Thinking Recursively

Part III

Assignment 3

Assignment 3

- Assignment 3 (***Recursion!***) goes out today. It's due one week from today at the start of class.
  - ***You are permitted to work with a partner on this assignment***. Please make sure you understand the requirements for doing so before beginning. They're on the website.
  - There are two optional warm-up problems. We'll release solutions on Wednesday.
- Anton is holding YEAH hours (Your Early Assignment Help hours) tonight in 420-040 from 7PM – 8PM. Highly recommended!

Tracing the Recursion

{ A, H, I }	`{{A, H, I}, {A, H}, {A, I}, {A},` `{H, I}, {H}, {I}, { }}`
{ H, I }	`{ {H, I}, {H}, {I}, { } }`
{ I }	`{ {I}, { } }`
{ }	`{ { } }`

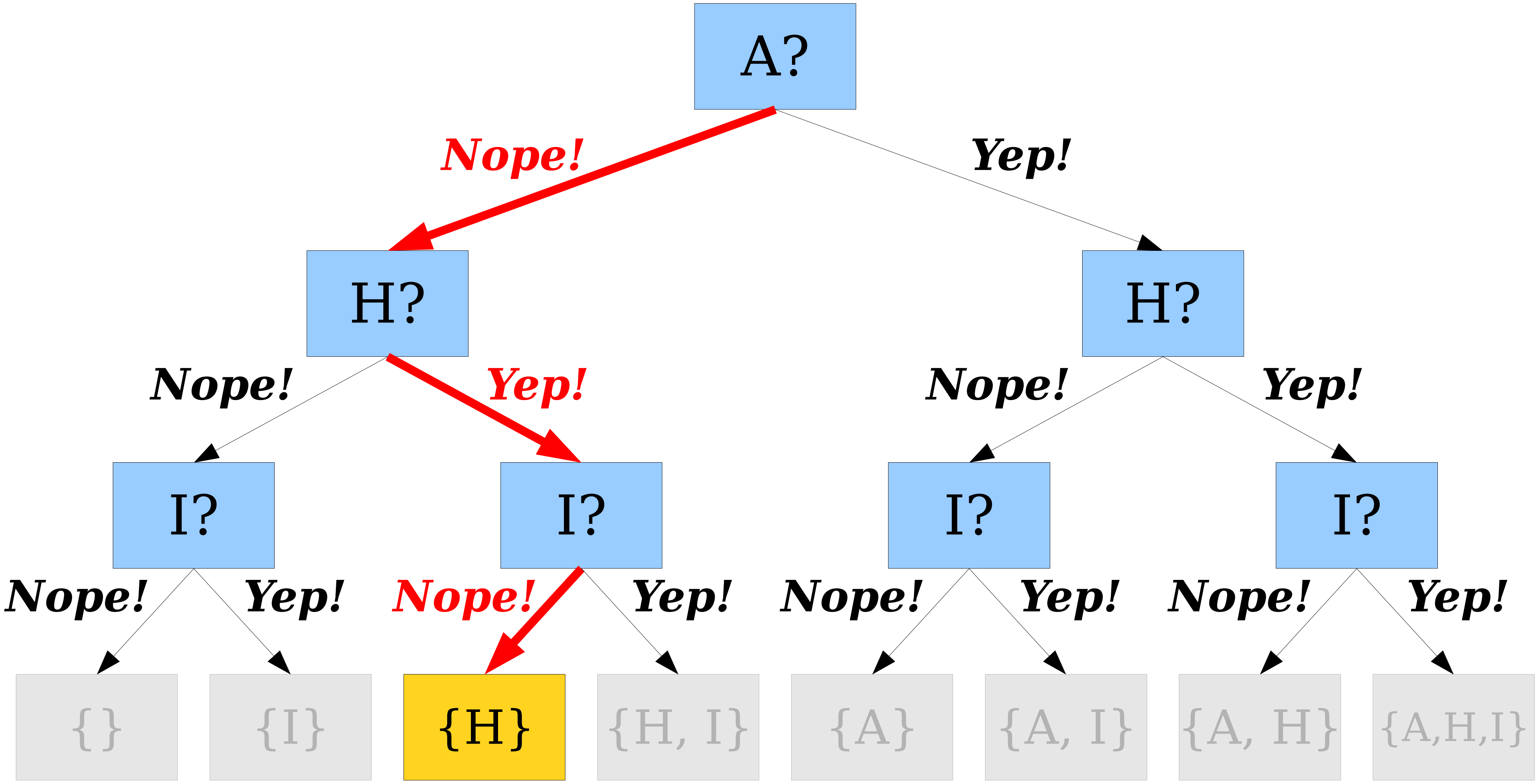
Analyzing Our Function

- ***Useful fact***: Given any *n*-element set, there are **2^n** subsets of that set.
- The returned collection of sets will need to have space for at least 2^n sets.
- For a modest value of *n* (say, $n = 50$), this will completely exceed system resources!

Reducing Memory Usage

- In many cases, we need to perform some operation on each subset, but don't need to actually store those subsets.
- ***Idea:*** Generate each subset, process it, and then discard it.
- ***Question:*** How do we do this?

A Decision Tree



The Template

The Present

The Past

```
void exploreFrom(current state, decisions made) {
    if (all decisions have been made) {
        output the result of the decisions we've made;
    } else {
        for (each decision we can make) {
            exploreFrom(result of making that decision,
                        decisions made + this decision);
        }
    }
}

void exploreAllTheThings(initial state) {
    exploreFrom(initial state, {});
}
```

The Future!

You own a classy print shop.

You've got a list of jobs you print.

Each job requires some amount of time and has a hard deadline.

Which jobs should you pick to maximize your profit?

Permutations

- A ***permutation*** of a sequence is a sequence with the same elements, though possibly in a different order.
- For example:
  - E Pluribus Unum
  - E Unum Pluribus
  - Pluribus E Unum
  - Pluribus Unum E
  - Unum E Pluribus
  - Unum Pluribus E

Generating Permutations

x_1	x_2	x_3	x_4

x_1	x_2	x_3	x_4
x_1	x_2	x_4	x_3
x_1	x_3	x_2	x_4
x_1	x_3	x_4	x_2
x_1	x_4	x_2	x_3
x_1	x_4	x_3	x_2

x_2	x_1	x_3	x_4
x_2	x_1	x_4	x_3
x_2	x_3	x_1	x_4
x_2	x_3	x_4	x_1
x_2	x_4	x_1	x_3
x_2	x_4	x_3	x_1

x_3	x_1	x_2	x_4
x_3	x_1	x_4	x_2
x_3	x_2	x_1	x_4
x_3	x_2	x_4	x_1
x_3	x_4	x_1	x_2
x_3	x_4	x_2	x_1

x_4	x_1	x_2	x_3
x_4	x_1	x_3	x_2
x_4	x_2	x_1	x_3
x_4	x_2	x_3	x_1
x_4	x_3	x_1	x_2
x_4	x_3	x_2	x_1

Generating Permutations

x_1	x_2	x_3	x_4

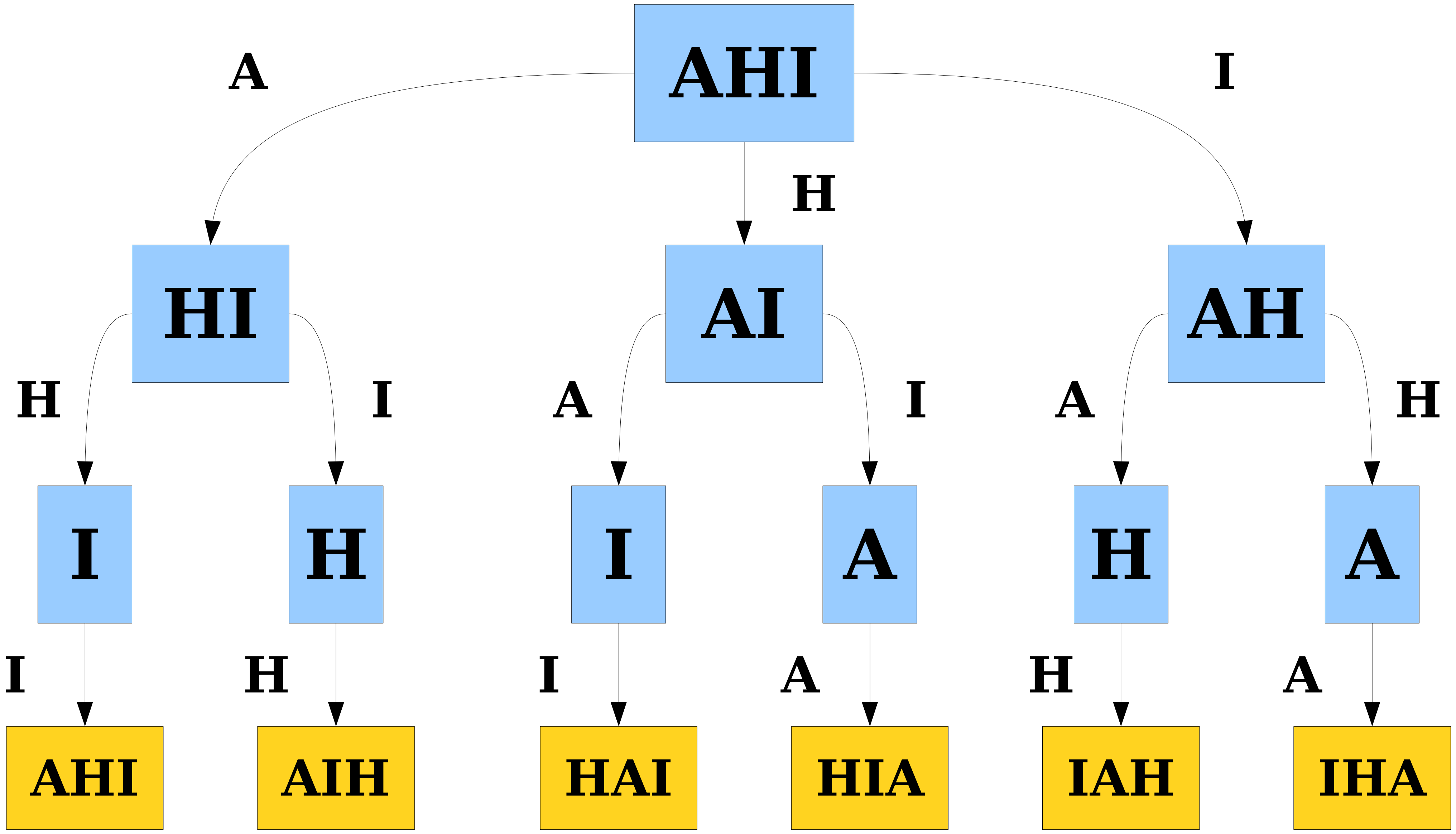
x_1	x_2	x_3	x_4
x_1	x_2	x_4	x_3
x_1	x_3	x_2	x_4
x_1	x_3	x_4	x_2
x_1	x_4	x_2	x_3
x_1	x_4	x_3	x_2

x_2	x_1	x_3	x_4
x_2	x_1	x_4	x_3
x_2	x_3	x_1	x_4
x_2	x_3	x_4	x_1
x_2	x_4	x_1	x_3
x_2	x_4	x_3	x_1

x_3	x_1	x_2	x_4
x_3	x_1	x_4	x_2
x_3	x_2	x_1	x_4
x_3	x_2	x_4	x_1
x_3	x_4	x_1	x_2
x_3	x_4	x_2	x_1

x_4	x_1	x_2	x_3
x_4	x_1	x_3	x_2
x_4	x_2	x_1	x_3
x_4	x_2	x_3	x_1
x_4	x_3	x_1	x_2
x_4	x_3	x_2	x_1

A Decision Tree



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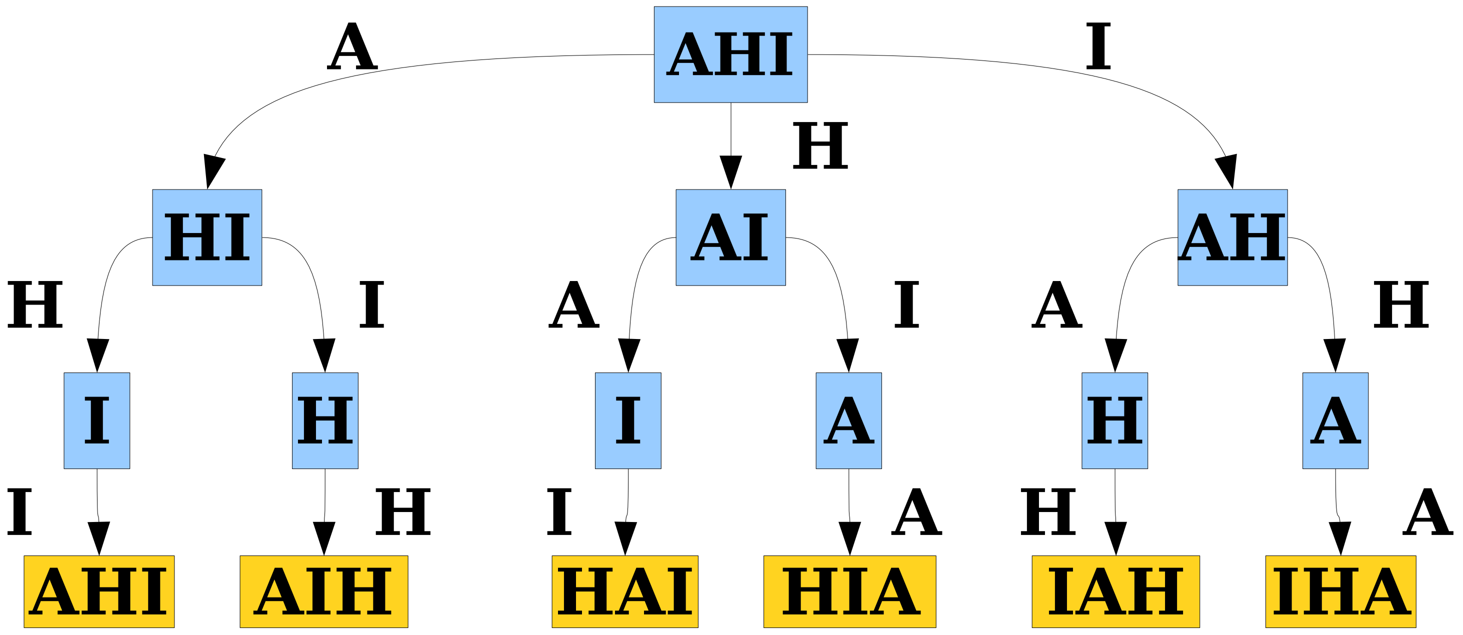

Your Action Items

- Start working on Assignment 3.
  - ***Don’t put this one off!*** It’s going to require some thought.
- Stop by YEAH Hours to get some help on how to get started on this assignemtn.
- Read Chapter 8, if you haven’t yet done so.
- Start reading Chapter 9 in preparation for Wednesday’s lecture.

Next Time

- ***Generating Combinations***
  - How do we find the best group of people to pick for a task?
- ***Recursive Backtracking***
  - How do we determine whether something is feasible?