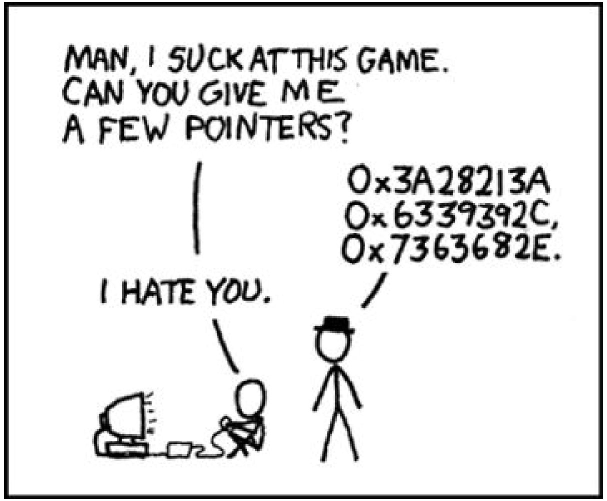
YEAH - Priority Queue

Anton Apostolatos



Source: XKCD

Queue: order items by when they were placed - first in, first out (*FIFO*)

Functions

```
void enqueue(string s)   // Inserts an element into the queue
string dequeue()         // Returns and removes the first element placed
string peek()            // Returns the first element placed
int size()               // Returns the number of elements
bool isEmpty()           // Returns whether the queue is empty
```

PriorityQueue: order items by **rank**

Functions

```
void enqueue(string s)   // Inserts an element into the priority queue
string dequeueMin()      // Returns and removes the highest-ranked item
string peek()            // Returns the highest-ranked item
int size()               // Returns the number of elements
bool isEmpty()           // Returns whether the queue is empty
```

Order is lexicographic/alphabetic!

`“Albus” < “Ginny” < “Harry” < “Hermione” < “Ronald” < “Tom Marvolo”`

```
PQueue pq;

pq.enqueue(“There”);
pq.enqueue(“And”);
pq.enqueue(“Back”);
pq.enqueue(“Again”);

cout << pq.dequeue << endl;
cout << pq.dequeue << endl;
```

Console

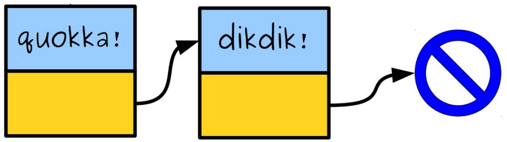
```
Again
And
```

A5: PQueue

Unsorted Vector

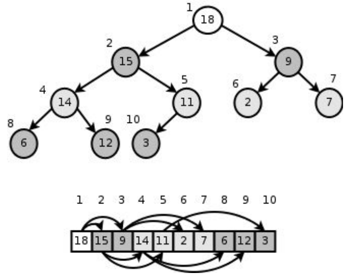
"Little"	"Teresa"	"Kevin"	"Paula"

Sorted Singly-Linked List



Unsorted Doubly-Linked List

Binary Heap



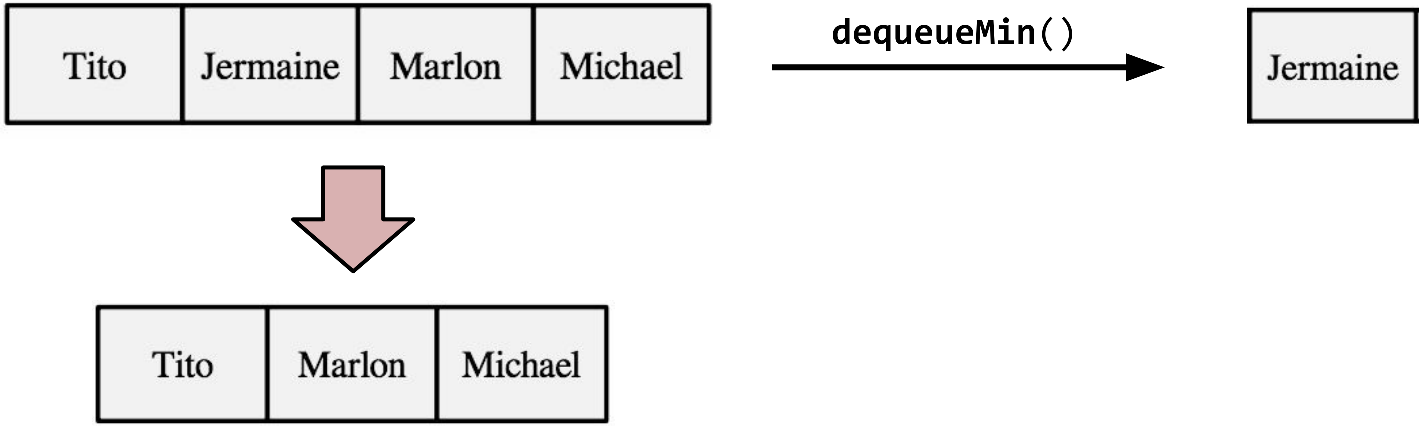
Unsorted Vector

"Little"	"Teresa"	"Kevin"	"Paula"

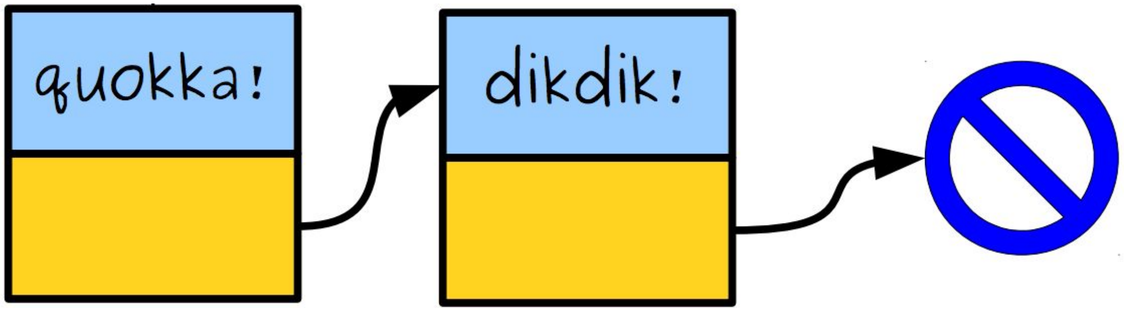
Unsorted and `Vector` wrapper - Simplest to implement and think about!

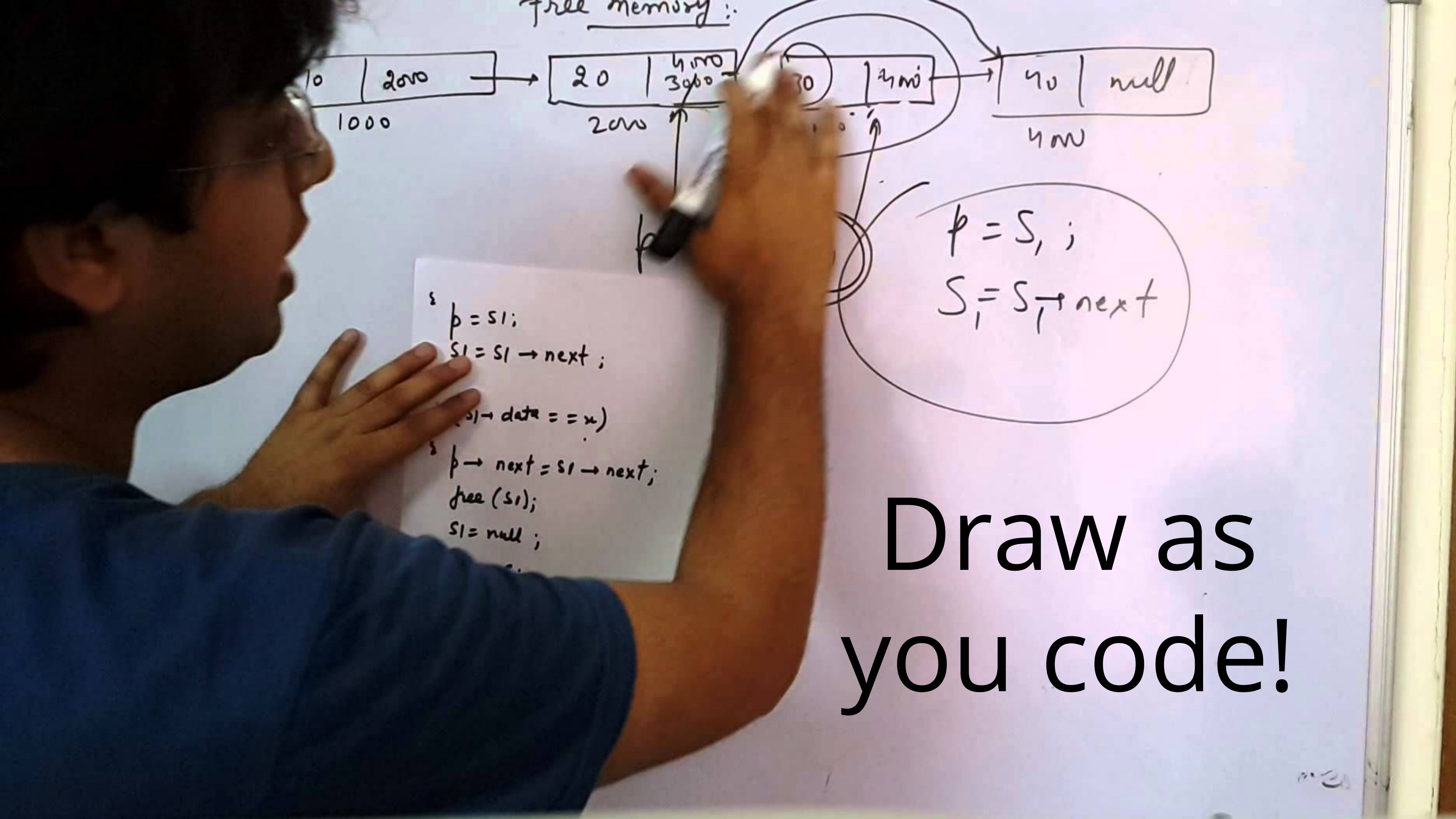
`->` **Enqueue**: append to a vector!

`->` **`Dequeue/peek`**: scan the vector and find the smallest element



Sorted Singly-Linked List





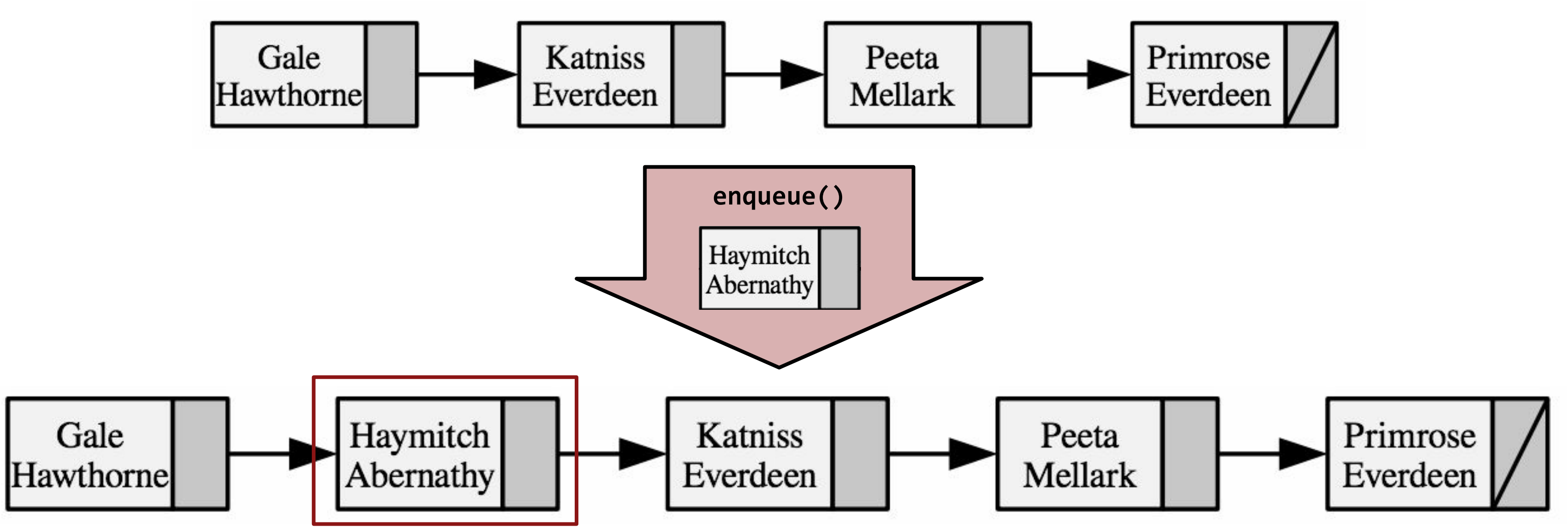
```
{
p = s1;
s1 = s1 → next;

(s1→ data = = x)
{
p→ next = s1 → next;
free (s1);
s1 = null;
```

Draw as you code!

You need to create a Linked List and enforce that all elements are stored in lexicographic order

-> **Enqueue**: look for its place in the list and place it there

-> **Dequeue/peek**: first element!

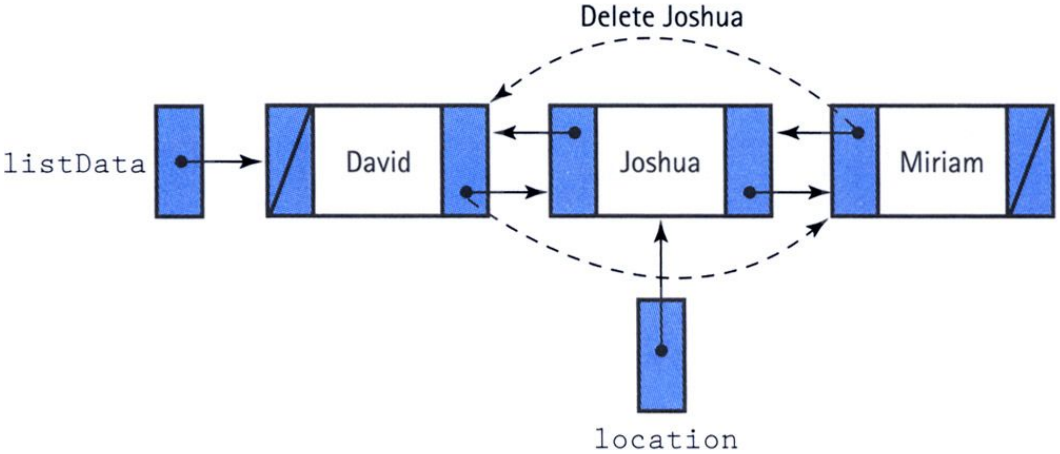


Unsorted Doubly-Linked List

Unsorted, but every cell now has a **`next`** and **`prev`** pointer

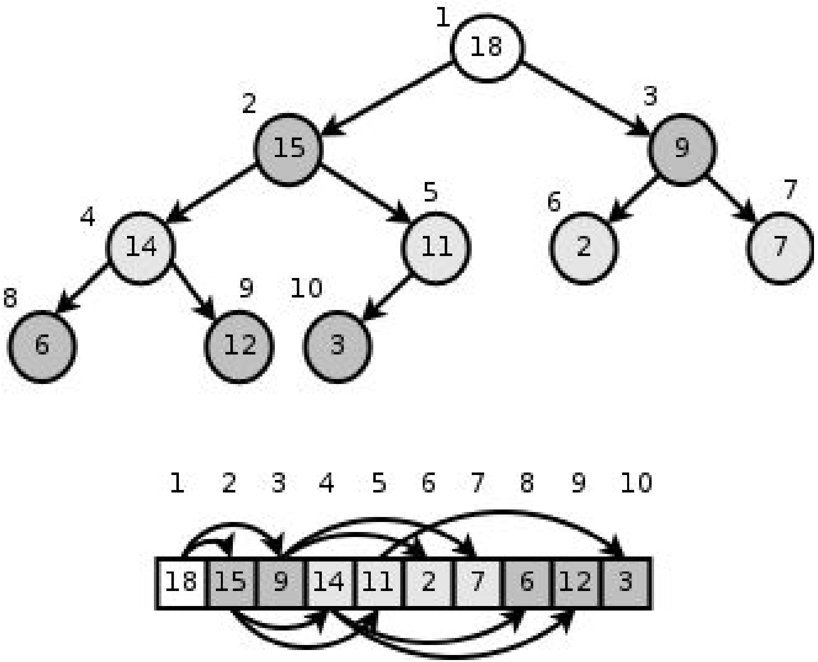
New functionality: You can splice (remove) a cell without needing to keep a second pointer!

- -> **Enqueue**: prepend new item to the list
- -> **Dequeue/peek**: loop through list to find smallest element



Binary Heap

Slides by Chris Gregg!



1	2	3	4	5	6	7	8	9	10
18	15	9	14	11	2	7	6	12	3

Binary Heaps

A heap is a *tree-based* structure that satisfies the heap property:

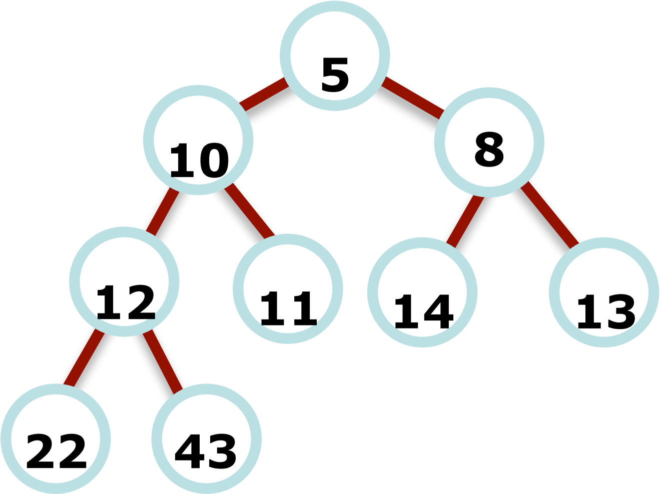
Parents have a higher priority than any of their children.

Binary Heaps

- There are two types of heaps:

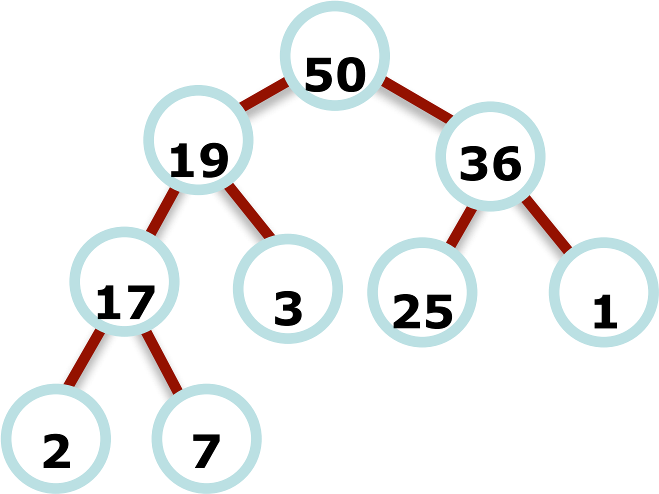
Min Heap

(root is the smallest element)



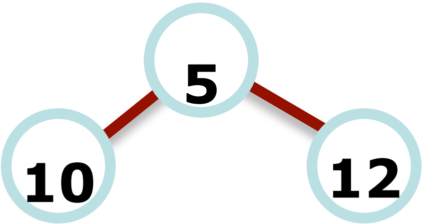
Max Heap

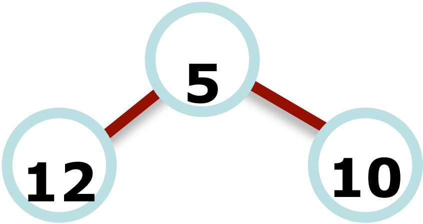
(root is the largest element)



Binary Heaps

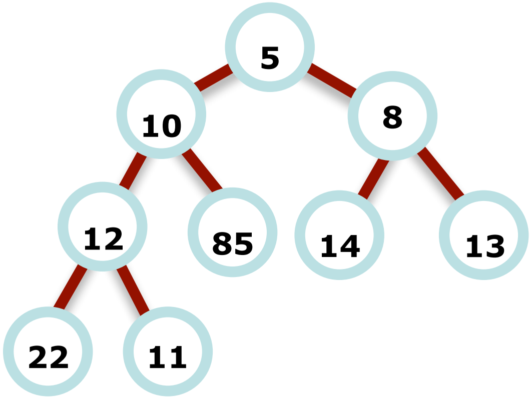
- There are no implied orderings between siblings, so both of the trees below are min-heaps:

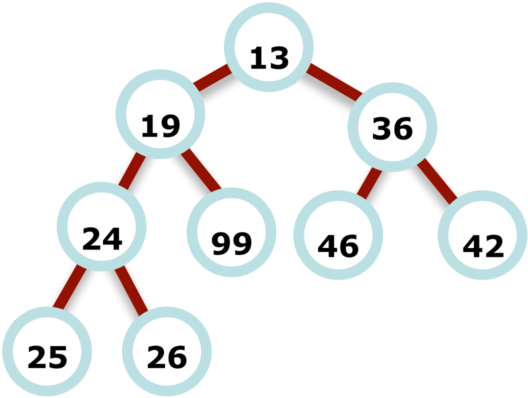




Binary Heaps

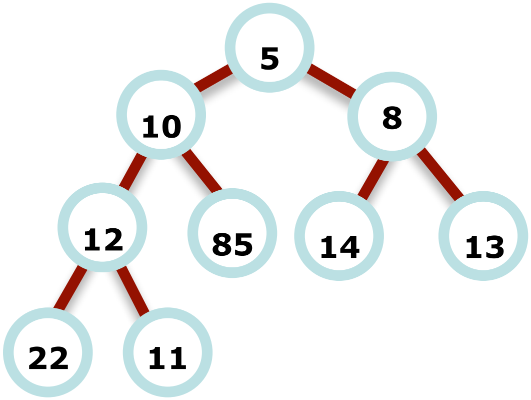
- Circle the min-heap(s):

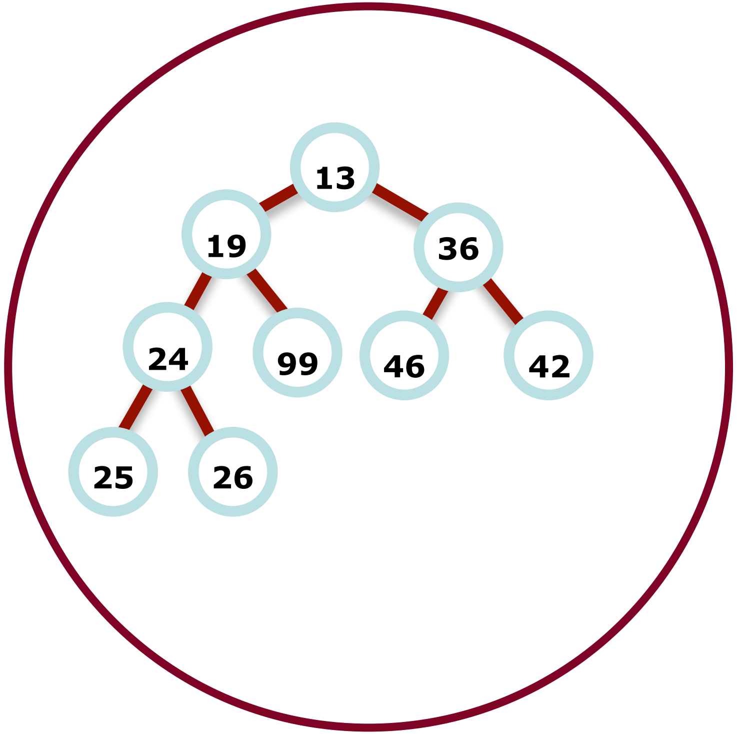




Binary Heaps

- Circle the min-heap(s):





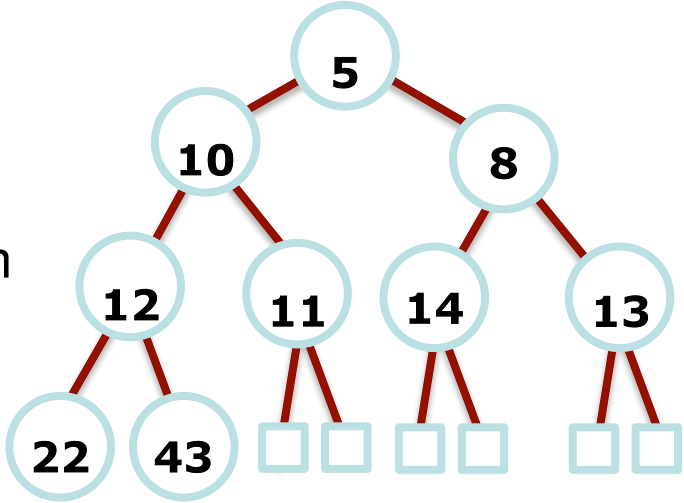
Binary Heaps

Heaps are completely filled, with the exception of the bottom level. They are, therefore, "**complete binary trees**":

complete: all levels filled except the bottom

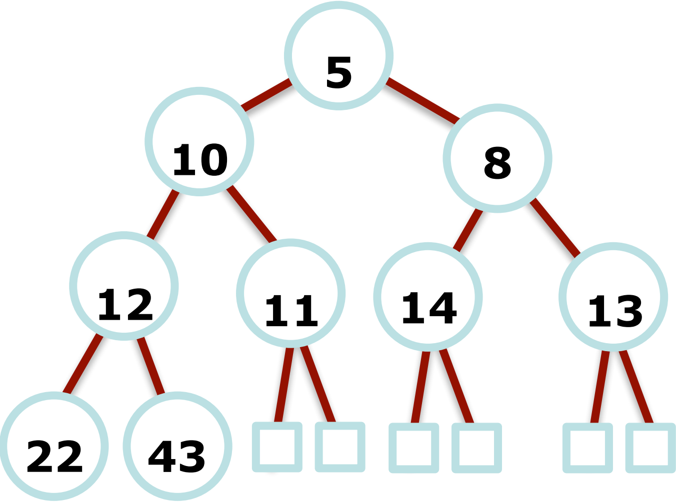
binary: two children per node (parent)

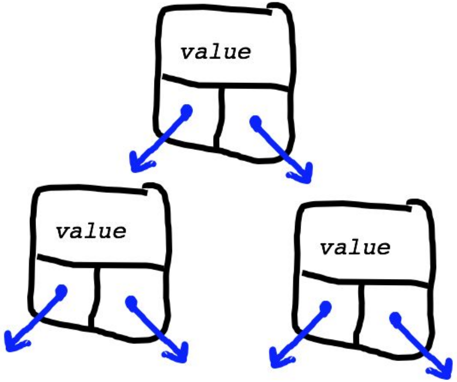
height? log(n)



Binary Heaps

What is the best way to store a heap?



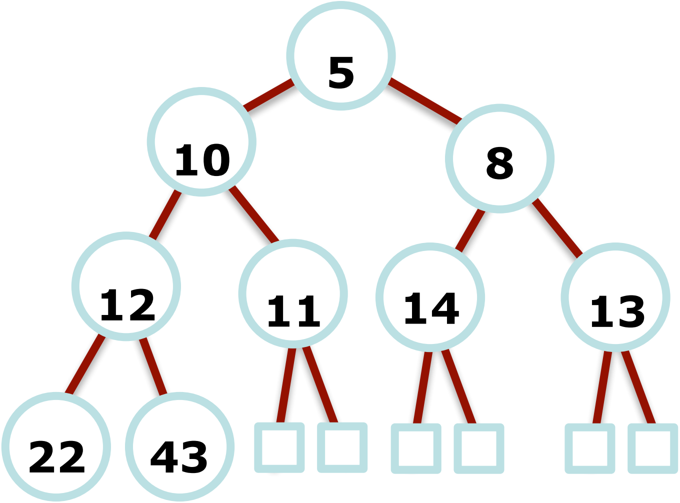


We could use a node-based solution, but…

Binary Heaps

It turns out that an array works **great** for storing a binary heap!

We will put the root at index 1 instead of index 0 (this makes the math work out just a bit nicer).

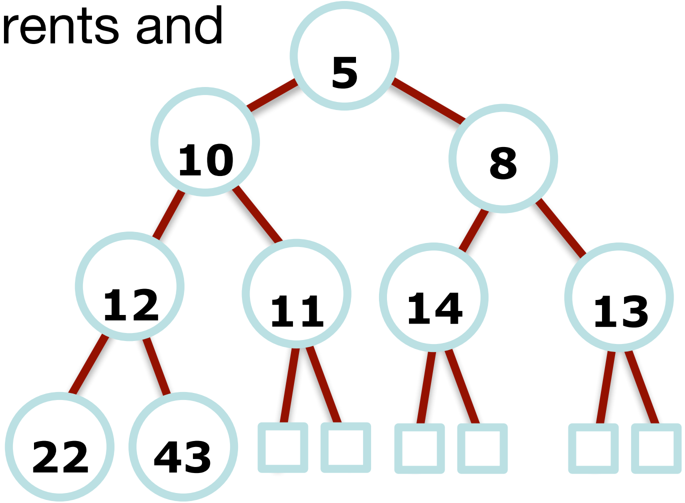


	5	10	8	12	11	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Binary Heaps

The array representation makes determining parents and children a matter of simple arithmetic:

For an element at position ***i***:
- left child is at $2i$
- right child is at $2i+1$
- parent is at $\lfloor i/2 \rfloor$



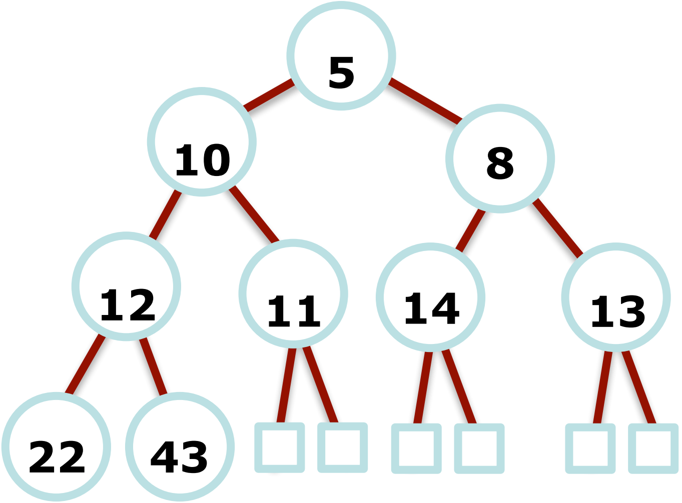
	5	10	8	12	11	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations

Remember that there are three important priority queue operations:

- **`peek()`**: return an element of h with the smallest key.
- **`enqueue(e)`**: insert element e into the heap.
- **`dequeueMin()`**: removes the smallest element from h.

We can accomplish this with a heap!

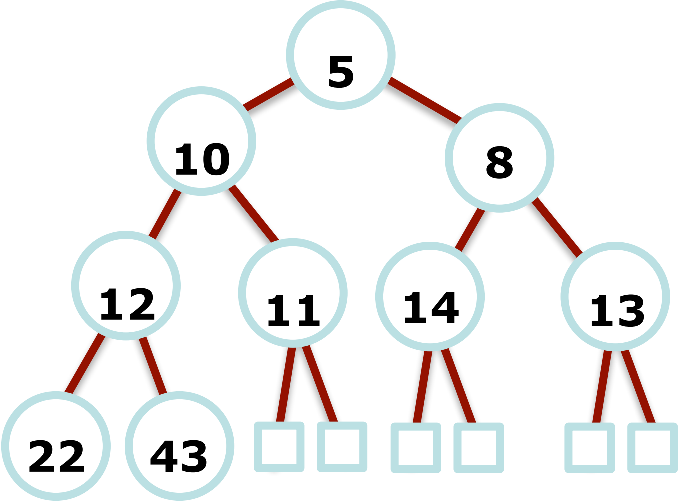


Heap Operations: peek()

`peek()`

Just return the root!

`return heap[1]`

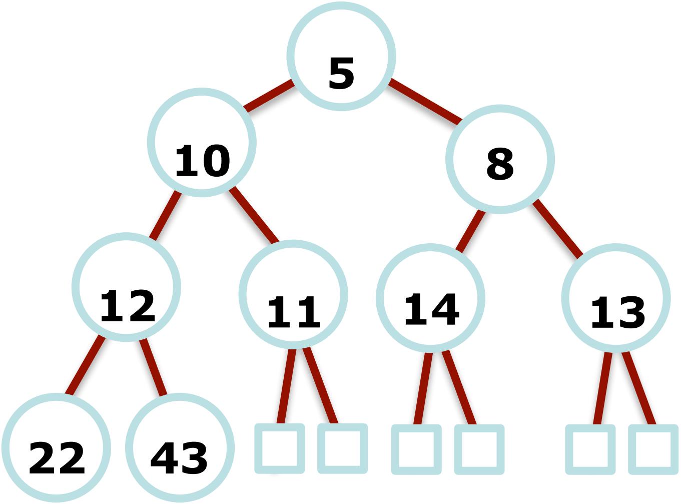


	5	10	8	12	11	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: enqueue(k)

`enqueue(k)`

How might we go about inserting into a binary heap?



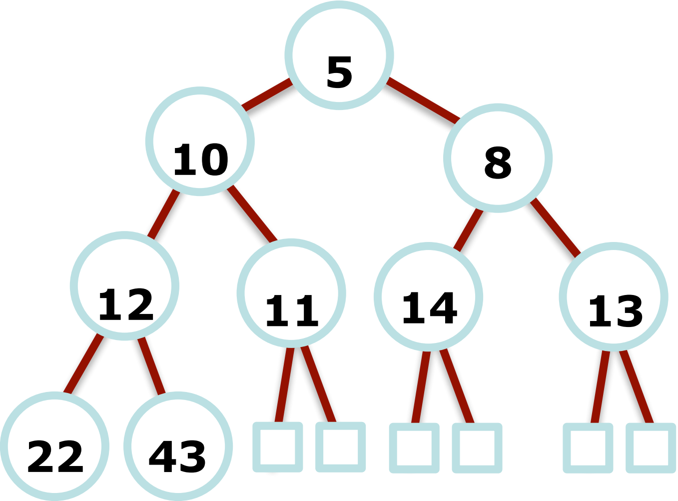
	5	10	8	12	11	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: enqueue(k)

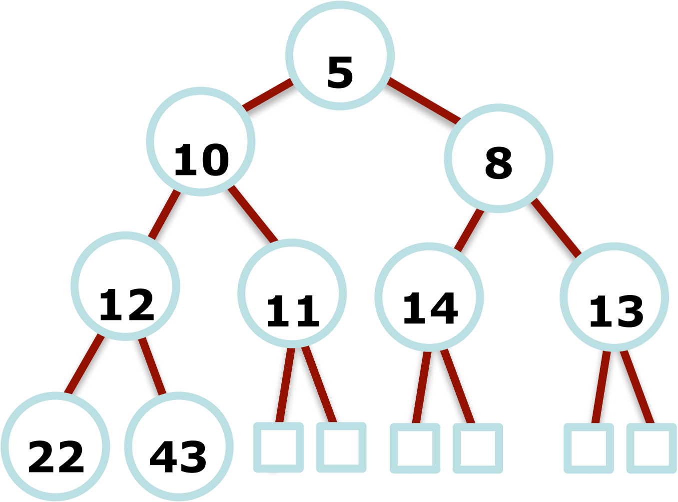
Heap Operations: **`enqueue(k)`**

Insert item at element **`array[heap.size()+1]`**
(this probably destroys the heap property)

Perform a "**bubble up**" operation:
- Compare the added element with its parent
  - if in correct order, stop
  - If not, swap and repeat



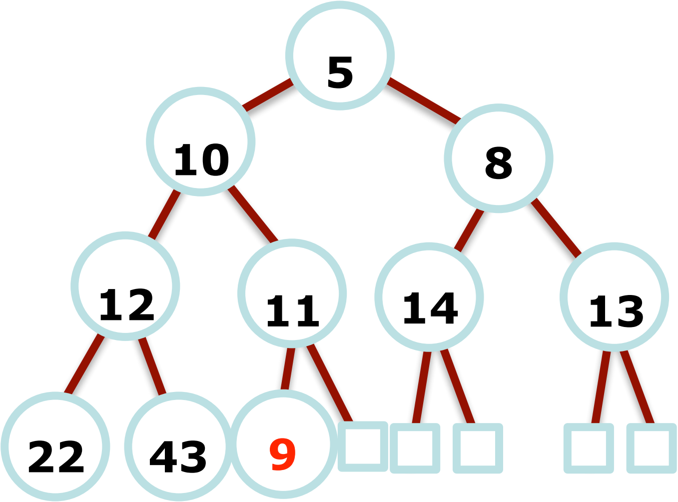
Heap Operations: enqueue(9)



	5	10	8	12	11	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Start by inserting the key at the first empty position. This is always at index `heap.size()+1`.

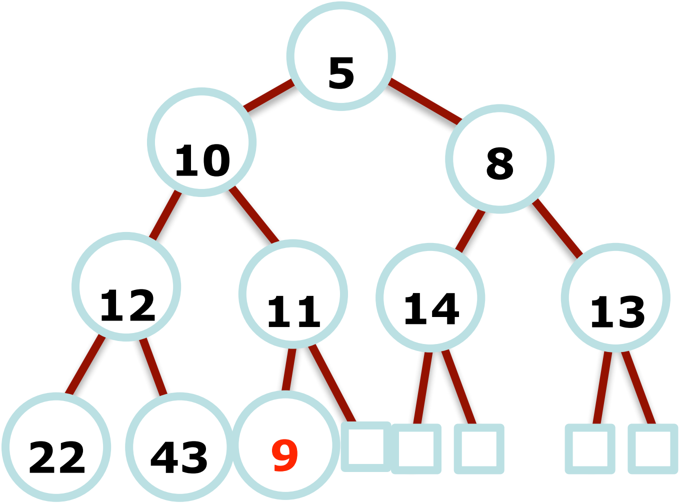
Heap Operations: enqueue(9)



	5	10	8	12	11	14	13	22	43	9	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Start by inserting the key at the first empty position. This is always at index `heap.size()+1`.

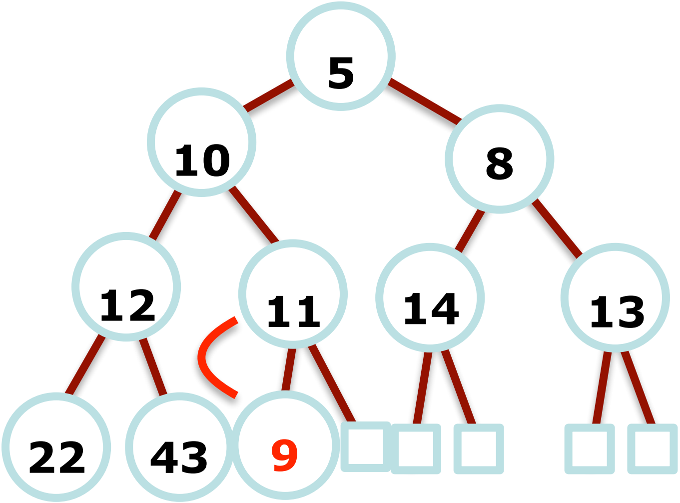
Heap Operations: enqueue(9)



	5	10	8	12	11	14	13	22	43	9	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Look at parent of index 10, and compare: do we meet the heap property requirement?

Heap Operations: enqueue(9)

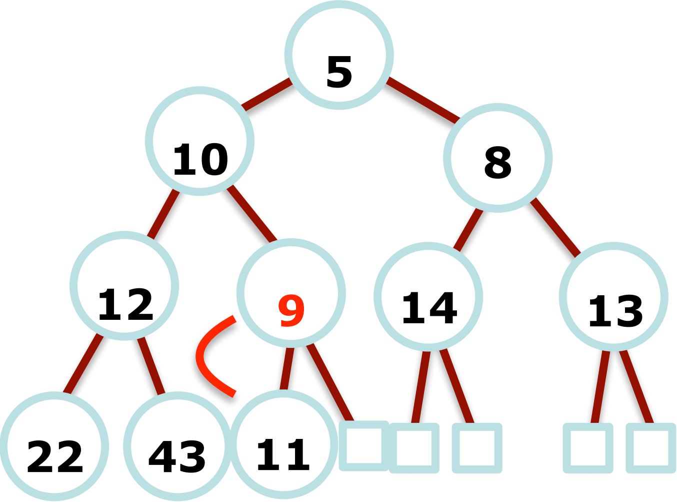


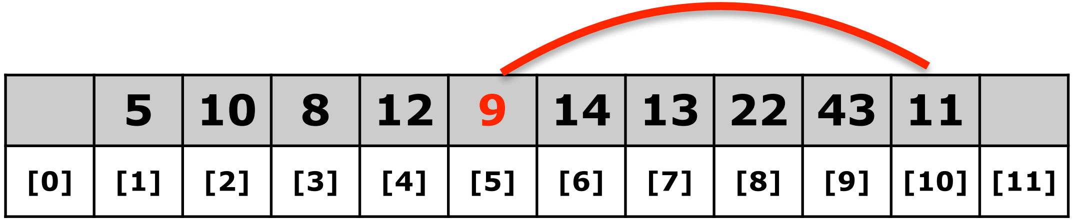
	5	10	8	12	11	14	13	22	43	9	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Look at parent of index 10, and compare:
do we meet the heap property
requirement?

No -- we must swap.

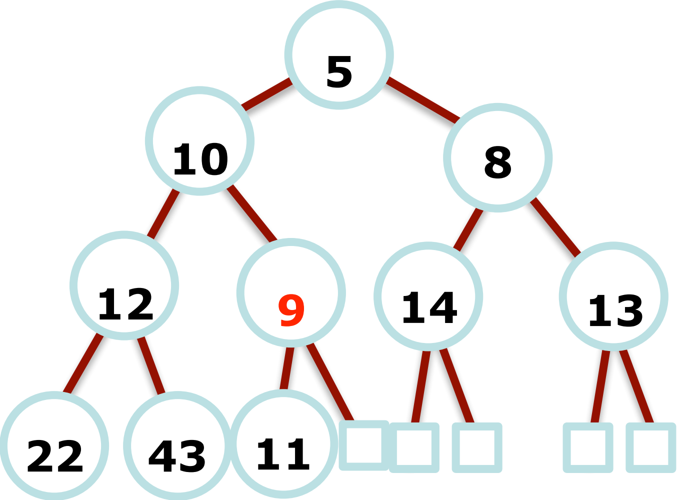
Heap Operations: enqueue(9)





	5	10	8	12	9	14	13	22	43	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: enqueue(9)

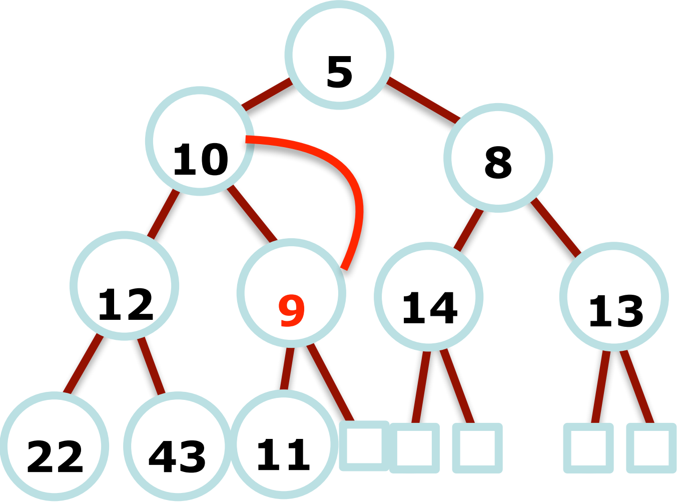


	5	10	8	12	9	14	13	22	43	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Look at parent of index 5, and compare: do we meet the heap property requirement?

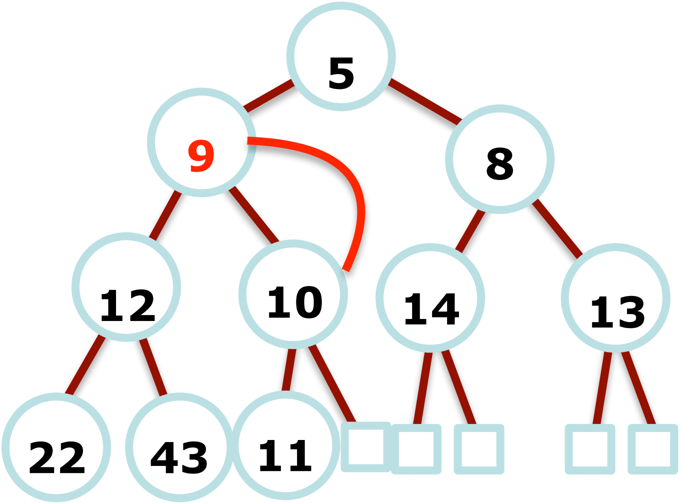
No -- we must swap.

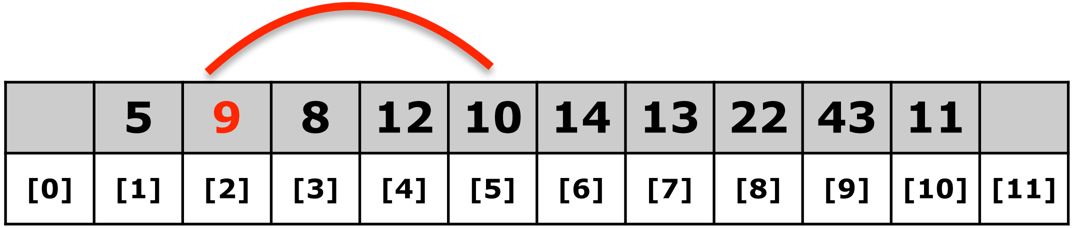
Heap Operations: enqueue(9)



	5	10	8	12	9	14	13	22	43	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: enqueue(9)

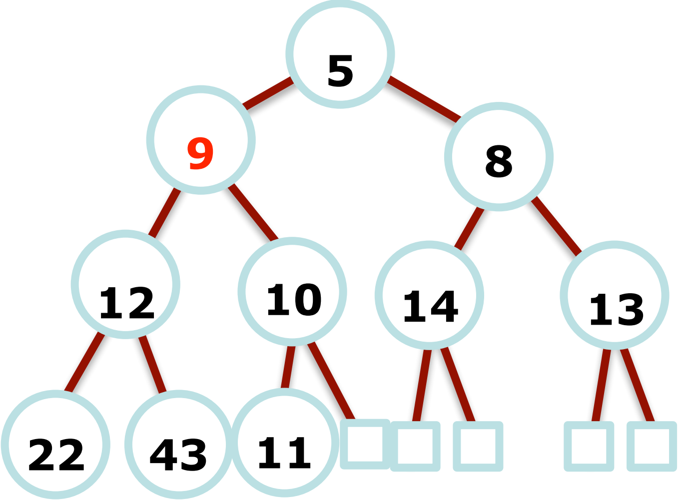




	5	9	8	12	10	14	13	22	43	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: enqueue(9)

No swap necessary between index 2 and its parent.
We're done bubbling up!



	5	9	8	12	10	14	13	22	43	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Demo!

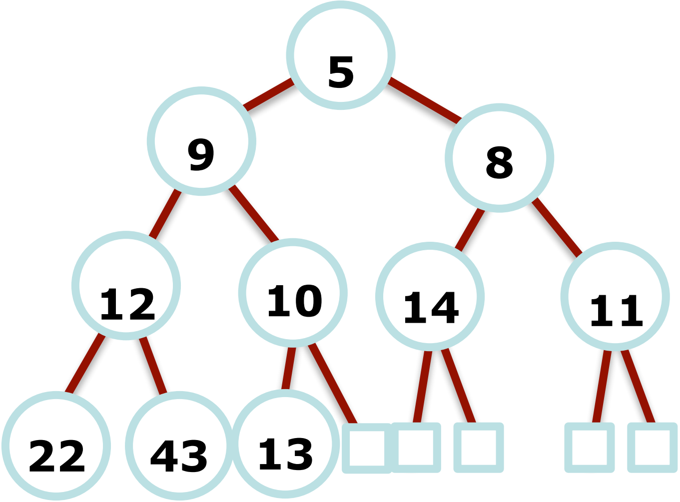
http://www.cs.usfca.edu/~galles/visualization/Heap.html

Heap Operations: dequeue()

- How might we go about removing the minimum?

```
dequeue()
```

	5	9	8	12	10	14	11	22	43	13	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

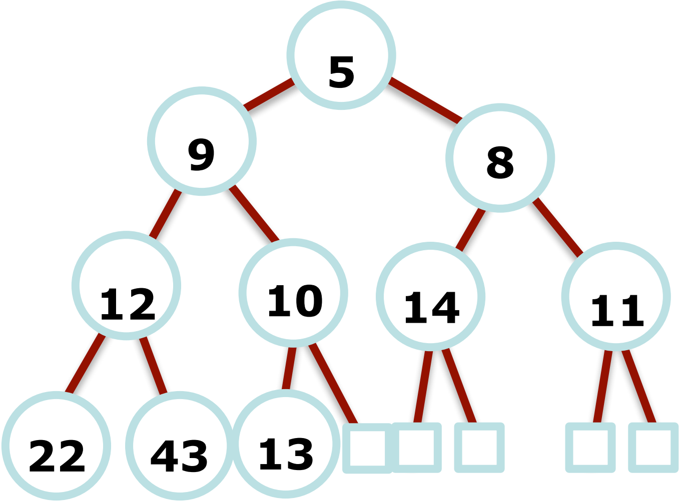


Heap Operations: dequeue()

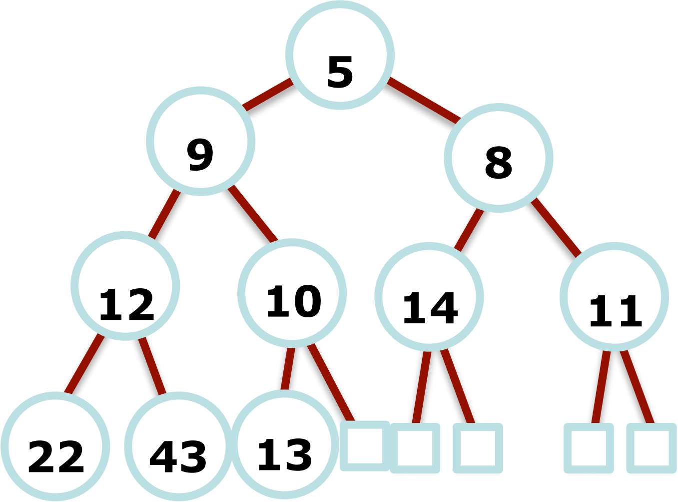
We are removing the root, and we need to retain a complete tree: replace root with last element.

"**bubble-down**" or "down-heap" the new root:

- Compare the root with its children:
  - if in correct order, stop.
  - if not, swap with smallest child, and repeat



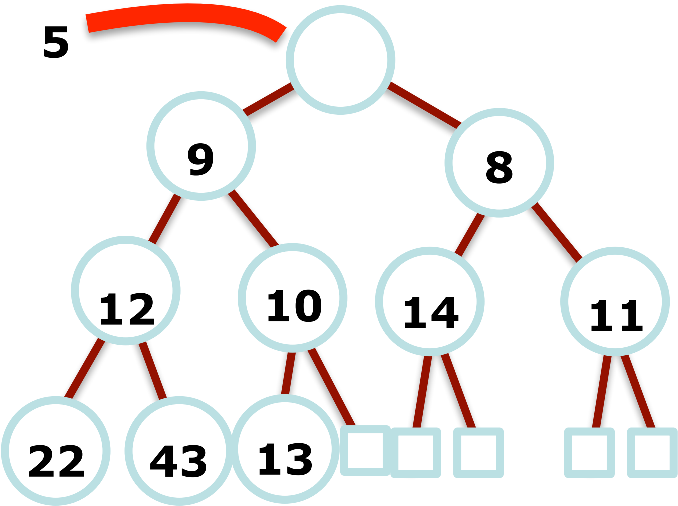
Heap Operations: dequeue()



	5	9	8	12	10	14	11	22	43	13	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: dequeue()

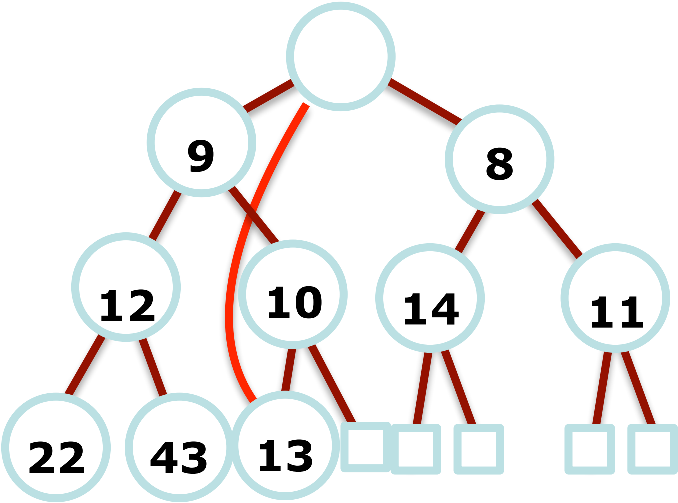
Remove root (will return at the end)

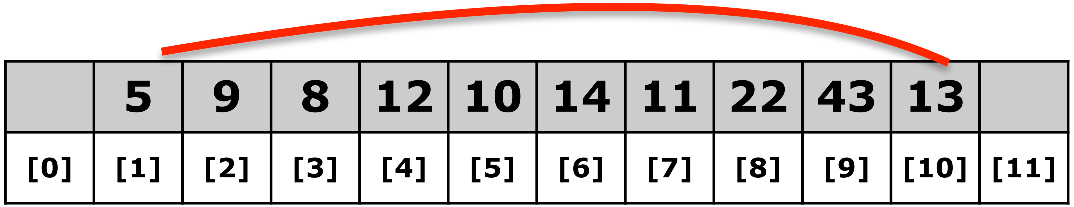


	5	9	8	12	10	14	11	22	43	13	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: dequeue()

Move last element (at `heap[heap.size()]`) to the root (this may be unintuitive!) to begin bubble-down

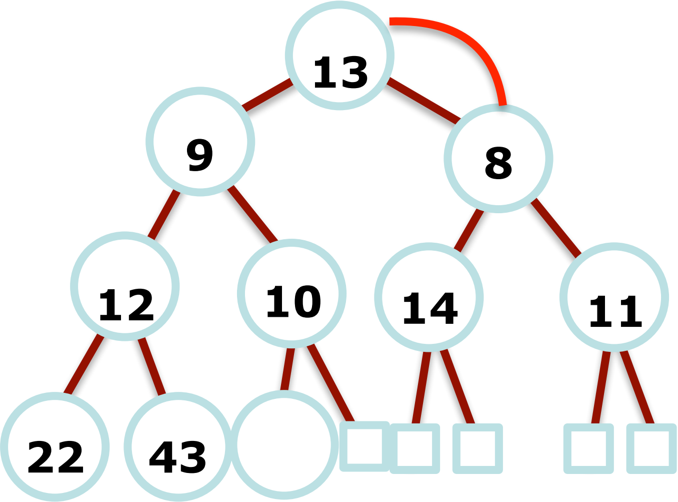




	5	9	8	12	10	14	11	22	43	13	
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: dequeue()

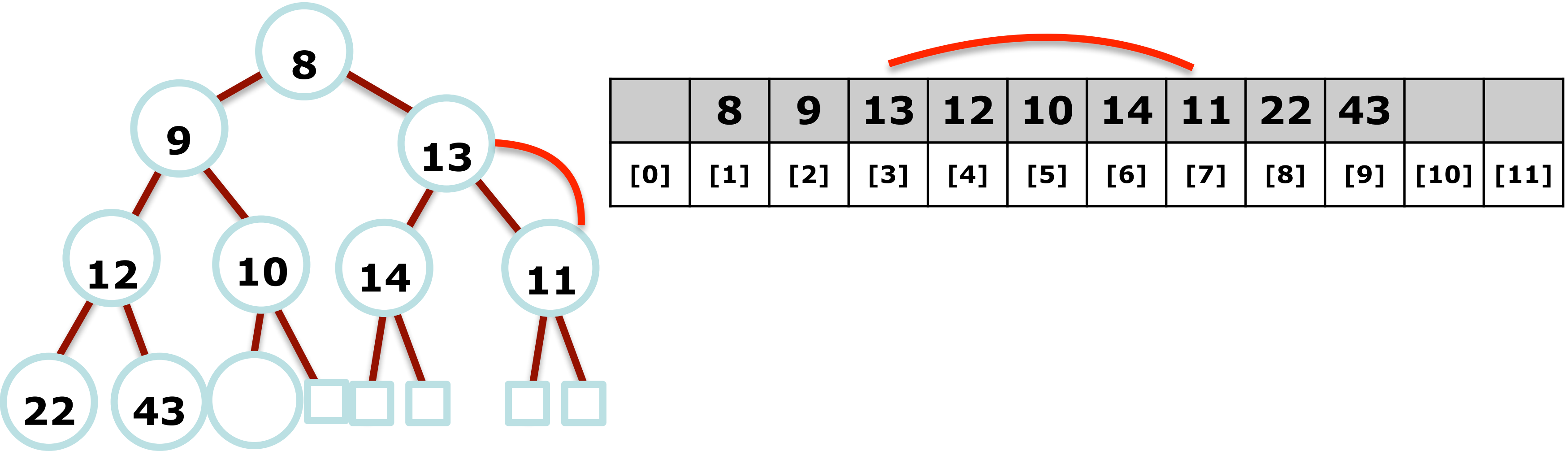
Compare children of root with root: swap root with the smaller one (why?)



	13	9	8	12	10	14	11	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: dequeue()

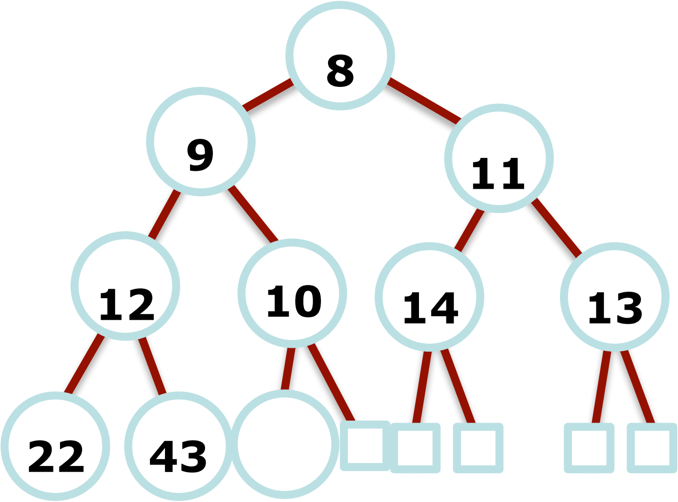
Keep swapping new element if necessary. In this case: compare 13 to 11 and 14, and swap with smallest (11).



	8	9	13	12	10	14	11	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Heap Operations: dequeue()

13 has now bubbled down until it has no more children, so we are done!



	8	9	11	12	10	14	13	22	43		
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

Questions?

Tips and Tricks

- Height of a binary heap is O(log(n))
- Before writing any code, go through simple toy examples by hand to make sure your proposed solution’s logic is sound
- Don’t forget the semicolon after a struct or class definition!
- Bad idea to declare multiple pointers on the same line:

```
Node * head, tail;
```

Tips and Tricks: Continued

- Nested structs are weird. If we create a cell inside of PQueue then a helper function that returns a Cell* would be *declared* as:

```
Cell* helperFunction(Cell* ptr);
```

- And would be *implemented* as

```
PQueue::Cell* PQueue::helperFunction(Cell* ptr);
```

- Do your best to make your size functions not O(n)! → how?

- We'll ask you for Big-O of every function you write!

General questions?