## Shortest Paths Part One

## Recap from Last Time

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## Breadth-First Search

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Yellow nodes are in the queue.

Green nodes have had the best path discovered.

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## BFS Pseudocode

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## The Limits of Breadth-First Search



## The Model

- We have a graph in which each edge has a nonnegative cost or weight associated with it.
- We want to find the lowest-cost path from point $A$ to point $B$.
- BFS does not take edge weights into account.
- How might we go about solving this problem?


## Option 1: Brute-Force!

- We could conceivably solve this problem using brute force and a backtracking recursion.
- Problem: There can be a lot of different paths in a graph!

- This is way too inefficient to use in practice.


## Option 2: Expand the Graph

- BFS works in the case where each edge has equal weight.
- Idea: What if we split each edge of length $k$ into $k$ smaller edges?



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- Idea: What if we
2.71828 smaller edges?
- What if there are fractional edges? Or large weights?


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- BFS works in the case where each edge has equal weight.
- Idea: What if we split each edge of length $k$ into $k$ smaller edges?
- What if there are fractional edges? Or large weights?

Option 3: Look at the problem more closely


























3 ?
















































## The Pattern



All yellow nodes
(nodes we've seen, but don't
know the distance to.)

## The Pattern



## The Pattern



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## At a Glance

- The approach suggested here gives rise to Dijkstra's algorithm, a fast, powerful, and famous algorithm for computing shortest paths.
- Key idea: As in BFS, split nodes into
- gray nodes we haven't seen,
- yellow nodes that are on the frontier, and
- green nodes we have the best path to, then repeatedly turn the lowest-cost yellow node into a green node.


## Implementing Dijkstra’s Algorithm

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    }
    \ Still under construction!^
} Don't use this as a reference!
```



5

3
breadth-first-search() \{
make a queue of nodes.
enqueue start node.
color the start node yellow.
while (the queue is not empty) \{ dequeue a node from the queue.
color that node green.
for (each neighboring node) \{ if (that node is gray) \{ color the node yellow. enqueue it.
\}
\}
\}
\}

```
dijkstra's-algorithm() {
    make a priority queue of nodes.
    enqueue start node at distance 0.
    color the start node yellow.
    while (the queue is not empty) {
        dequeue a node from the queue.
        if (that node isn't green) {
        color that node green.
        for (each neighboring node) {
            if (that node is not green) {
                        color the node yellow.
                        enqueue it at the new distance.
                }
        }
    }
    }
}
\ Still under construction! \
    Don't use this as a reference!
```


## The Finished Product

```
dijkstra's-algorithm() {
    make a priority queue of nodes.
    enqueue start node at distance 0.
    color the start node yellow.
    while (the queue is not empty) {
        dequeue a node from the queue.
        if (that node isn't green) {
            color that node green.
            for (each neighboring node) {
            if (that node is not green) {
                color the node yellow.
                enqueue it at the new distance.
            }
            }
        }
    }
}
```


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    _(em (each neighboring node){
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```




```
    _(em (each neighboring node){
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## The Finished Product

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        if (that node isn't green) {
            color that node green.
            for (each neighboring node) {
            if (that node is not green) {
                color the node yellow.
                enqueue it at the new distance.
            }
        }
        }
    }
}
```

As a consequence,
when dequeuing
nodes, make sure
we're not visiting
something we've
already processed.

