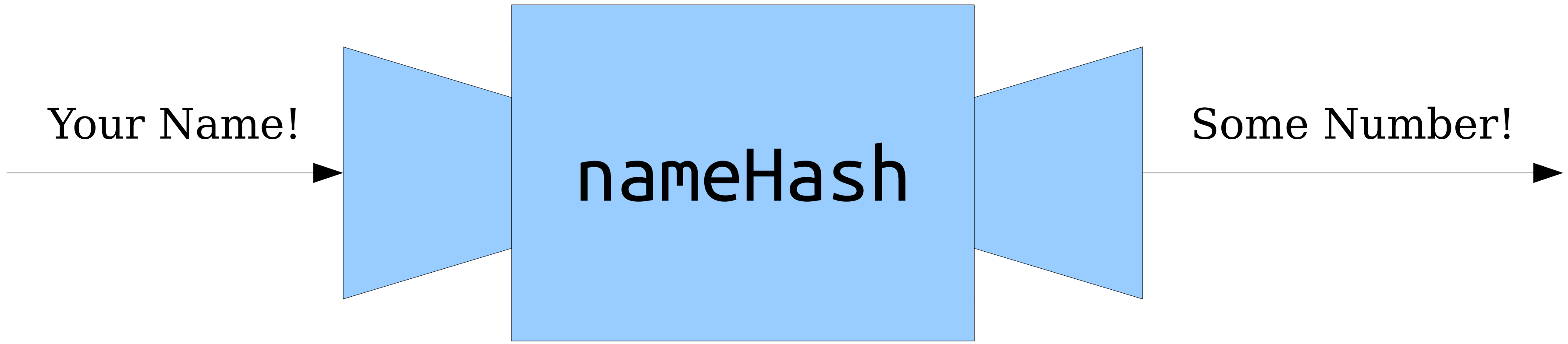
Hashing

Way Back When…

```
int nameHash(string first, string last){
    /* This hashing scheme needs two prime numbers, a large prime and a small
     * prime. These numbers were chosen because their product is less than
     * 2^31 - kLargePrime - 1.
     */
    static const int kLargePrime = 16908799;
    static const int kSmallPrime = 127;

    int hashVal = 0;

    /* Iterate across all the characters in the first name, then the last
     * name, updating the hash at each step.
     */
    for (char ch: first + last) {
        /* Convert the input character to lower case. The numeric values of
         * lower-case letters are always less than 127.
         */
        ch = tolower(ch);
        hashVal = (kSmallPrime * hashVal + ch) % kLargePrime;
    }
    return hashVal;
}
```



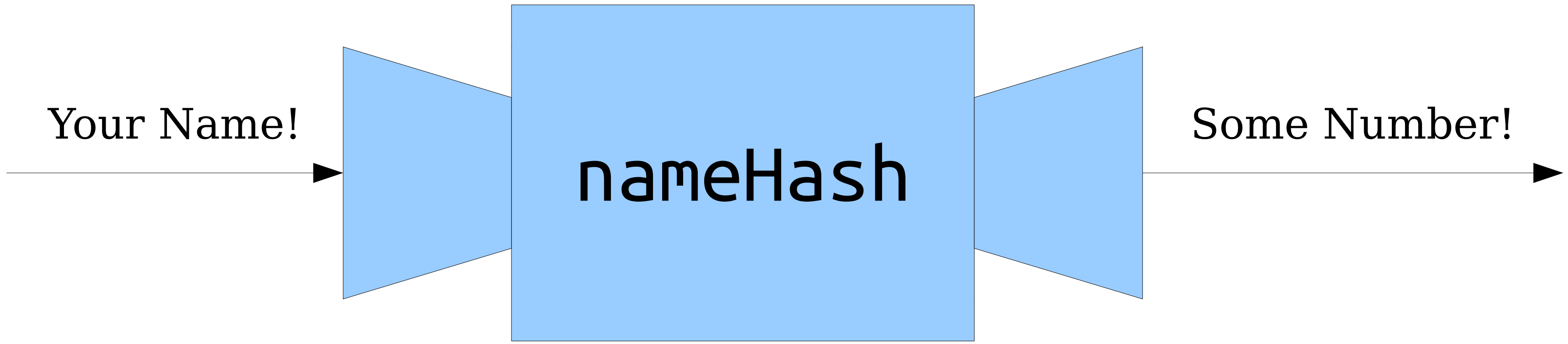
A hash function is a function

```
int hashCode(Type arg);
```

that is

1. ***deterministic*** (the same input always produces the same output) and
2. ***well-distributed*** (The numbers produced are as spread out as possible.)

I’ve got a secret!



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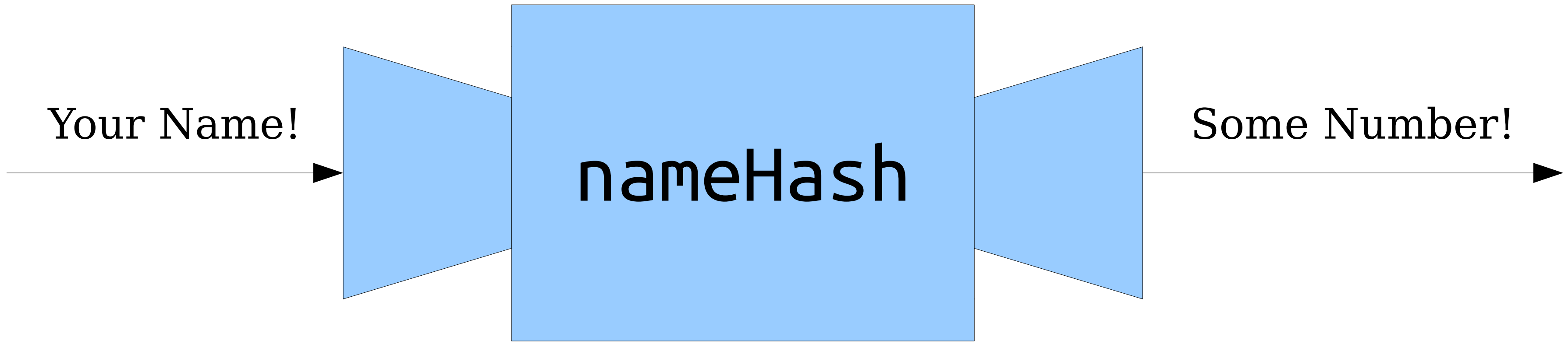
This is how passwords are typically stored. Look up ***salting and hashing*** for more details!

And look up ***commitment schemes*** if you want to see some even cooler things!

Did I hear that correctly?



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This is done in practice!

Look up ***SHA-256***, the ***Luhn algorithm***, and ***CRC32*** for some examples!

And, of course, something to do with data structures.

HashMap and HashSet

HashMap and HashSet

- The `HashMap` and `HashSet` types work just like `Map` and `Set`, except that they do *not* store their keys/elements in sorted order.
- In practice, they are *much* faster than `Map` and `Set`, and they should likely be your defaults going forward.
- ***Recall:*** all the major operations (insertions, deletions, lookups) on `Map` and `Set` run in time O(log *n*).
- So how on earth are these things faster?

The Juicy Details

An Example: Clothes

For Large Values of *n*

Our Strategy

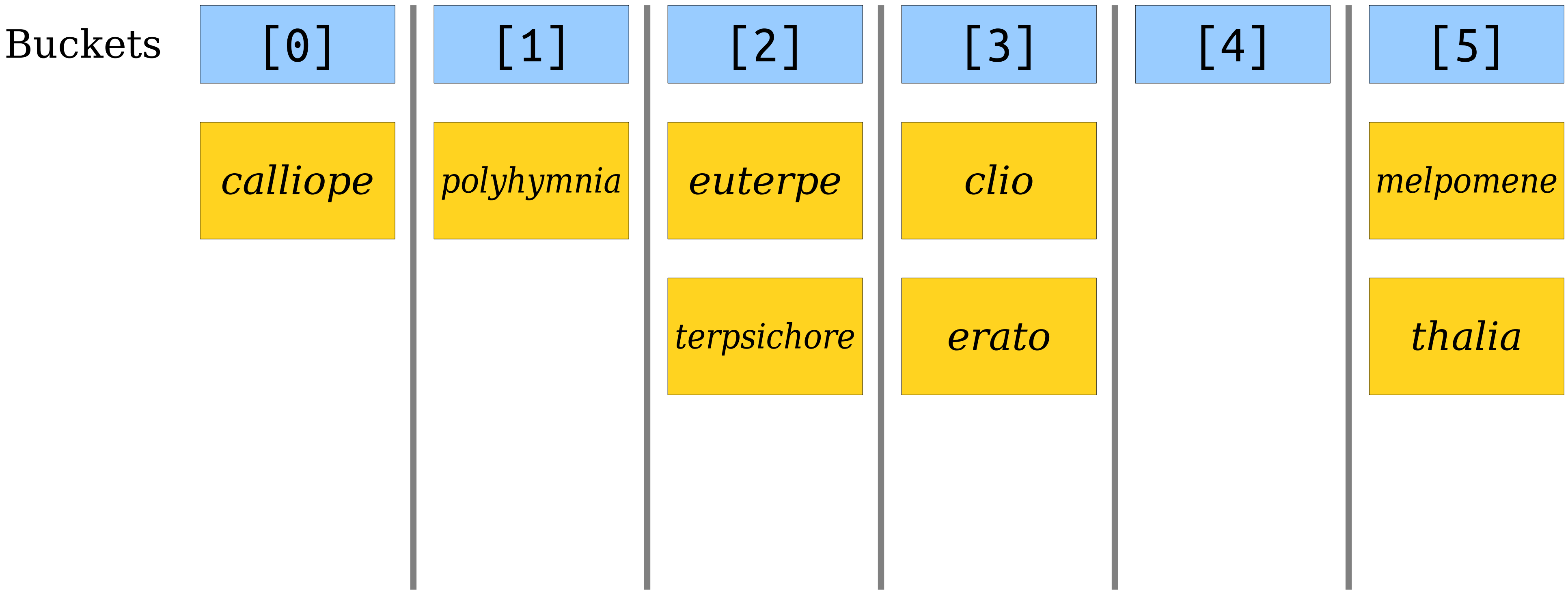
- Maintain a large number of small collections called ***buckets*** (think drawers).
- Find a ***rule*** that lets us tell where each object should go (think knowing which drawer is which.)
- To find something, only look in the bucket assigned to it (think looking for socks.)

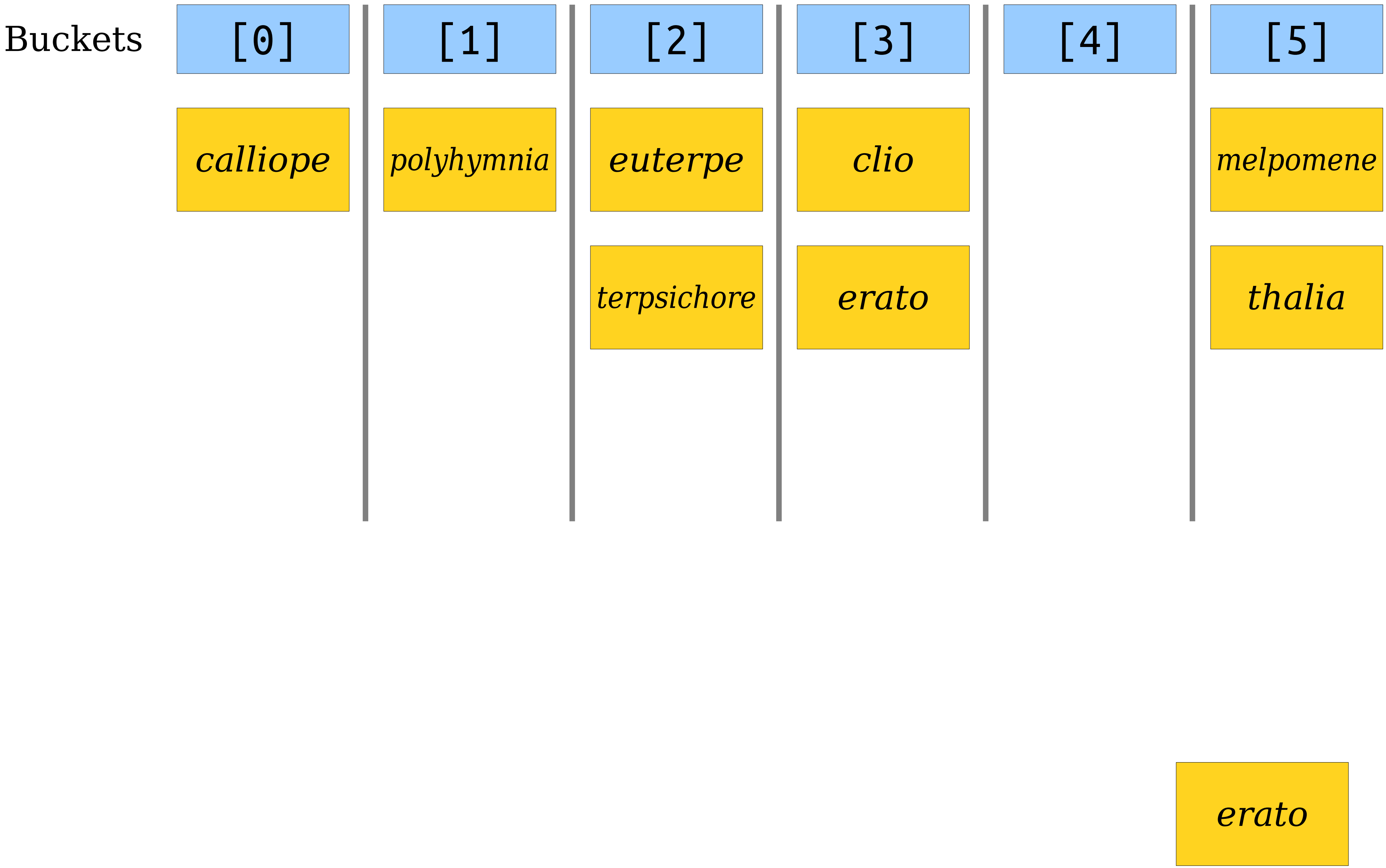
Our Strategy

- Maintain a large number of small collections called ***buckets*** (think drawers).
- Find a ***rule*** that lets us tell where each object should go (think knowing which drawer is which.)

Use a hash function!

- To find something, onl e bucket assigned to it (think looking for socks.)


Buckets
[0]
[1]
[2]
[3]
[4]
[5]
calliope
polyhymnia
euterpe
clio
melpomene
terpsichore
erato
thalia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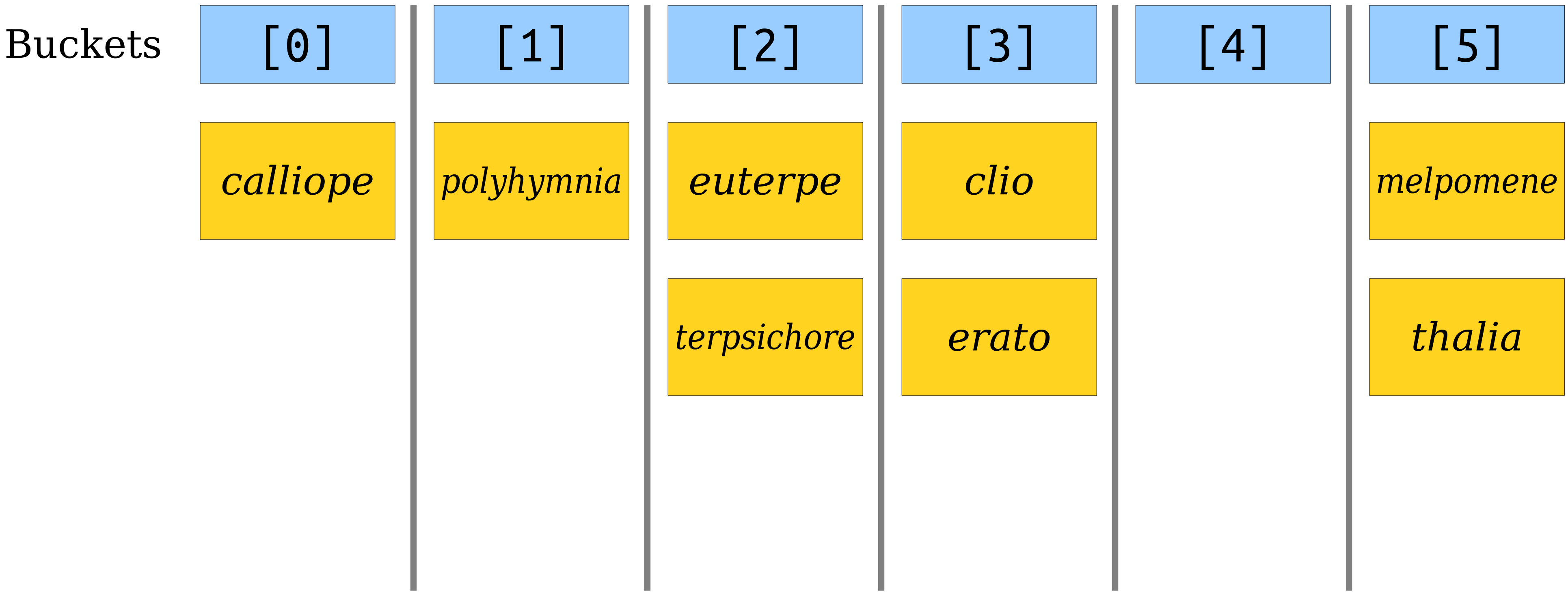
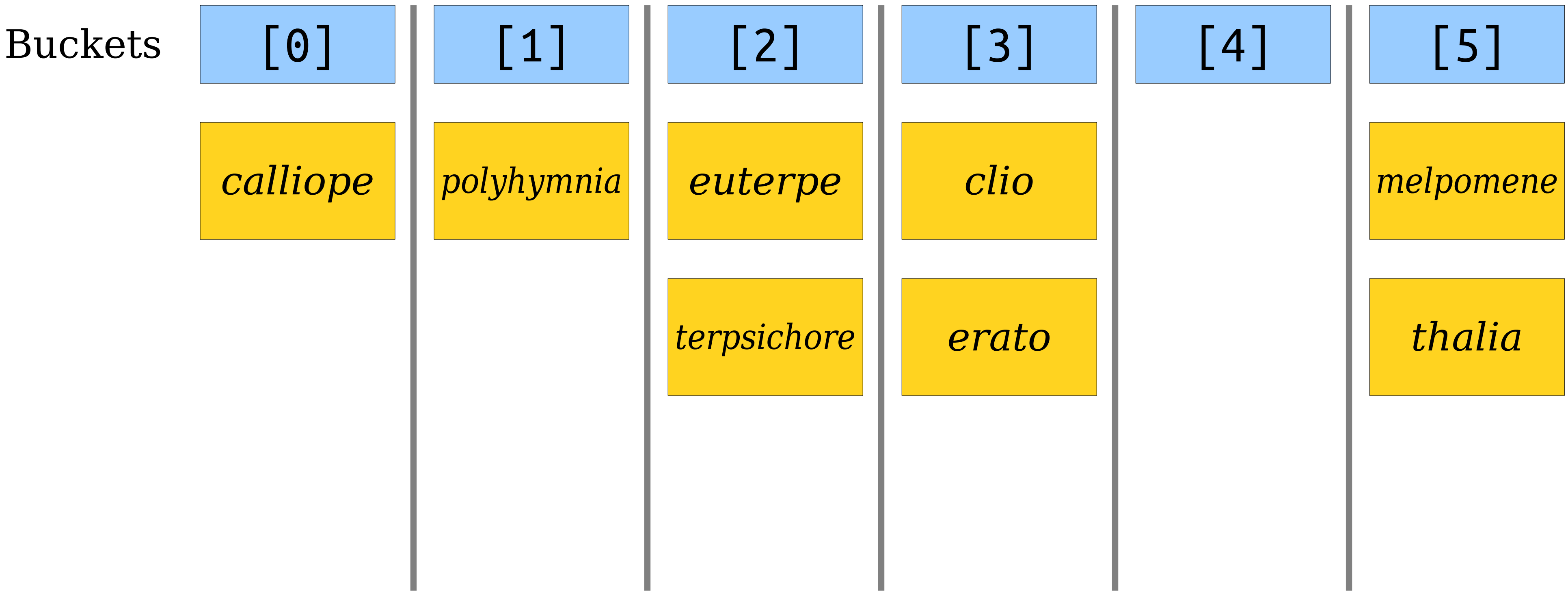



```
bool OurHashSet::contains(const string& value) const {




}
```




```
bool OurHashSet::contains(const string& value) const {
    int bucket = hashCode(value) % buckets.size();




}
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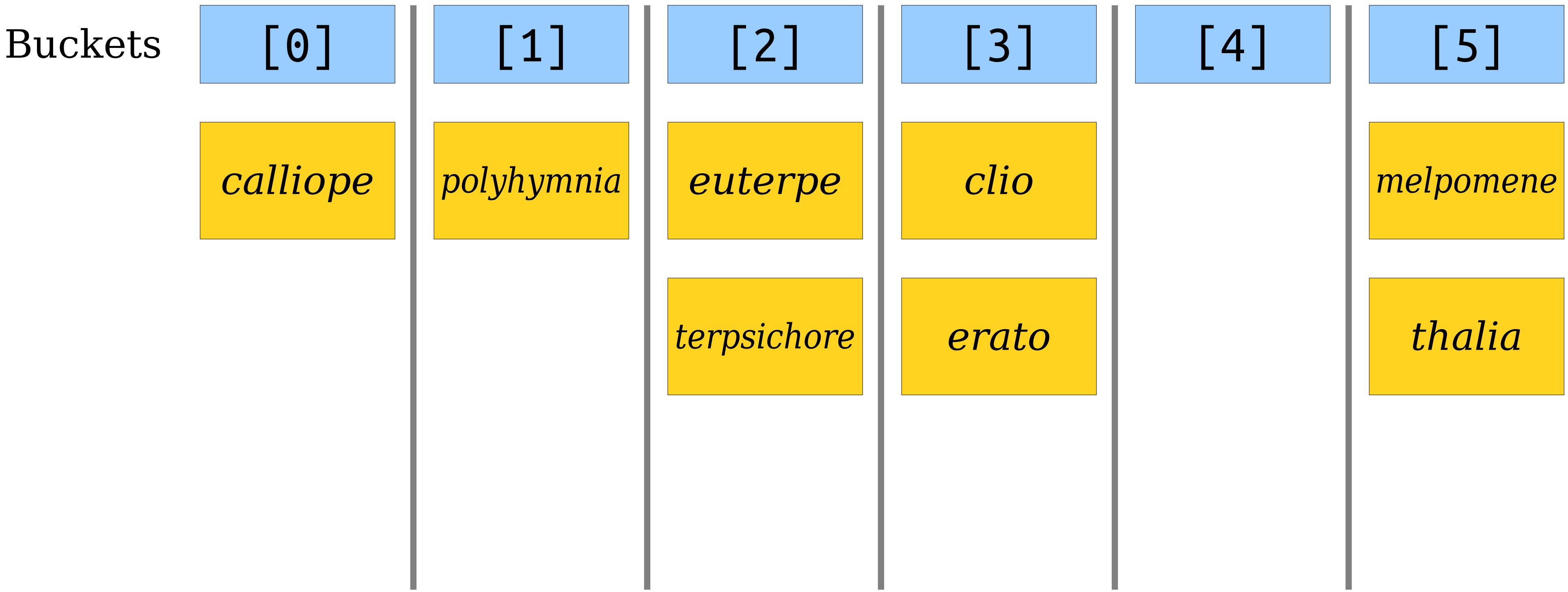


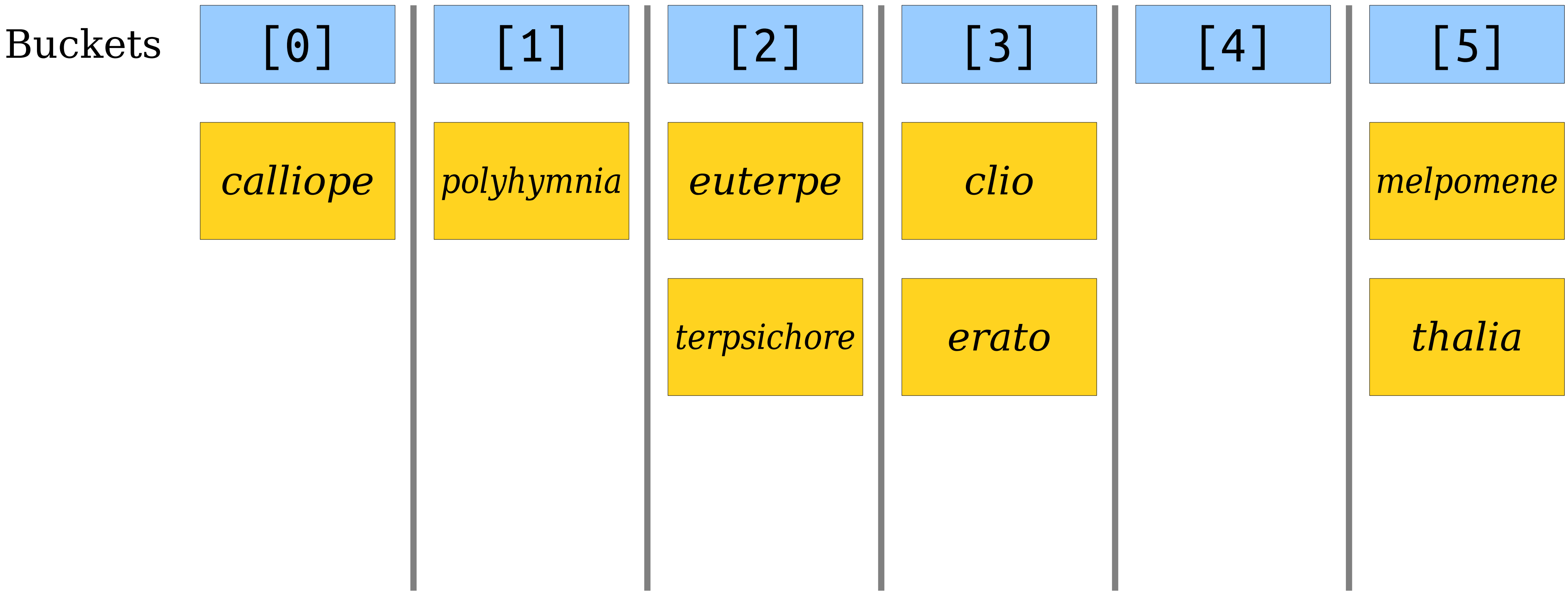

```
bool OurHashSet::contains(const string& value) const {
    int bucket = hashCode(value) % buckets.size();





}
```

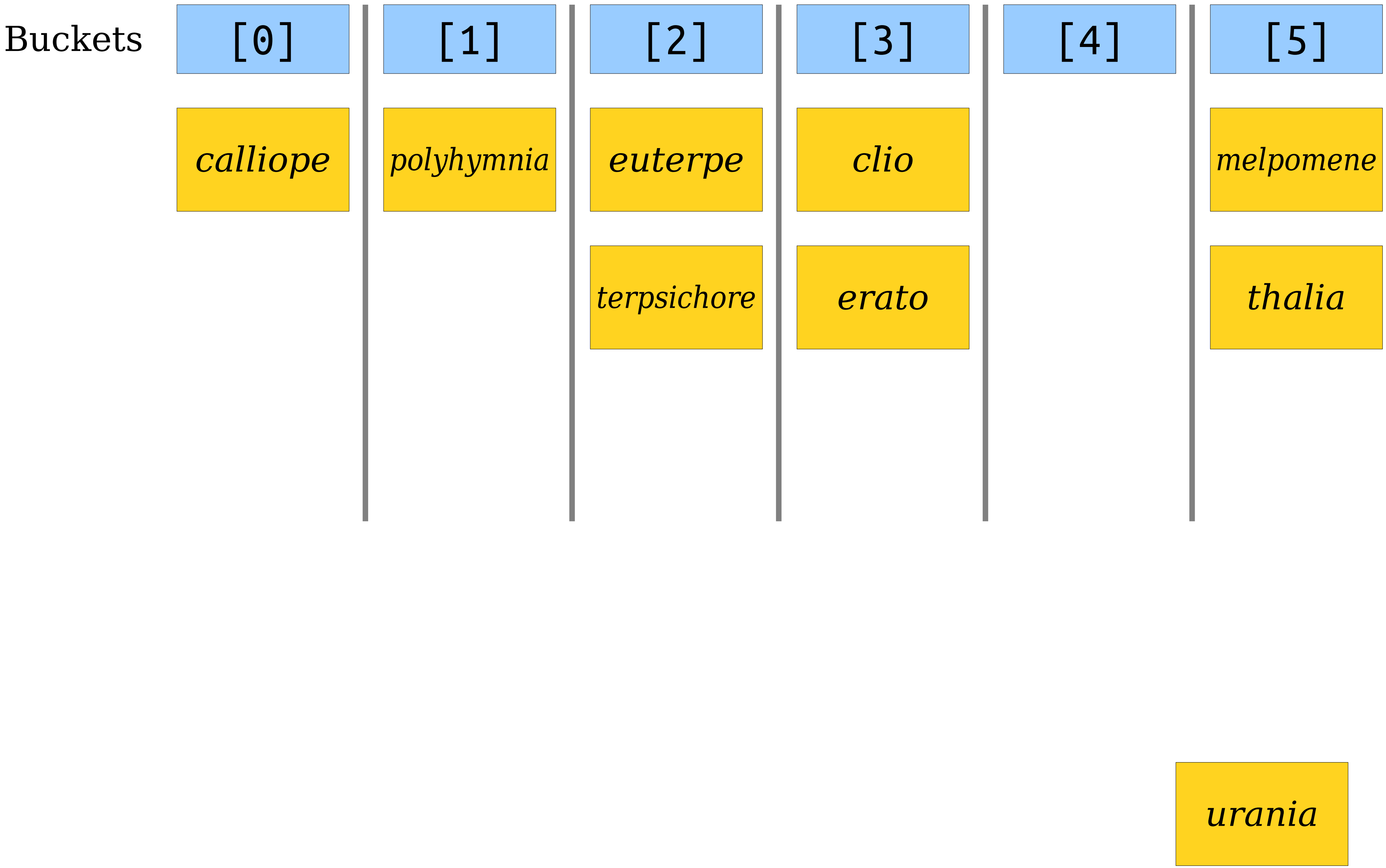
(bucket 3)




```
bool OurHashSet::contains(const string& value) const {
    int bucket = hashCode(value) % buckets.size();

    for (string elem: buckets[bucket]) {
        if (elem == value) return true;
    }

    return false;
}
```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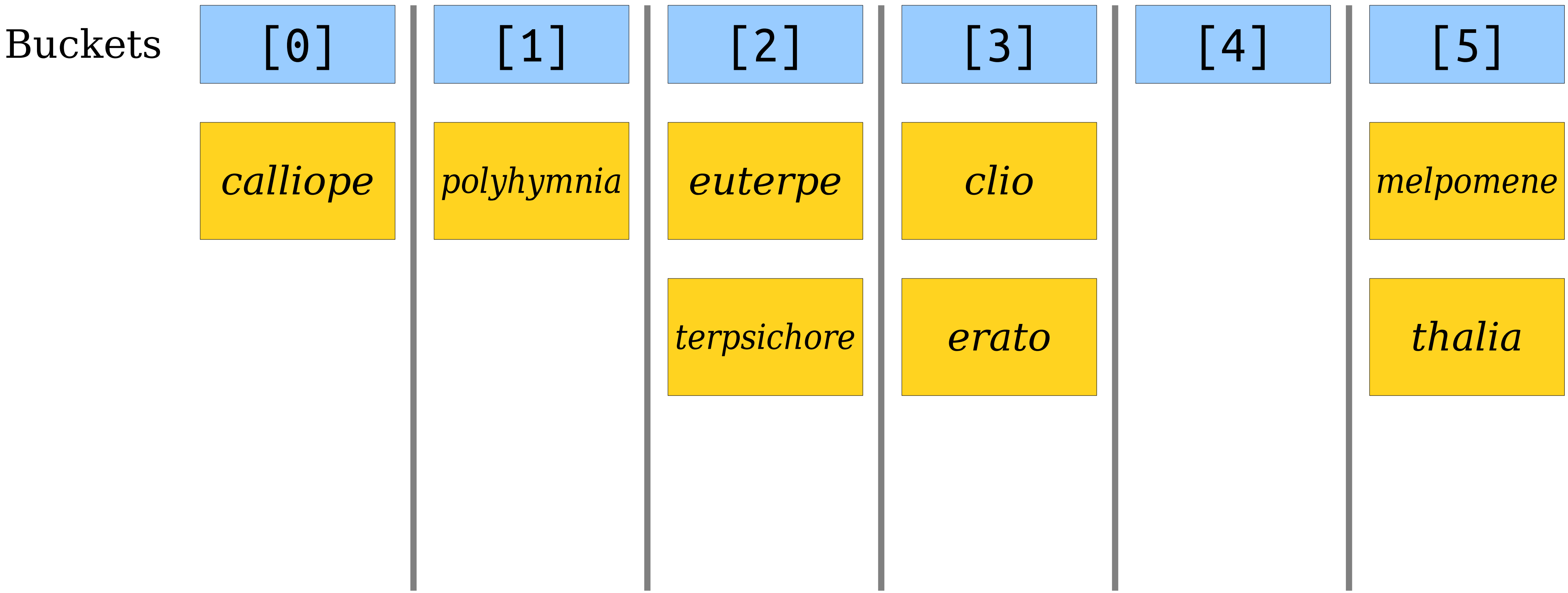
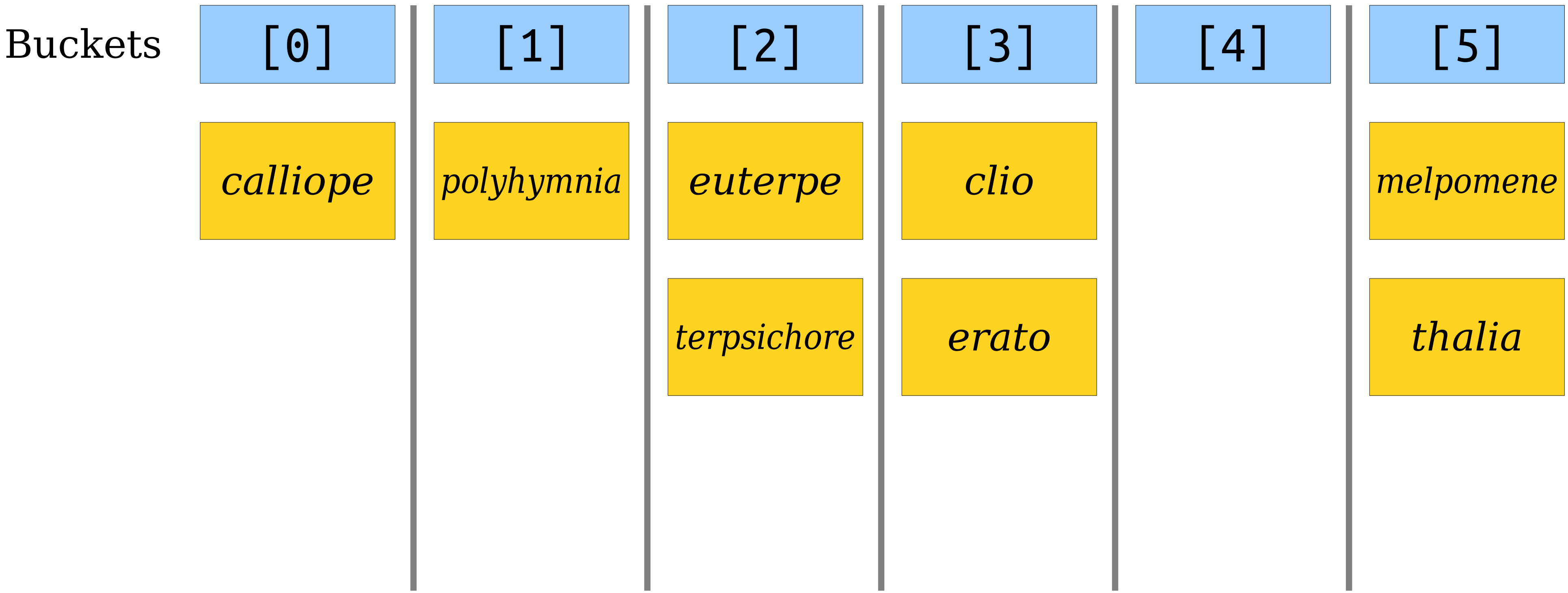



```
void OurHashSet::add(const string& value) {


}
```

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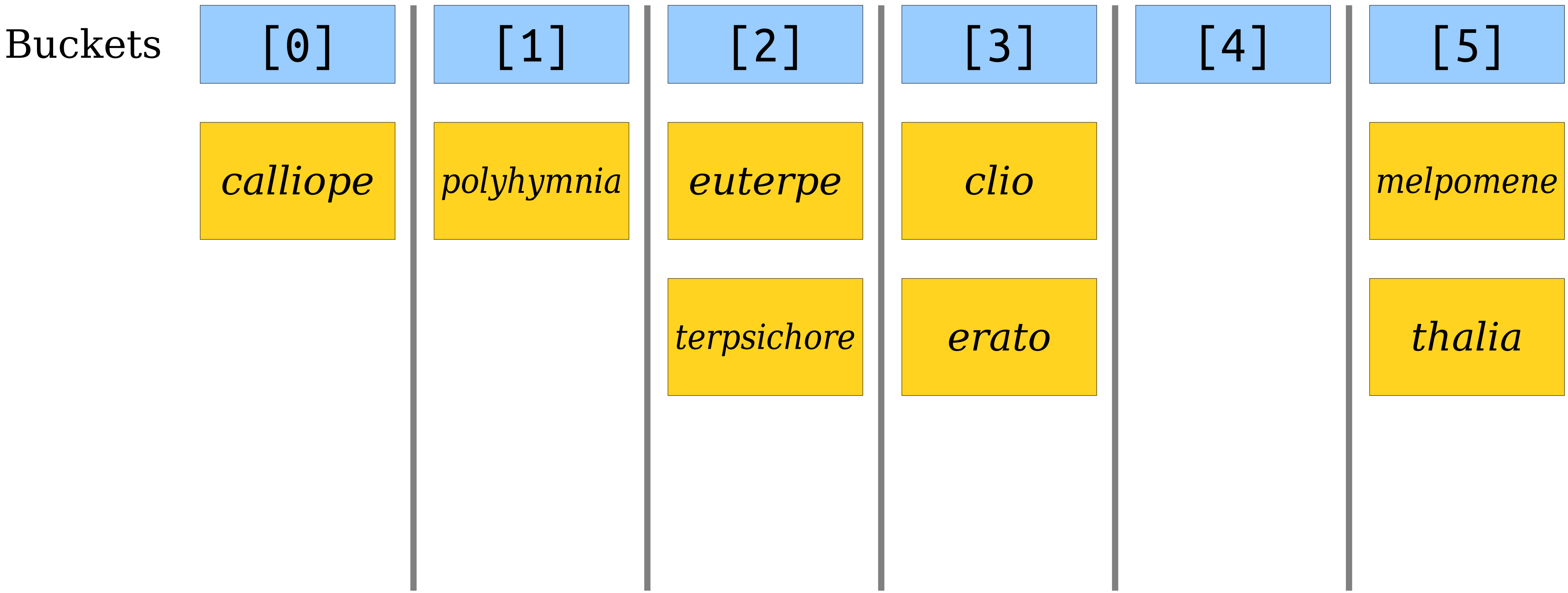
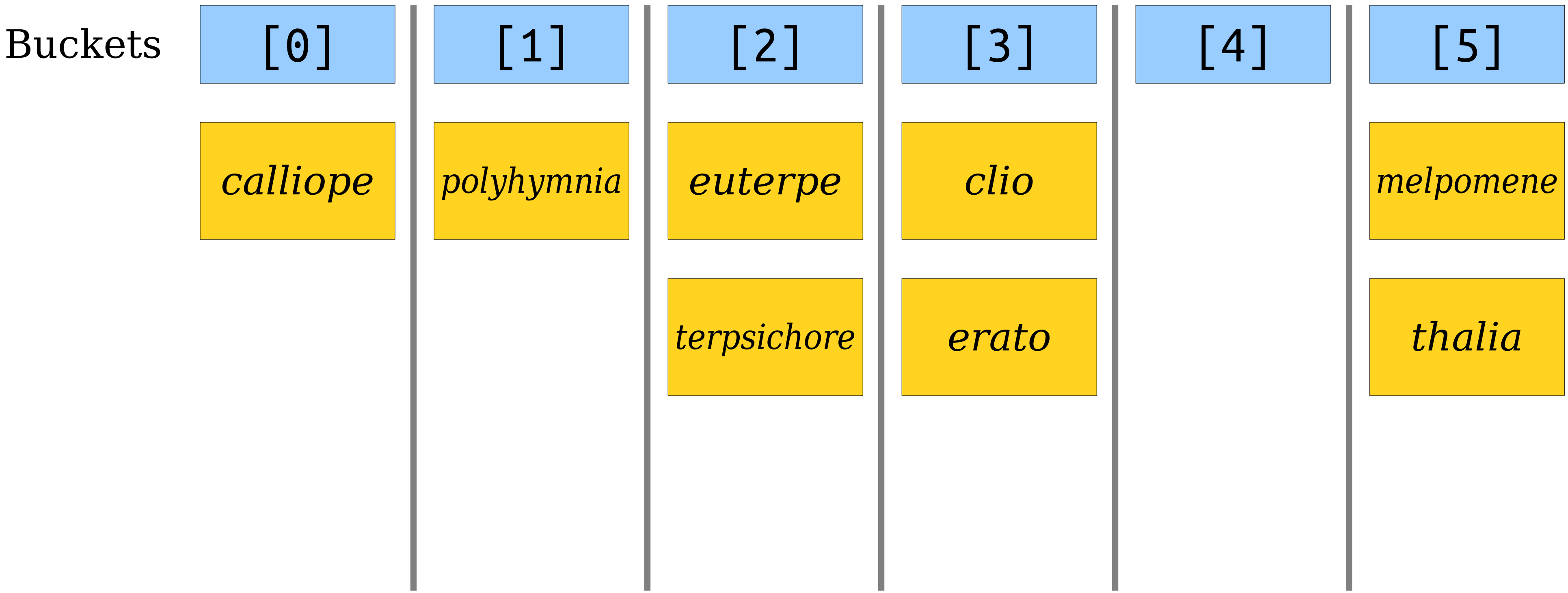


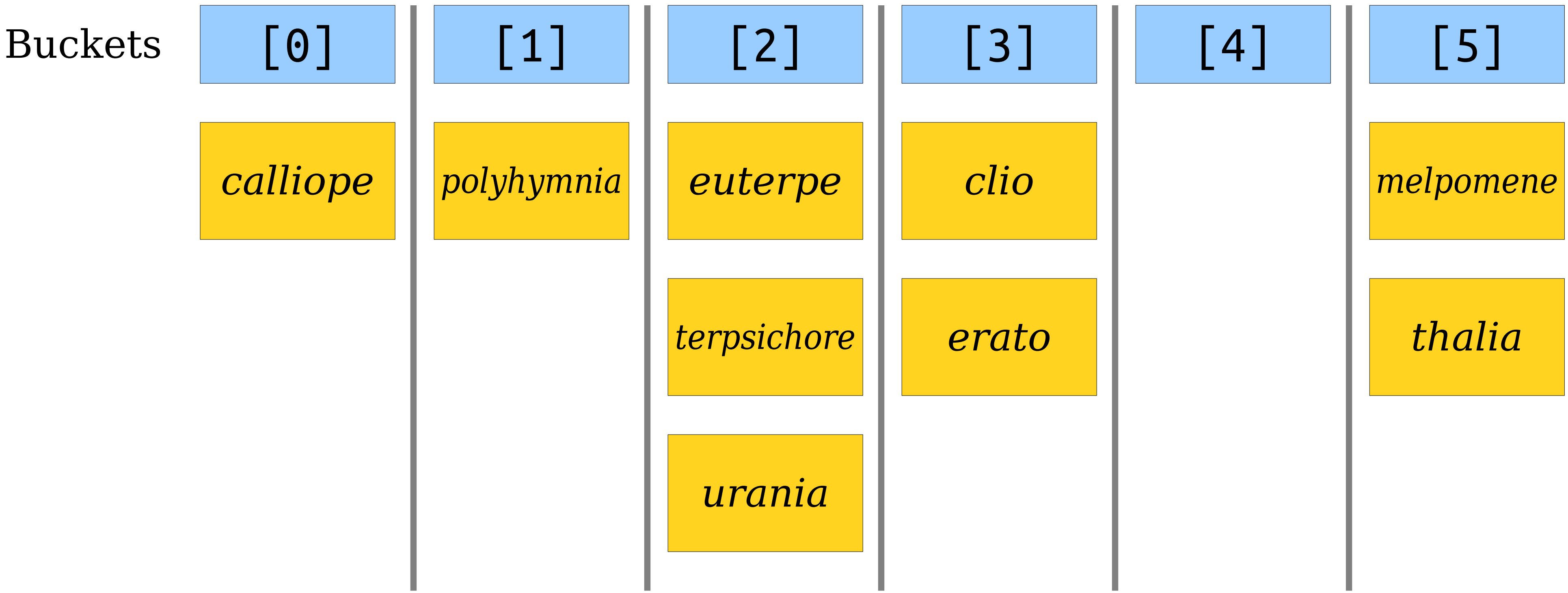

```
void OurHashSet::add(const string& value) {
    int bucket = hashCode(value) % buckets.size();

}
```

(bucket 2)




```
void OurHashSet::add(const string& value) {
    int bucket = hashCode(value) % buckets.size();
    buckets[bucket] += value;
}
```

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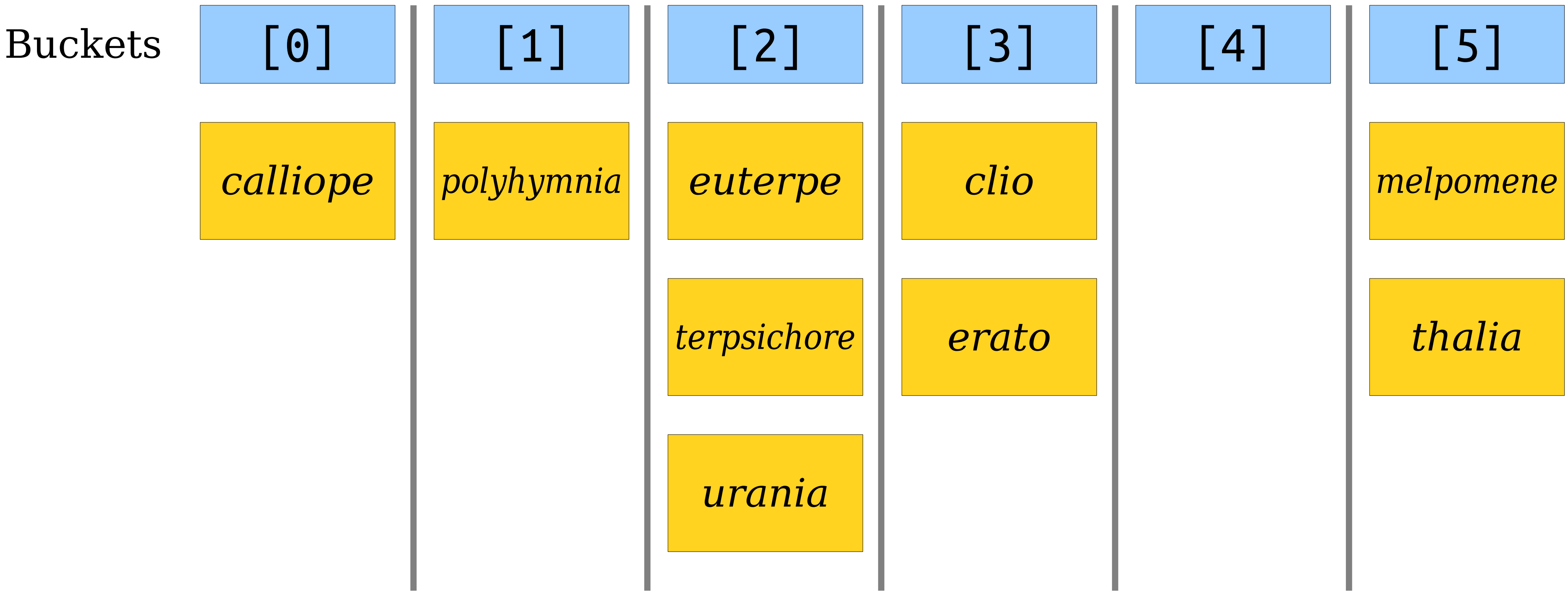



```
void OurHashSet::add(const string& value) {
    int bucket = hashCode(value) % buckets.size();

    for (string elem: buckets[bucket]) {
        if (elem == value) return;
    }

    buckets[bucket] += value;
}
```

(bucket 2)

Time-Out for Announcements!

Assignment 6

- Assignment 5 was due today at the start of class.
  - Using a late day? Turn it in by Monday of next week!
- Assignment 6 (Huffman Encoding) goes out today. It's due next Friday, March 10.
  - Play around with binary trees in a whole new way!
  - Get some practice with tree recursion!
  - And make your files smaller!
- Anton is holding YEAH hours ***today*** at 3PM in room 420-041.
- ***This assignment must be completed individually***. We've broken it down into a bunch of independent, easily-testable, bite-size pieces and included a lot of guidance in the assignment handout.

Need More Practice?

- Many of you have asked about where to go to get extra practice with the material.
- Up on the course website, we have
  - all three versions of the midterm exam (the main exam plus the two alternates), plus
  - section handouts with way more problems on them than anyone could reasonably expect to do in section.
- Feel free to take advantage of these resources, and let us know if you need more!

Change of Grading Basis

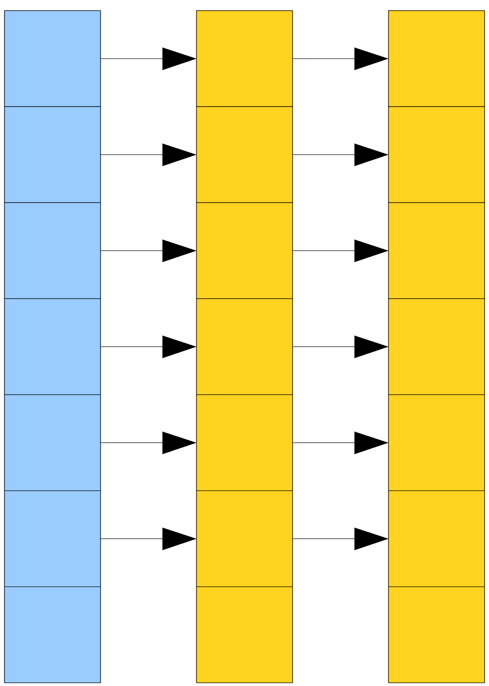
- A number of you have asked about the change of grading basis deadline today.
- ***Before you make a decision, work out the math on your grade.***
  - Assignments are 40% of your grade. If you're averaging a ✓+, figure you've got roughly a 95%. With a ✓ average, figure you've got roughly 85%. With a ✓- average, figure you've got roughly a 75%.
  - The midterm is 25% of your grade.
  - The final is the remaining 35%.
- ***Unless you earned a low-single-digit score on the midterm <u>and</u> have extremely low assignment grades, it is absolutely still possible to pass this class and do well in it.*** A single bad midterm score will not cause you to fail the class, though it may knock you out of contention for a solid A grade.
- ***We never curve grades down.*** A raw score of 90% is never lower than an A-, a raw score of 80% is never lower than a B-, and a raw score of 70% is never lower than a C-.
- ***Compute your raw score before making a switch.*** Every quarter I give CR grades to a bunch of folks who earn raw A's, A-'s, B+'s, and B's, and I always feel bad when that happens.

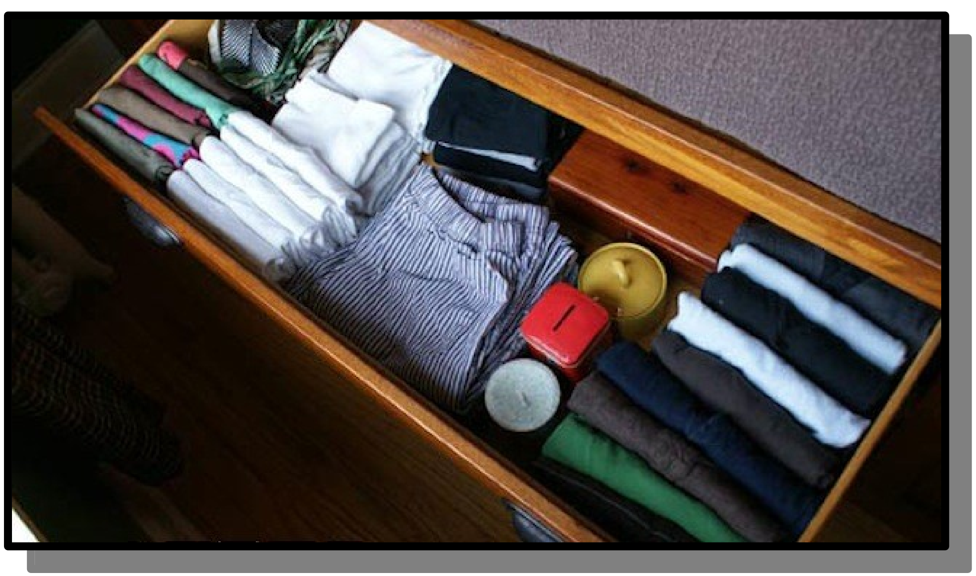
Back to CS106B!

So how efficient is our solution?

Efficiency Concerns

- Each hash table operation
  - chooses a bucket and jumps there, then
  - potentially scans everything in the bucket.
- ***Claim:*** The efficiency of our hash table depends on how well-spread the elements are.





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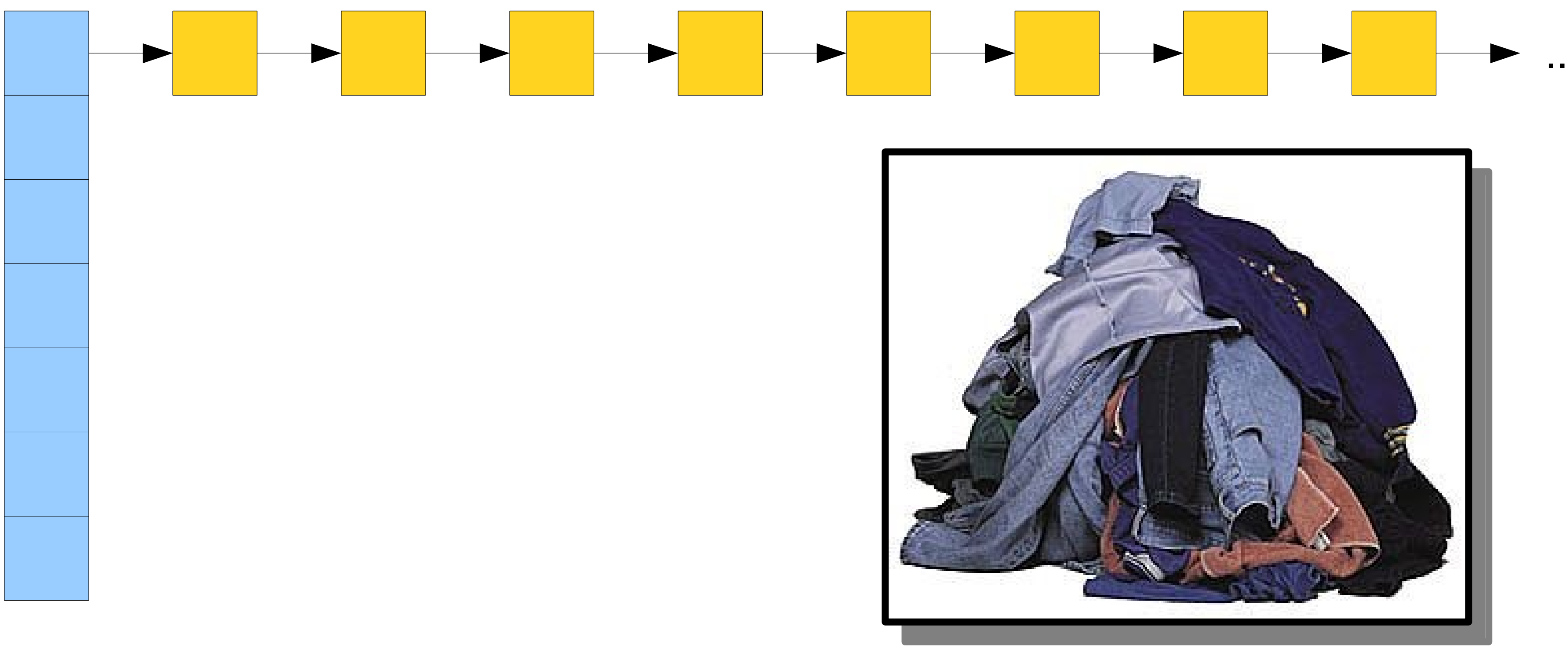


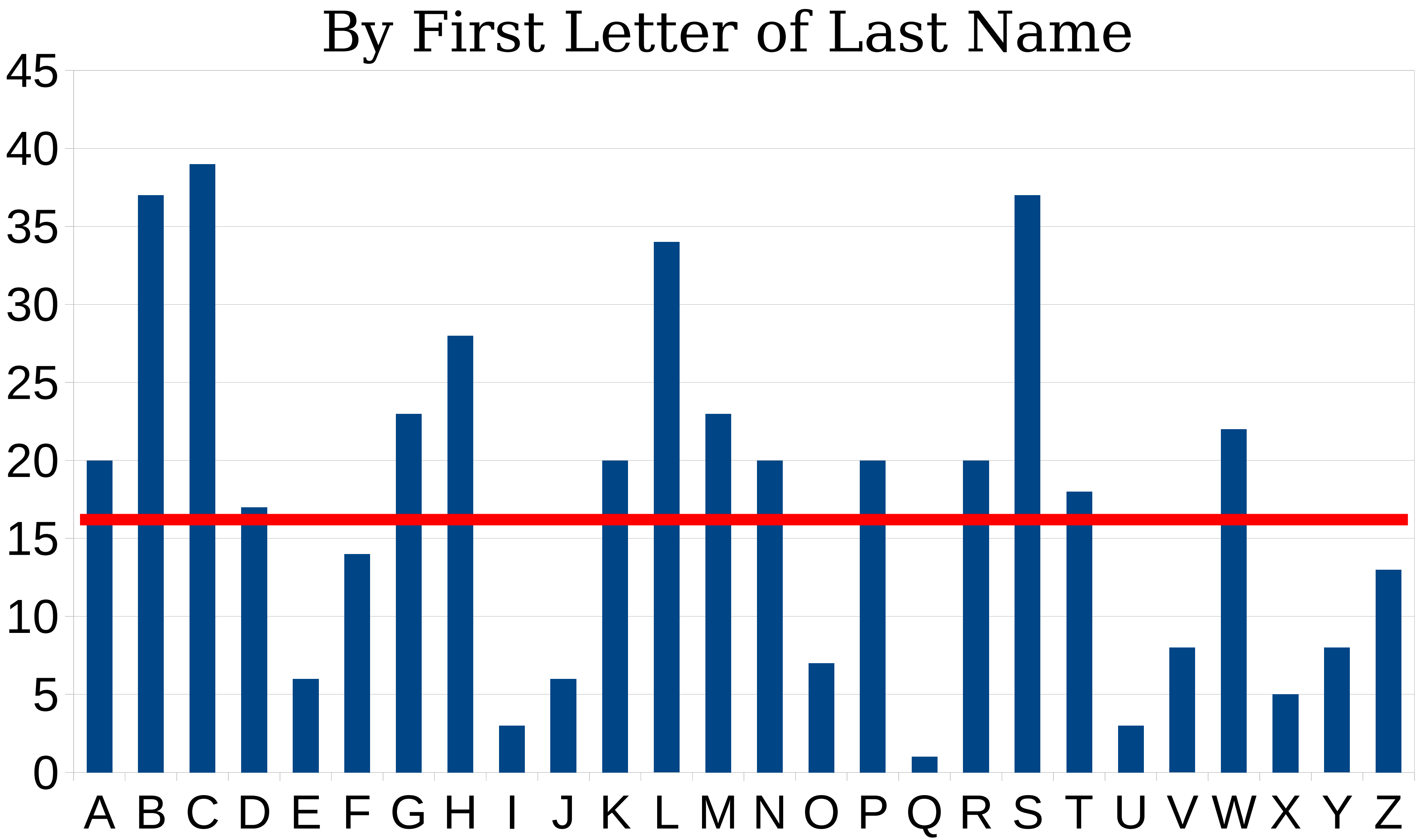
Hash Table Performance

- Suppose that we have n elements and b buckets.
- If the elements are distributed as evenly as possible over the buckets as possible, the cost of an operation is $O(1 + n / b)$.
  - The ratio n / b is called the ***load factor*** and is sometimes denoted $\boldsymbol{\alpha}$.
- If the elements are all distributed into a single bucket, the cost of an operation is $O(n)$.
- ***It's really important to choose a good hash function!***

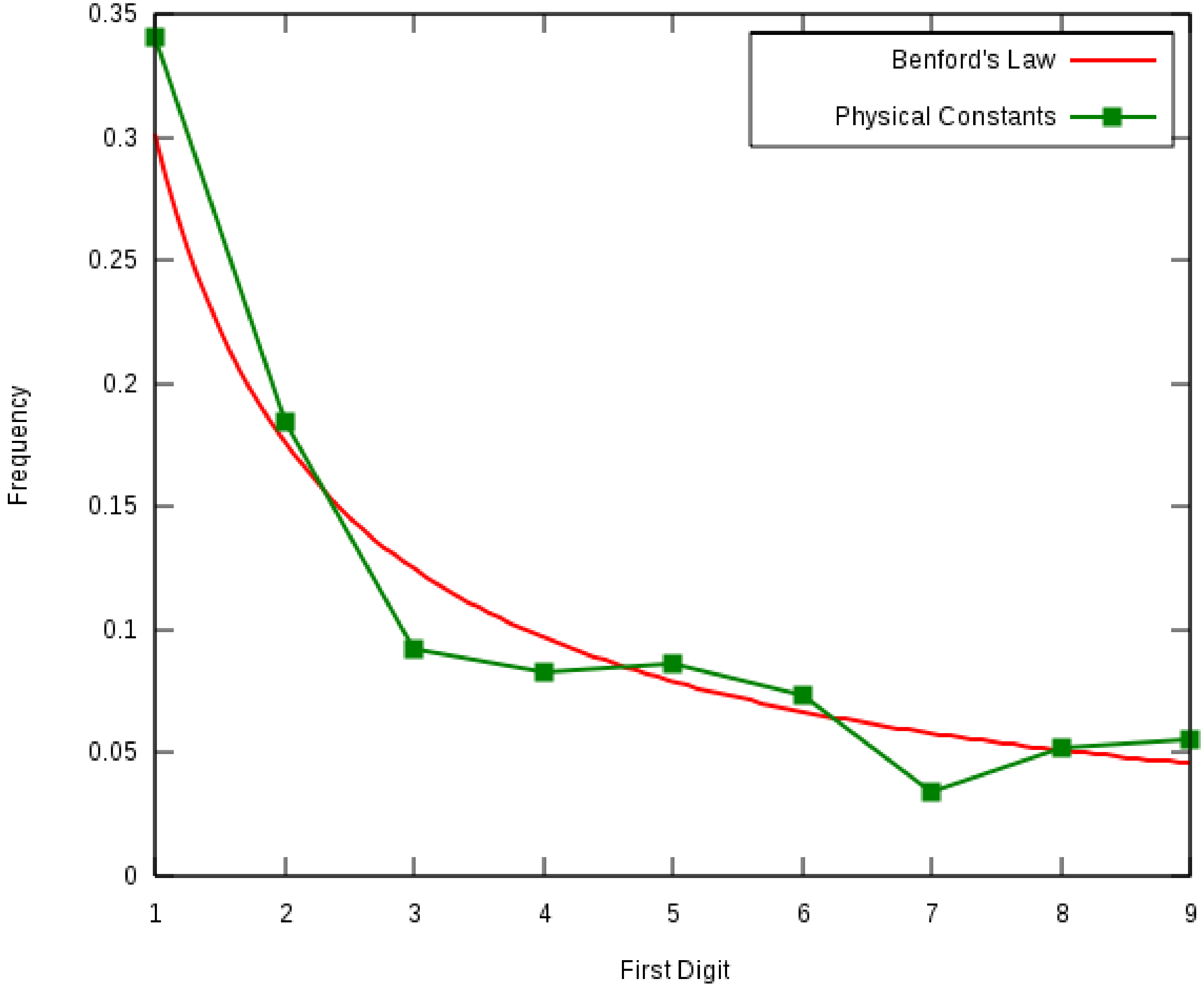
Distributing Keys

- We want to choose a hash function that will distribute elements as evenly as possible to try to guarantee a nice, even spread.
- Suppose you want to build a hash function for names.
- ***Idea:*** Hash each last name to the first letter of that last name.
- How well will this distribute elements?

CS106B Name Distributions

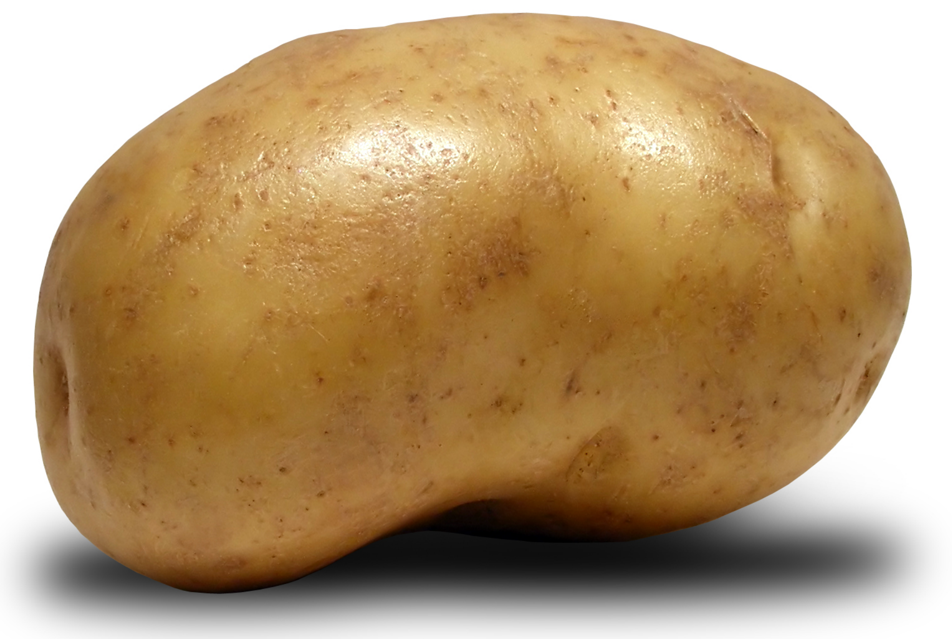


Benford's Law



Good Hash Functions

- A good hash function typically will scramble all of the bits of the input together in a way that appears totally random.
- Hence the name “hash function.”



Implementing a Hash Code

- There's a lot of *beautiful* mathematical theory behind the design of hash functions.
  - Take CS109, CS161, CS166, or CS255 for details!
  - Or come talk to me after class – this stuff is *super cool!*
- ***Claim:*** With well-chosen and well-implemented hash functions, you can assume the *expected* cost of an operation in a hash table is $O(1 + \alpha)$.
  - α is the load factor, the ratio of the number of elements to the number of buckets.

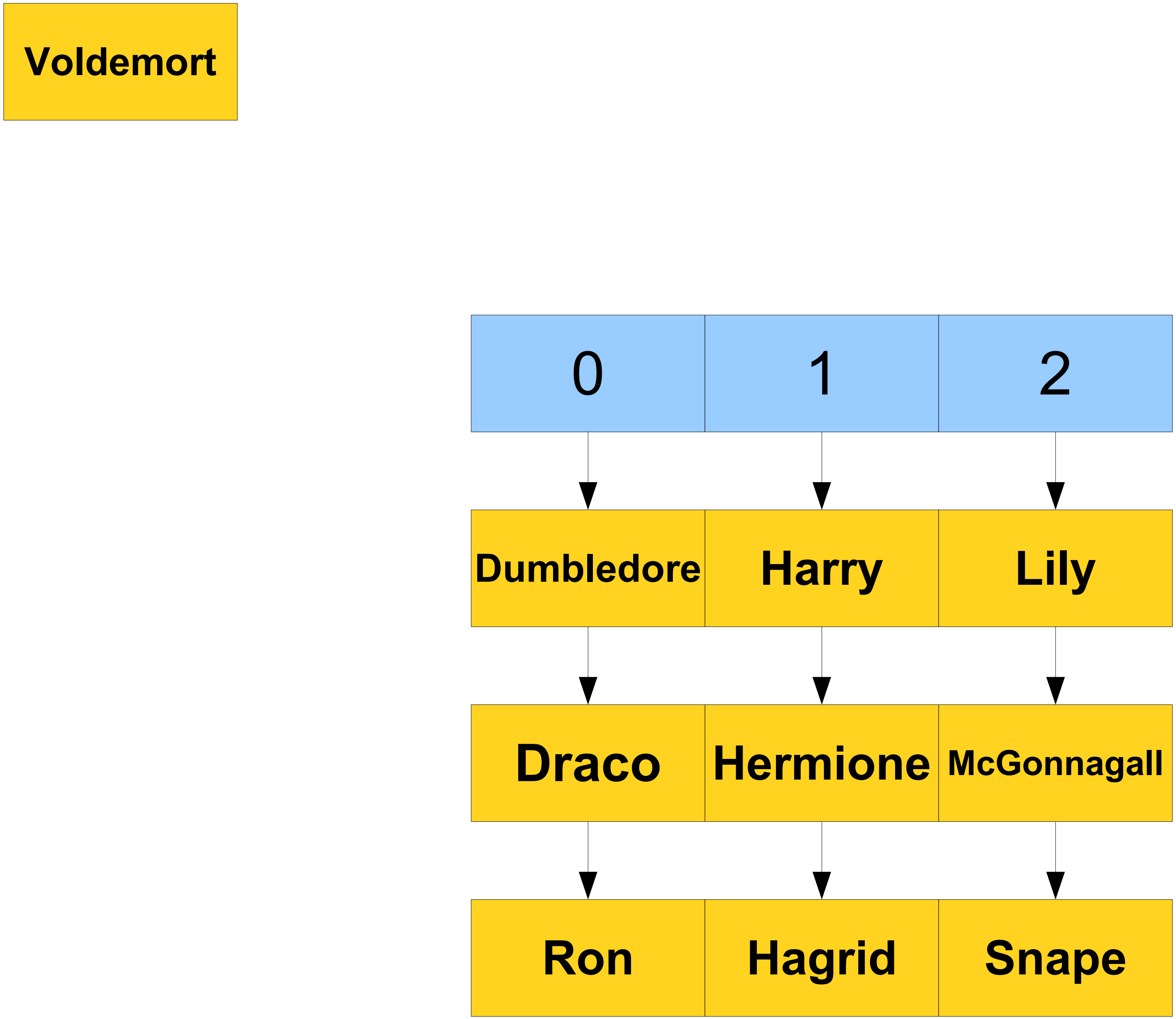
What does $O(1 + \alpha)$ mean?

O(1 + α)

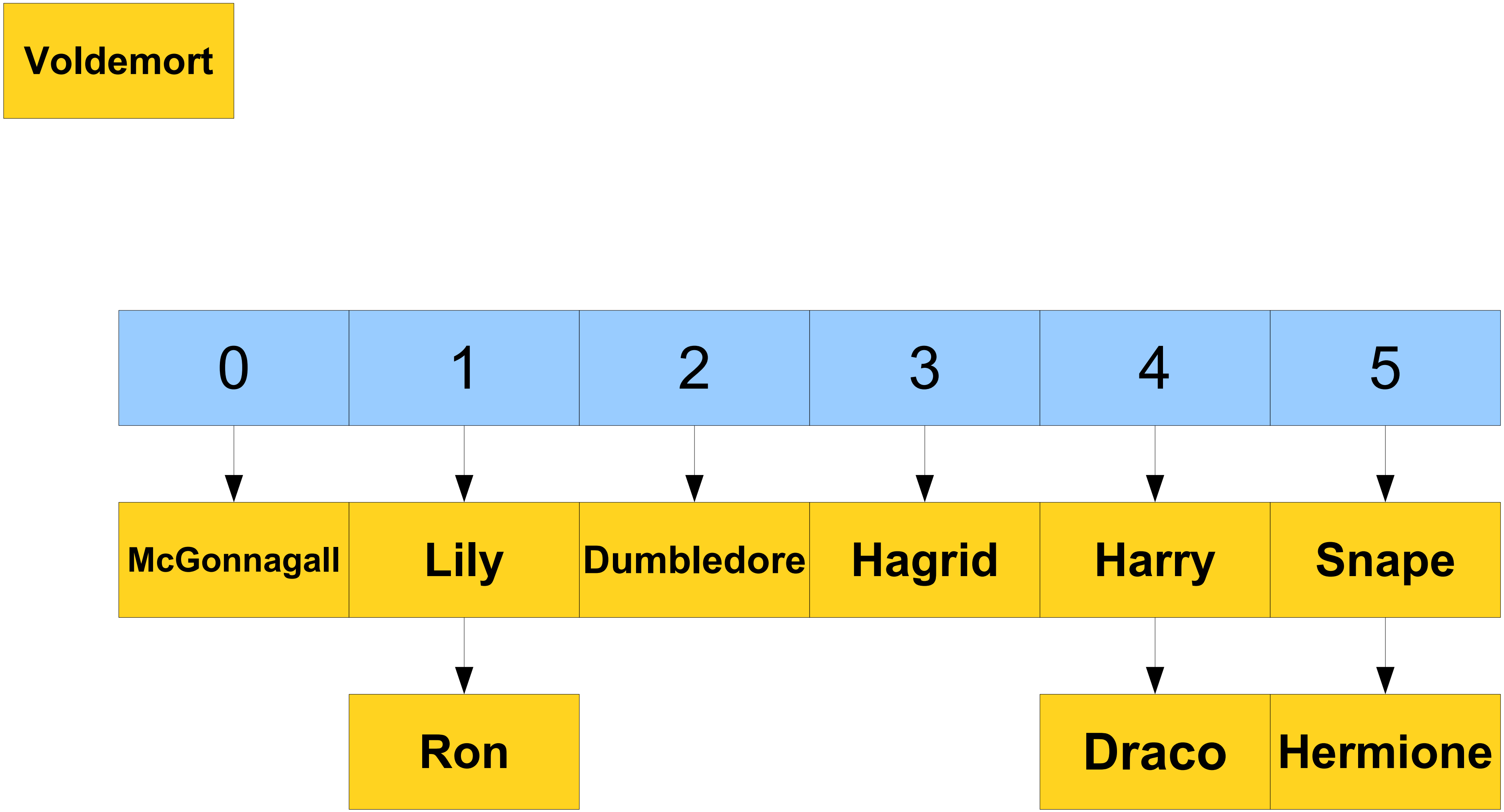
- The expected cost of an operation on a hash table is O(1 + α), where α is the ratio of the number of elements (*n*) to the number of buckets (*b*).
- ***Observation:*** If we can keep α small – say, at most two – then this cost is O(1)!
- ***Claim:*** The *expected* cost of an operation on a well-implemented hash table is O(1).
- But how do we keep α small?

Hashing and Rehashing

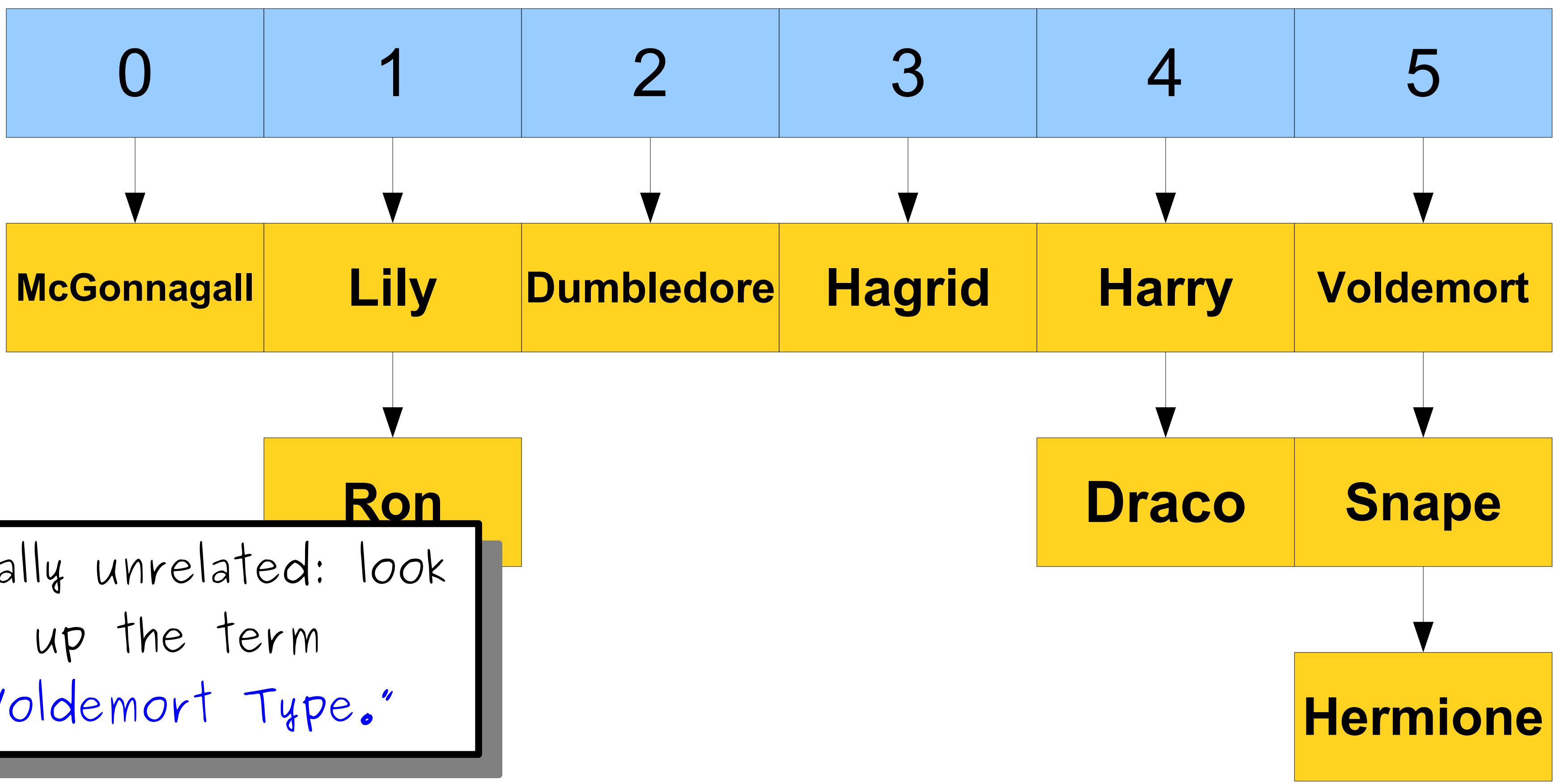
Hashing and Rehashing



Hashing and Rehashing



Hashing and Rehashing



Totally unrelated: look up the term "Voldemort Type."

Hashing and Rehashing

- ***Idea:*** Track the number of buckets *b* and the number of total elements *n*.
- When inserting, if *n* / *b* exceeds some small constant (say, 2), double the number of buckets and redistribute the elements into the new table.
  - As with `Stack`, this rehashing happens so infrequently that it's extremely fast on average.
- This makes $\alpha \leq 2$, so the expected lookup time in a hash table is **O(1)**.
- On average, the lookup time is *independent* of the total number of elements in the table!

To Summarize

- The cost of an insertion, lookup, or deletion in a hash table is, on average, **O(1)**.
  - This assumes you have a good choice of hash function. Unless you have a background in abstract algebra, just follow the template we'll provide in a second. ☺
- This is why hash tables are one of the single most common data structures used today!

Custom Types in Hash Tables

Custom Types in Hash Tables

- In order to store a custom type in a hash table, you need to be able to
  - get a hash code for it, and
  - compare whether two objects of that type are equal.
- This first task is handled by writing

```
int hashCode(const Type& value);
```

- This second task is handled by writing

```
bool operator== (const Type& lhs, const Type& rhs);
```

Implementing a Hash Code

- ***Implementing a good hash function is hard. It's better to follow a template than to try to be creative.***
- Best advice we can offer: write your hash function by combining a bunch of smaller hash functions together.
- One technique:

```
int hashCode(const Type& value) {
    int result = hashCode(value.field1);
    result = 31 * result + hashCode(value.field2);
    result = 31 * result + hashCode(value.field3);
        …
    result = 31 * result + hashCode(value.fieldN);
    return result & 0x7FFFFFFF;
}
```

- Come talk to me after class for a discussion of why this works!

Implementing Equality

- To implement an equality operator, you typically just return whether all the fields are equal:

```
bool operator== (const Type& lhs, const Type& rhs) {
    return lhs.field1 == rhs.field1 &&
           lhs.field2 == rhs.field2 &&
           …
           lhs.fieldN == rhs.fieldN;
}
```

To Summarize

- Hash tables are very fast! You should use them.
- They're powered by hash functions, which are the Cool Kids at the Function Party.
- Writing your own hash function is hard. Follow a template.
- Don't forget to implement `operator==`!

Next Time