#### Shortest Paths Part One

#### Recap from Last Time





A graph consists of a set of *nodes* connected by *edges*.



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### BFS Pseudocode

breadth-first-search() {
 make a queue of nodes.
 enqueue start node.
 color the start node yellow.

while (the queue is not empty) {
 dequeue a node from the queue.
 color that node green.

for (each neighboring node) {
 if (that node is gray) {
 color the node yellow.
 enqueue it.
 }
}

#### The Limits of Breadth-First Search



### The Model

- We have a graph in which each edge has a nonnegative *cost* or *weight* associated with it.
- We want to find the lowest-cost path from point *A* to point *B*.
- BFS does not take edge weights into account.
- How might we go about solving this problem?



### Option 1: Brute-Force!

- We could conceivably solve this problem using brute force and a backtracking recursion.
- **Problem:** There can be a *lot* of different paths in a graph!
- This is way too inefficient to use in practice.



- BFS works in the case where each edge has equal weight.
- **Idea:** What if we split each edge of length k into k smaller edges?



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- Idea: What if we split each edge of length k into k smaller edges?
- What if there are fractional edges? Or large weights?



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- **Idea:** What if we split each edge of length k into k smaller edges?
- What if there are fractional edges? Or large weights?



#### Option 3: Look at the problem more closely















































































































































## The Pattern



All yellow nodes (nodes we've seen, but don't know the distance to.)














# At a Glance

- The approach suggested here gives rise to *Dijkstra's algorithm*, a fast, powerful, and famous algorithm for computing shortest paths.
- *Key idea:* As in BFS, split nodes into
  - gray nodes we haven't seen,
  - **yellow nodes** that are on the frontier, and
  - green nodes we have the best path to,

then repeatedly turn the lowest-cost yellow node into a green node.

#### Implementing Dijkstra's Algorithm

```
breadth-first-search() {
 make a queue of nodes.
 enqueue start node.
 color the start node yellow.
 while (the queue is not empty) {
   dequeue a node from the queue.
   color that node green.
   for (each neighboring node) {
      if (that node is gray) {
        color the node yellow.
        enqueue it.
     }
    }
```



```
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  enqueue start node.
                                             enqueue start node.
  color the start node yellow.
                                             color the start node yellow.
  while (the gueue is not empty) {
                                             while (the queue is not empty) {
    dequeue a node from the queue.
                                                dequeue a node from the queue.
                                                color that node green.
    color that node green.
    for (each neighboring node) {
                                                for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
        enqueue it.
                                                    enqueue it.
      }
                                                  }
    }
                                                }
                                             }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



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      if (that node is gray) {
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        color the node yellow.
                                                    color the node yellow.
        enqueue it.
                                                    enqueue it.
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                                                  }
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```



```
tra's algorithm()
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  make a queue of nodes.
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                                             enqueue start node.
  color the start node yellow.
                                              color the start node yellow.
  while (the queue is not empty) {
                                             while (the queue is not empty) {
    dequeue a node from the queue.
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                                                color that node green.
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                                                     <del>(cach neighboring node</del>)
      if (that node is gray) {
                                                   if (that node is gray) {
        color the node yellow.
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        enqueue it.
                                                     enqueue it.
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                                                    \triangle Still under construction! \triangle
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                                                for (each neighboring node) {
                                                  if (that node is gray) {
      if (that node is gray) {
                                                    color the node yellow.
        color the node yellow.
        enqueue it.
                                                    enqueue ii.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
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                                               color that node green.
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                                                  if (that node is gray) {
      if (that node is gray) {
                                                    color the node yellow.
        color the node yellow.
        enqueue it.
                                                   enqueue ii.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



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                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
                                                    color the node vellow
        color the node yellow.
        enqueue it.
                                                    enqueue it.
      }
    }
                                                   \triangle Still under construction! \triangle
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```



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   color that node green.
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    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                 if (that node is gray) {
                                                   color the node vellow
        color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```









```
breadth-first-search() {
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               1?
```





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               1?
```





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        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
  make a queue of nodes.
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  enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
  while (the queue is not empty) {
                                                    (the queue is not empty)
    dequeue a node from the queue.
                                                dequeue a node from the queue.
                                                color that node green.
    color that node green.
    for (each neighboring node) {
                                                for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
  make a queue of nodes.
                                             make a priority queue of nodes.
  enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
  while (the queue is not empty) {
                                                    (the queue is not empty)
                                               dequeue a node from the queue.
    dequeue a node from the queue.
                                               color that node green.
    color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
   color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
   color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                            dijkstra's-algorithm() {
  make a queue of nodes.
                                              make a priority queue of nodes.
  enqueue start node.
                                              enqueue start node at distance 0.
                                              color the start node yellow.
  color the start node yellow.
  while (the queue is not empty) {
                                              while (the queue is not empty) {
    dequeue a node from the queue.
                                                dequeue a node from the queue.
    color that node green.
                                                color that node green.
                                                for (each neighboring node)
    for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is gray) {
        color the node yellow.
                                                     <del>color the node yellow.</del>
                                                     enqueue it at the new distance.
        enqueue it.
      }
    }
                                                    \triangle Still under construction! \triangle
                                            }
                                                    Don't use this as a reference!
```





```
breadth-first-search() {
                                            dijkstra's-algorithm() {
  make a queue of nodes.
                                              make a priority queue of nodes.
  enqueue start node.
                                              enqueue start node at distance 0.
                                              color the start node yellow.
  color the start node yellow.
  while (the queue is not empty) {
                                              while (the queue is not empty) {
    dequeue a node from the queue.
                                                dequeue a node from the queue.
    color that node green.
                                                color that node green.
                                                for (each neighboring node)
    for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                     <del>color the node yellow.</del>
                                                     enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                    \triangle Still under construction! \triangle
                                            }
                                                    Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
 while (the queue is not empty) {
                                             while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
   color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
                                                  if (that node is not green)
      if (that node is gray) {
        color the node yellow.
                                                    color the node yellow.
        enqueue it.
                                                    enqueue it at the new distance.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
                                             enqueue start node at distance 0.
 enqueue start node.
 color the start node yellow.
                                             color the start node yellow.
 while (the queue is not empty) {
                                             while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
   color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
                                                    color the node vellow
        color the node yellow.
        enqueue it.
                                                    enqueue it at the new distance.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
                                             enqueue start node at distance 0.
 enqueue start node.
 color the start node yellow.
                                             color the start node yellow.
 while (the queue is not empty) {
                                             while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
   color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
                                                    color the node vellow
        color the node yellow.
        enqueue it.
                                                    enqueue it at the new distance.
      }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                 if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty)
 while (the queue is not empty) {
                                               dequeue a node from the queue.
    dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
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                                             color the start node yellow.
                                             while (the queue is not empty)
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
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 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                            dijkstra's-algorithm() {
  make a queue of nodes.
                                              make a priority queue of nodes.
  enqueue start node.
                                              enqueue start node at distance 0.
  color the start node yellow.
                                              color the start node yellow.
                                              while (the queue is not empty) {
  while (the queue is not empty) {
    dequeue a node from the queue.
                                                dequeue a node from the queue.
    color that node green.
                                                color that node green.
    for (each neighboring node) {
                                                for (each neighboring node) {
      if (that node is gray) {
                                                   <del>if (that node is not green) {</del>
        color the node yellow.
                                                     color the node yellow.
                                                     enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                    \triangle Still under construction! \triangle
                                            }
                                                    Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                 if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty)
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```





```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty)
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               color that node green.
   color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                 }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
  make a queue of nodes.
                                             make a priority queue of nodes.
  enqueue start node.
                                             enqueue start node at distance 0.
  color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
  while (the queue is not empty) {
    dequeue a node from the queue.
                                                dequeue a node from the
    color that node green.
                                               color that node green.
    for (each neighboring node) {
                                               for (each neighboring node) {
      if (that node is gray) {
                                                  if (that node is not green) {
        color the node yellow.
                                                    color the node yellow.
                                                    enqueue it at the new distance.
        enqueue it.
      }
                                                  }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
                                               dequeue a node from the queue.
    dequeue a node from the queue.
                                               if (that node isn't green) {
   color that node green.
                                                 color that node green.
    for (each neighboring node) {
                                                 for (each neighboring node) {
      if (that node is gray) {
                                                    if (that node is not green) {
        color the node yellow.
                                                      color the node yellow.
        enqueue it.
                                                      enqueue it at the new distance.
      }
                                                    }
    }
                                                   \triangle Still under construction! \triangle
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
                                               dequeue a node from the queue.
    dequeue a node from the queue.
                                               if (that node isn't green) {
                                                 color that node green.
   color that node green.
    for (each neighboring node) {
                                                 for (each neighboring node) {
                                                   if (that node is not green) {
      if (that node is gray) {
        color the node yellow.
                                                      color the node yellow.
        enqueue it.
                                                      enqueue it at the new distance.
      }
                                                    }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
breadth-first-search() {
                                           dijkstra's-algorithm() {
 make a queue of nodes.
                                             make a priority queue of nodes.
 enqueue start node.
                                             enqueue start node at distance 0.
 color the start node yellow.
                                             color the start node yellow.
                                             while (the queue is not empty) {
 while (the queue is not empty) {
    dequeue a node from the queue.
                                               dequeue a node from the queue.
                                               if (that node isn't green) {
                                                 color that node green.
   color that node green.
    for (each neighboring node) {
                                                 for (each neighboring node) {
                                                   if (that node is not green) {
      if (that node is gray) {
        color the node yellow.
                                                      color the node yellow.
                                                      enqueue it at the new distance.
        enqueue it.
      }
                                                    }
    }
                                                   \triangle Still under construction! \triangle
                                           }
                                                   Don't use this as a reference!
```



```
dijkstra's-algorithm() {
  make a priority queue of nodes.
  enqueue start node at distance 0.
  color the start node yellow.
```

```
while (the queue is not empty) {
   dequeue a node from the queue.
   if (that node isn't green) {
      color that node green.
```

```
for (each neighboring node) {
    if (that node is not green) {
        color the node yellow.
        enqueue it at the new distance.
    }
    }
}
```



```
dijkstra's-algorithm() {
 make a priority queue of nodes.
  enqueue start node at distance 0.
  color the start node yellow.
 while (the queue is not empty) {
    dequeue a node from the queue.
    if (that node isn't green) {
      color that node green.
      for (each neighboring node) {
        if (that node is not green) {
          color the node yellow.
                                              Allow nodes to be
          enqueue it at the new distance.
                                              enqueued multiple
        }
                                             times. The first time
                                               we find the node
                                               might not be the
                                                 best option.
```

```
dijkstra's-algorithm() {
 make a priority queue of nodes.
  enqueue start node at distance 0.
  color the start node yellow.
 while (the queue is not empty) {
    dequeue a node from the queue -
    if (that node isn't green) {
      color that node green.
      for (each neighboring node) {
        if (that node is not green) {
                                                As a consequence,
          color the node yellow.
                                                 when dequeuing
          enqueue it at the new distance.
                                                nodes, make sure
        }
                                                we're not visiting
                                                 something we've
                                                already processed.
```