Thinking Recursively

Part V

A Little Word Puzzle

“What nine-letter word can be reduced to a single-letter word one letter at a time by removing letters, leaving it a legal word at each step?”

Shrinkable Words

- Let's define a ***shrinkable word*** as a word that can be reduced down to one letter by removing one character at a time, leaving a word at each step.
- ***Base Cases*:**
  - A string that is not a word is not a shrinkable word.
  - Any single-letter word is shrinkable (A, I, and O).
- ***Recursive Step*:**
  - A multi-letter word is shrinkable if you can remove a letter to form a shrinkable word.
  - A multi-letter word is not shrinkable if no matter what letter you remove, it’s not shrinkable.

Recursive Backtracking

- This code is an example of ***recursive backtracking***.
- At each step, we try one of many possible options.
- If *any* option succeeds, that's great! We're done.
- If *none* of the options succeed, then this particular problem can't be solved.

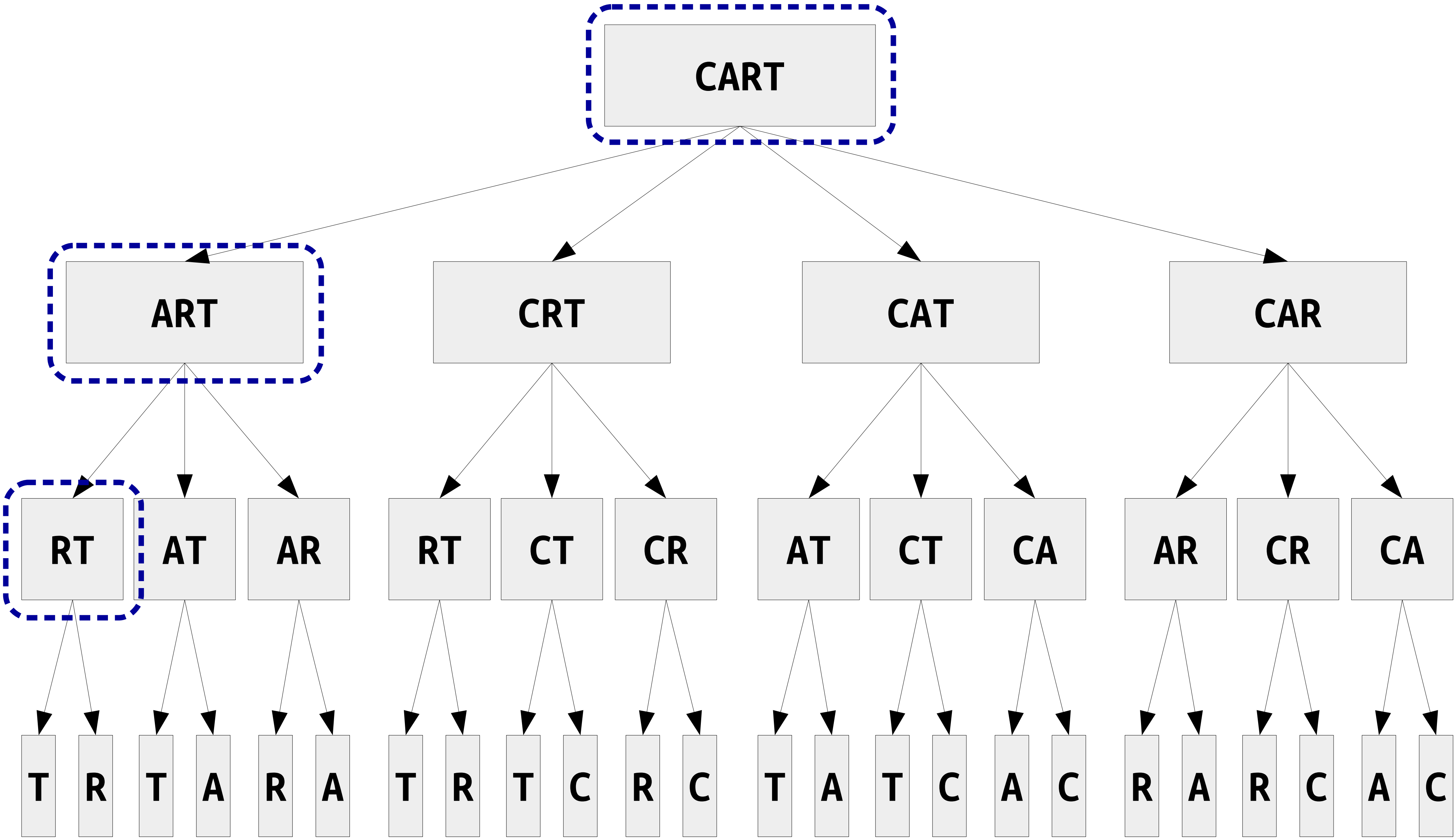
```
bool isShrinkable(const string& word, const Lexicon& english) {
    if (!english.contains(word)) return false;
    if (word.length() == 1) return true;

    for (int i = 0; i < word.length(); i++) {
        string shrunken = word.substr(0, i) + word.substr(i + 1);
        if (isShrinkable(shrunken, english)) {
            return true;
        }
    }
    return false;
}
```

```cpp
bool isShrinkable(const string& word, const Lexicon& english) {
    if (!english.contains(word)) return false;
    if (word.length() == 1) return true;

    for (int i = 0; i < word.length(); i++) {
        string shrunken = word.substr(0, i) + word.substr(i + 1);
        return isShrinkable(shrunken, english); // ⚠ Bad Idea ⚠
    }
    return false;
}
```

Tenacity is a Virtue



When backtracking recursively,
don’t give up if your first try fails!

Hold out hope that something else will work out. It very well might!

Recursive Backtracking

```
if (problem is sufficiently simple) {
    return whether or not the problem is solvable
} else {
    for (each choice) {
        try out that choice
        if (that choice leads to success) {
            return success;
        }
    }
    return failure;
}
```

Note that *if* the recursive call succeeds, *then* we return success. If it doesn't succeed, that doesn't mean we've failed – it just means we need to try out the next option.

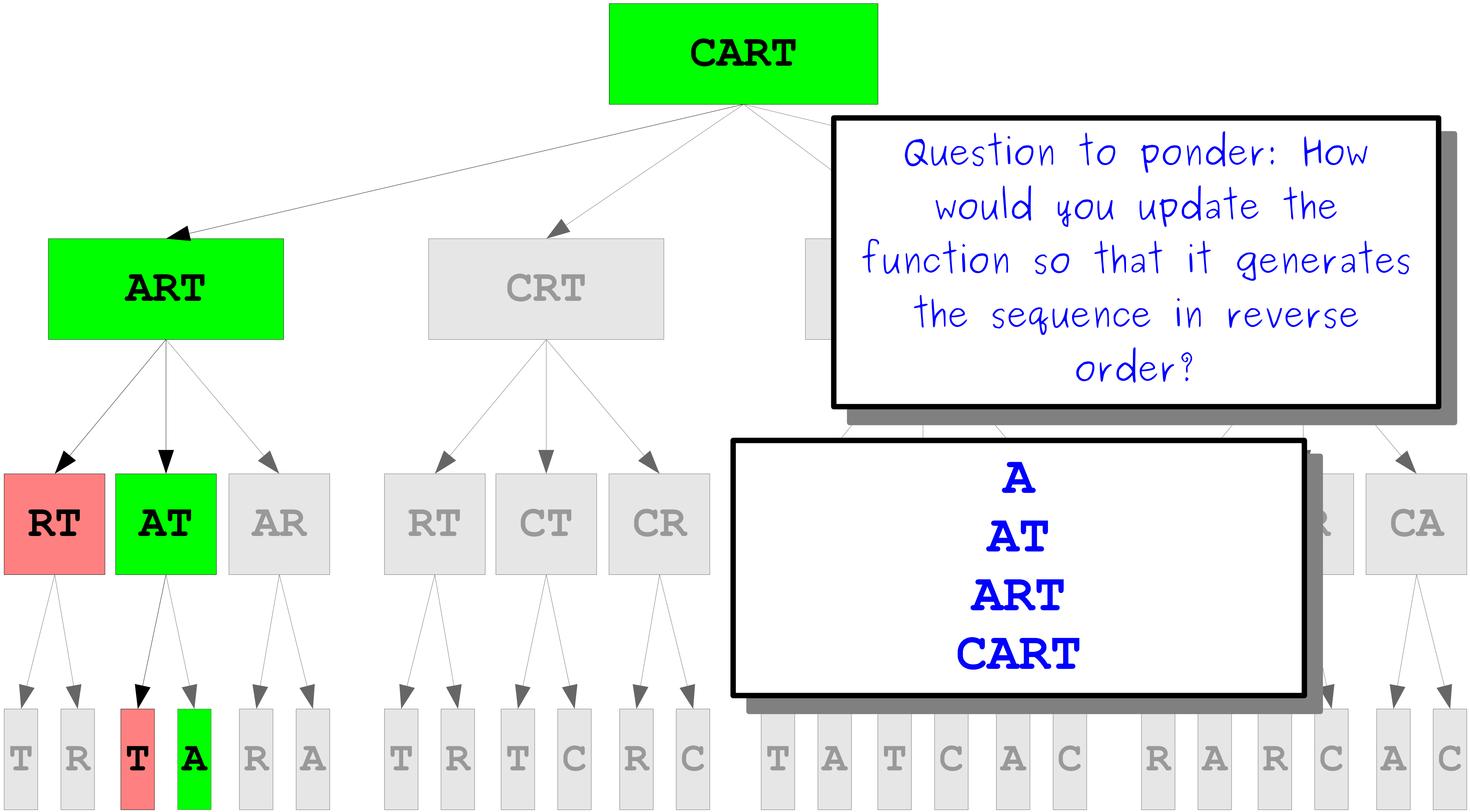
Extracting a Solution

- We now have a list of words that allegedly are shrinkable, but we don't actually know how to shrink them!
- Can the function tell us *how* to shrink the word?

Output Parameters

- An ***output parameter*** (or ***outparam***) is a parameter to a function that stores the result of that function.
- Caller passes the parameter by reference, function overwrites the value.
- Often used with recursive backtracking:
  - The return value says whether a solution exists.
  - If one does, it's loaded into the outparameter.

Generating the Answer



Question to ponder: How would you update the function so that it generates the sequence in reverse order?

A
AT
ART
CART

Dense Crosswords

aahs

abet

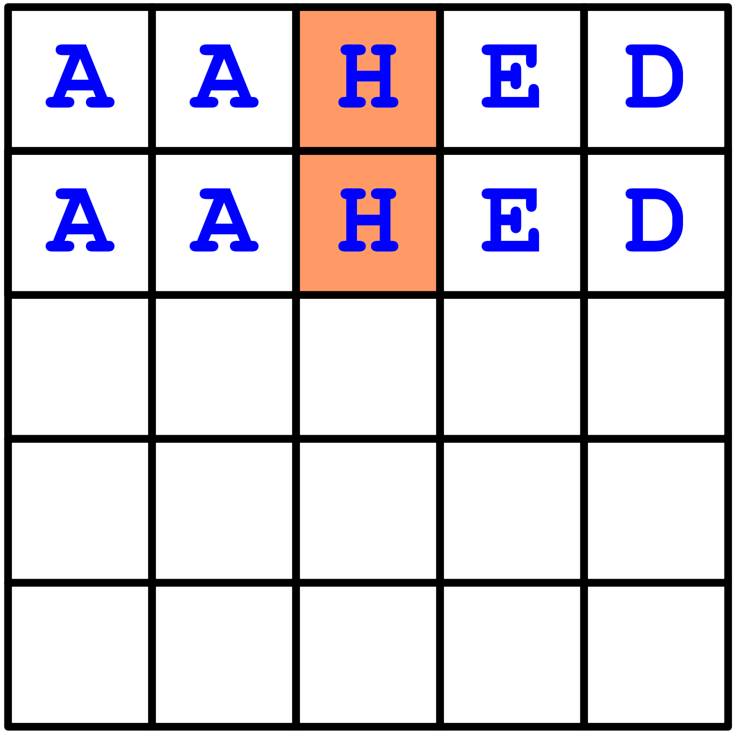
heme

stem

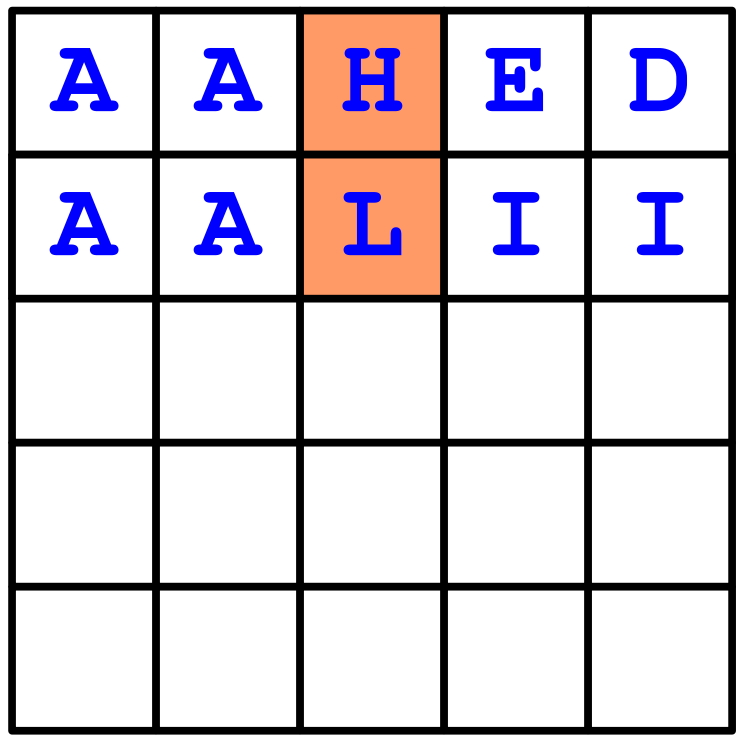
Generating Dense Crosswords

- ***Idea:*** Solve the problem "is there a way to extend this partial crossword into a full one?"
- ***Base Case:***
  - If the crossword is already filled in, then we just check whether it's legal.
- ***Recursive Step:***
  - For each possible word that can go in the current row, try extending the crossword with that word.
  - If the remainder can be extended to a full crossword, we're done!
  - If no matter what word we put in that row, we can't complete the crossword, there's no way to extend what we have.

Generating Dense Crosswords



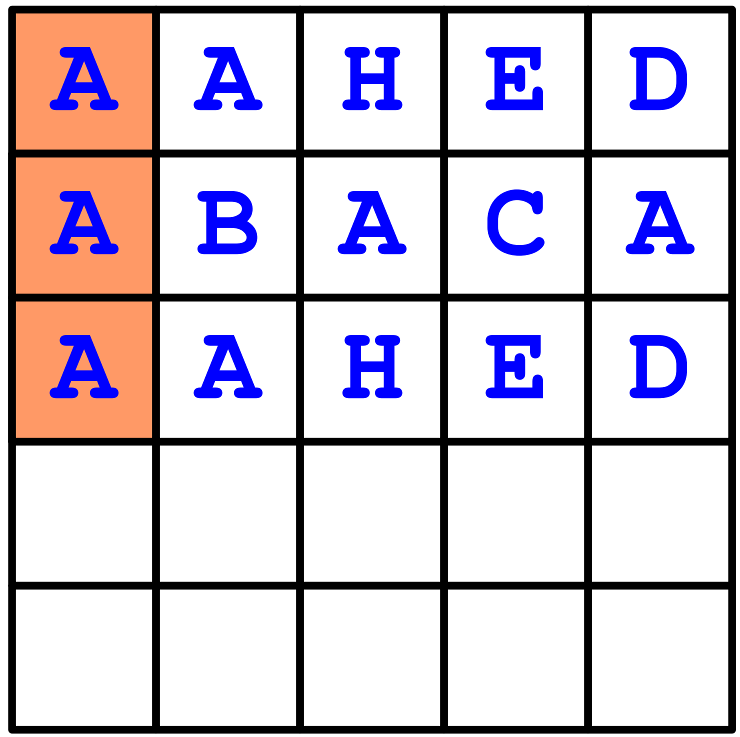
Generating Dense Crosswords



Generating Dense Crosswords

A	A	H	E	D
A	B	A	C	A

Generating Dense Crosswords



Generating Dense Crosswords

- ***Idea:*** Solve the problem "is there a way to extend this partial crossword into a full one?"
- ***Base Case:***
  - If the crossword is already filled in, then we just check whether it's legal.
  - If any column contains a string that isn't a prefix of any English word, report a failure without checking anything else.
- ***Recursive Step:***
  - For each possible word that can go in the current row, try extending the crossword with that word.
  - If the remainder can be extended to a full crossword, we're done!
  - If no matter what word we put in that row, we can't complete the crossword, there's no way to extend what we have.

Closing Thoughts on Recursion

You now know how to use recursion to ***view problems from a different perspective*** that can lead to ***short and elegant solutions***.

You’ve seen how to use recursion to ***enumerate all objects of some type***, which you can use to find the ***optimal solution to a problem***.

You’ve seen how to use recursive backtracking to ***determine whether something is possible*** and, if so to ***find some way to do it***.

You've seen that ***optimizing code*** is more about ***changing strategy*** than writing less code.

Next Time

- ***Algorithmic Analysis***
  - How do we formally analyze the complexity of a piece of code?