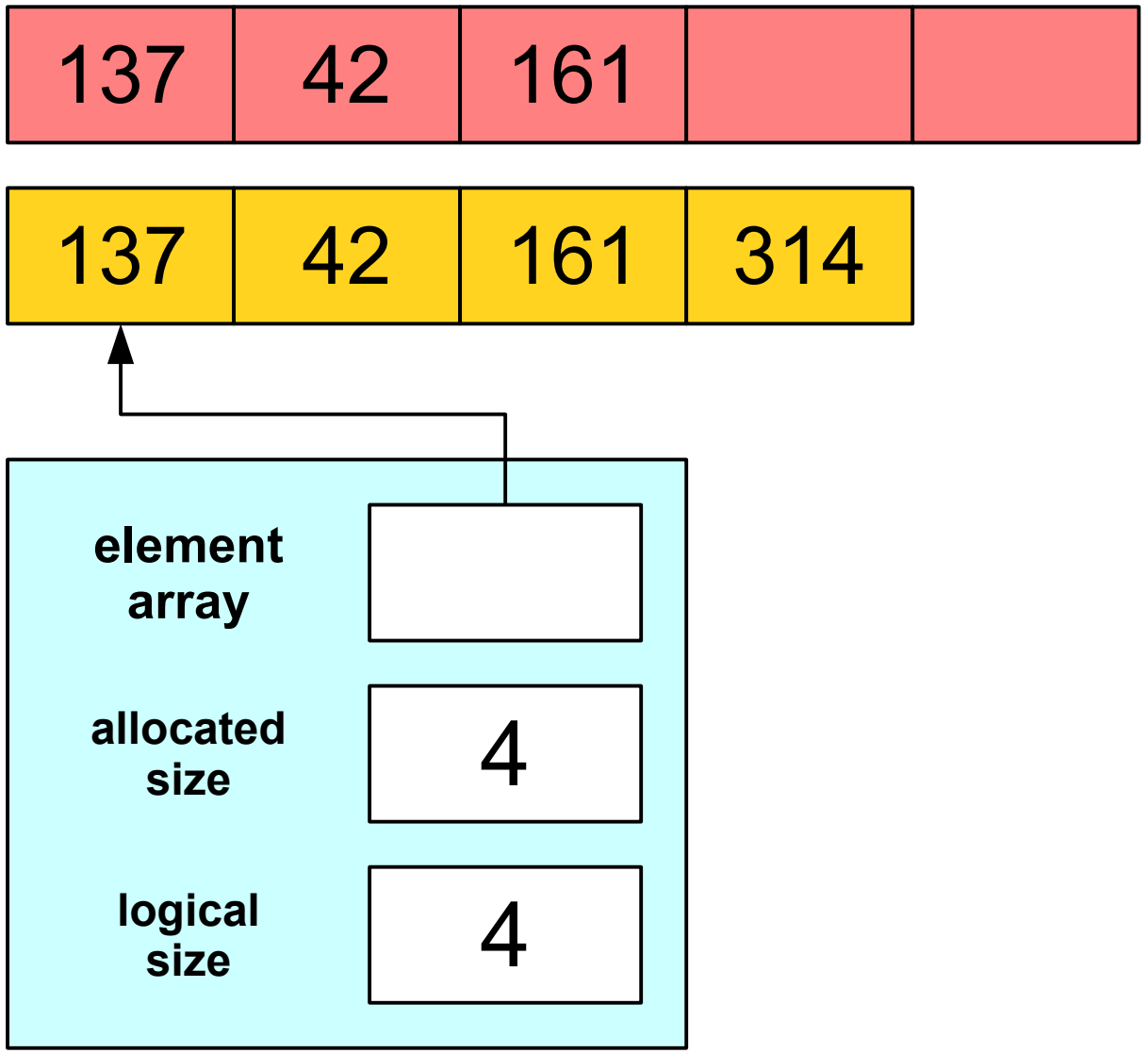

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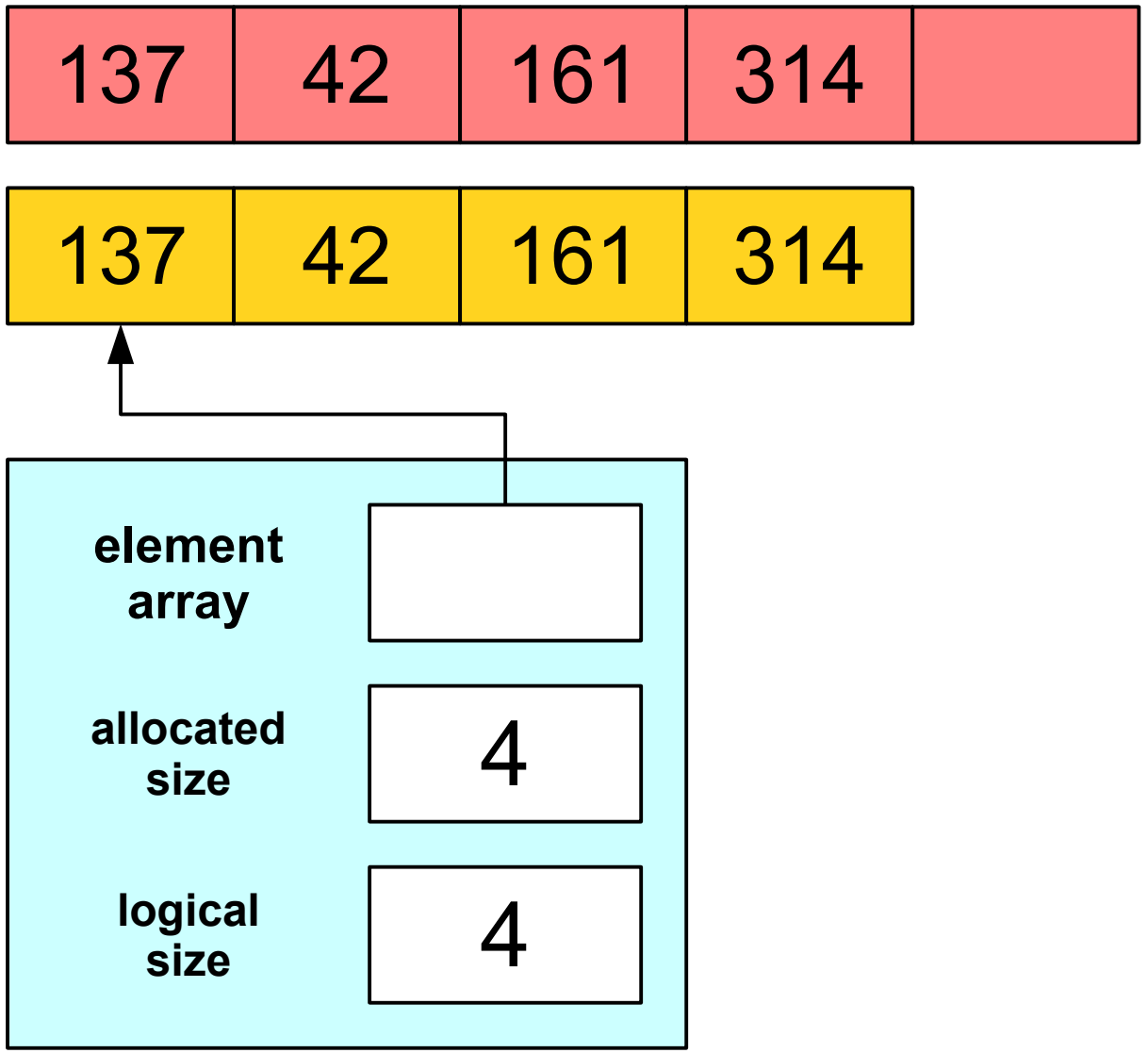

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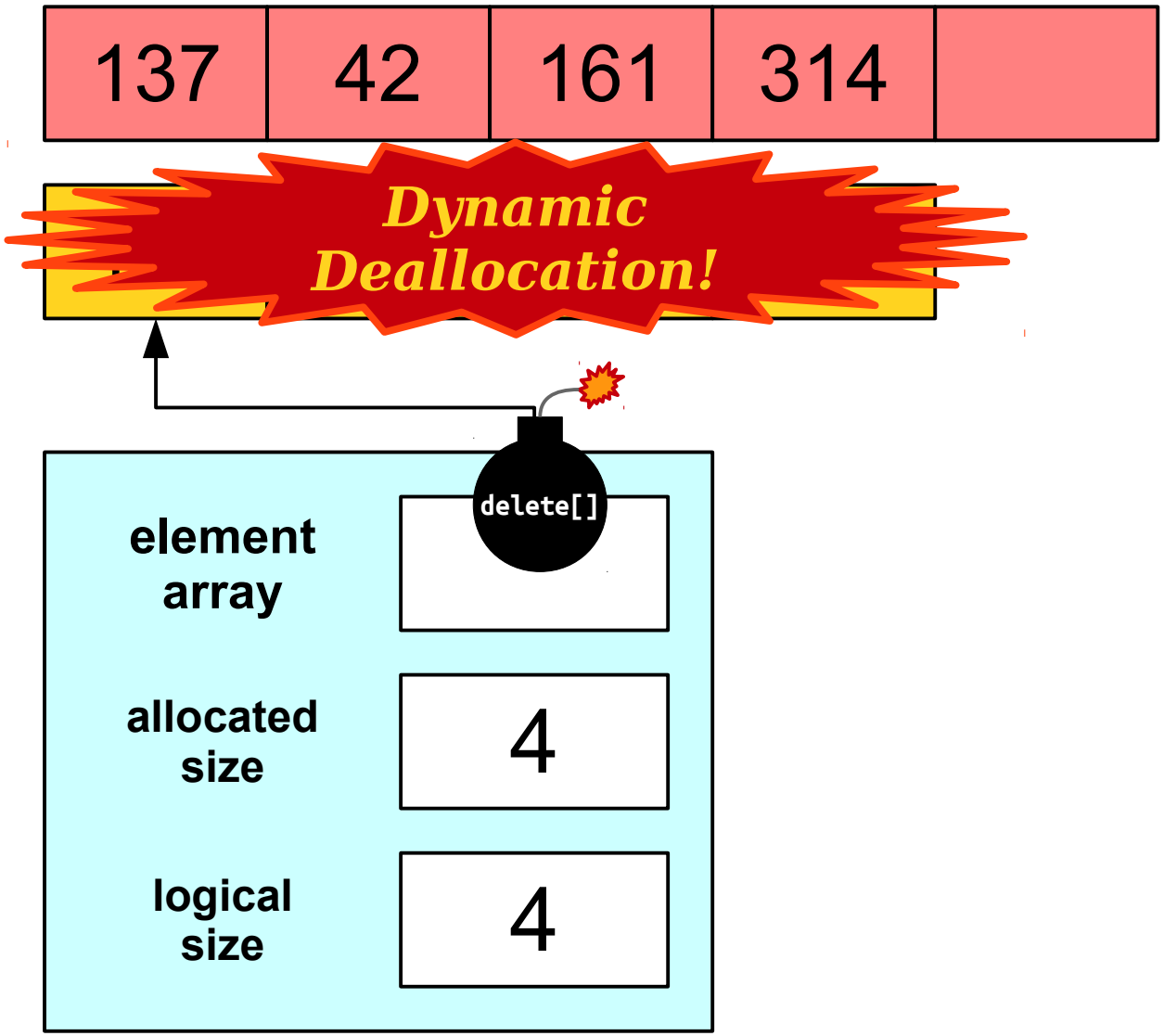

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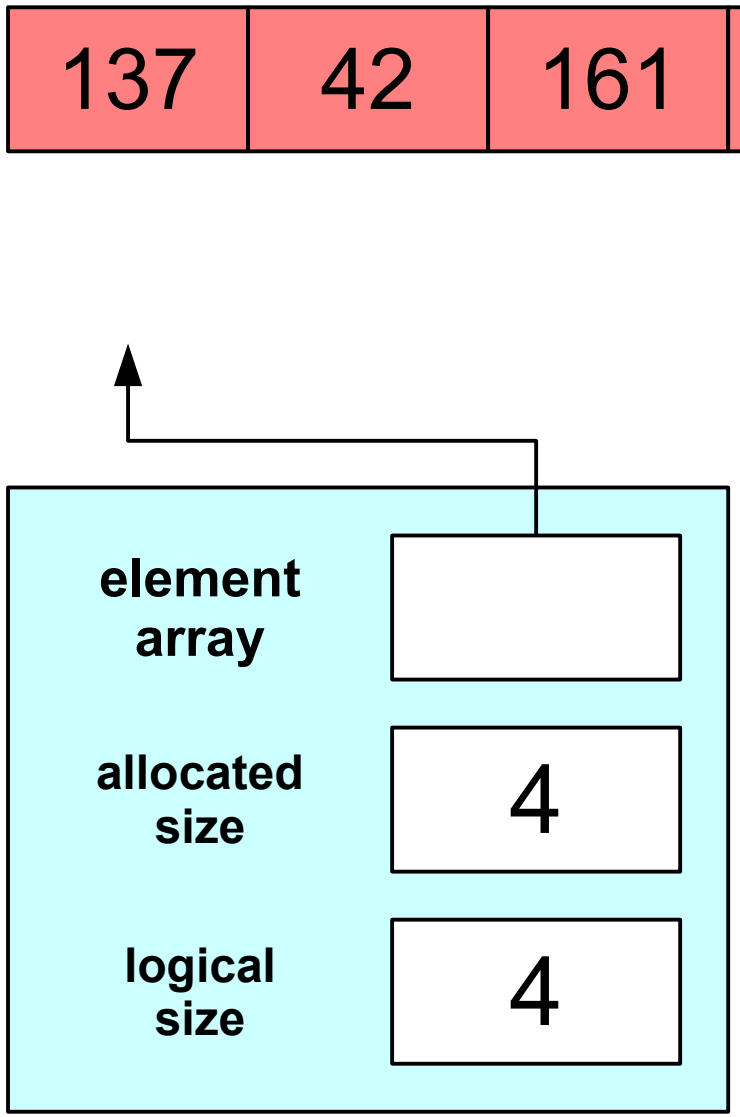

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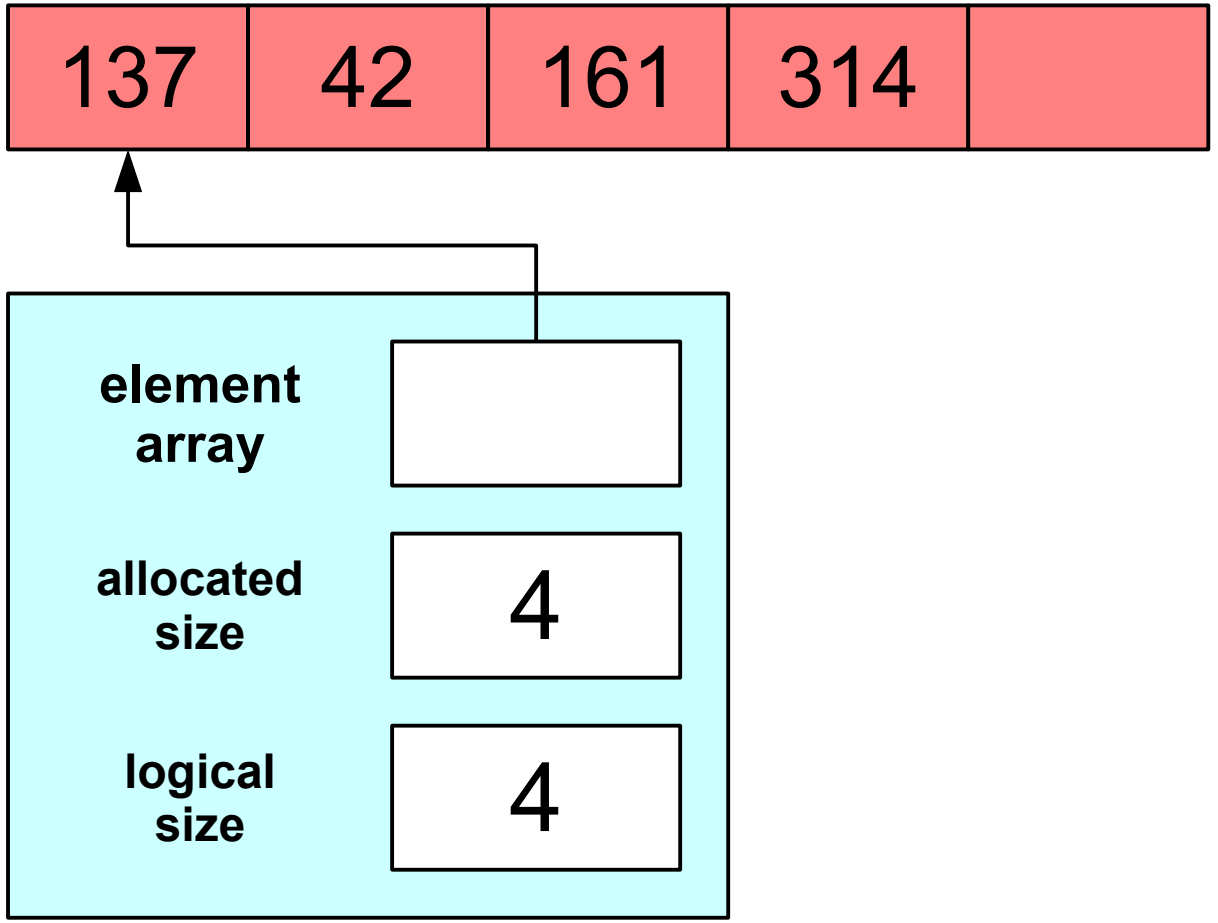

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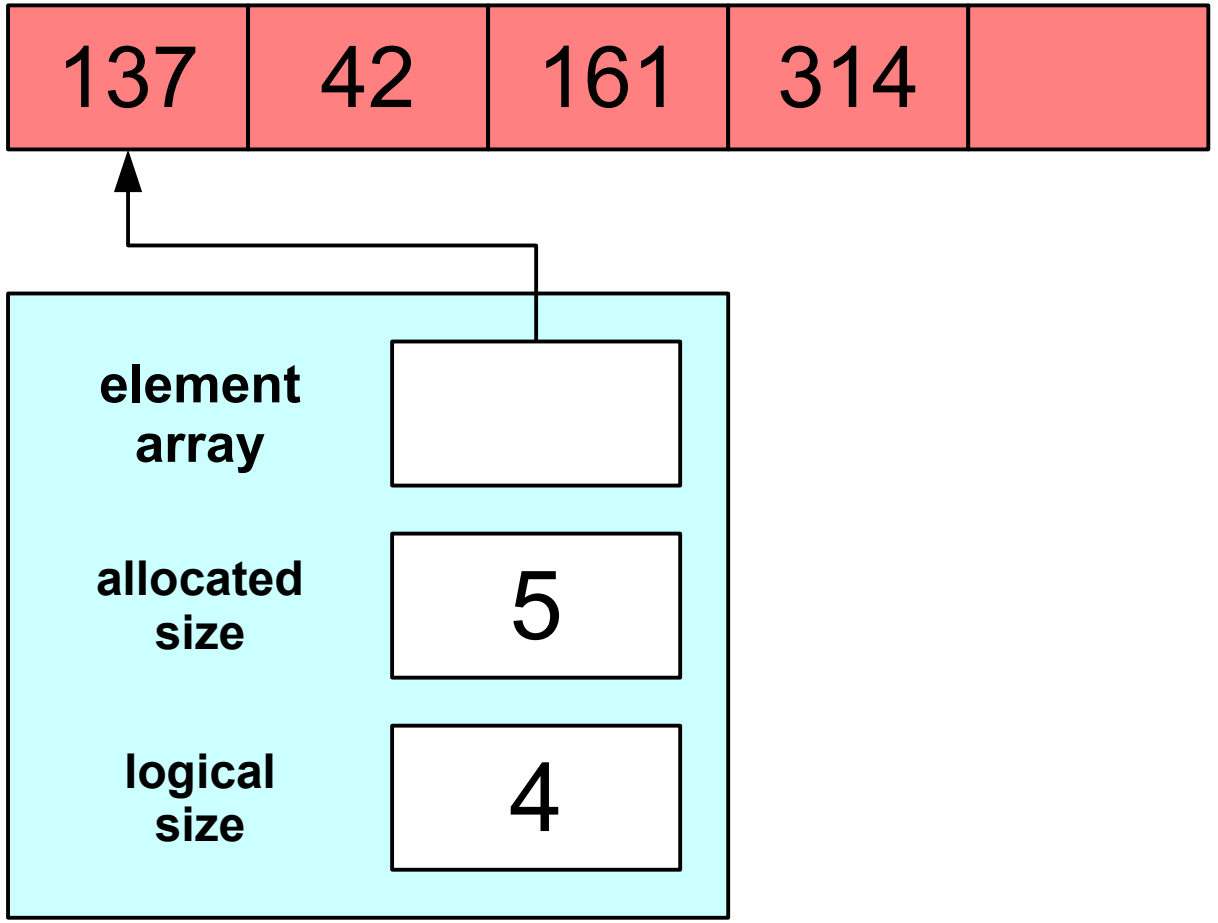

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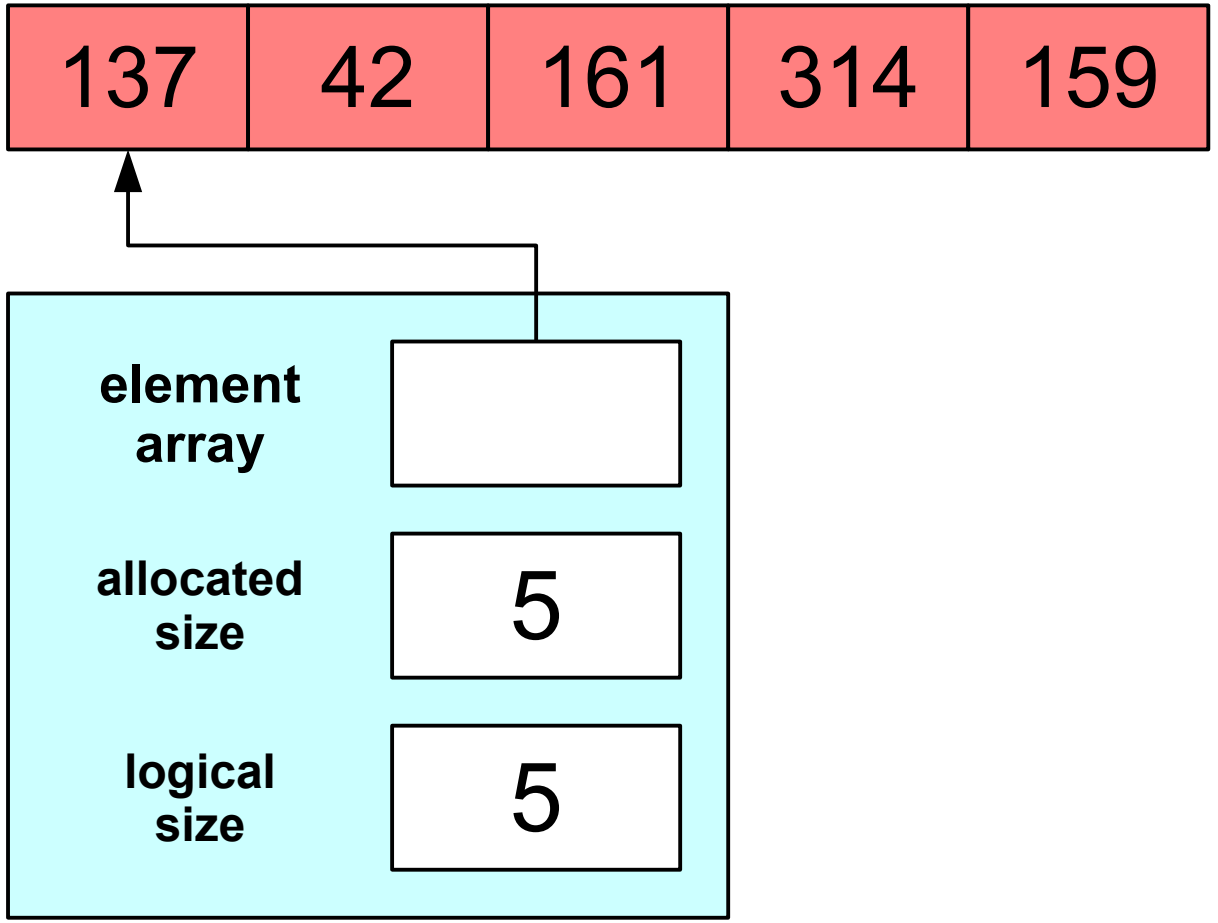

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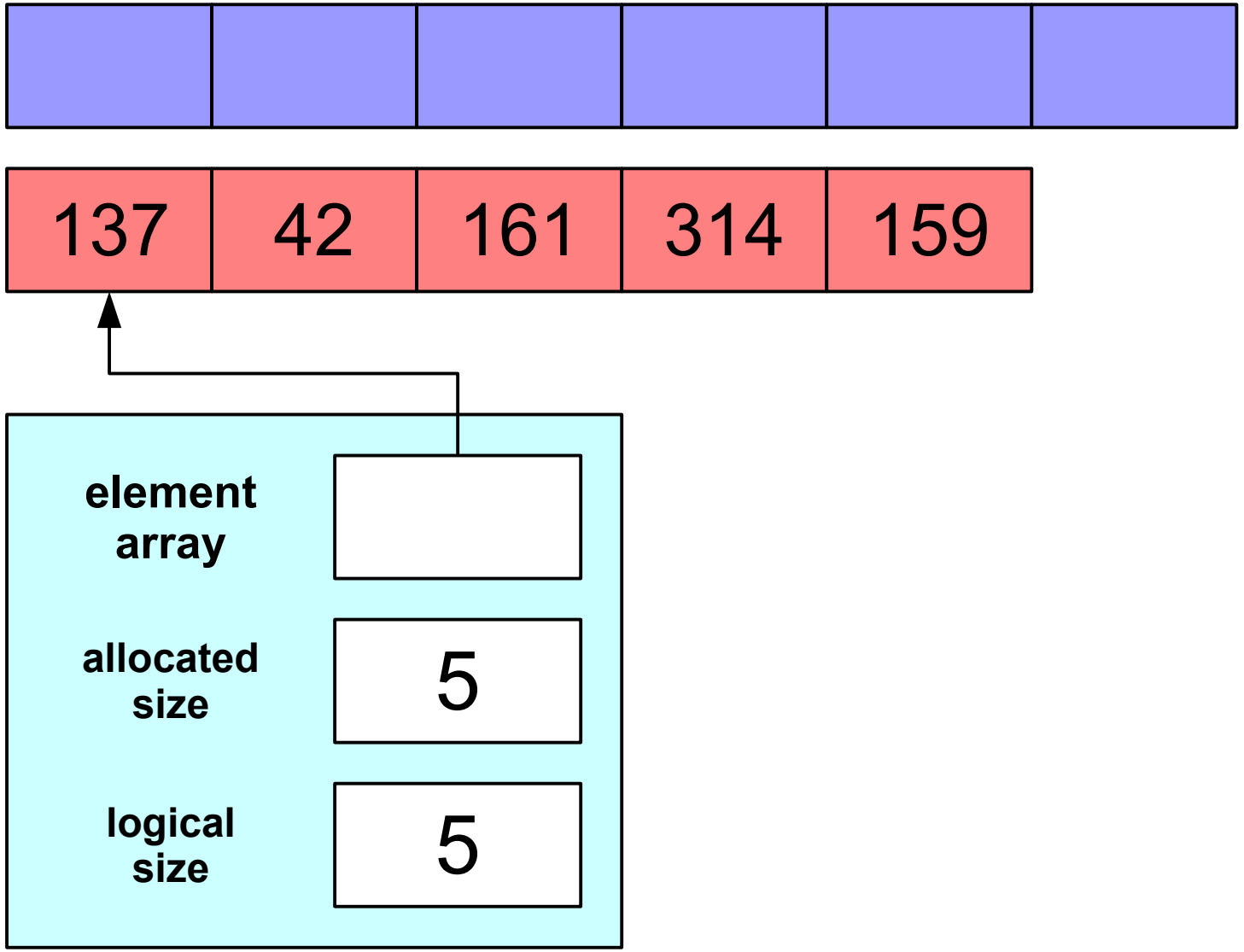

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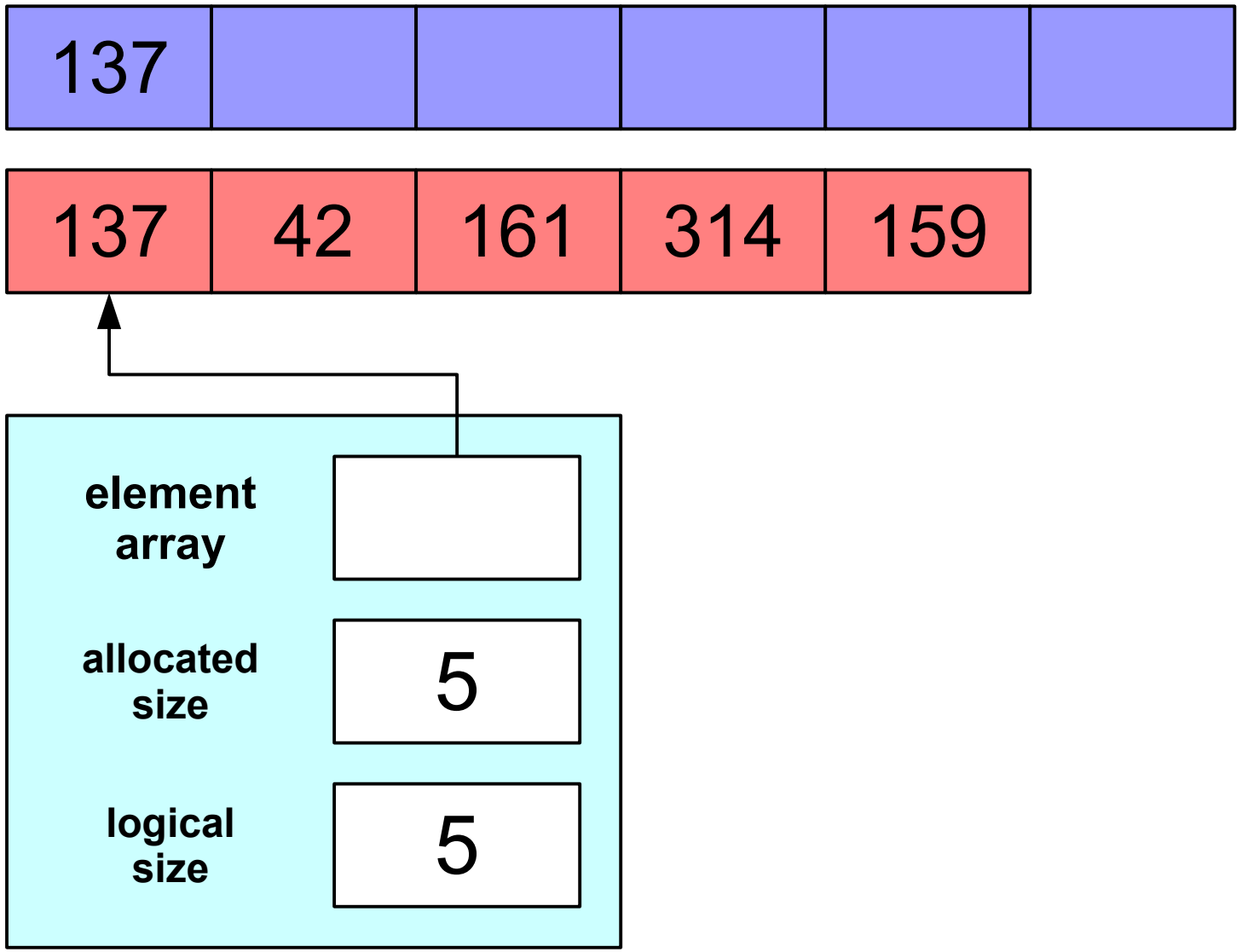

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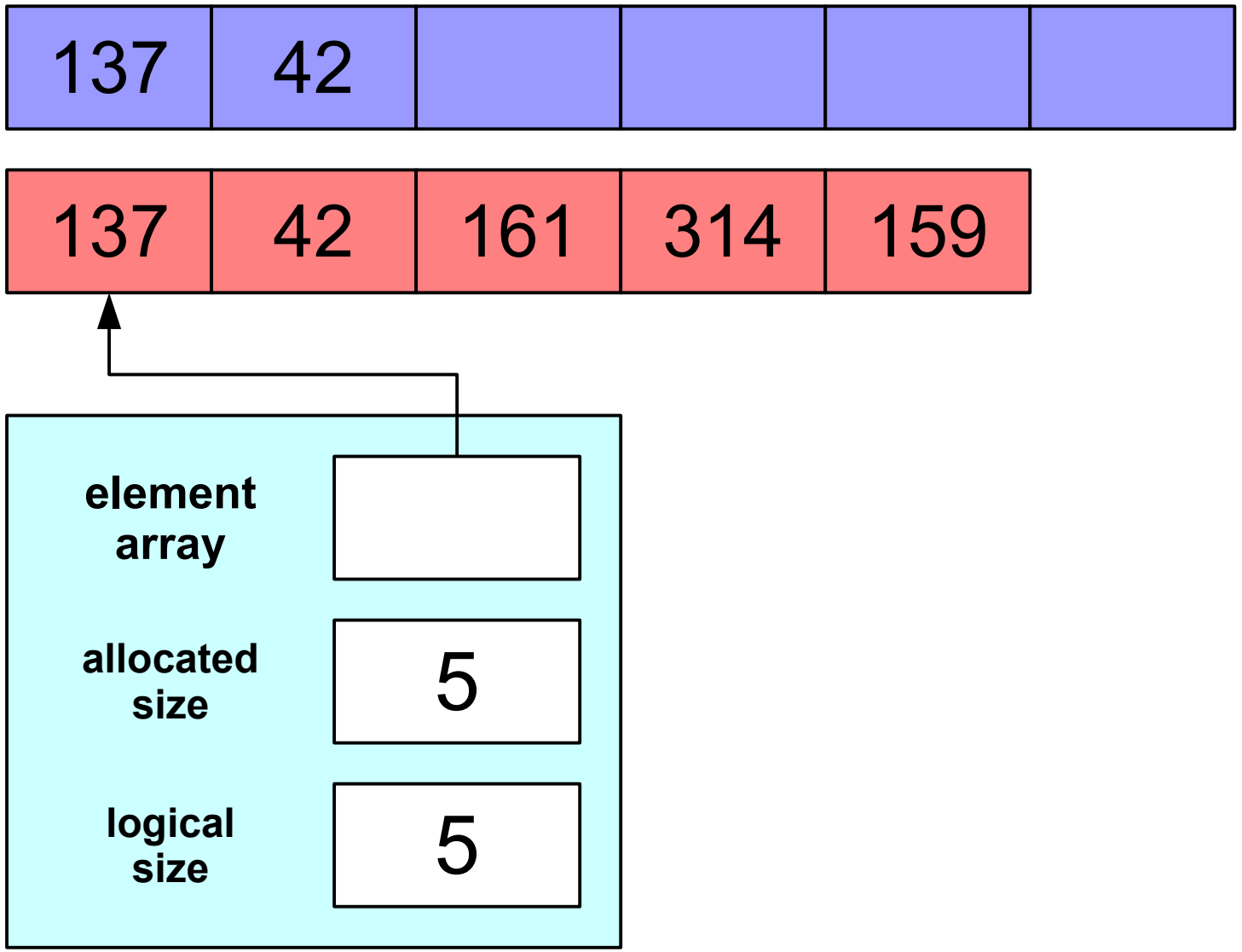

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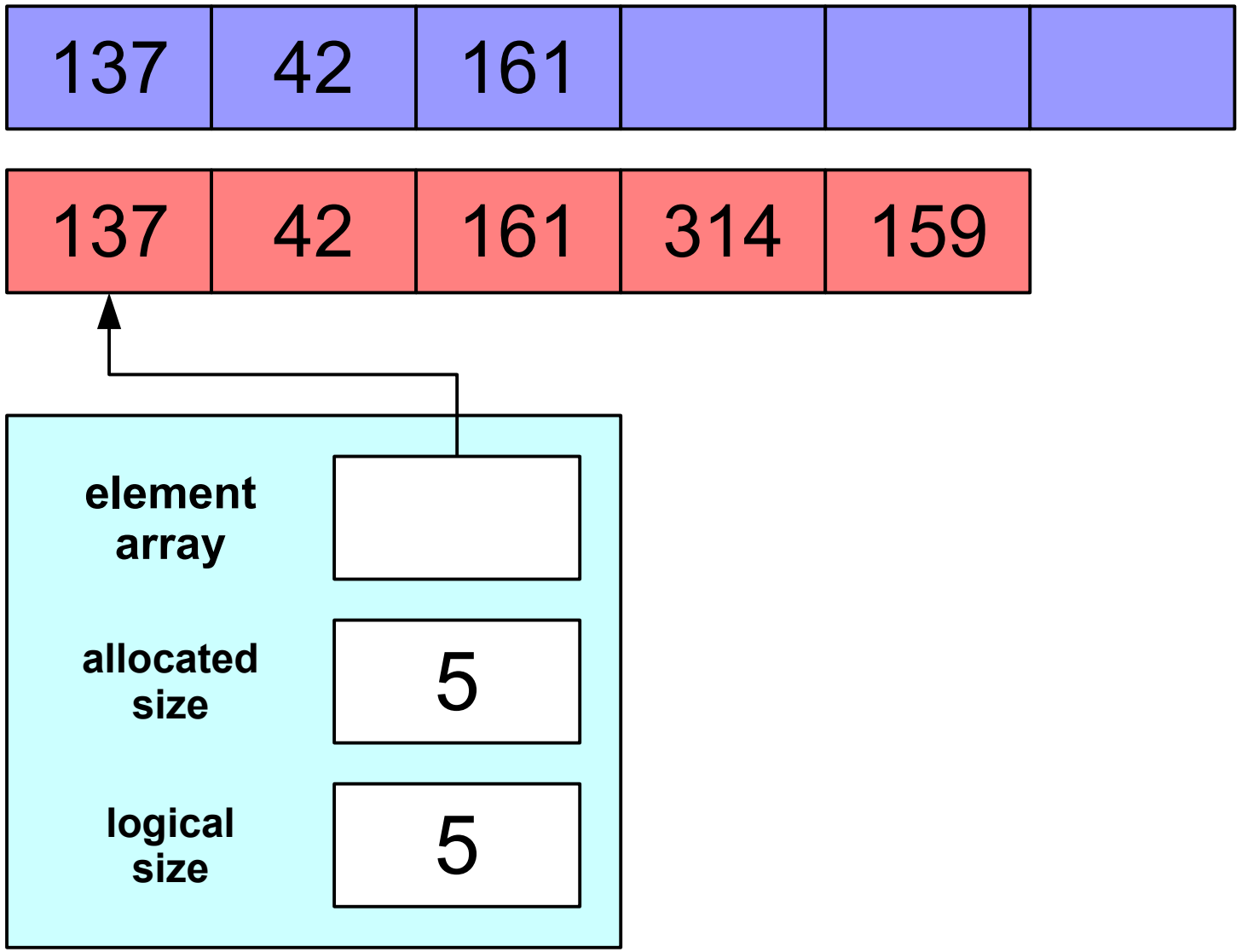

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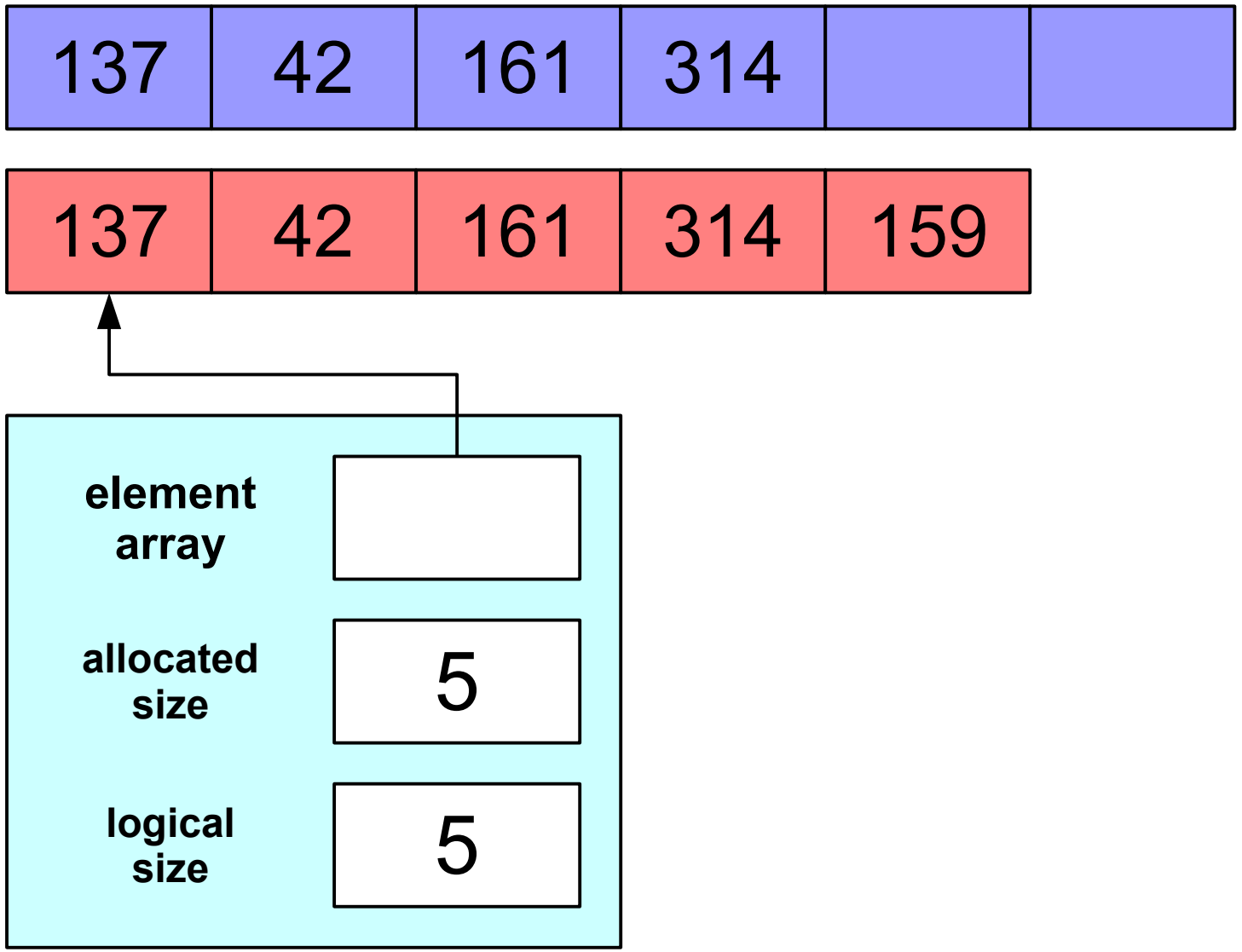

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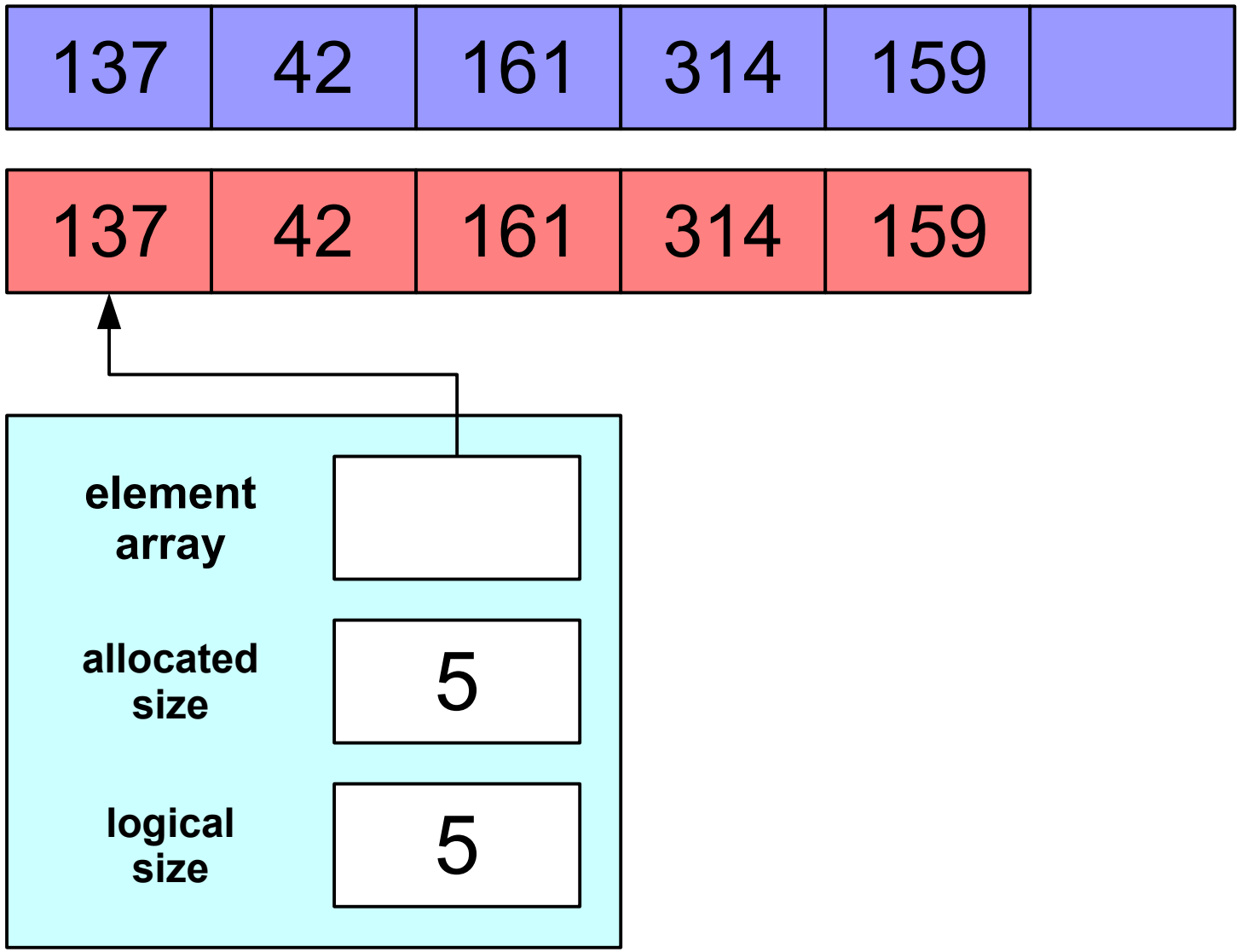

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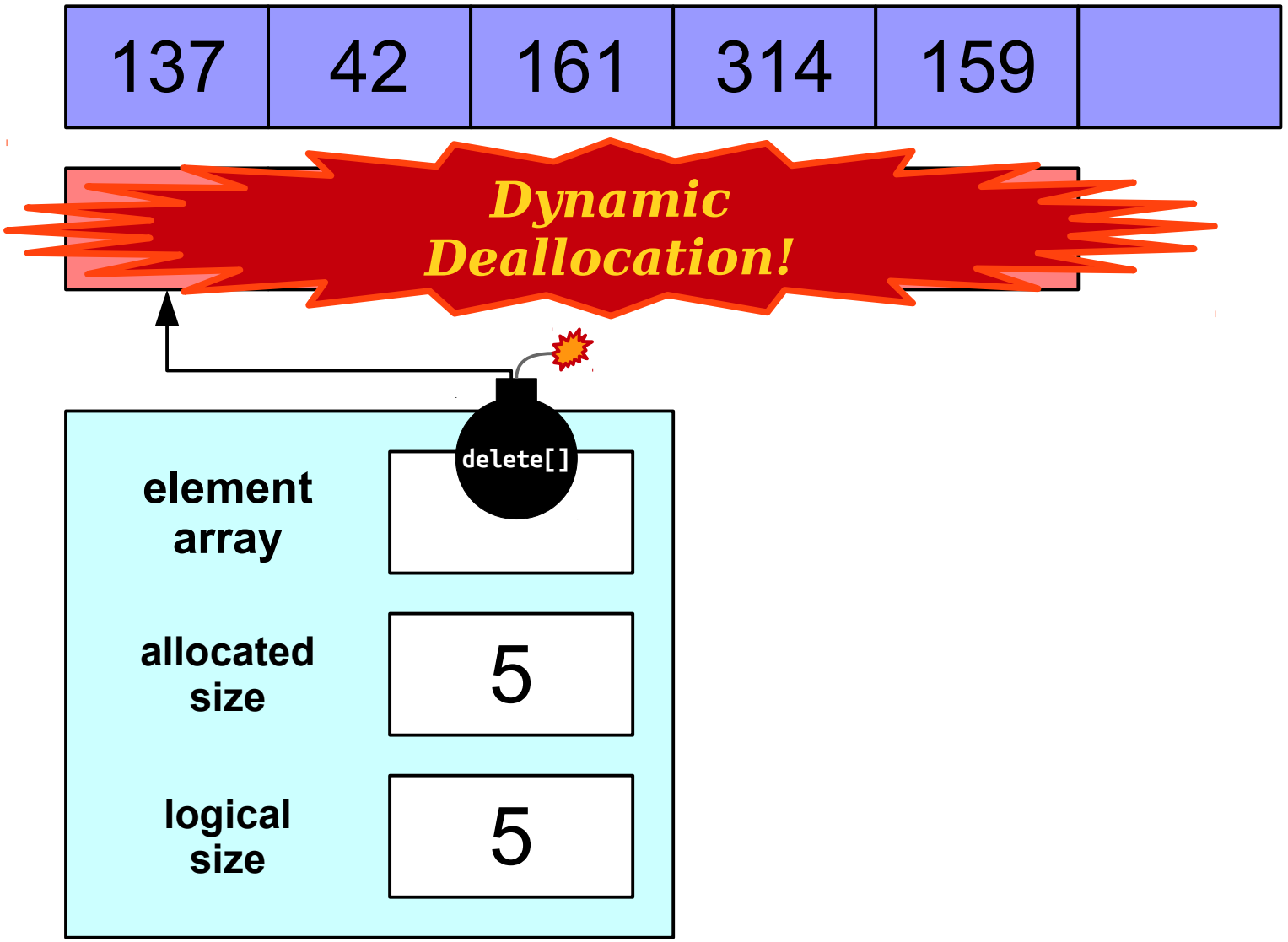

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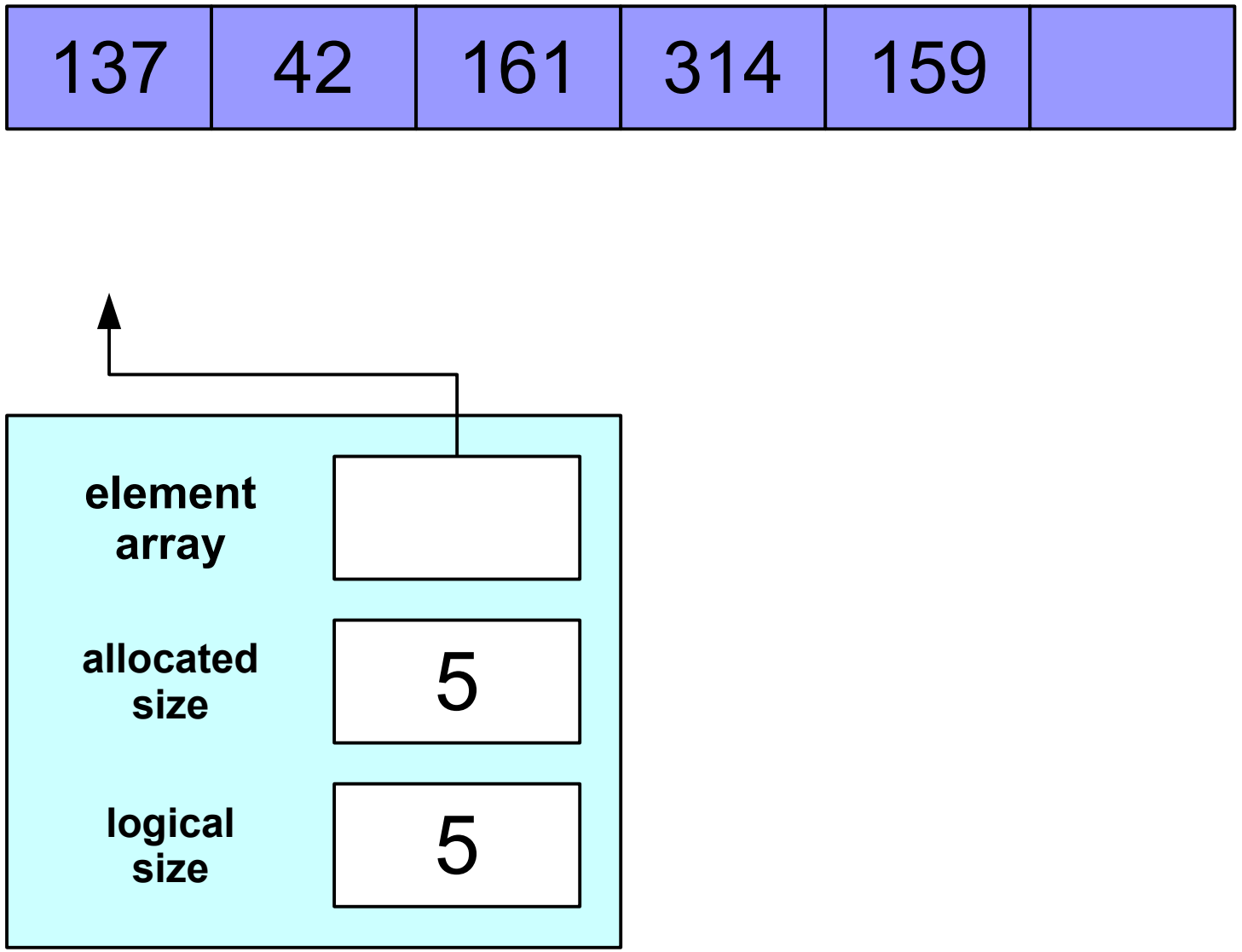

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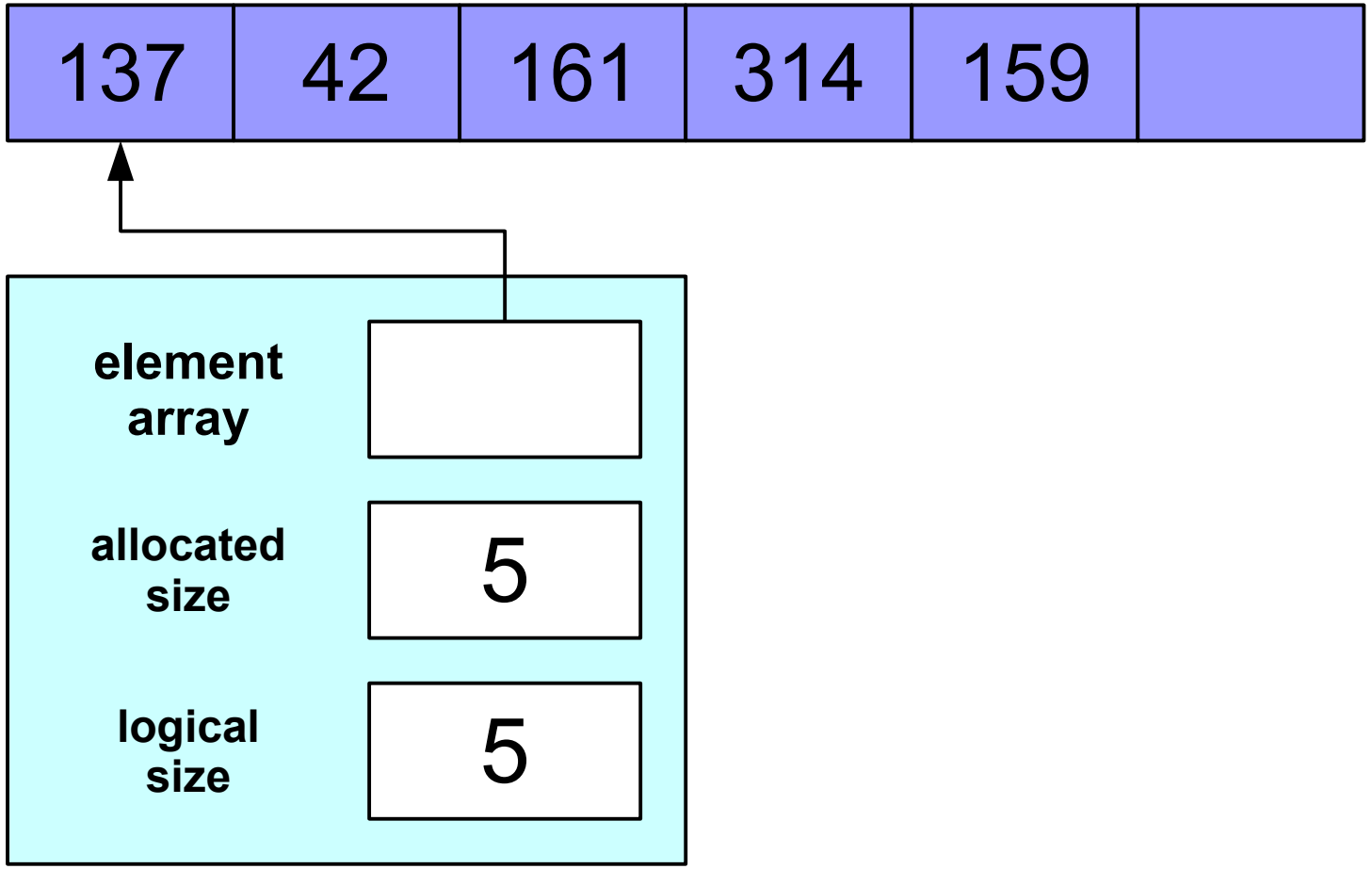

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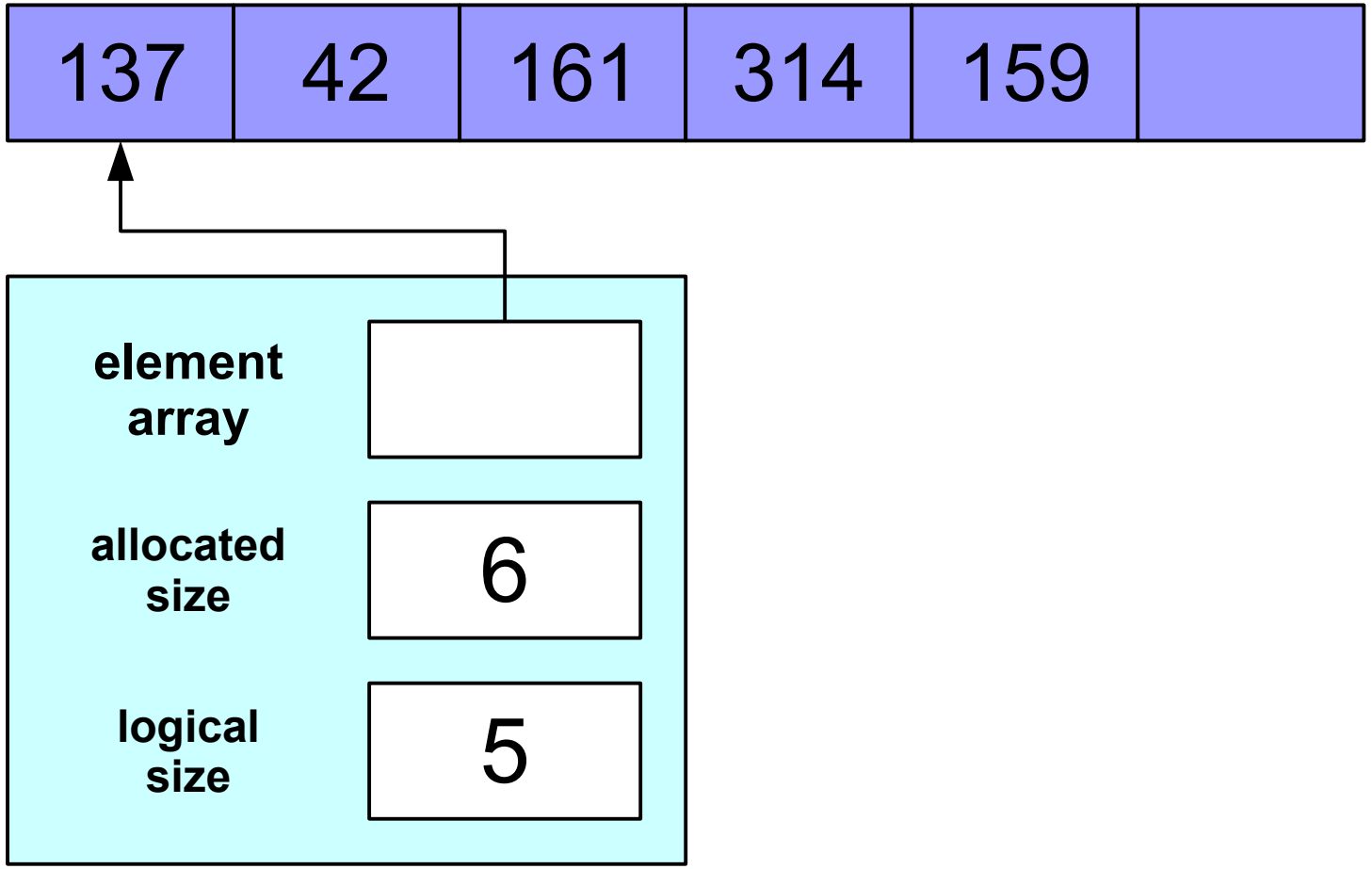

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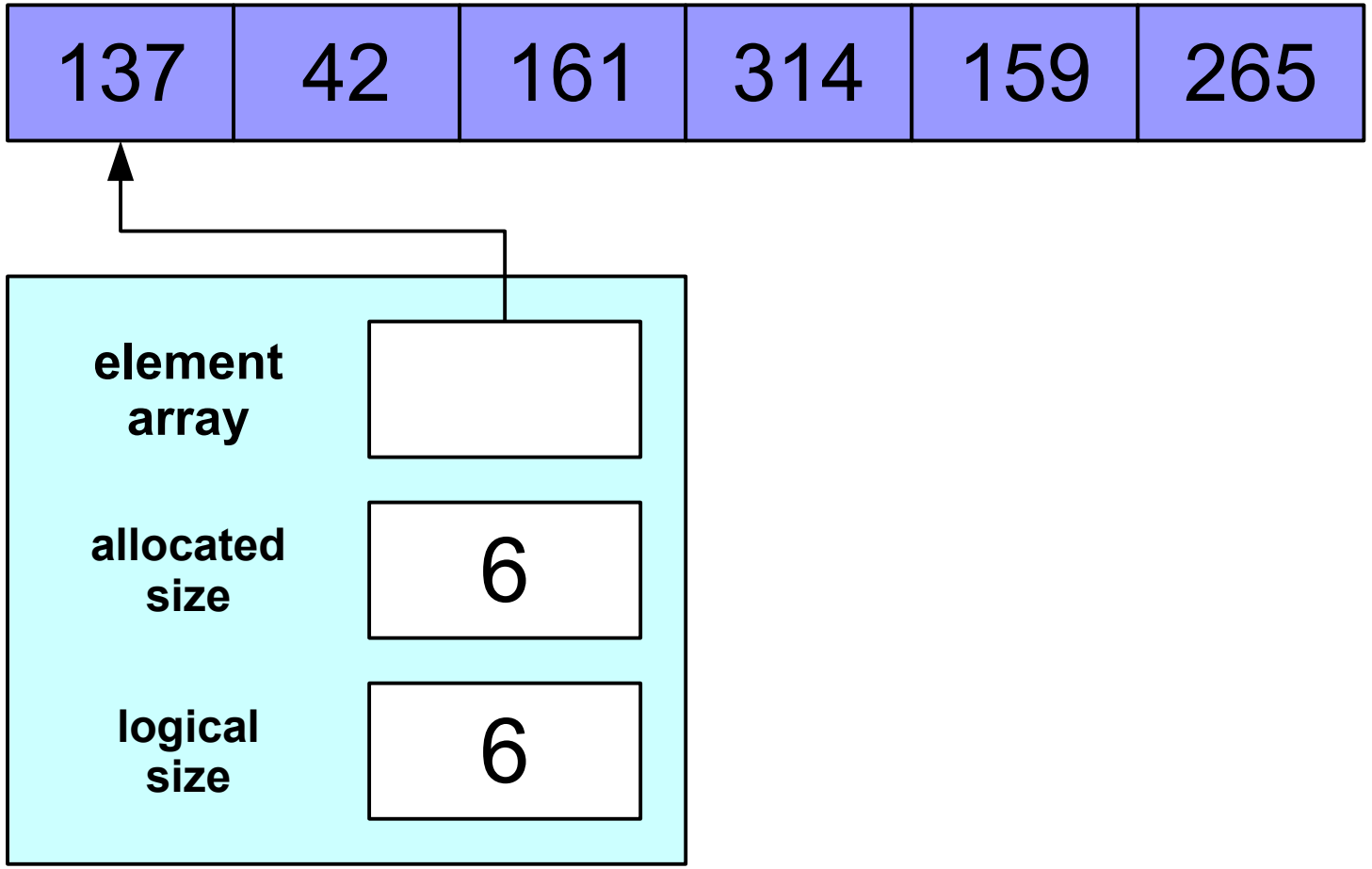

Ready... set... grow!

```
class OurStack {
public:
    OurStack();
    ~OurStack();

    void push(int value);
    int  pop();
    int  peek() const;

    int  size() const;
    bool isEmpty() const;

private:


    int* elems;
    int  allocatedSize;
    int  logicalSize;
};
```

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    void grow();

    int* elems;
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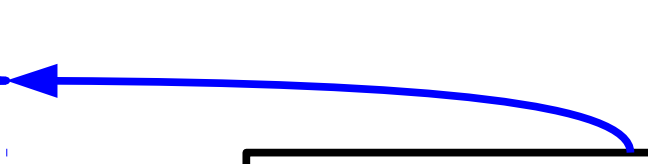

This is a ***private member function***. It's a helper function only the implementation can call.

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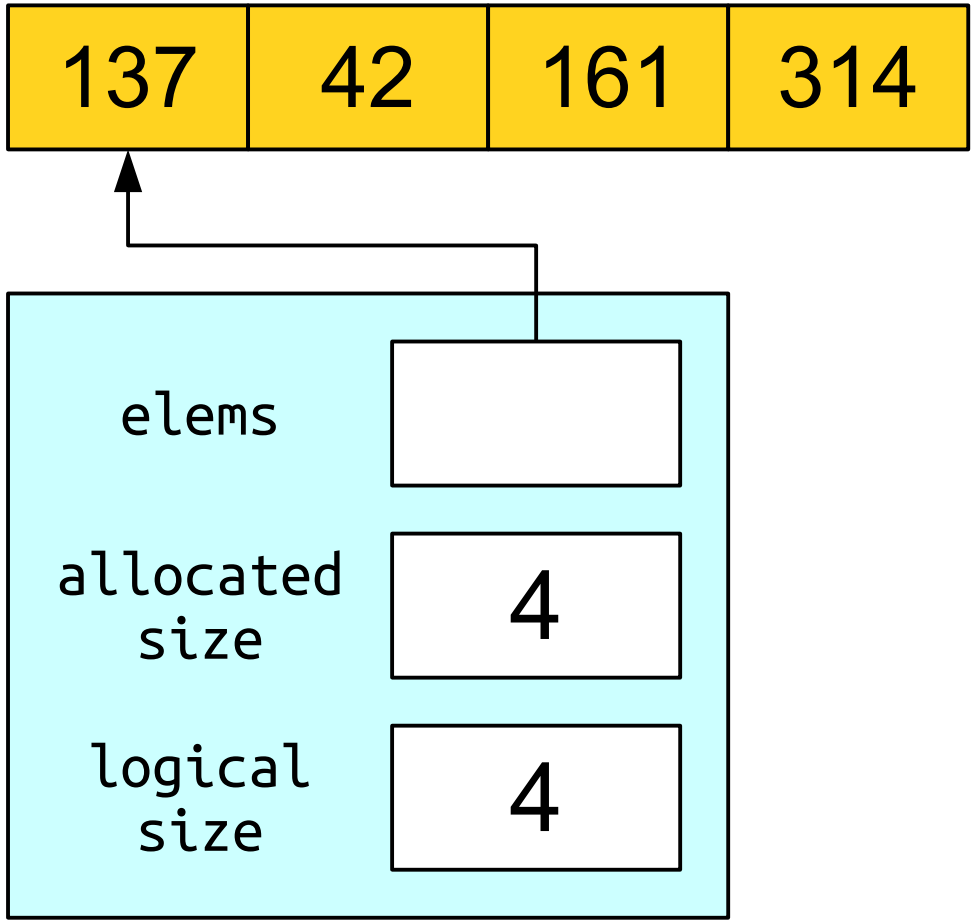

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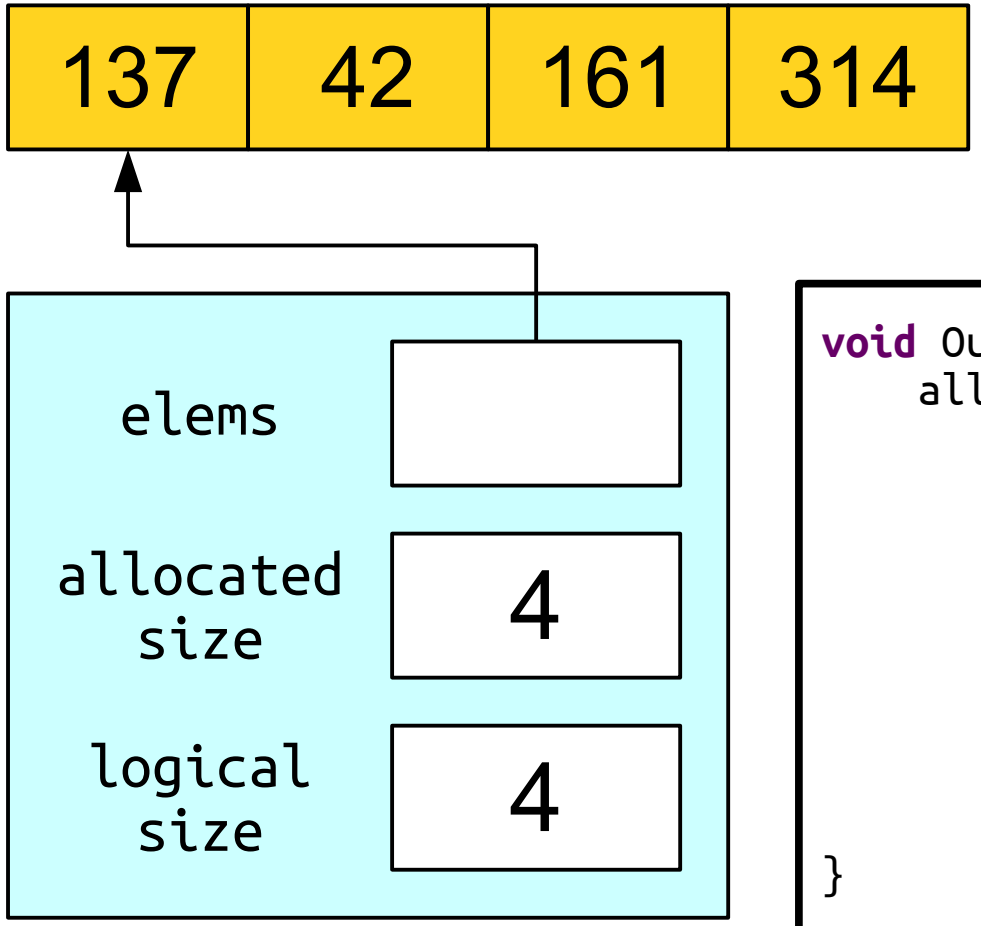


```
void OurStack::grow() {
    allocatedSize++;



}
```

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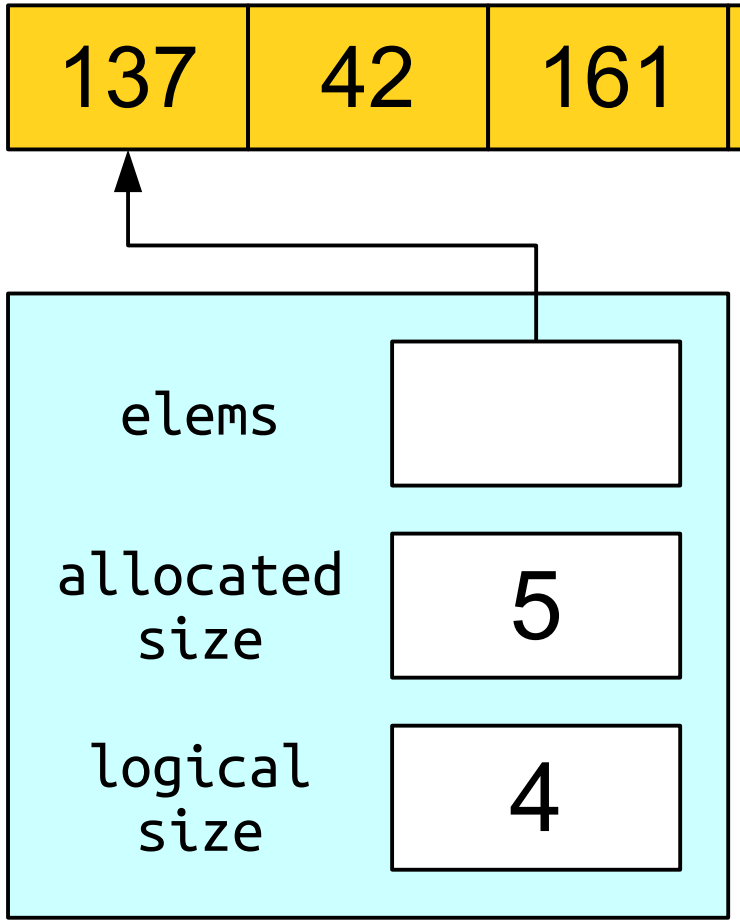


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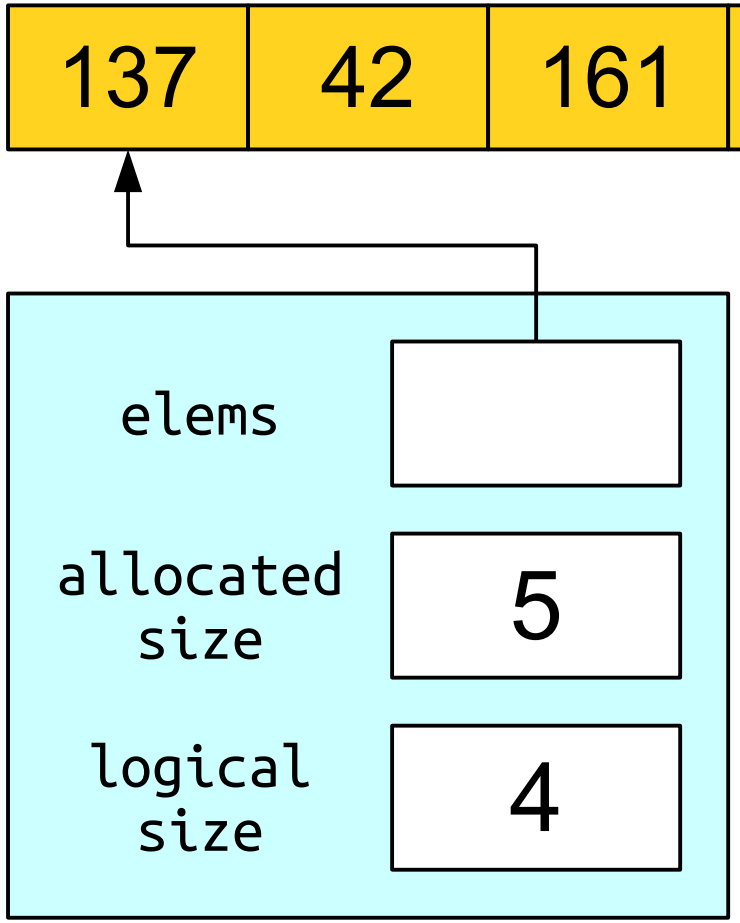


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void OurStack::grow() {
    allocatedSize++;

    int* newElems = new int[allocatedSize];




}
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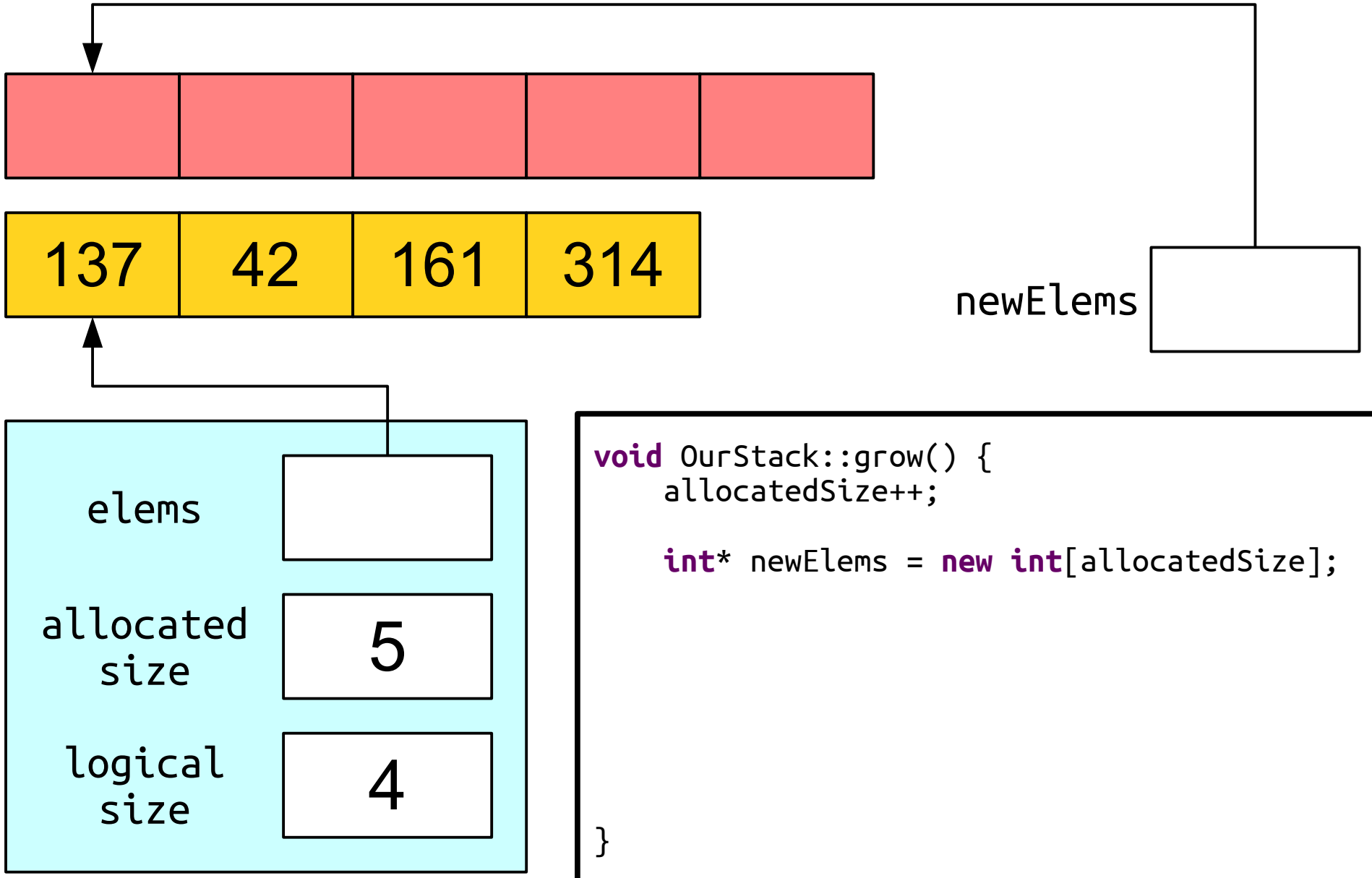


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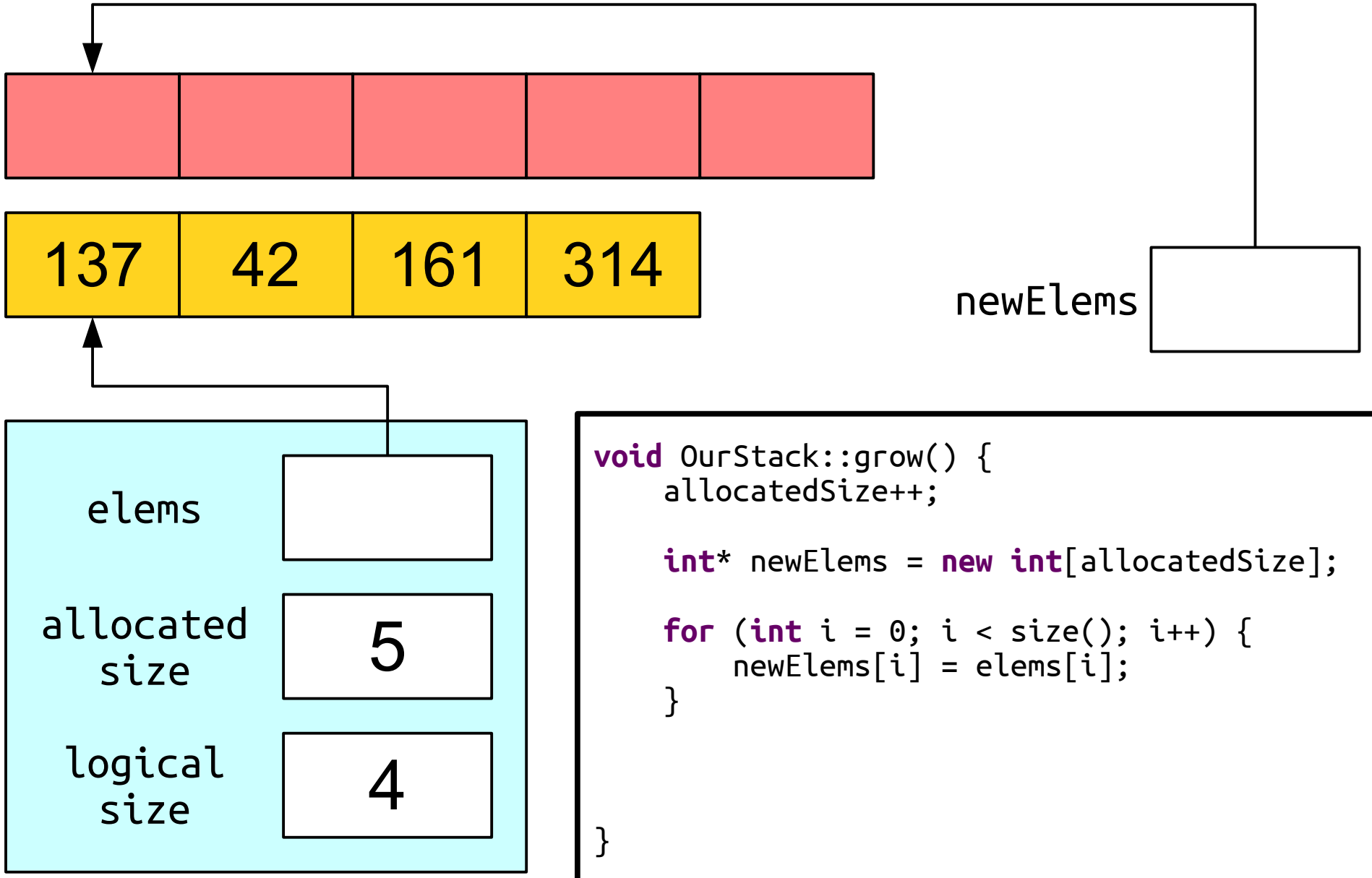


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    allocatedSize++;

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    for (int i = 0; i < size(); i++) {
        newElems[i] = elems[i];
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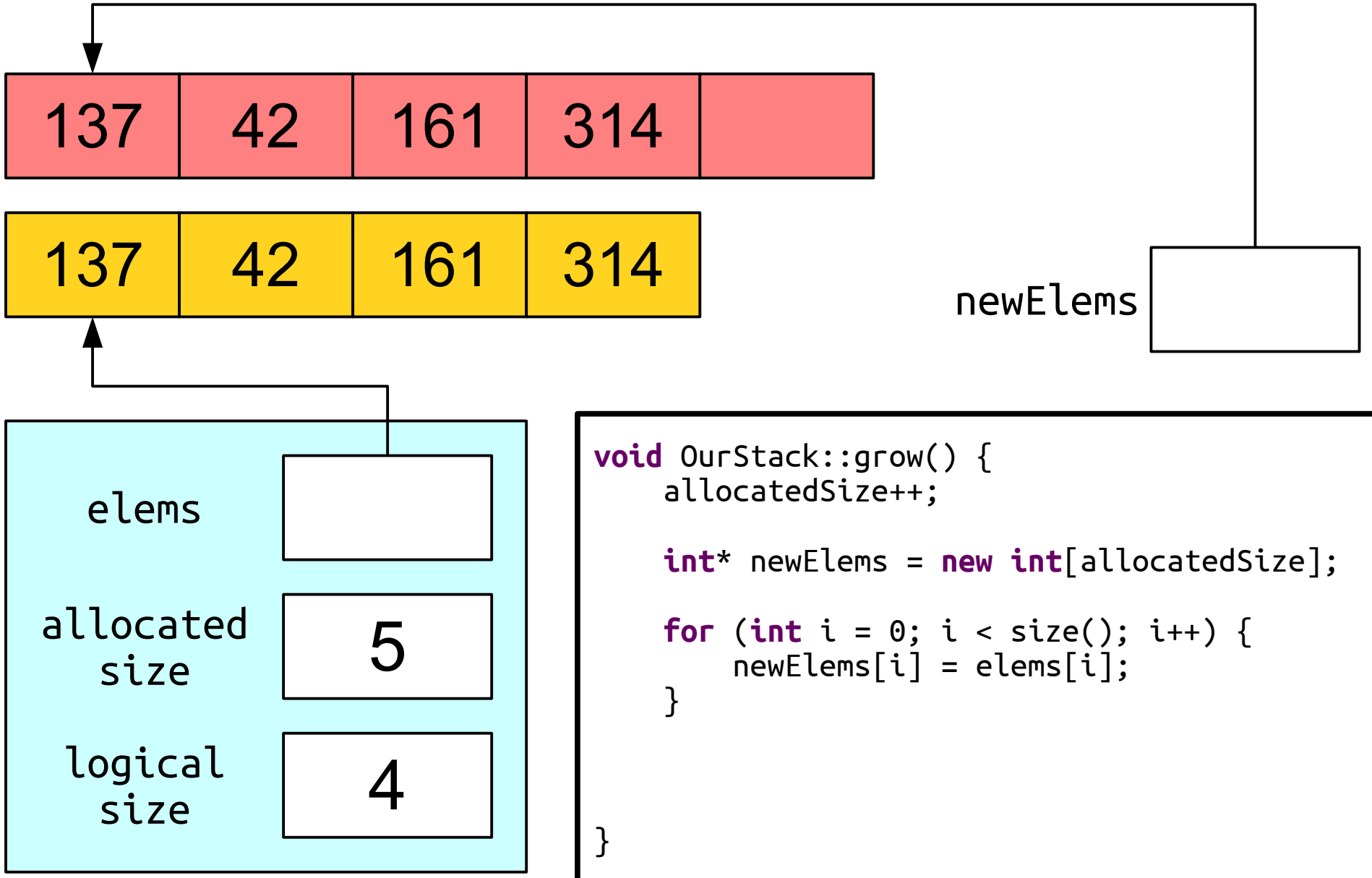


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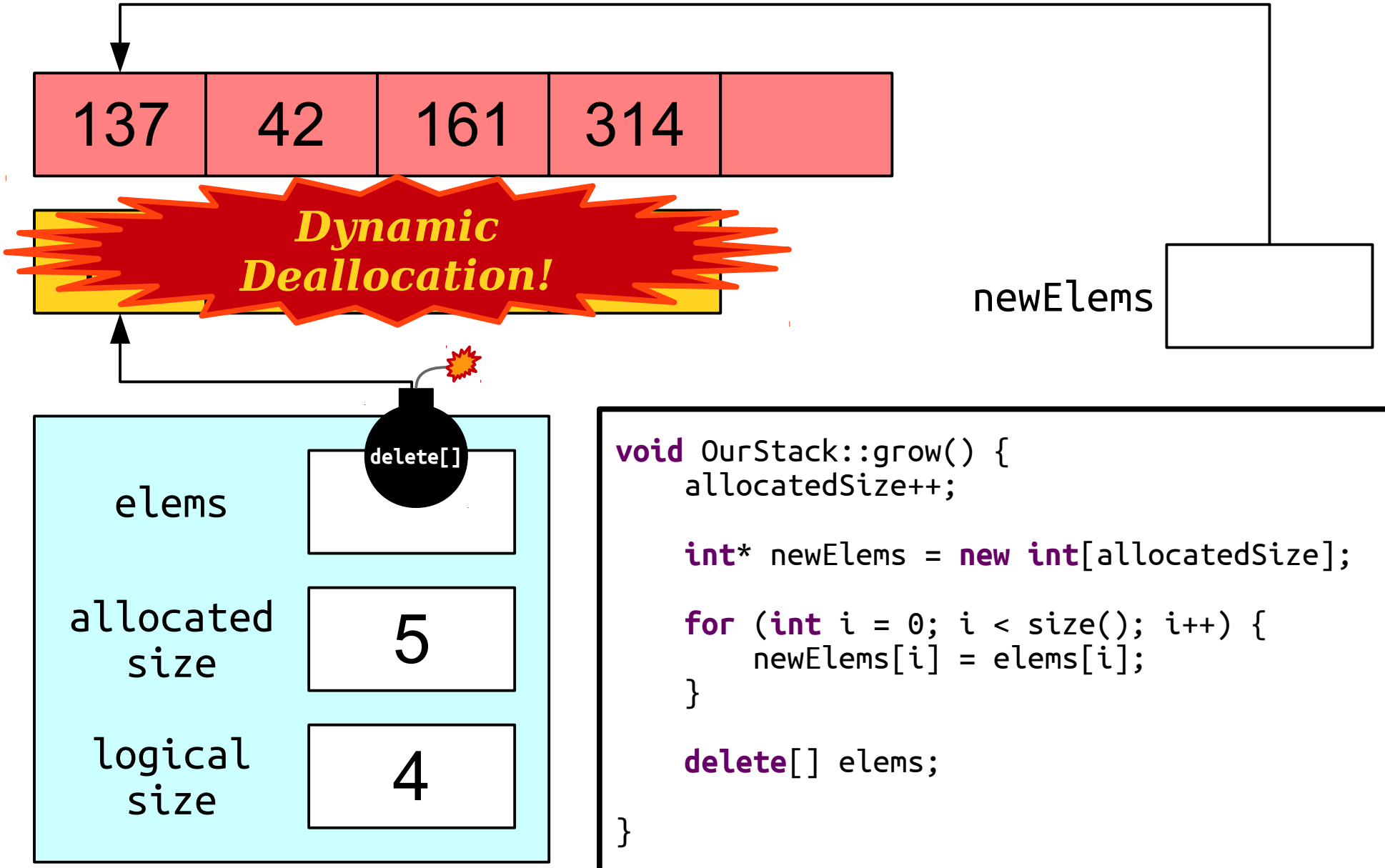


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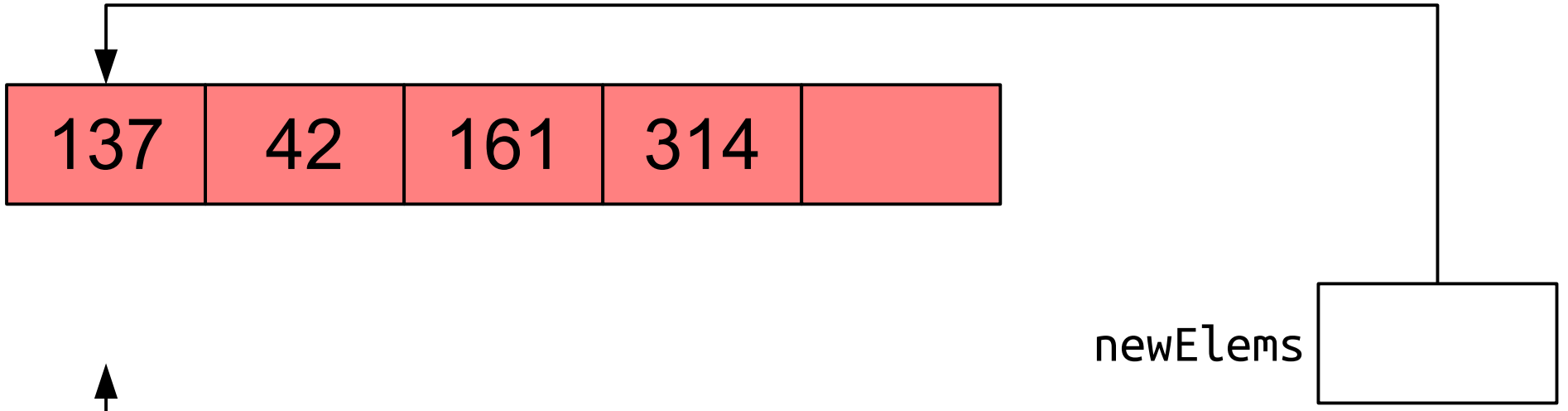

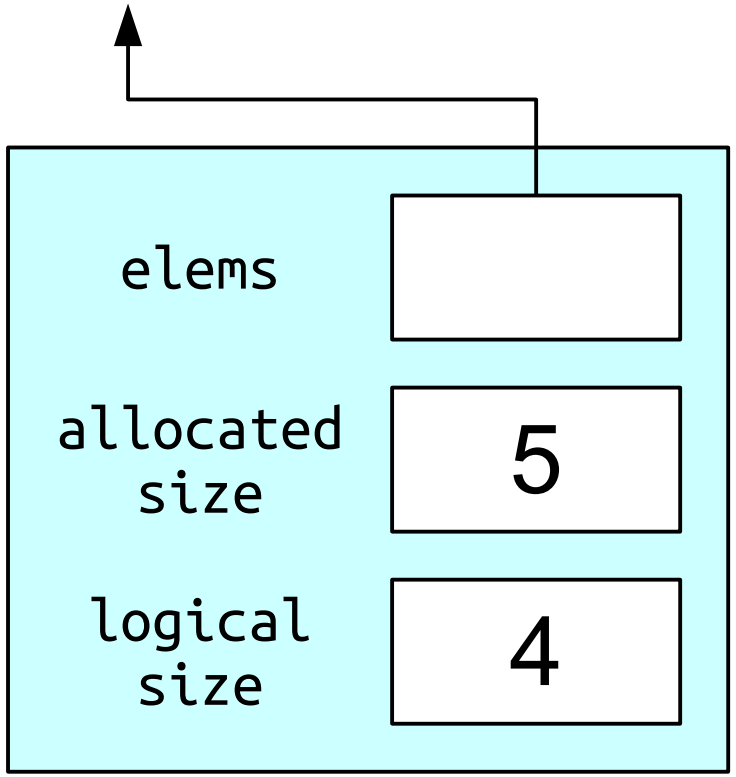


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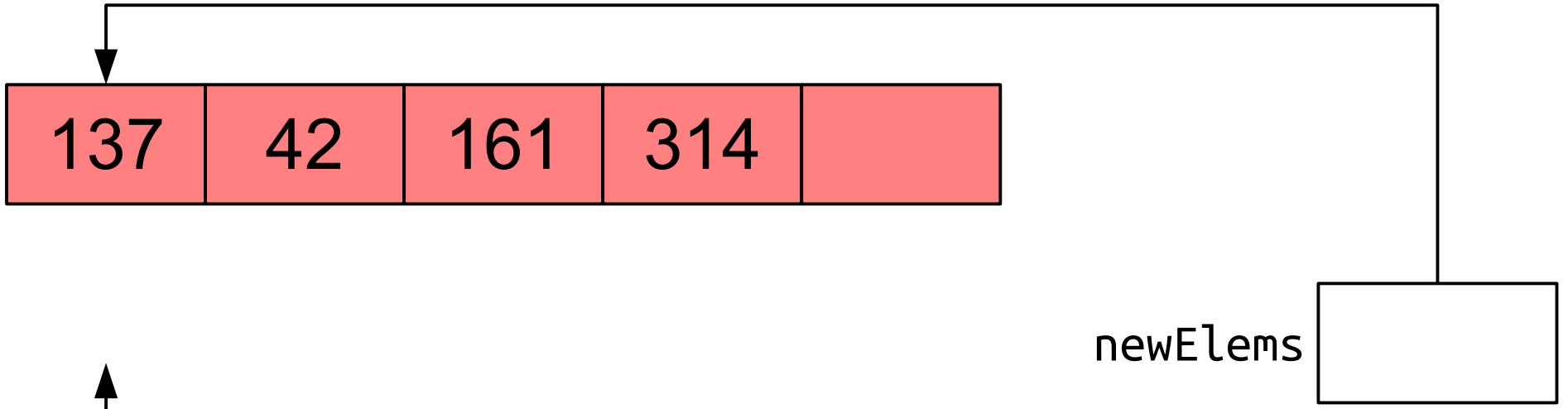

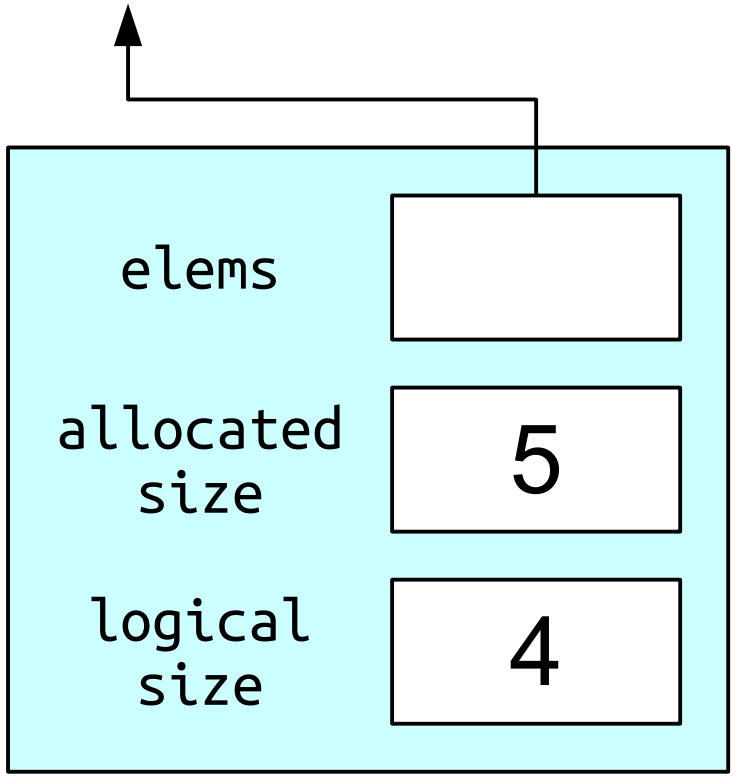


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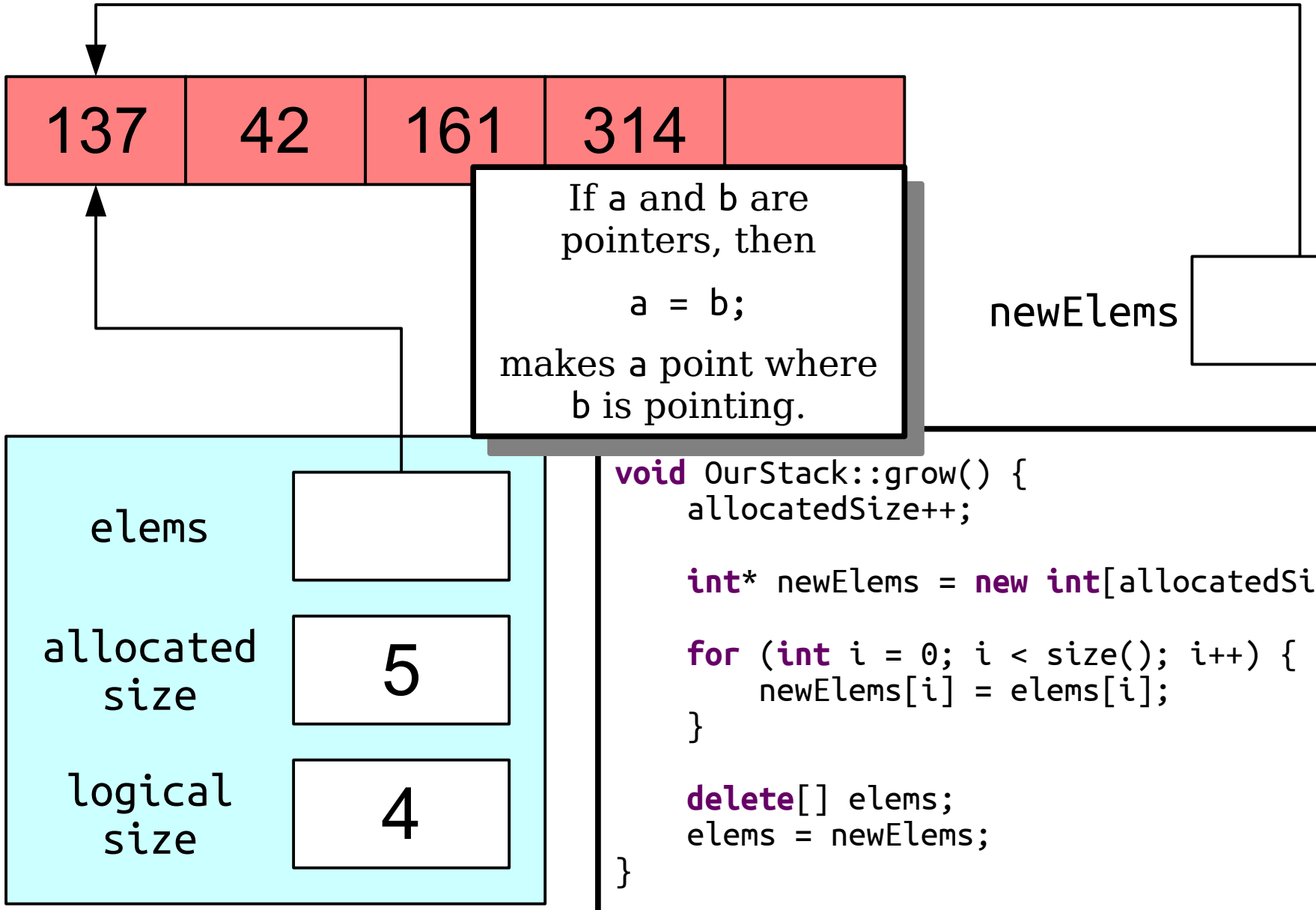


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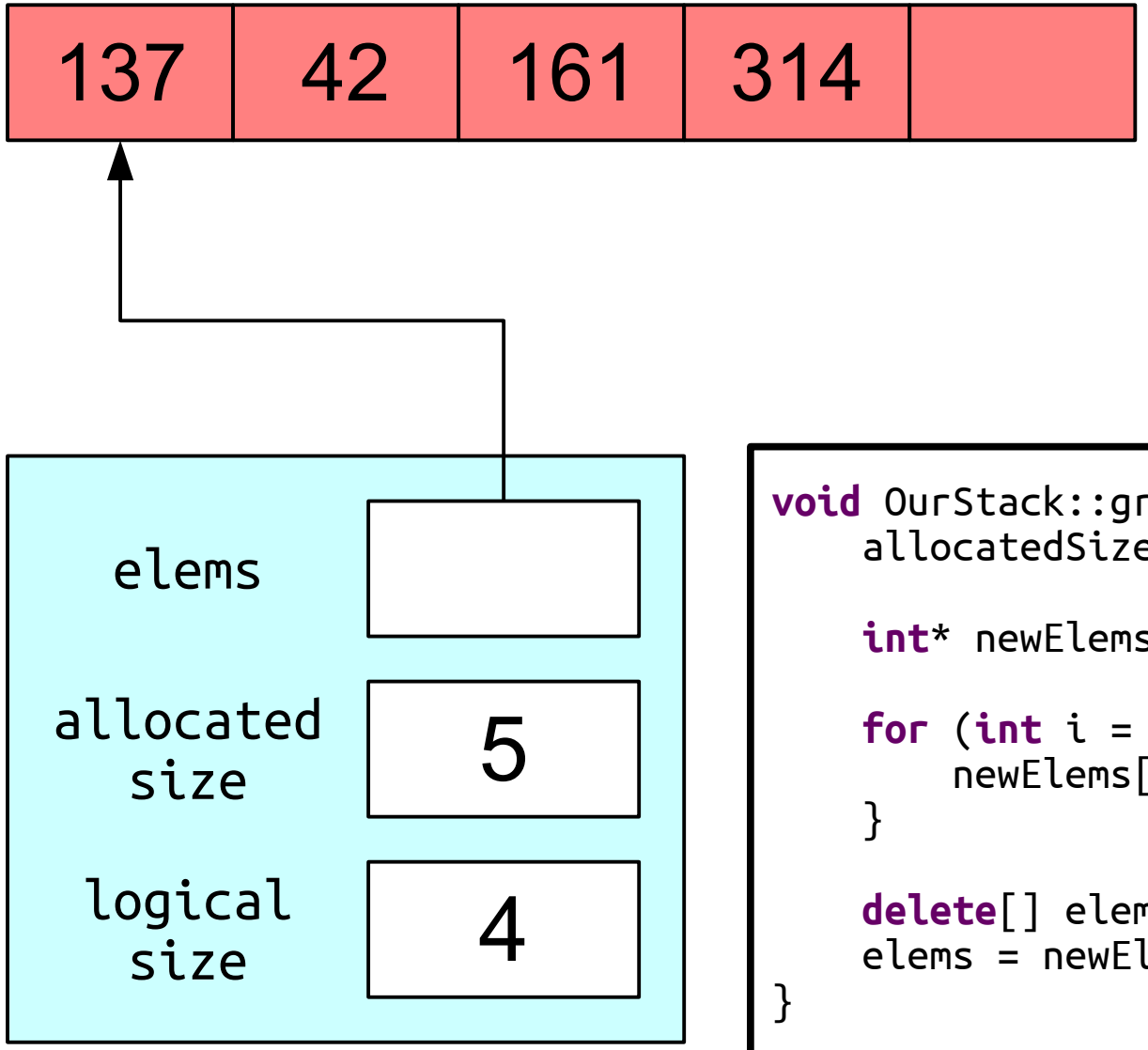


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Analyzing Our Approach

- We now have a working solution, but is it an *efficient* solution?
- Let's analyze the big-O complexity of the five operations.
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  - `isEmpty`: **O(1)**
  - `push`: **O(*n*)**
  - `pop`:
  - `peek`:

Analyzing Our Approach

- We now have a working solution, but is it an *efficient* solution?
- Let's analyze the big-O complexity of the five operations.
  - `size`: **O(1)**
  - `isEmpty`: **O(1)**
  - `push`: **O(*n*)**
  - `pop`: **O(1)**
  - `peek`: **O(1)**

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- Total cost: $\mathbf{O(n^2)}$

Validating Our Model

Time-Out for Announcements!

Assignment 4

- Assignment 4 is due on Friday.
- You can use a late day to extend the deadline to Wednesday (there’s no class on Monday), but we don’t recommend this.
  - That will eat into your time for studying for the exam.
  - Topics from Assignment 4 are fair game for the exam.
- YEAH Hours for Assignment 5 will be on Friday at 3:30PM in 380-380Y.

Midterm Exam

- The midterm exam is next ***Tuesday, February 19*** from ***7:00PM – 10:00PM***. Locations are divvied up by last (family) name:
  - A – K: Go to ***Bishop Auditorium***
  - L – Z: Go to ***Hewlett 200***
- It covers topics from Lectures 01 – 12 (up through and including big-O notation) and Assignments 0 – 4.
- The exam is closed-book and limited-note. You may bring one double-sided sheet of 8.5” × 11” of notes to the exam with you.

Midterm Exam

- We will be administering the exam using a software tool called ***BlueBook***.
- Visit the CS106B website, click the “BlueBook” link under the “Resources” tab, then download the BlueBook software.
- This week’s section handout will be done through BlueBook so that you get a chance to test it out.
- Need a laptop for the exam? We can help out with that. Please contact us ASAP so we can make appropriate arrangements.

Practice Midterm Exam

- There's a practice midterm exam up on the course website. It's a minimally-modified version of the exam we gave out in Winter 2017.
- The password is

  `maplesyrup`

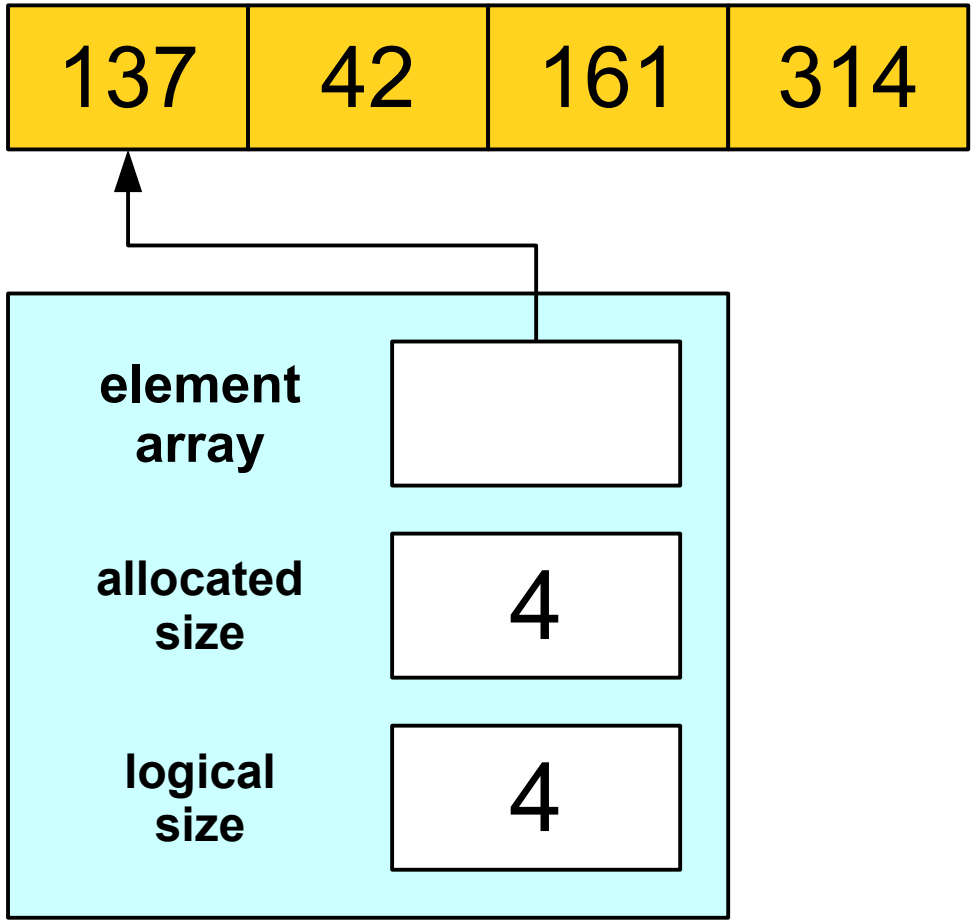
  and you'll see why when you start the exam. 😀

Back to the Stack!

Speeding up the Stack

Key Idea: ***Plan for the Future***

A Better Idea



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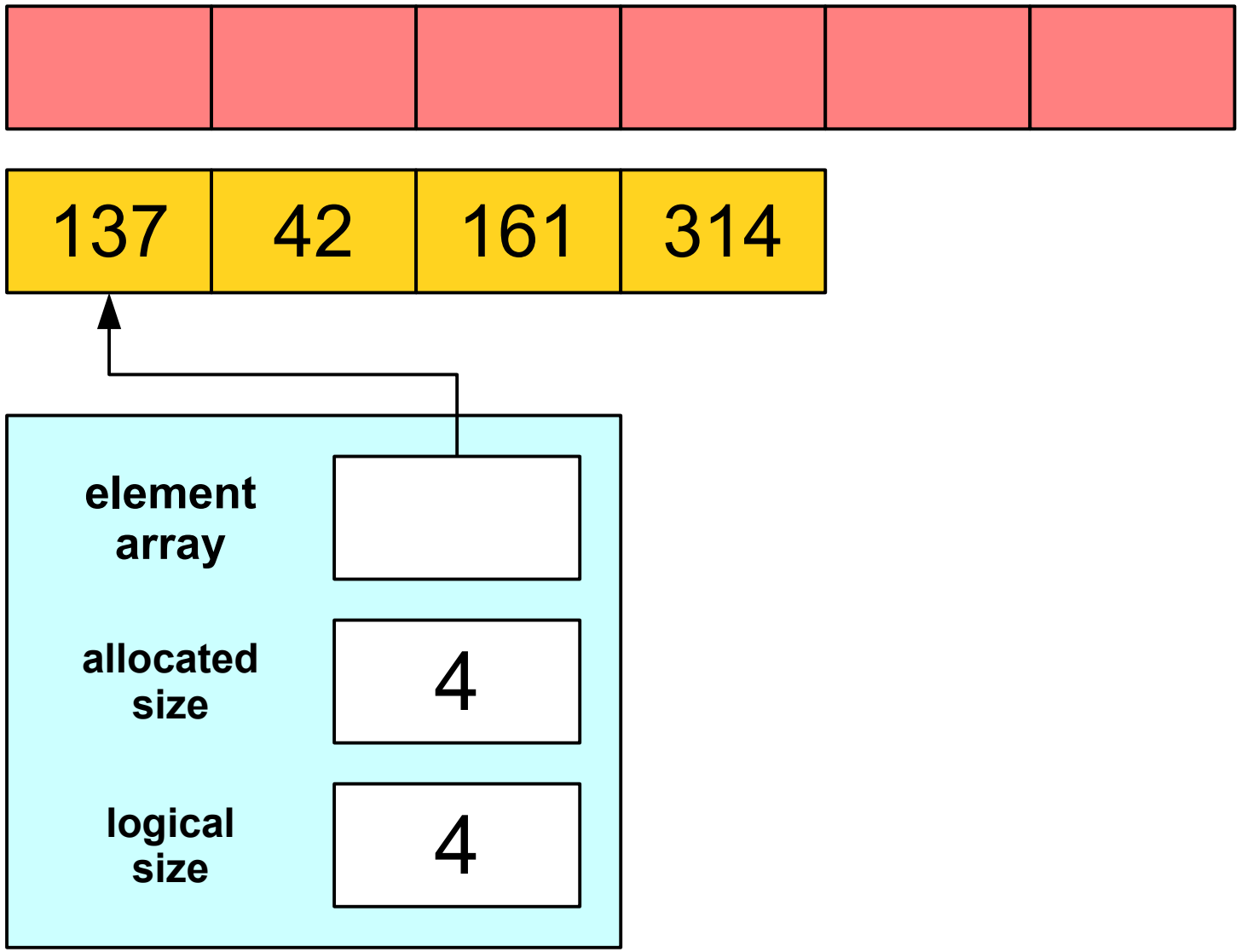

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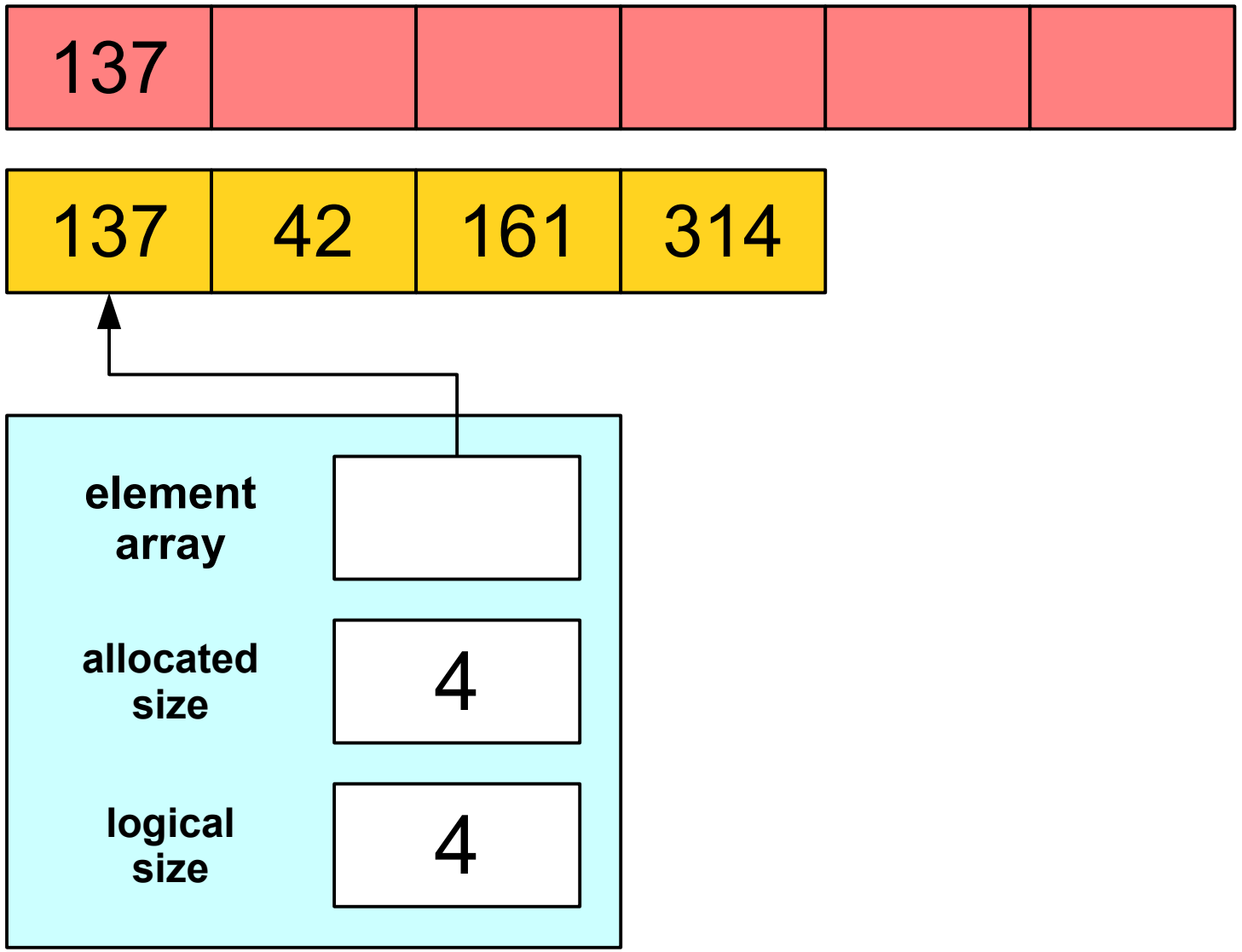

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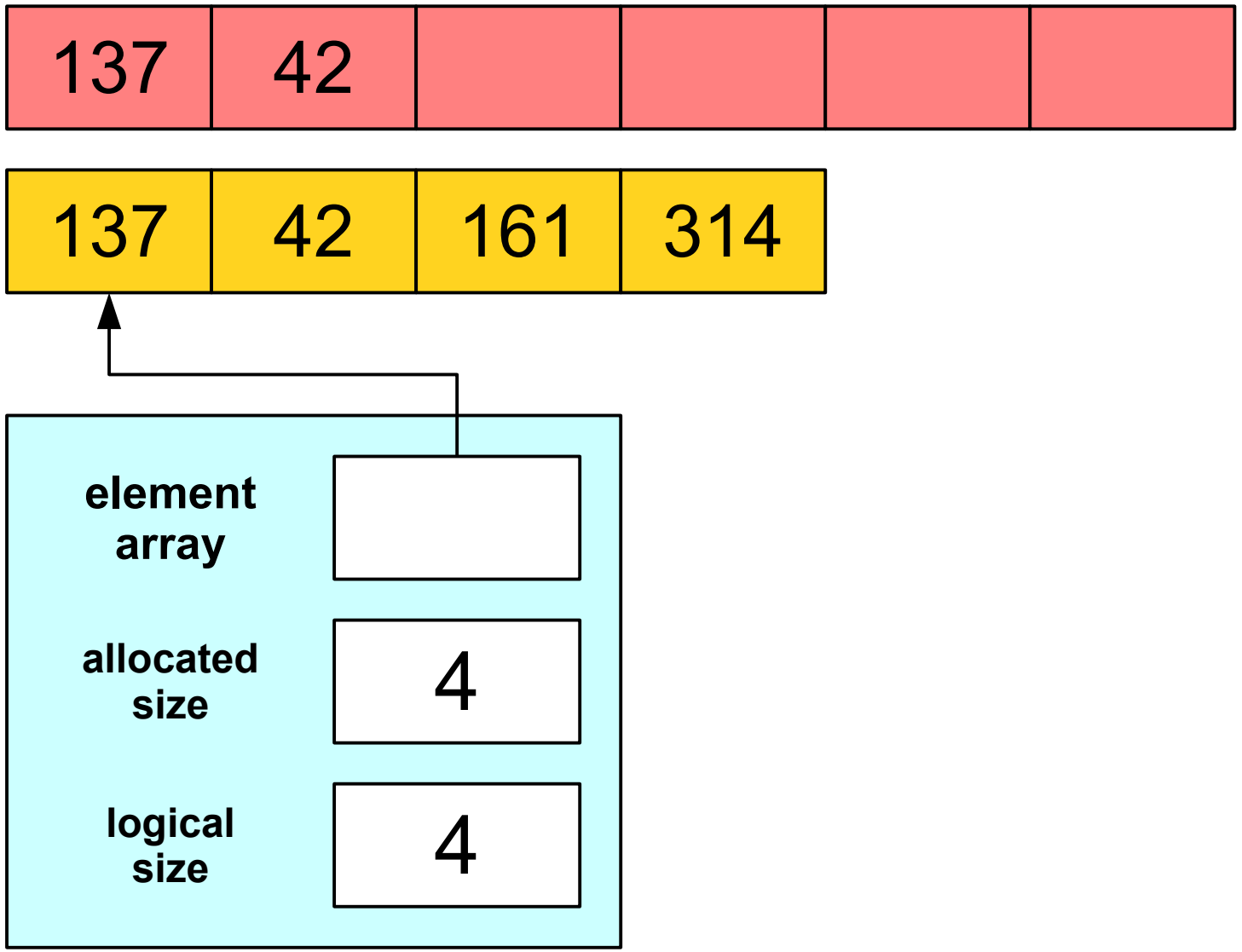

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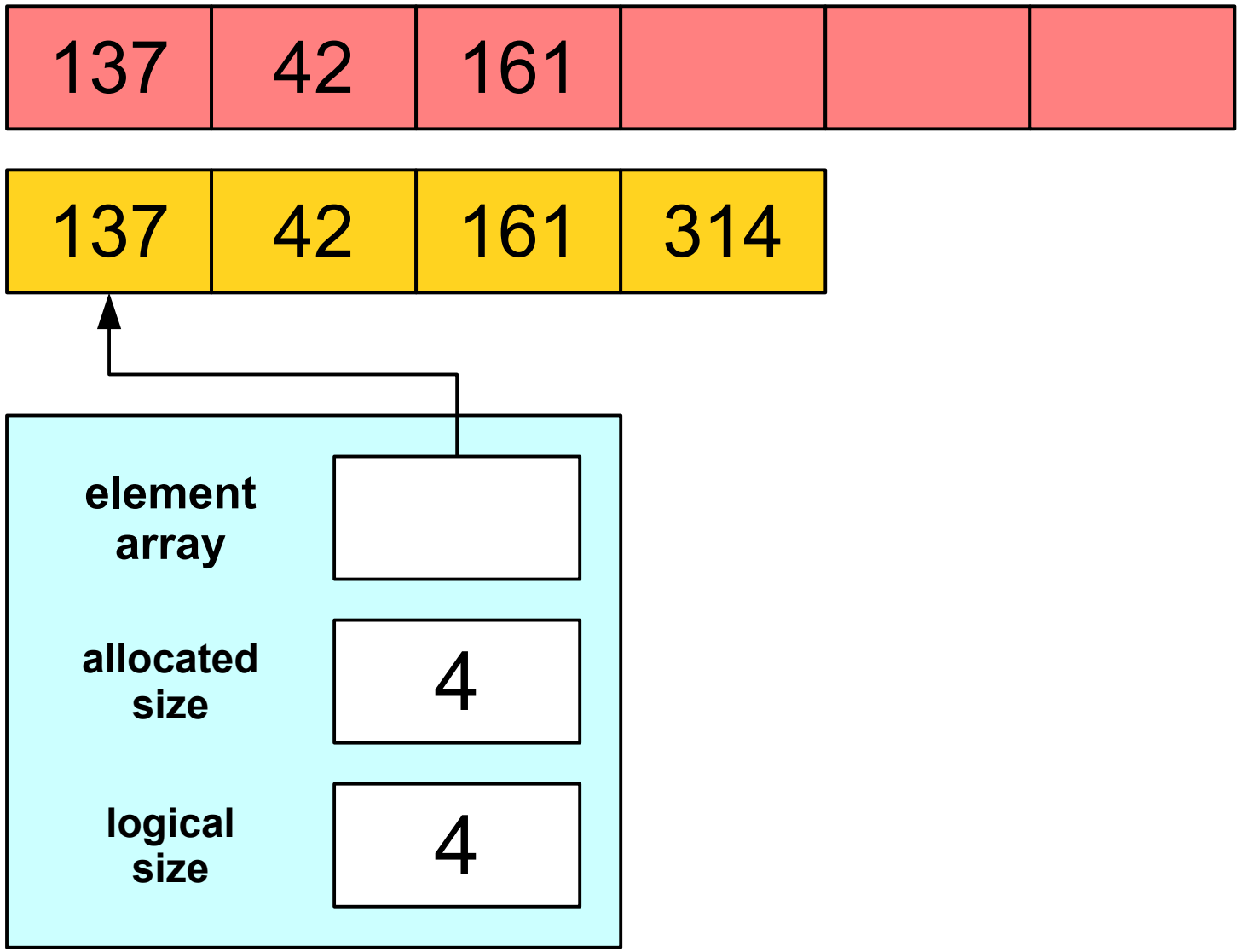

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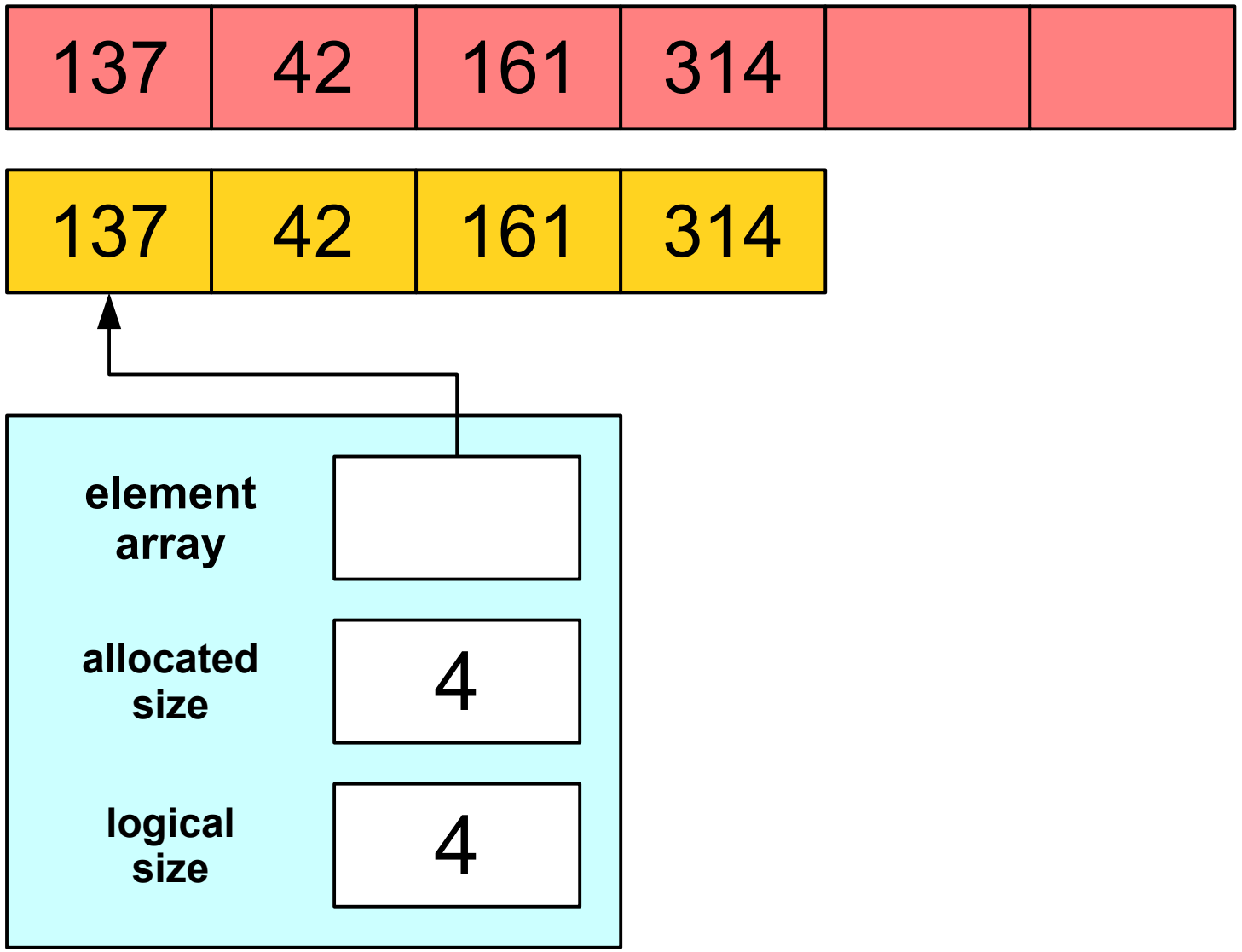

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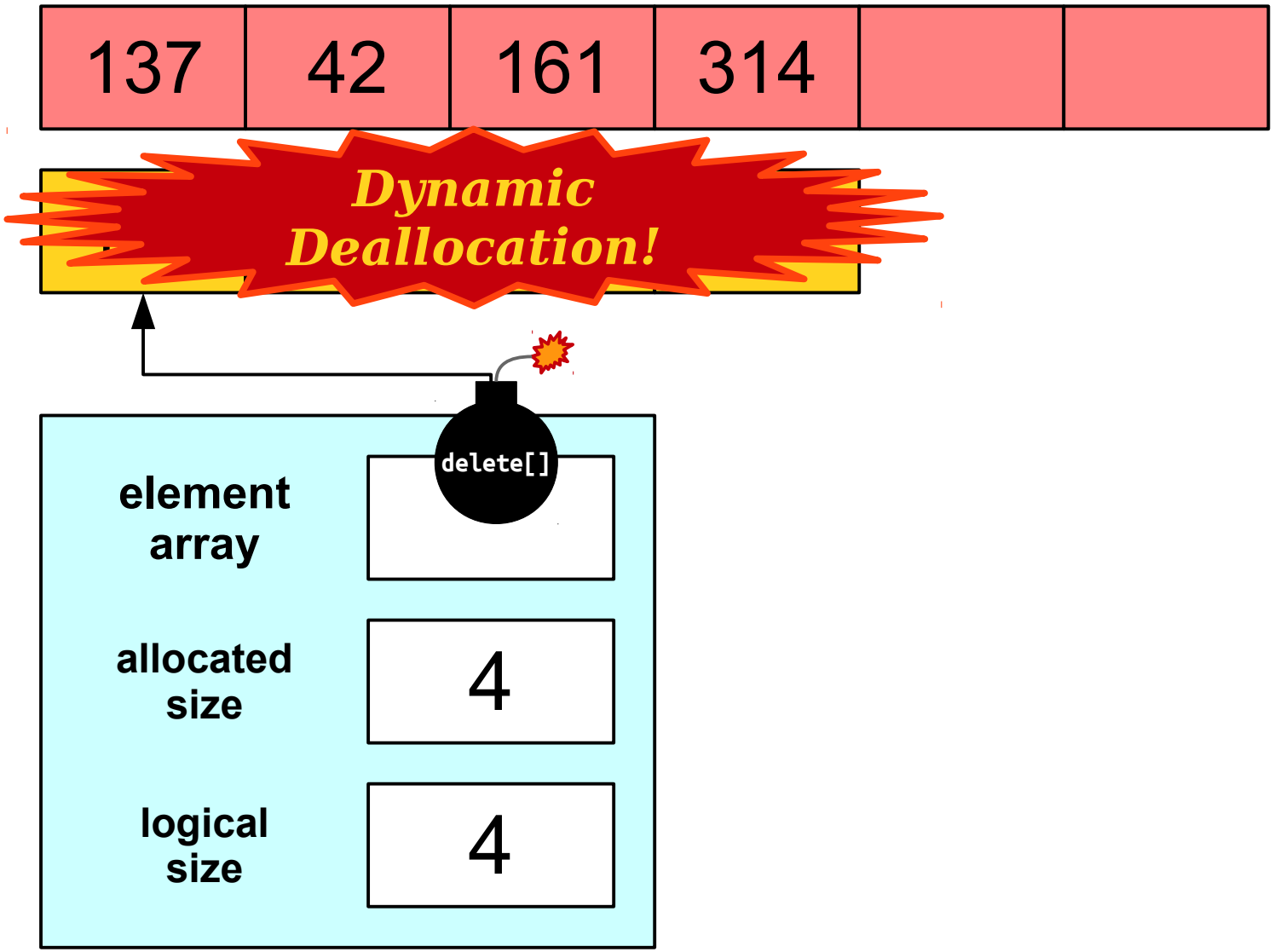

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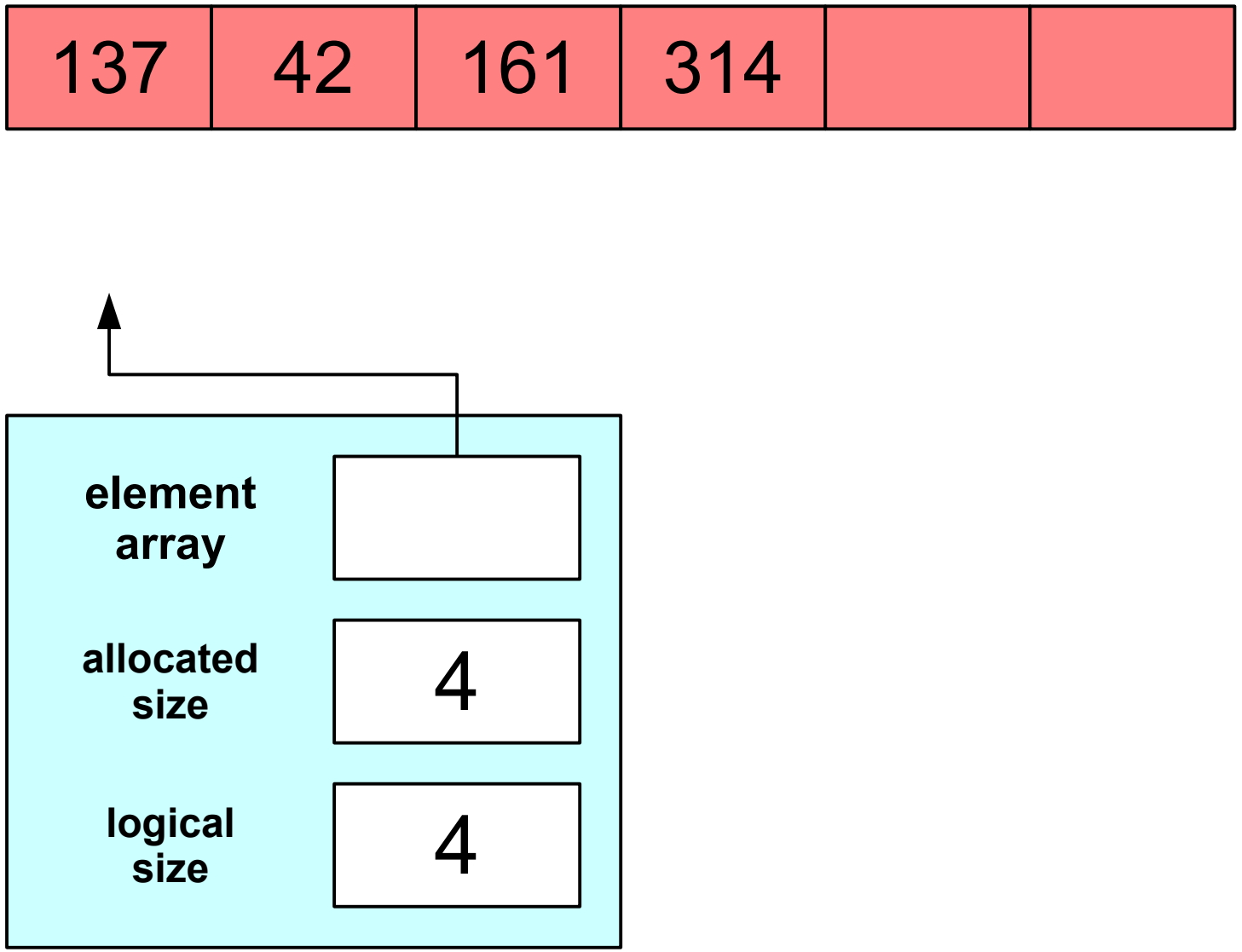

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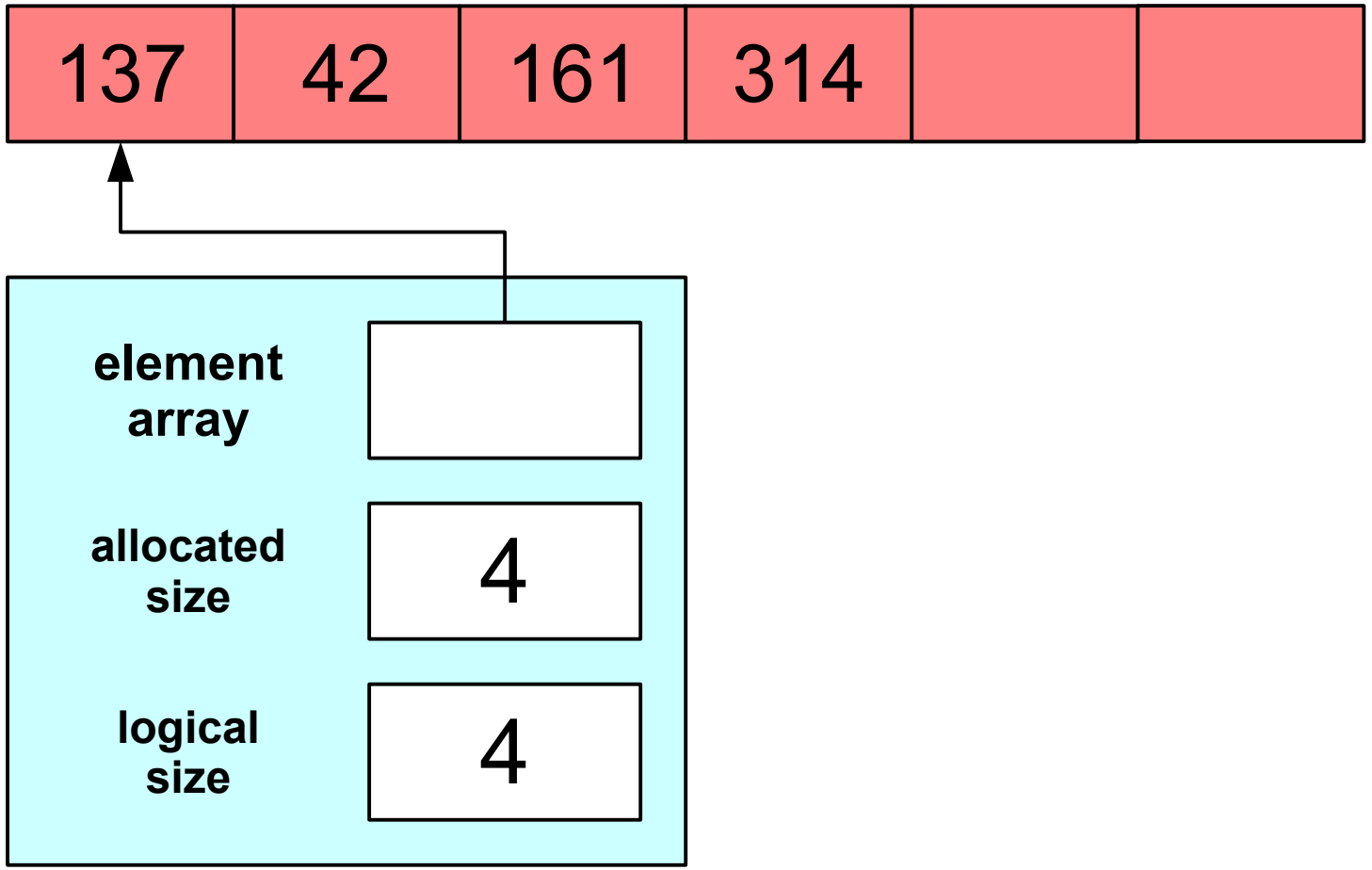

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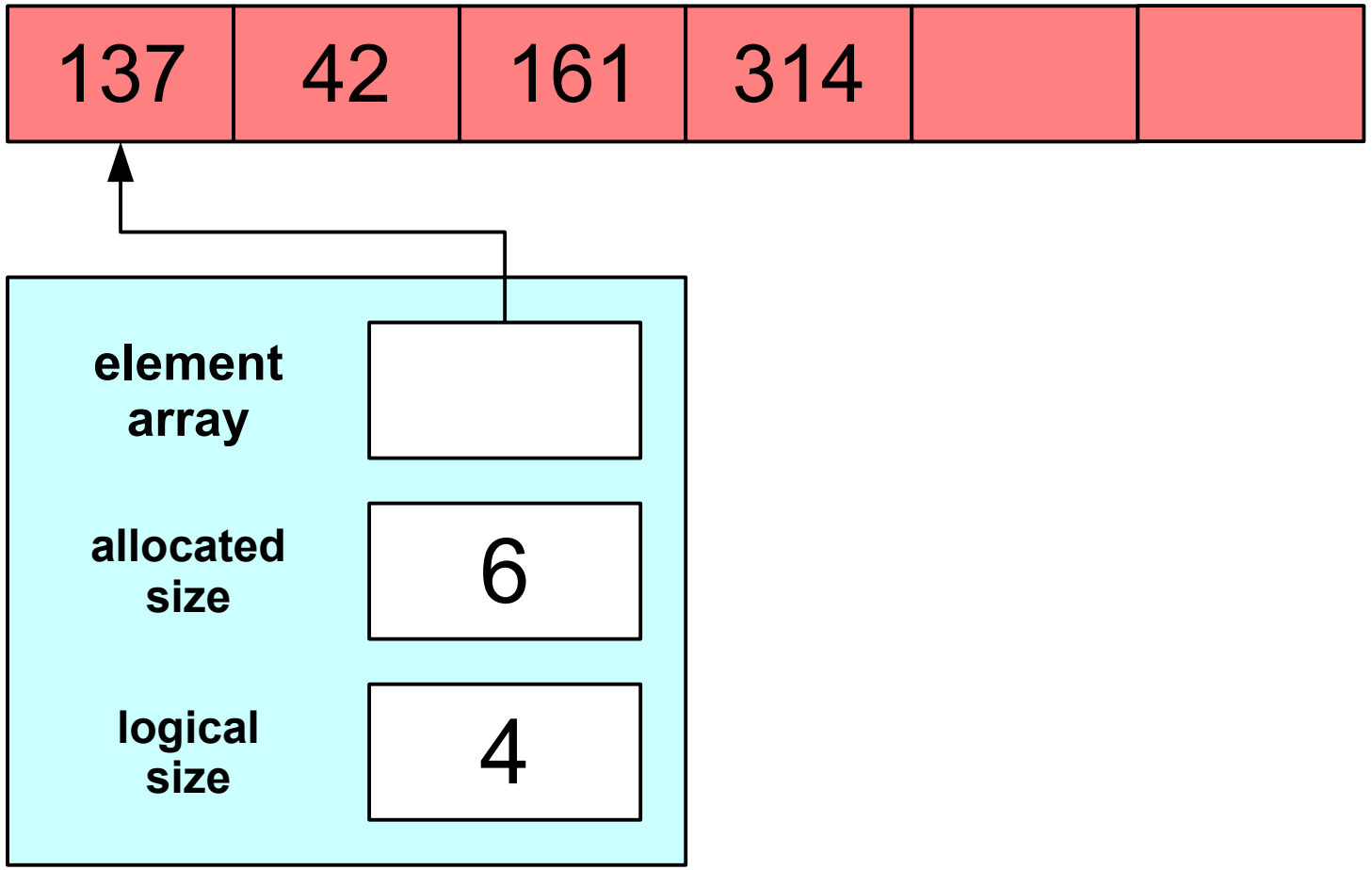

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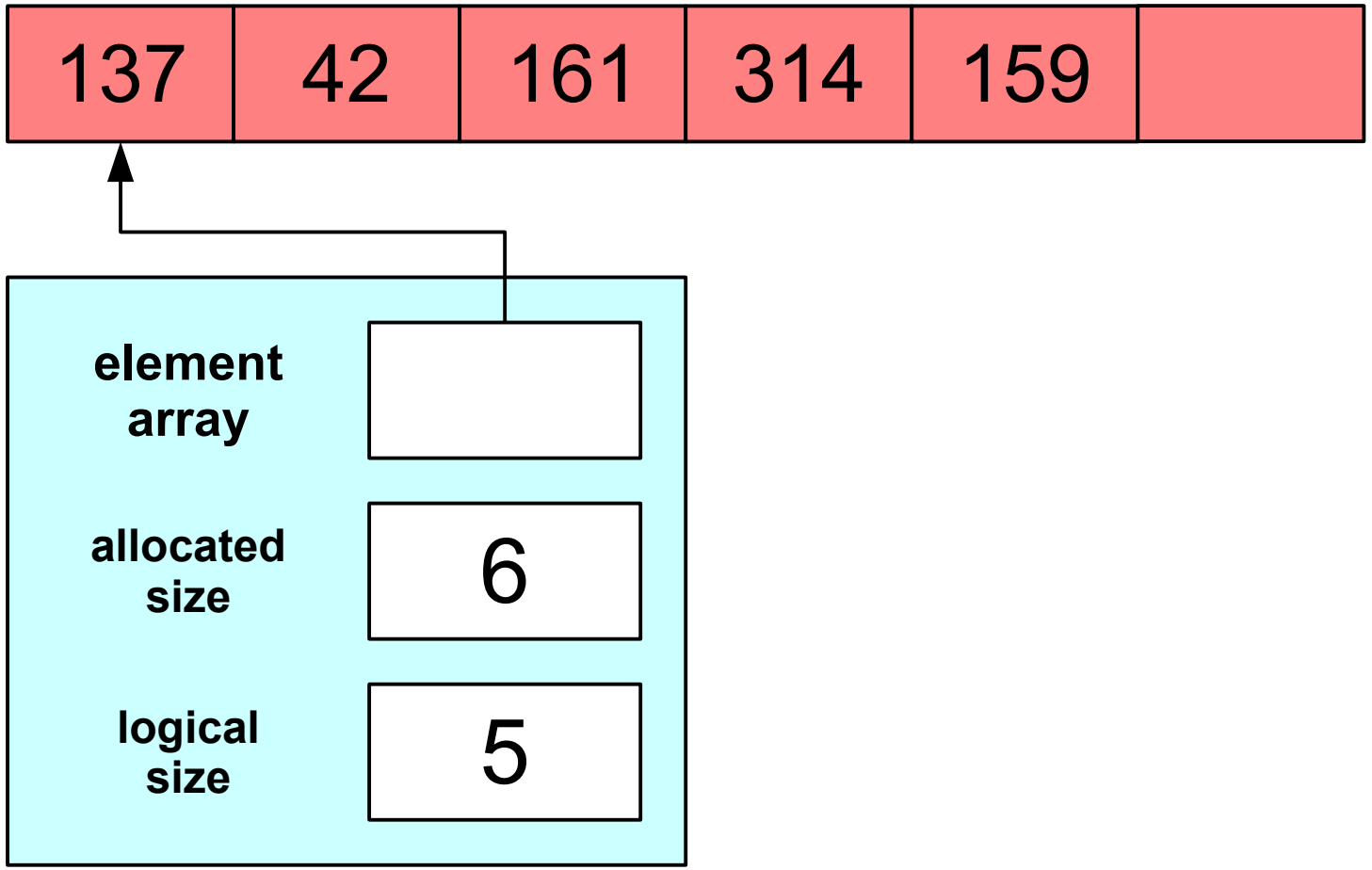

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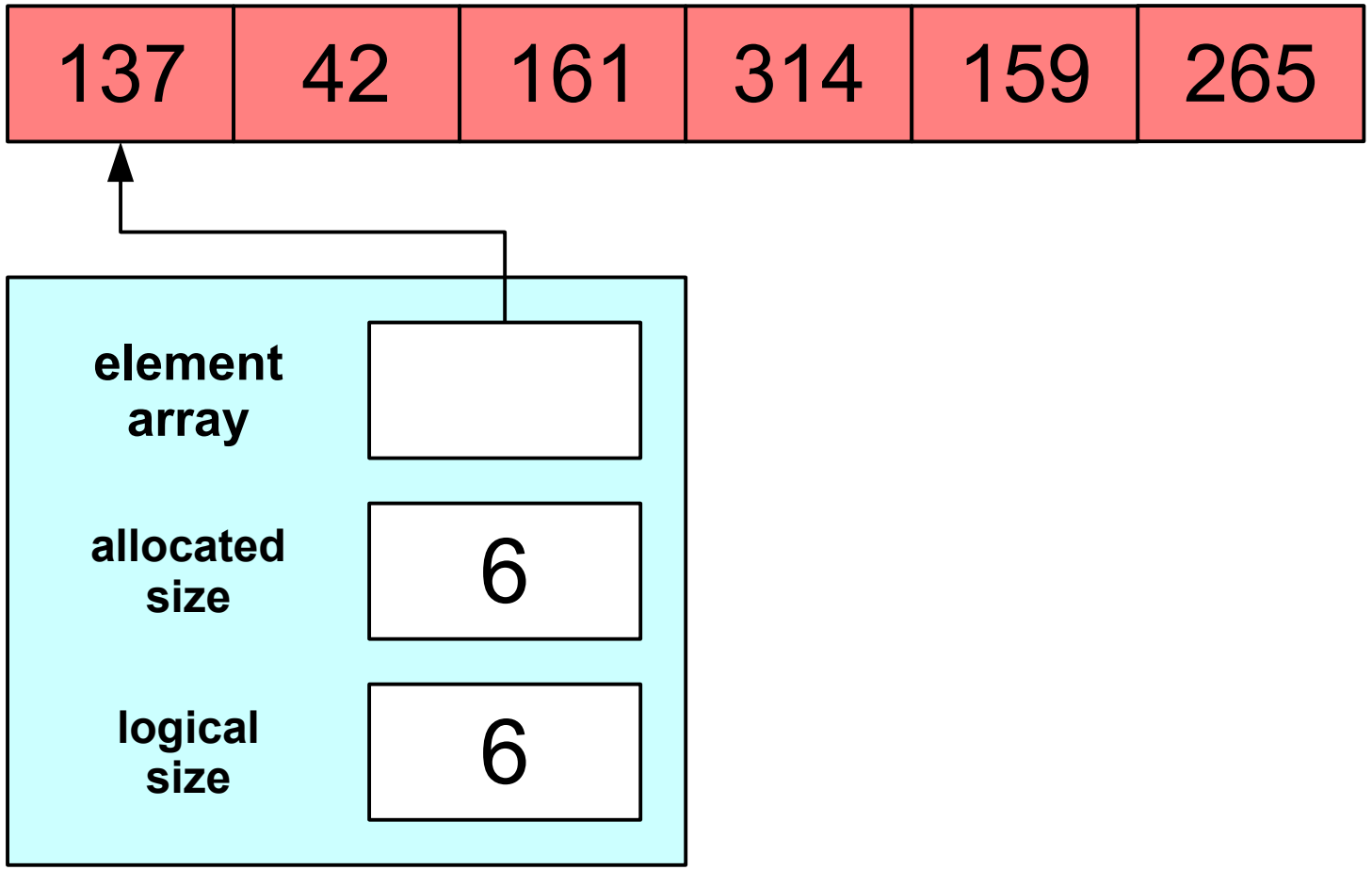
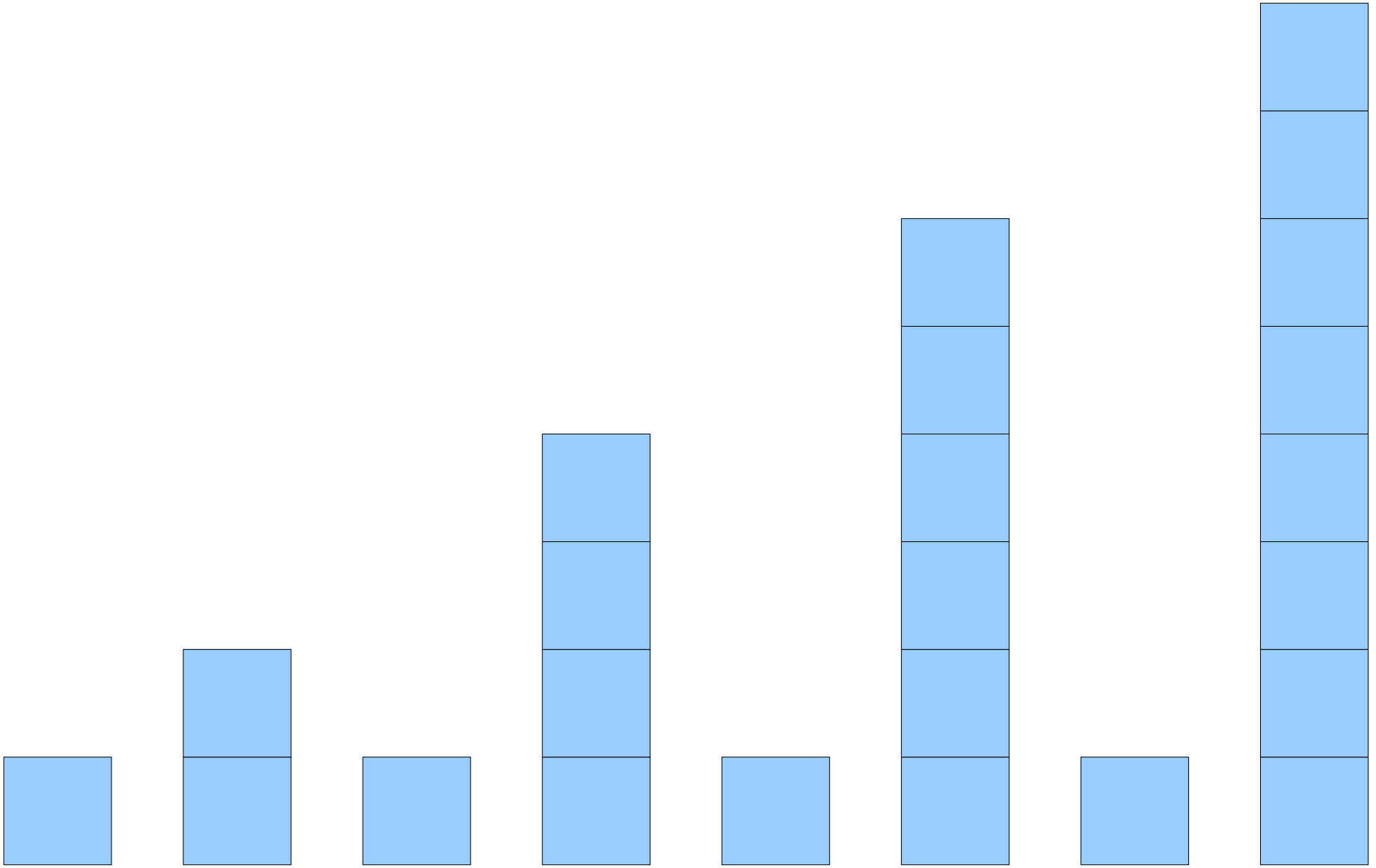


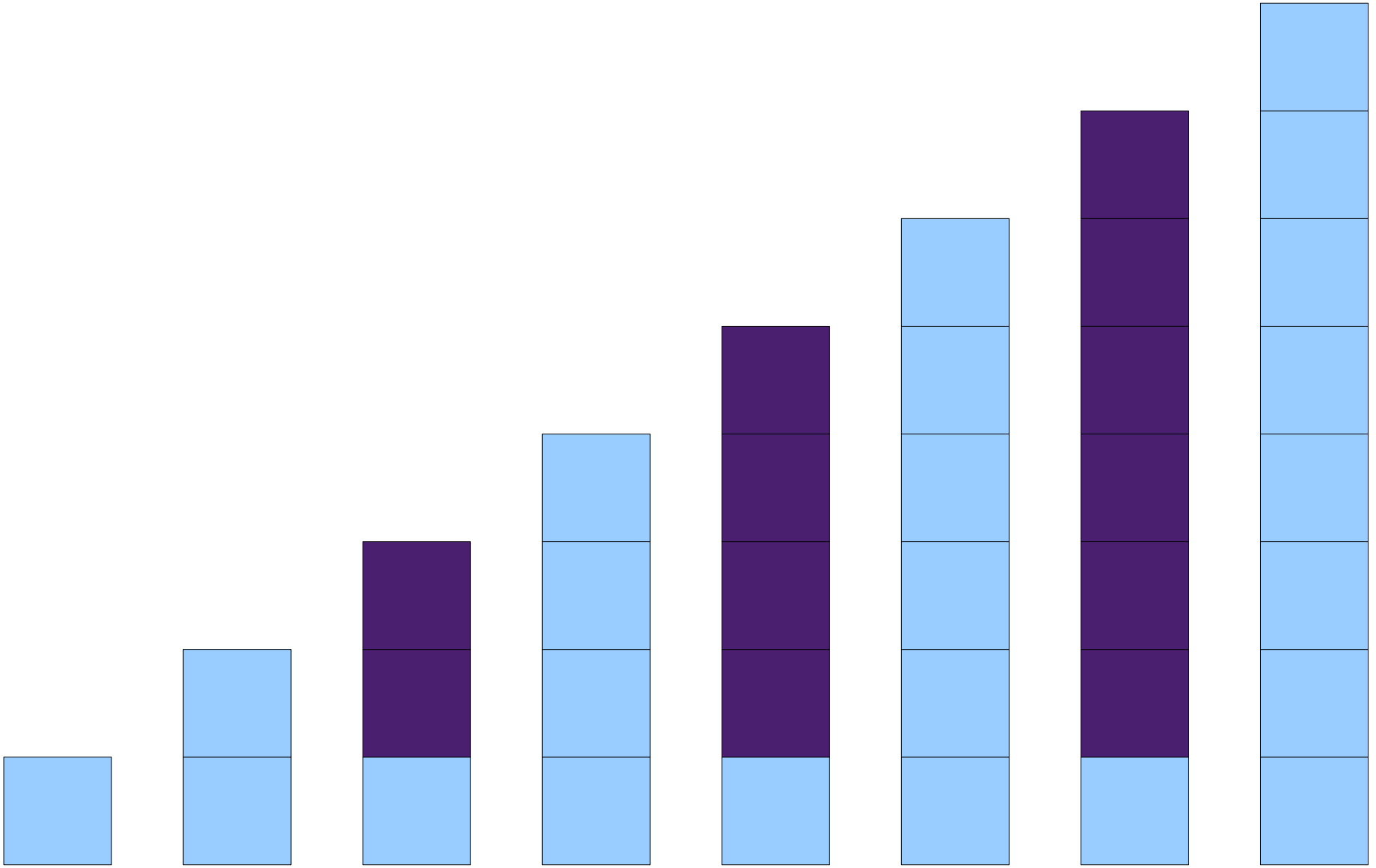
What Just Happened?

- Half of our pushes are now “easy” pushes, and half of our pushes are now “hard” pushes.
- Hard pushes still take time O(n).
- Easy pushes only take time O(1).
- Worst-case is still O(n).
- What about the average case?

Analyzing the Work

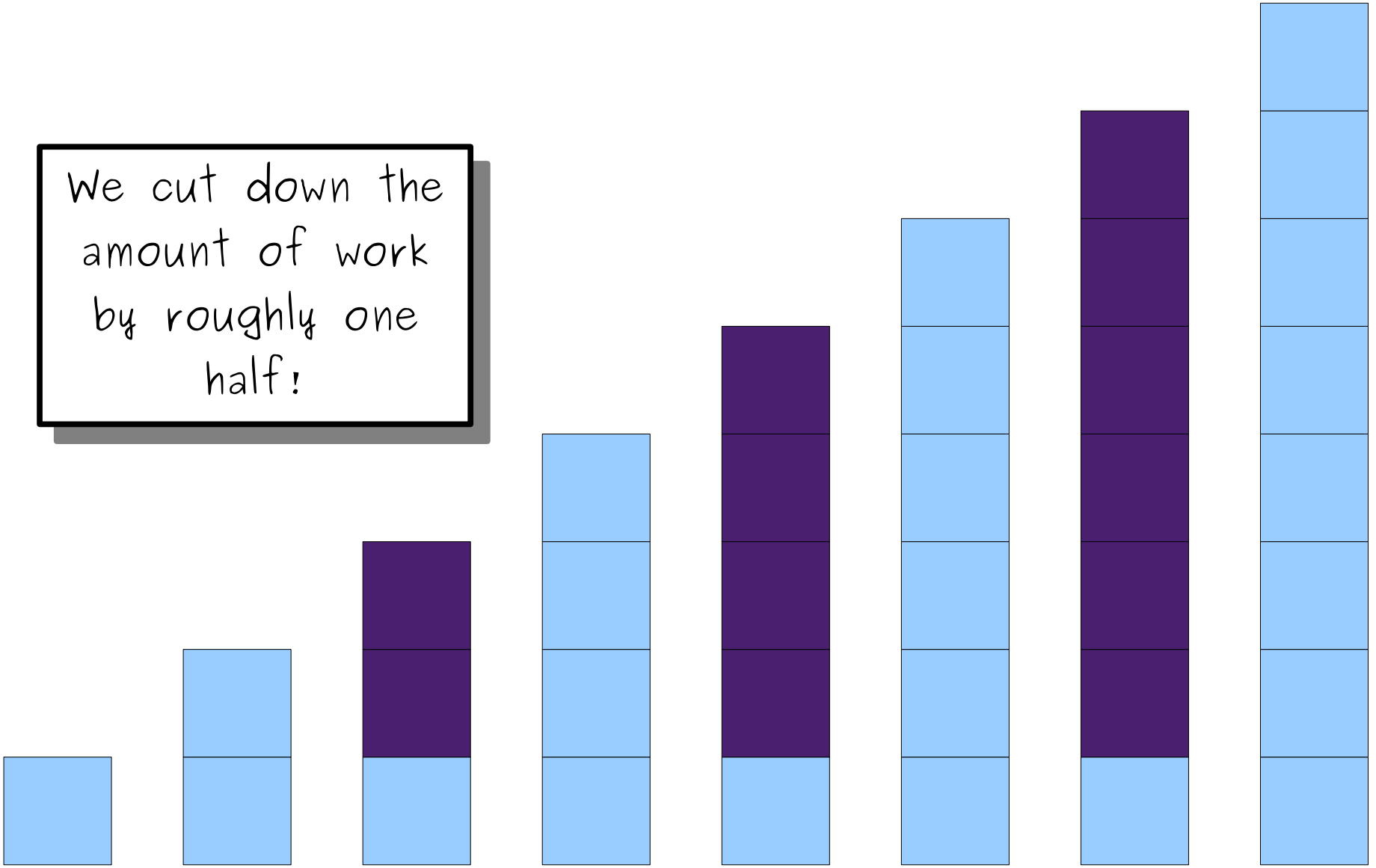


Analyzing the Work



Analyzing the Work

We cut down the amount of work by roughly one half!



A Different Analysis

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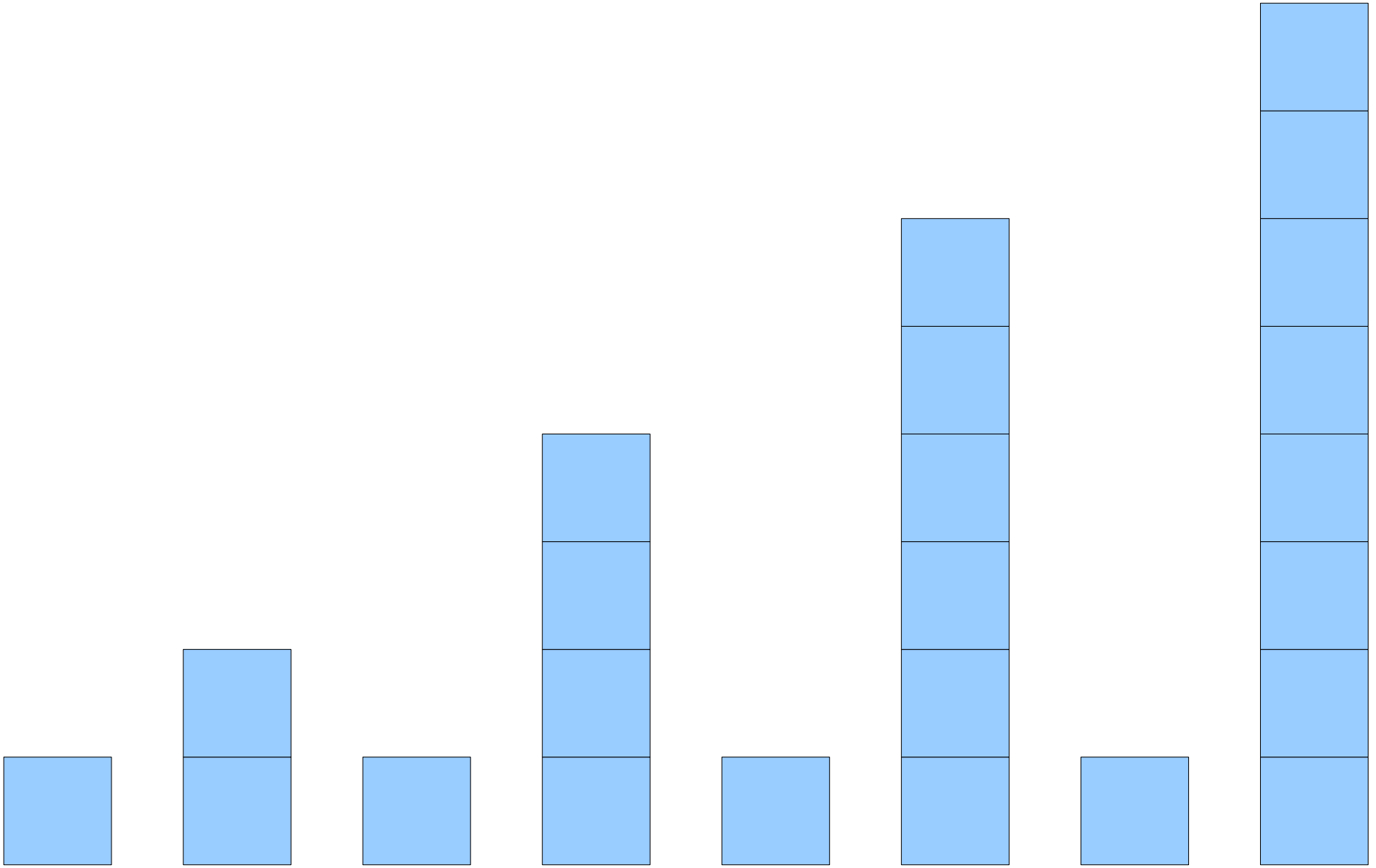

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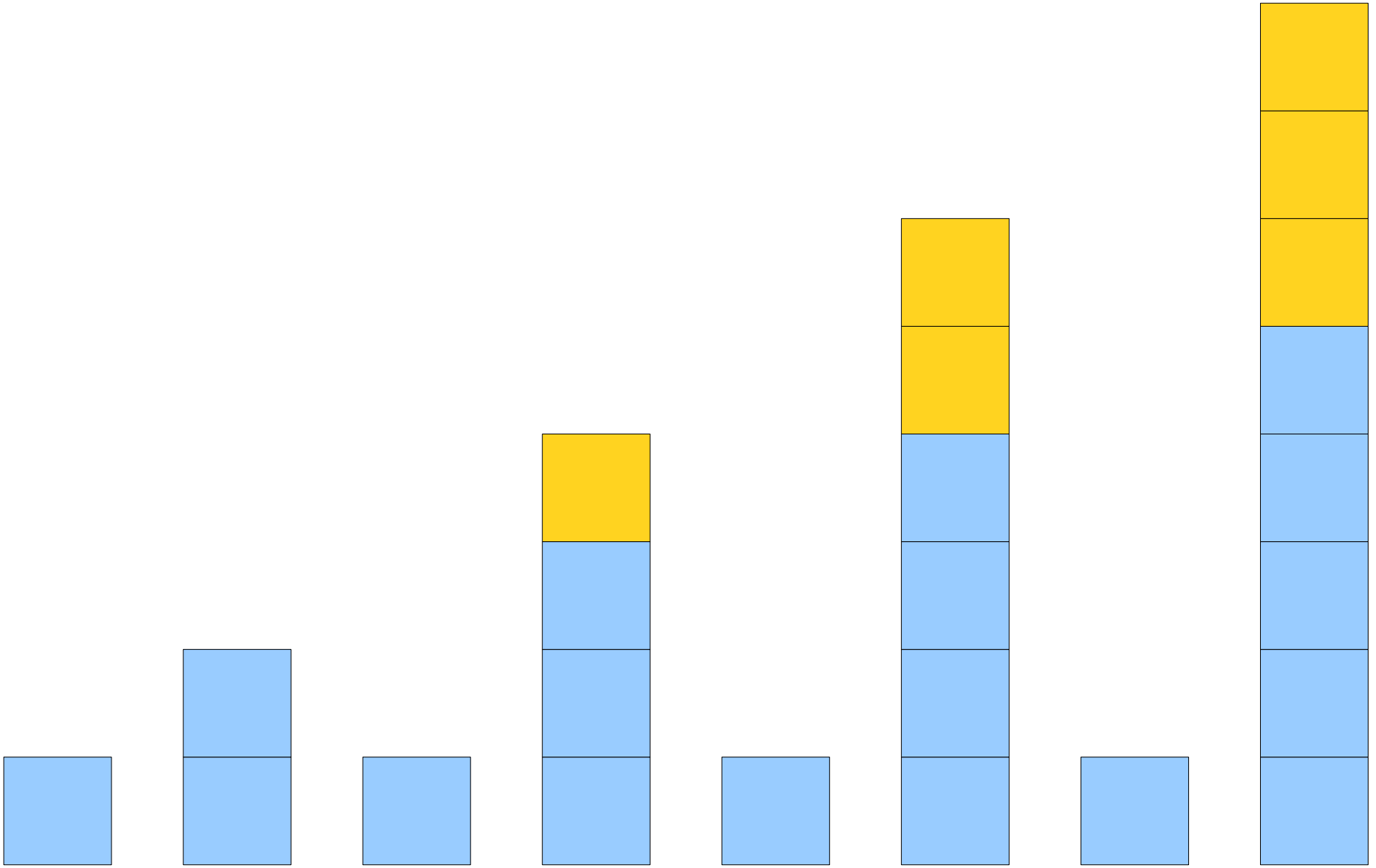

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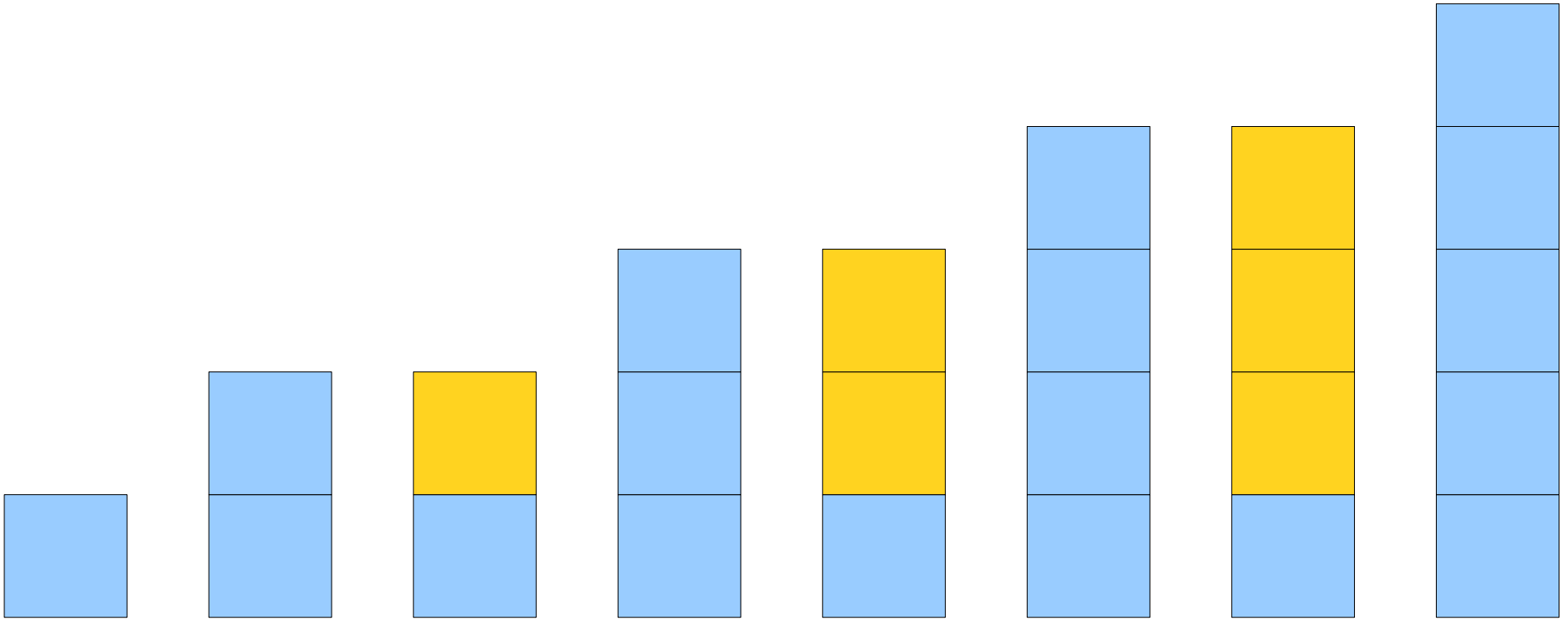

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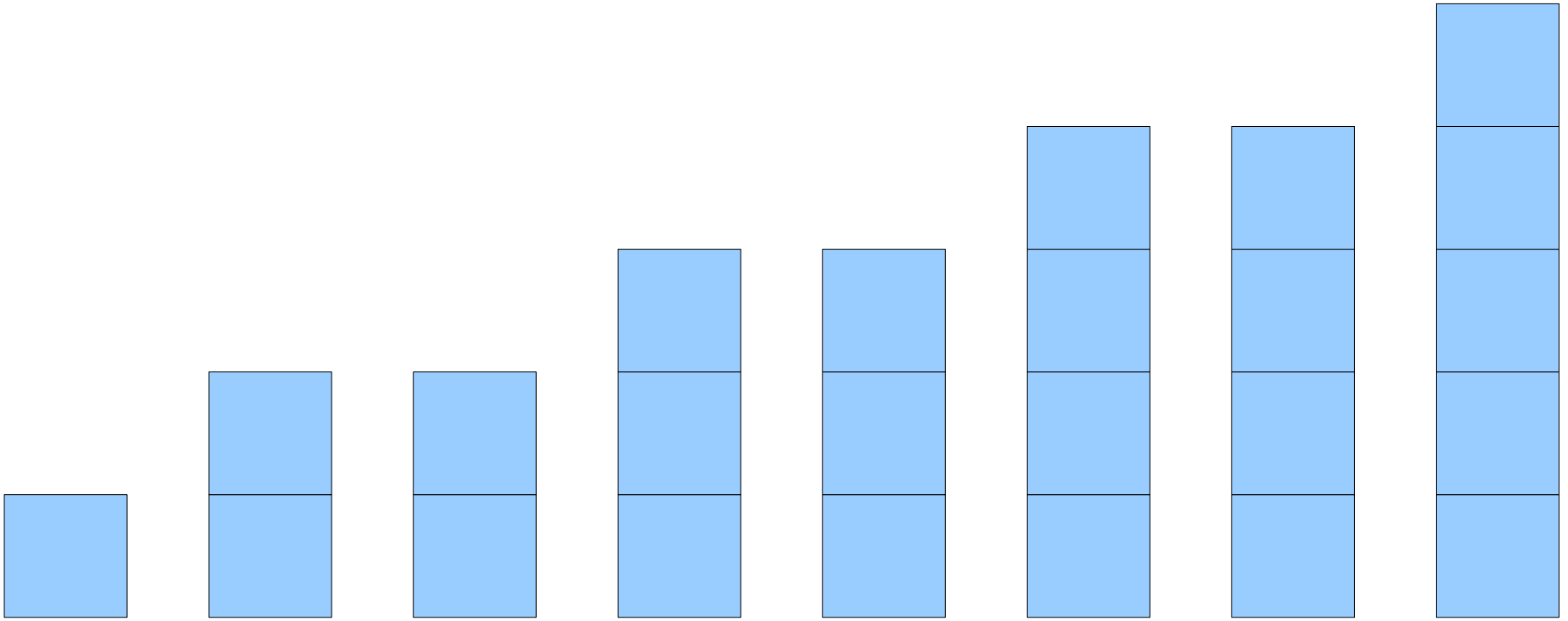

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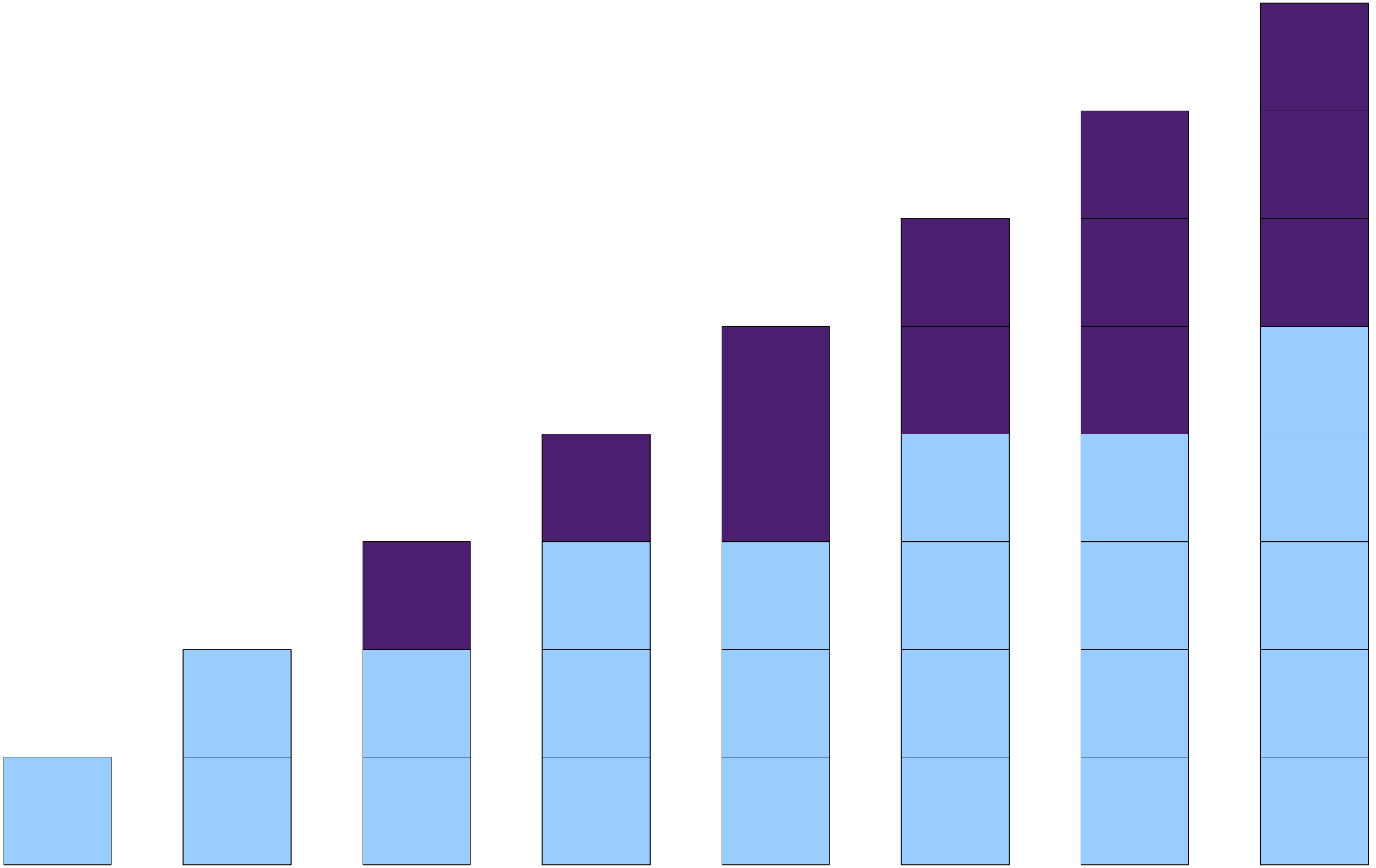

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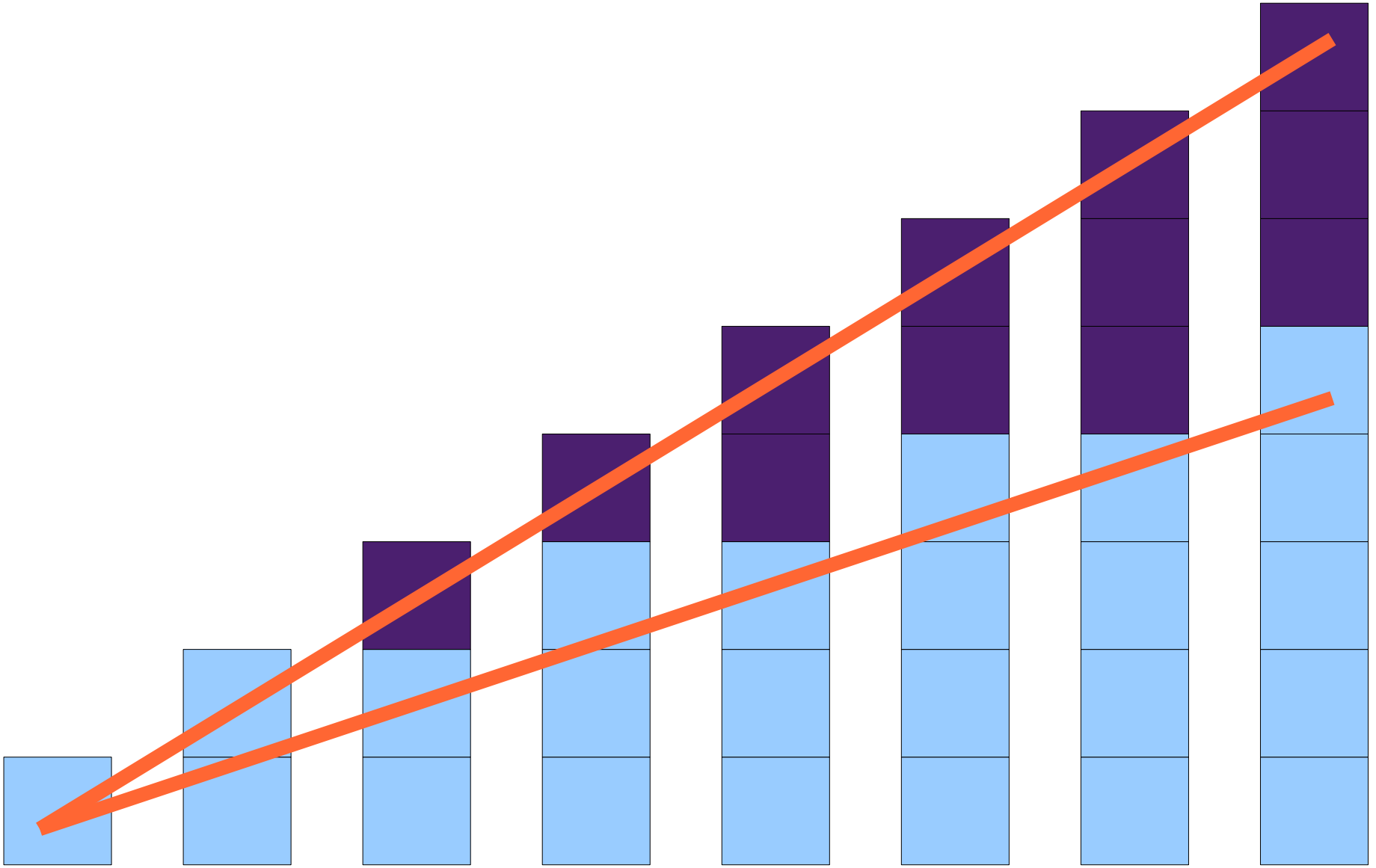

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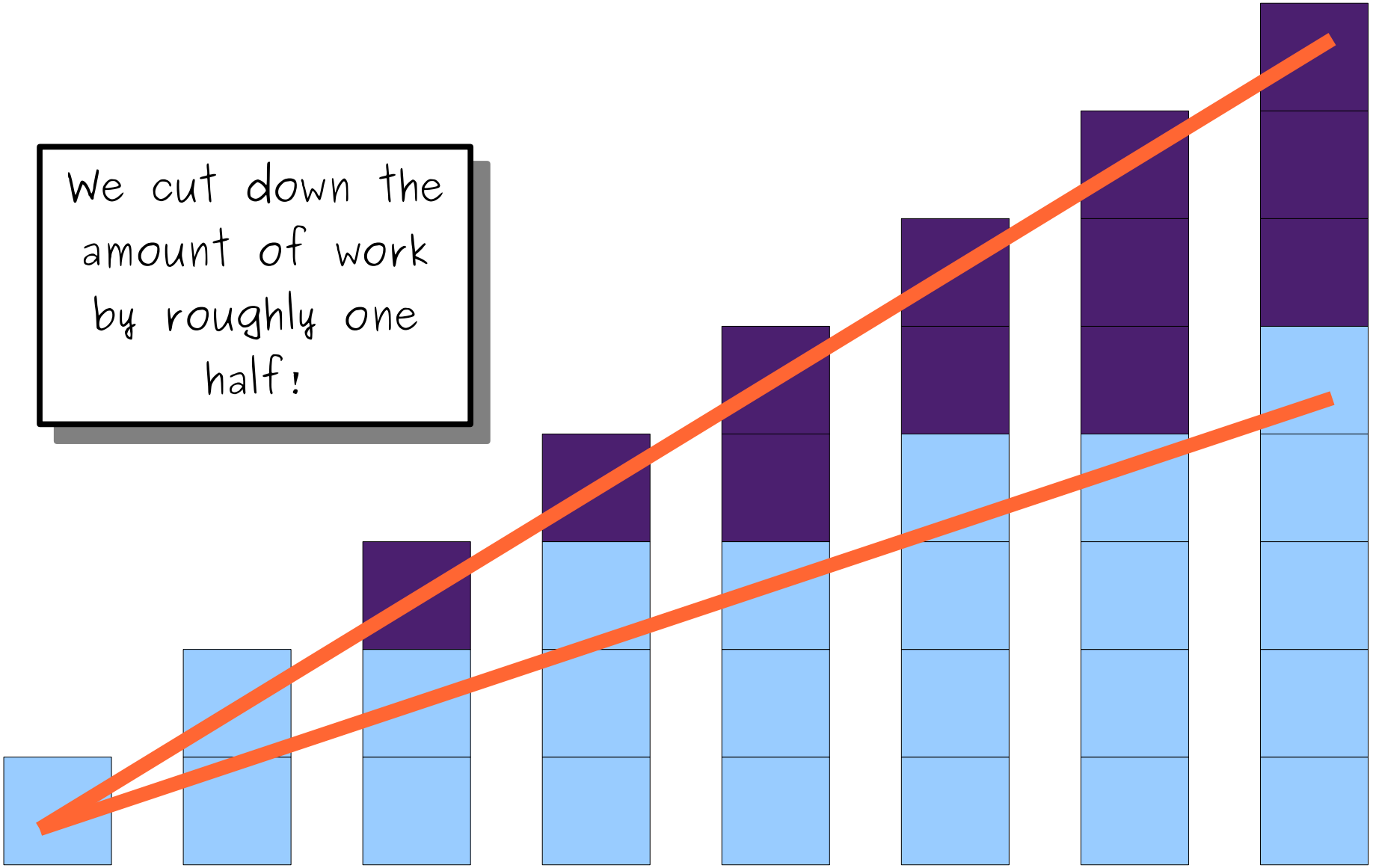
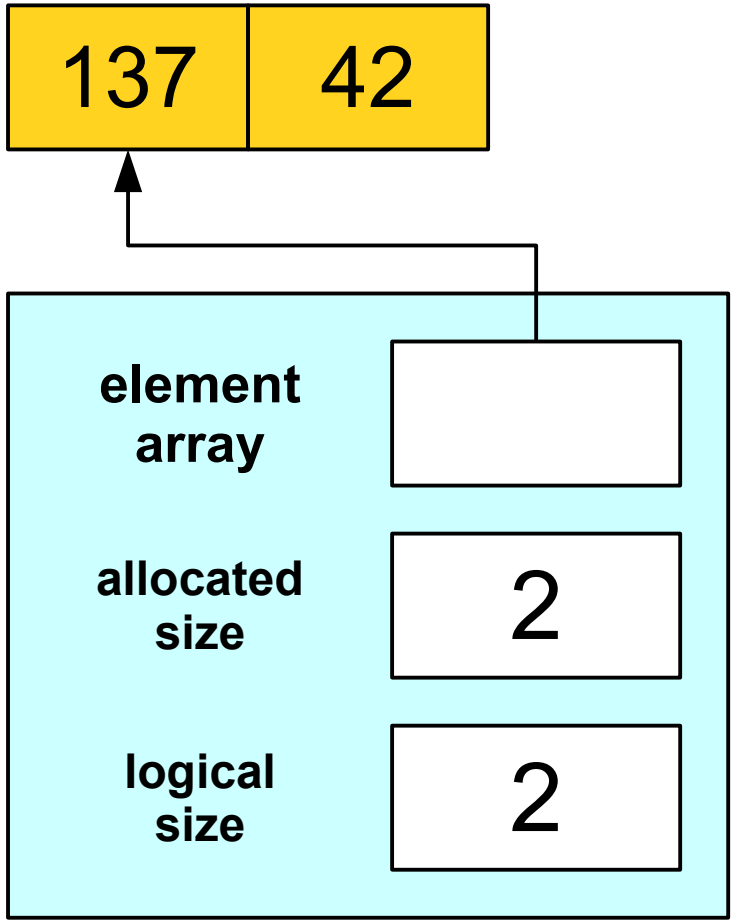


How does it stack up?

A Much Better Idea



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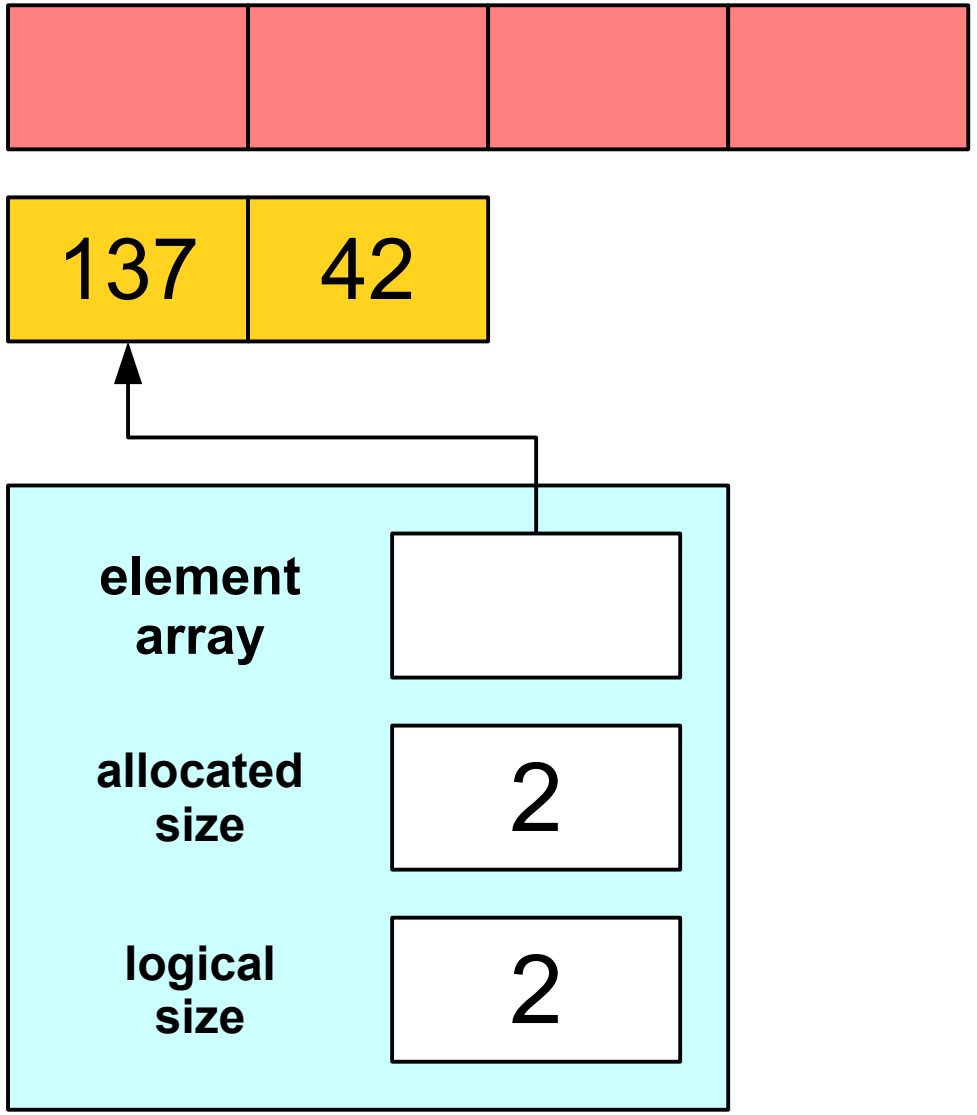

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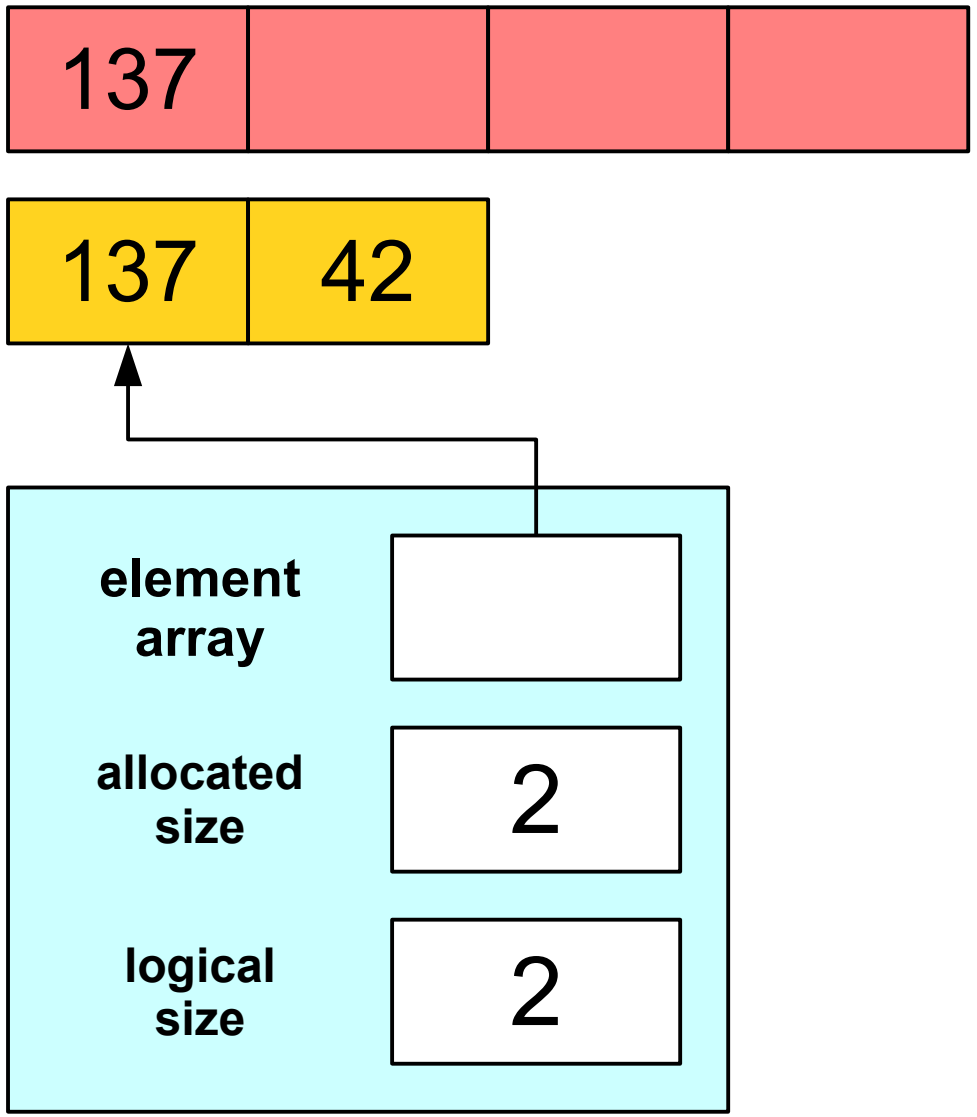

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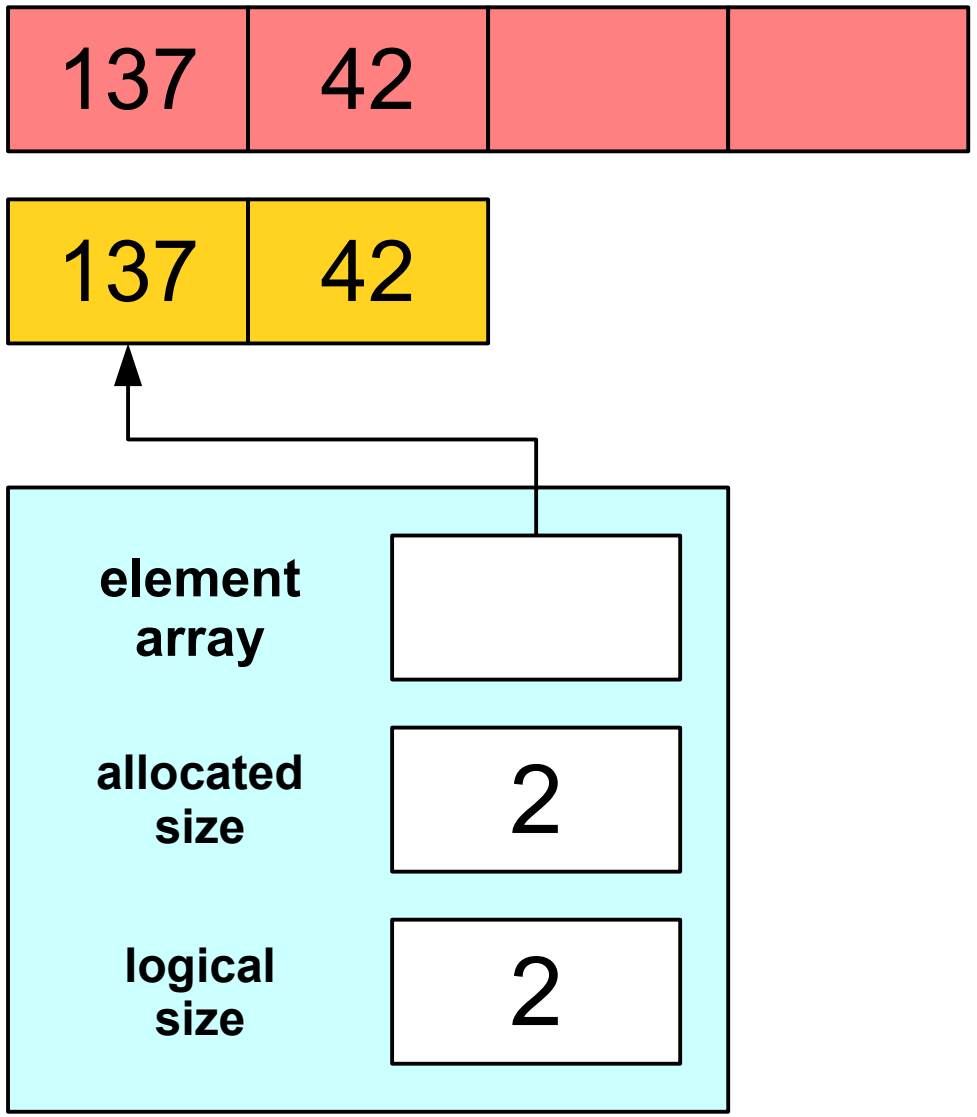

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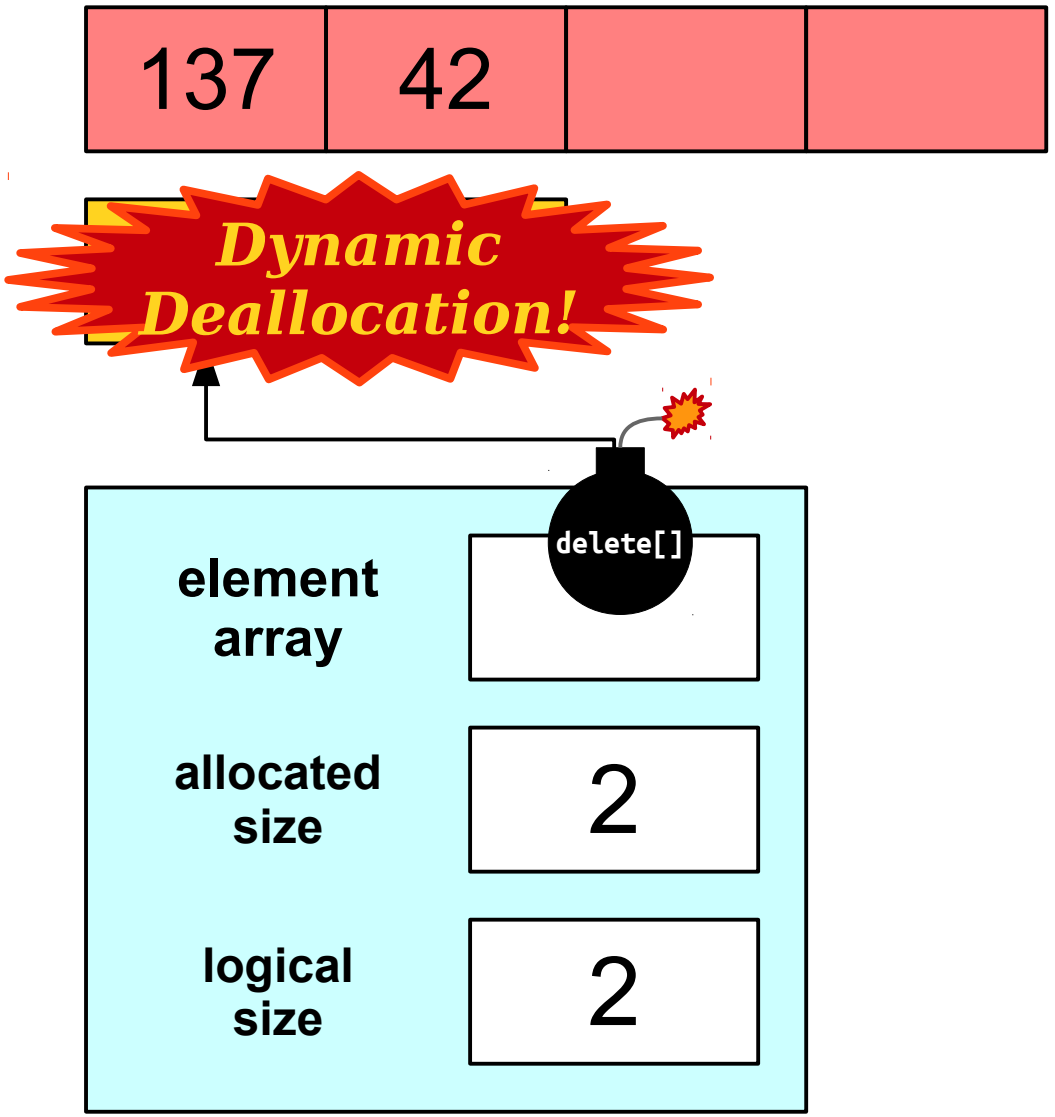

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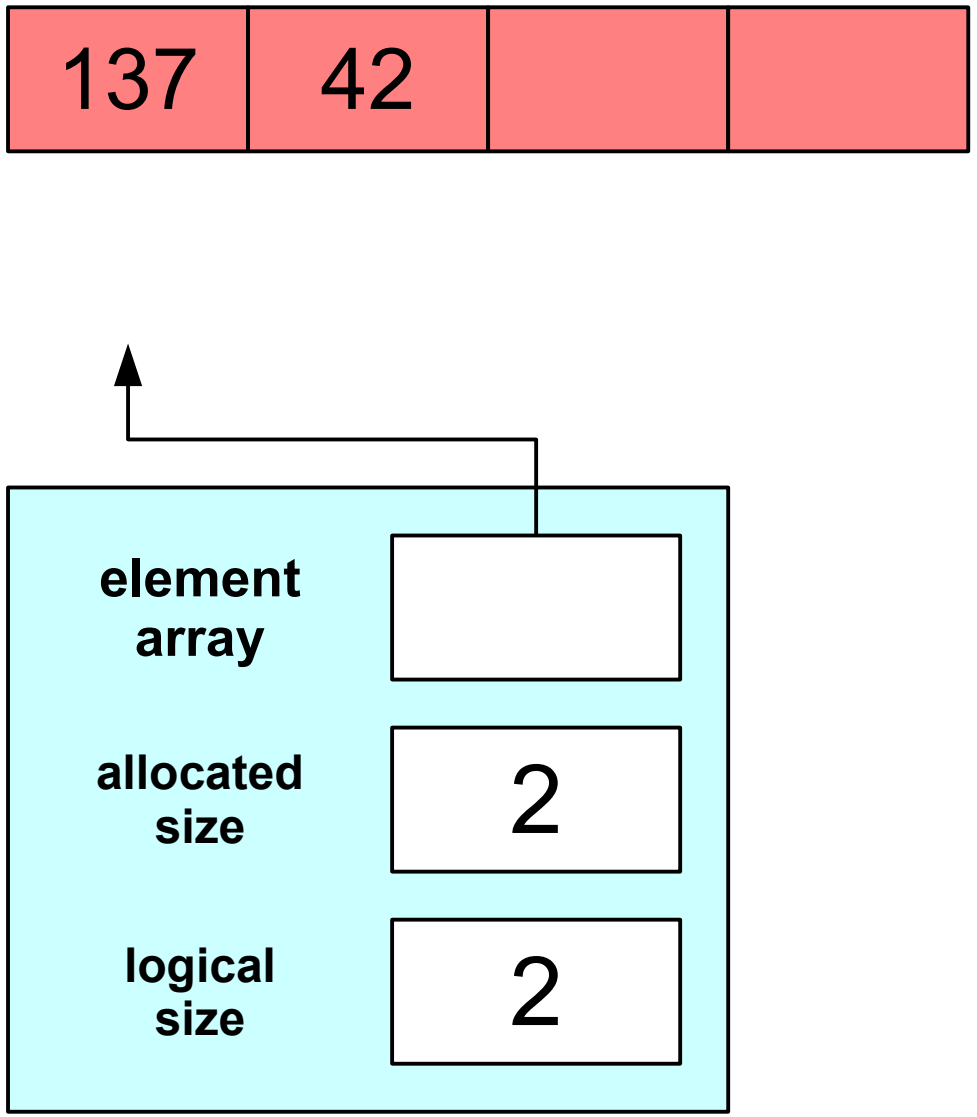

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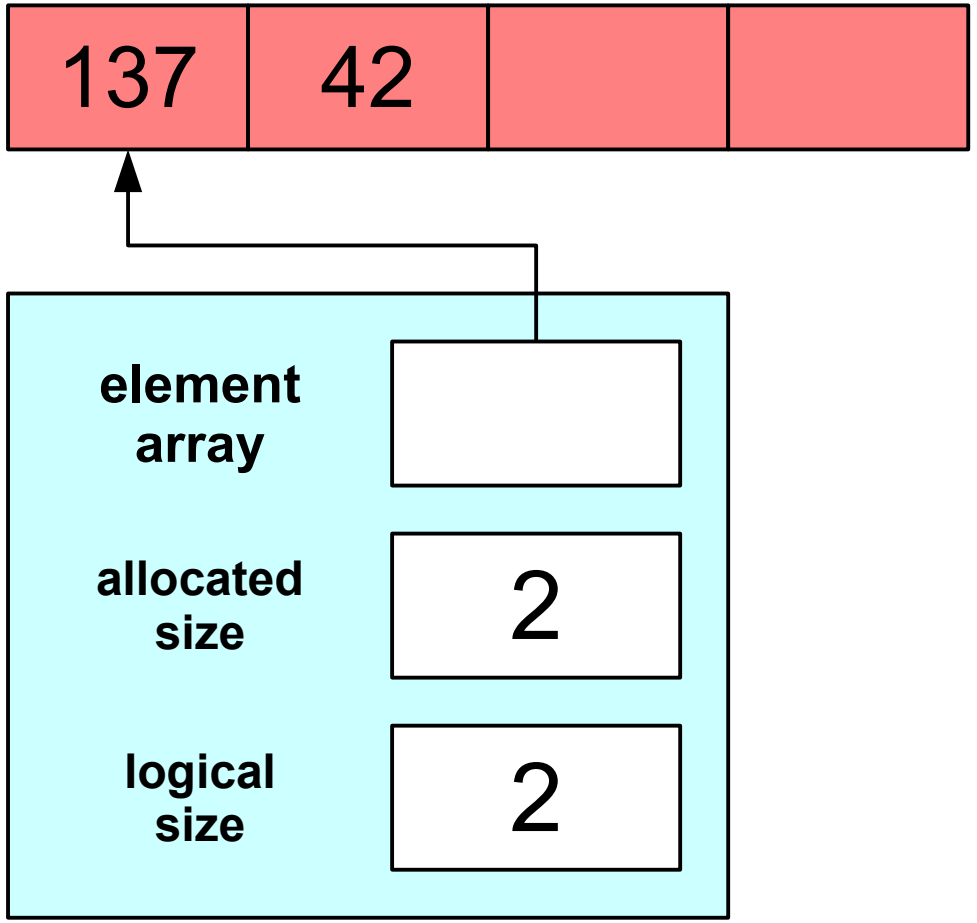

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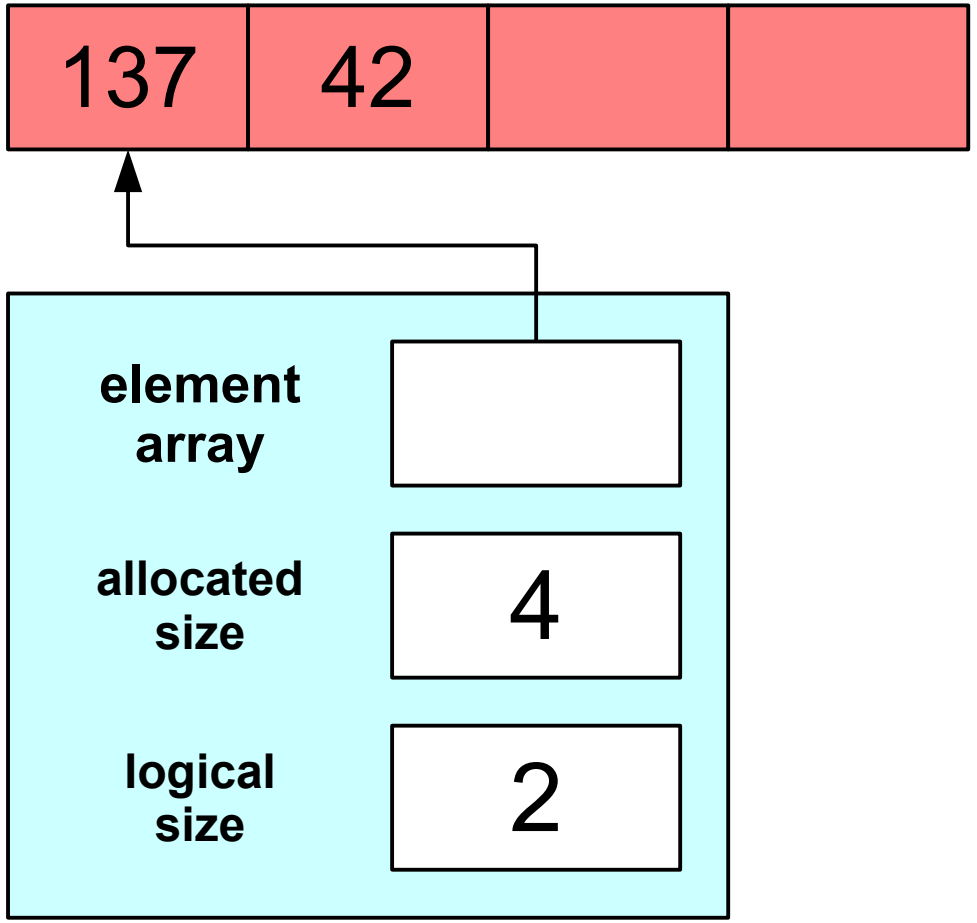

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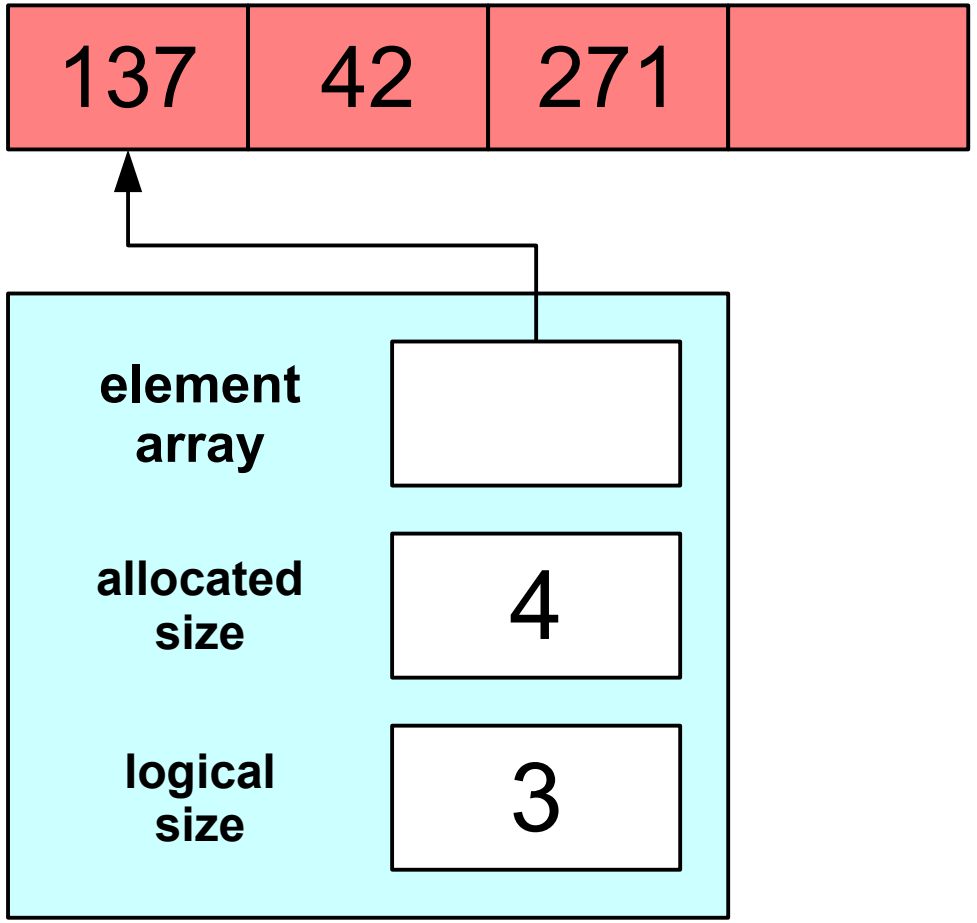

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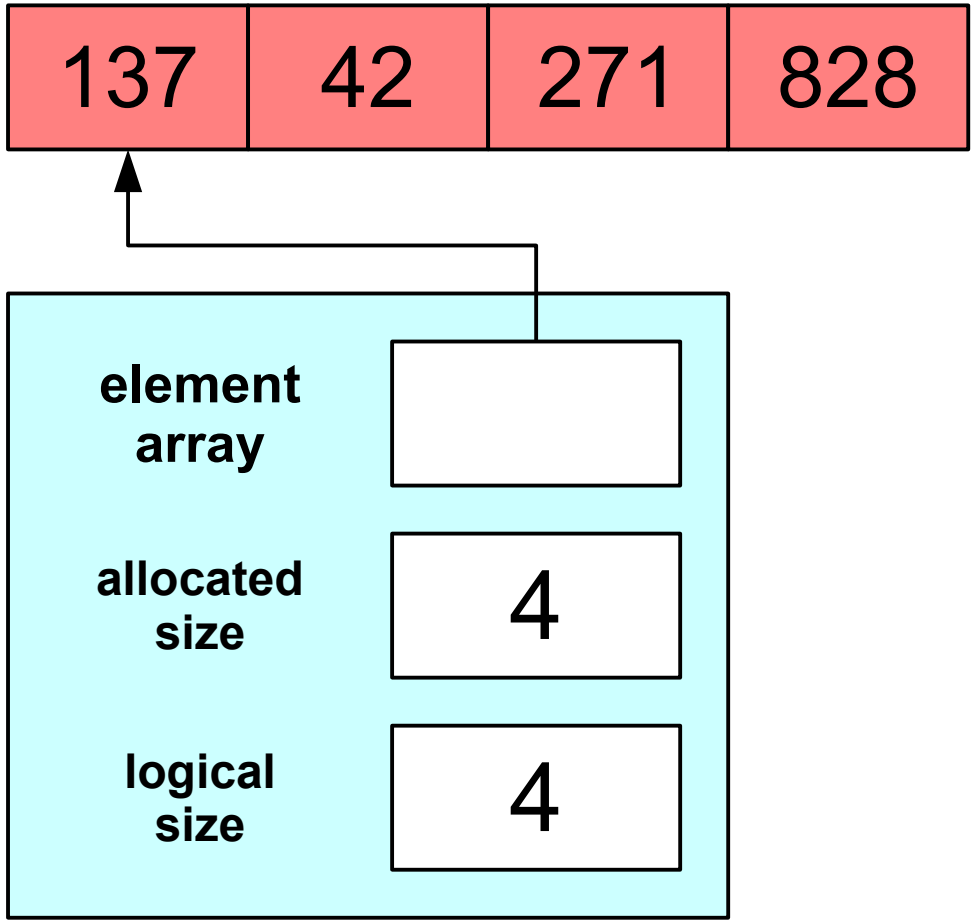

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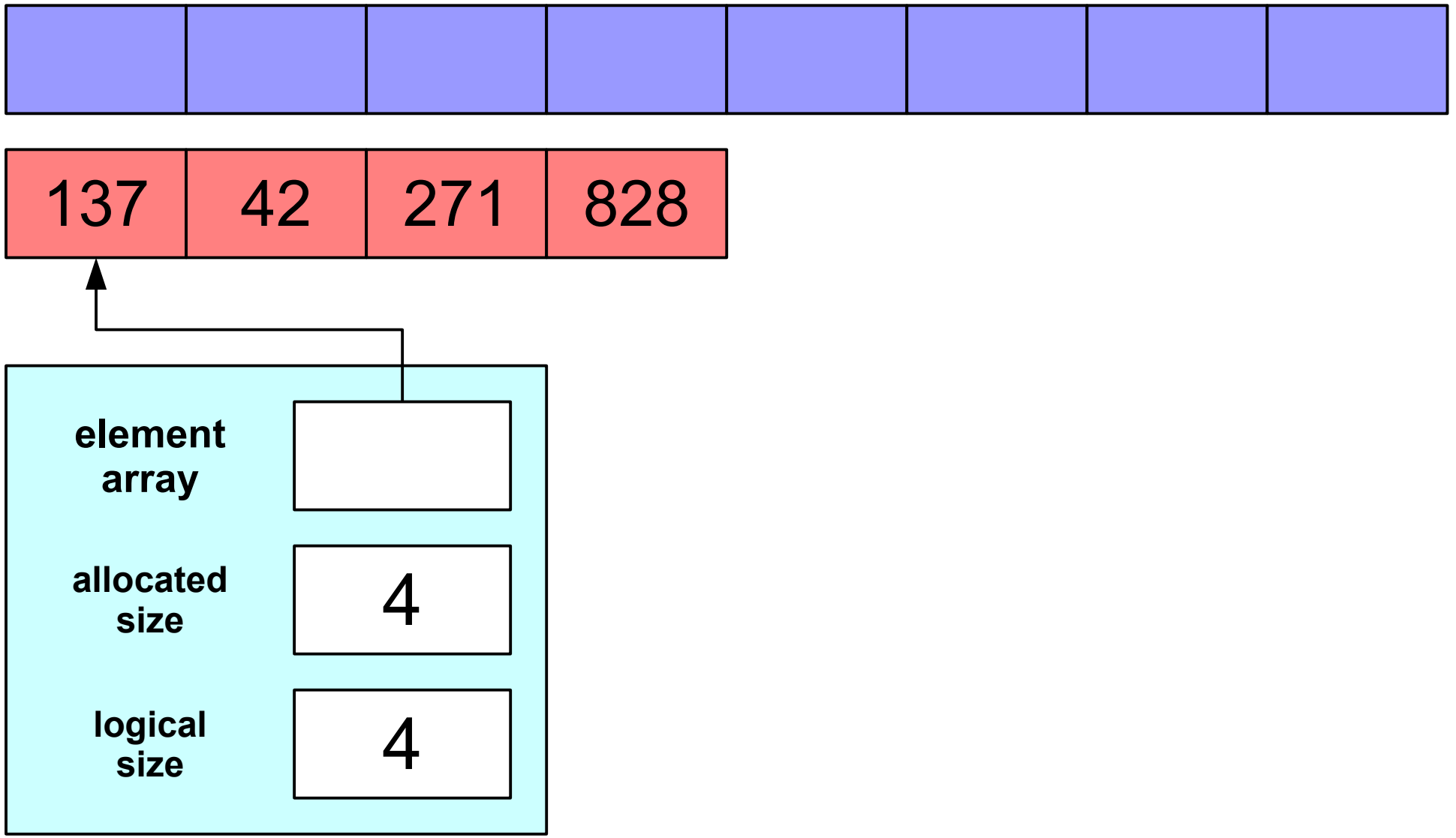

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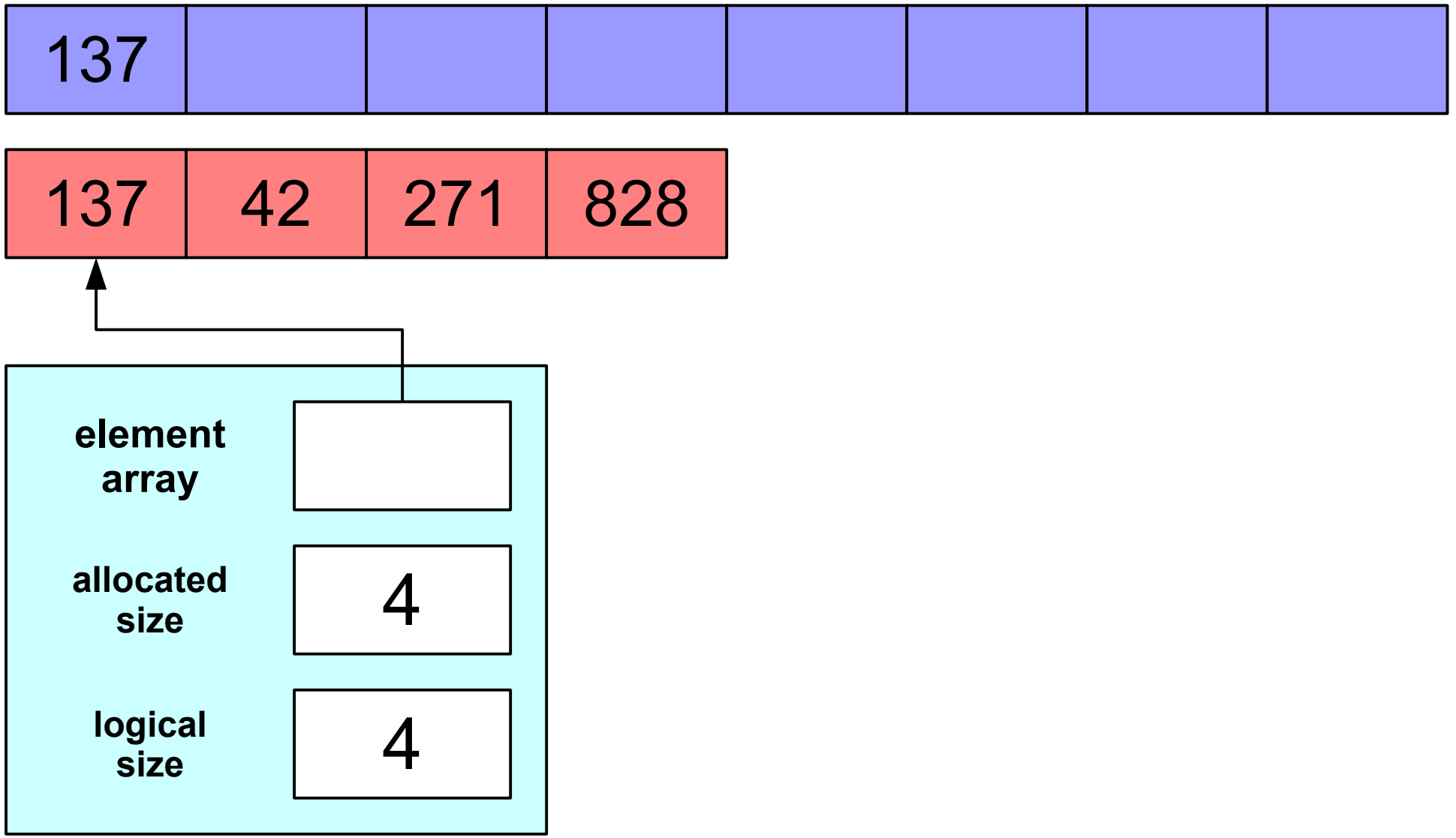

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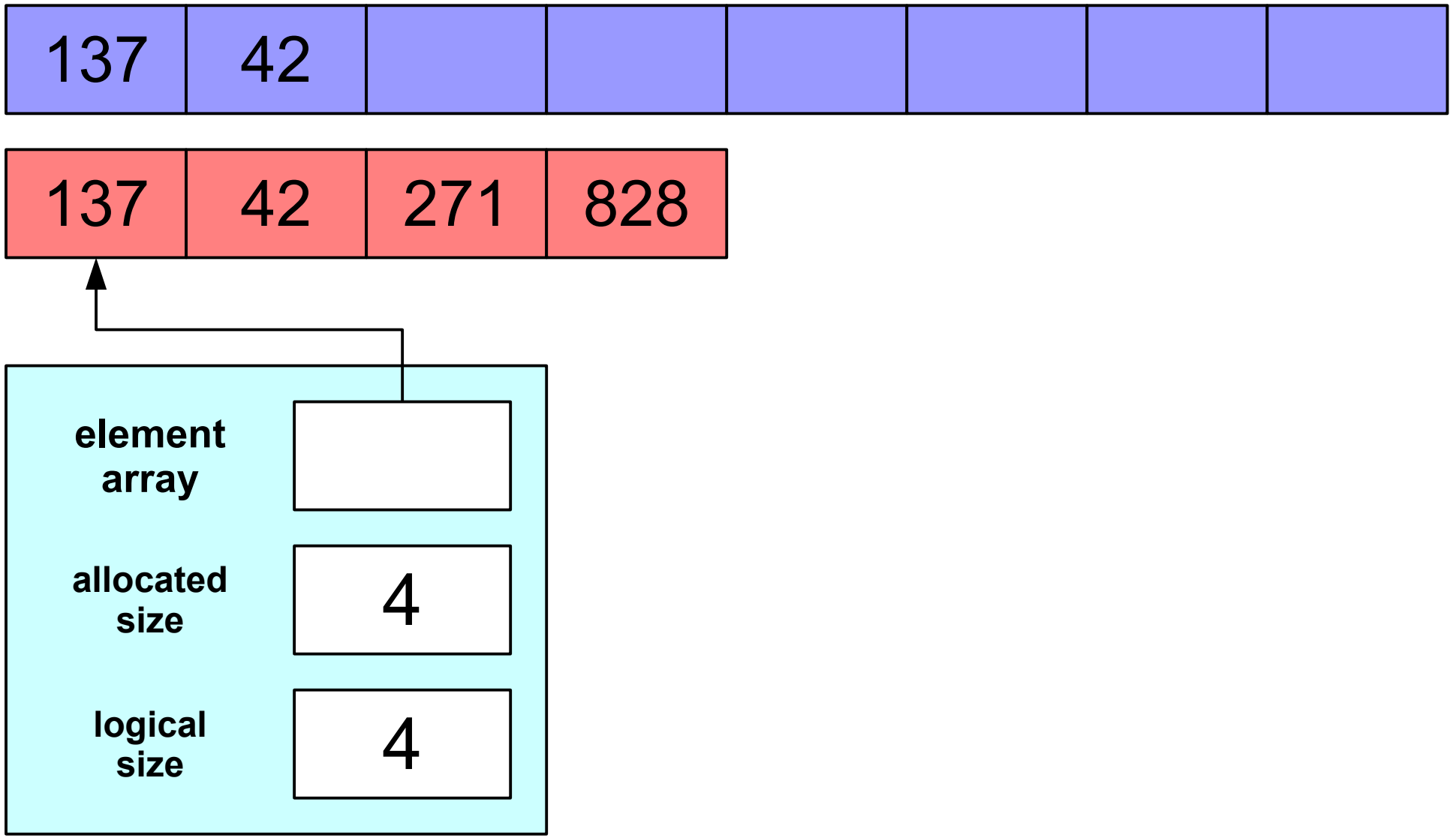

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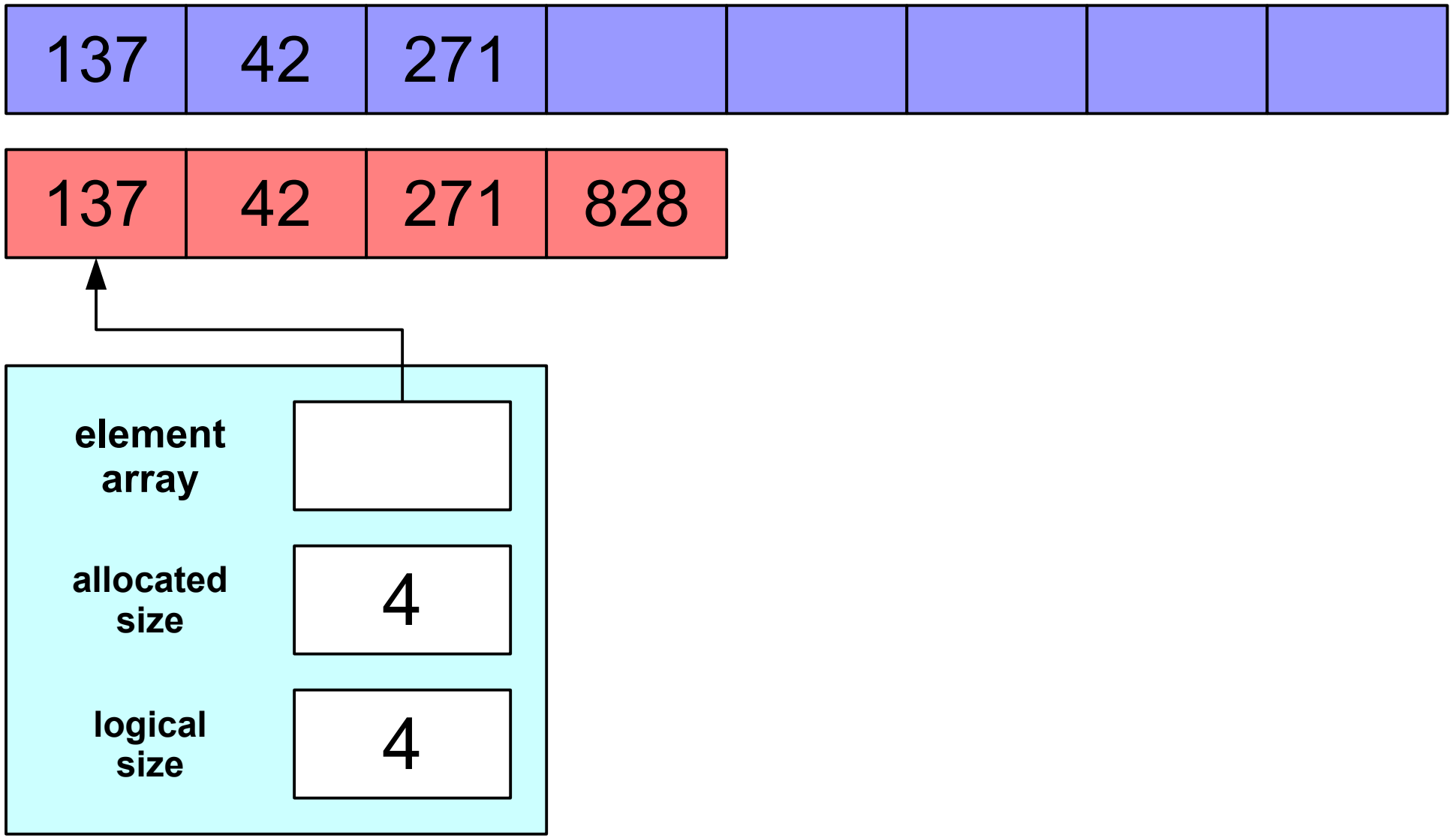

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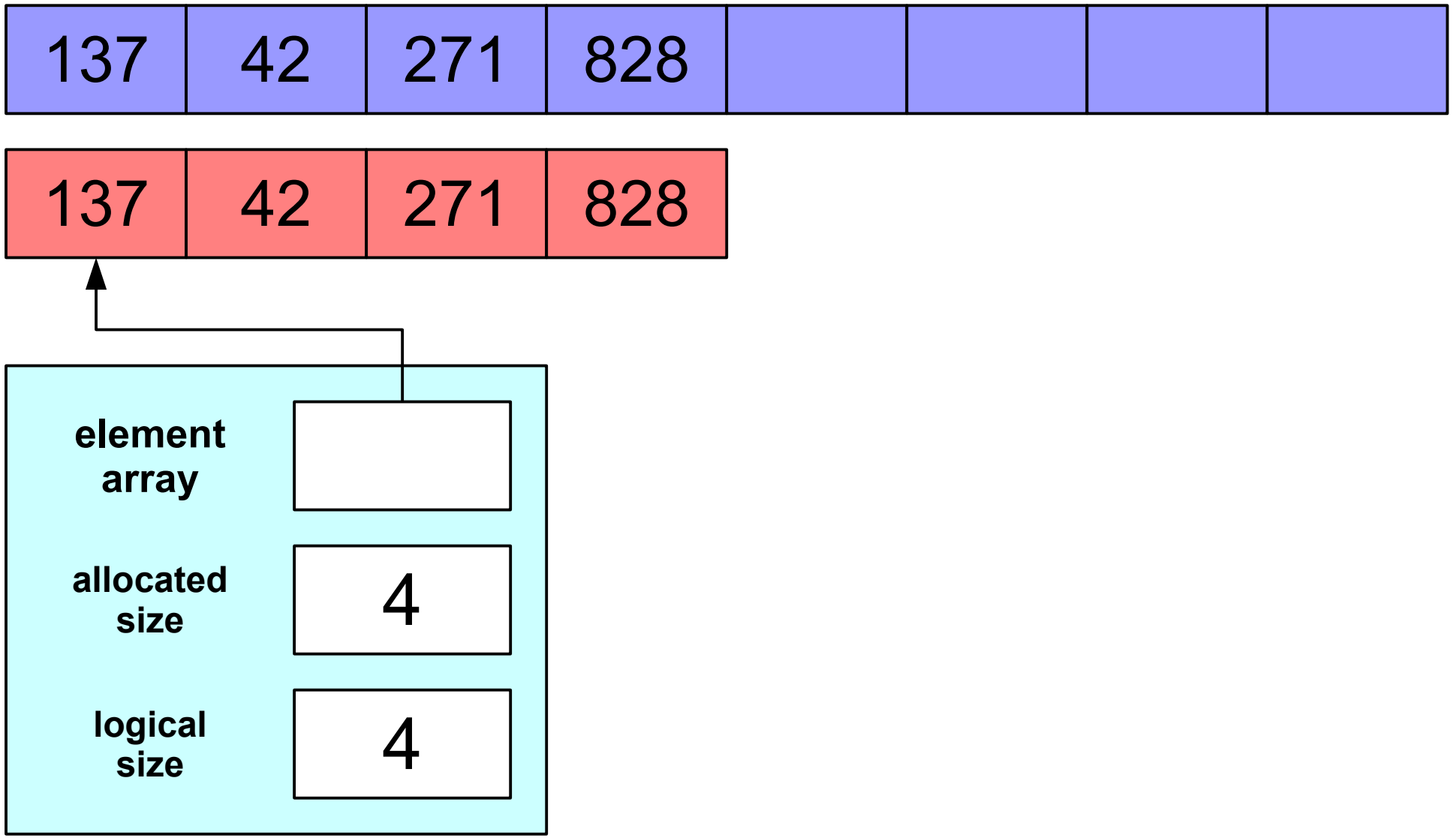

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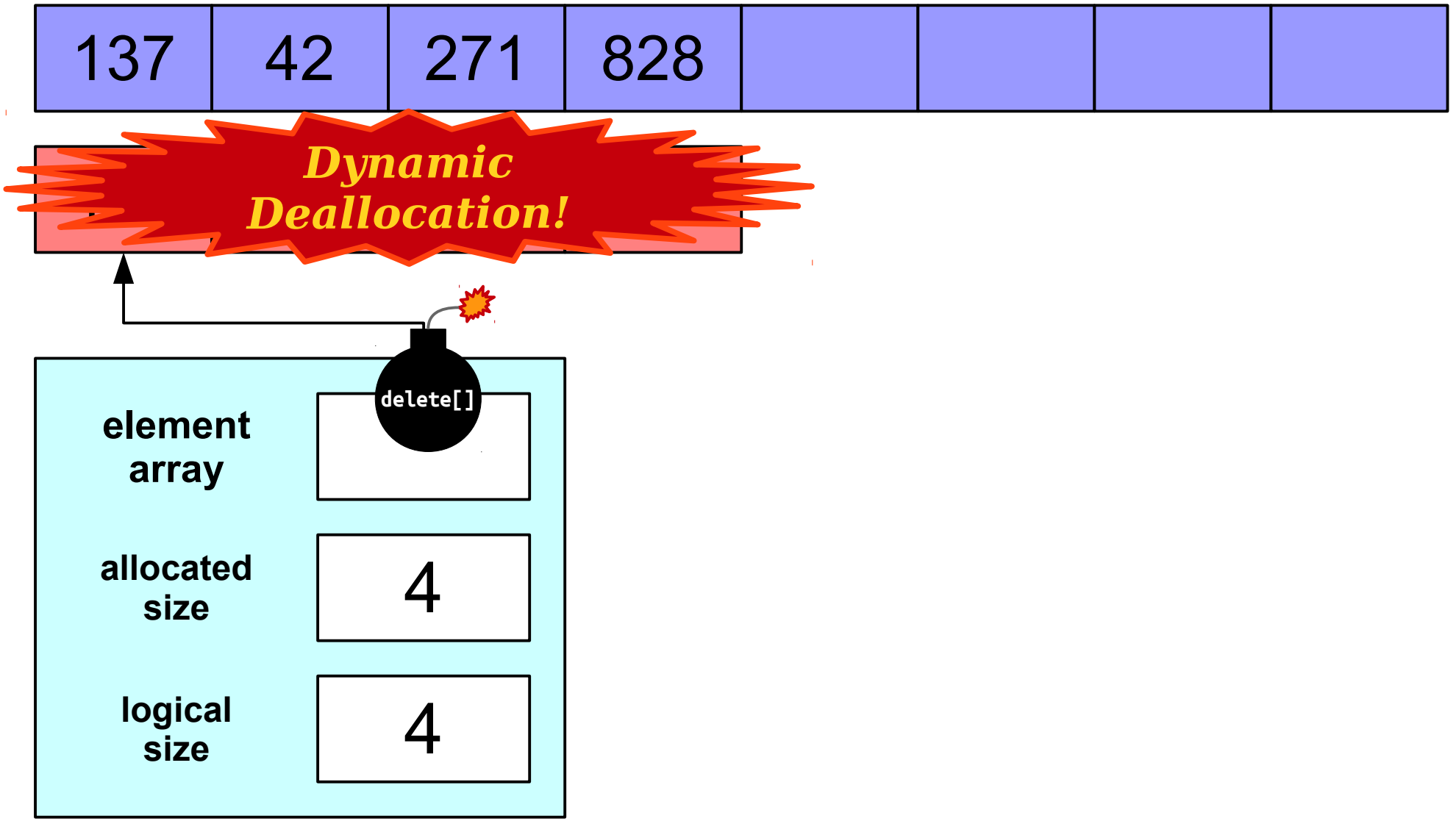

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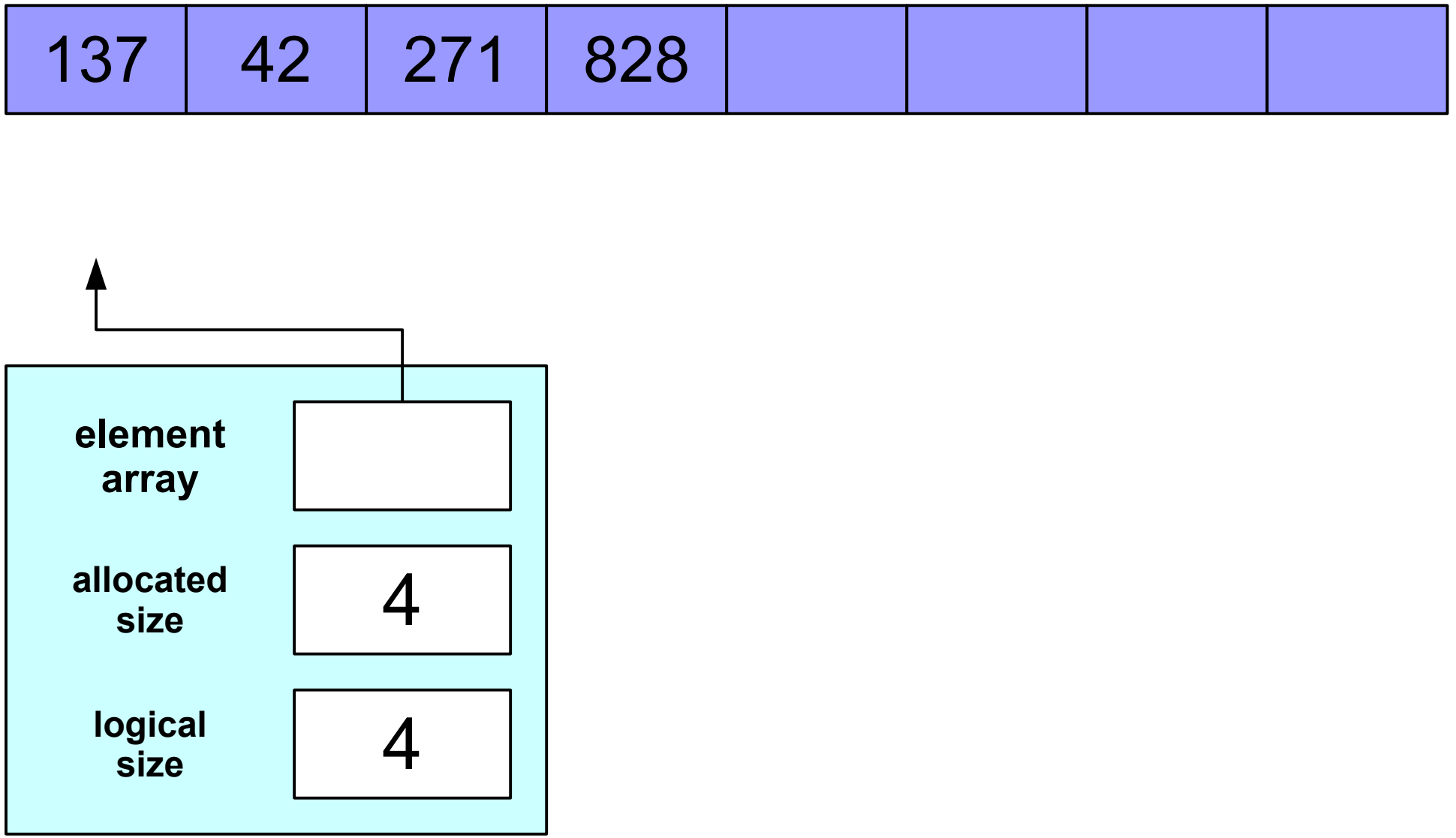

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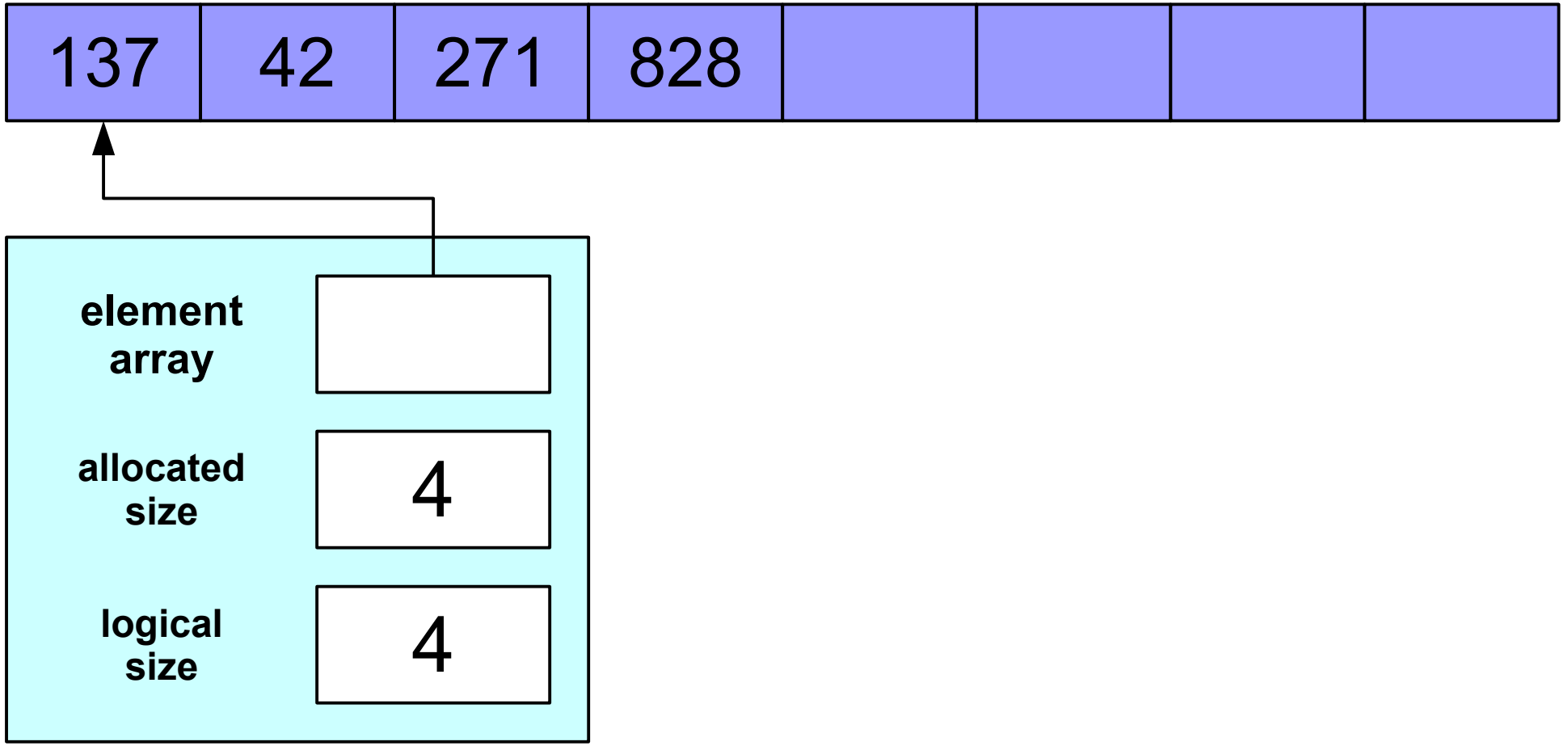

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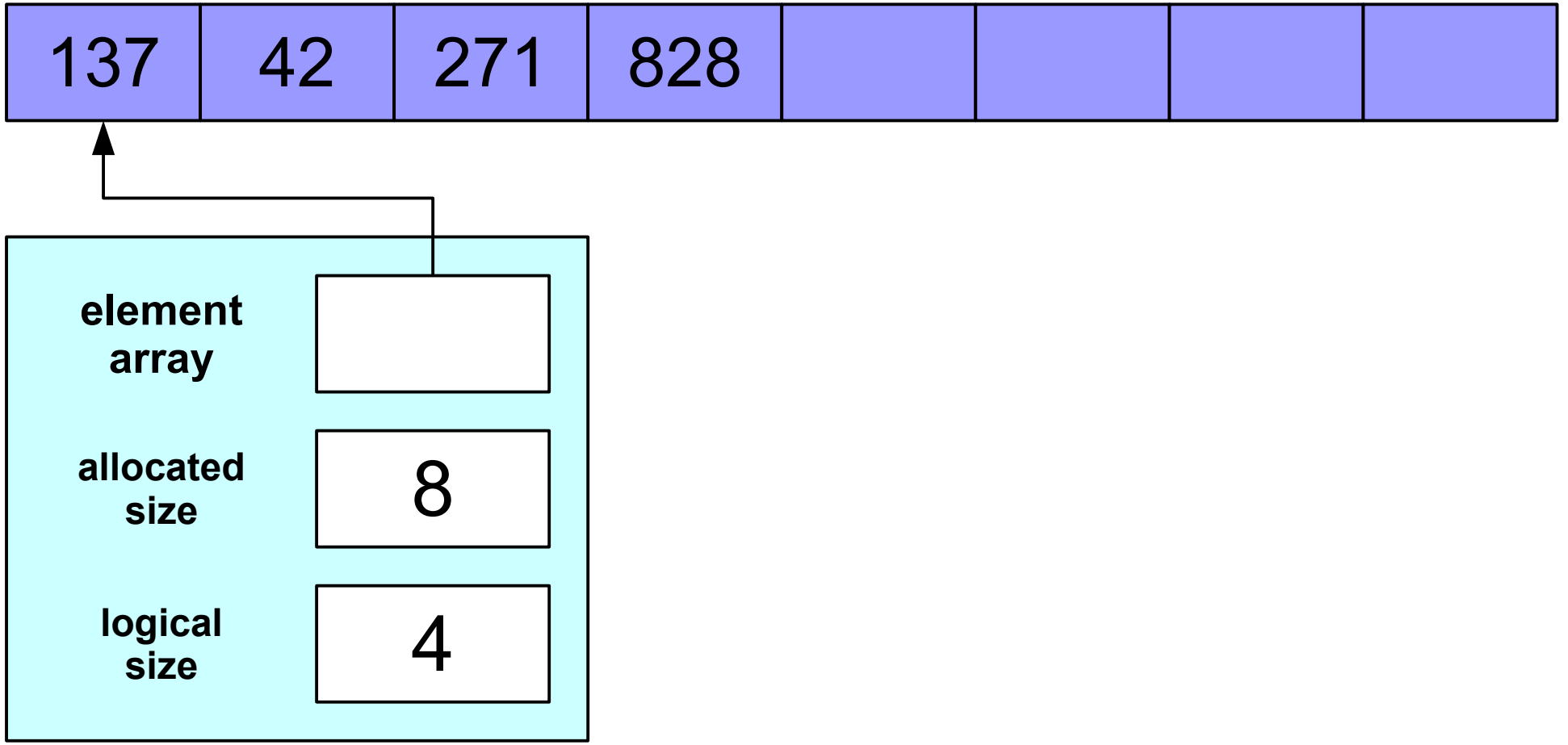

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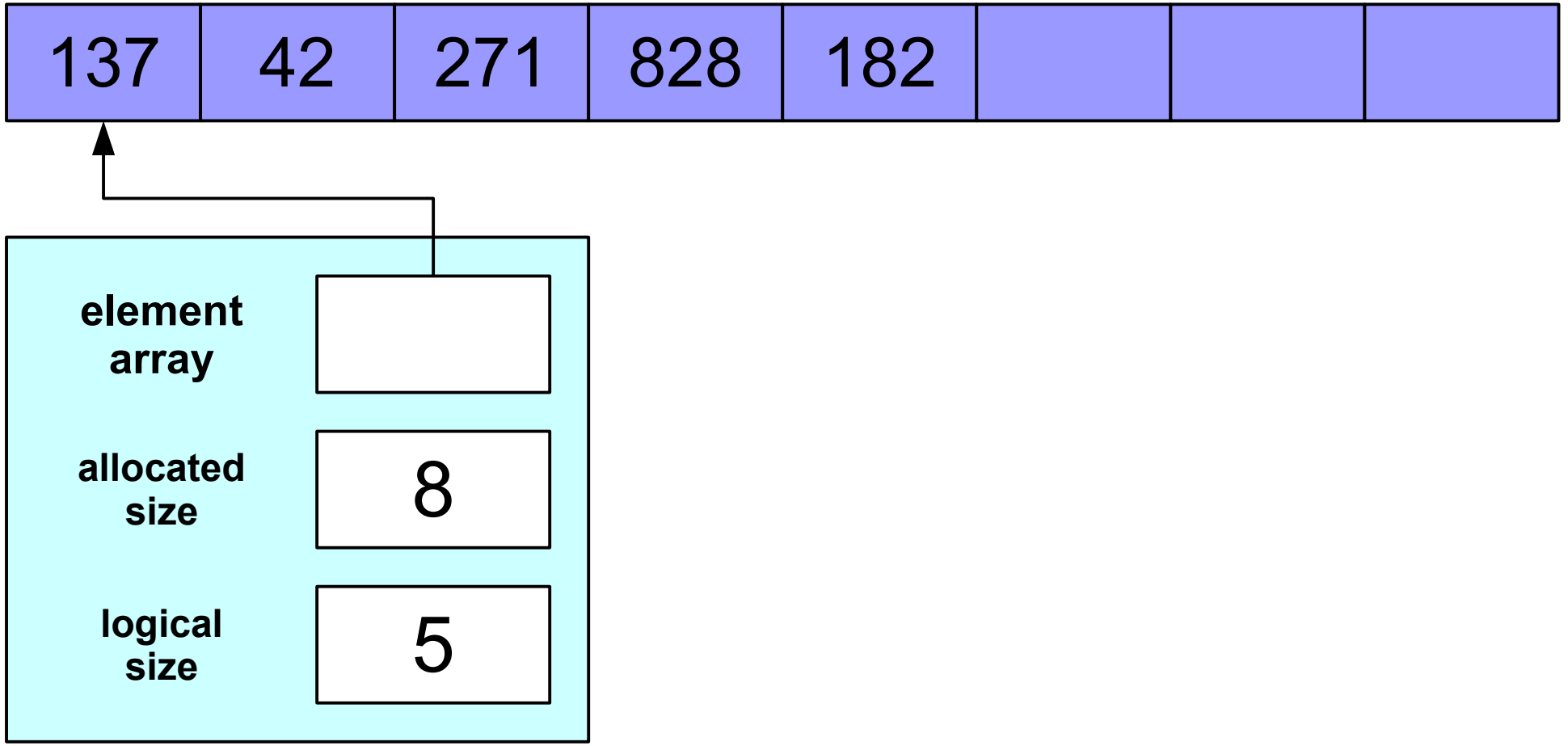

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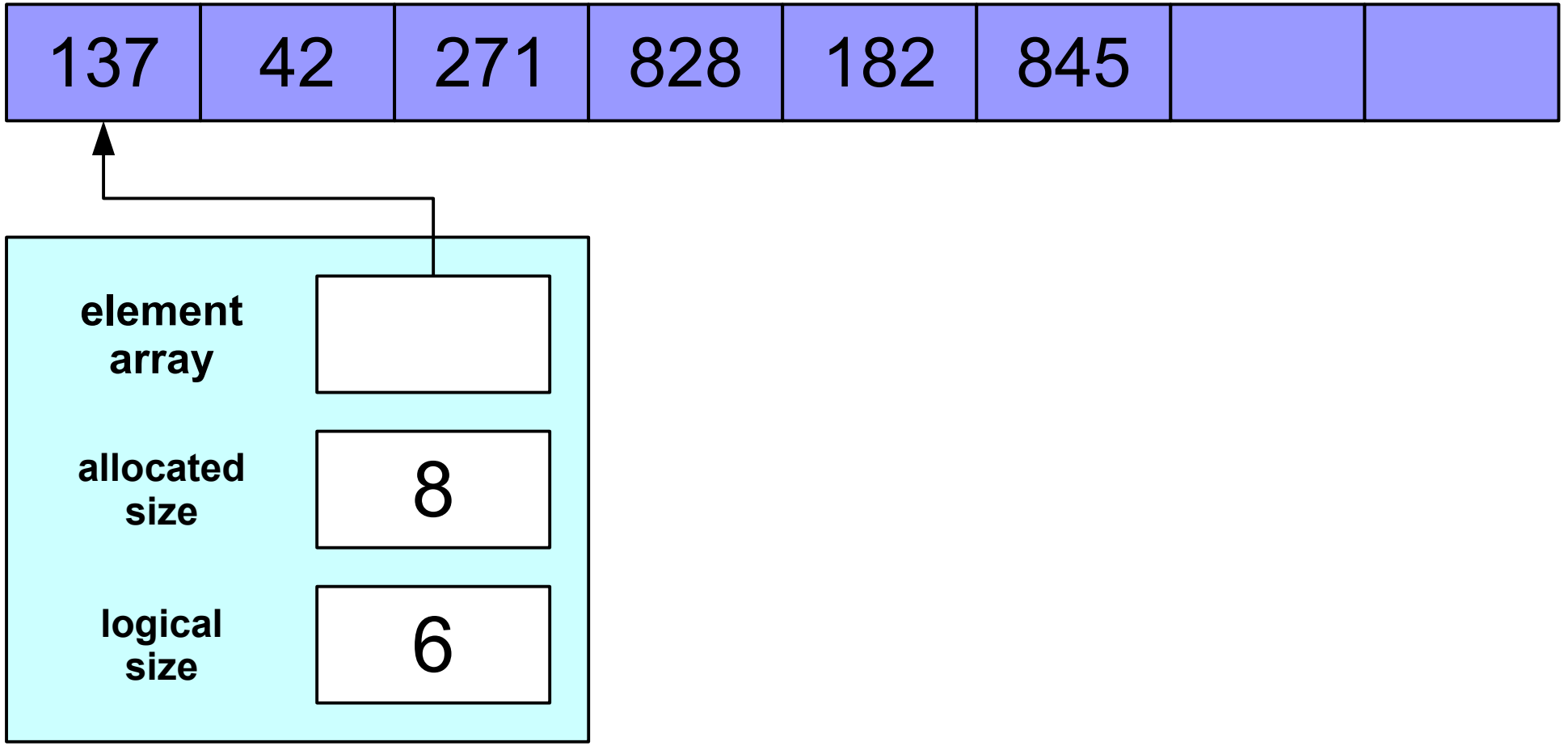

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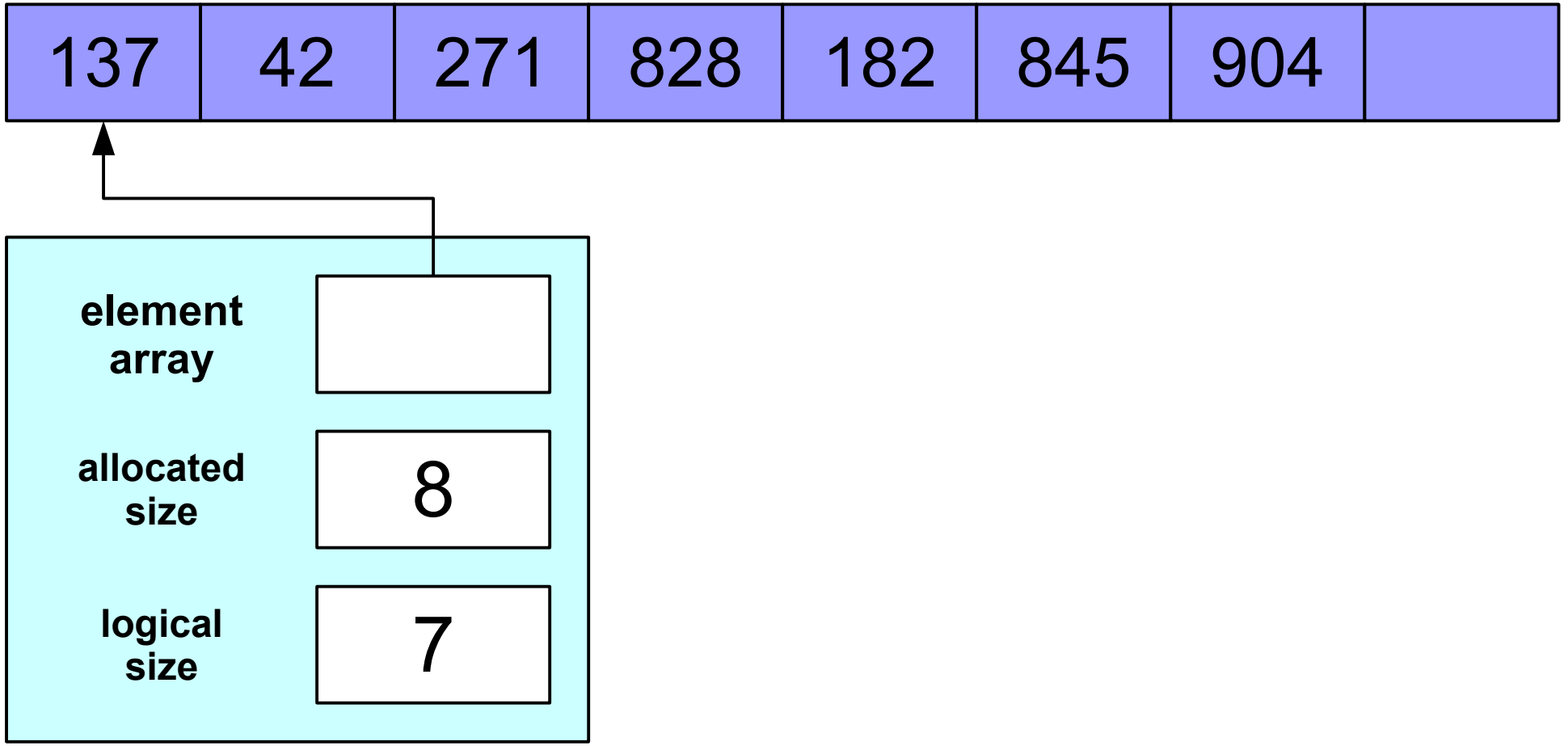

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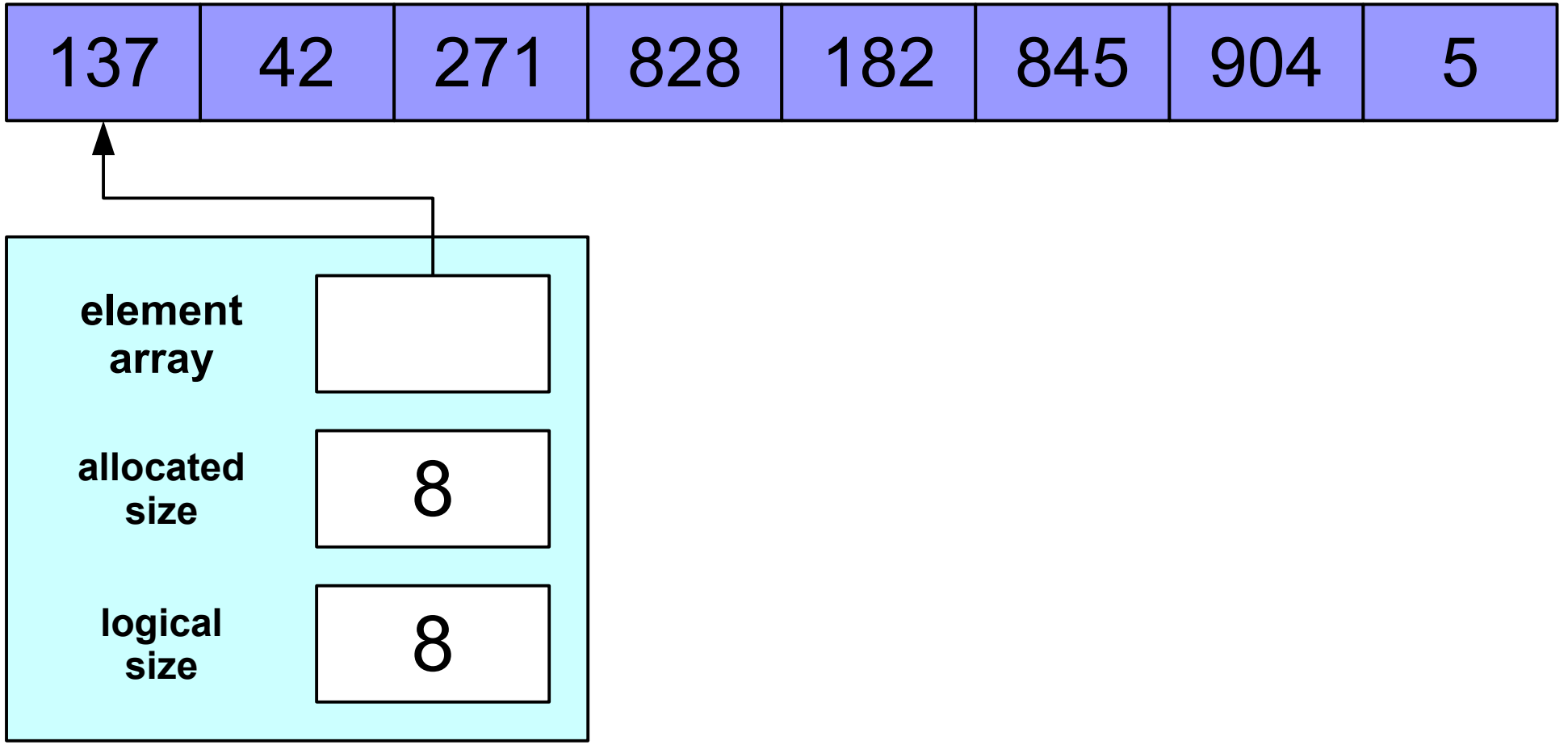
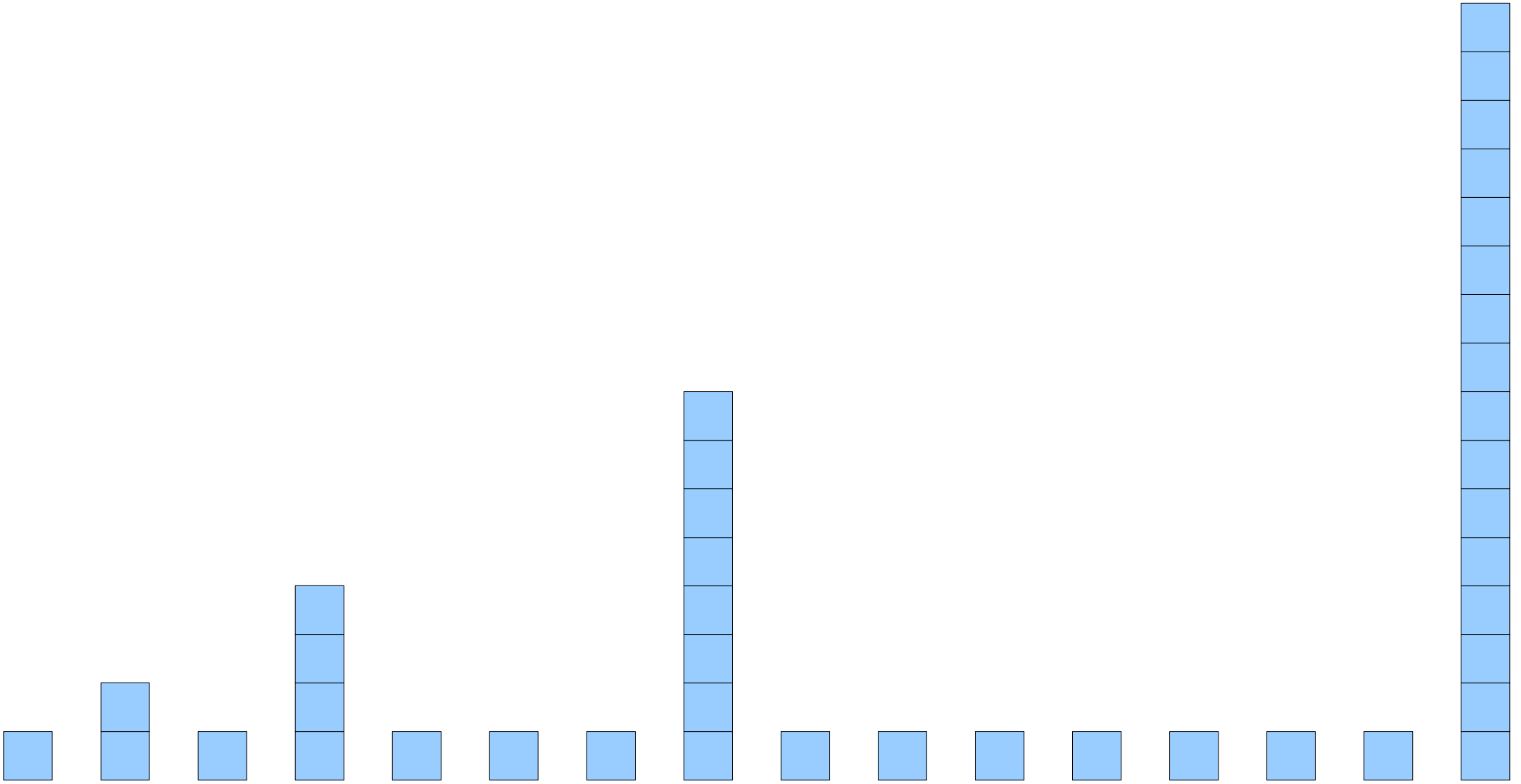


Let's Give it a Try!

How do we analyze this?

Spreading the Work



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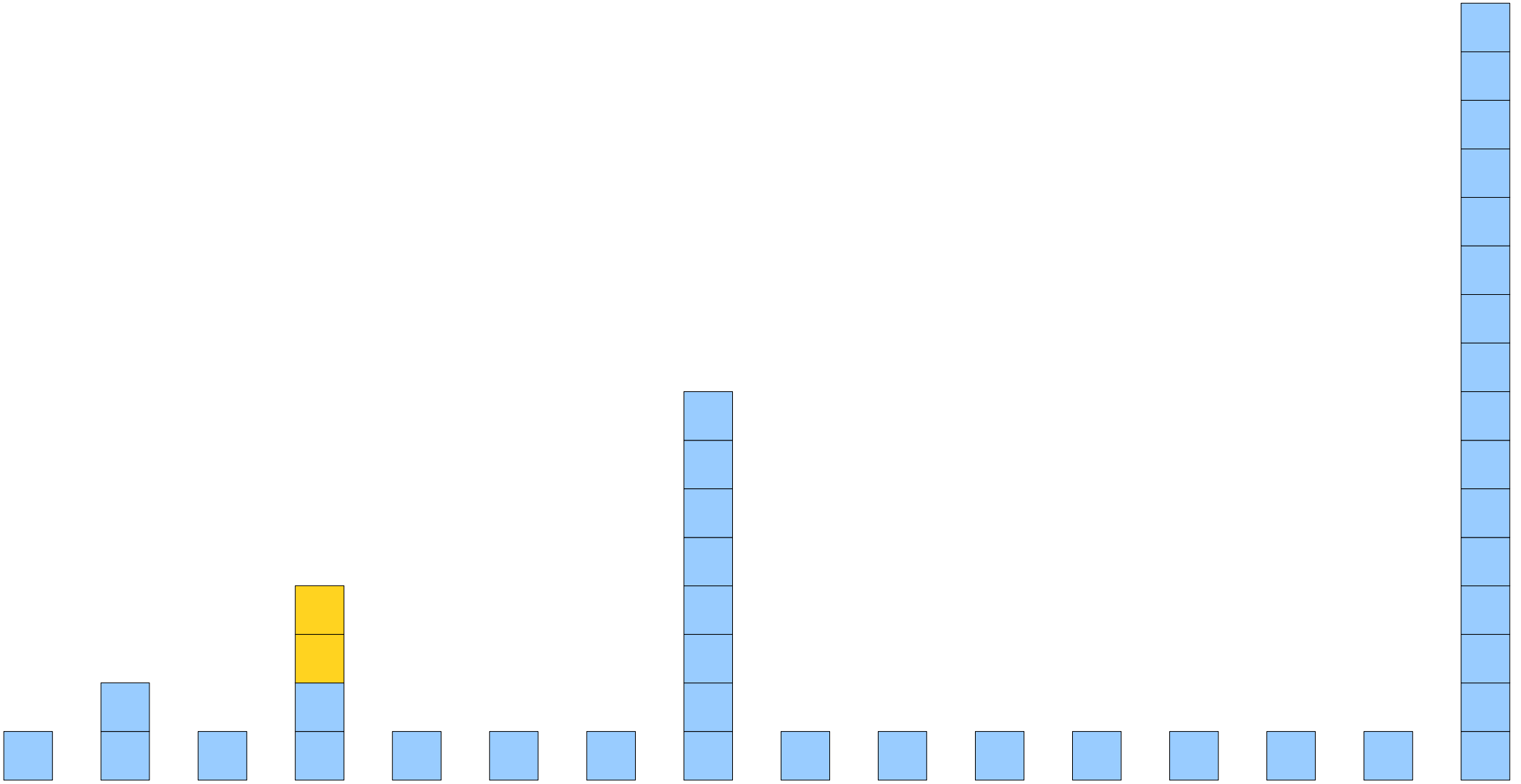

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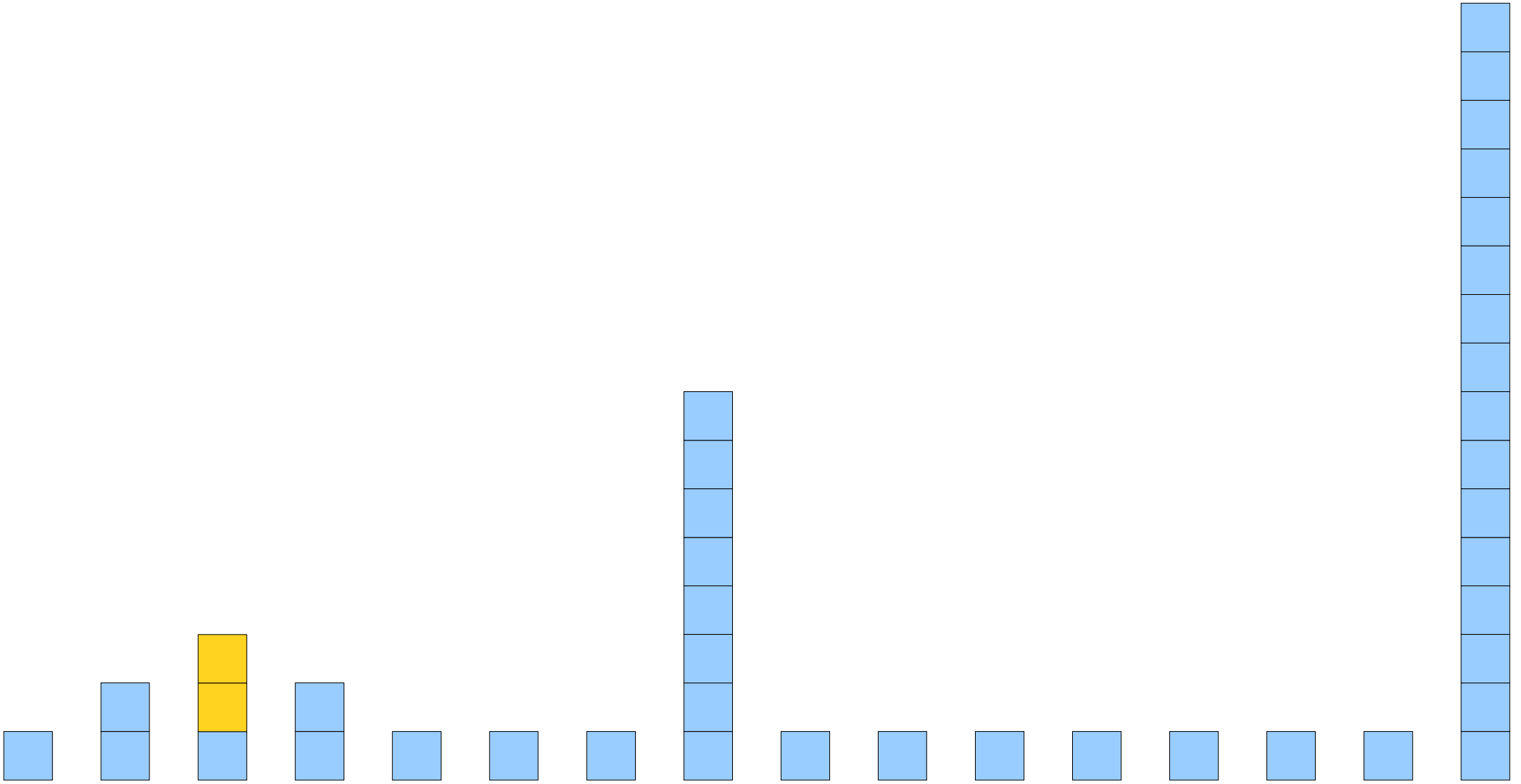

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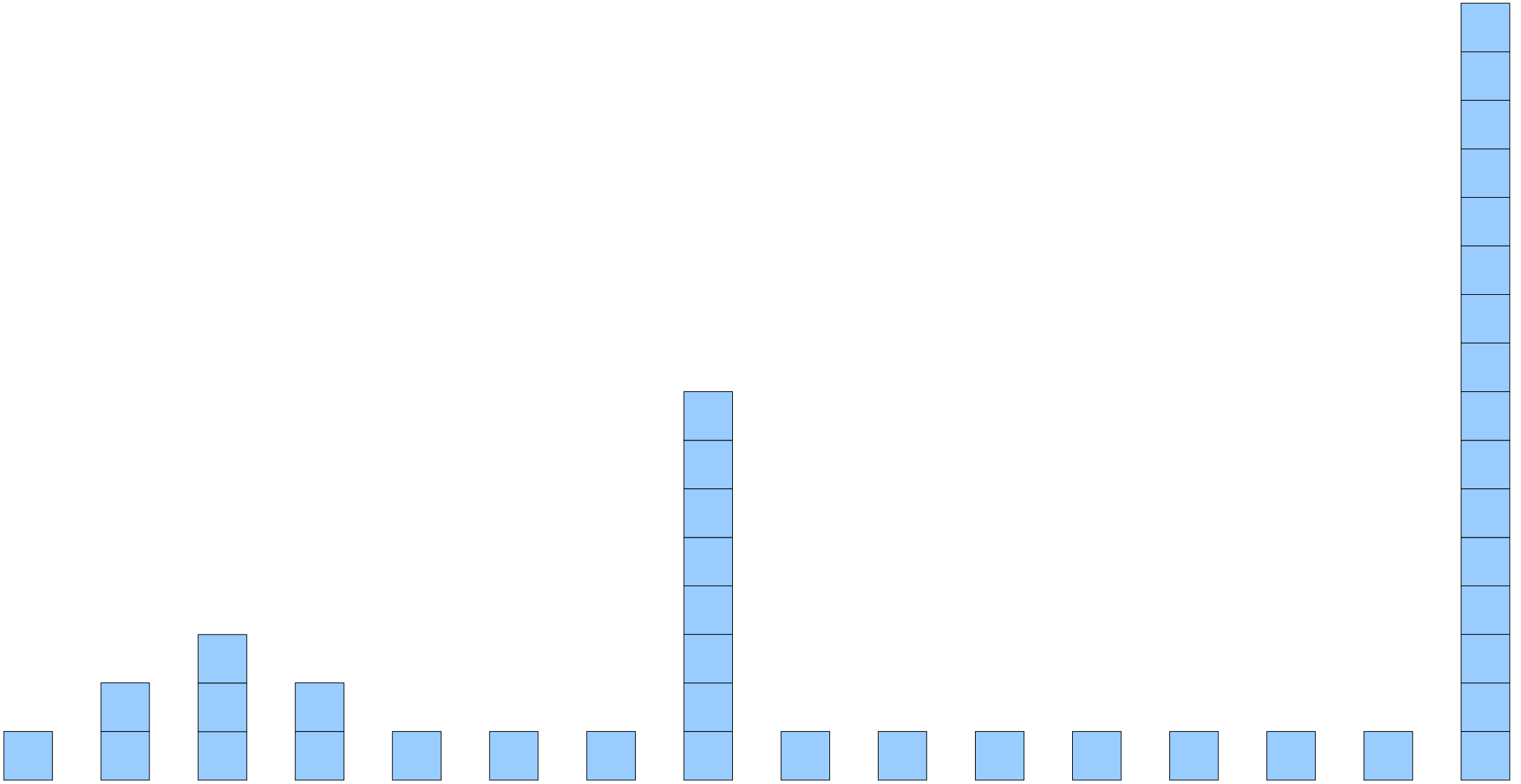

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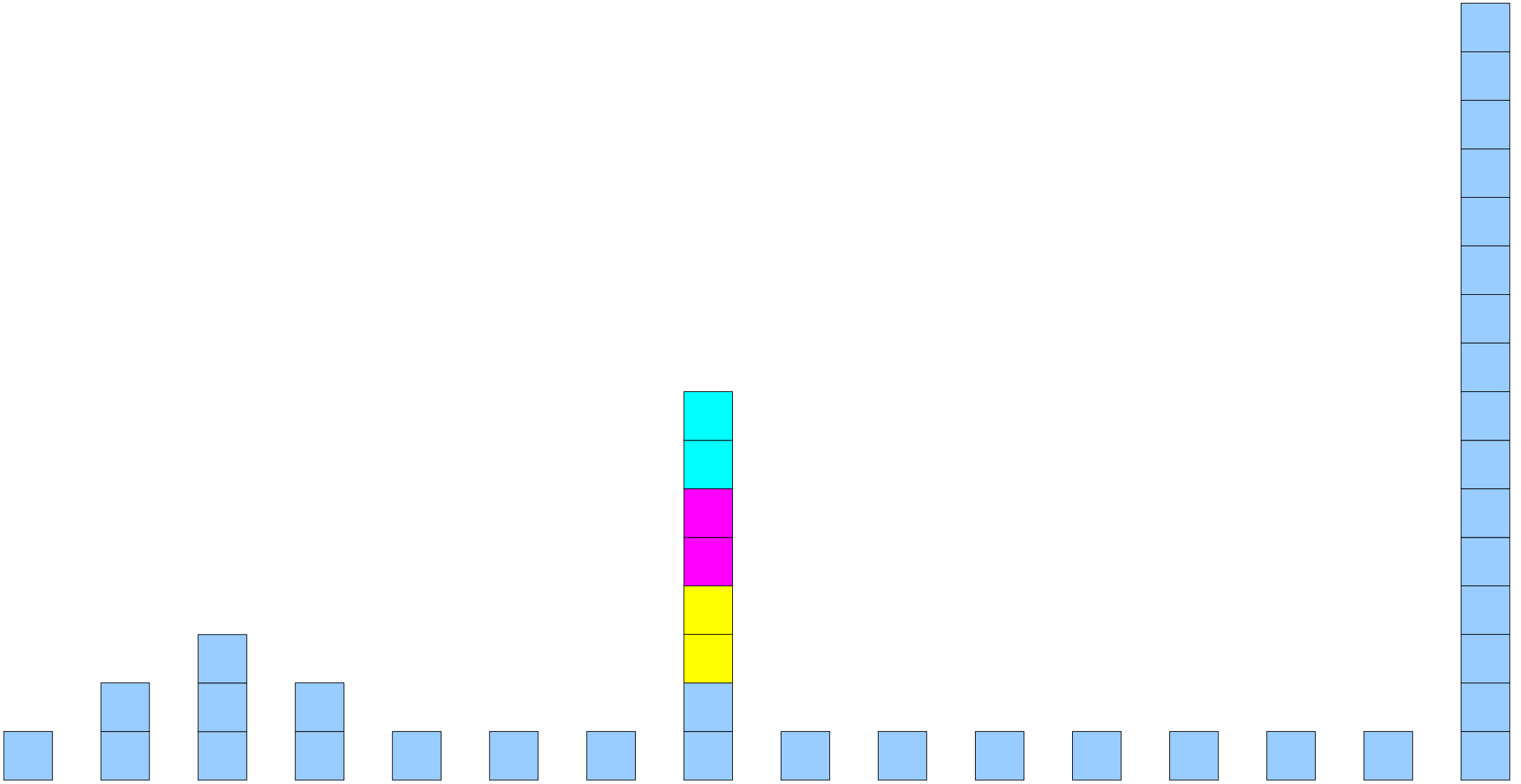
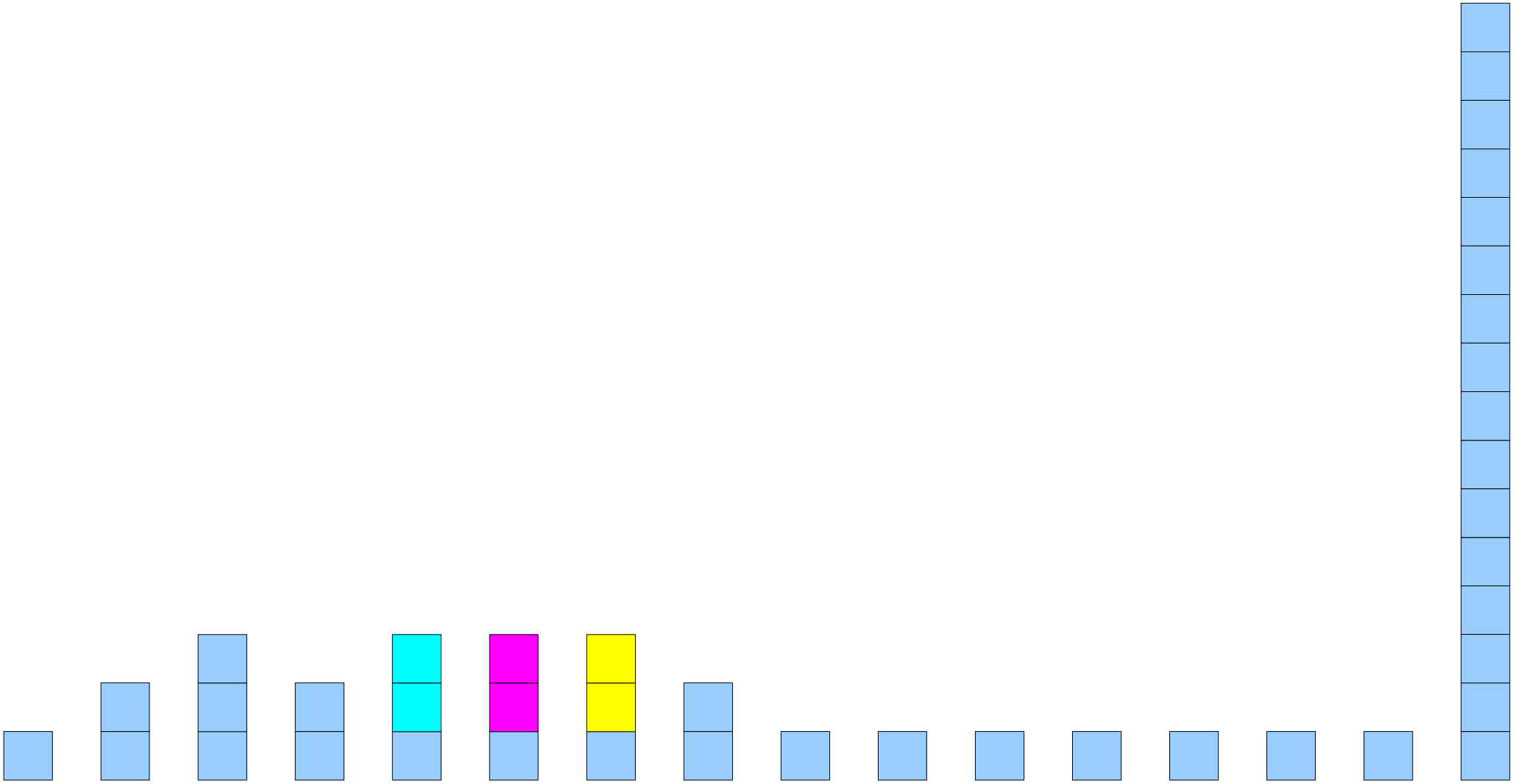
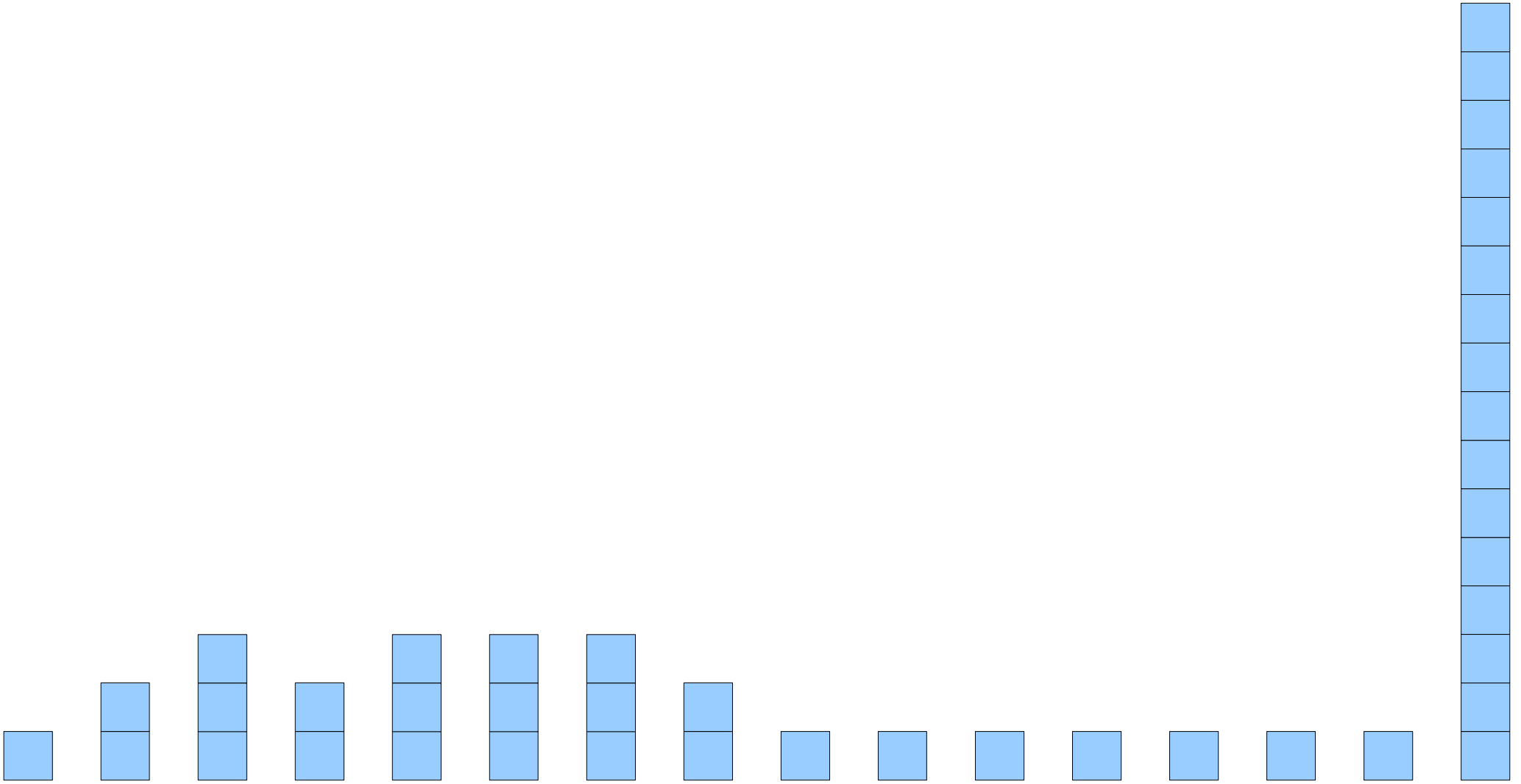


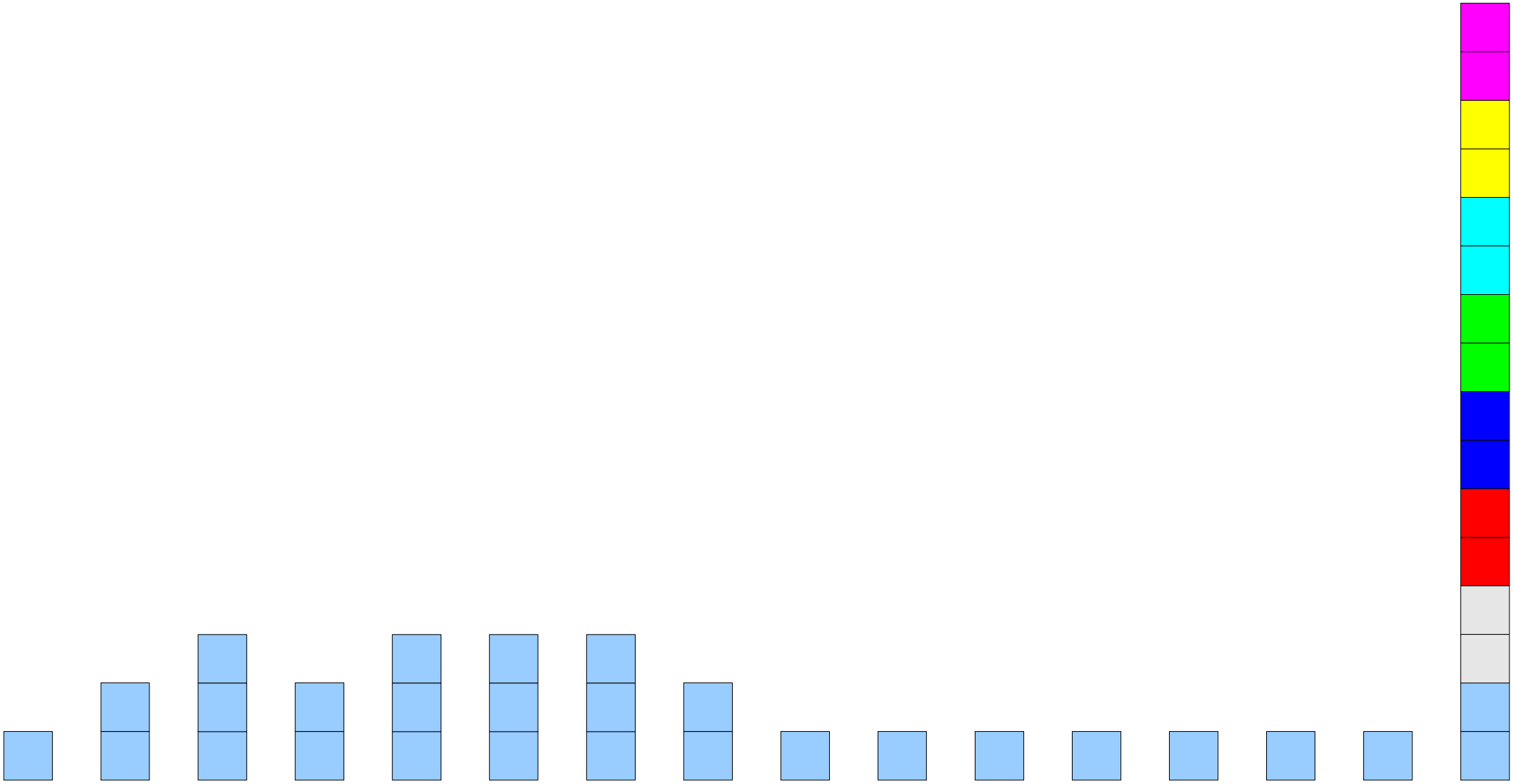
Spreading the Work



Spreading the Work



Spreading the Work



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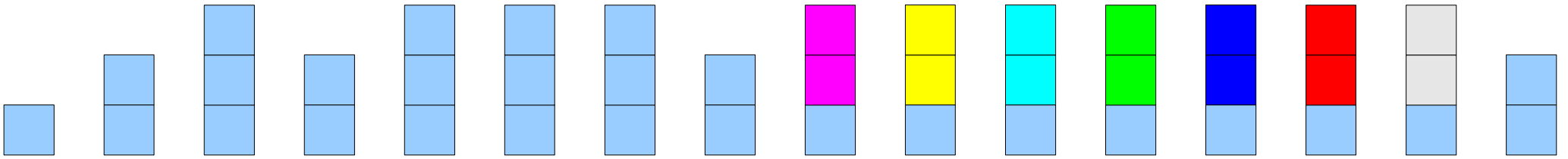
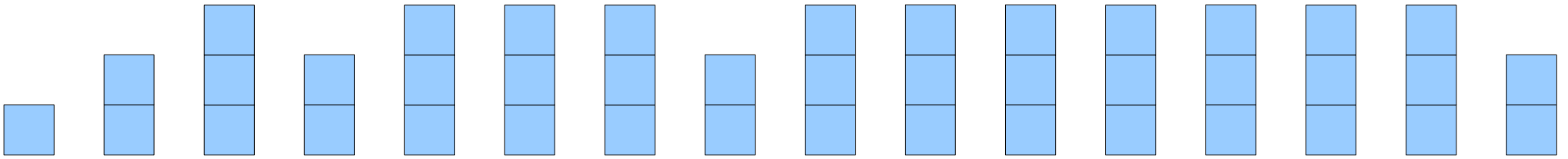


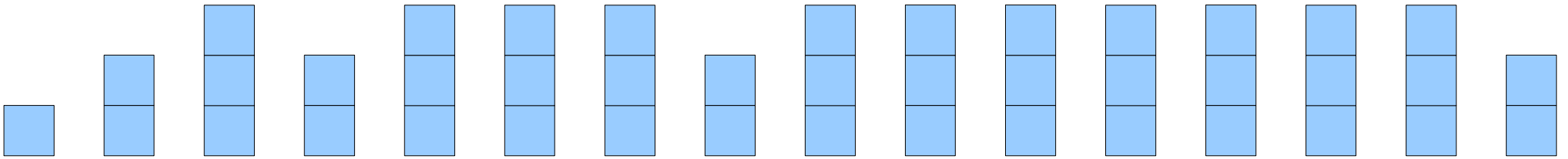
Spreading the Work



Spreading the Work

On average, we do just 3 units of work!

This is O(1) work on average!



Sharing the Burden

- We still have “heavy” pushes taking time O(n) and “light” pushes taking time O(1).
- Worst-case time for a push is O(n).
- Heavy pushes become so rare that the ***average*** time for a push is O(1).
- Can we confirm this?

Amortized Analysis

- The analysis we have just done is called an ***amortized analysis***.
- We reason about the total work done, not the word done per operation.
- In an amortized sense, our implementation of the stack is extremely fast!
- This is one of the most common approaches to implementing `Stack`.

Your Action Items

- ***Download BlueBook***
  - Hopefully you've already done this; if not, please do that soon.
- ***Finish Assignment 4***
  - Need help? Stop by the LaIR!

Next Time

- ***Linked Lists***
  - A different way to represent sequences of elements.
- ***Dynamic Allocation Revisited***
  - What else can we allocate?