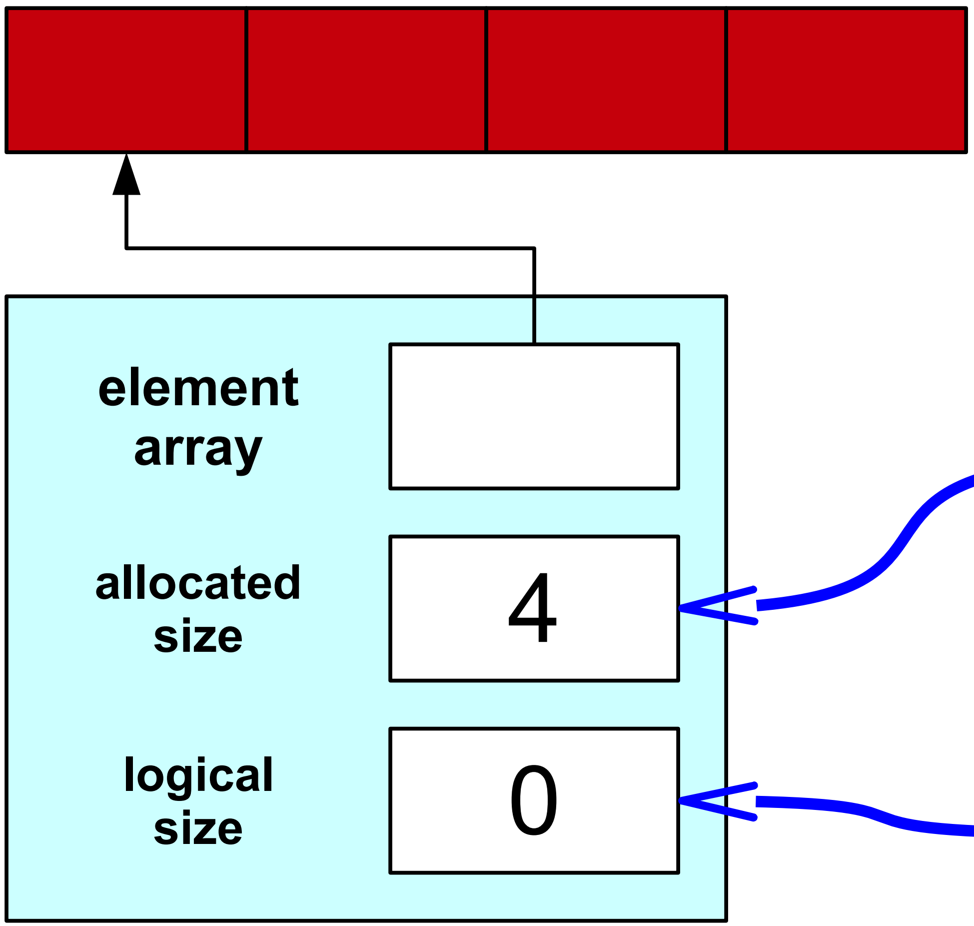
Implementing Abstractions

Part Two

Previously, on CS106B…

A Bounded Stack



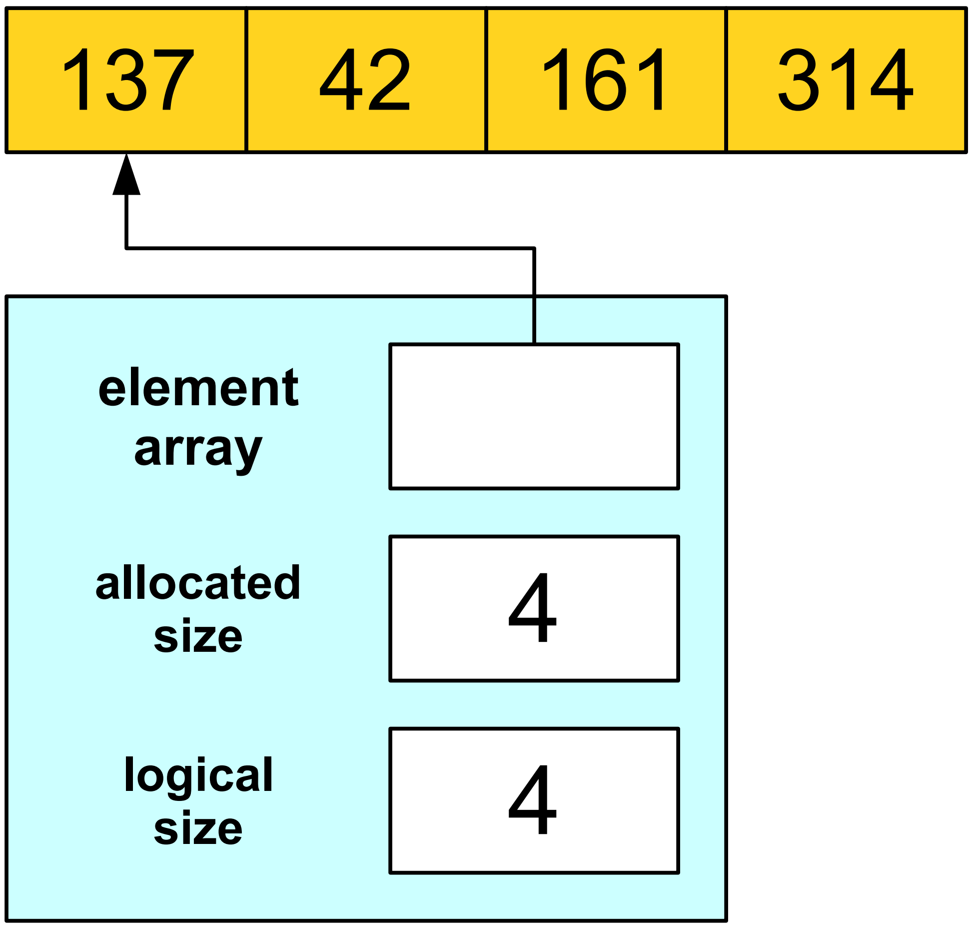
The stack's ***allocated size*** is the number of slots in the array. Remember – arrays in C++ cannot grow or shrink.

The stack's ***logical size*** is the number of elements actually stored in the stack. This lets us track how much space we're actually using.

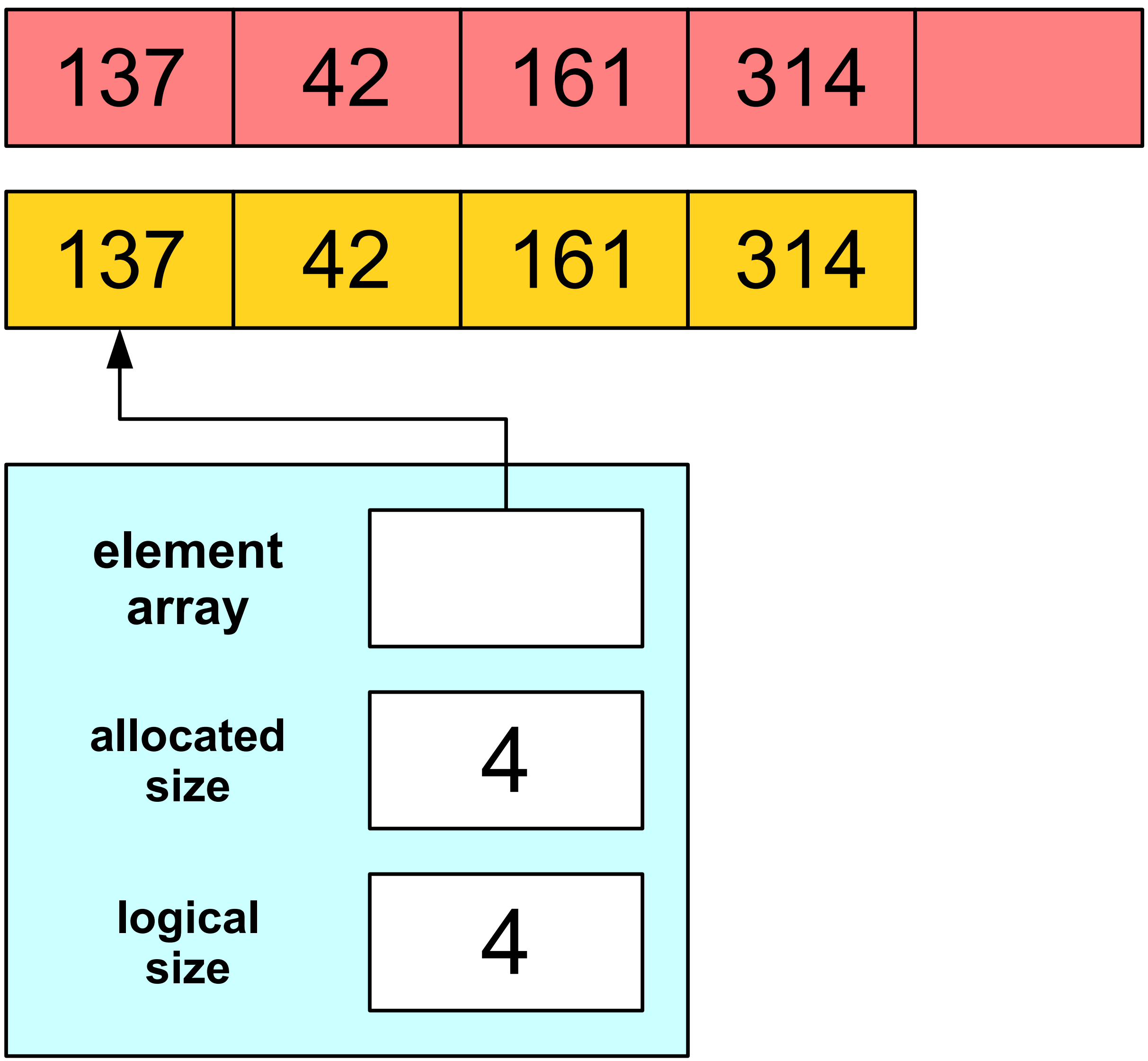
Running out of Space

- Our current implementation very quickly runs out of space to store elements.
- What should we do when this happens?

An Initial Idea



An Initial Idea



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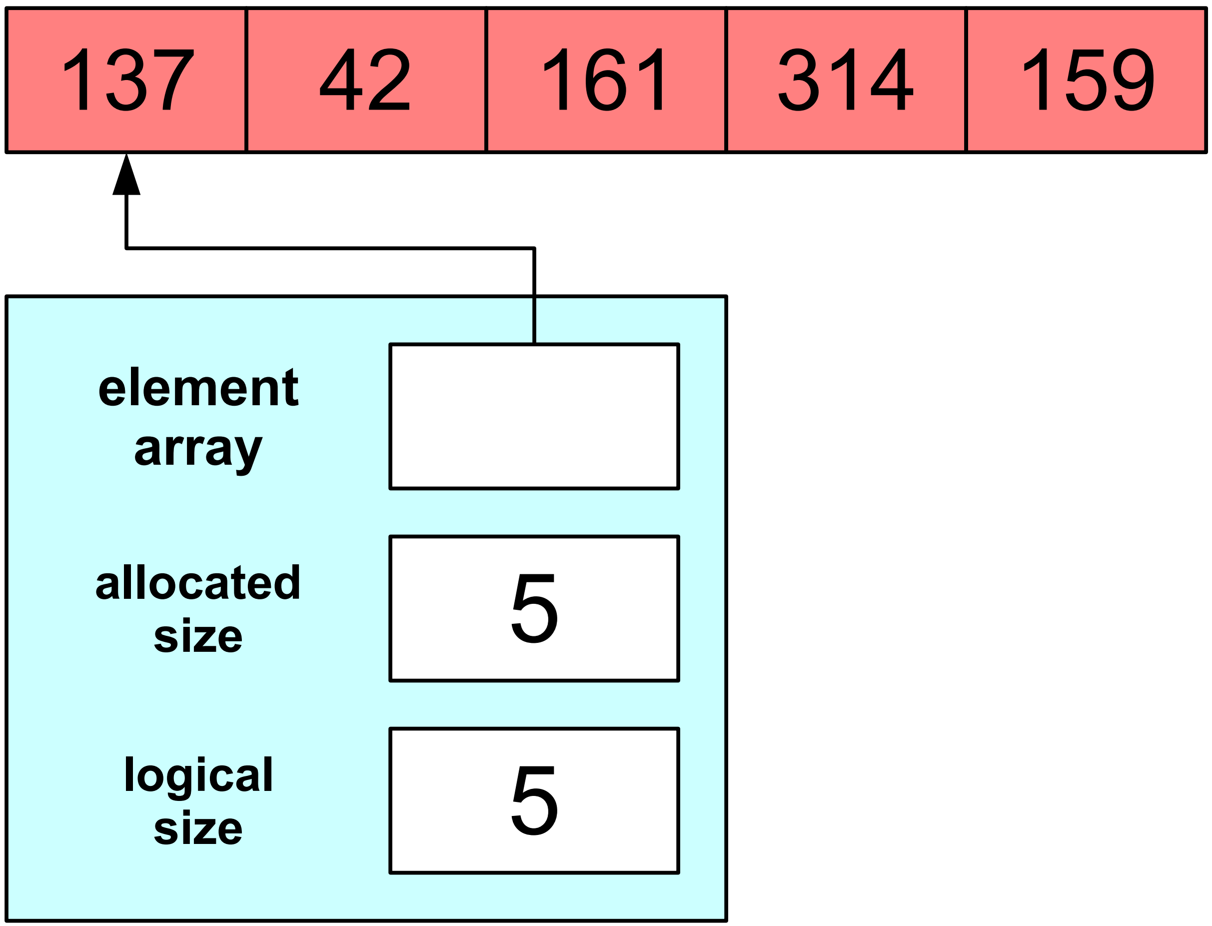
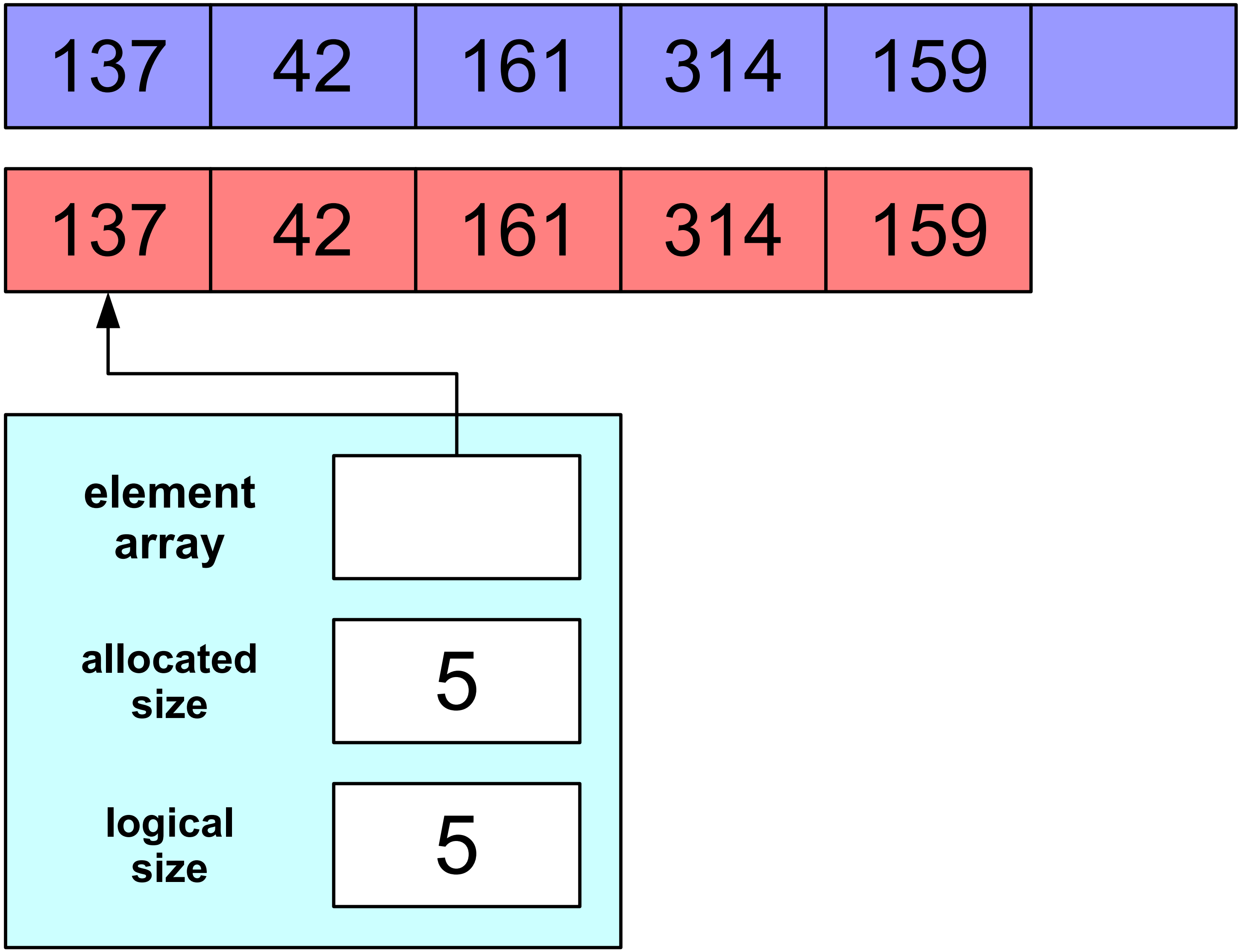
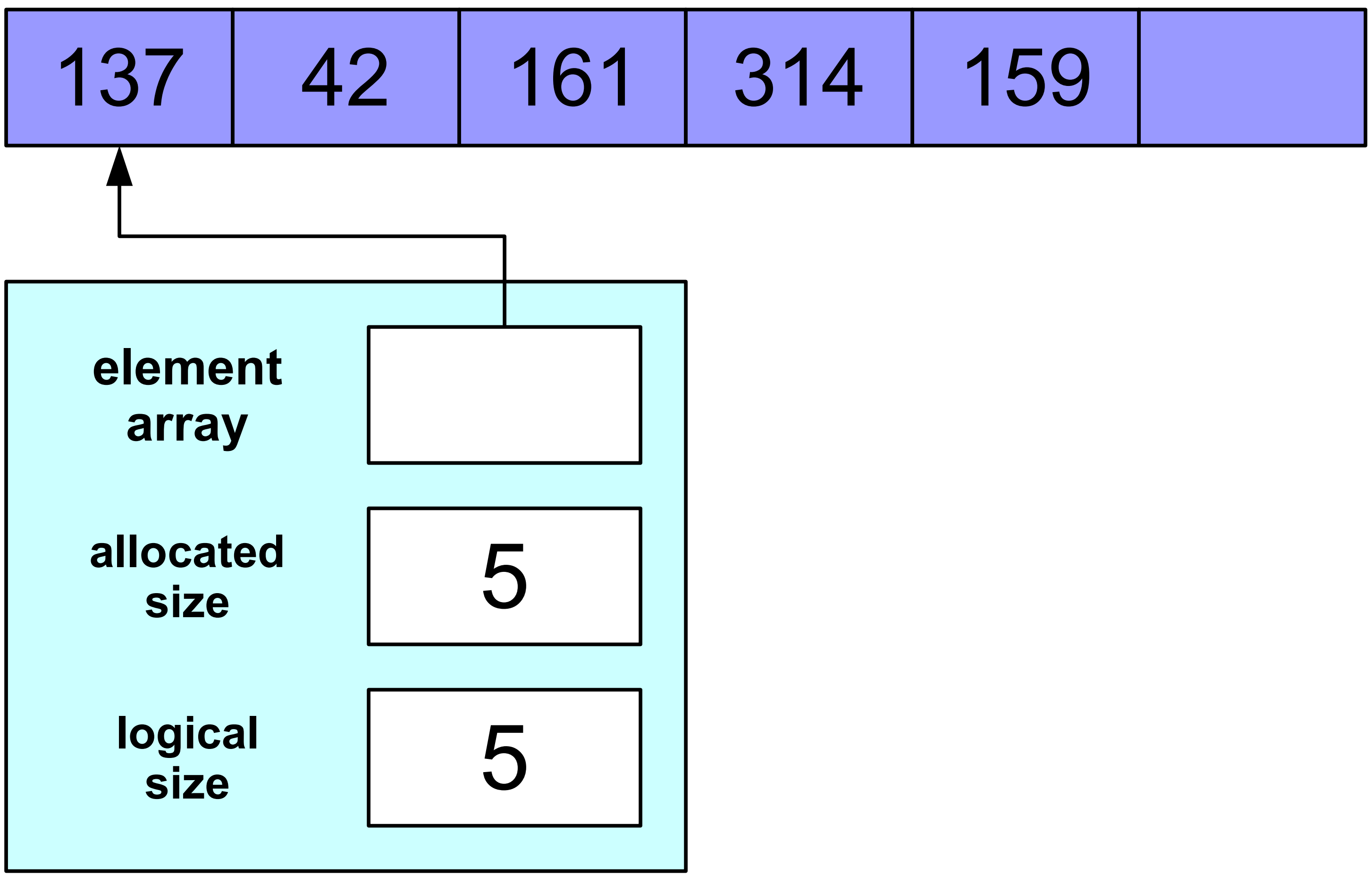


An Initial Idea

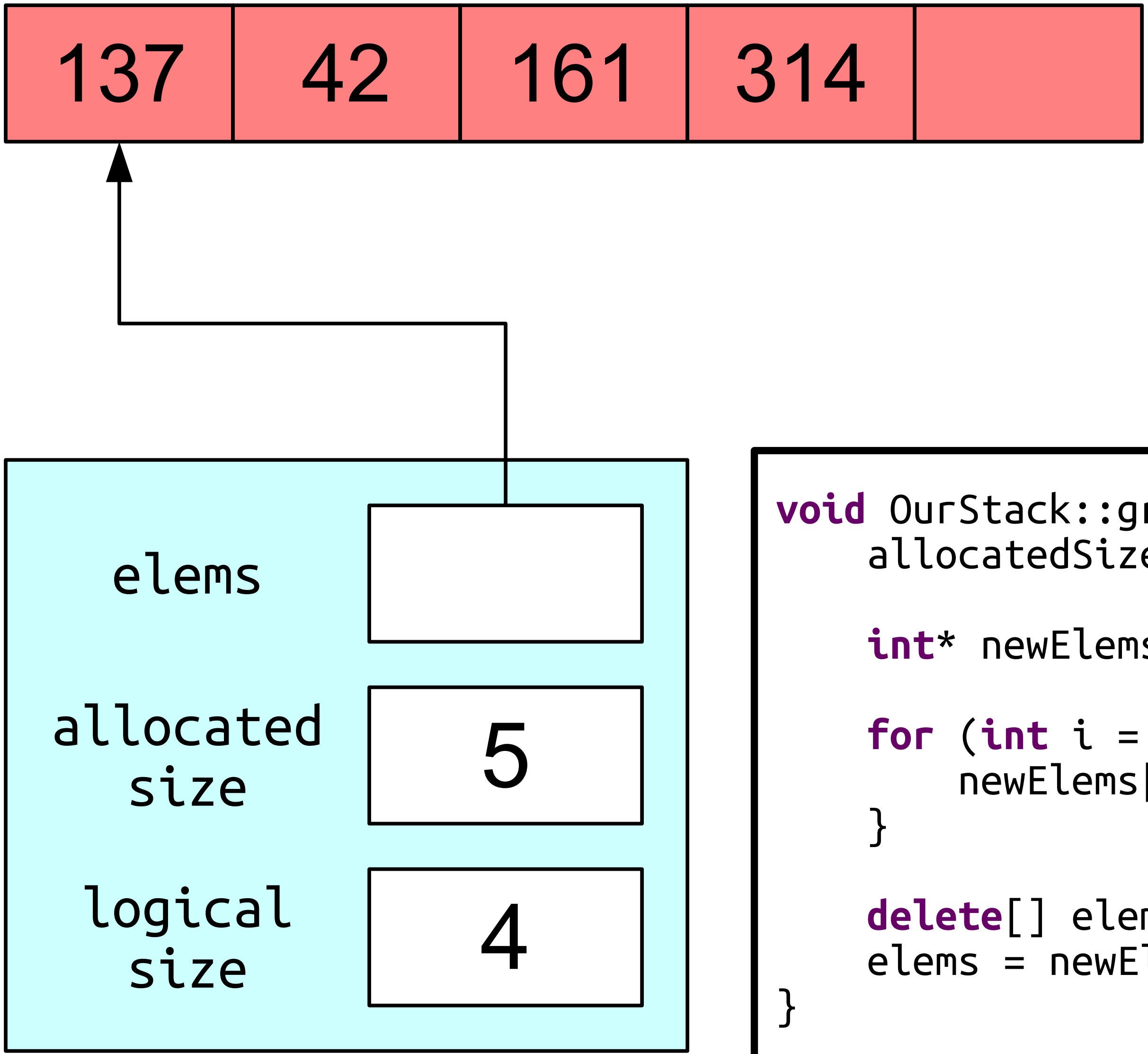


An Initial Idea



Ready… set… grow!

An Initial Idea




```
void OurStack::grow() {
    allocatedSize++;

    int* newElems = new int[allocatedSize];

    for (int i = 0; i < size(); i++) {
        newElems[i] = elems[i];
    }

    delete[] elems;
    elems = newElems;
}
```

Analyzing Our Approach

- We now have a working solution, but is it an *efficient* solution?
- Let's analyze the big-O complexity of the five operations.
  - `size`: **O(1)**
  - `isEmpty`: **O(1)**
  - `push`: **O(*n*)**
  - `pop`: **O(1)**
  - `peek`: **O(1)**

What This Means

- What is the complexity of pushing n elements and then popping them?
- Cost of the pushes:
  - $1 + 2 + 3 + 4 + \ldots + n =$ **$O(n^2)$**
- Cost of the pops:
  - $1 + 1 + 1 + 1 + \ldots + 1 =$ **$O(n)$**
- Total cost: **$O(n^2)$**

Validating Our Model

Time-Out for Announcements!

Assignment 4

- Assignment 4 is due on Friday.
- Recommendation: Aim to complete all the parts of the assignment by the end of this evening.
- We've posted a handy ***Assignment Submission Checklist*** up on the course website. Work through this before submitting – it'll help make sure your code is ready to go!

Midterm Exam

- The midterm exam is next Tuesday, February 21 from 7:00PM – 10:00PM.
  - Location TBA
- Covers topics up through and including big-O notation, plus Assignments 0 – 4.
- Closed-book, closed-computer, limited-note. You get one double-sided sheet of 8.5” × 11” notes when you take the exam. We also provide a library reference sheet.
- Practice exam posted on the course website.
- Need some practice? Work through the section handouts, the chapter exercises in the textbook, and revisit old assignments. Need more practice? Let us know!

Want to check out Treehacks?
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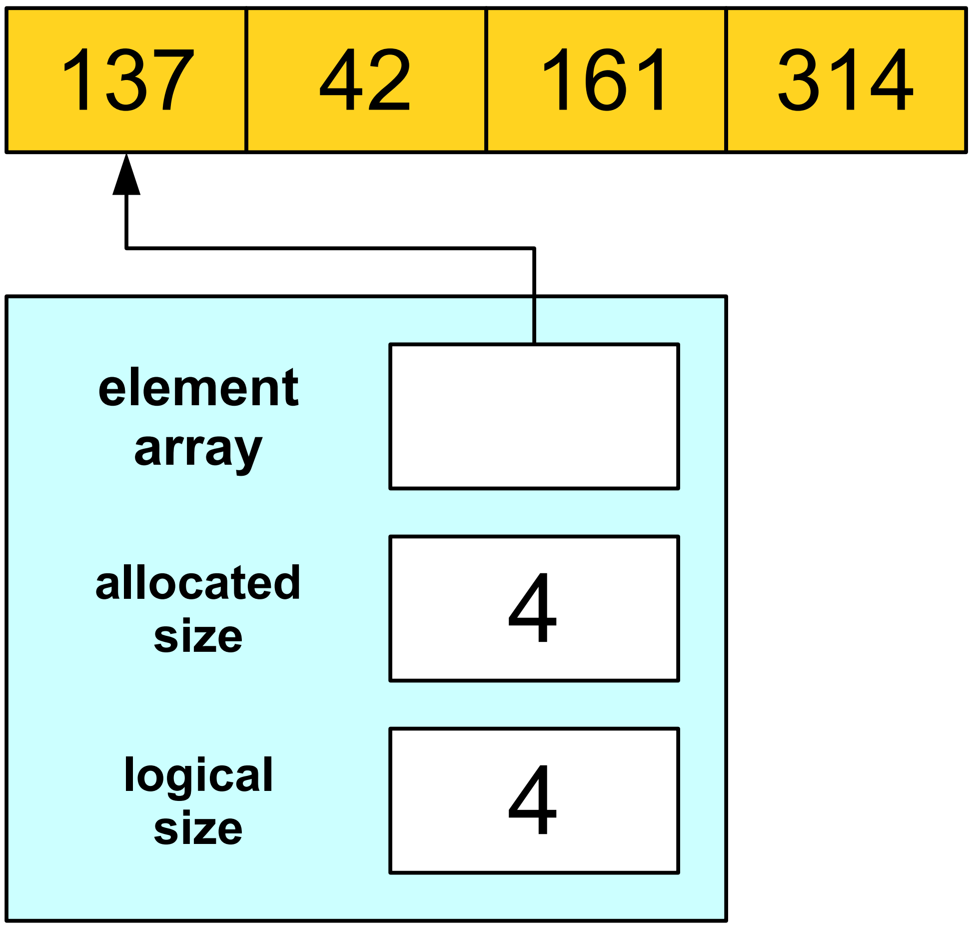
Back to the Stack!

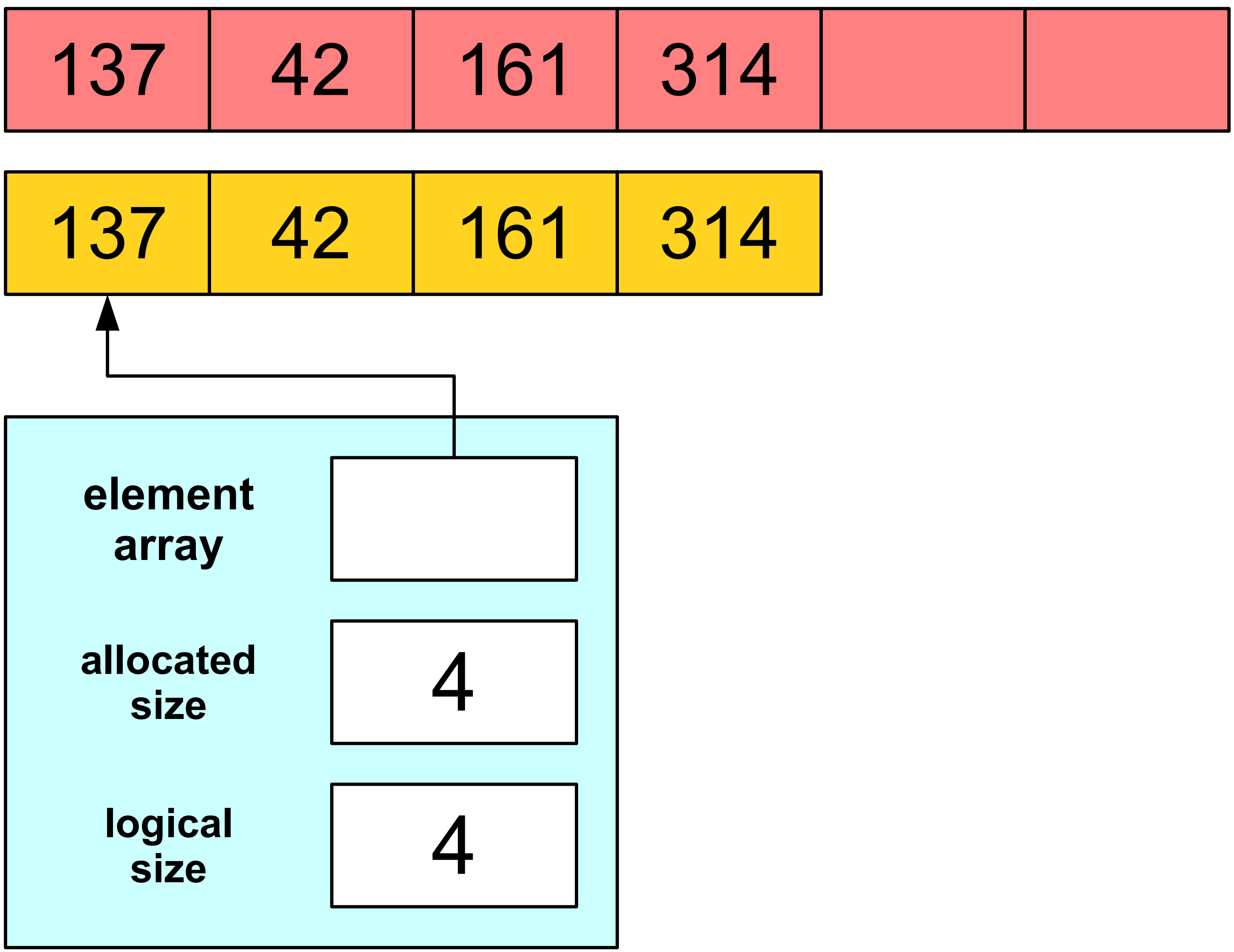
Speeding up the Stack

Key Idea: ***Plan for the Future***

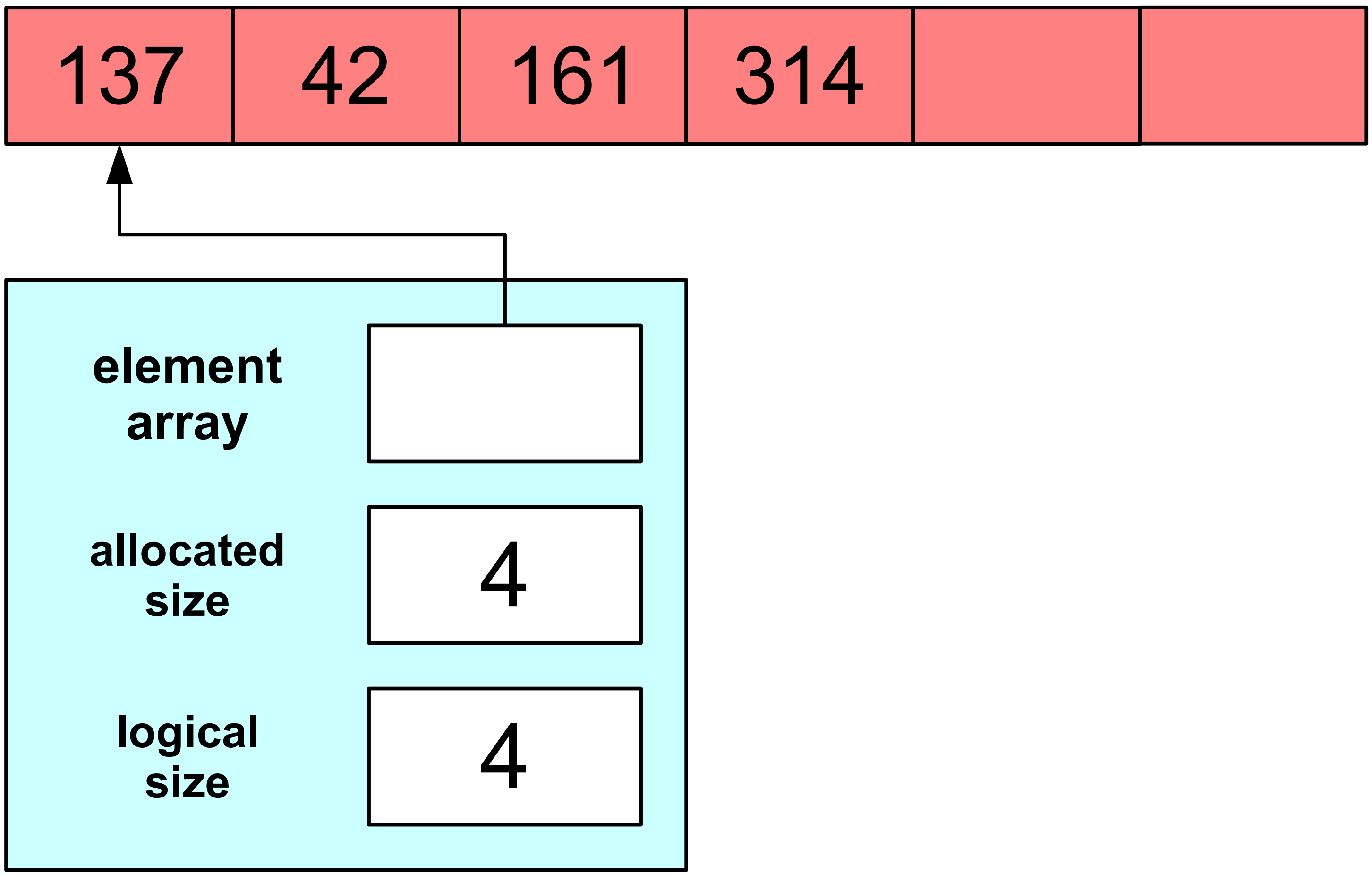
A Better Idea



A Better Idea



A Better Idea

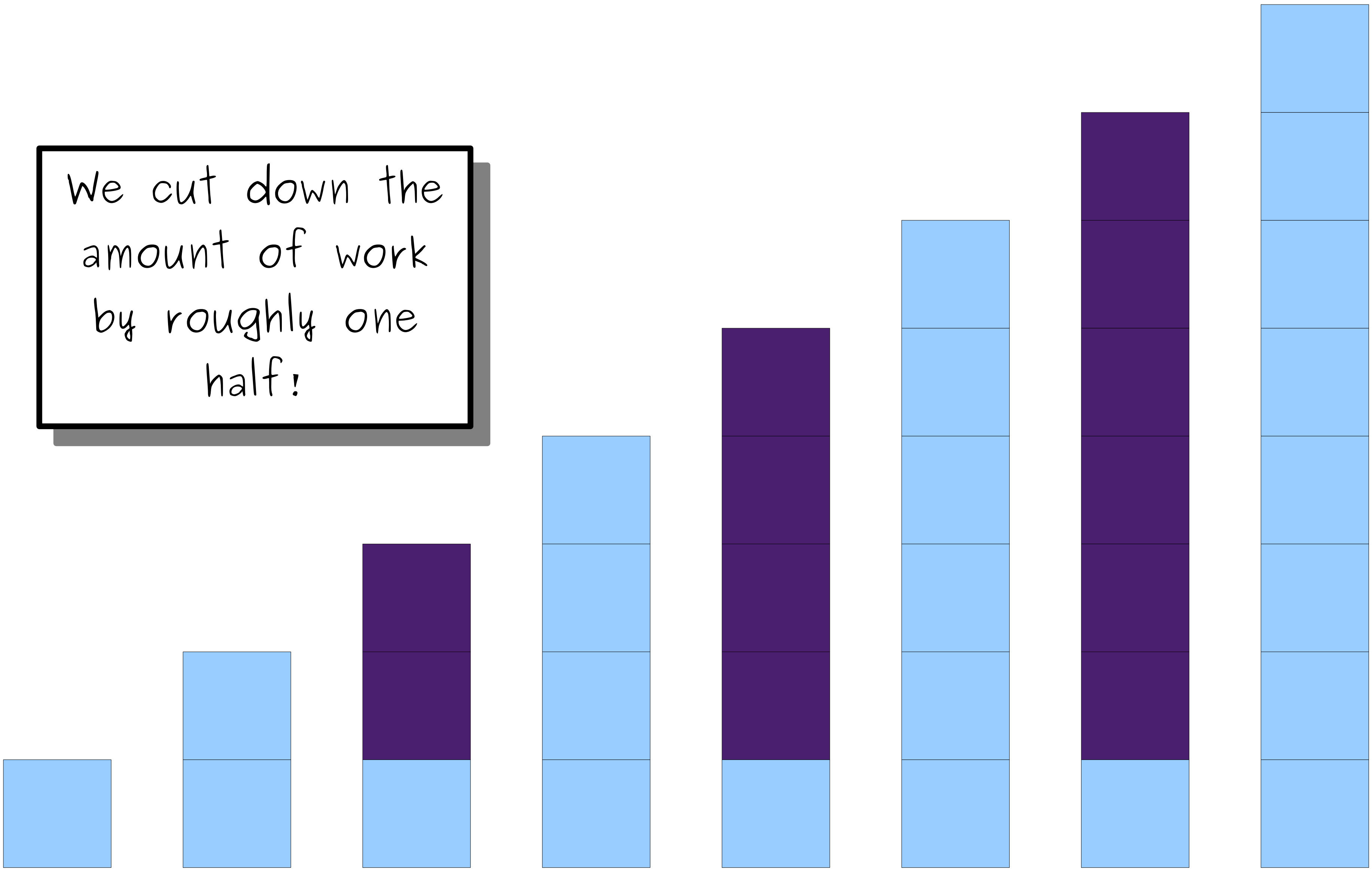


What Just Happened?

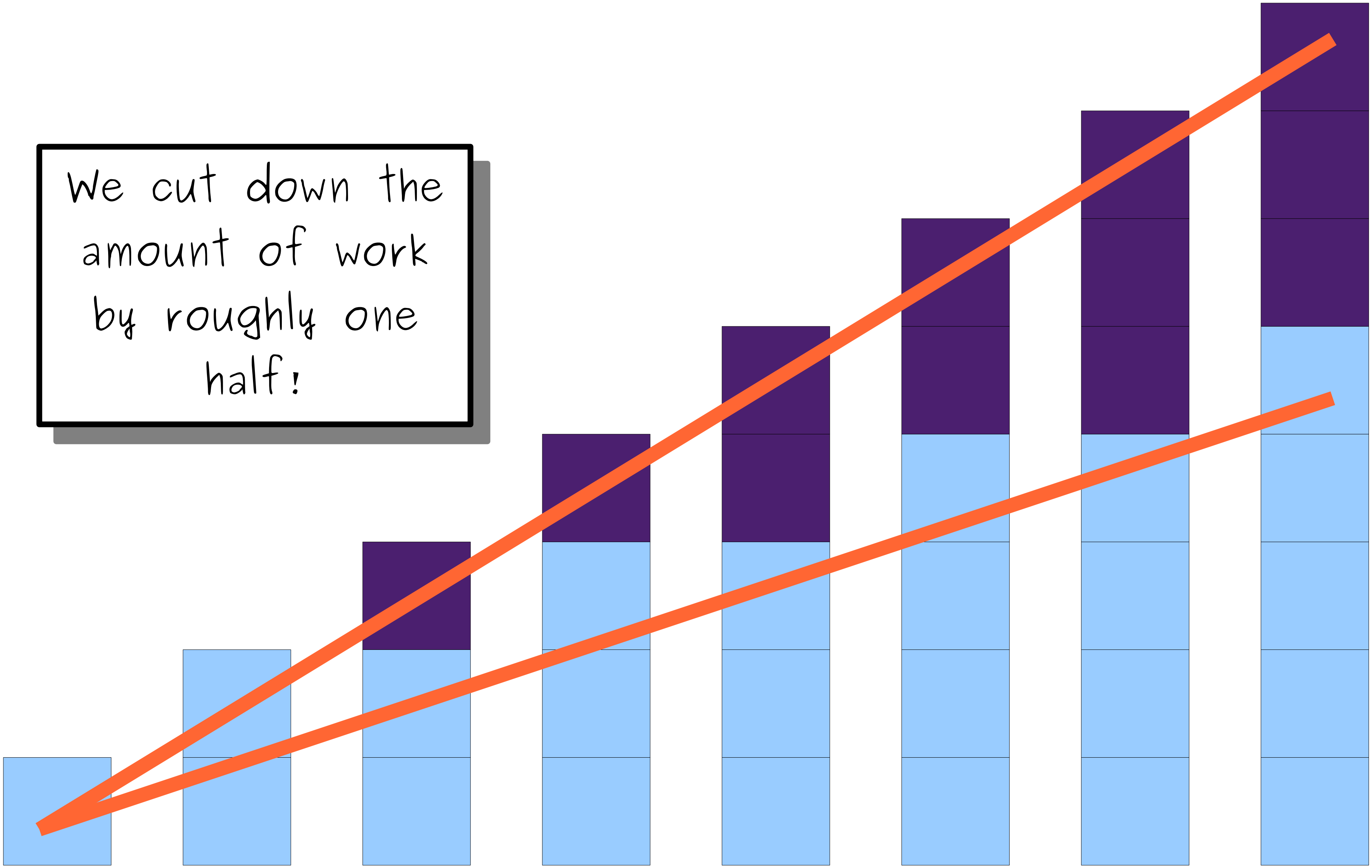
- Half of our pushes are now “easy” pushes, and half of our pushes are now “hard” pushes.
- Hard pushes still take time $O(n)$.
- Easy pushes only take time $O(1)$.
- Worst-case is still $O(n)$.
- What about the average case?

Analyzing the Work

We cut down the amount of work by roughly one half!

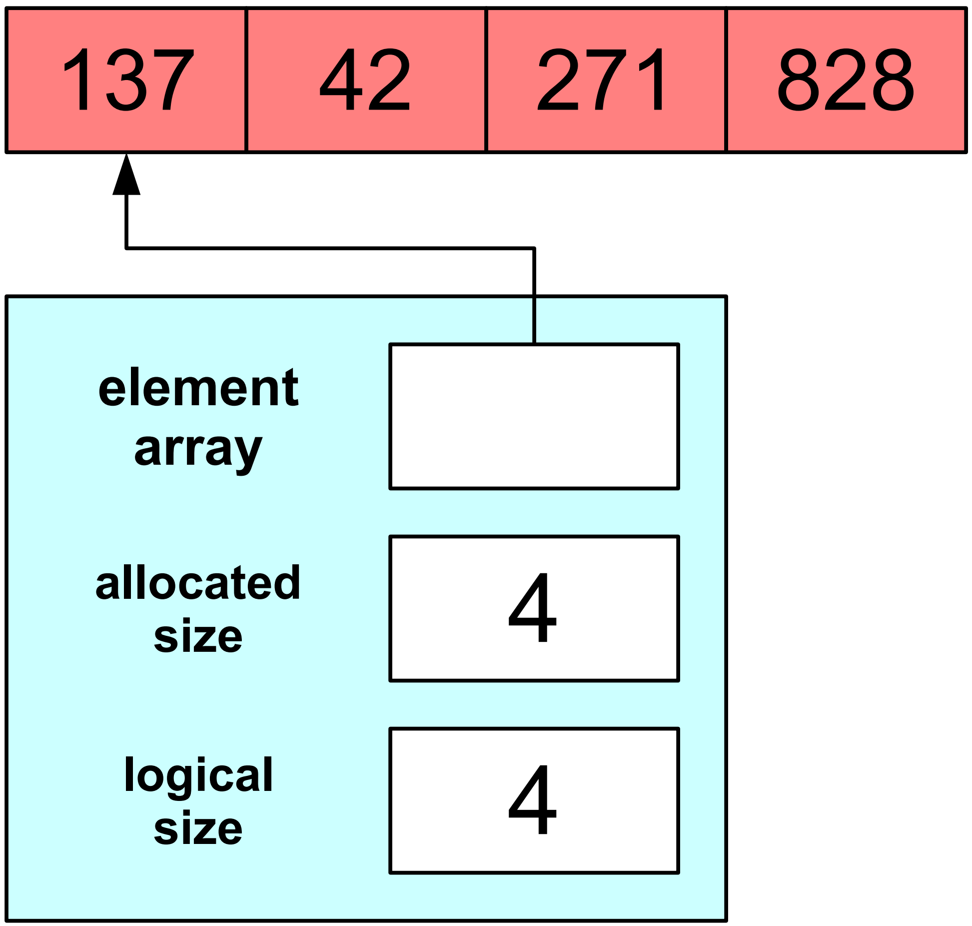


A Different Analysis

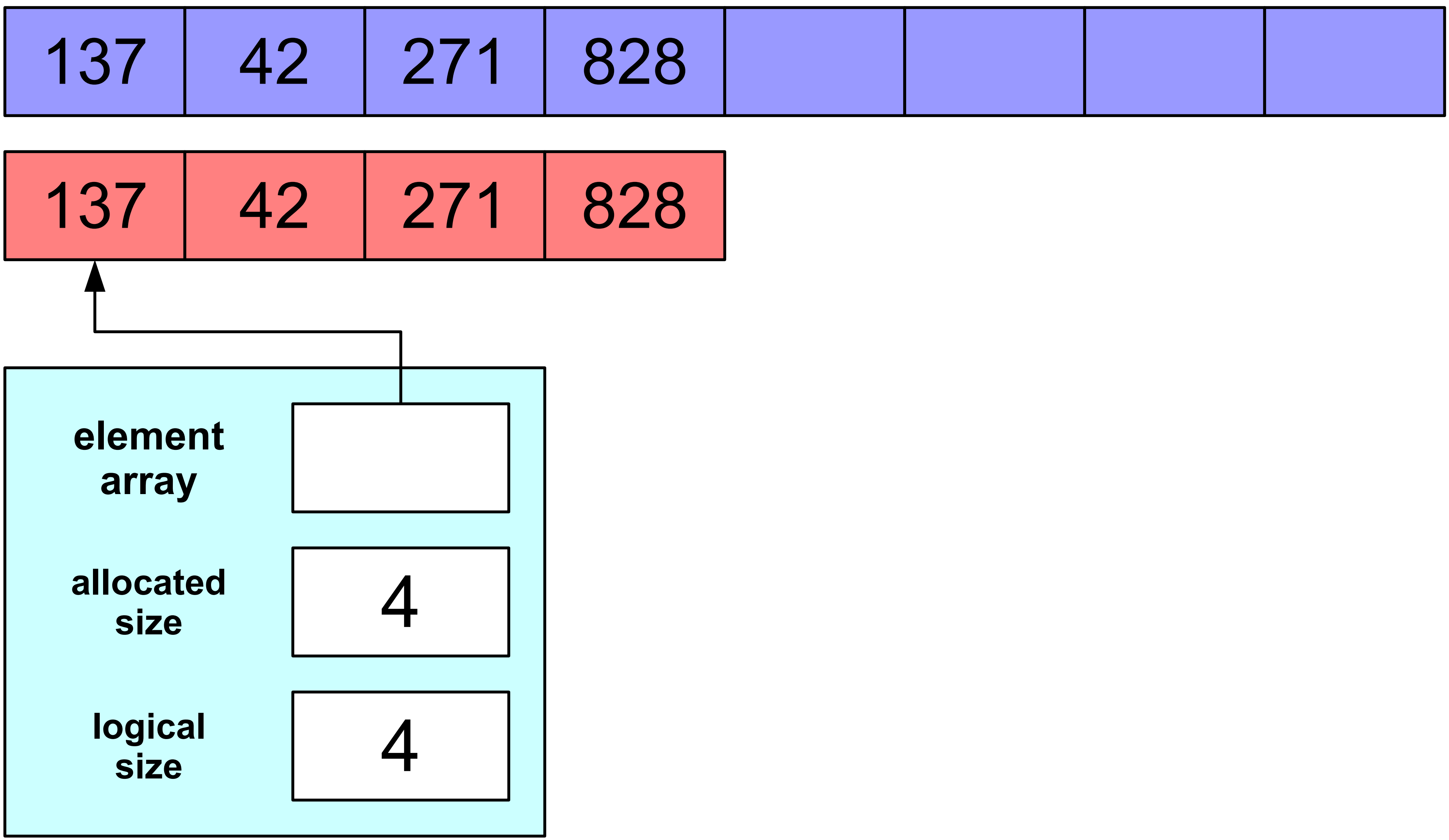


How does it stack up?

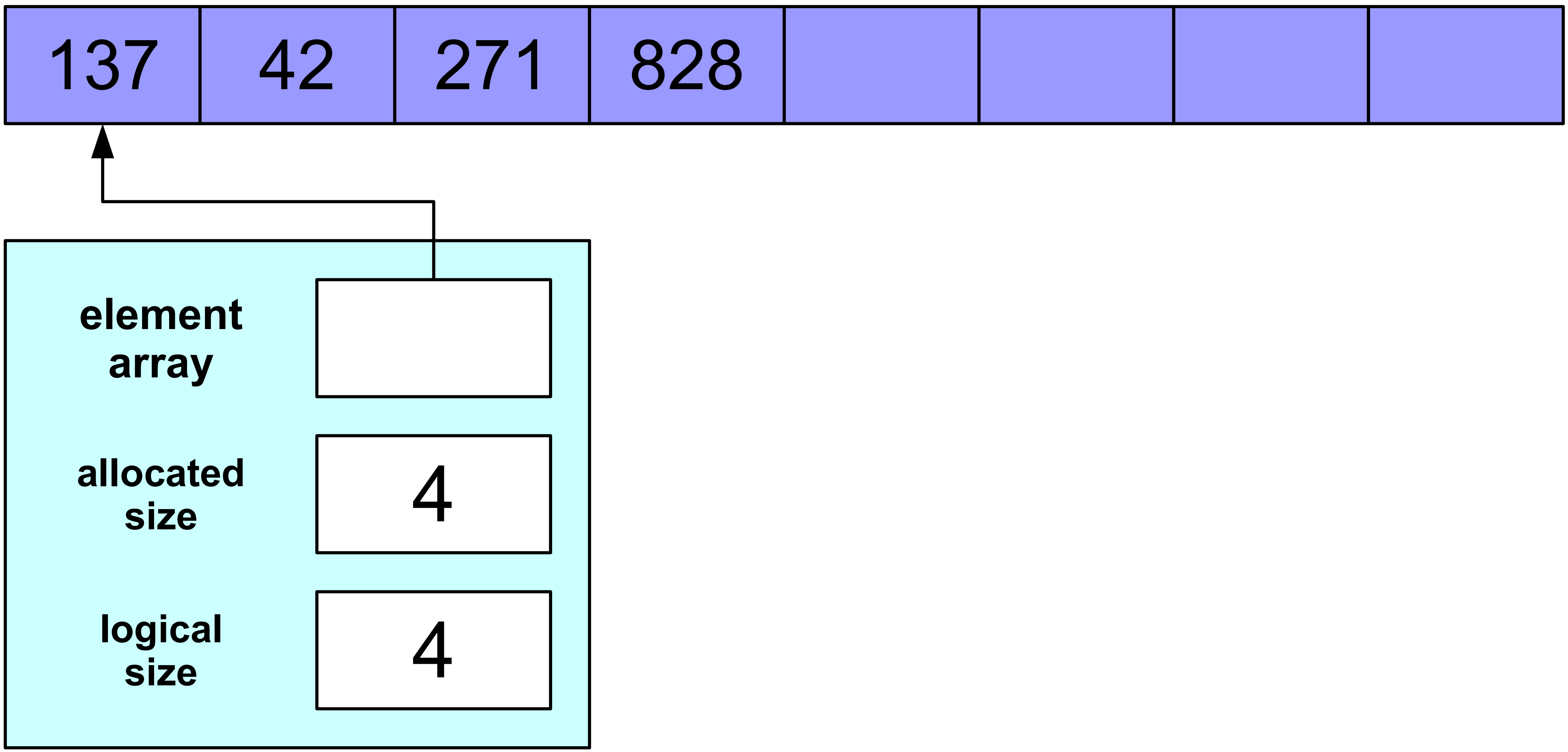
A Much Better Idea



A Much Better Idea



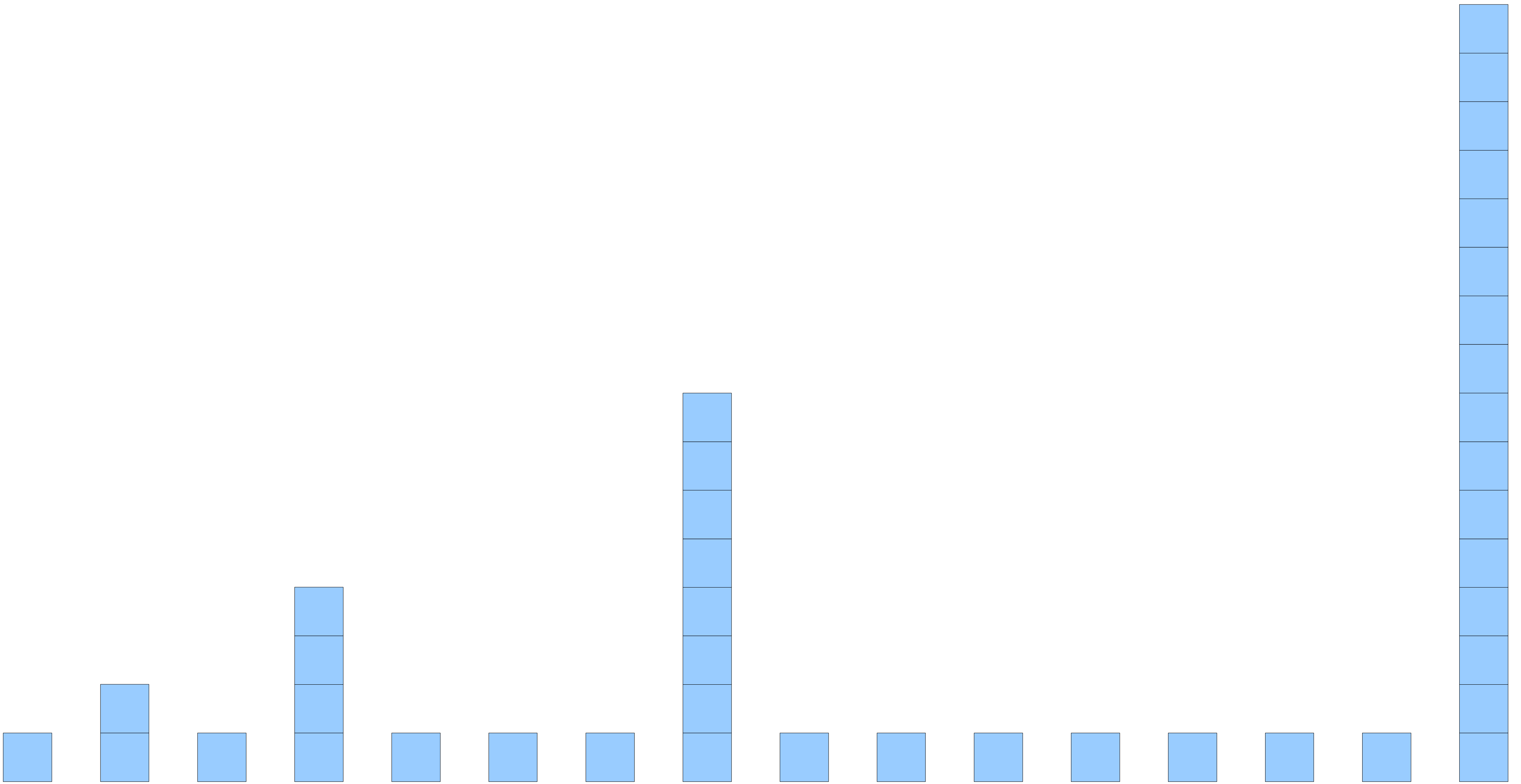
A Much Better Idea



Let's Give it a Try!

How do we analyze this?

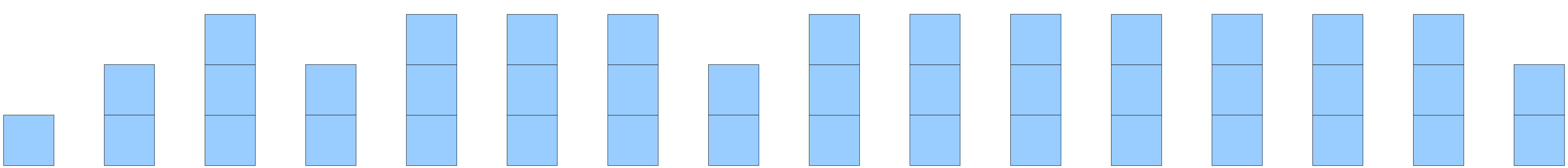
Spreading the Work



Spreading the Work

On average, we do just 3 units of work!

This is O(1) work on average!



Sharing the Burden

- We still have "heavy" pushes taking time O(n) and "light" pushes taking time O(1).
- Worst-case time for a push is O(n).
- Heavy pushes become so rare that the ***average*** time for a push is O(1).
- Can we confirm this?

Amortized Analysis

- The analysis we have just done is called an ***amortized analysis***.
- Reason about the total amount of work done, not the word done per operation.
- In an amortized sense, our implementation of the stack is extremely fast!
- This is one of the most common approaches to implementing `Stack`.

Implementing `Queue`

Implementing `Queue`

- We've just used dynamic arrays to implement a stack. Could we use them to implement a queue?
- Yes, but here's a better idea: ***could we use our stack to implement a queue?***

The Two-Stack Queue

- Maintain two stacks, an **In** stack and an **Out** stack.
- To enqueue an element, push it onto the **In** stack.
- To dequeue an element:
  - If the **Out** stack is empty, pop everything off the **In** stack and push it onto the **Out** stack.
  - Pop the **Out** stack and return its value.

Analyzing Efficiency

- How efficient is our two-stack queue?
- All enqueues just do one push.
- A dequeue might do a lot of pushes *and* a lot of pops.
- However, let's do an amortized analysis:
  - Each element is pushed at most twice and popped at most twice.
  - n enqueues and n dequeues thus do at most $4n$ pushes and pops.
  - Any $4n$ pushes / pops takes $O(n)$ amortized time.
  - Amortized cost: **O(1)** per operation.

Next Time

- ***Linked Lists***
  - A different way to represent sequences of elements.
- ***Dynamic Allocation Revisited***
  - What else can we allocate?