

Linked Lists

Part Two

Recursion is *Awesome*

<http://recursivedrawing.com/>

Friday Four Square!
4:15PM, Outside Gates

Apply to Section Lead!

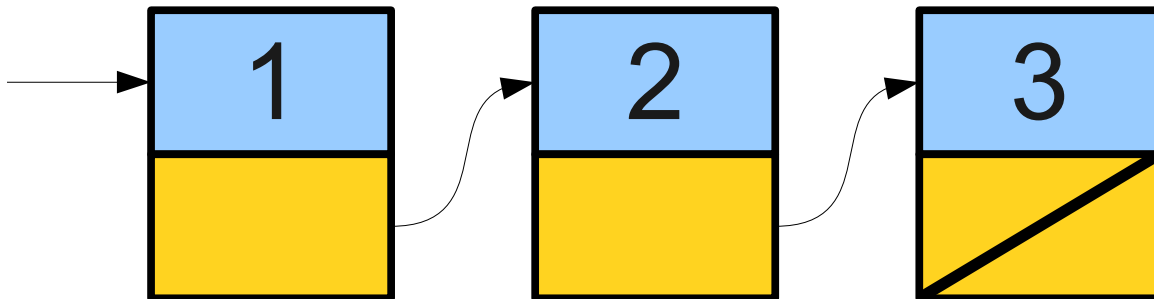
<http://cs198.stanford.edu>

Announcements

- Assignment 4 due right now.
- Assignment 5 (**Priority Queue**) out today, due Wednesday, May 23
 - Implement a powerful collection class.
 - Master dynamic allocation and linked lists.
 - YEAH hours next Tuesday from 4:15 – 5:45 in 380-380C.

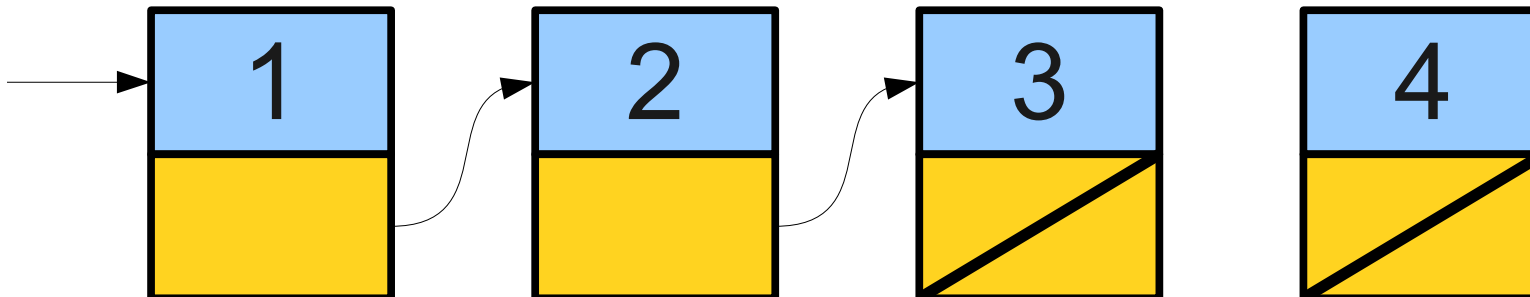
Linked Lists at a Glance

- A **linked list** is a data structure for storing a sequence of elements.
- Each element is stored separately from the rest.
- The elements are then chained together into a sequence.



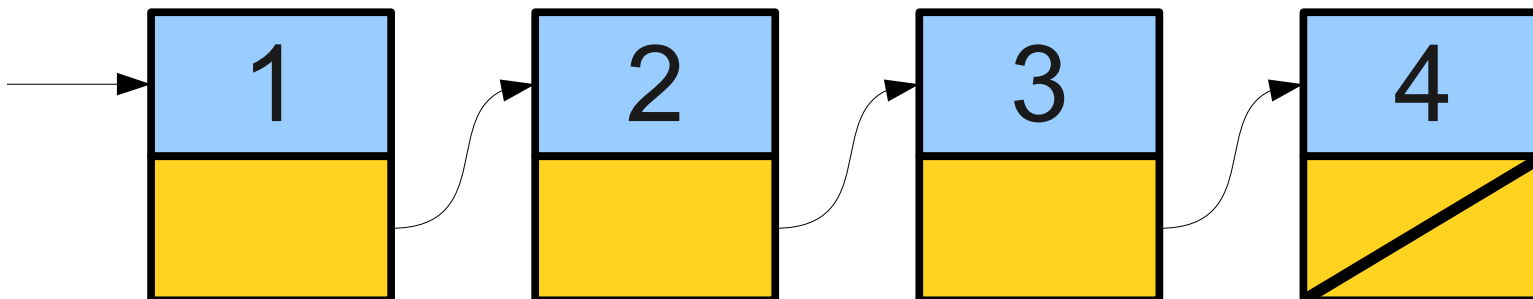
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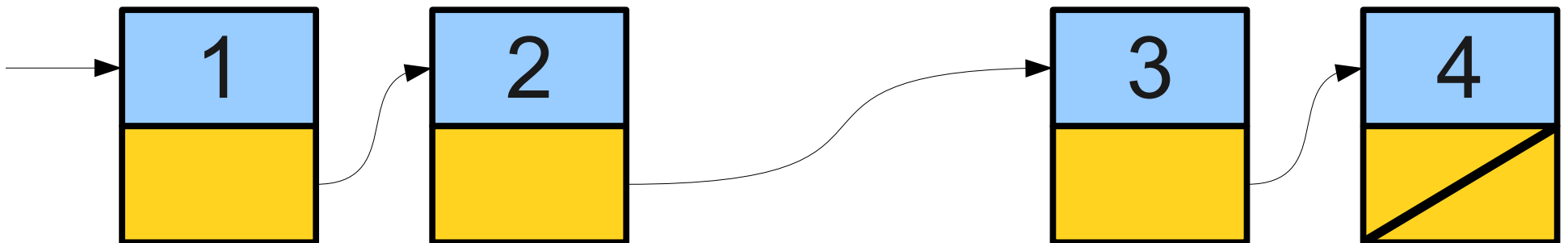
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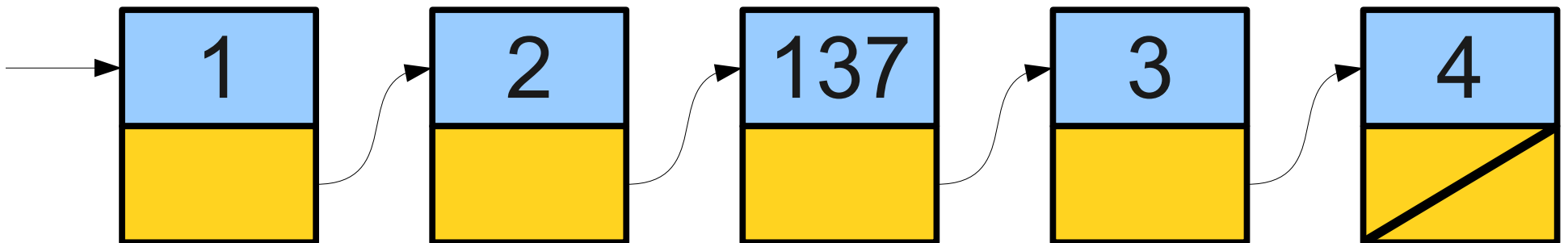
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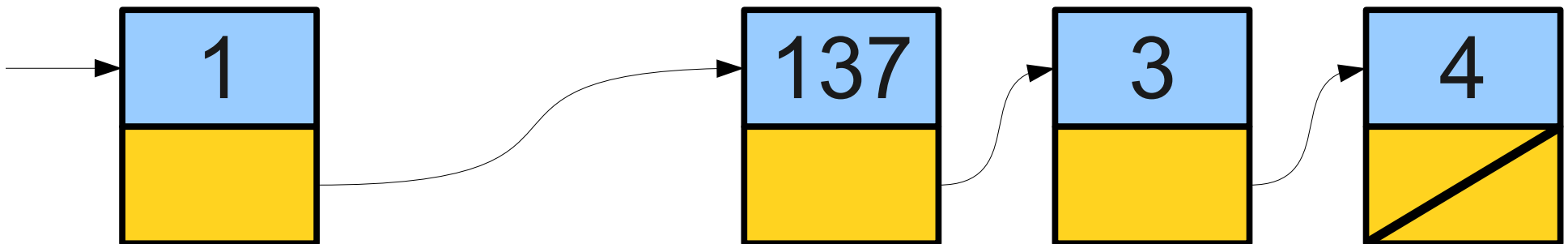
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Linked List Cells

- A linked list is a chain of **cells**.
- Each cell contains two pieces of information:
 - Some piece of data that is stored in the sequence, and
 - A **link** to the next cell in the list.
- We can traverse the list by starting at the first cell and repeatedly following its link.

Representing a Cell

- For simplicity, let's assume we're building a linked list of `strings`.
- We can represent a cell in the linked list as a structure:

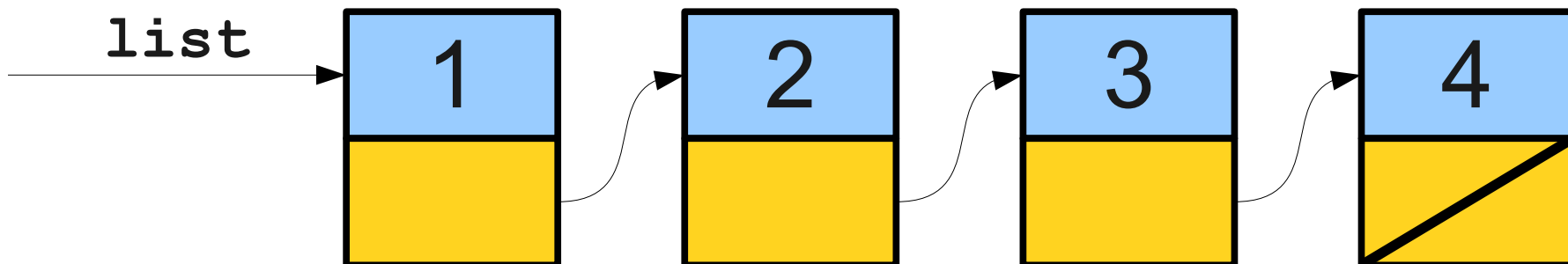
```
struct Cell {  
    string value;  
    Cell* next;  
};
```

- **The structure is defined recursively!**

Traversing a Linked List

- Once we have a linked list, we can traverse it by following the links one at a time.

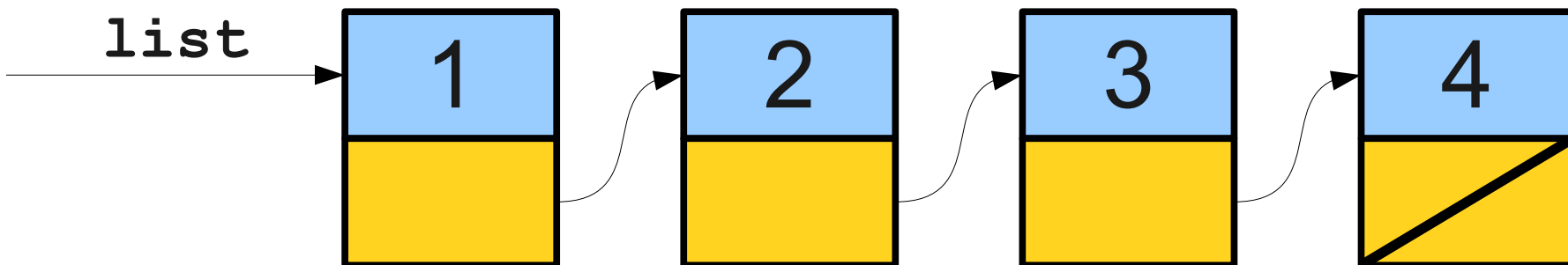
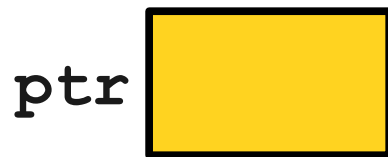
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for (Cell* ptr = list; ptr != NULL; ptr = ptr->next) {  
    /* ... use ptr ... */  
}
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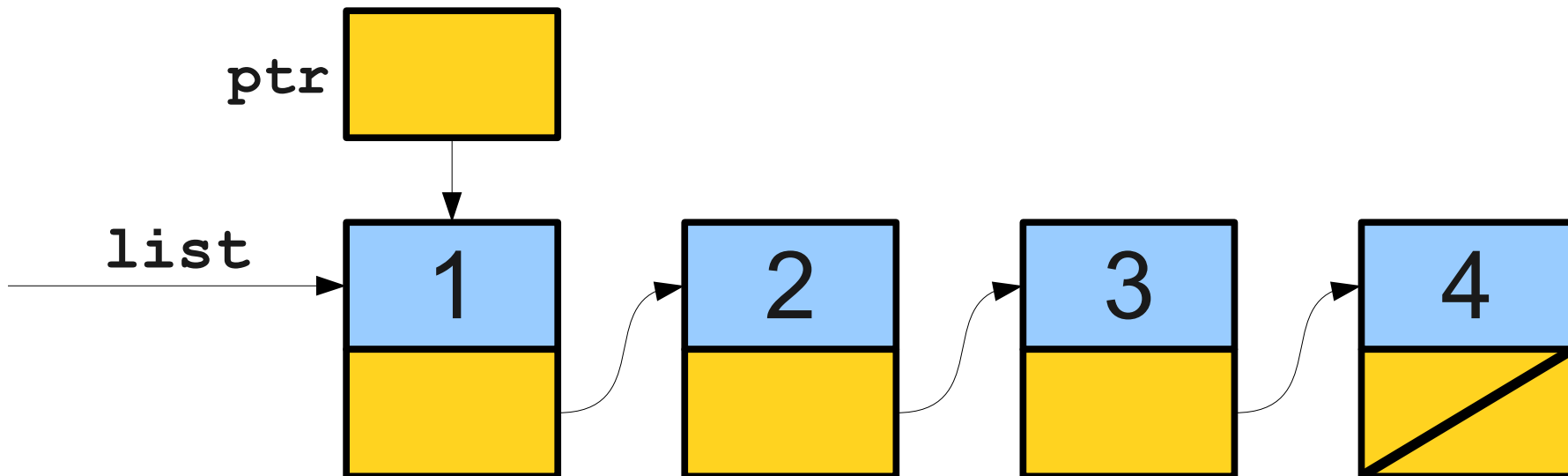
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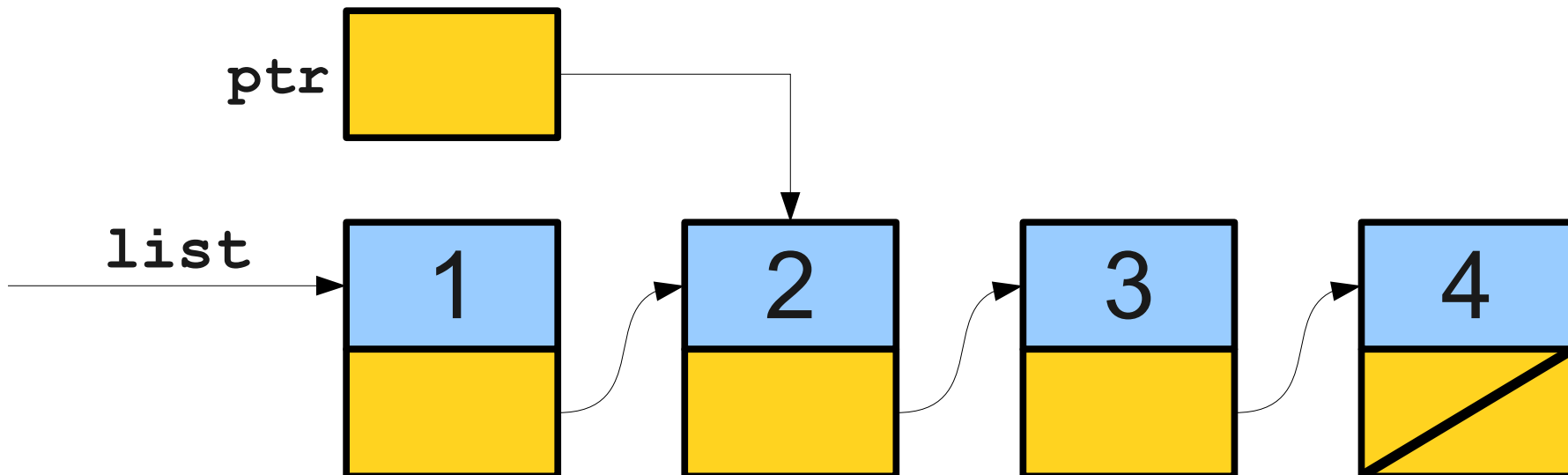
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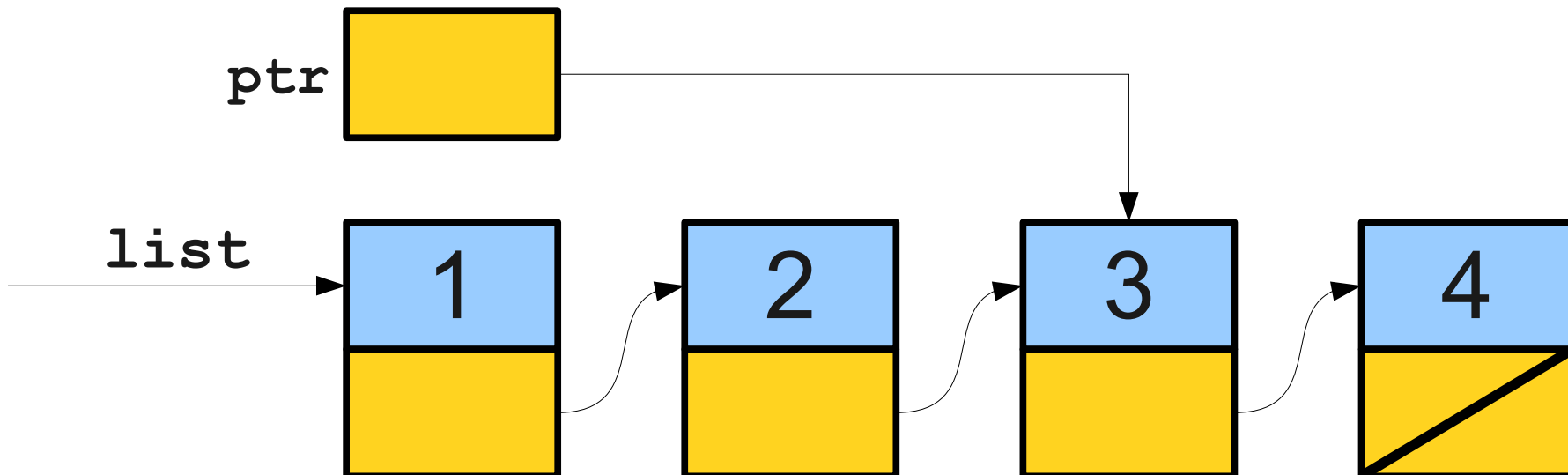
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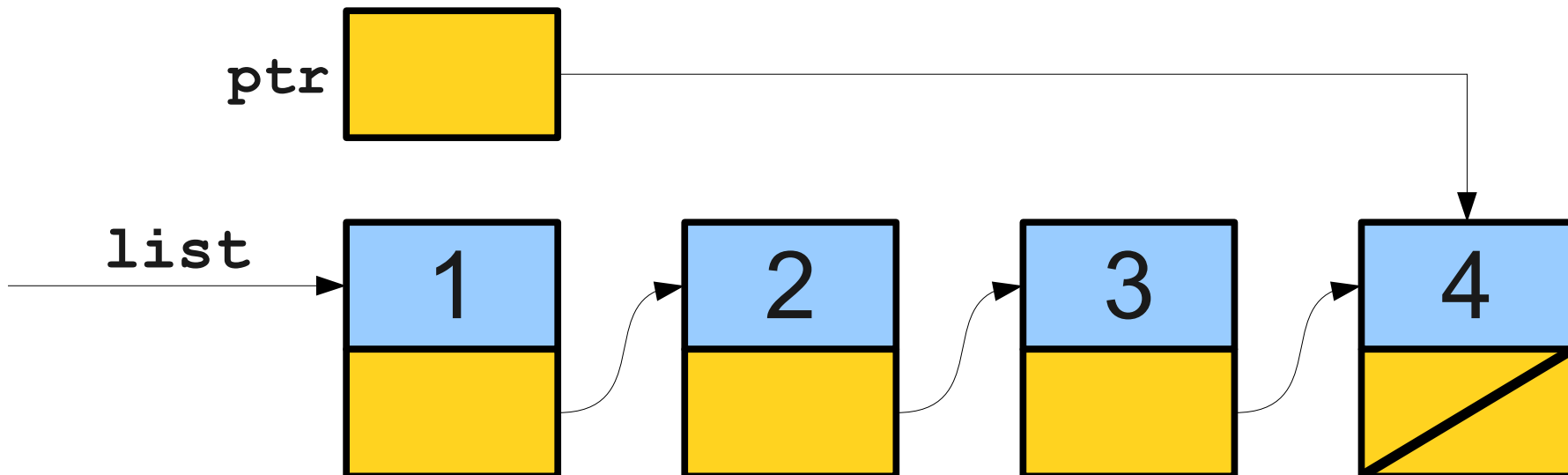
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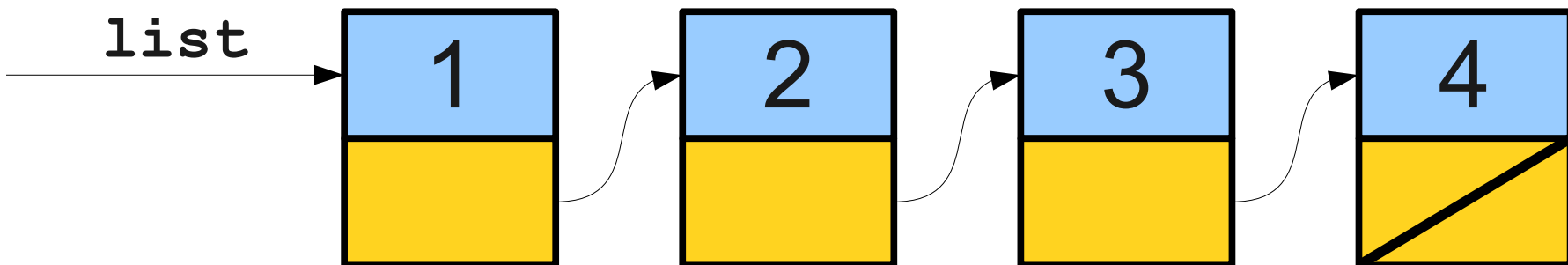
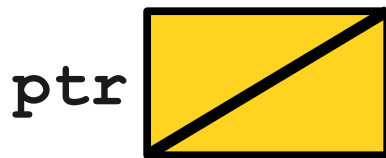
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Traversing a Linked List

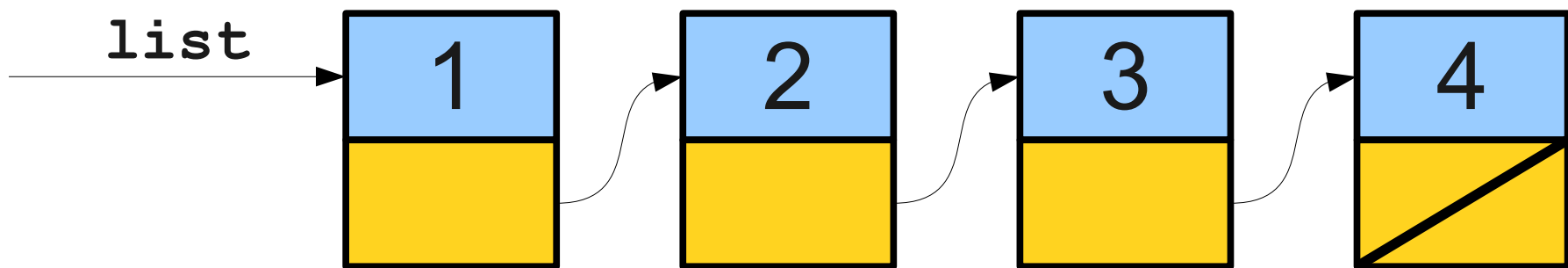
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```
for (Cell* ptr = list; ptr != NULL; ptr = ptr->next) {  
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}
```



A Recursive View of Linked Lists

- We can think of linked lists **recursively**.
- The empty list of no cells (represented by a **NULL** pointer) is a linked list.
- A linked list cell followed by a linked list is a linked list.



Once More With Recursion

- Linked lists are defined recursively, and we can traverse them using recursion!

```
void recursiveTraverse(Cell* list) {  
    if (list == NULL) return;  
    /* ... do something with list ... */  
    recursiveTraverse(list->next);  
}
```

Freeing a Linked List

- All good things must come to an end, and we eventually need to reclaim the memory for a linked list.
- The following is an Extremely Bad Idea:

```
for (Cell* ptr = list; ptr != NULL; ptr = ptr->next) {  
    delete ptr;  
}
```

Freeing a Linked List

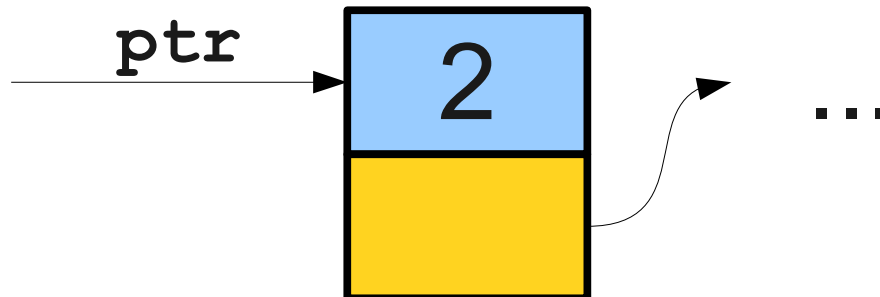
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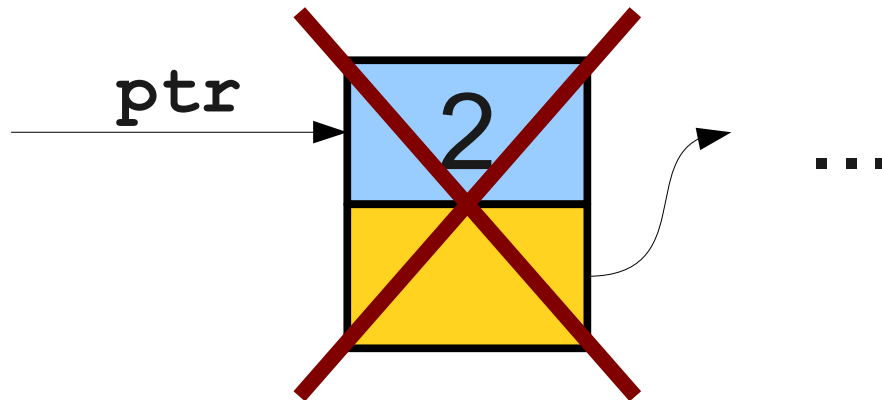
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    delete ptr;  
}
```

ptr → ???

Freeing a Linked List Properly

- To properly free a linked list, we have to be able to
 - Destroy a cell, and
 - Advance to the cell after it.
- How might we accomplish this?

Once More With Recursion

- We can also deallocate lists recursively!
- **Base Case:**
 - There is nothing to free in an empty list.
- **Recursive Case:**
 - Deallocate all cells after the current cell.
 - Deallocate the current cell.

Linked Lists: The Tricky Parts

- Suppose that we want to write a function that will add an element to a linked list.
- What might this function look like?

What went wrong?

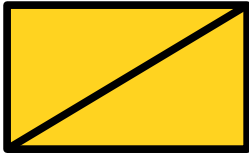
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    Cell* list = NULL;  
    ListInsert(list, 137);  
    ListInsert(list, 42);  
    ListInsert(list, 271);  
}
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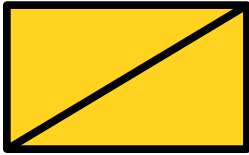
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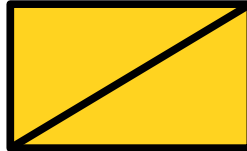
list



```
int main() {
```

```
void listInsert(Cell* list, int value) {  
    Cell* newCell = new Cell;  
    newCell->value = value;  
    newCell->next = list;  
    list = newCell;  
}
```

list



value

137

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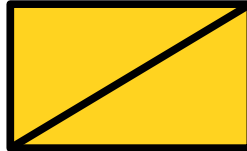
```
newCell->next = list;
```

```
list = newCell;
```

```
}
```

```
}
```

list



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137

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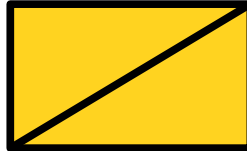
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}
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newCell



list



value

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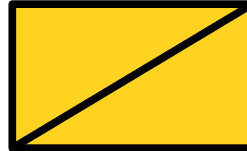
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newCell

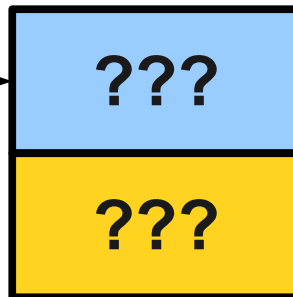
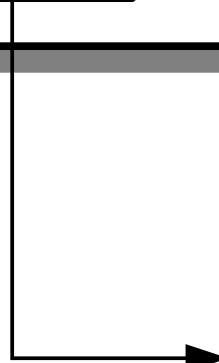


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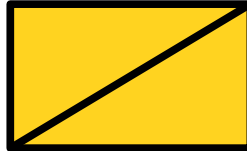
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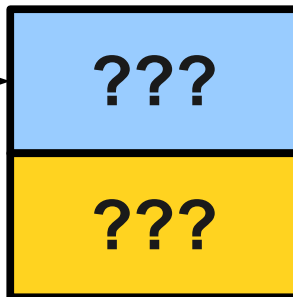


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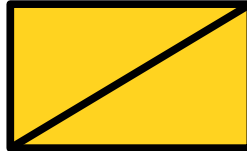
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newCell

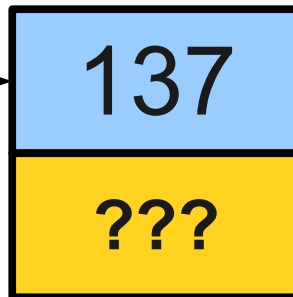


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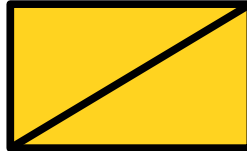
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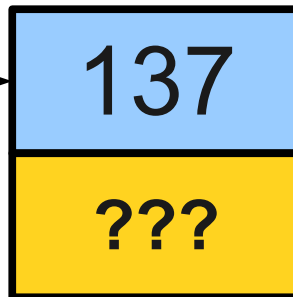


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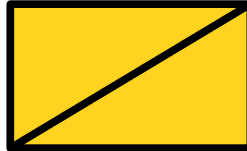
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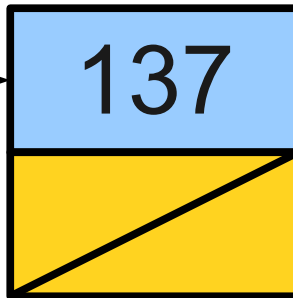


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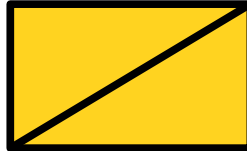
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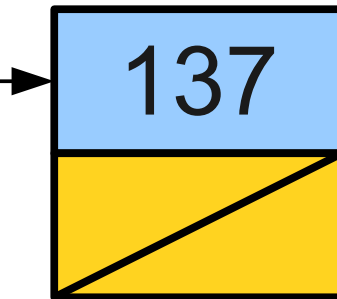


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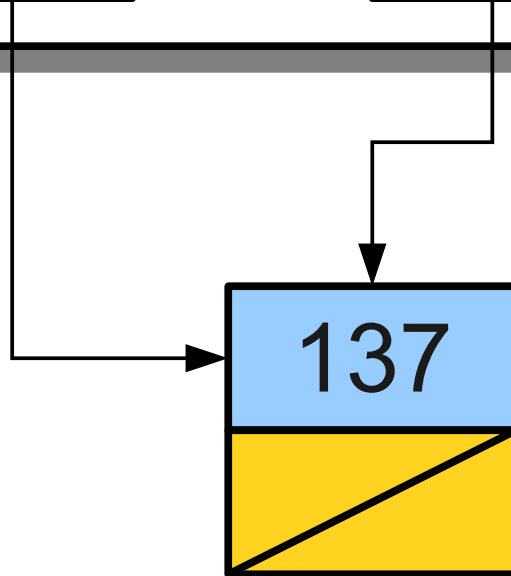


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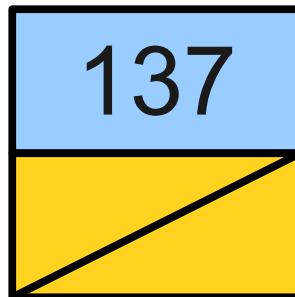
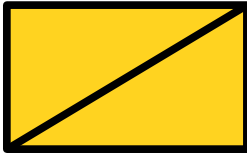
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Pointers by Reference

- In order to resolve this problem, we must pass the linked list pointer by reference.
- Our new function:

```
void listInsert(Cell*& list, int value) {  
    Cell* newCell = new Cell;  
    cell->value = value;  
    cell->next = list;  
    list = cell;  
}
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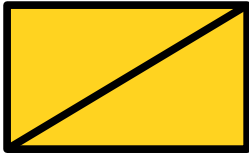


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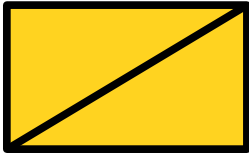
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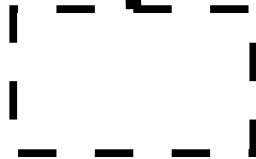
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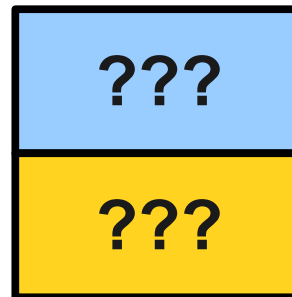
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newCell



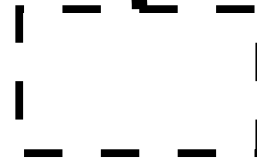

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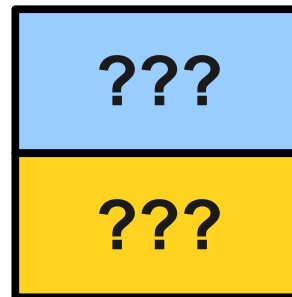
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137

newCell



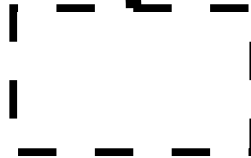
```
int main() {  
    Cell* list = ...  
    ListInsert(list, 137);  
    ListInsert(list, ...)  
}
```

```
void listInsert(Cell*& list, int value) {  
    Cell* newCell = new Cell;  
    newCell->value = value;  
    newCell->next = list;  
    list = newCell;  
}
```

list



list



value

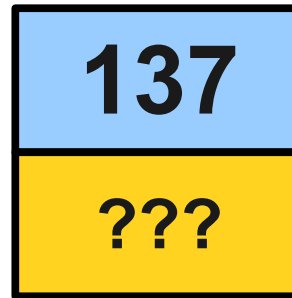
137

newCell



137

???



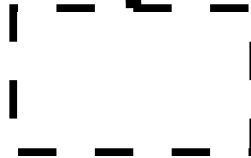
```
int main() {  
    Cell* list = ...  
    ListInsert(list, 137);  
    ListInsert(list, ...)  
    ListInsert(list, ...)  
}
```

```
void listInsert(Cell*& list, int value) {  
    Cell* newCell = new Cell;  
    newCell->value = value;  
    newCell->next = list;  
    list = newCell;  
}
```

list



list



value

137

newCell



137

???

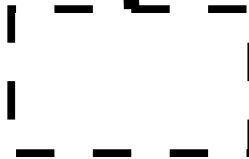
```
int main() {  
    Cell* list = ...  
    ListInsert(list, 137);  
    ListInsert(list, ...)  
    ListInsert(list, ...)  
}
```

```
void listInsert(Cell*& list, int value) {  
    Cell* newCell = new Cell;  
    newCell->value = value;  
    newCell->next = list;  
    list = newCell;  
}
```

list



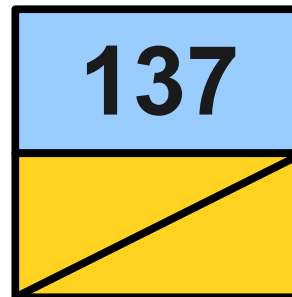
list



value

137

newCell



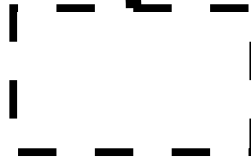
```
int main() {  
    Cell* list = ...  
    ListInsert(list, 137);  
    ListInsert(list, ...)  
    ListInsert(list, ...)  
}
```

```
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    list = newCell;  
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```

list



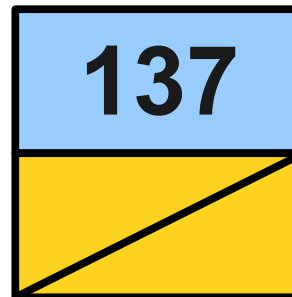
list



value

137

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```
int main() {  
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}
```

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void listInsert(Cell*& list, int value) {  
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    newCell->next = list;  
    list = newCell;  
}
```

list

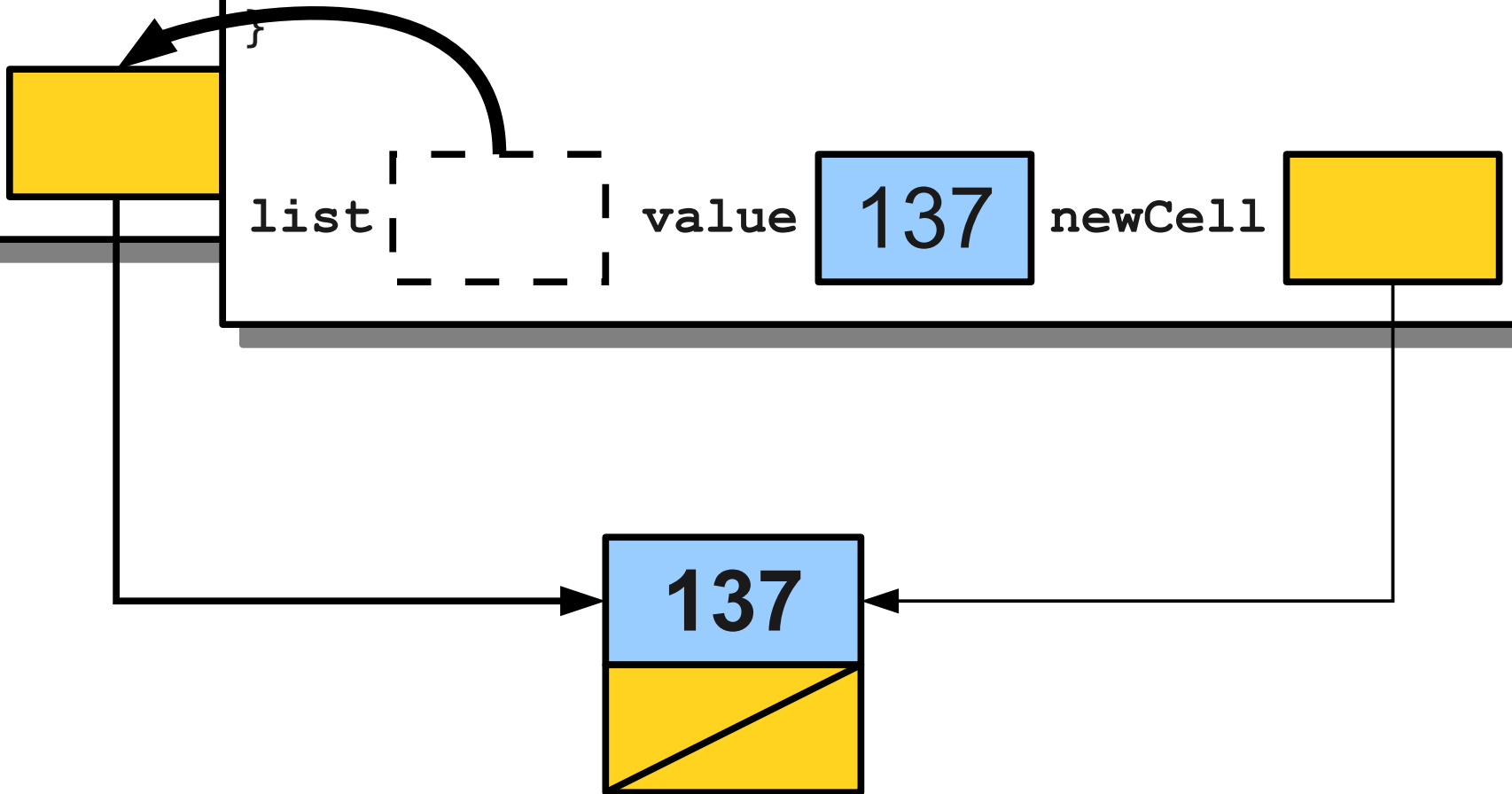
list

value

137

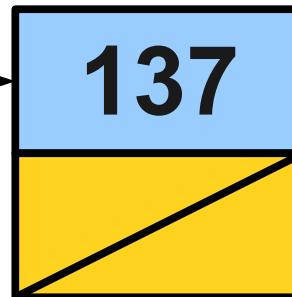
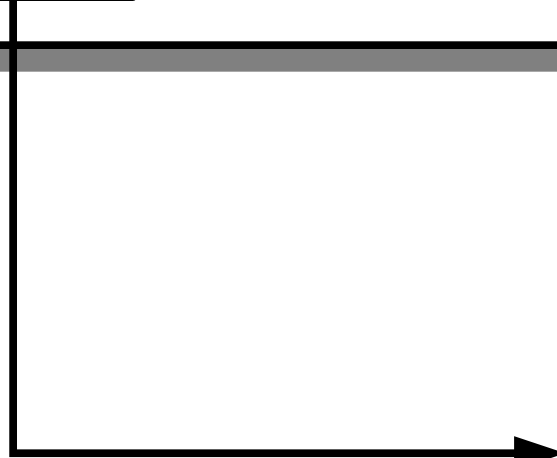
newCell

137



```
int main() {  
    Cell* list = NULL;  
    ListInsert(list, 137);  
    ListInsert(list, 42);  
    ListInsert(list, 271);  
}
```

list

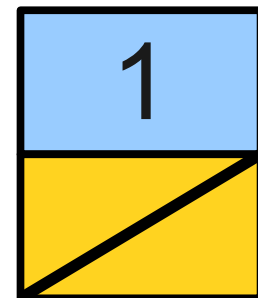


Reimplementing Stack

- We have already seen one way to implement the stack (dynamic arrays).
- We can also implement a stack efficiently using a linked list.
- **Push**: Prepend a new cell to the front of the list.
- **Pop**: Remove the first cell of the list.

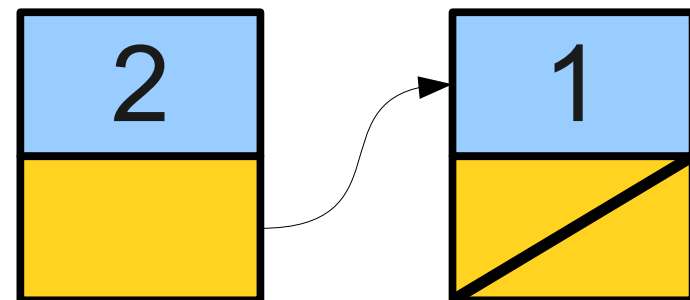
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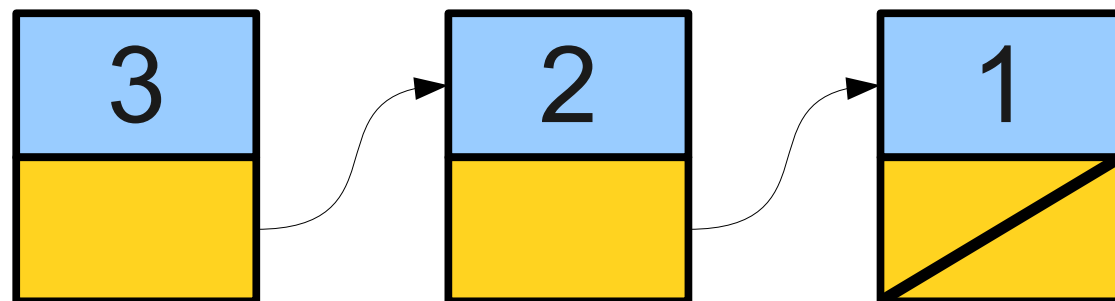
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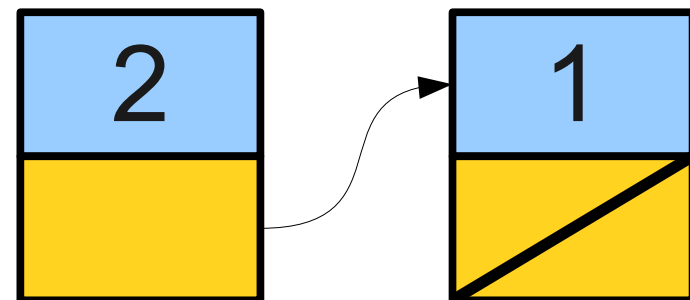
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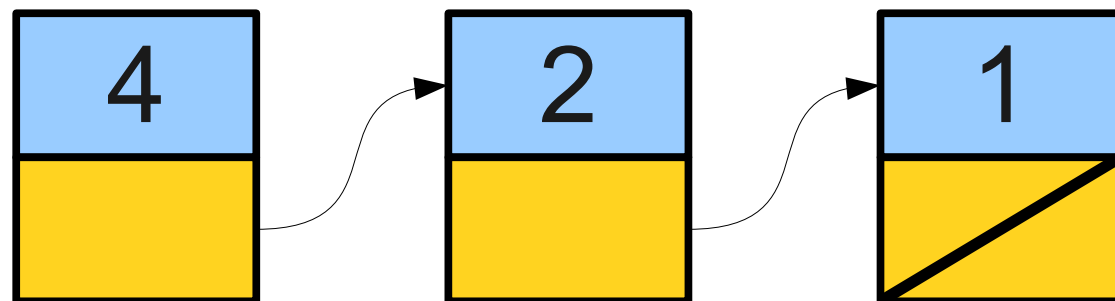
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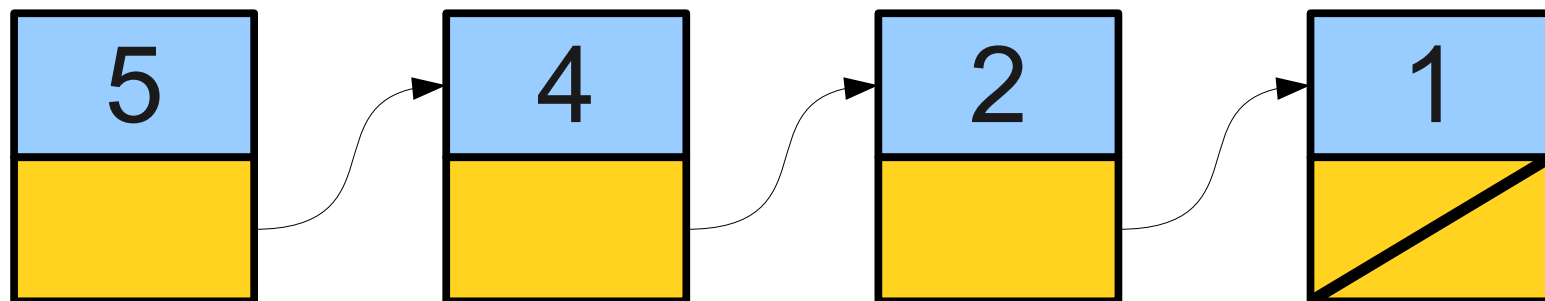
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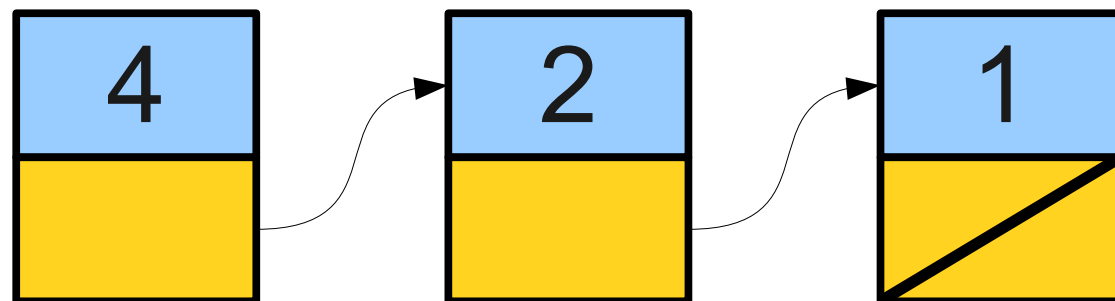
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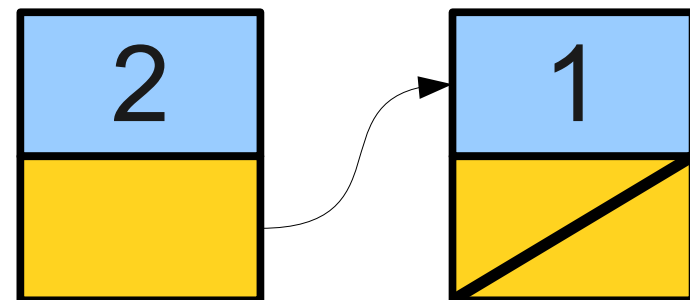
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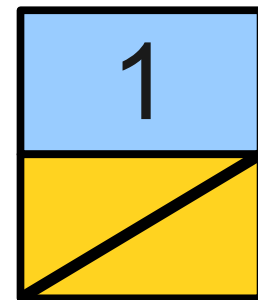
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Analyzing our Stack

- Push and pop are now **worst-case** $O(1)$ instead of **average-case** $O(1)$.
- What about the total runtime?

Analyzing our Stack

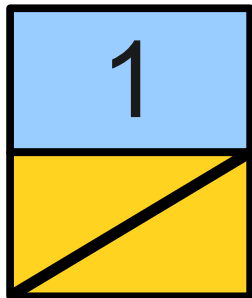
- Push and pop are now **worst-case** $O(1)$ instead of **average-case** $O(1)$.
- What about the total runtime?
- **Slower than before.**
- Why?
 - Cost of allocating individual linked list cells exceeds cost of allocating very few blocks and copying values over.
 - Trade average-case for worst-case speed.

Implementing Queue

- We can also implement the queue using a linked list.
- Idea:
 - To **enqueue**, append a new cell to the end of the list.
 - To **dequeue**, remove the first cell from the list.

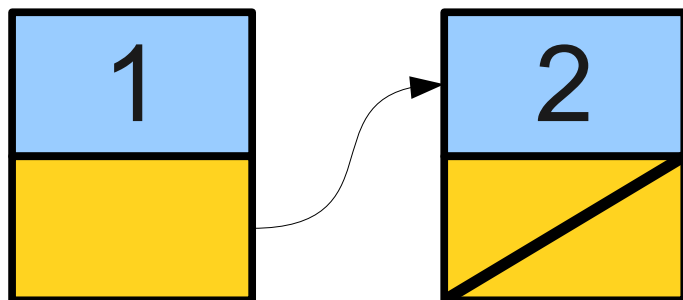
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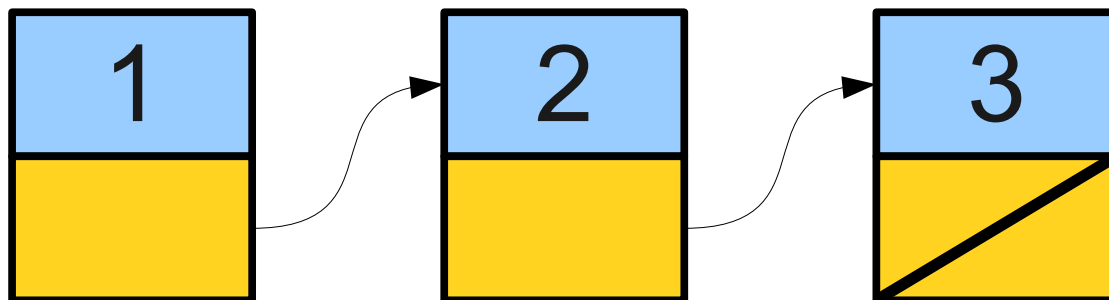
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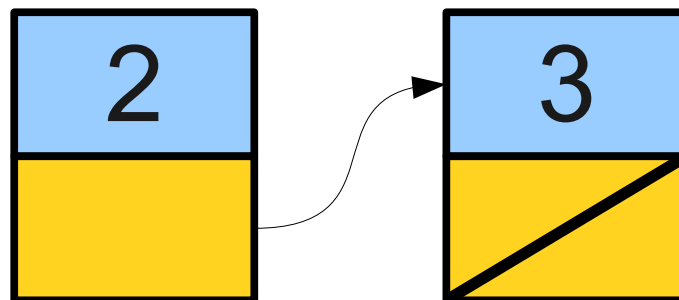
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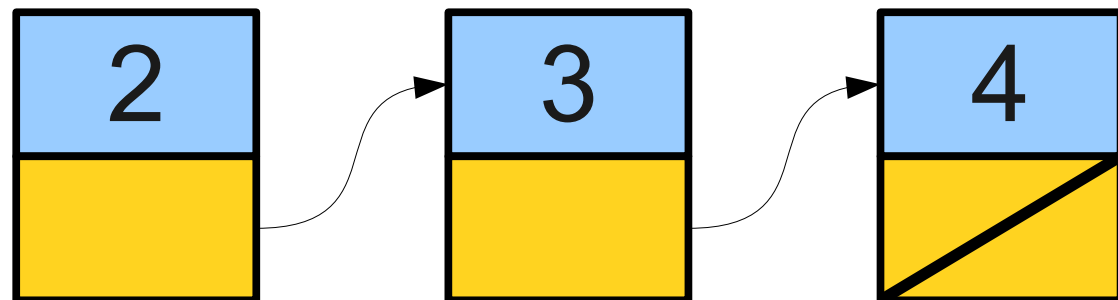
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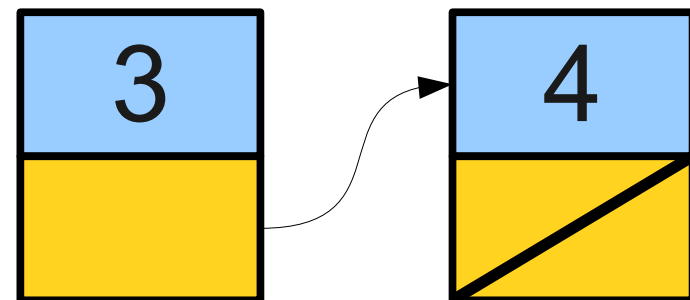
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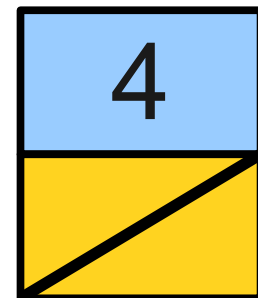
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Analyzing Efficiency

- What is the big-O complexity of a dequeue?
- Answer: **$O(1)$** .
- What is the big-O complexity of an enqueue?
- Answer: **$O(n)$** .

Improving Efficiency

- The $O(n)$ work in enqueue comes from scanning the list to find the end.
- **Idea:** What if we just stored a pointer to the very last cell in the list?
- Can immediately jump to the end to append a value.

Analyzing Efficiency

- What is the big-O complexity of a dequeue?
- Answer: **$O(1)$** .
- What is the big-O complexity of an enqueue?
- Answer: **$O(1)$** .

The Takeaway Point

- You can have multiple pointers into the same linked list.
- This makes it possible to efficiently insert values at multiple places in the list.

Next Time

- **Implementing Maps**
 - Implementation strategies
 - Hashing
 - Building a hash table