Searching and Sorting

Friday Four Square!
Outside Gates, 4:15PM

In The News

Using image processing to
amplify subtle motion in videos:

http://bits.blogs.nytimes.com/2013/02/27/scientists-uncover-invisible-motion-in-video/

Algorithms Revisited

- An **algorithm** is a procedure for effecting some result.
- There can be many different algorithms for solving the same problem.
- How can we compare algorithms against one another?
- What's the best algorithm for solving a given problem?

Two Famous Problems

- **Searching**
  - Given an array of values, determine whether some value is contained in that array.
  - Very important: finding medical records, determining if a bookstore has a copy of a book, etc.
- **Sorting**
  - Given an array of values, rearrange those values to put them in sorted order.
  - Enormously important: shows up in iTunes, Google, Facebook, etc.

Searching

Can I get some volunteers?

Linear Search

Linear Search

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private int linearSearch(int[] arr, int key) {
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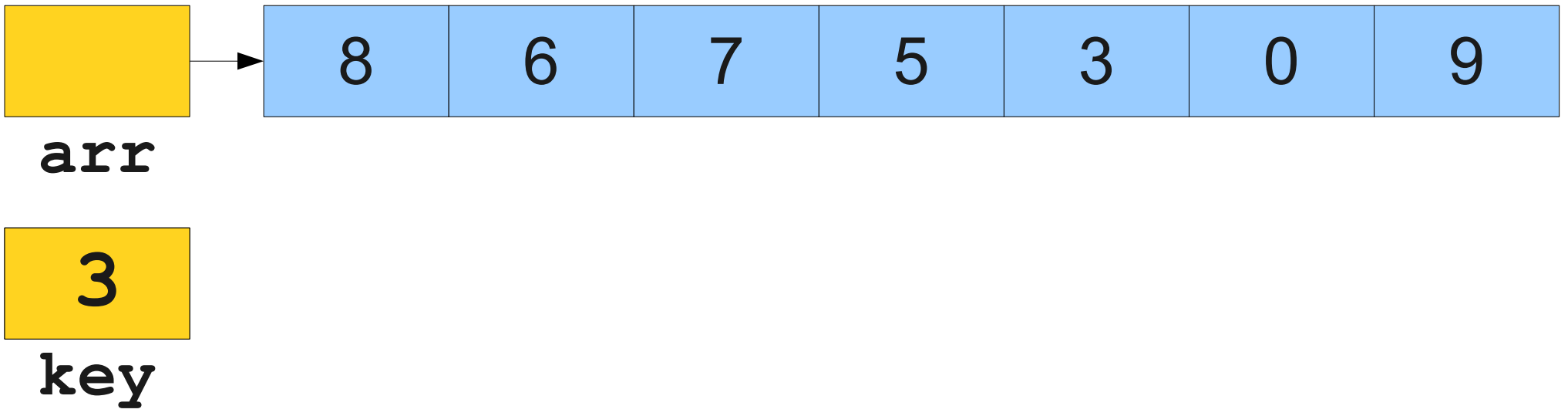

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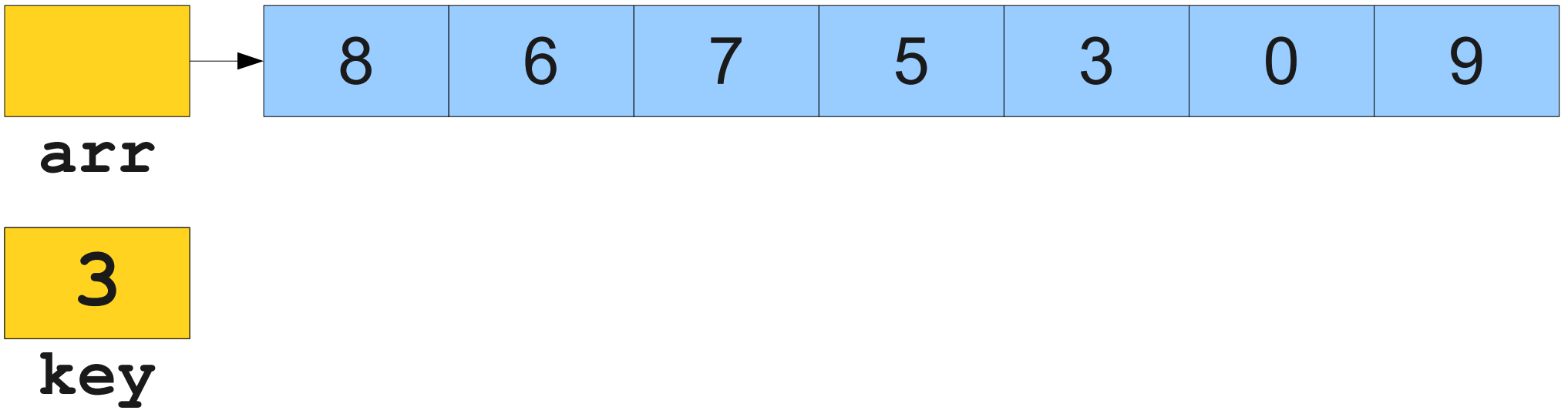

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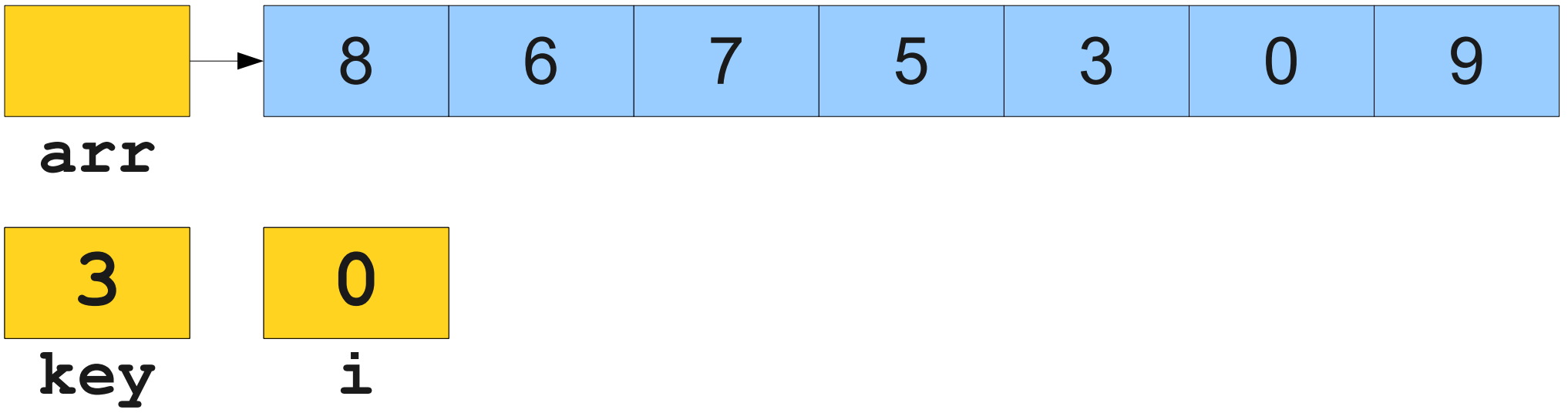

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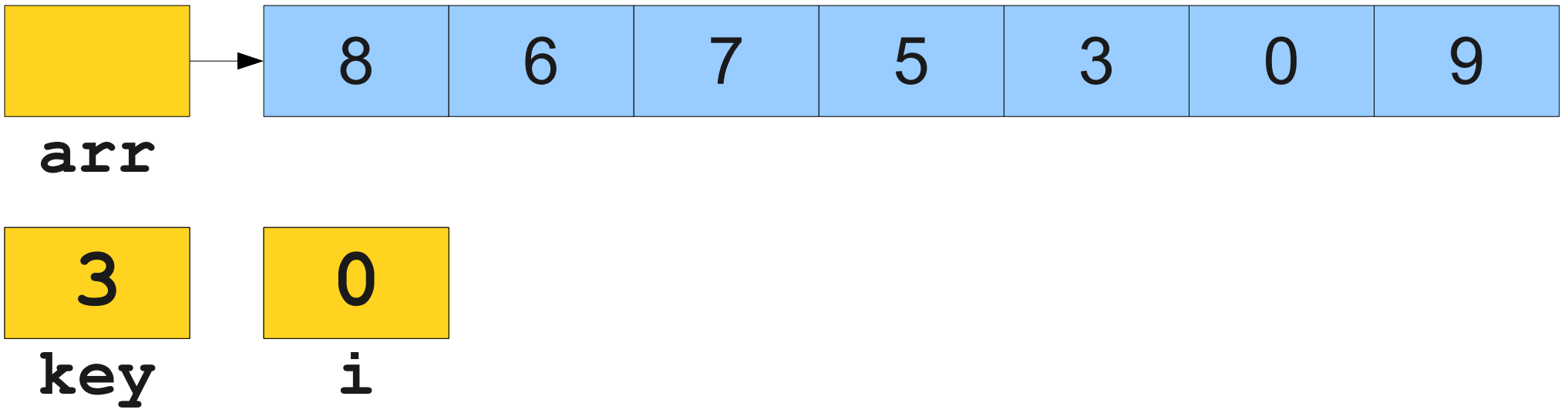

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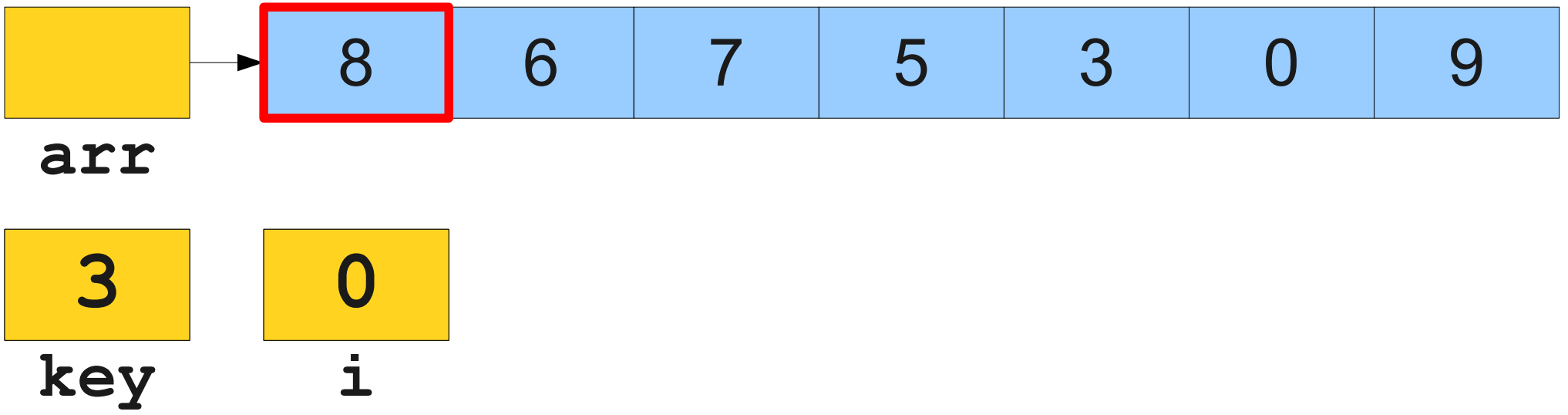

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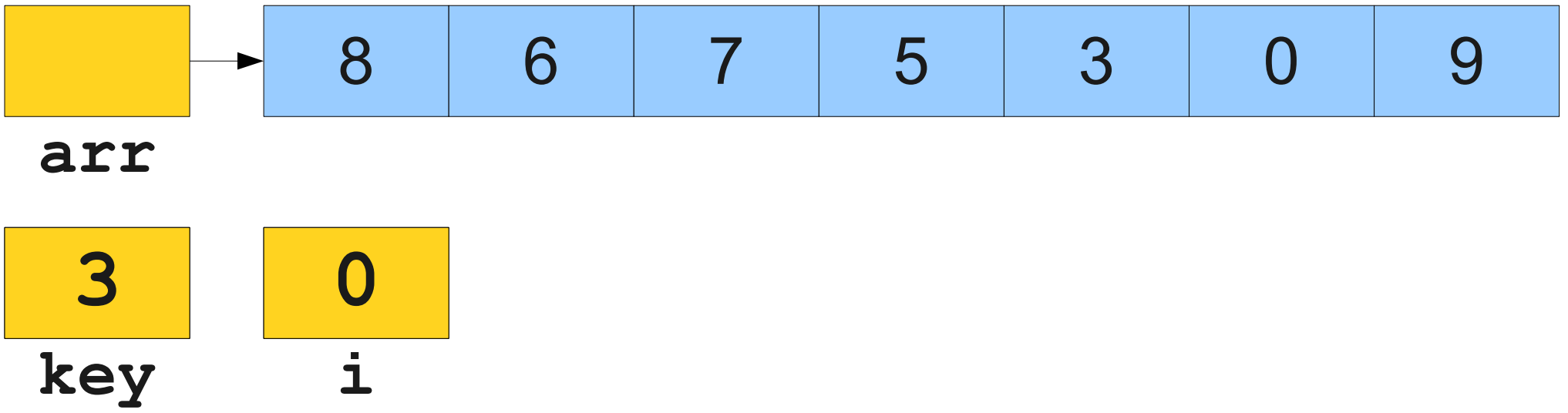

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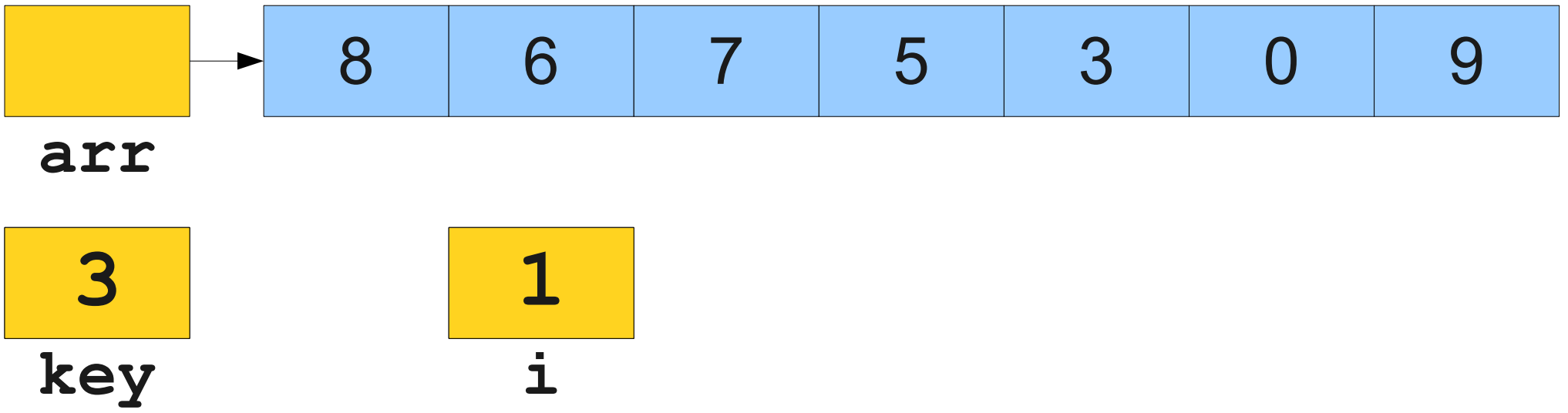

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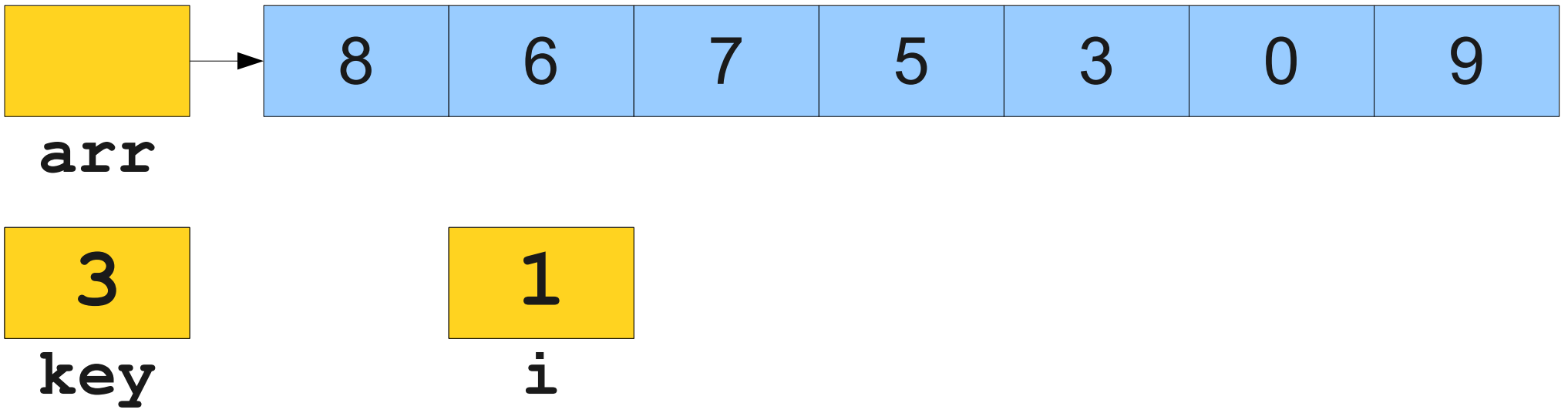

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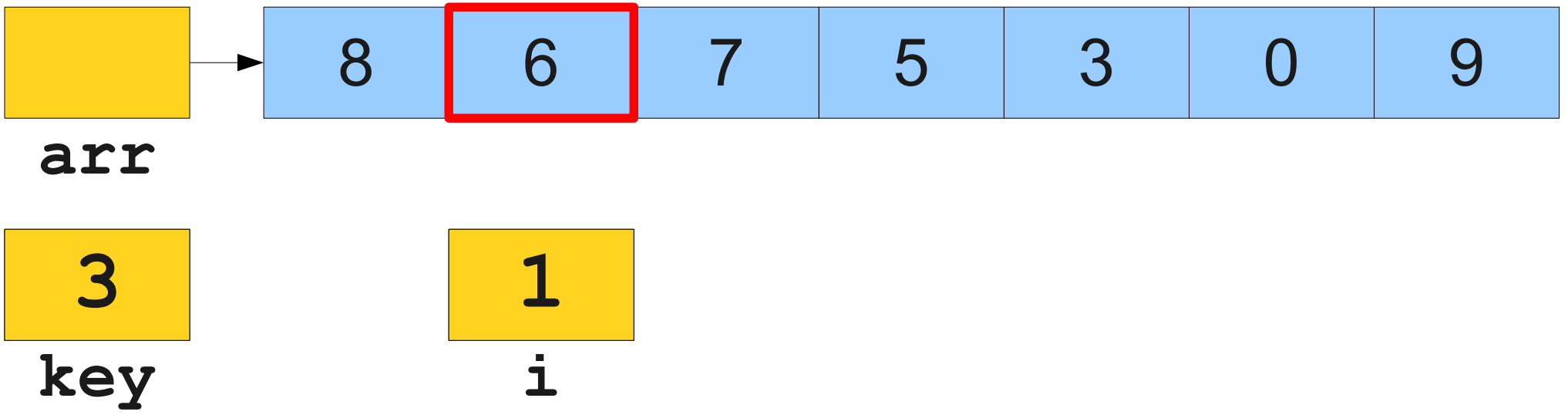

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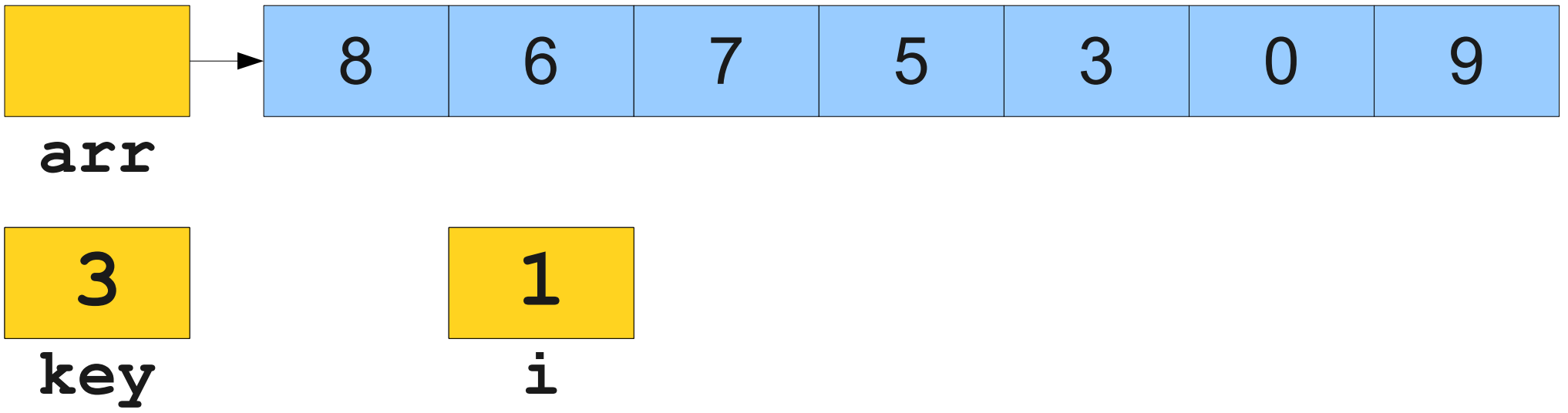

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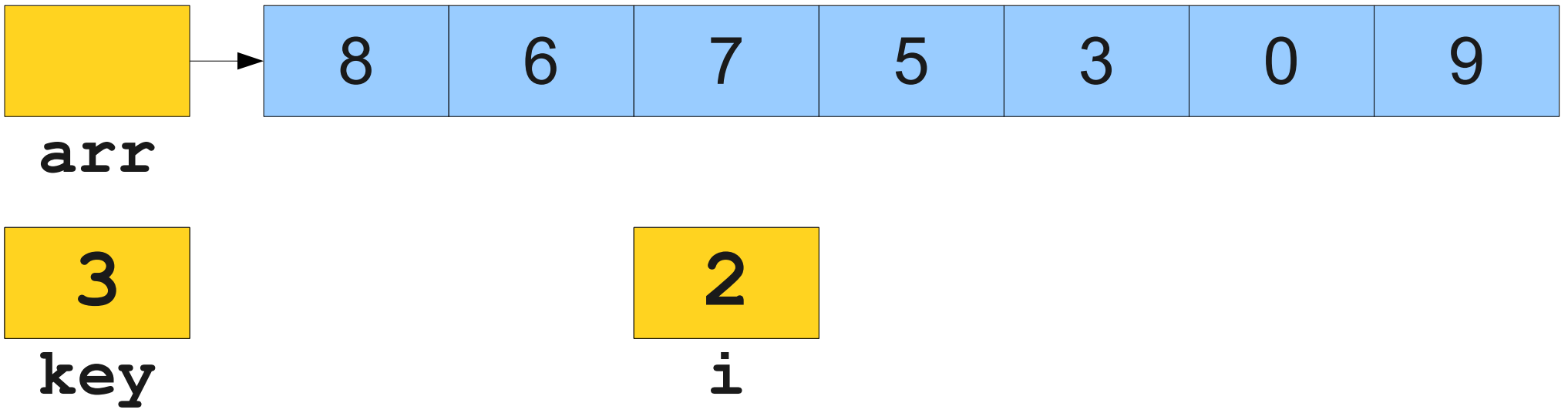

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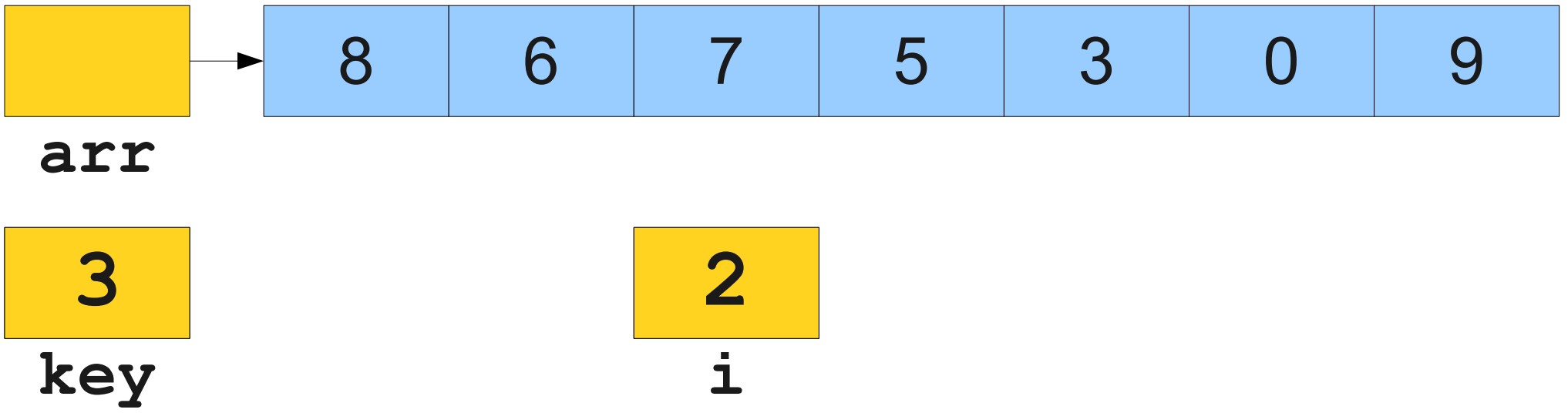

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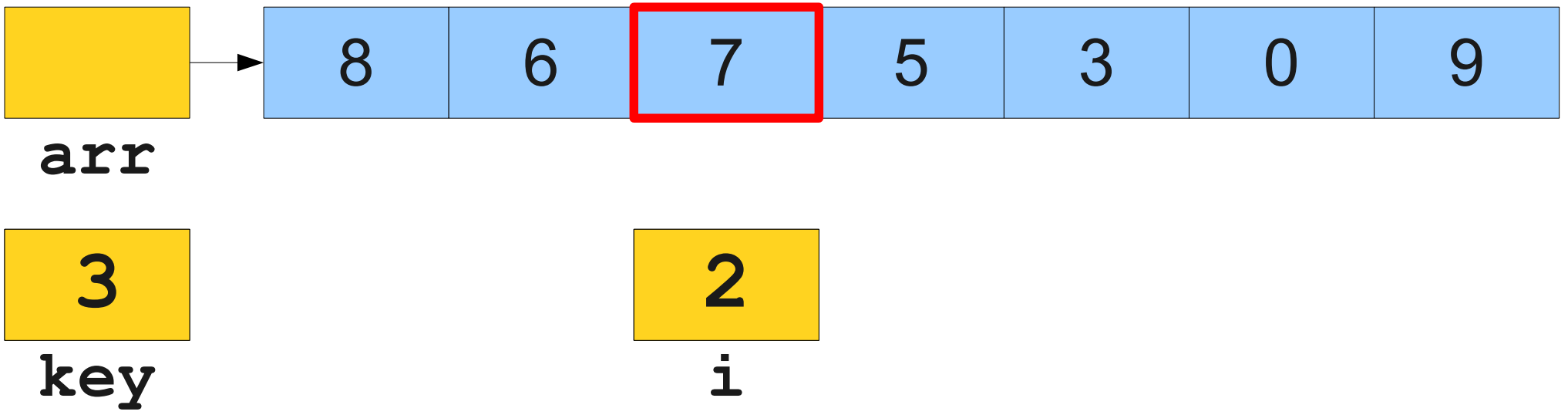

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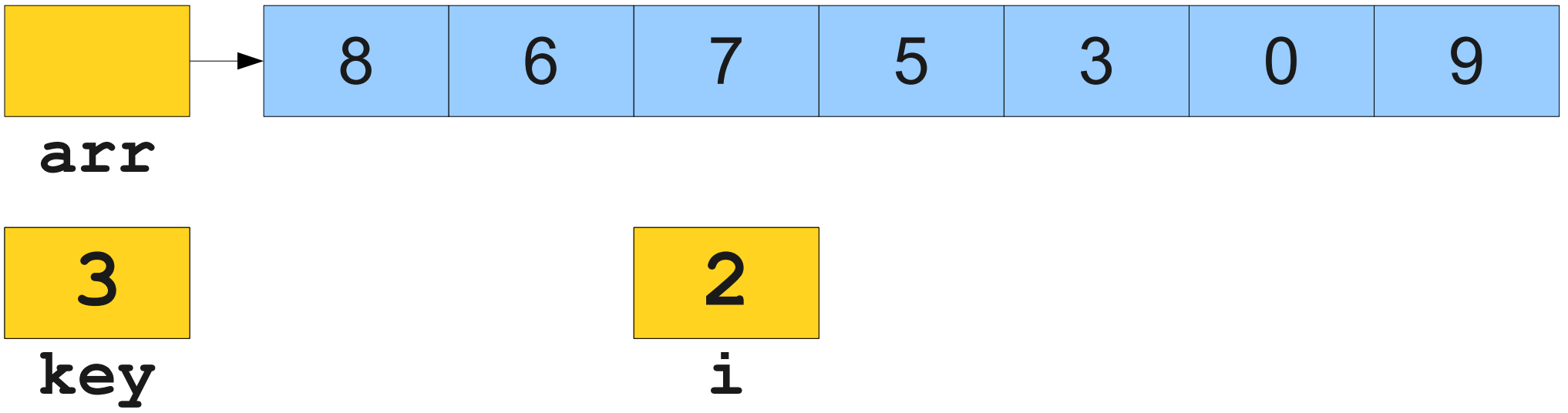

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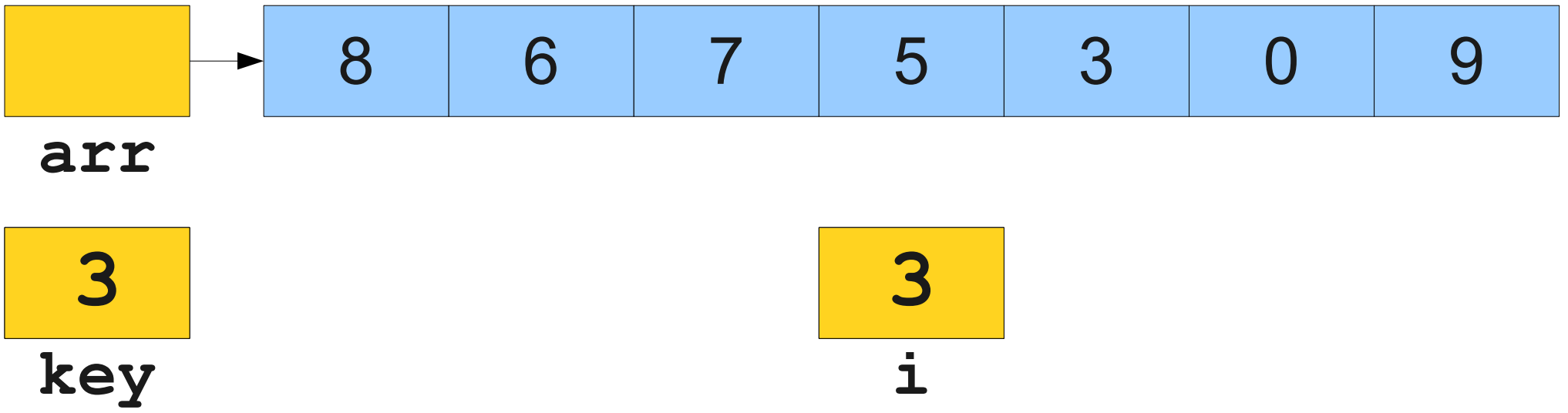

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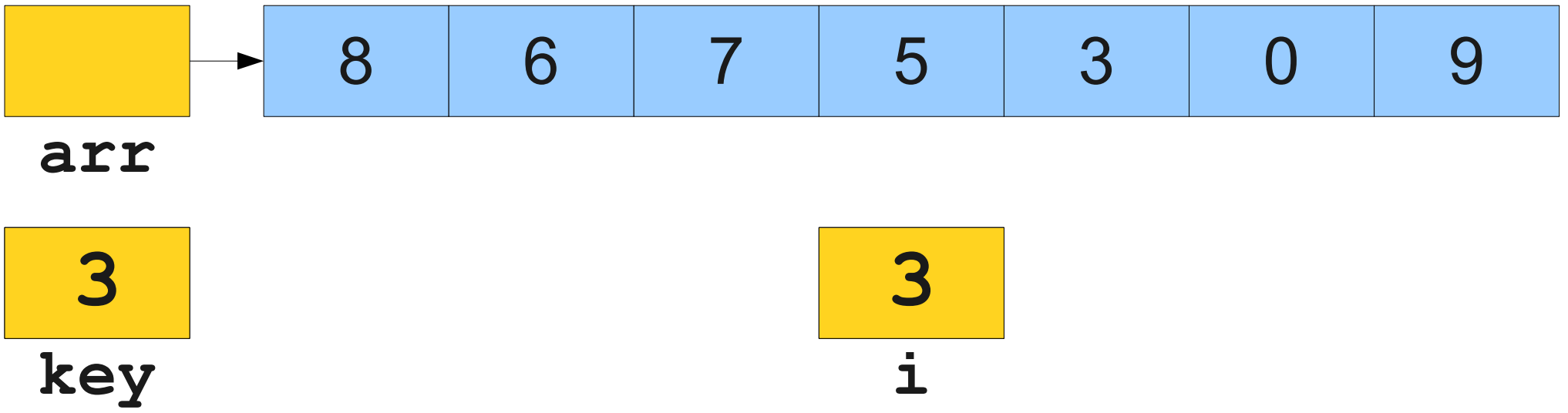

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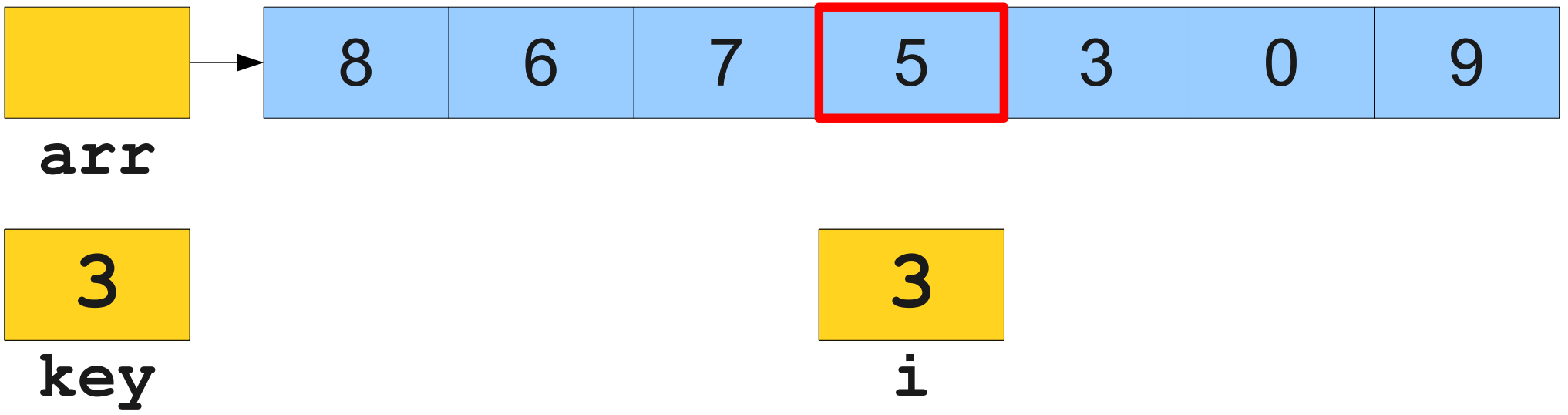

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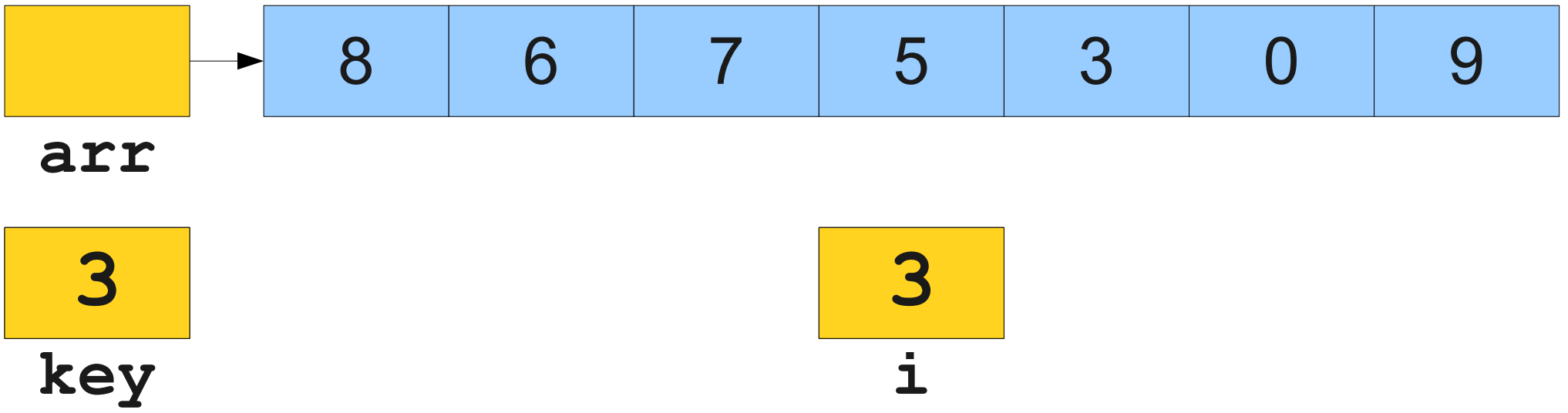

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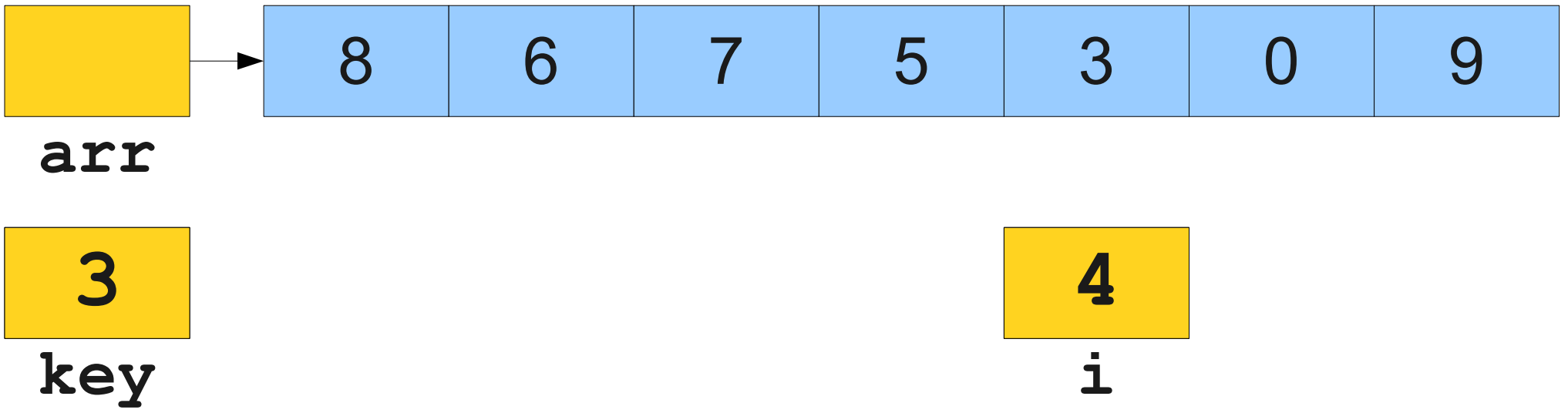

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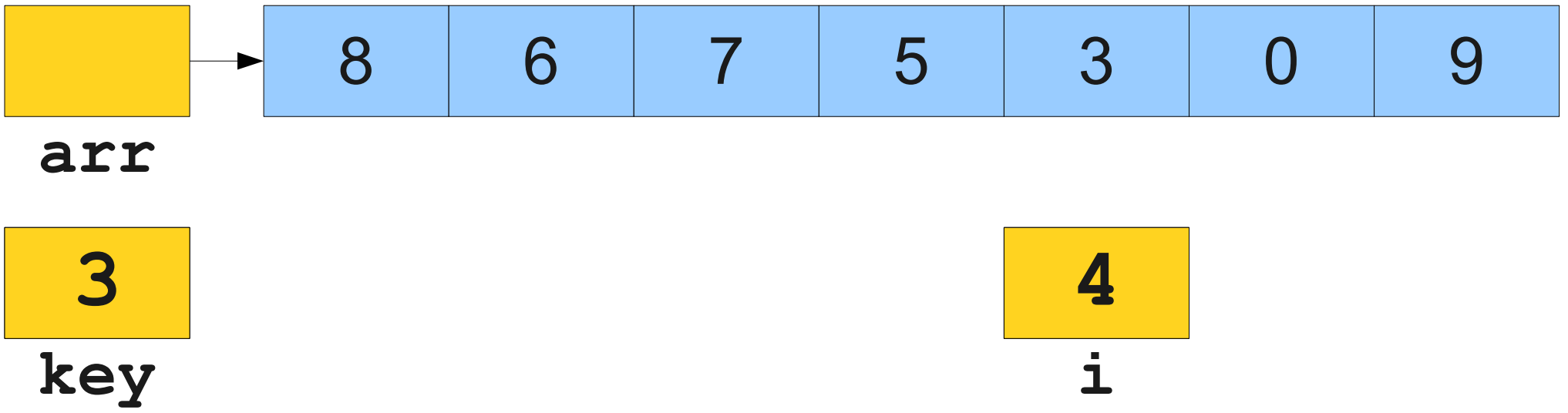

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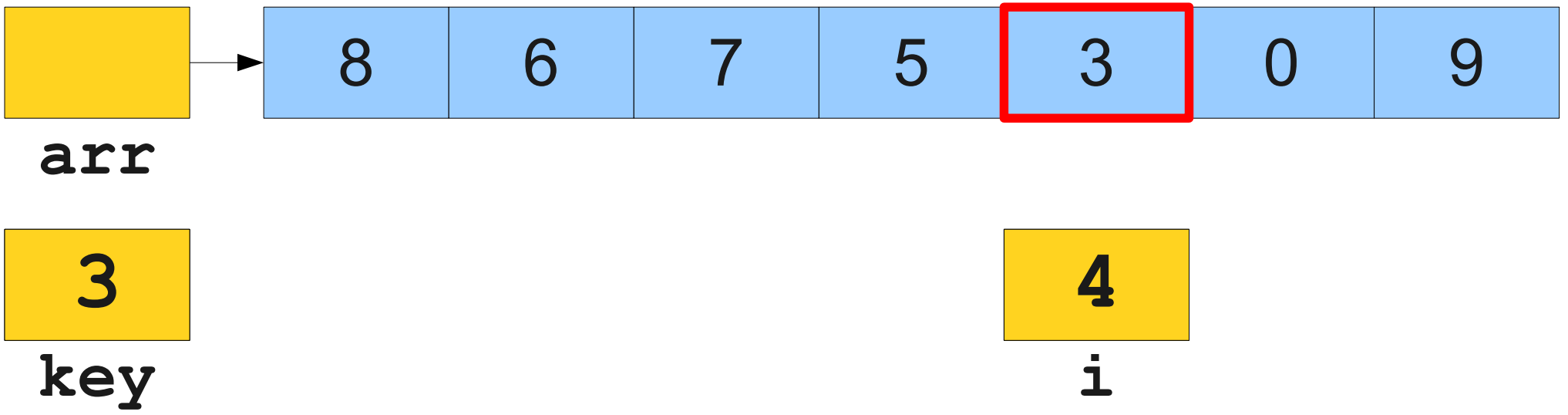

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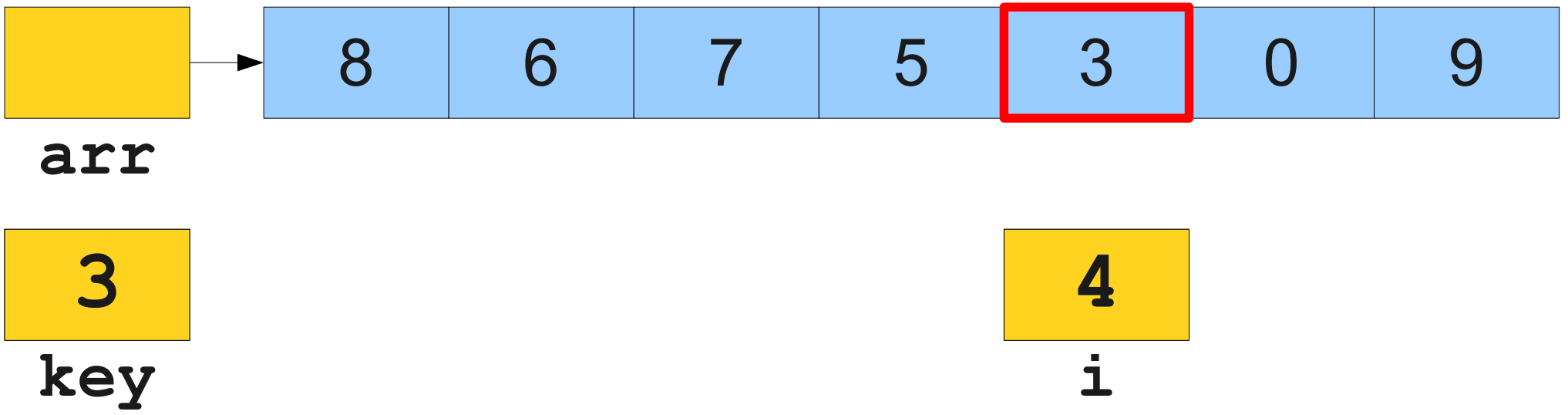

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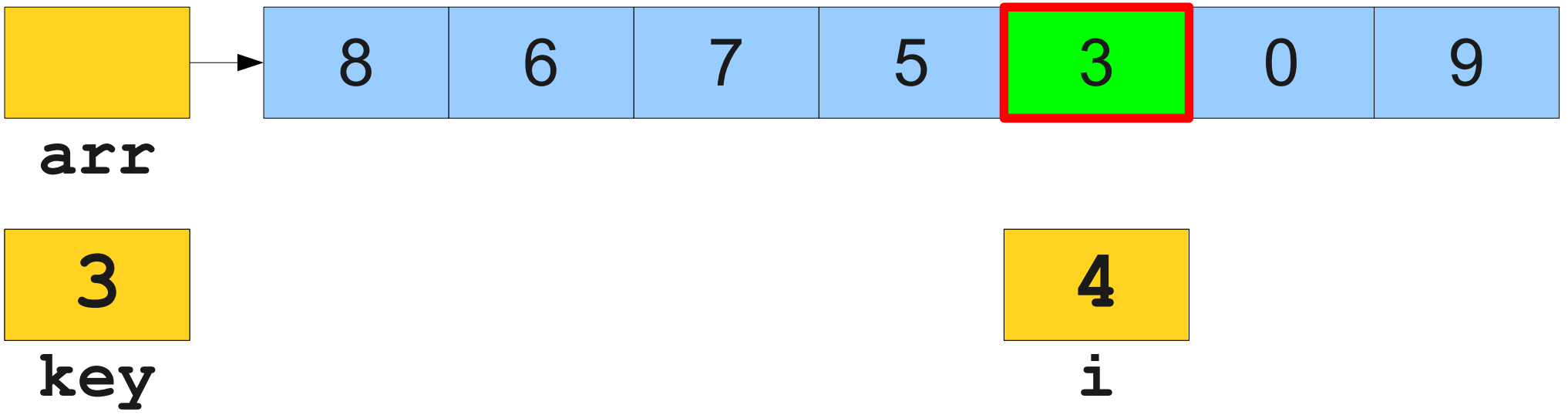

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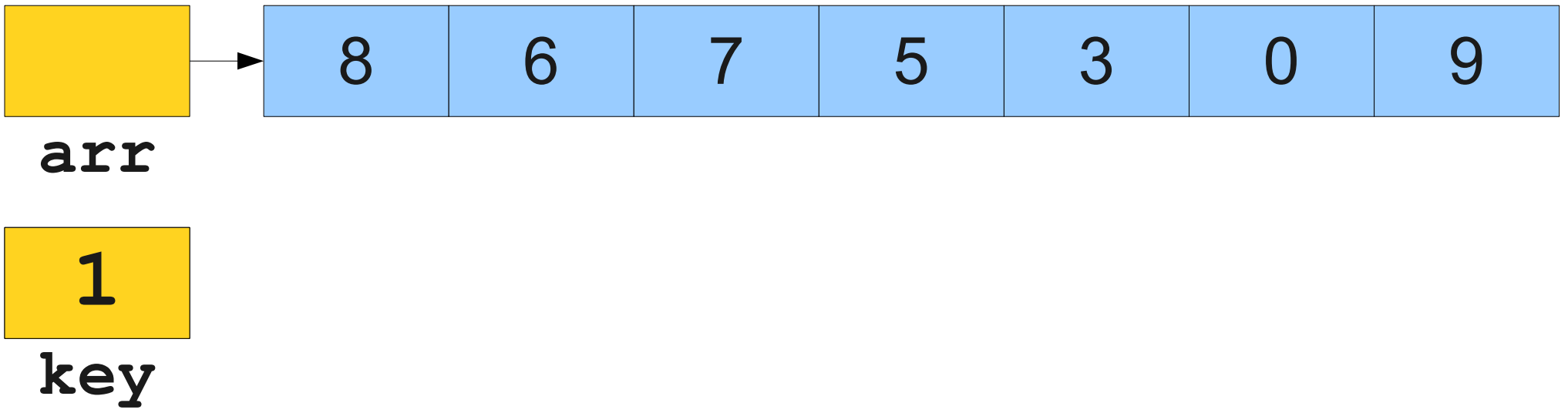

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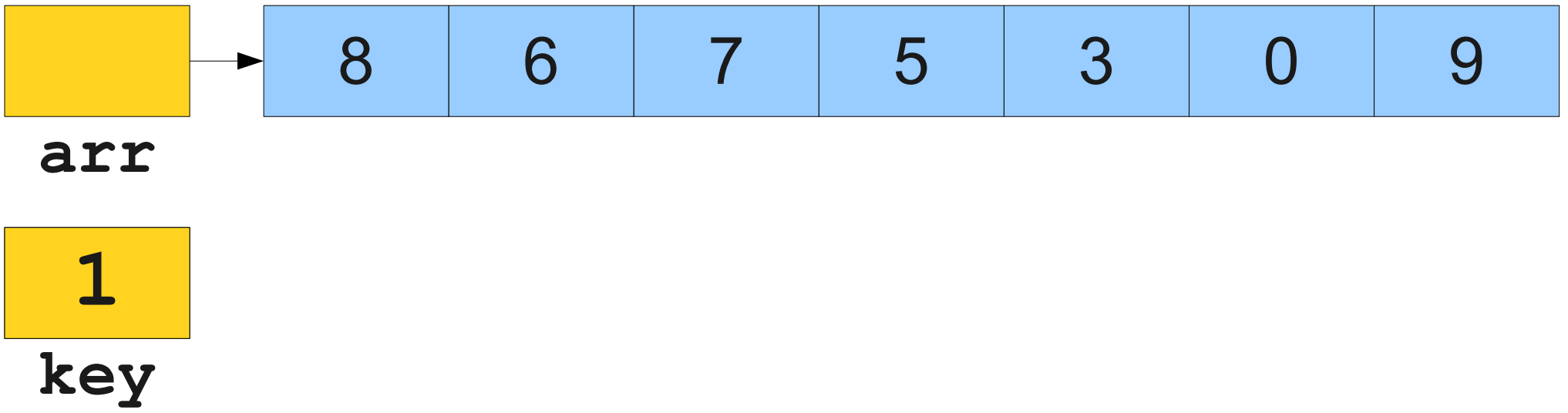

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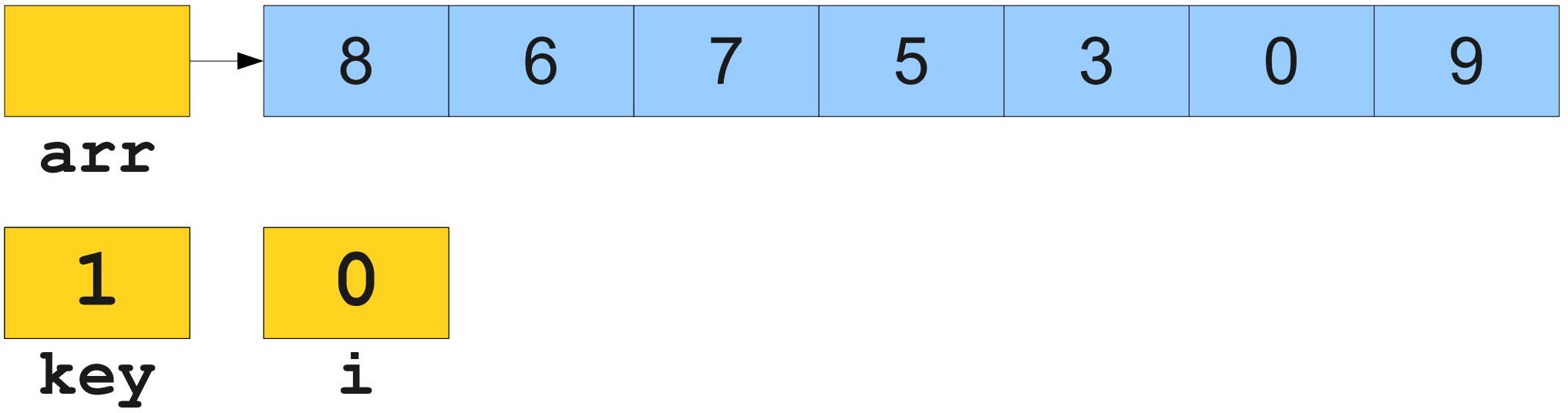

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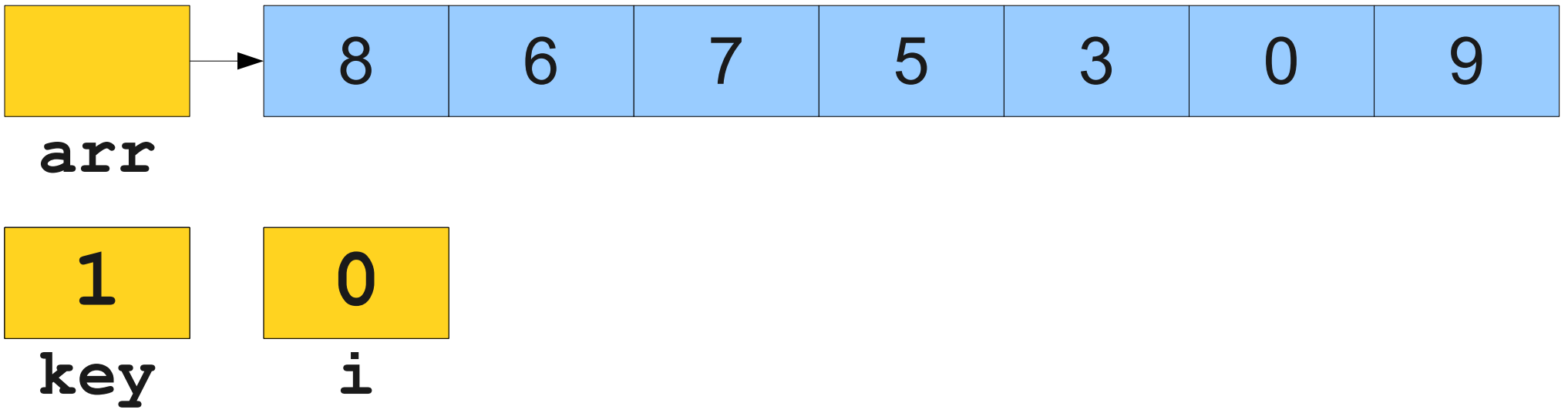

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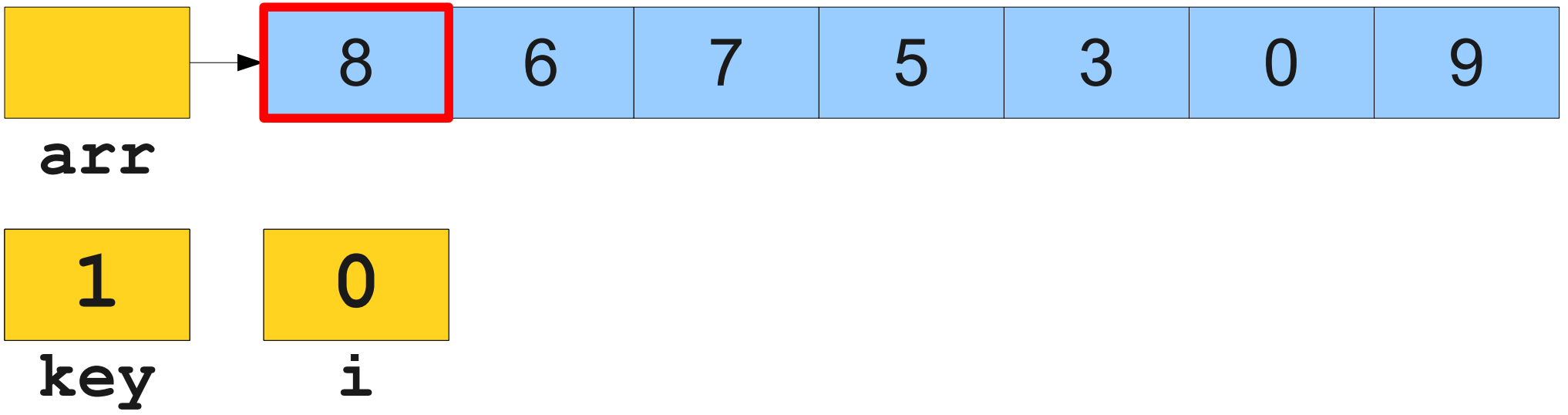

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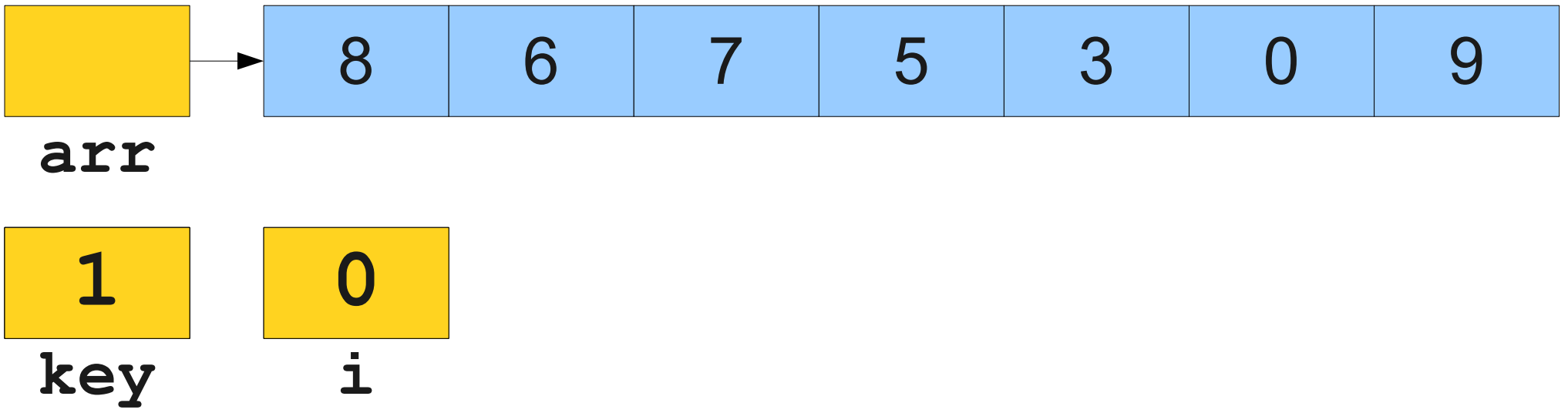

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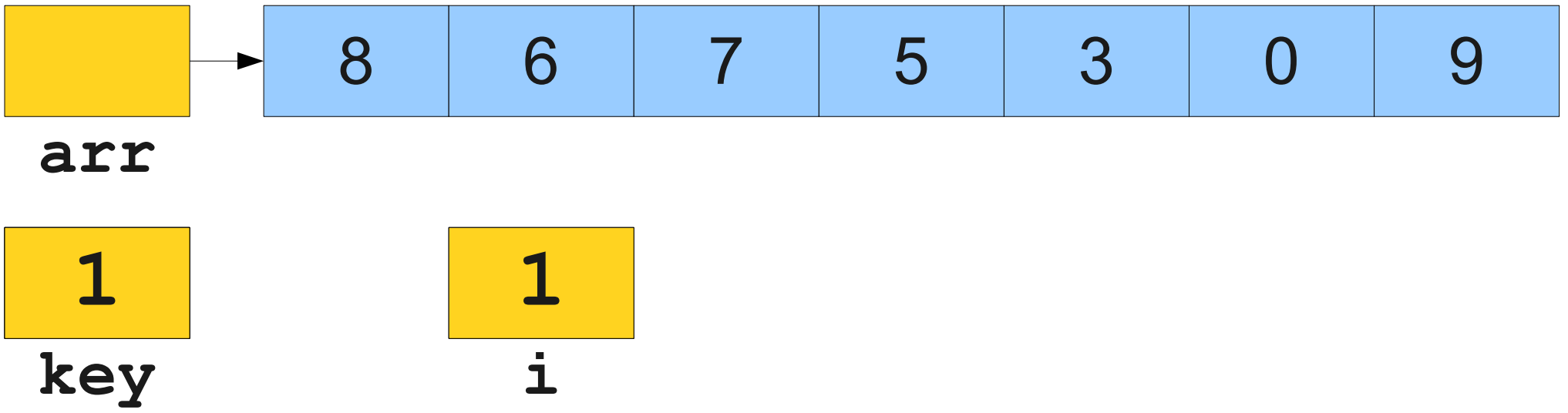

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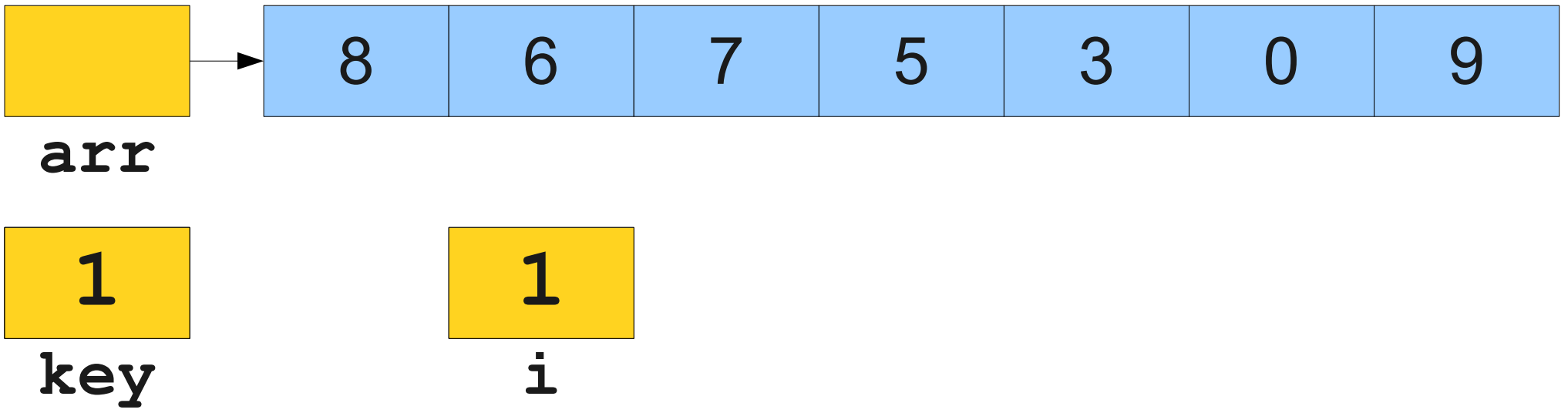

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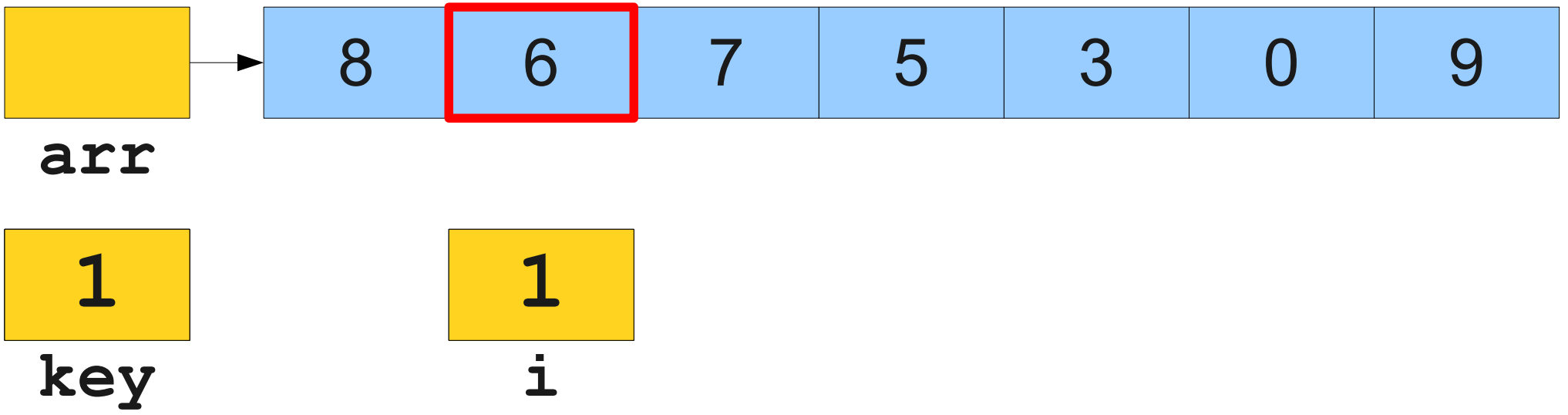

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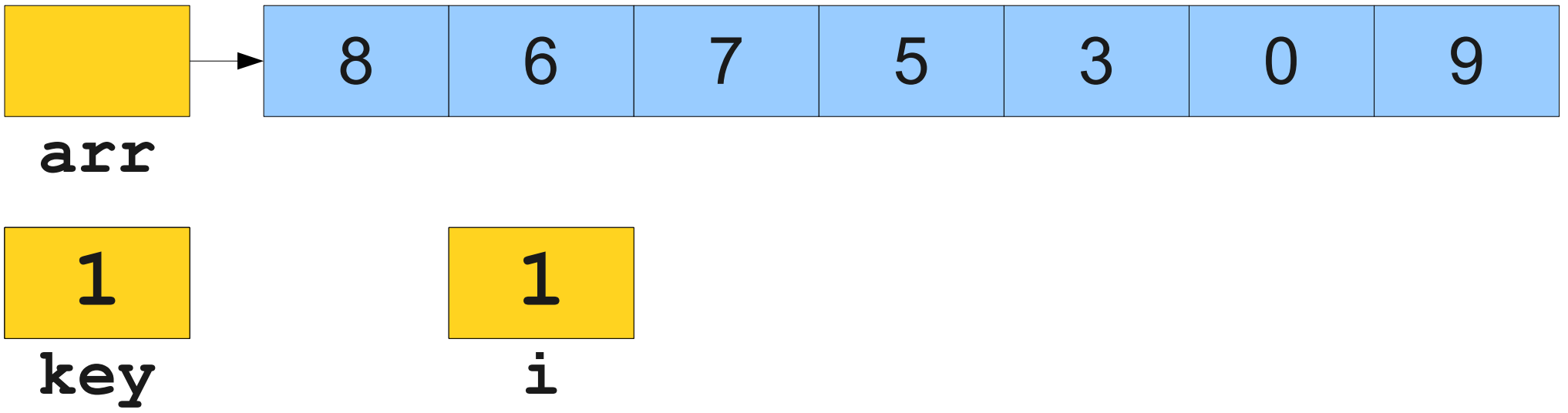

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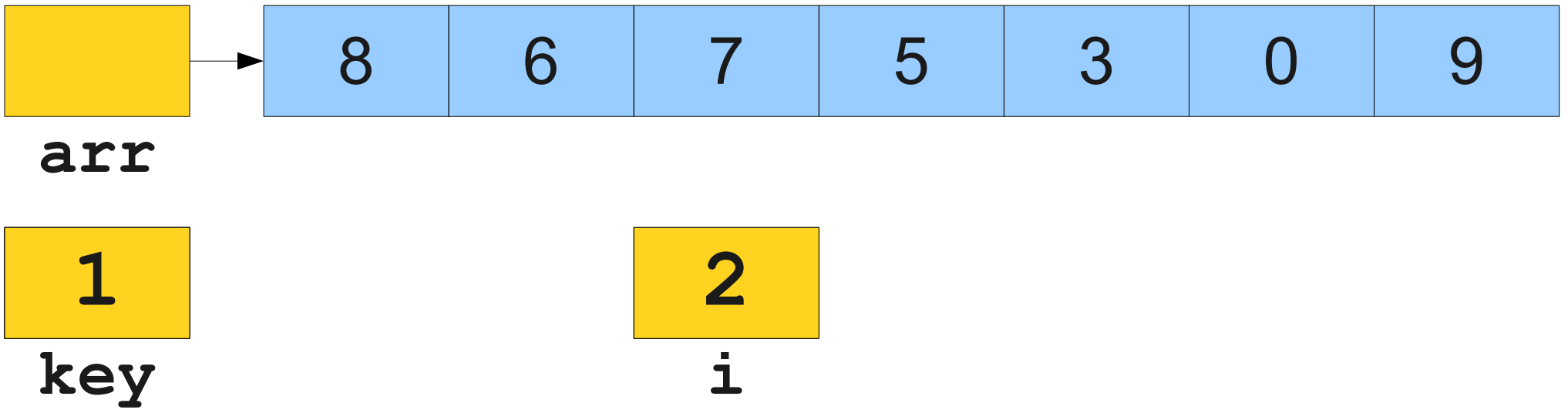

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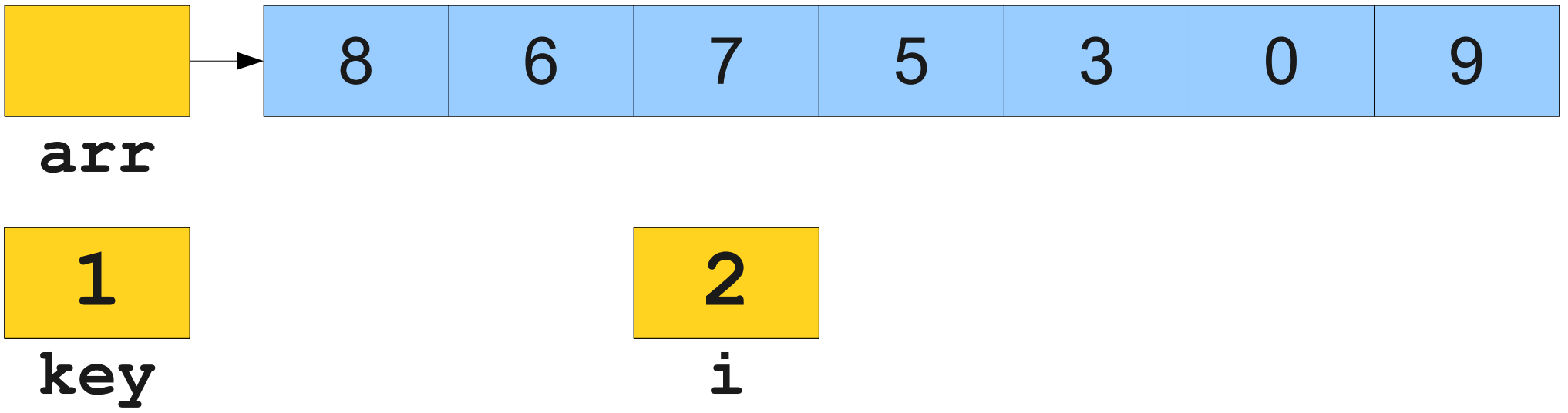

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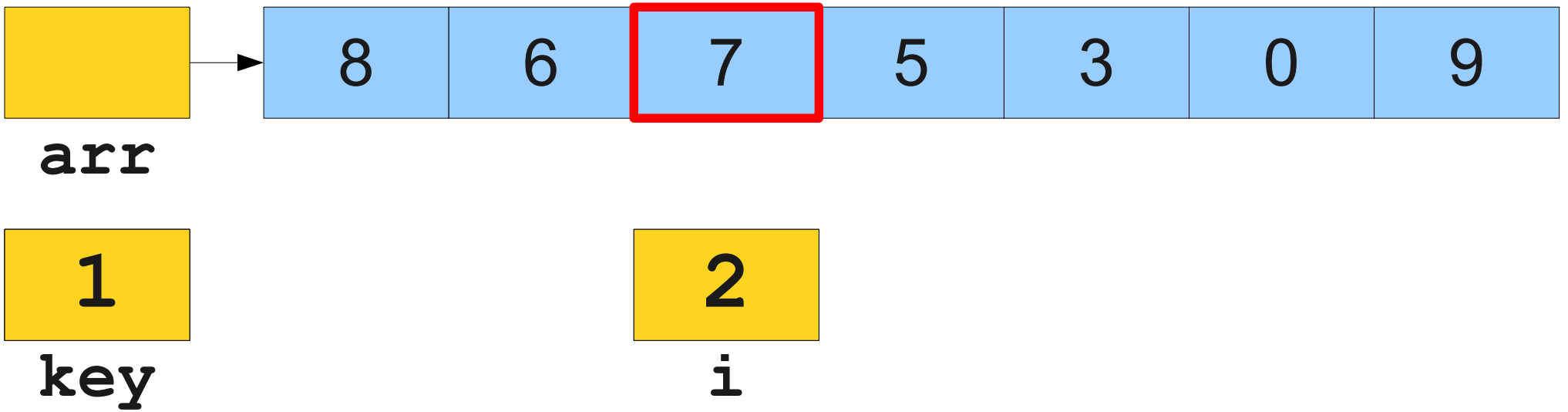

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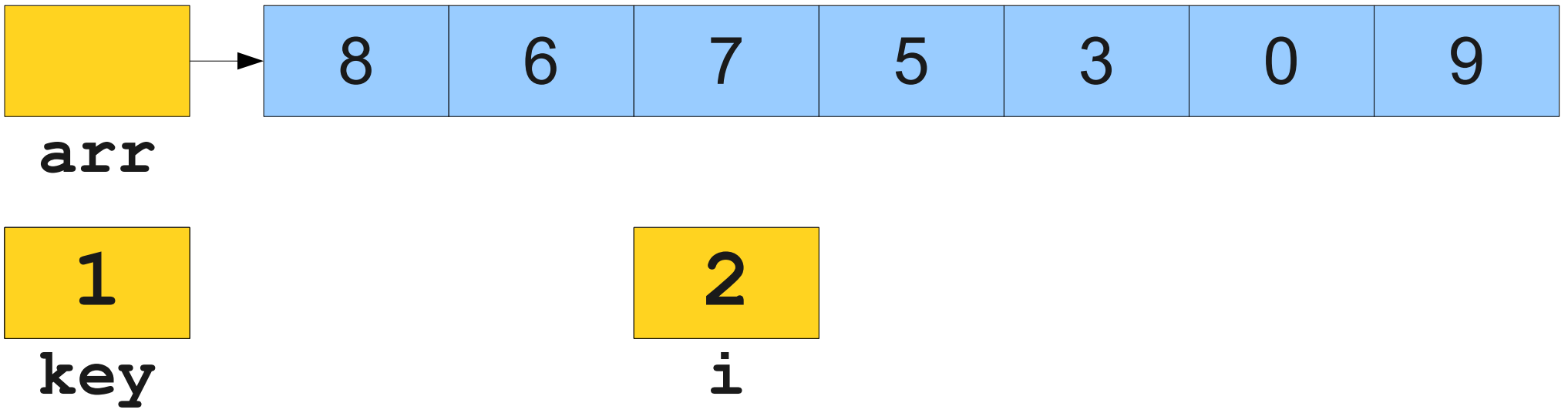

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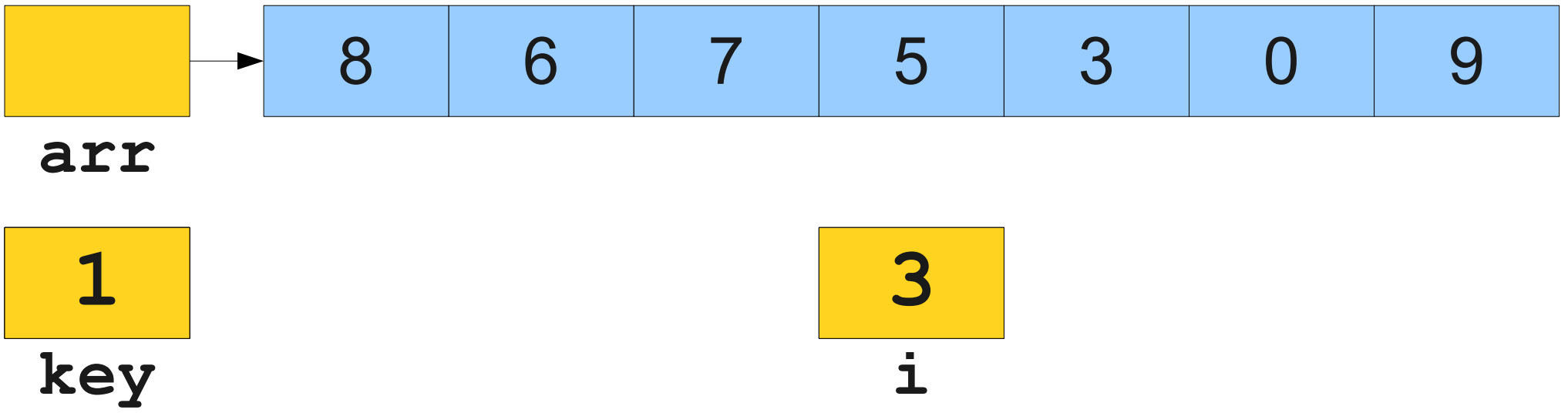

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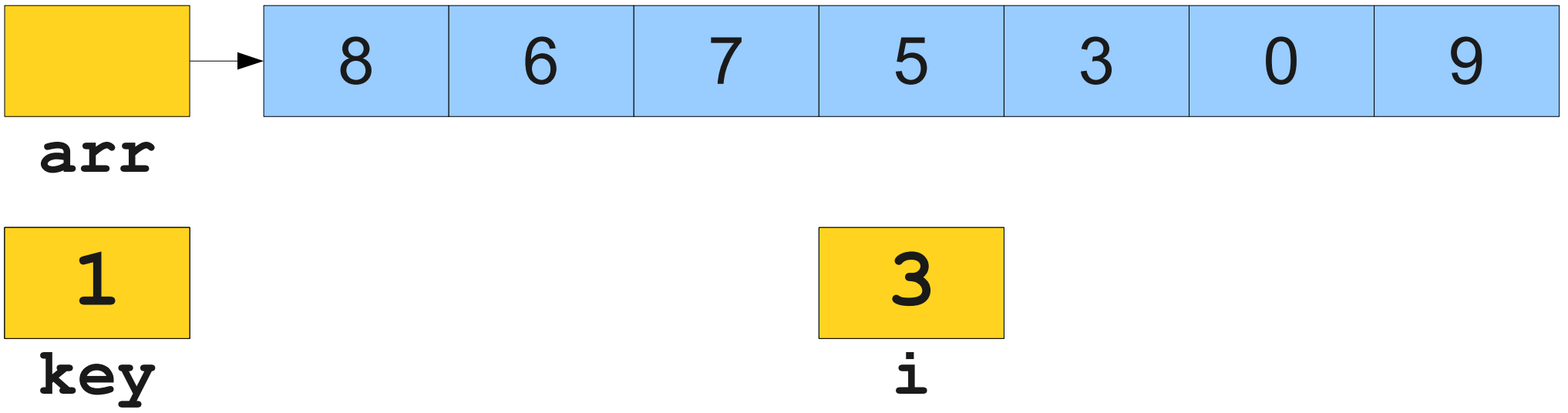

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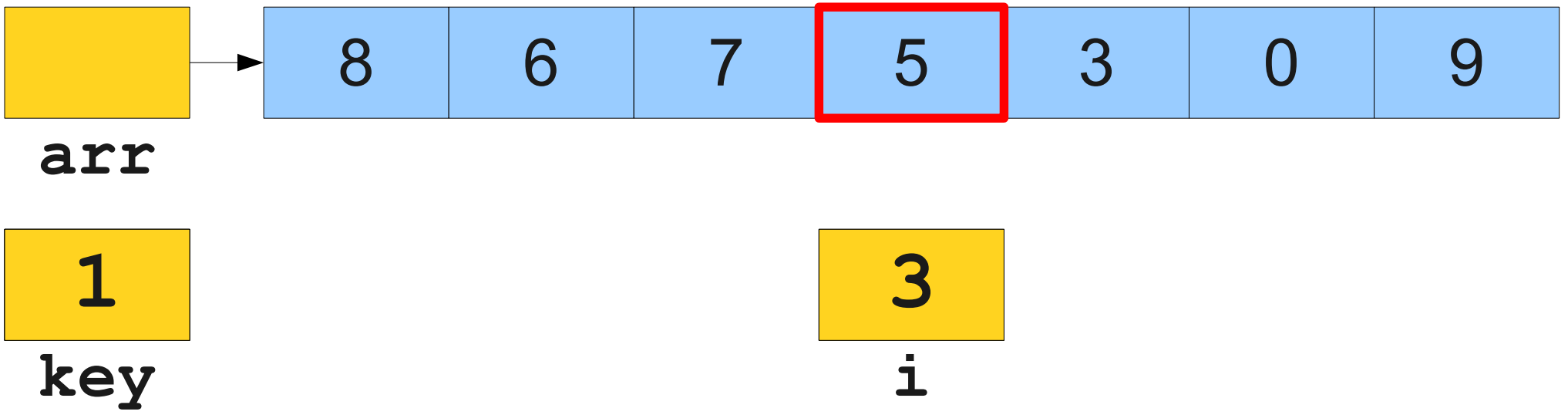

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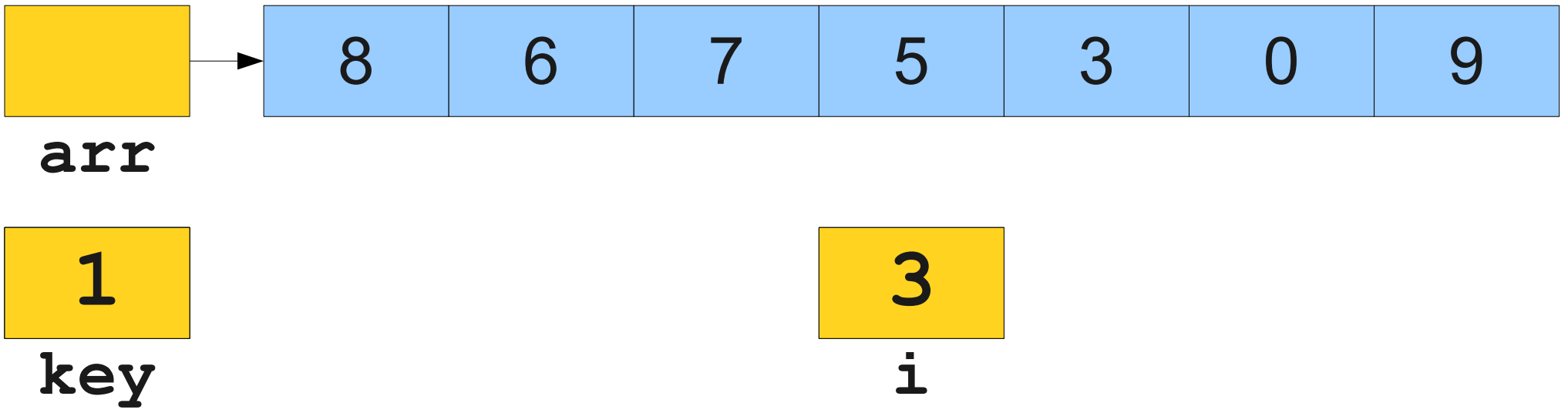

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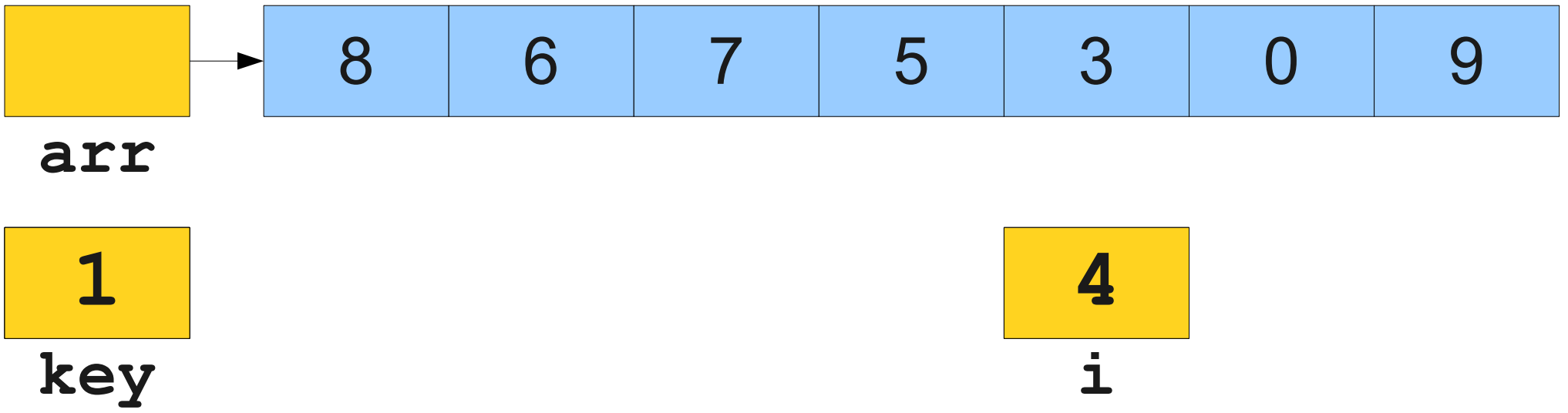

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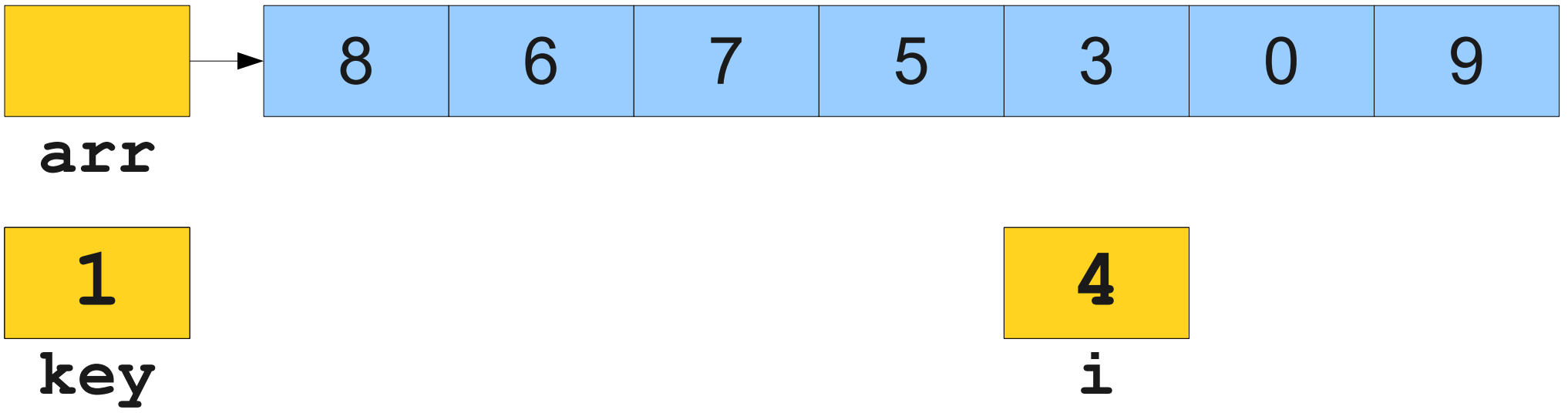

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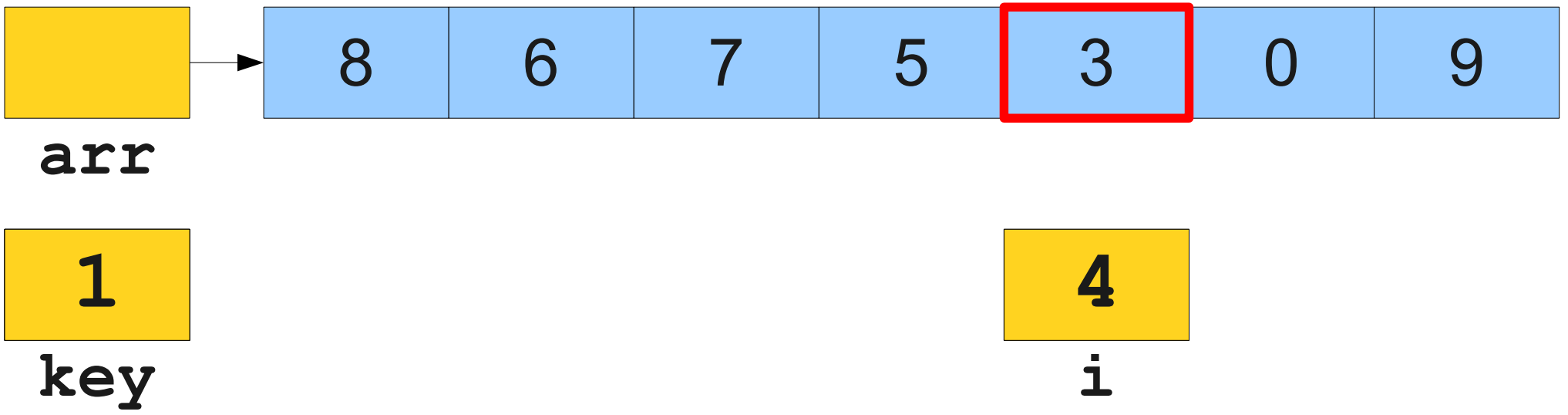

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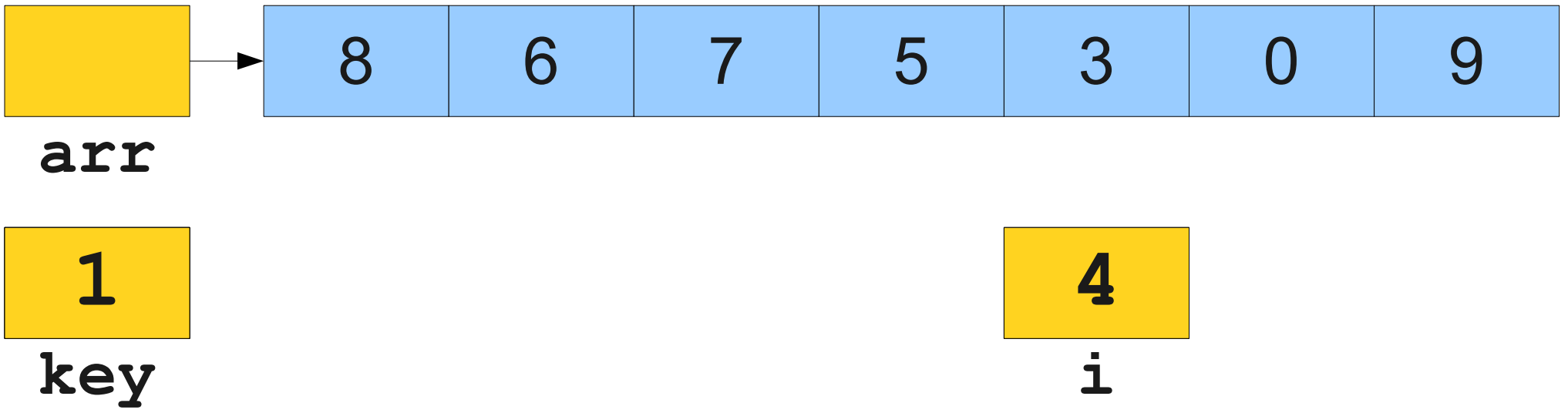

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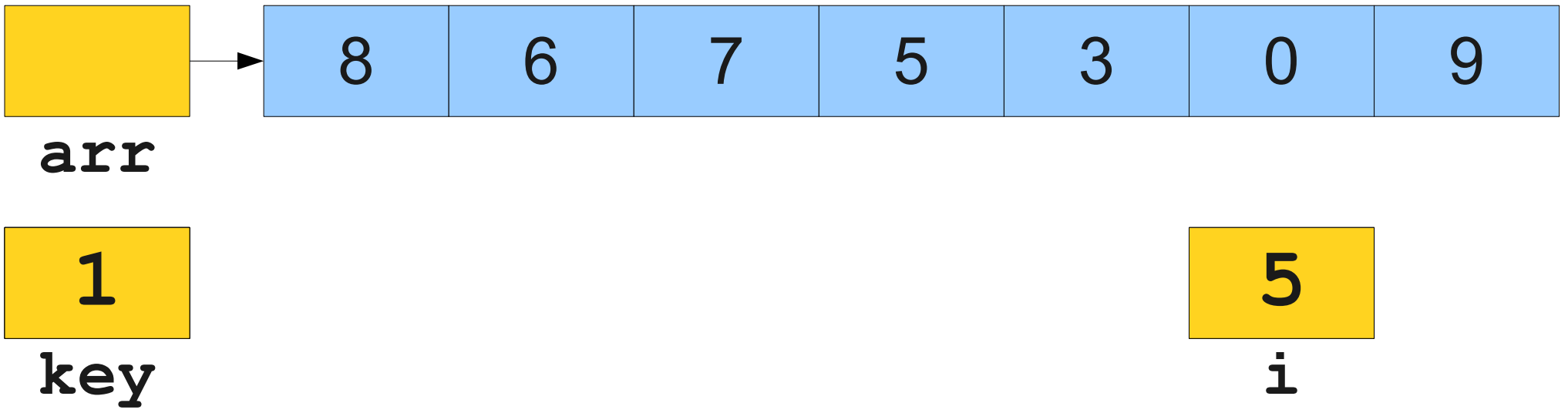

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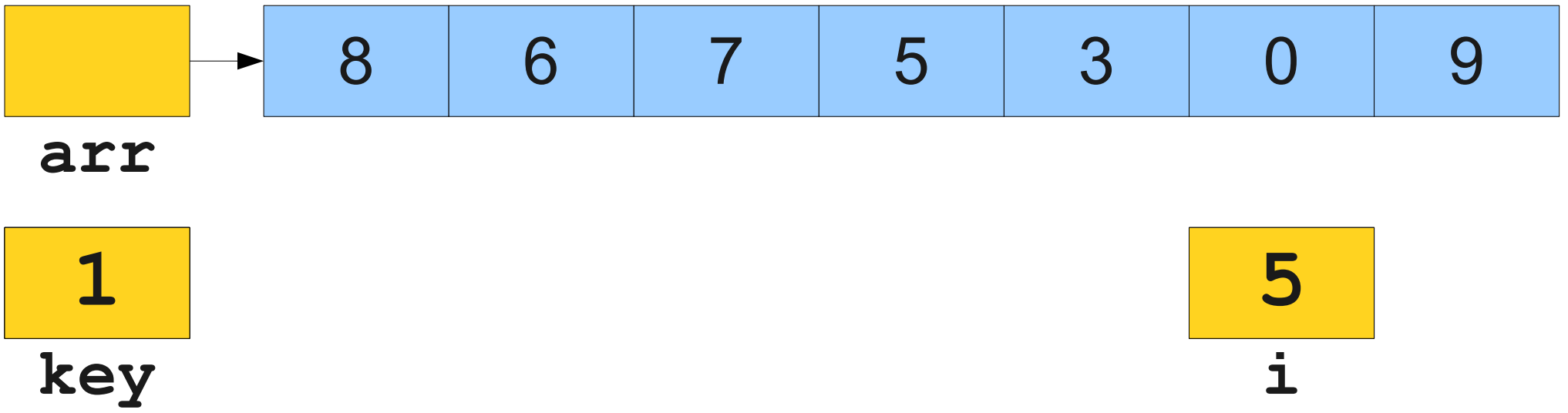

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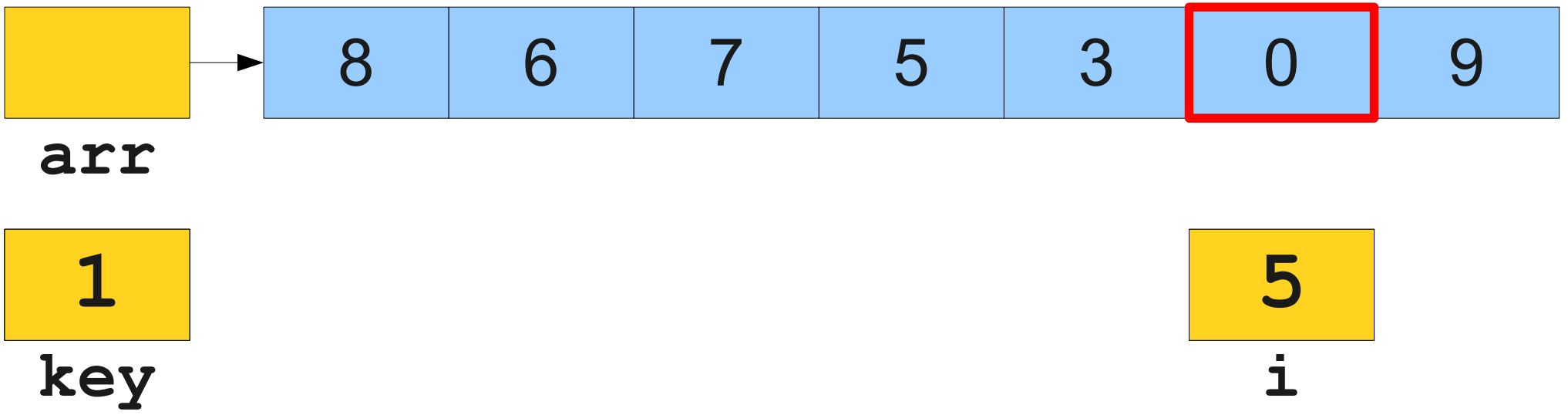

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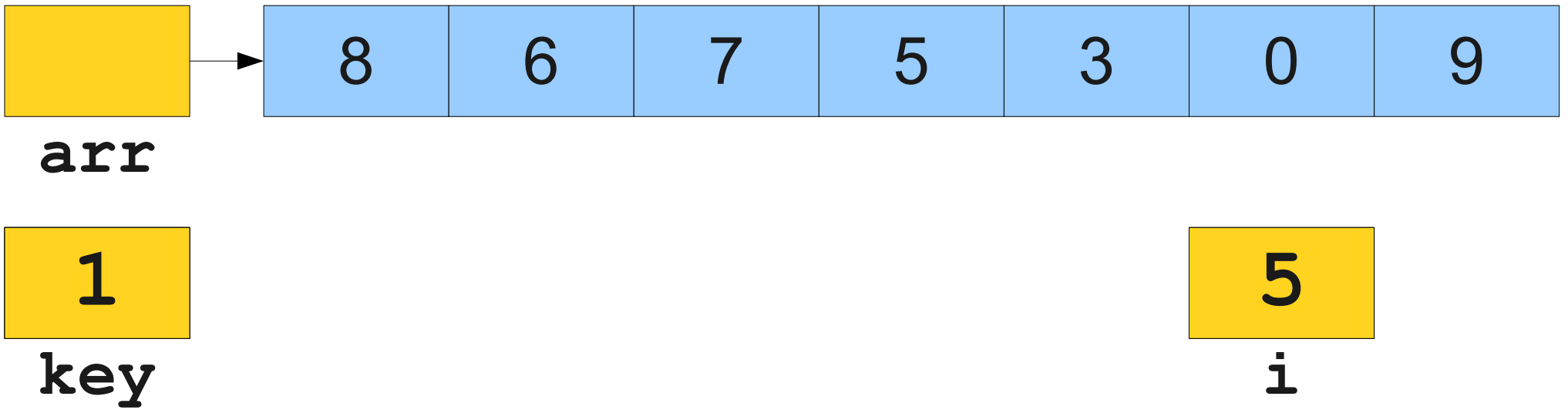

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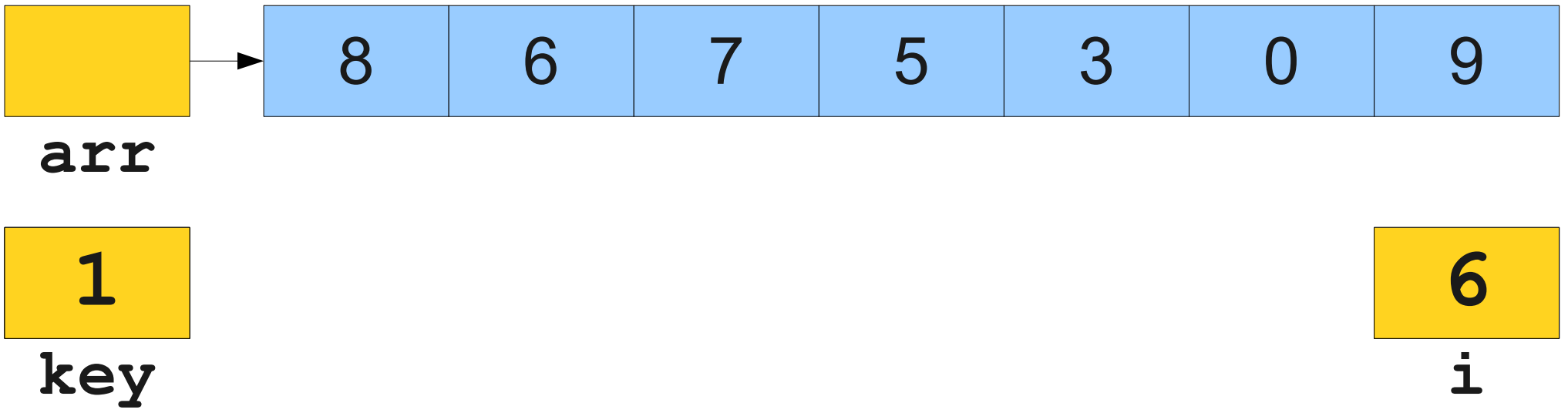

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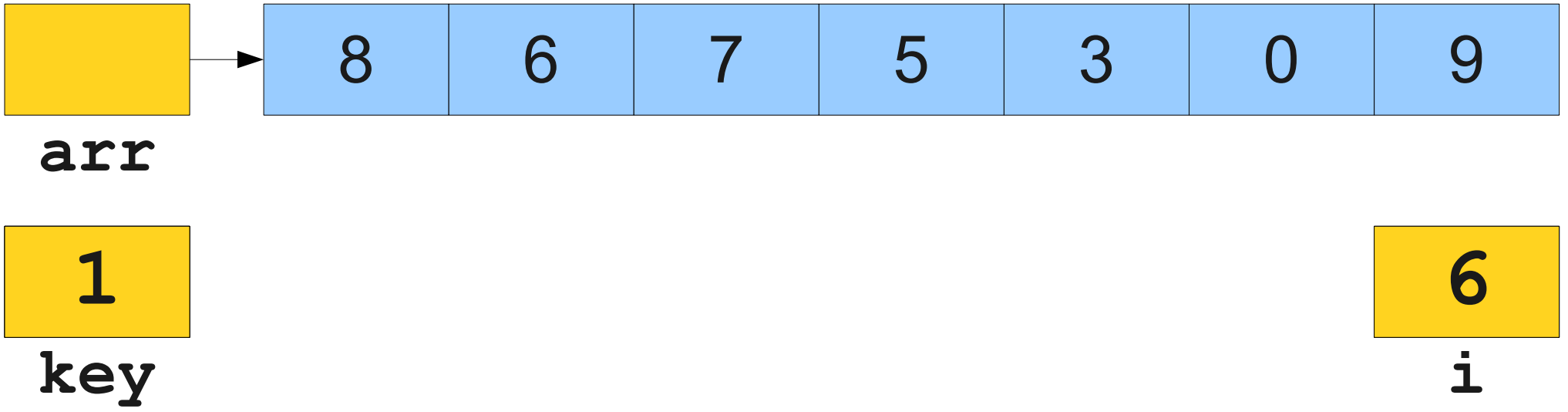

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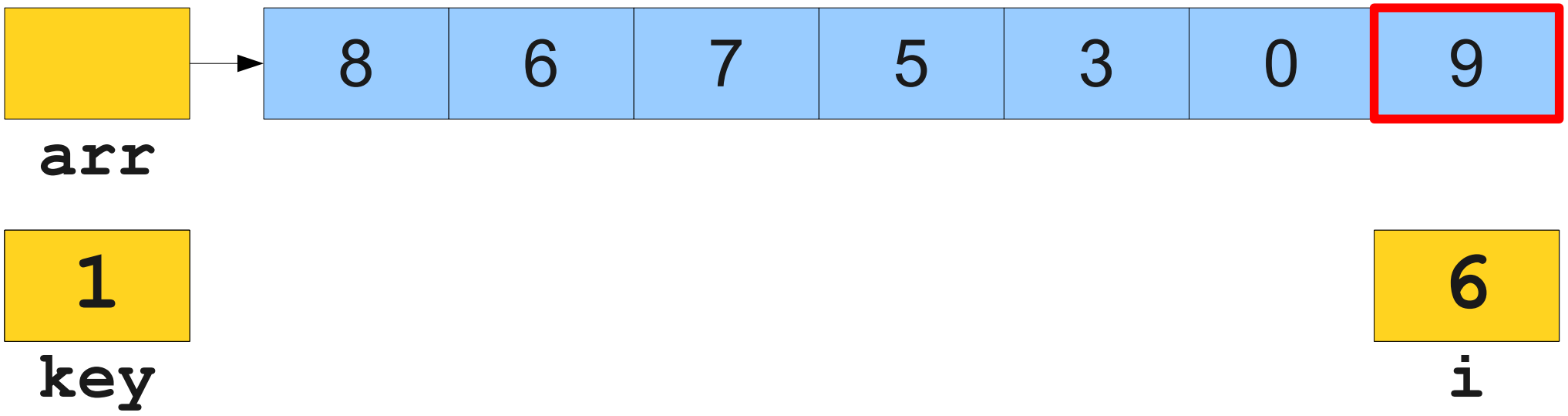

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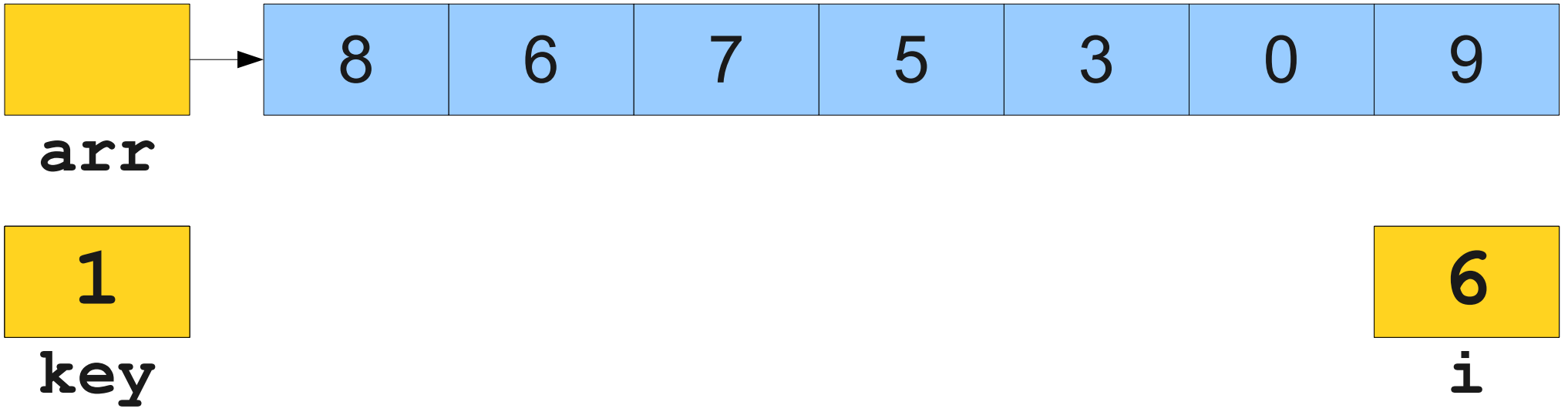

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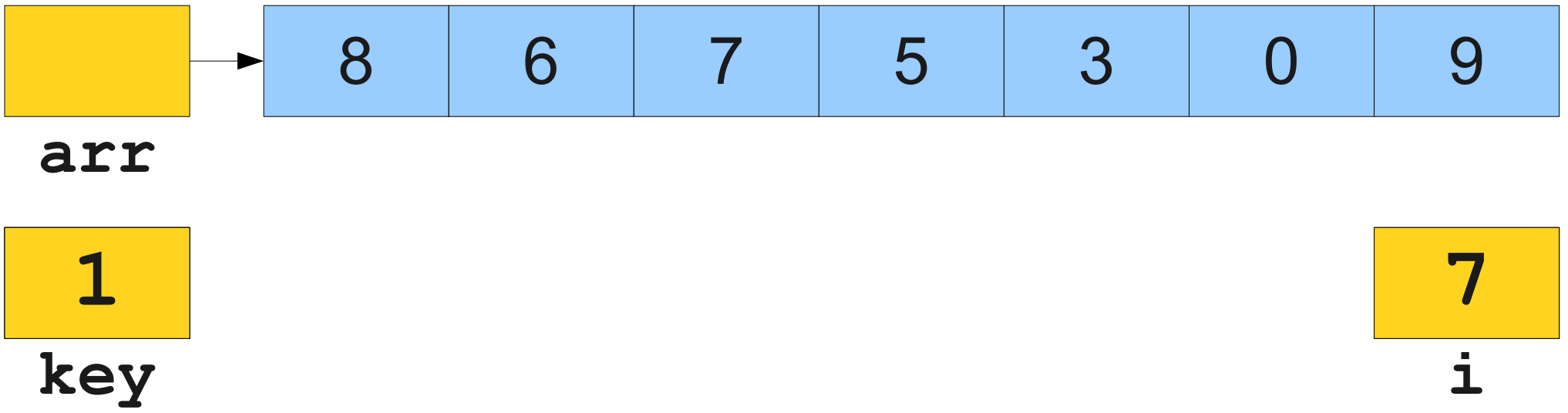

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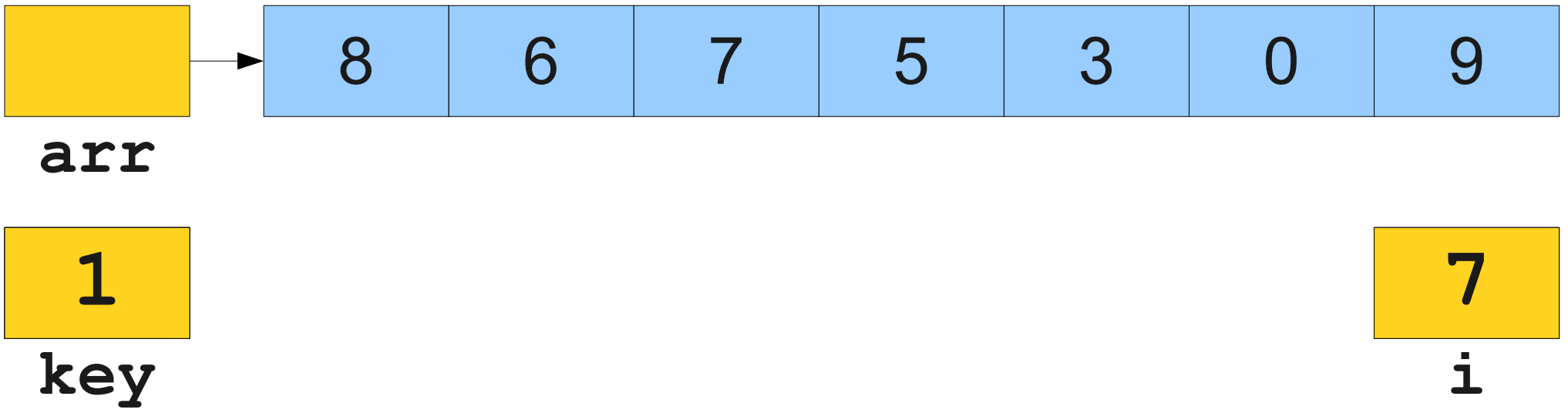

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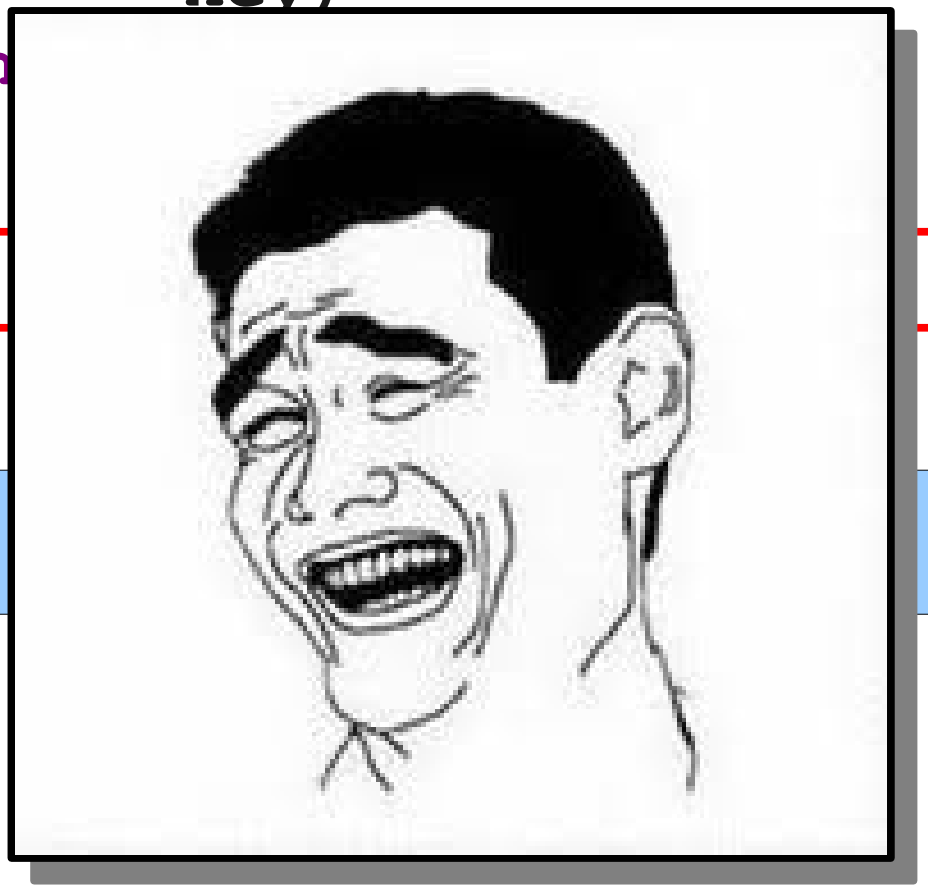

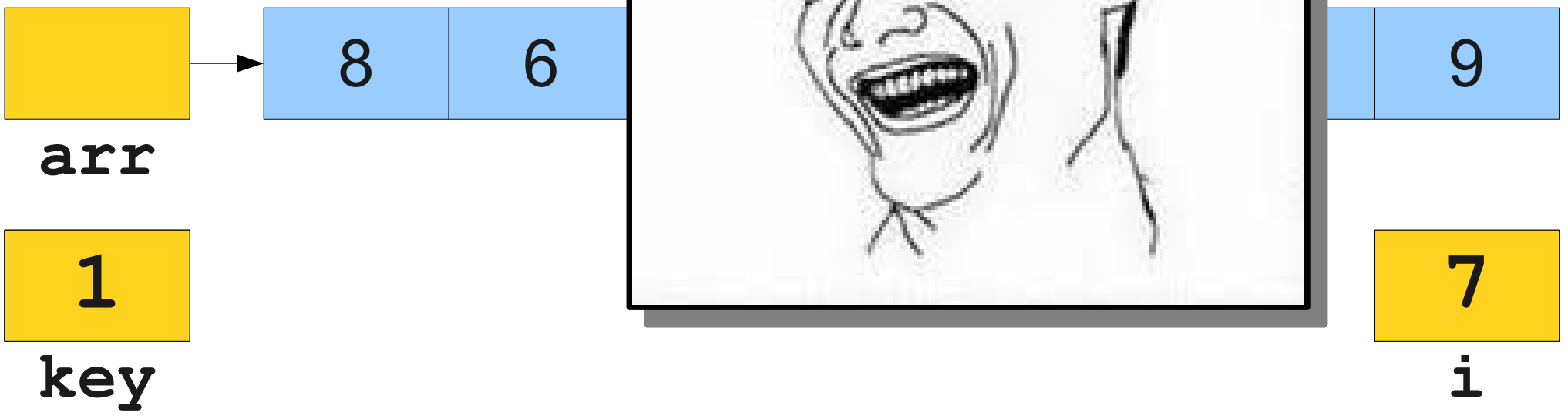

Searching II

Binary Search

2	3	5	7	11	13	17	19	23	29	31	37	41
43	47	53	59	61	67	71	73	79	83	89	97	101
103	107	109	113	127	131	137	139	149	151	157	163	167

Binary Search

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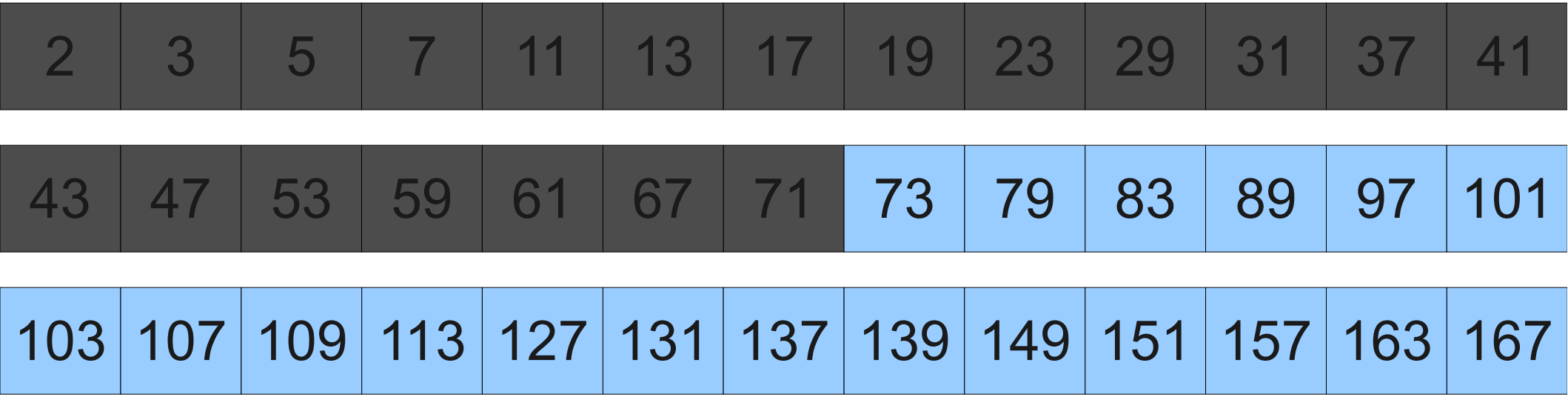

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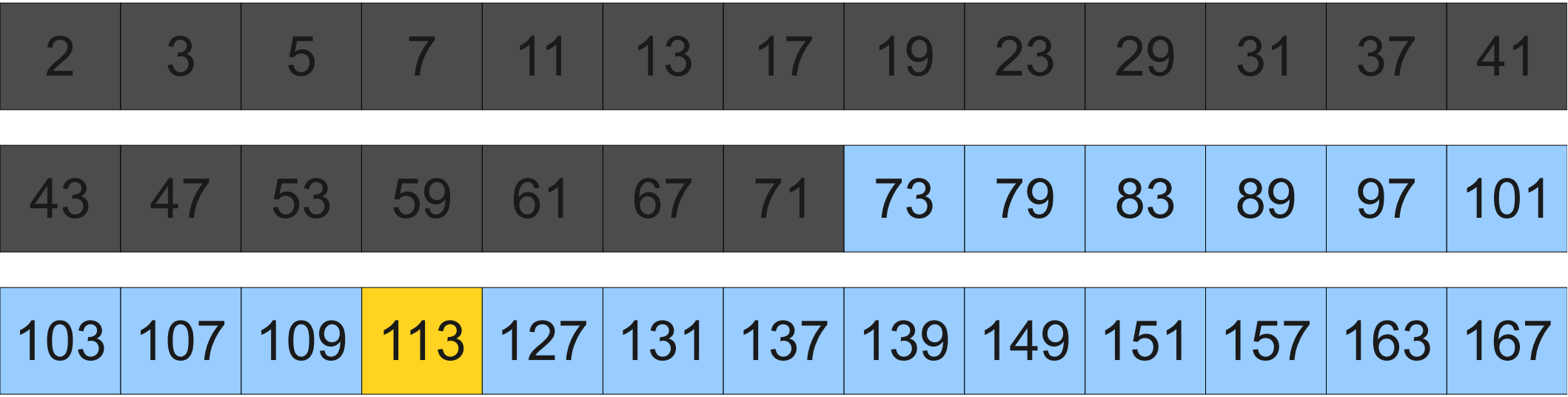

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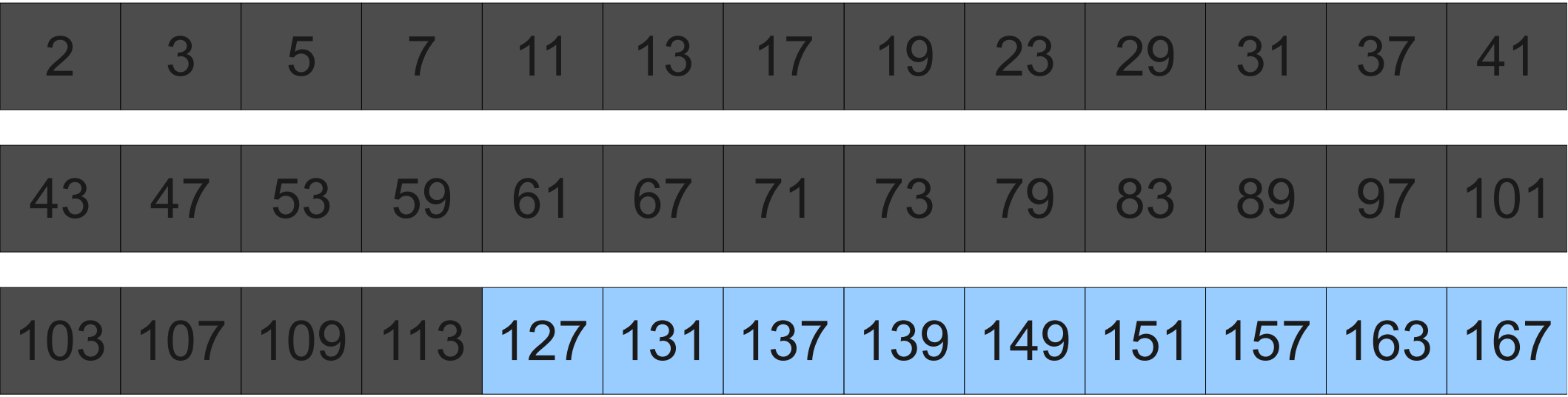

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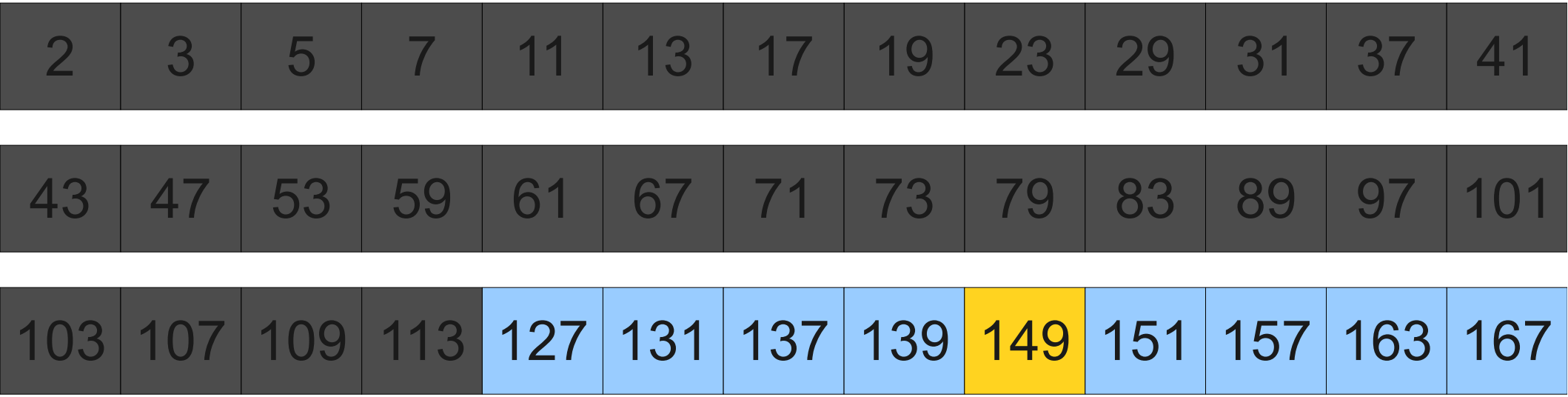

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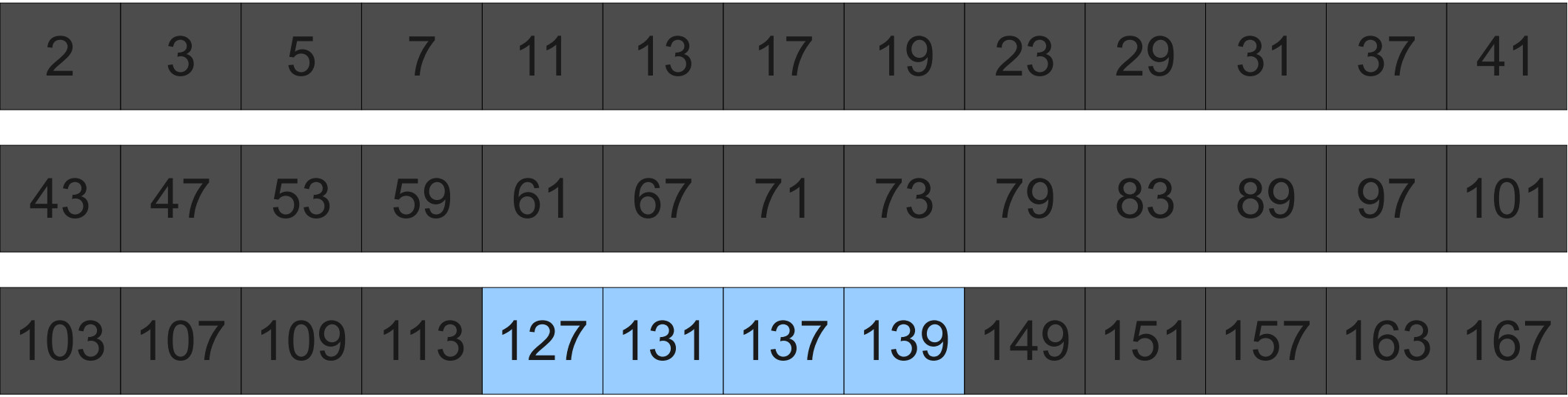

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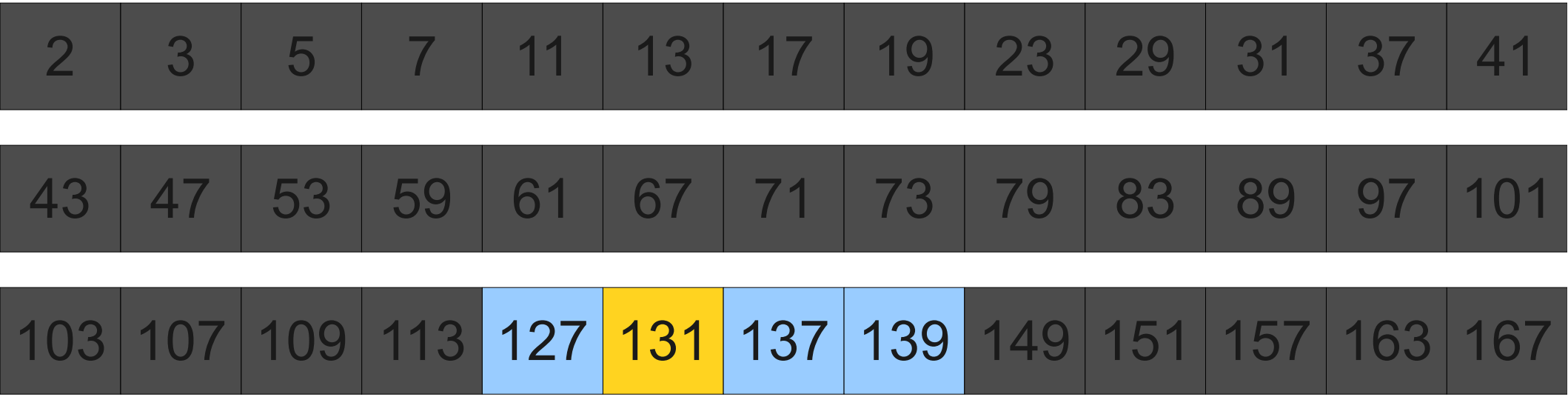

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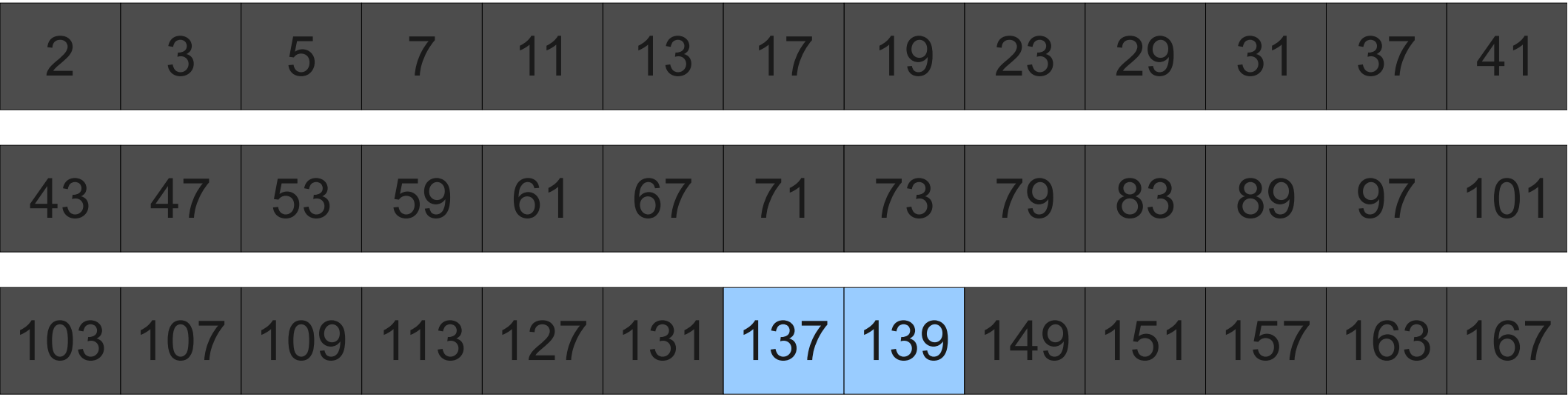

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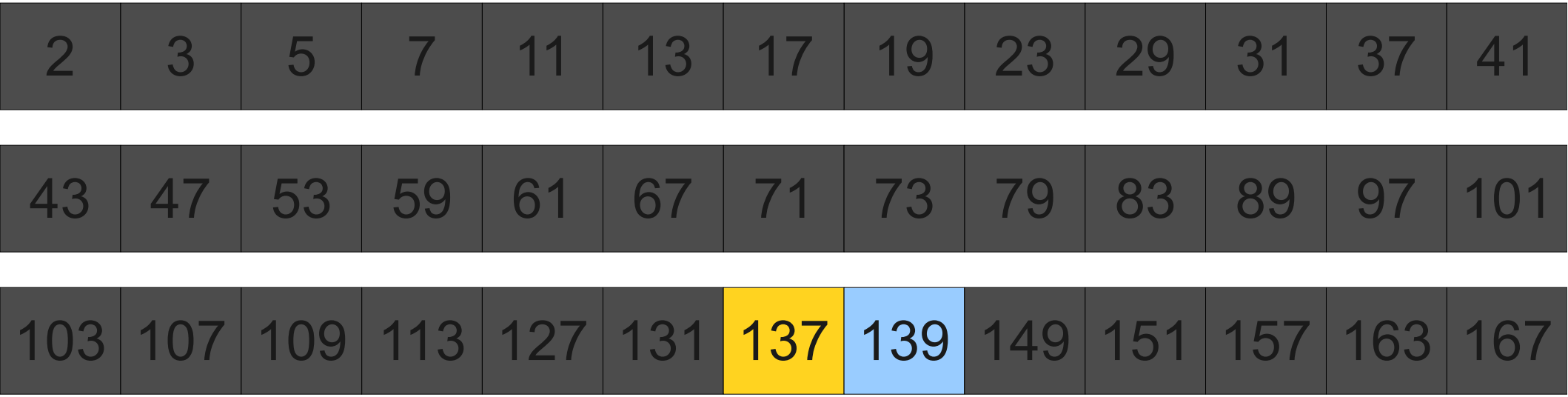

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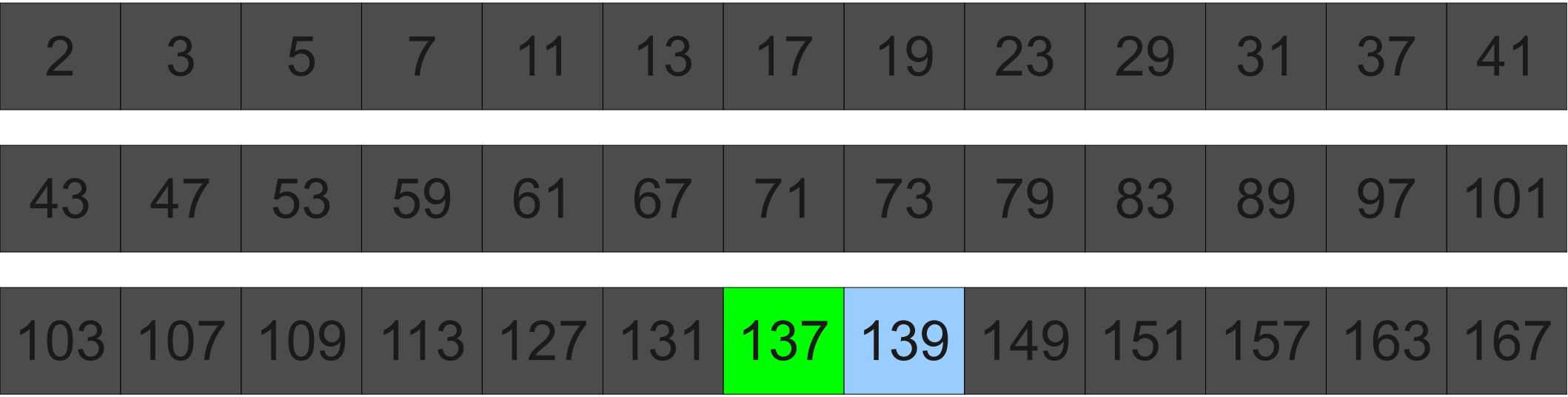

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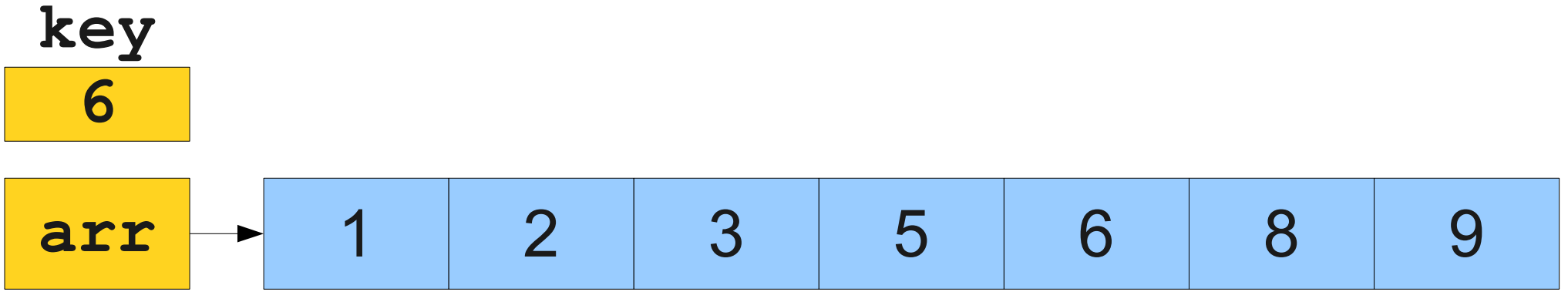

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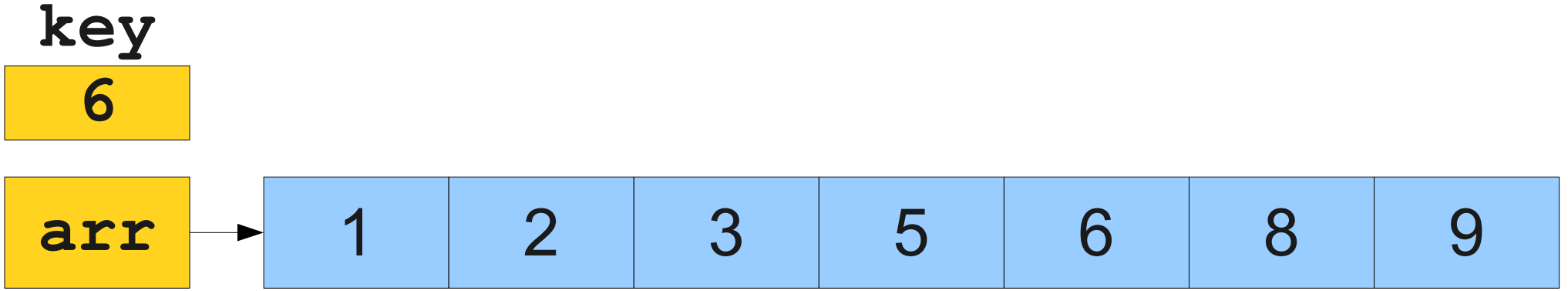

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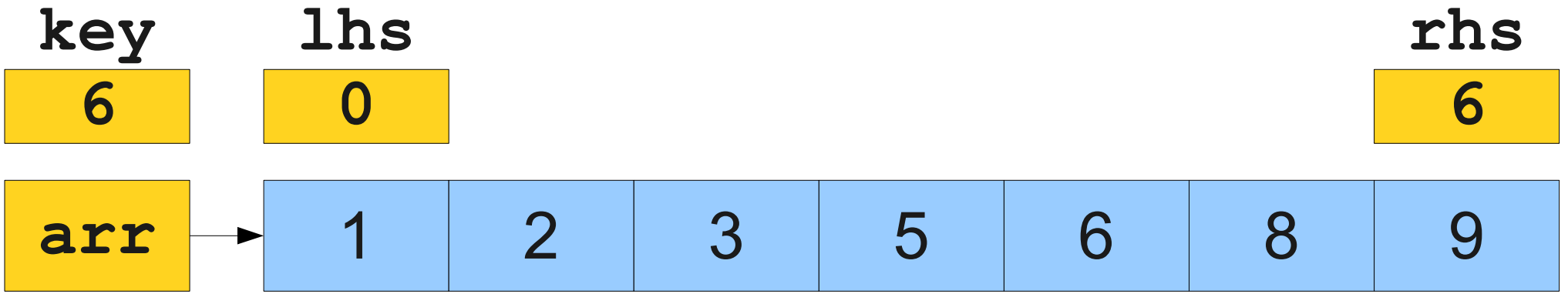

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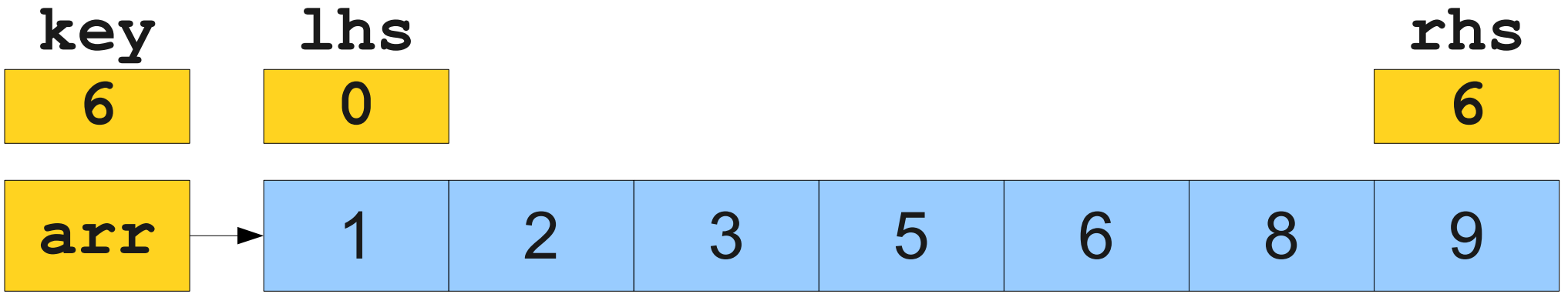

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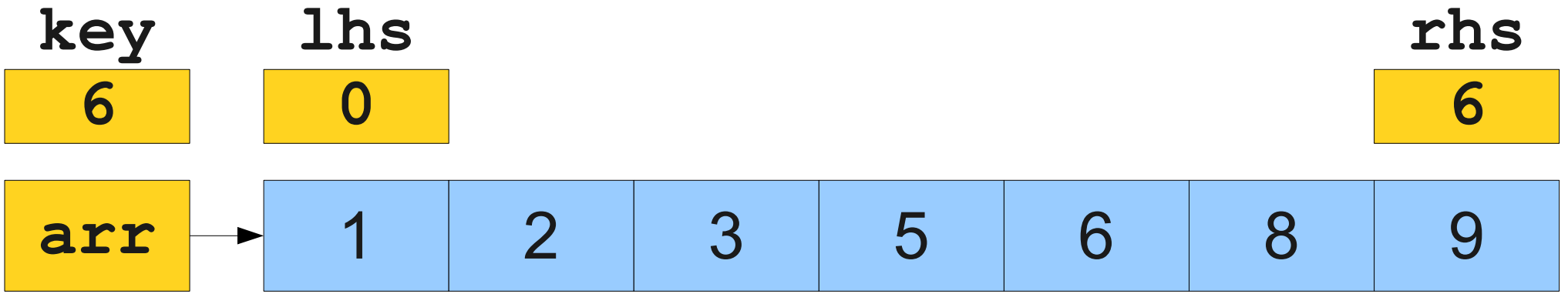

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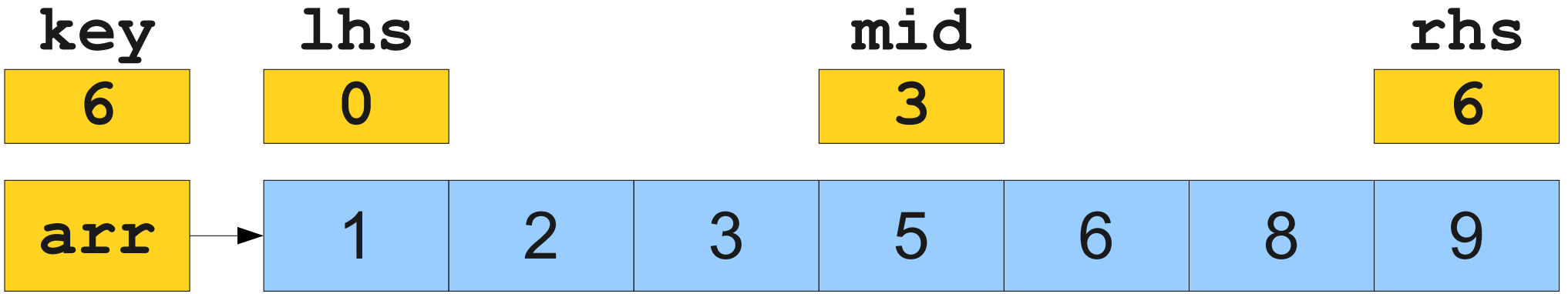

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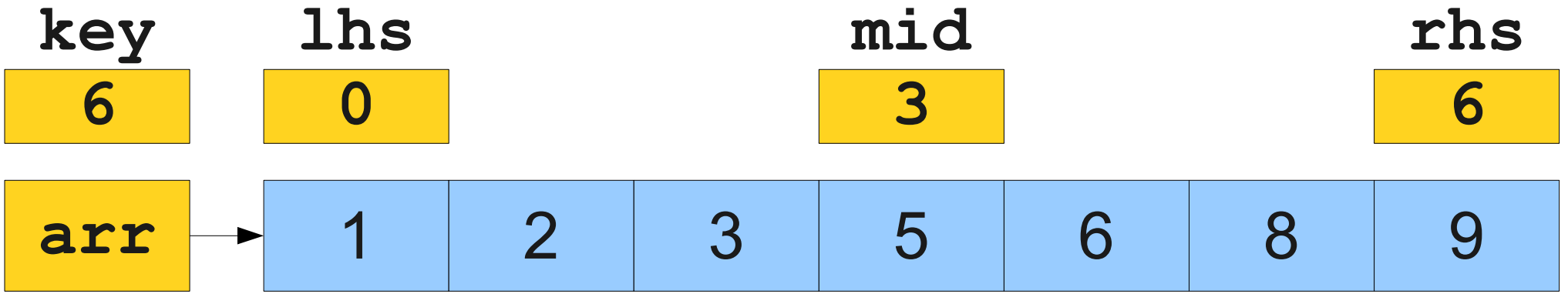

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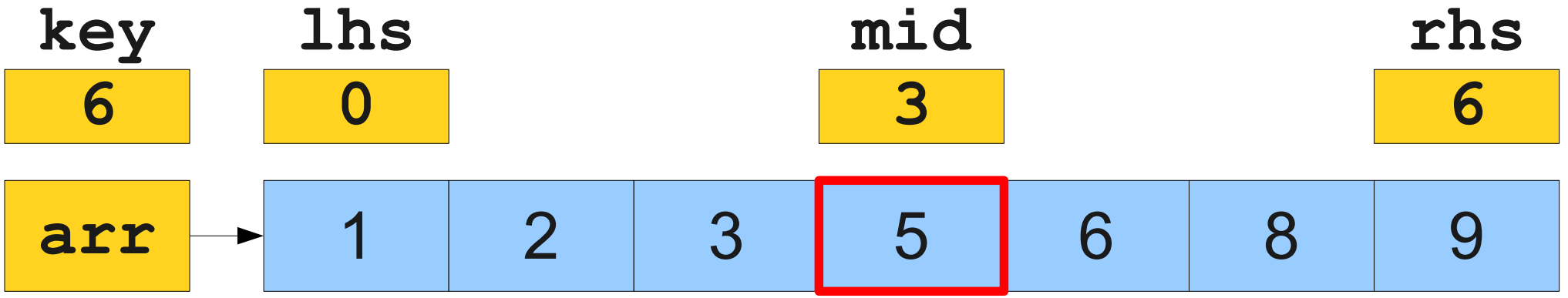

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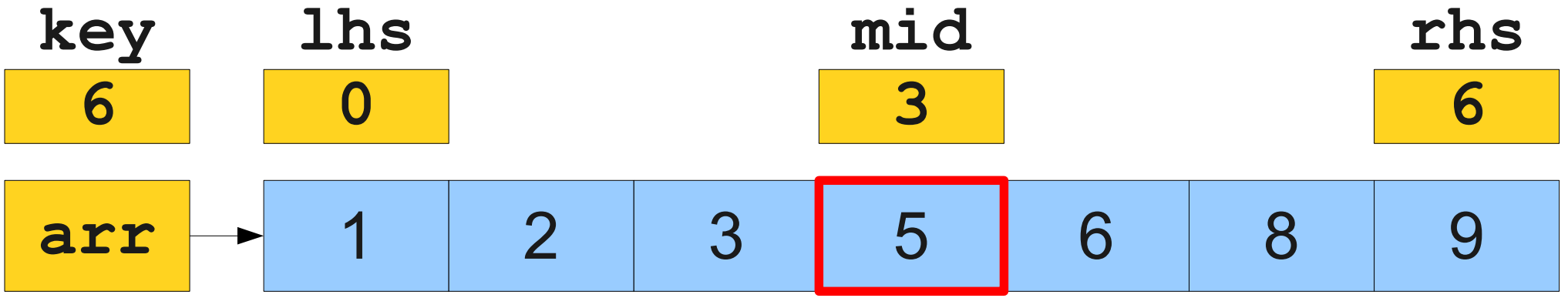

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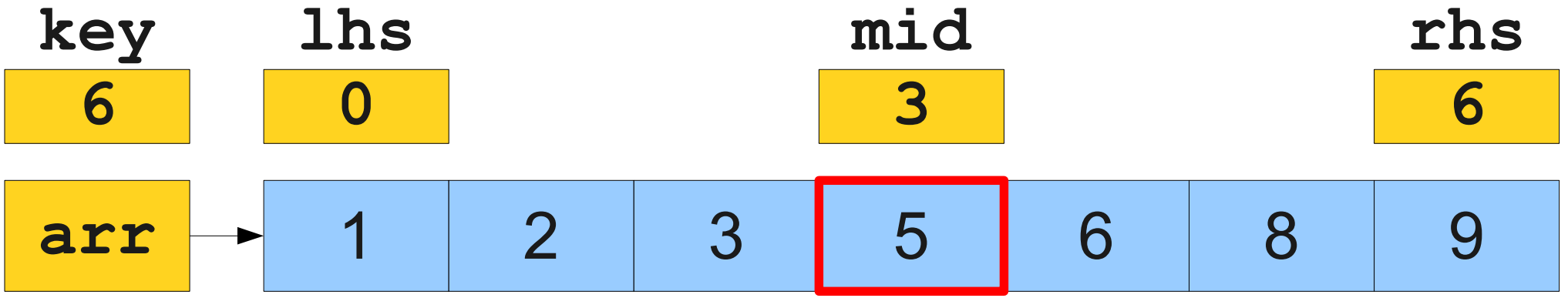

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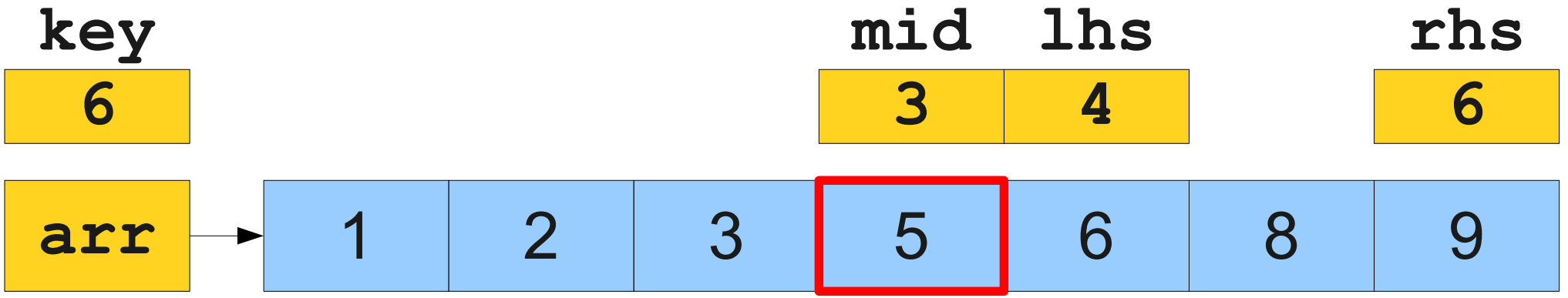

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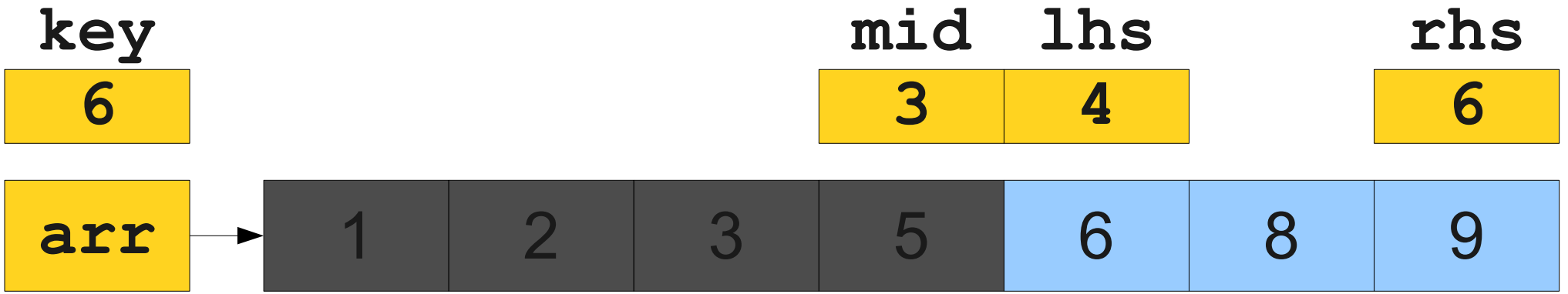

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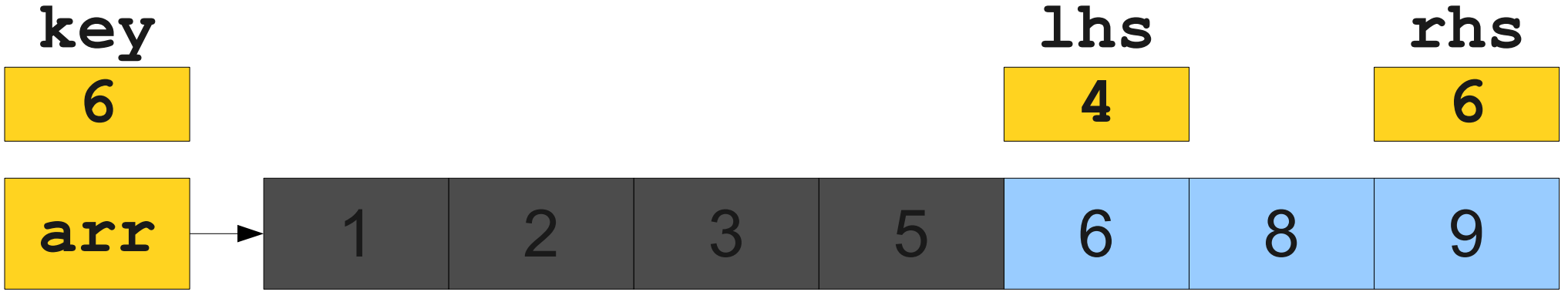

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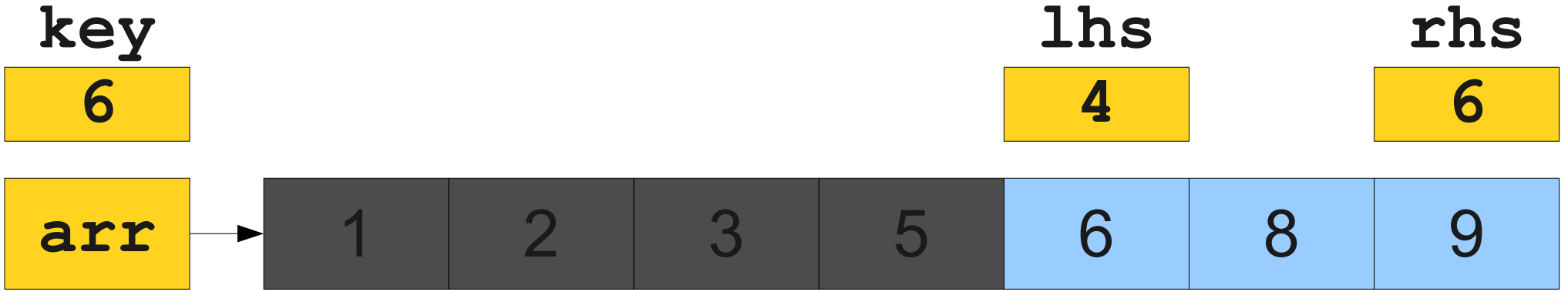

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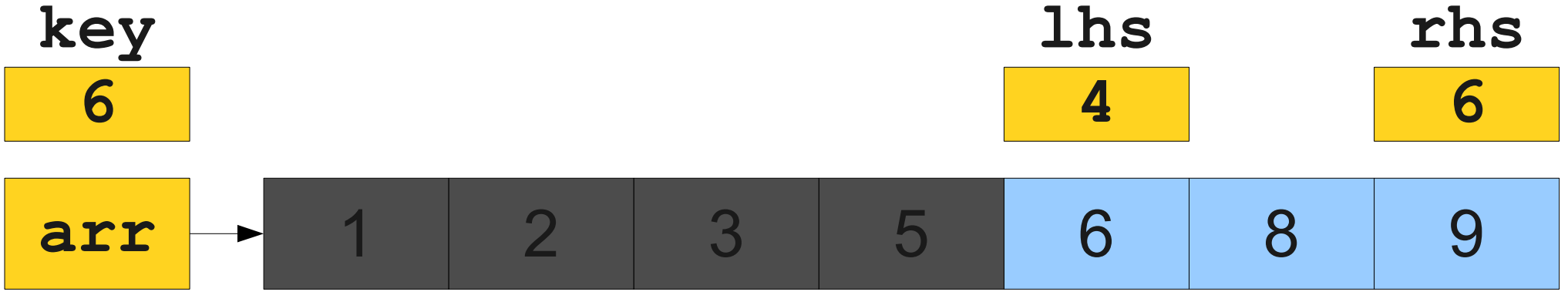

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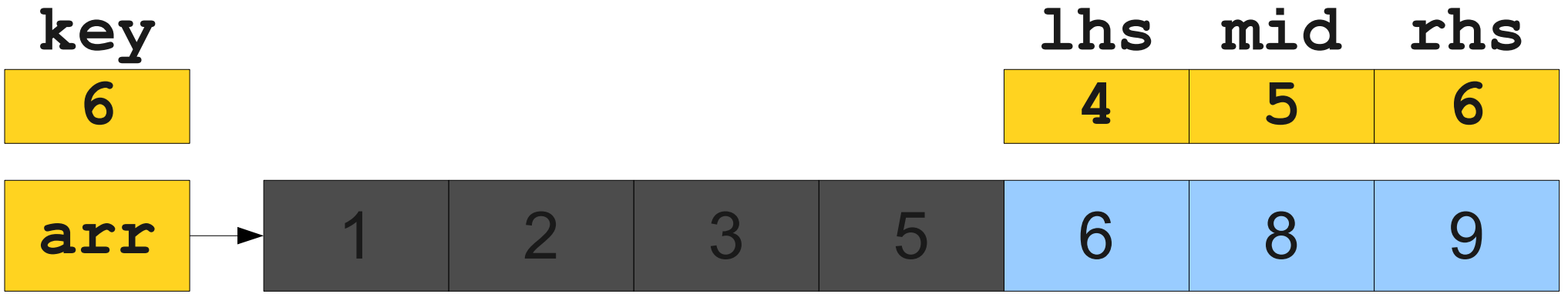

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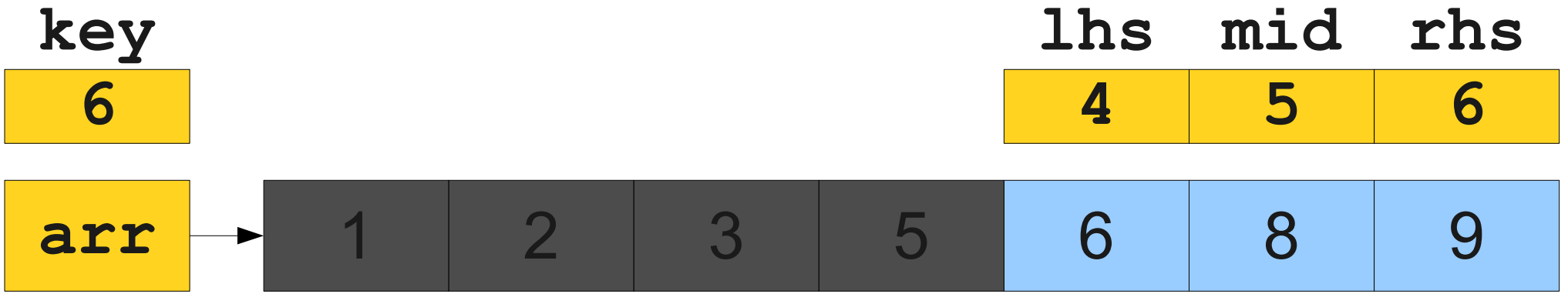

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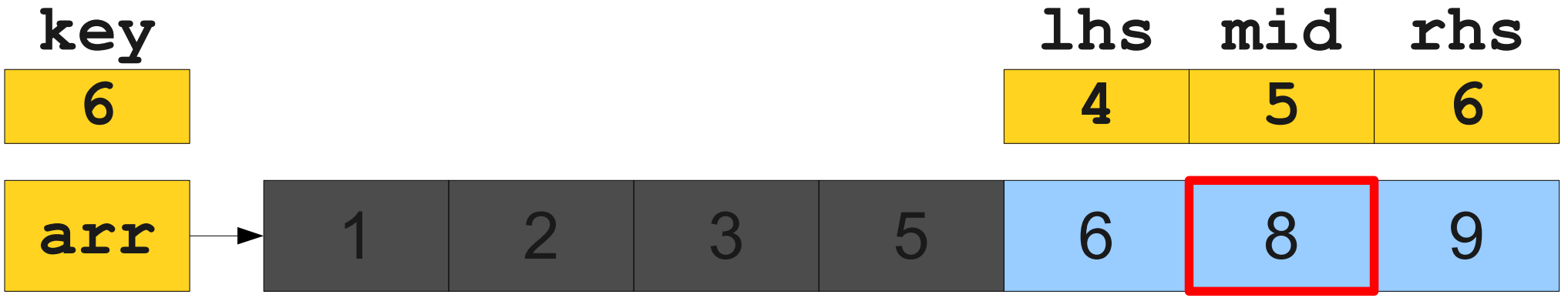

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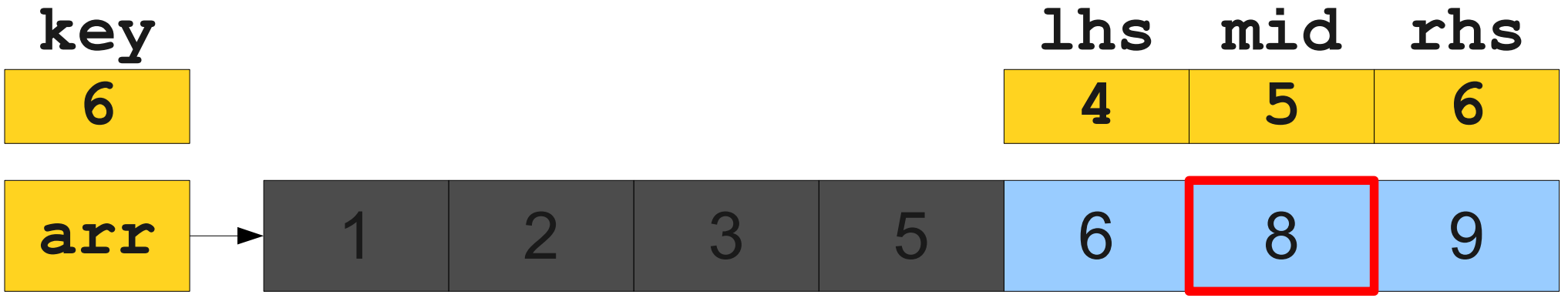

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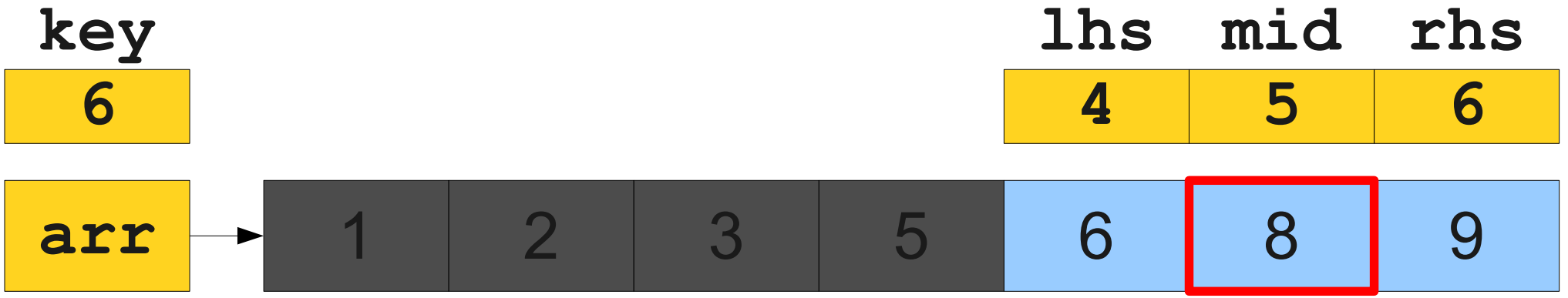

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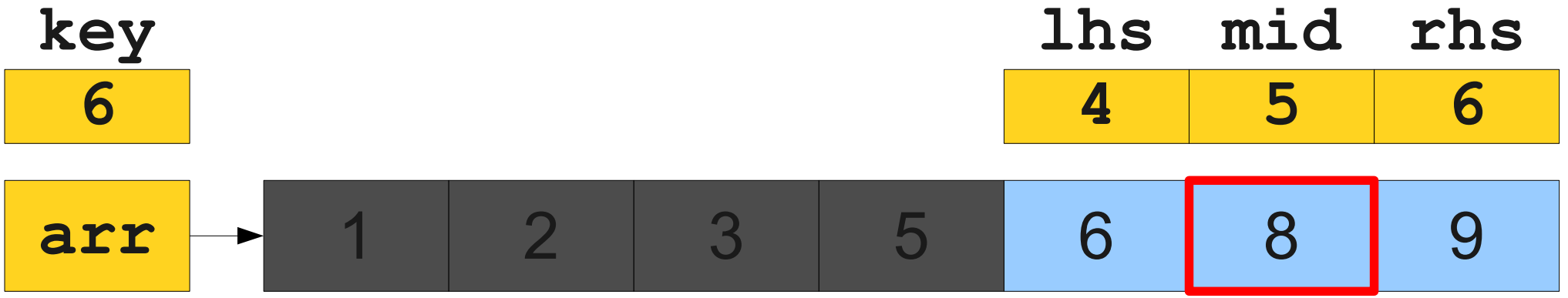

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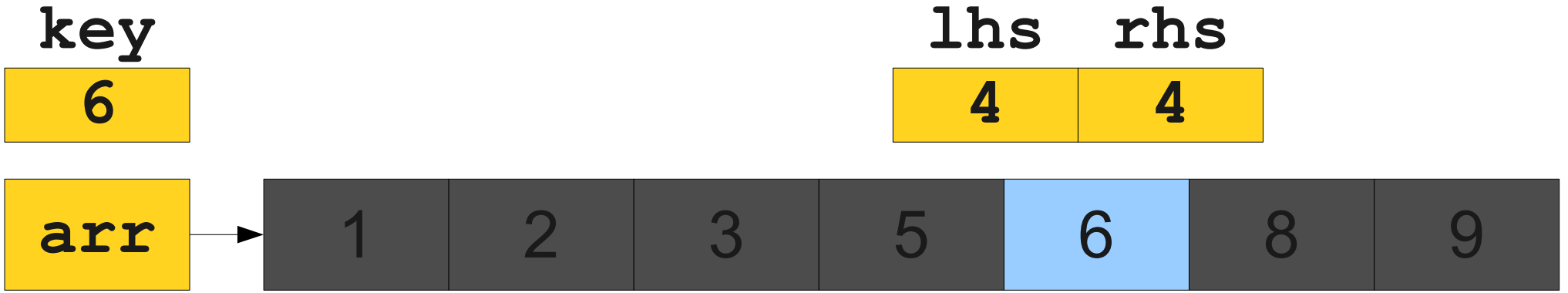

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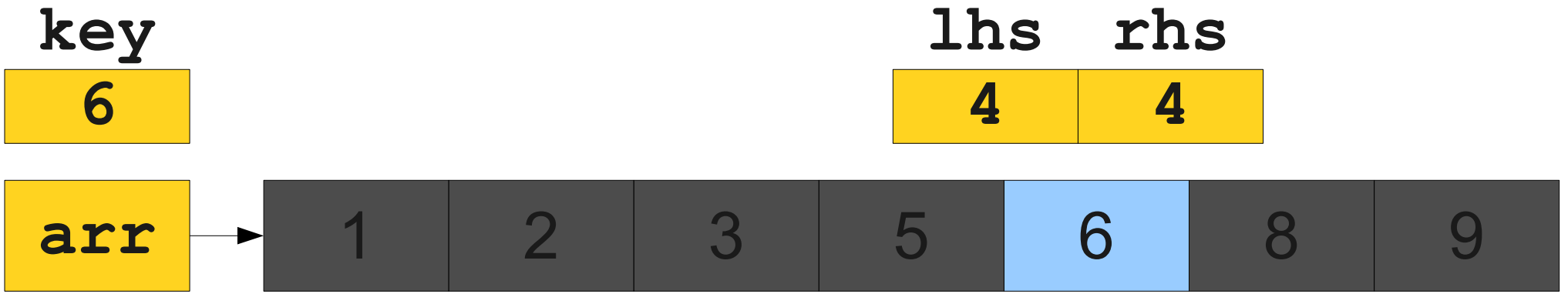

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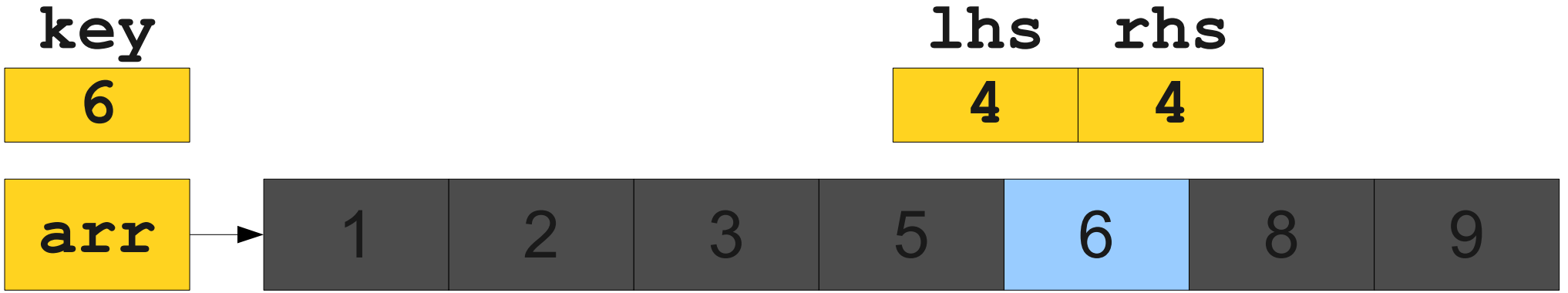

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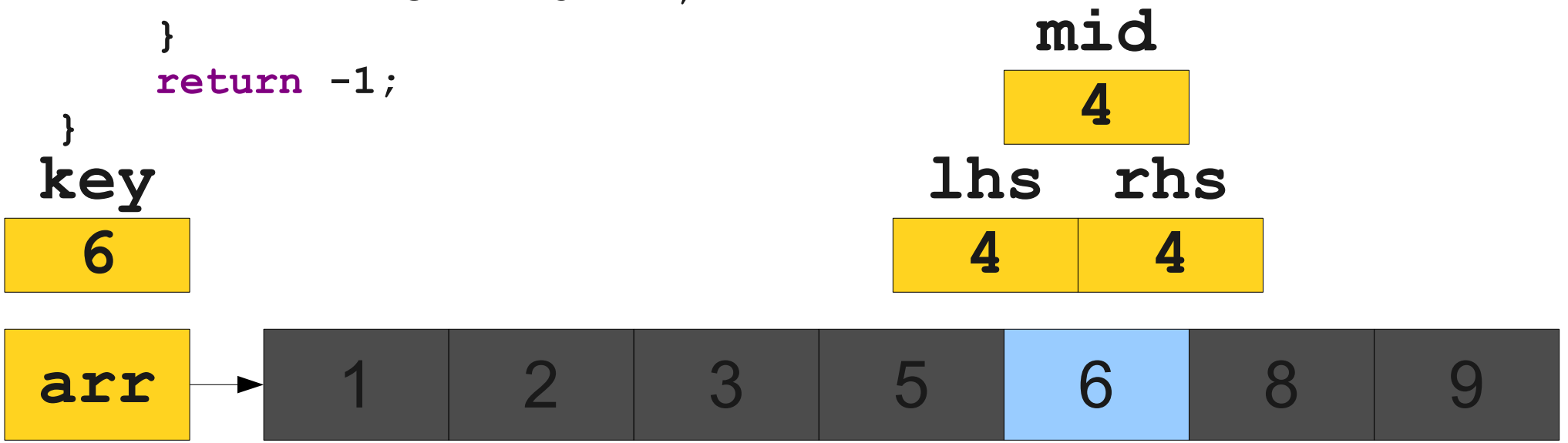

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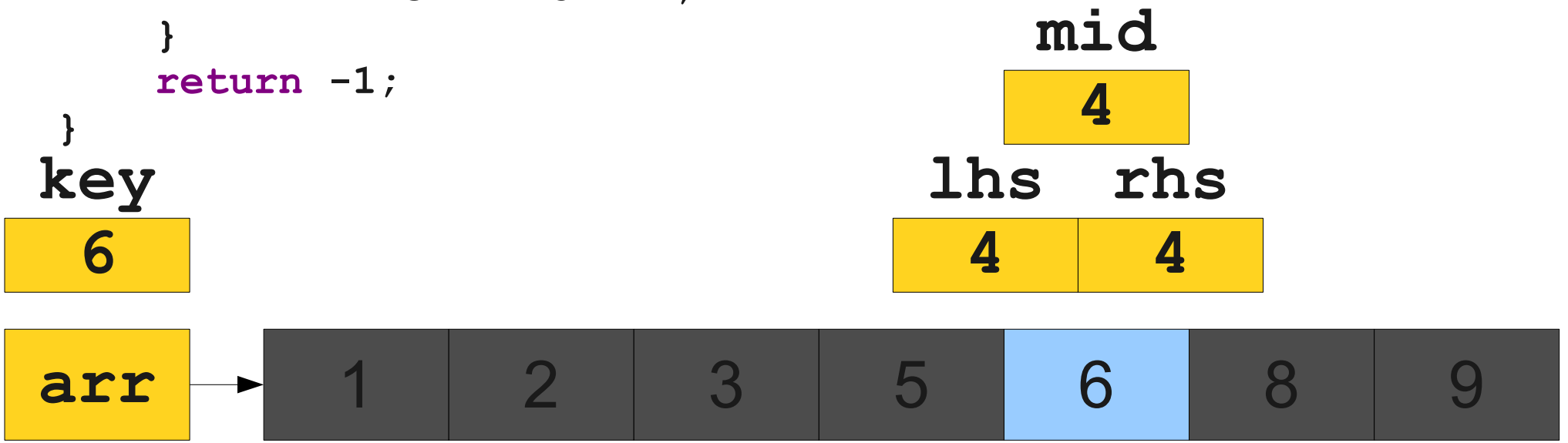

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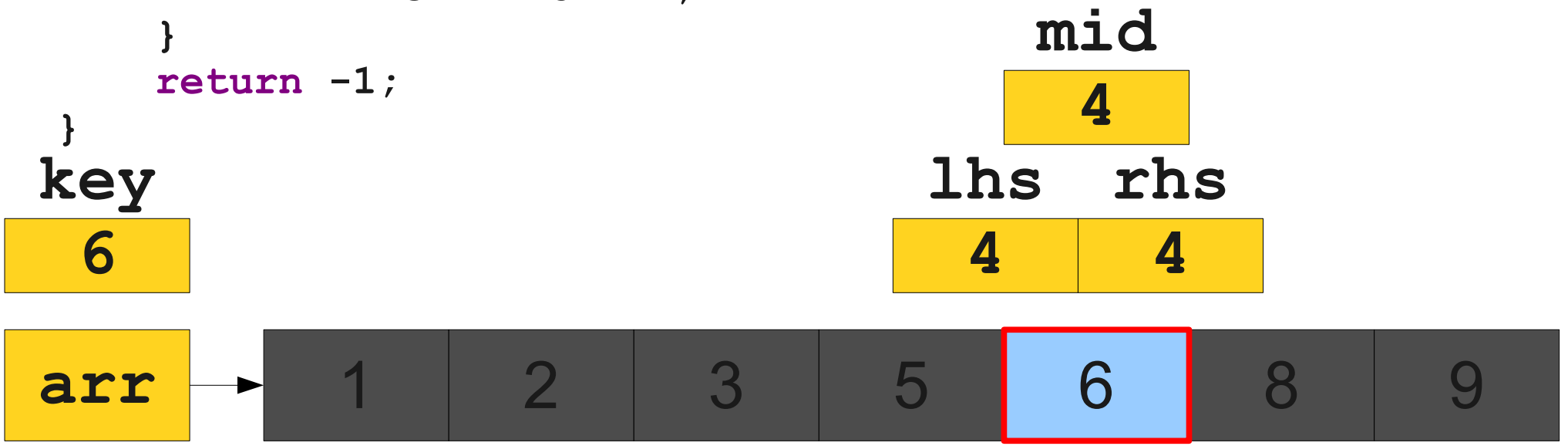

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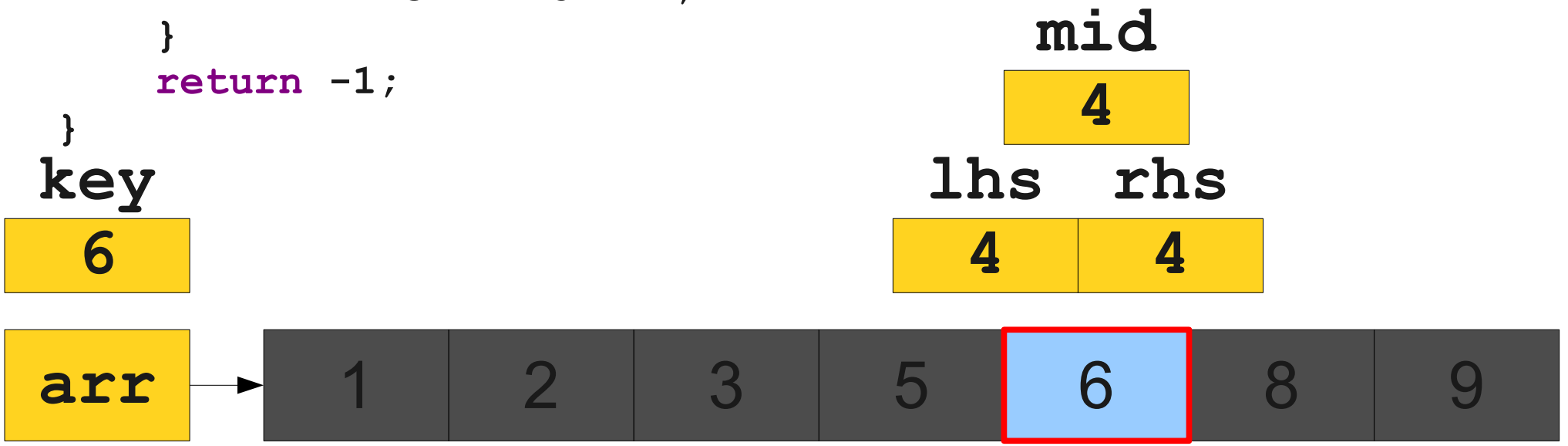

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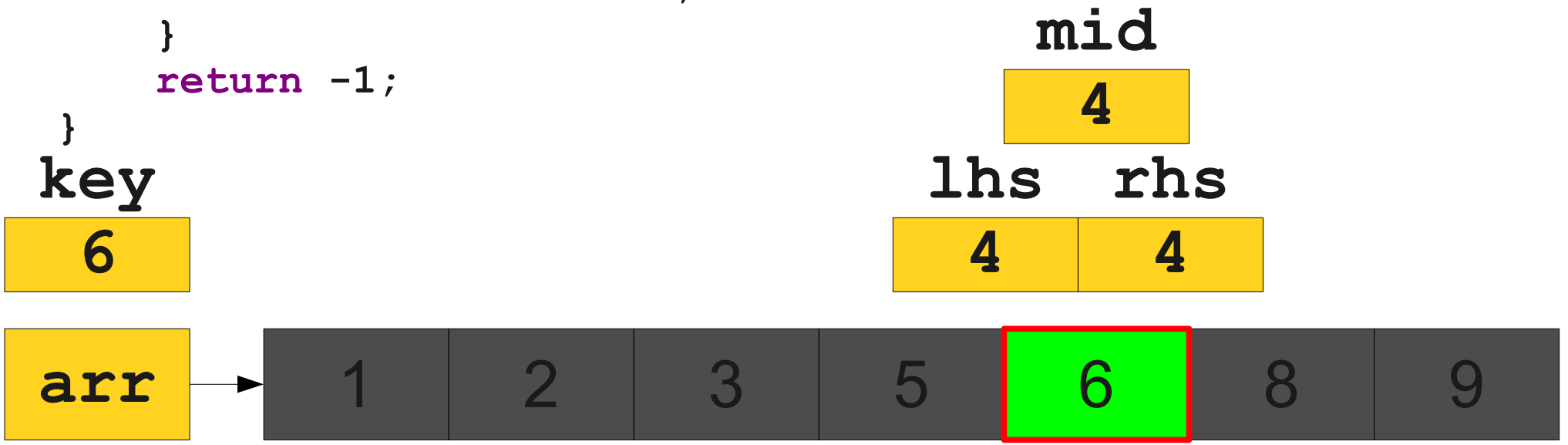

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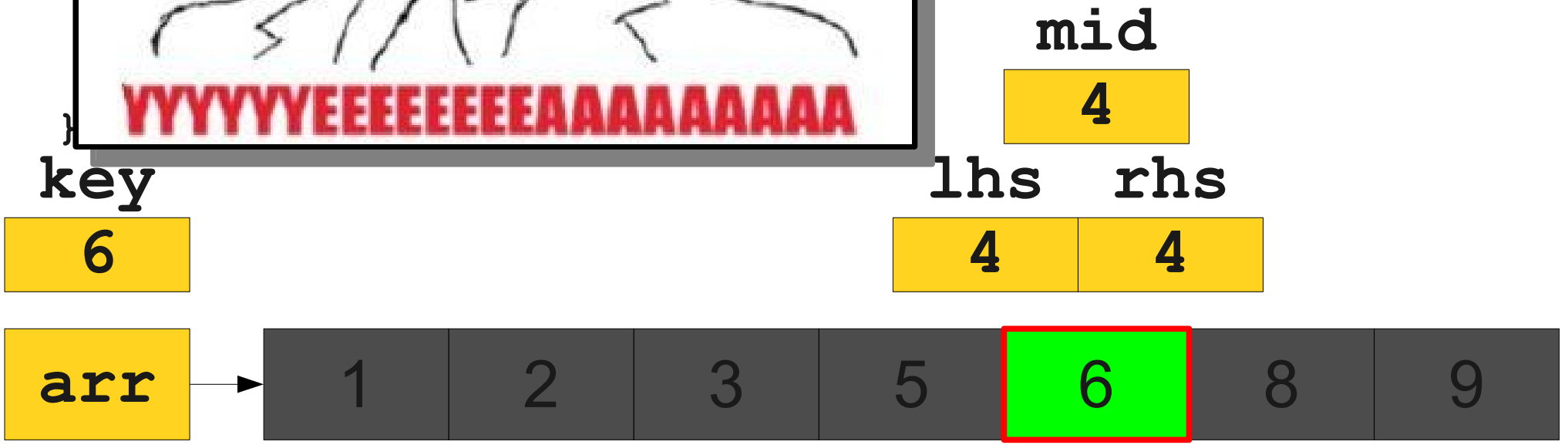

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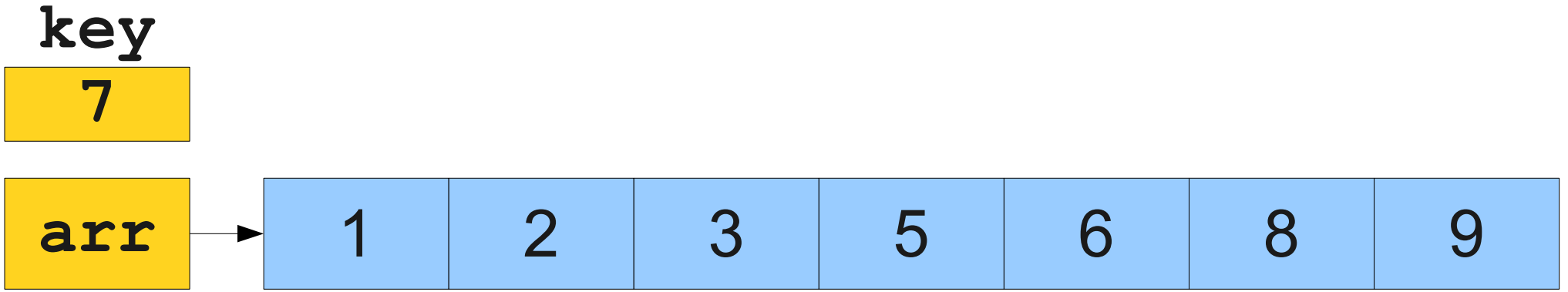

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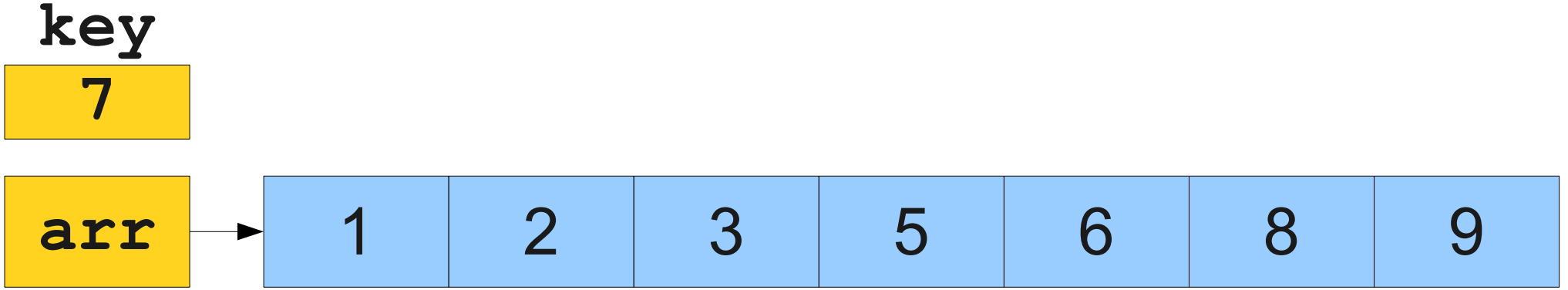

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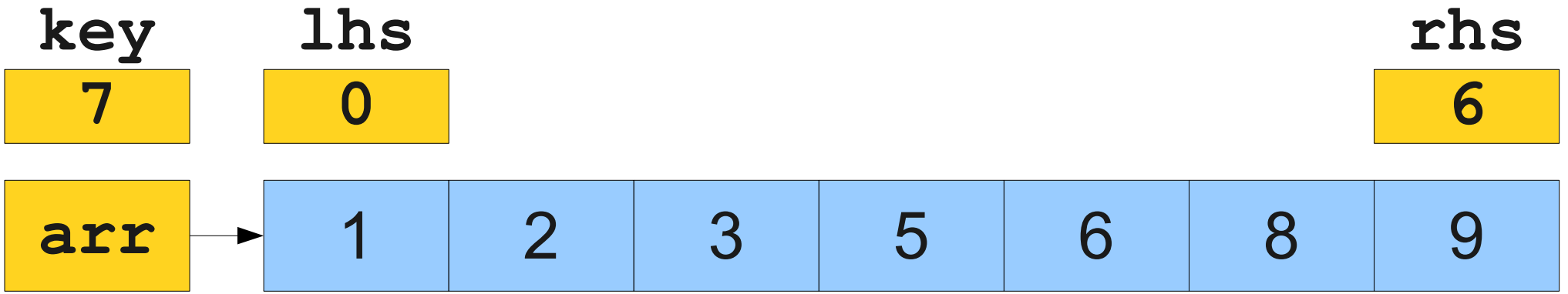

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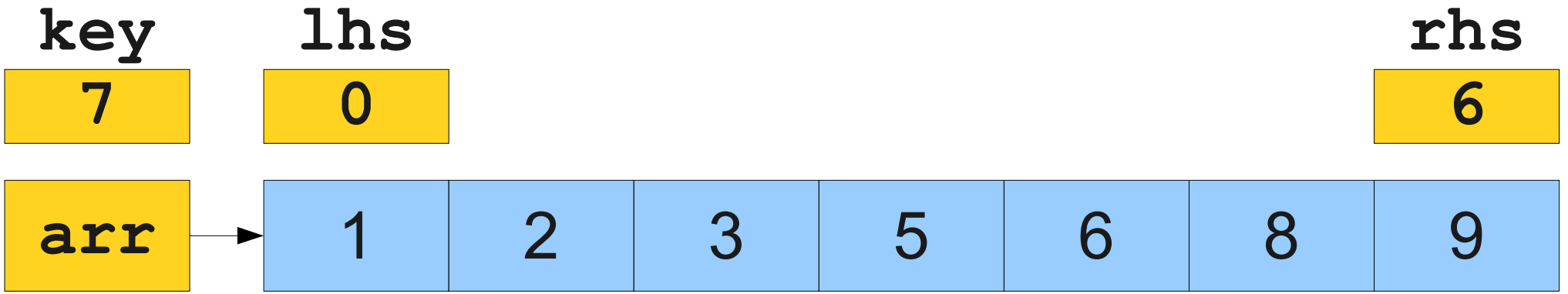

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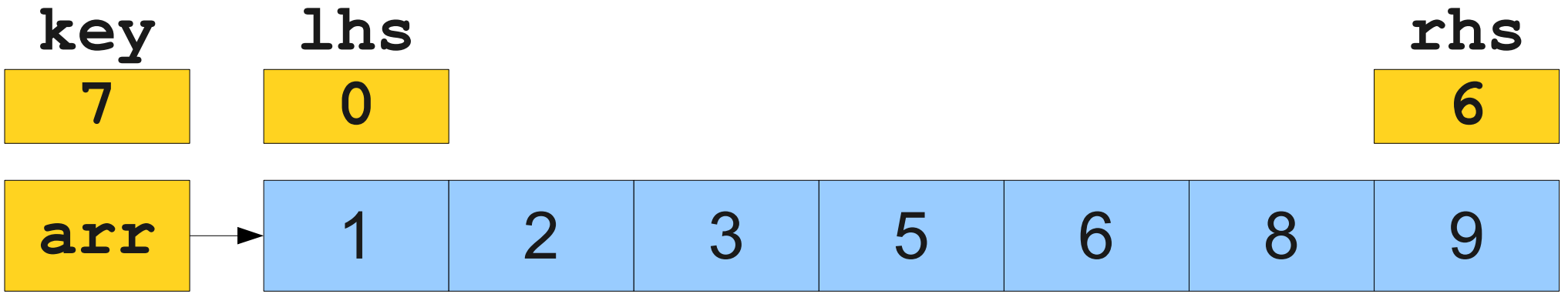

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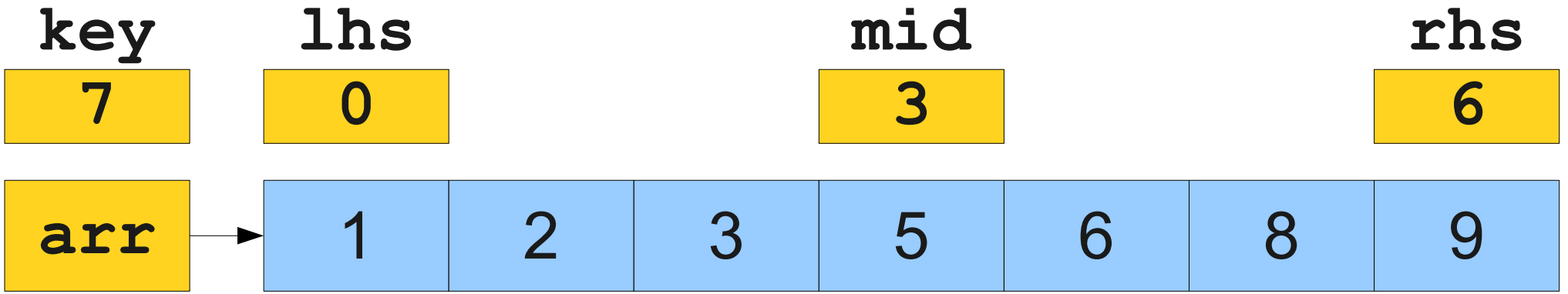

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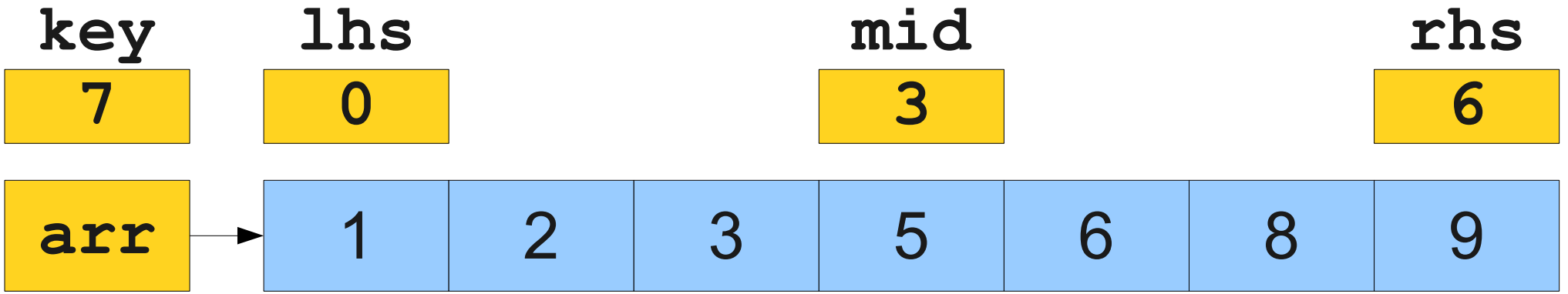

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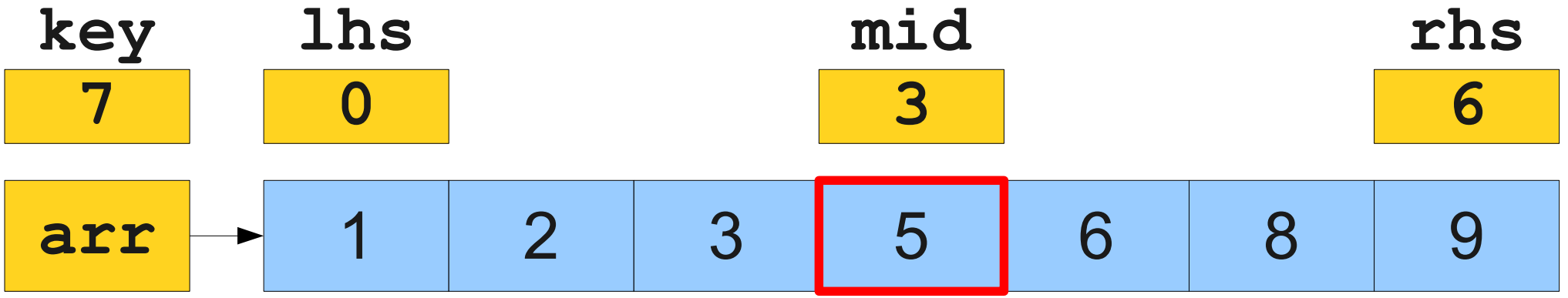

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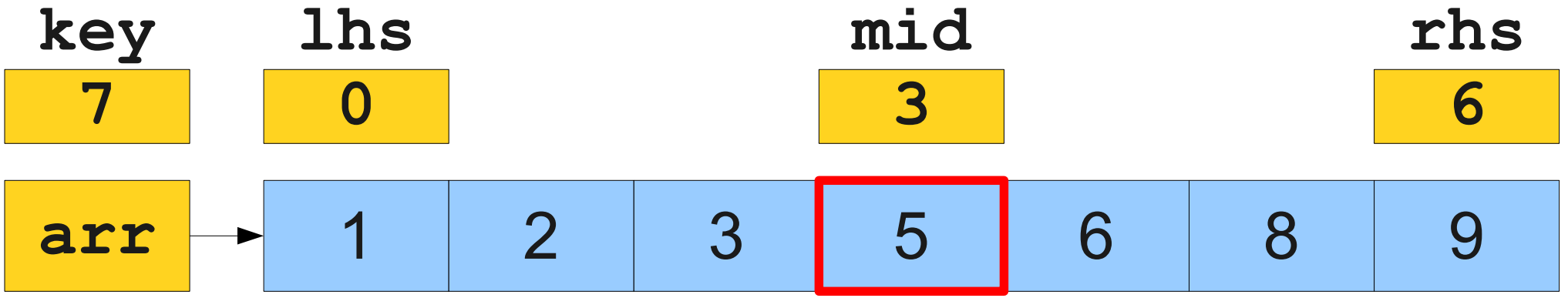

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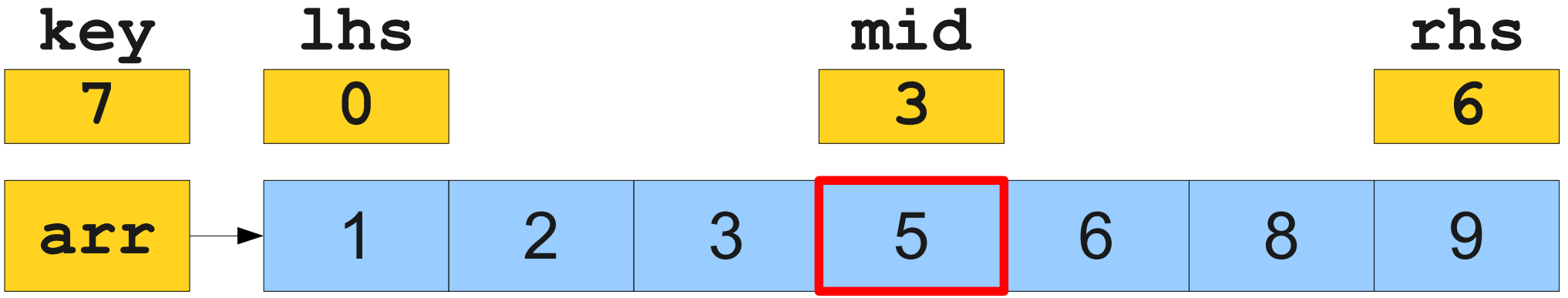

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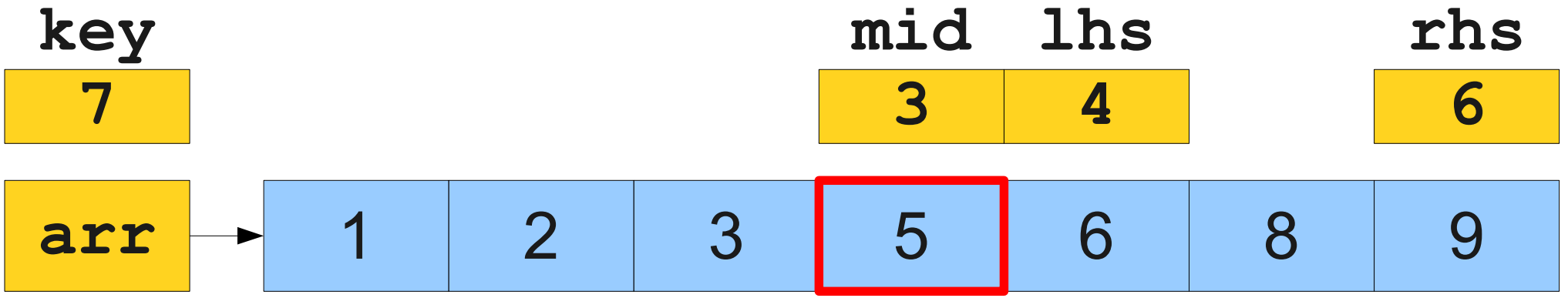

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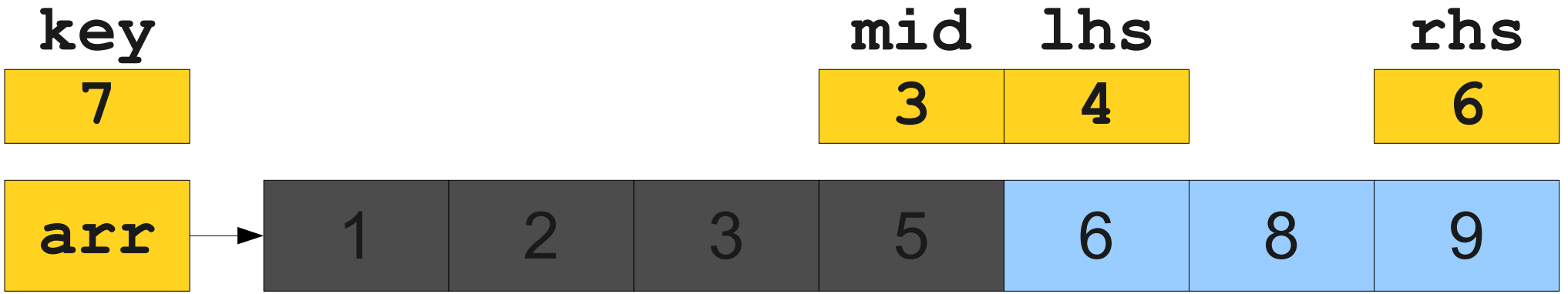

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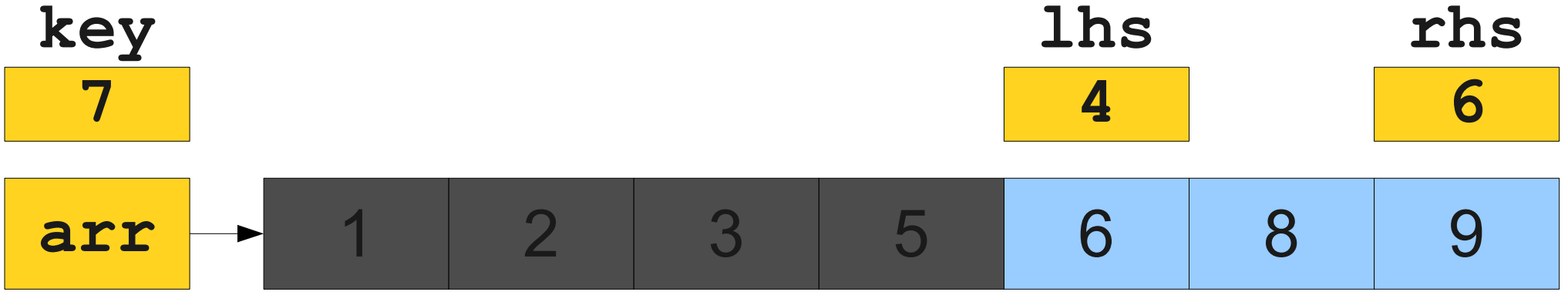

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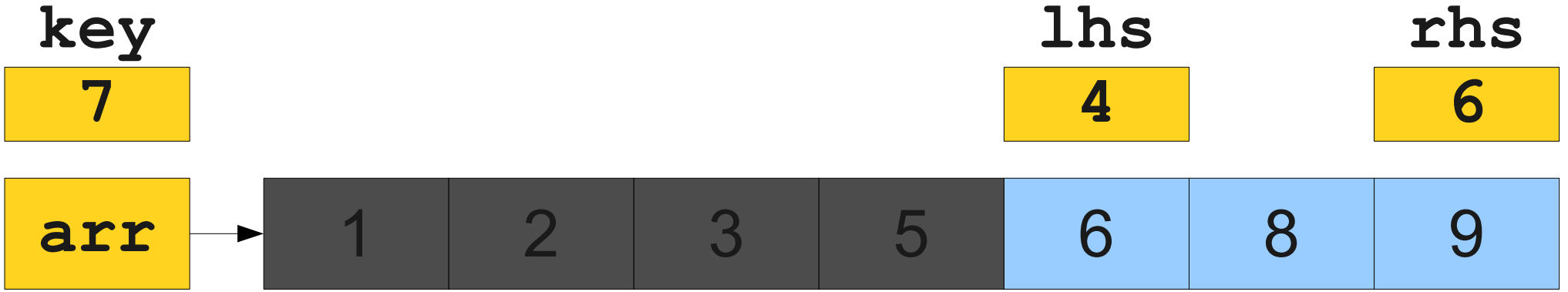

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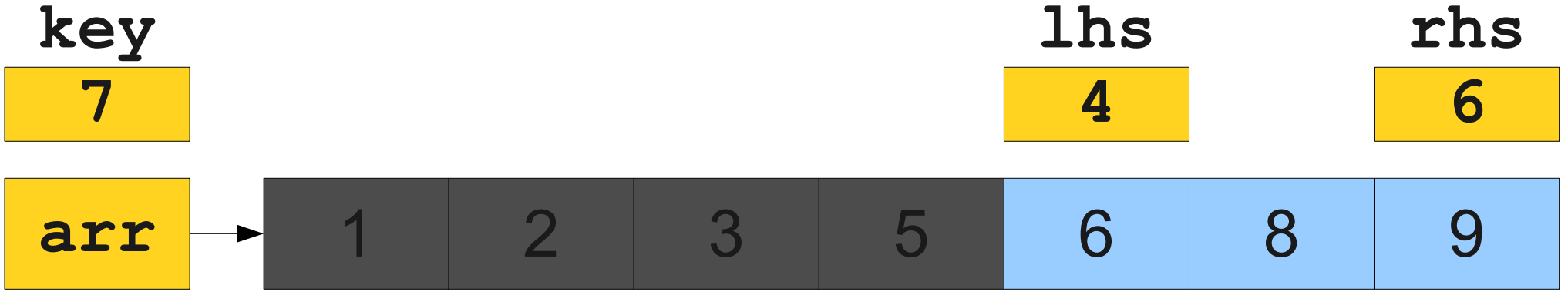

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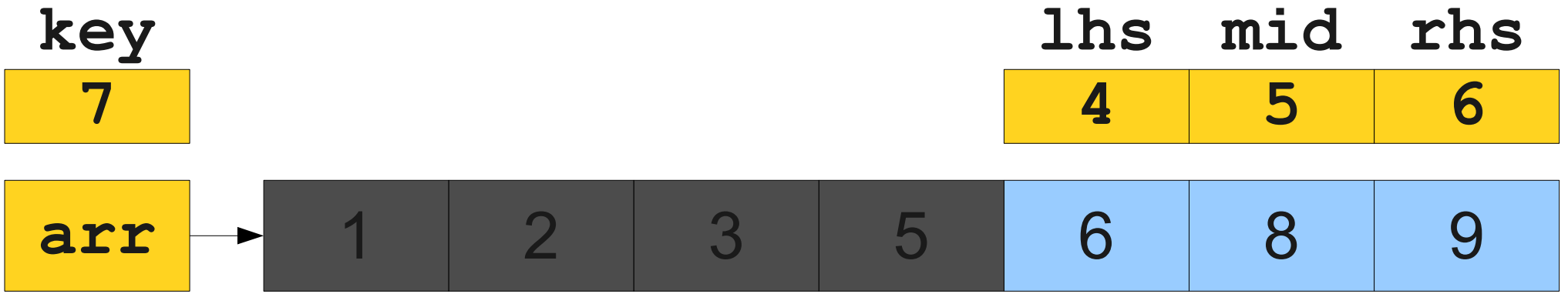

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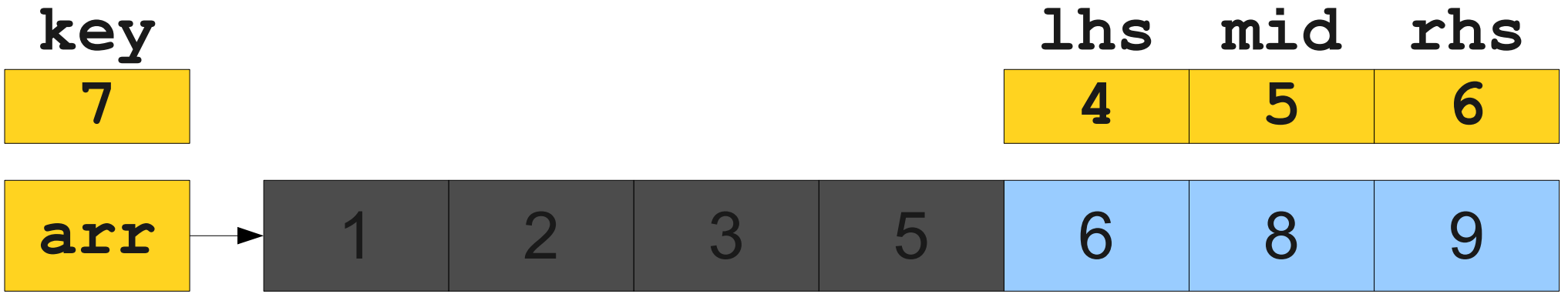

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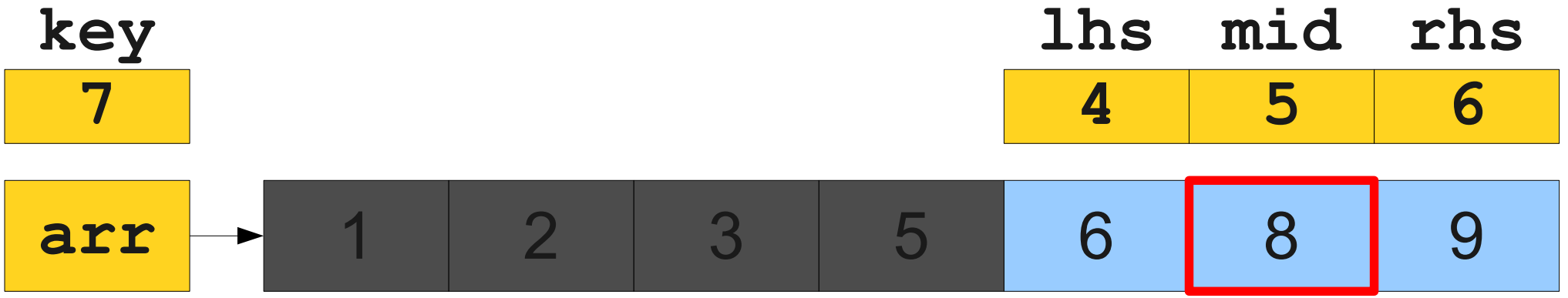

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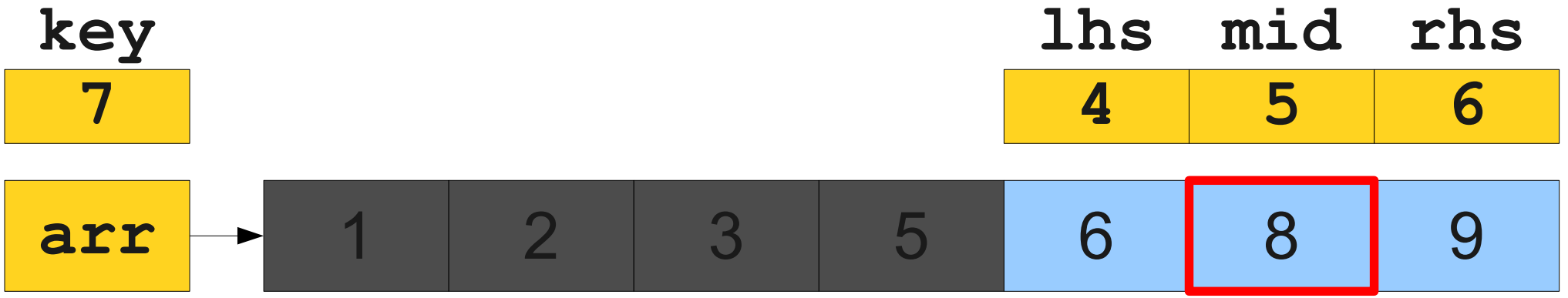

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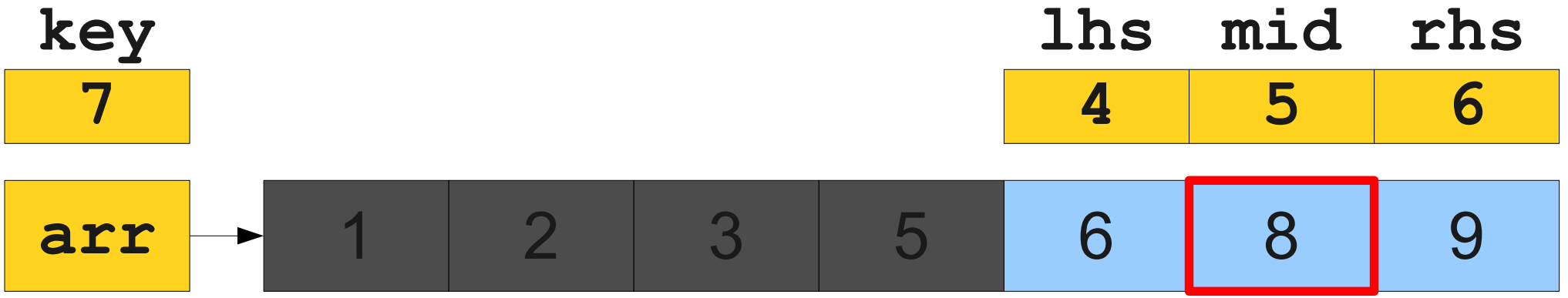

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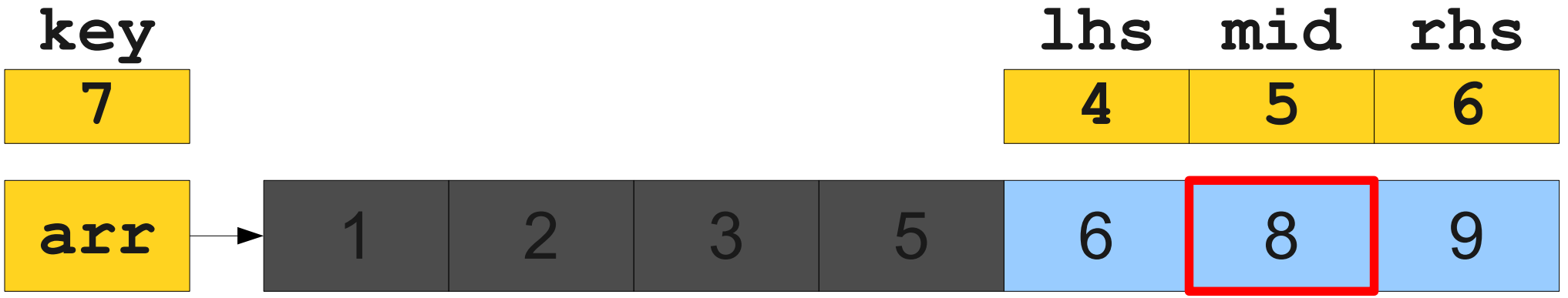

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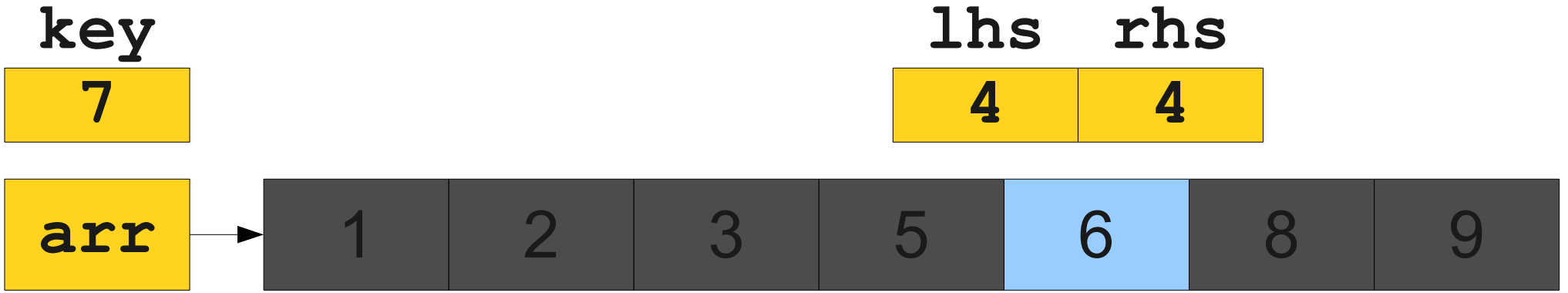

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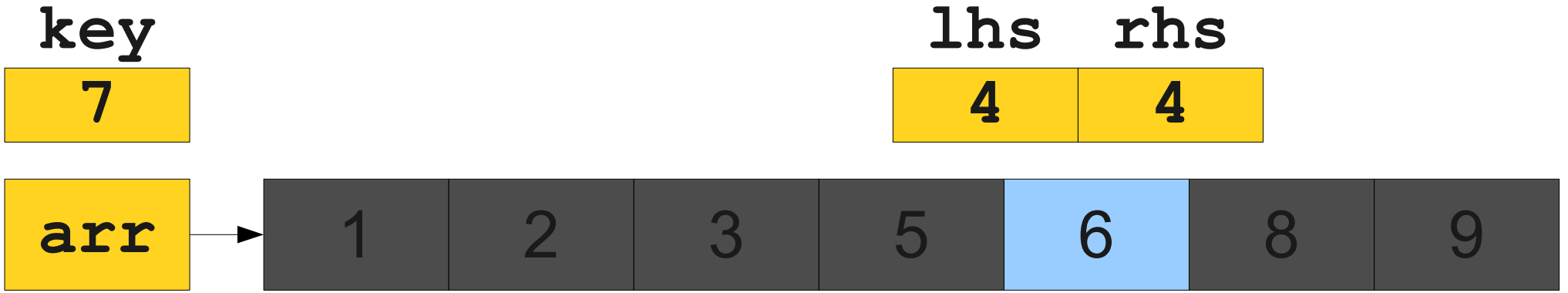

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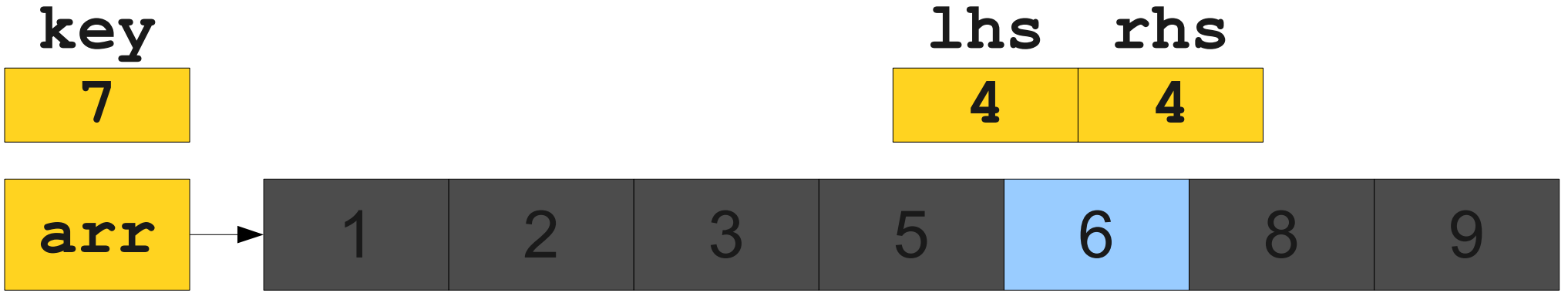

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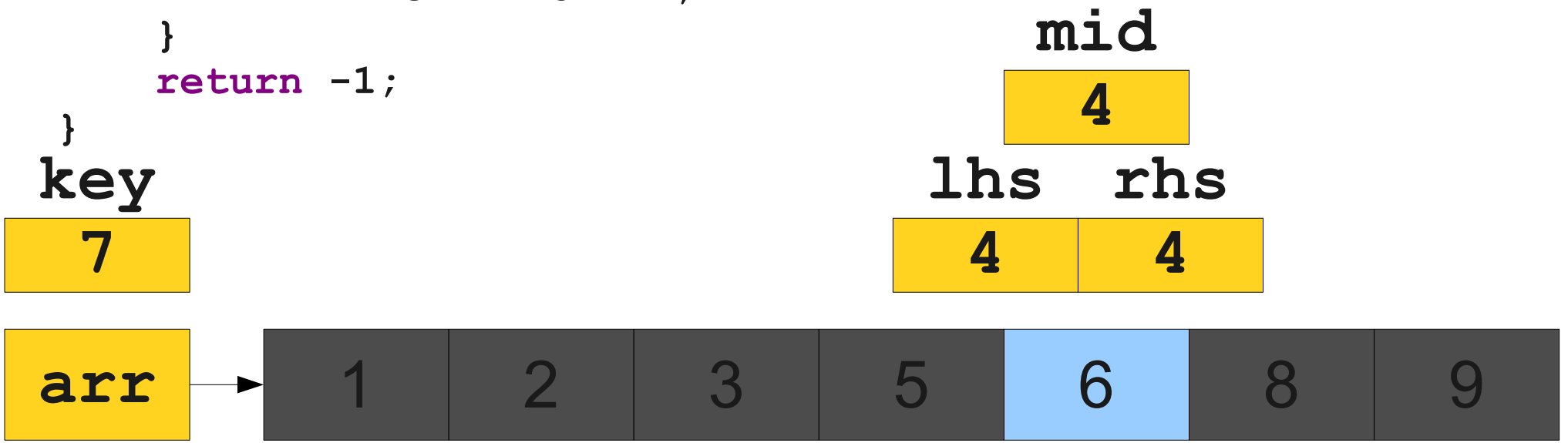

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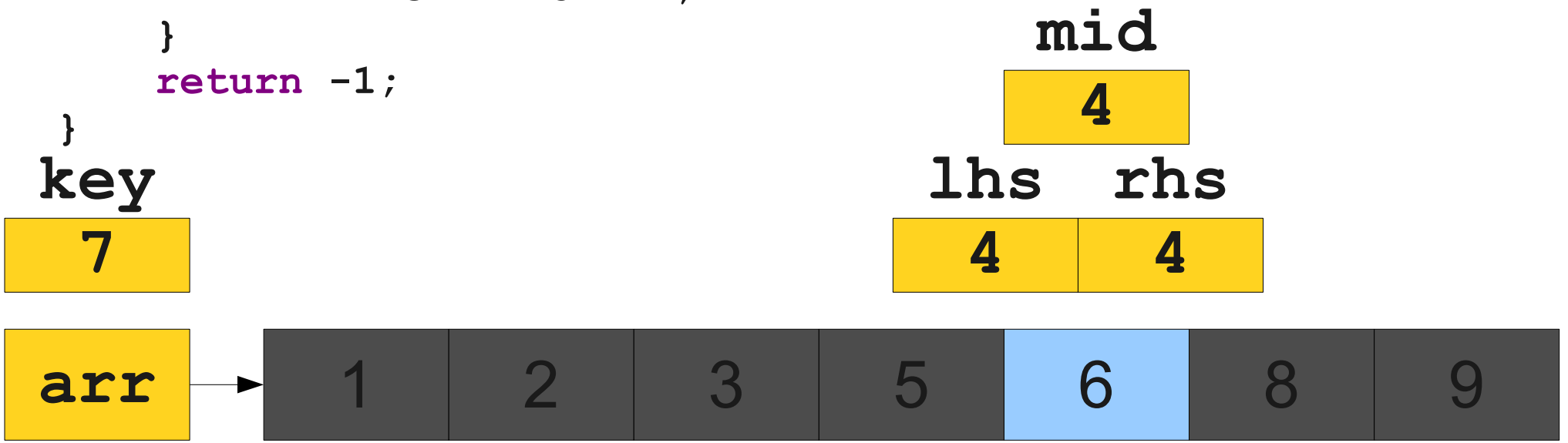

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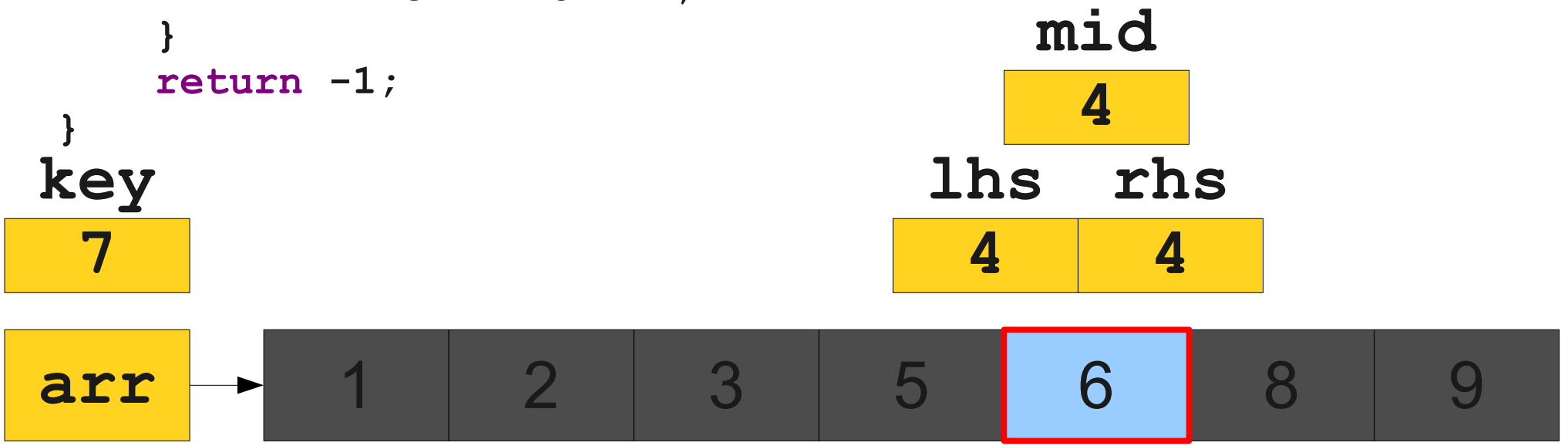

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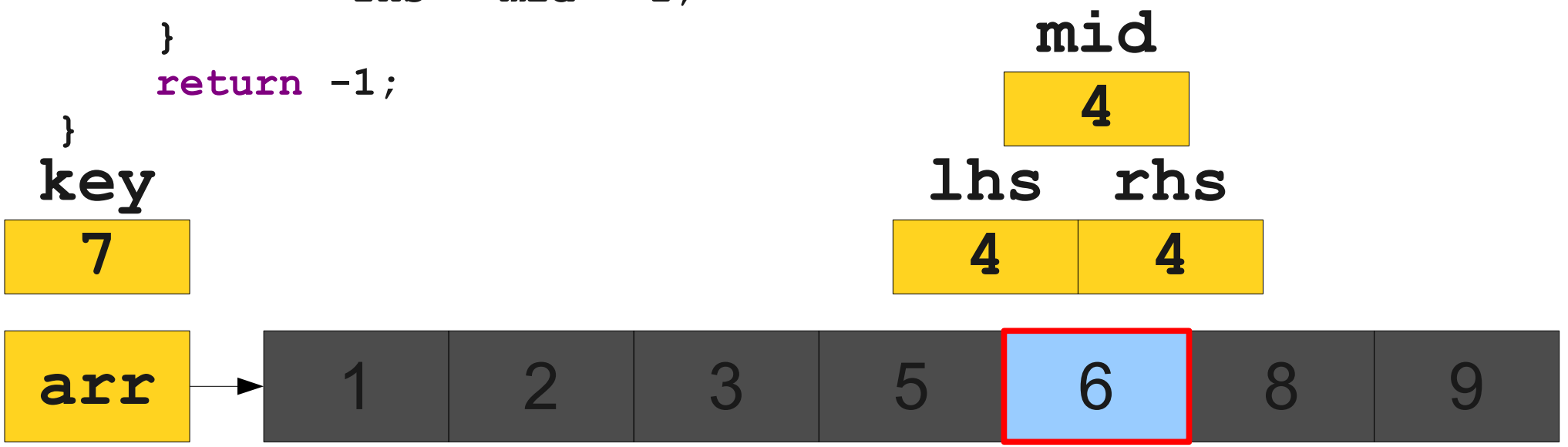

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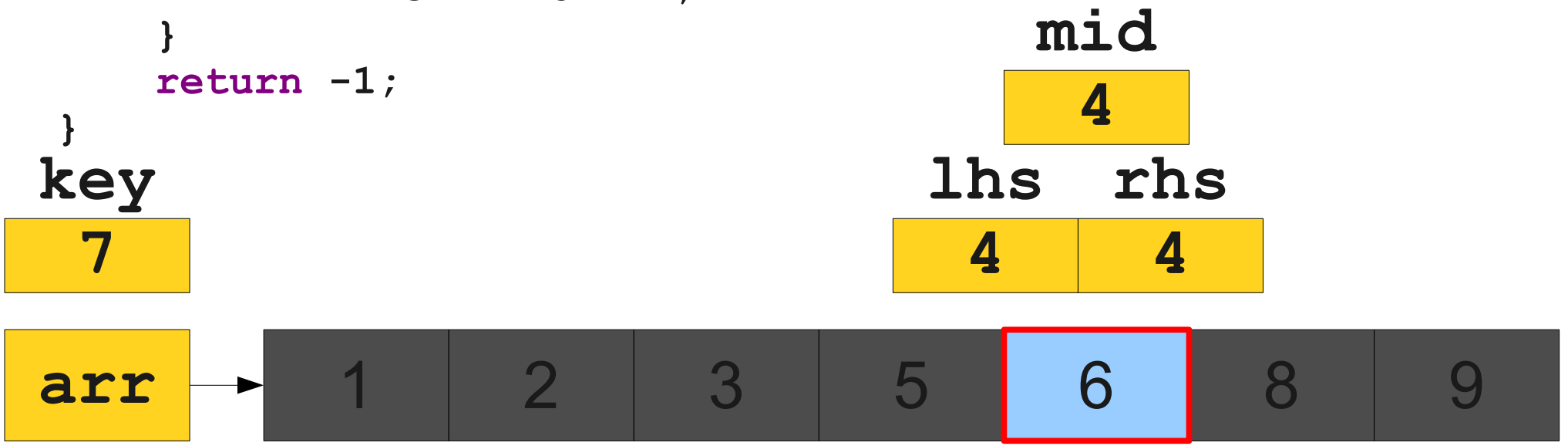

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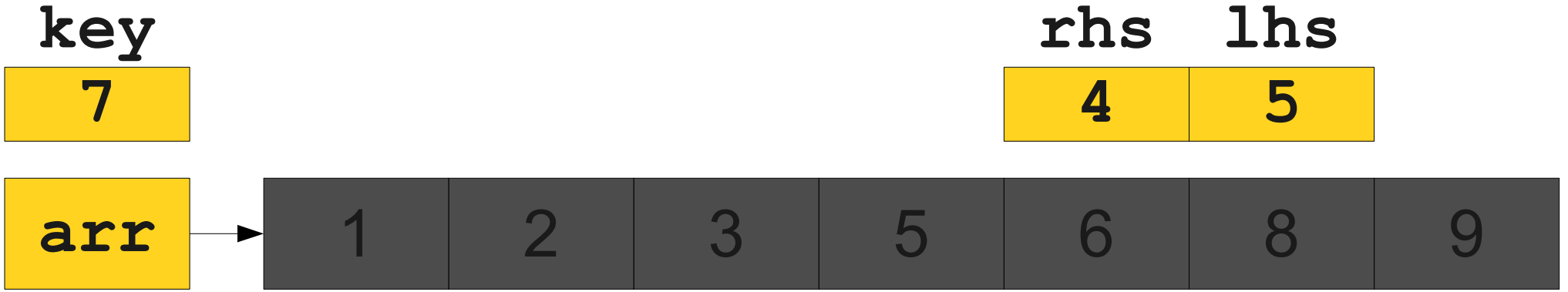

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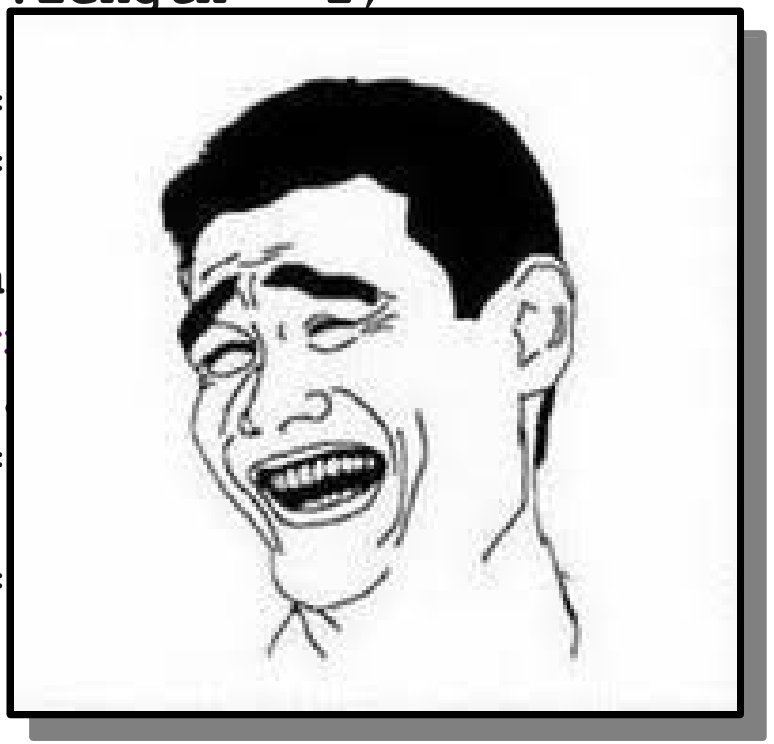

key

7

rhs lhs

4 5

arr → 1 2 3 5 6 8 9

Analyzing the Algorithms

For Comparison

```
private int linearSearch(int[] arr,
                         int key) {
    for (int i = 0; i < arr.length; i++) {
        if (arr[i] == key)
            return i;
    }

    return -1;
}
```

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Analyzing Linear Search

- How many elements of the array do we have to look at to do a linear search?
- Let's suppose that there are N elements in the array.
- We may have to look at each of them once.
- Number of lookups: N.

Analyzing Binary Search

- How many elements of the array do we have to look at to do a binary search?
- Let's suppose that there are N elements in the array.
- Each lookup cuts the size of the array in half.
- How many times can we cut the array in half before we run out of elements?

Slicing and Dicing

- After zero lookups: N
- After one lookup: $N / 2$
- After two lookups: $N / 4$
- After three lookups: $N / 8$

...

- After k lookups: $N / 2^k$

Cutting in Half

- After doing k lookups, there are $N / 2^k$ elements left.
- The algorithm stops when there is just one element left.
- Solving for the number of iterations:

$$N / 2^k = 1$$

$$N = 2^k$$

$$\log_2 N = k$$

- So binary search stops after **$\log_2 N$** lookups.

For Comparison

N	$\log_2 N$
10	3
100	7
1000	10
1,000,000	20
1,000,000,000	30

Binary search can check whether a value exists in an array of one billion elements in just 30 array accesses!

A Feel for $\log_2 N$

- It is conjectured that the number of atoms in the universe is 10^{100}.
- $\log_2 10^{100} \approx 300$.
- If you (somehow) listed all the atoms in the universe in sorted order, you would need to look at 300 before you found the one you were looking for.

An Interesting Lecture Excerpt

Admiral Grace Hopper on nanoseconds:

http://www.youtube.com/watch?v=JEpsKnWZrJ8

Sorting

Bubble Sort

- Until the array is sorted:
  - Look at each adjacent pair of elements.
  - If they are out of order, swap them.

Should we use bubble sort?

Should we use bubble sort?

Let's ask the President of the United States!

Barack Obama, in 2008.

Eric Schmidt then CEO of Google.

A Presidential Decree

"The bubble sort would be the wrong way to go."

- Barack Obama

What other algorithms could we use instead?

An Second Idea: **Selection Sort**

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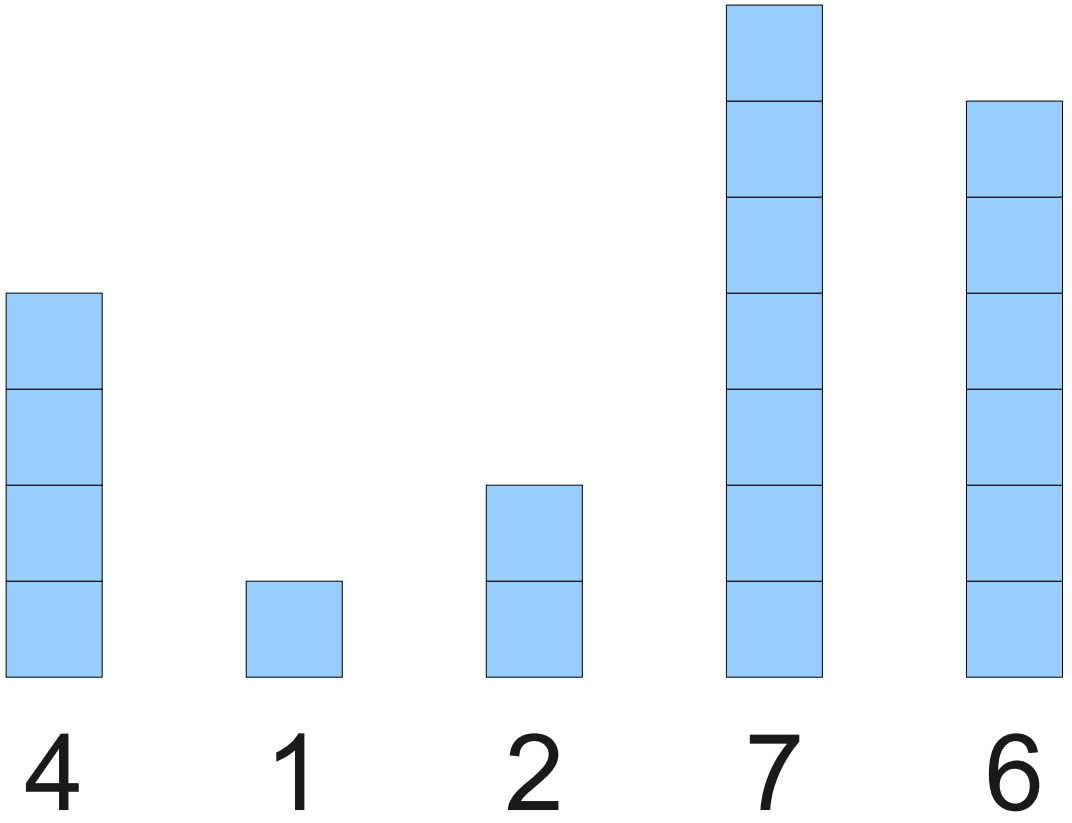

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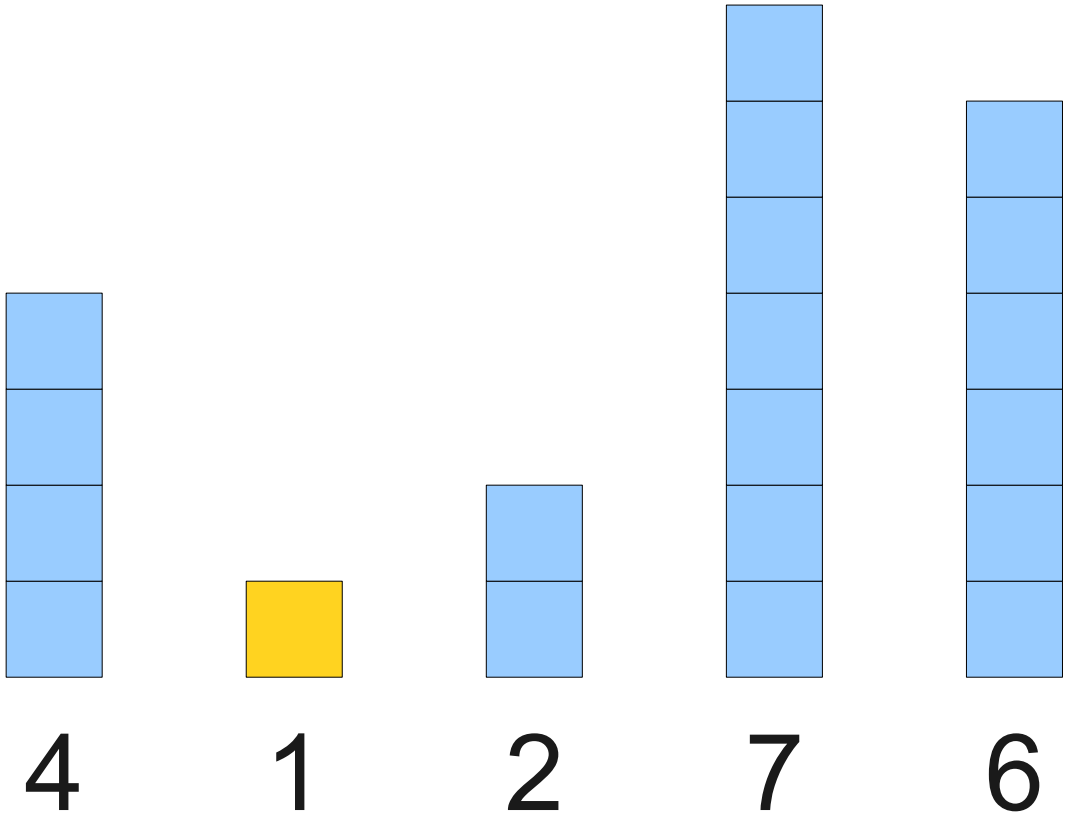

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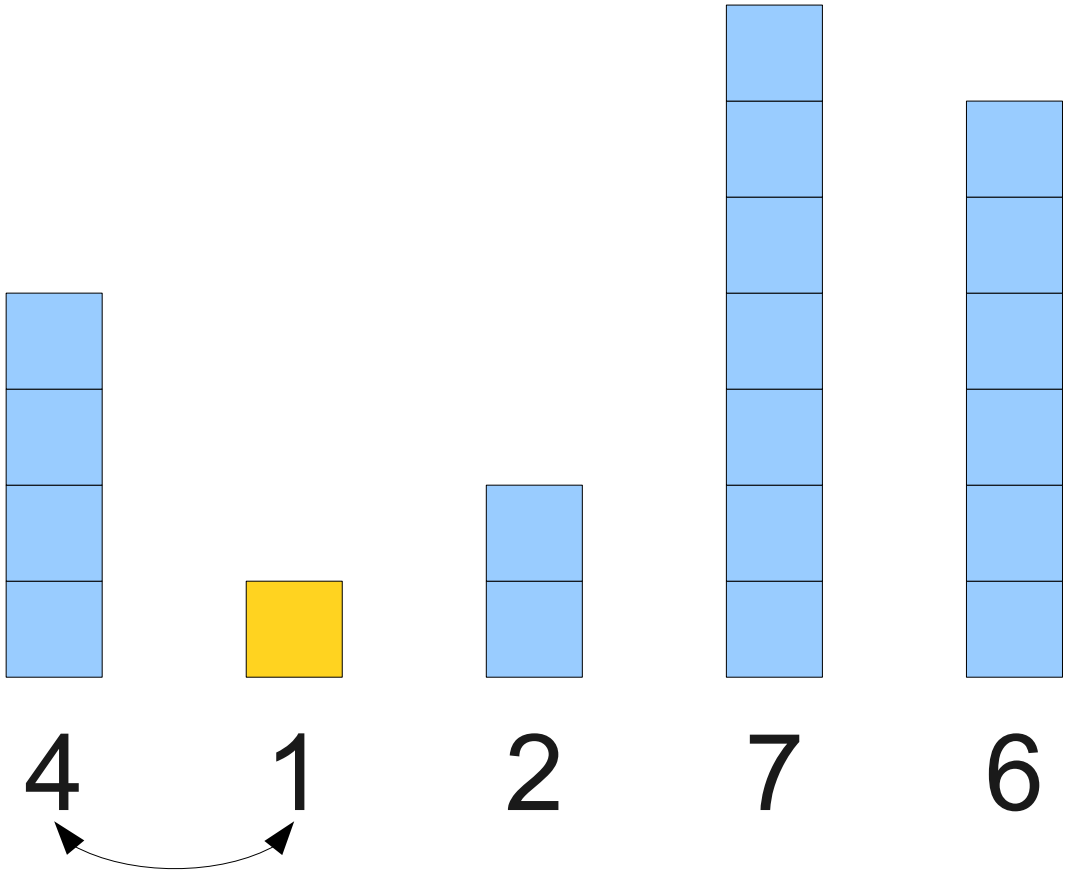

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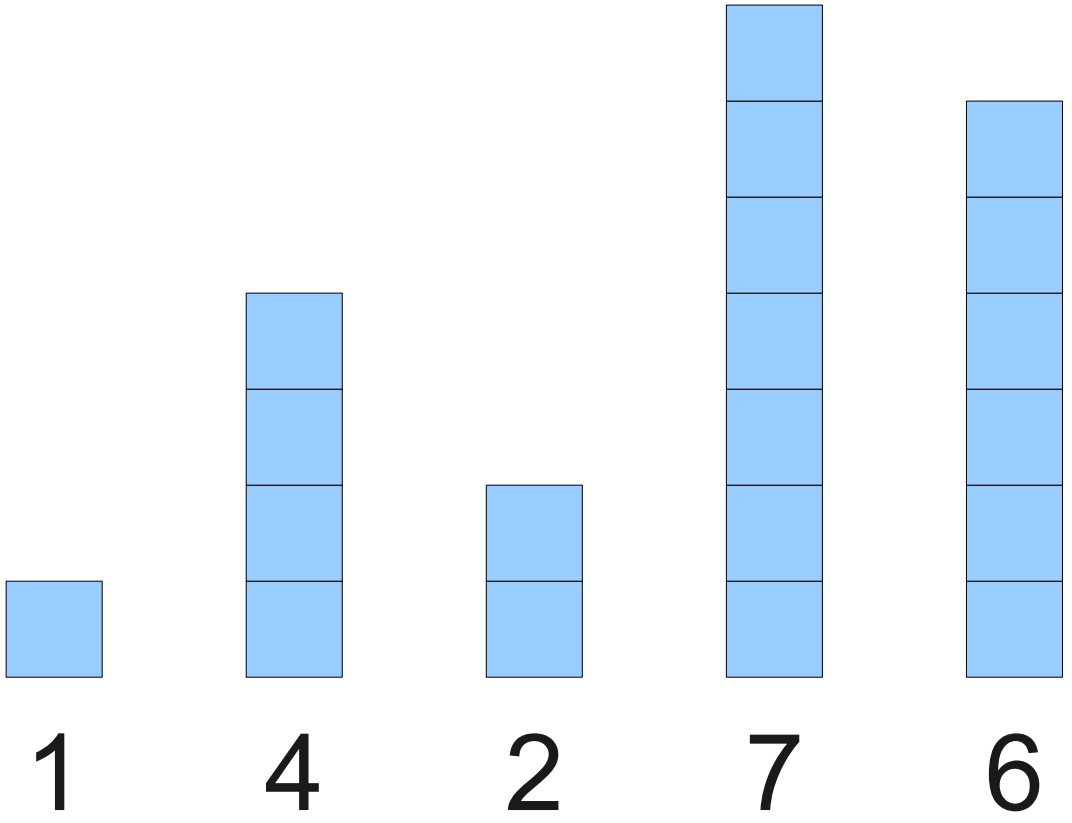

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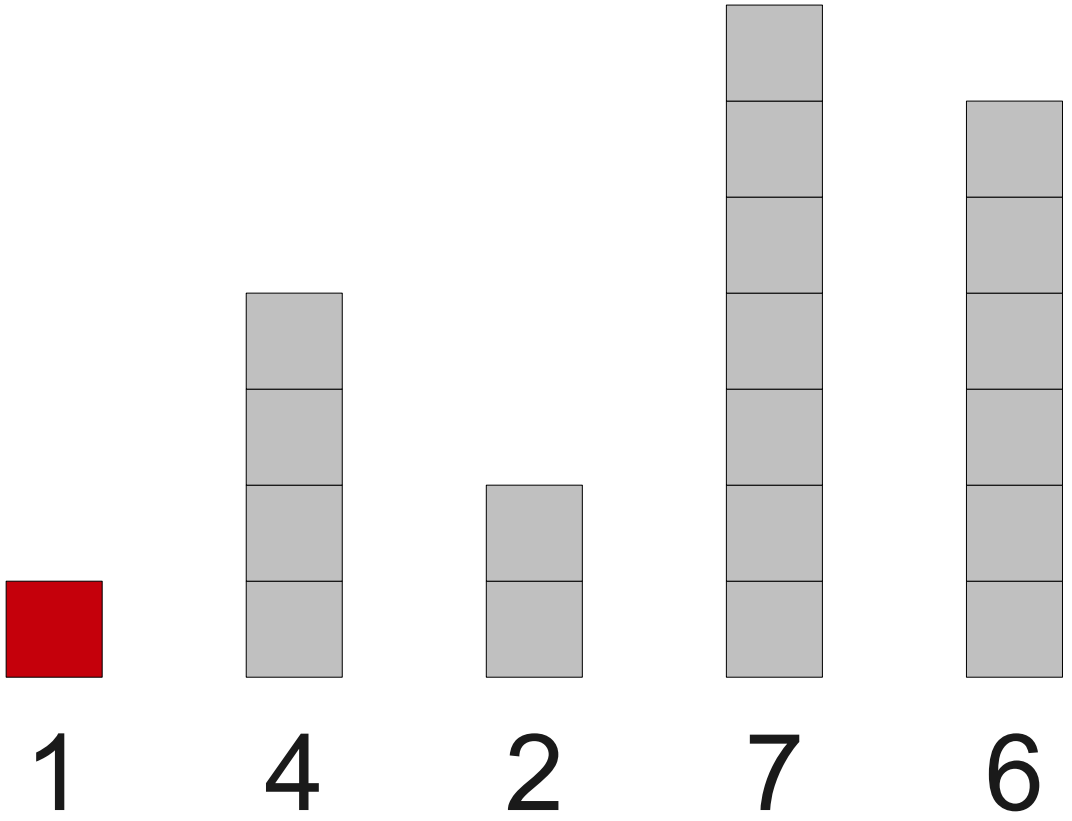

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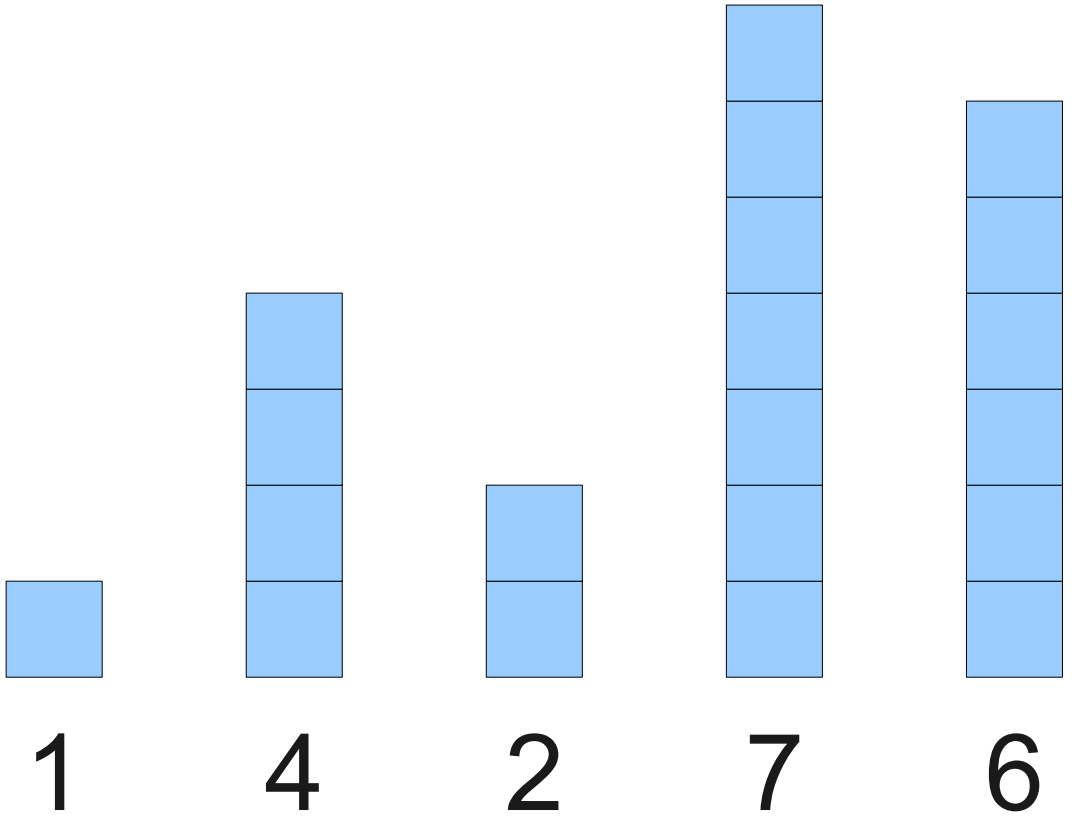

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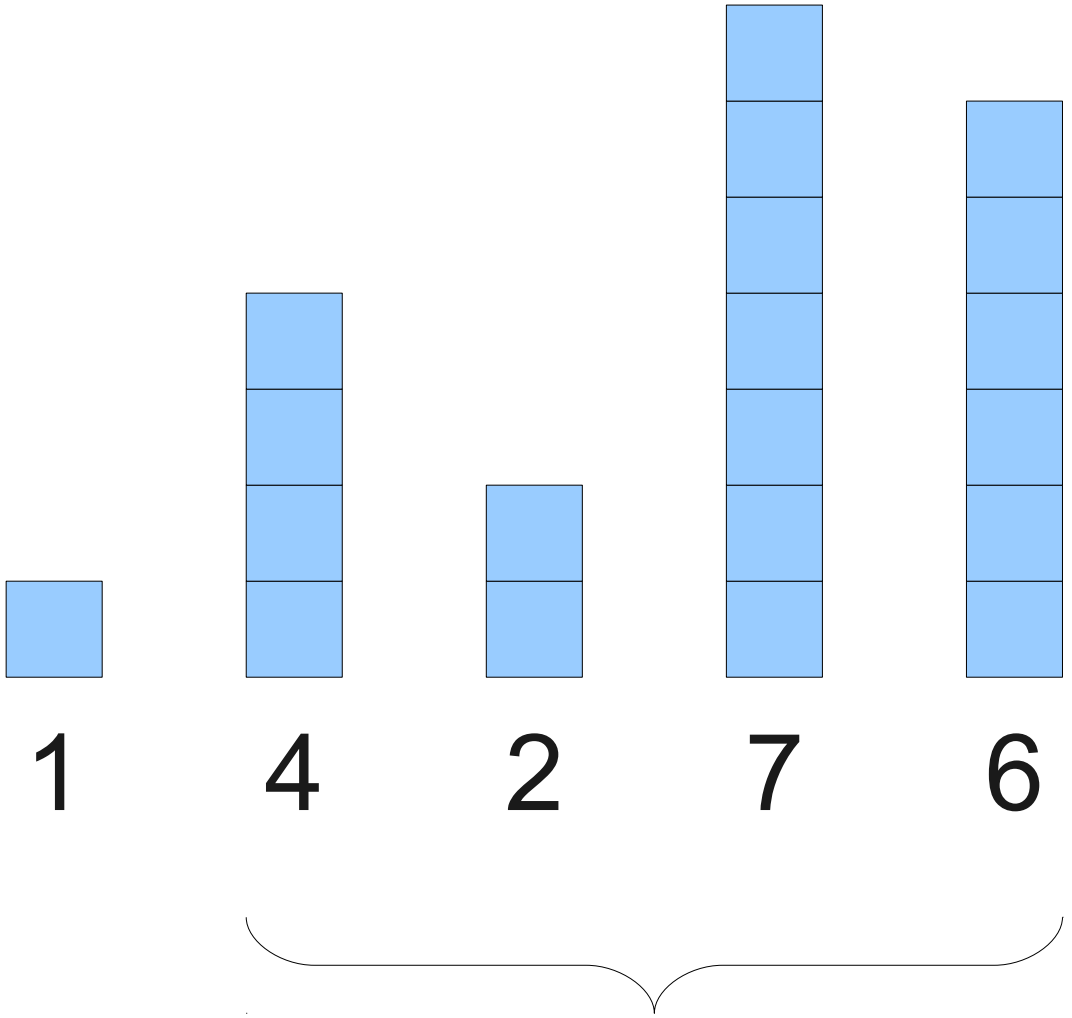

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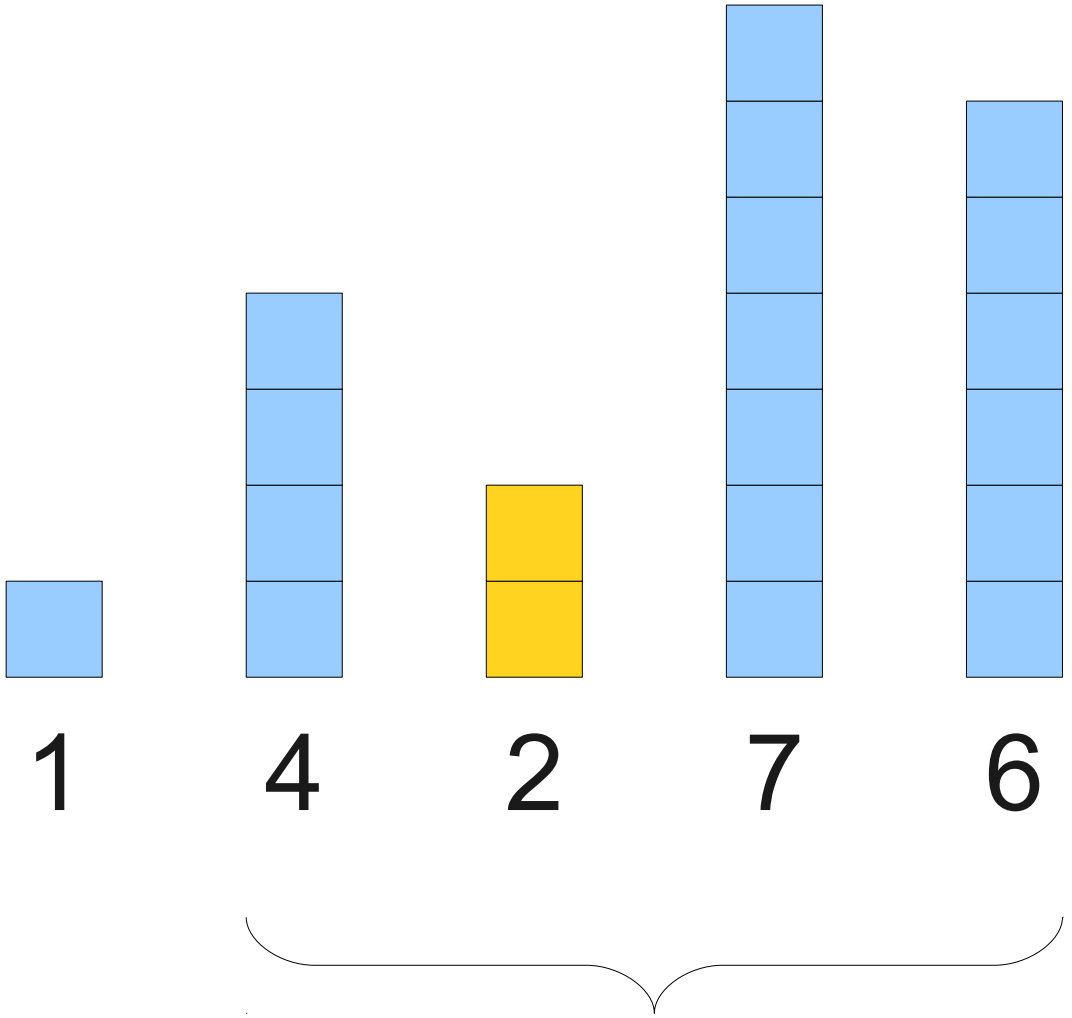

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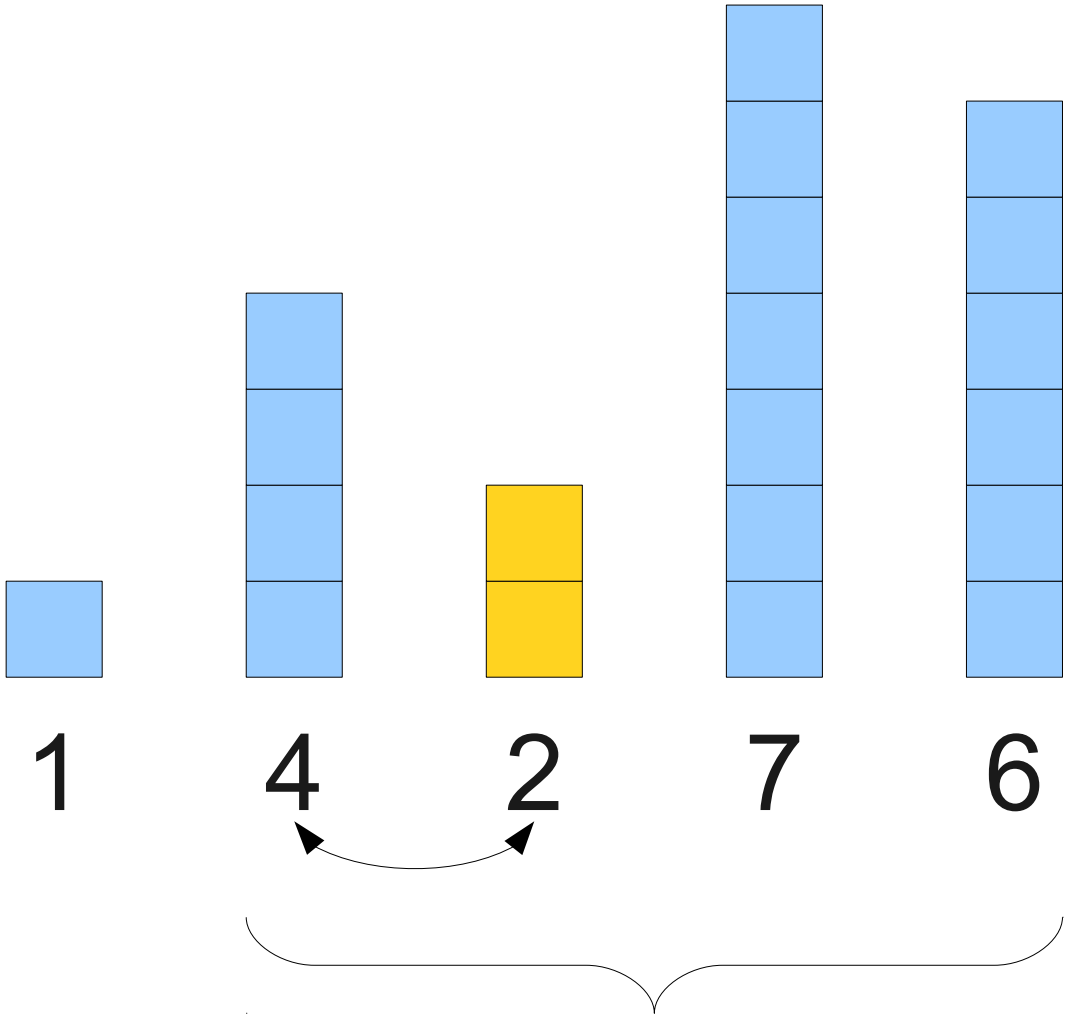

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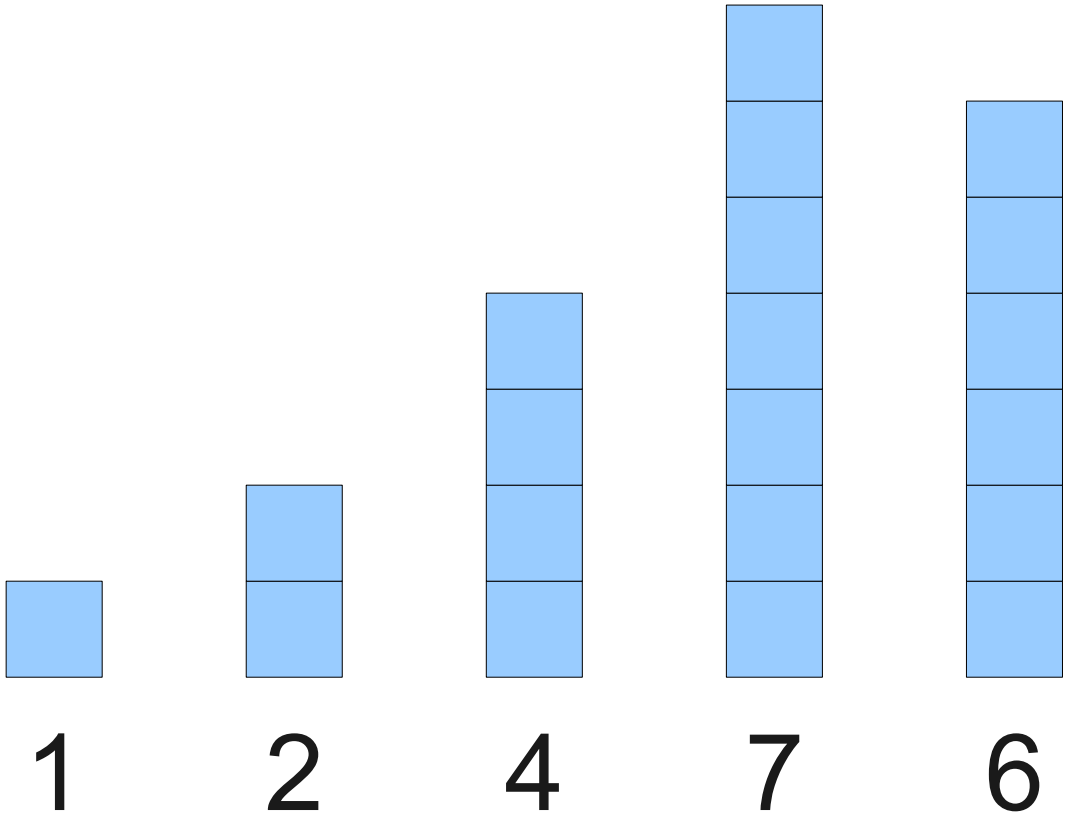

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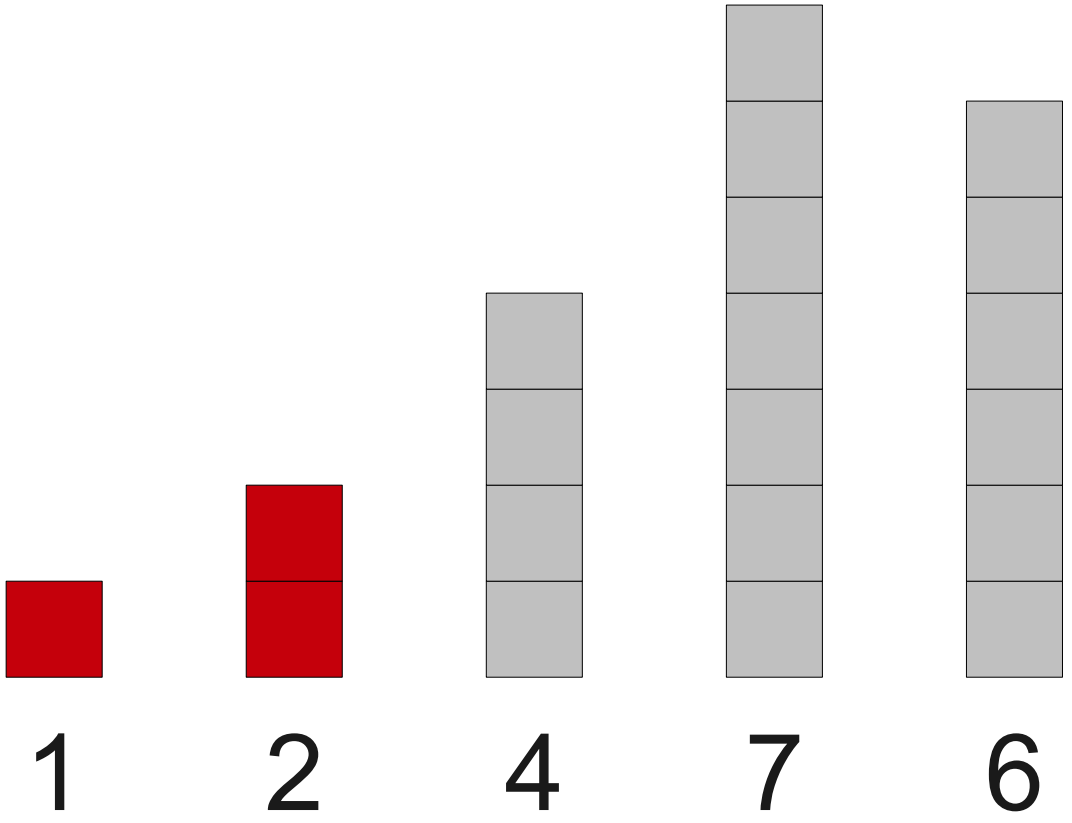

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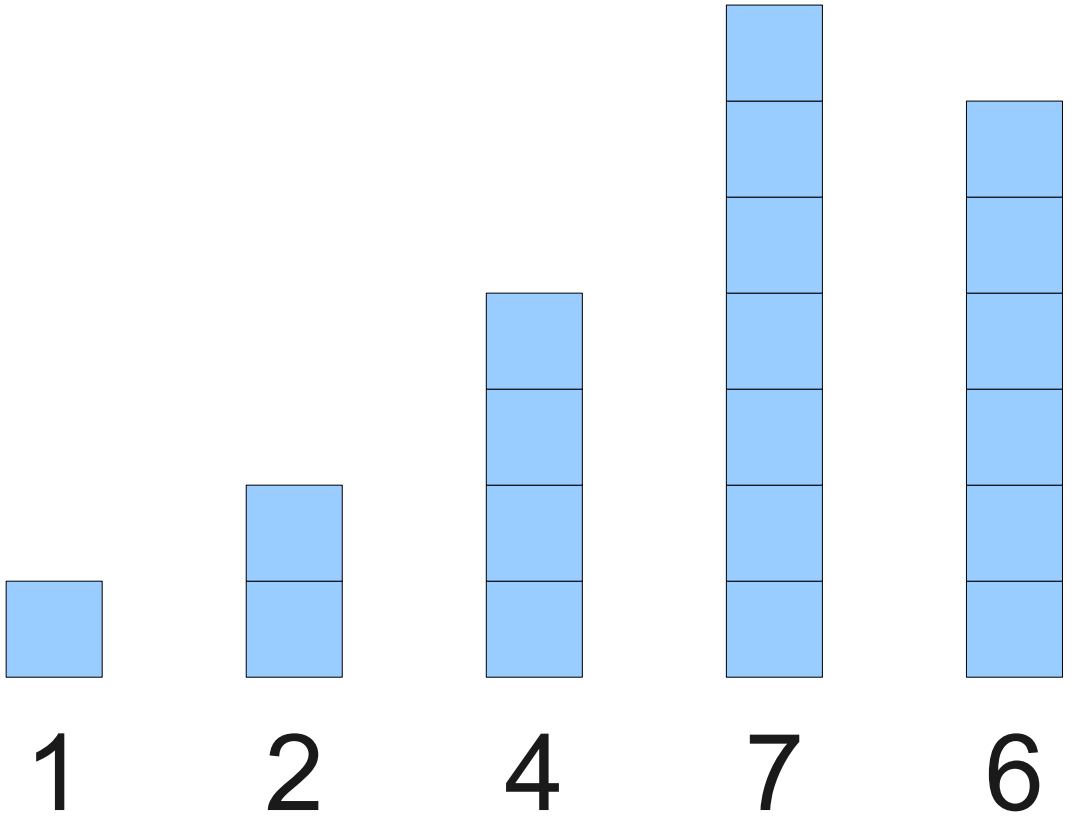

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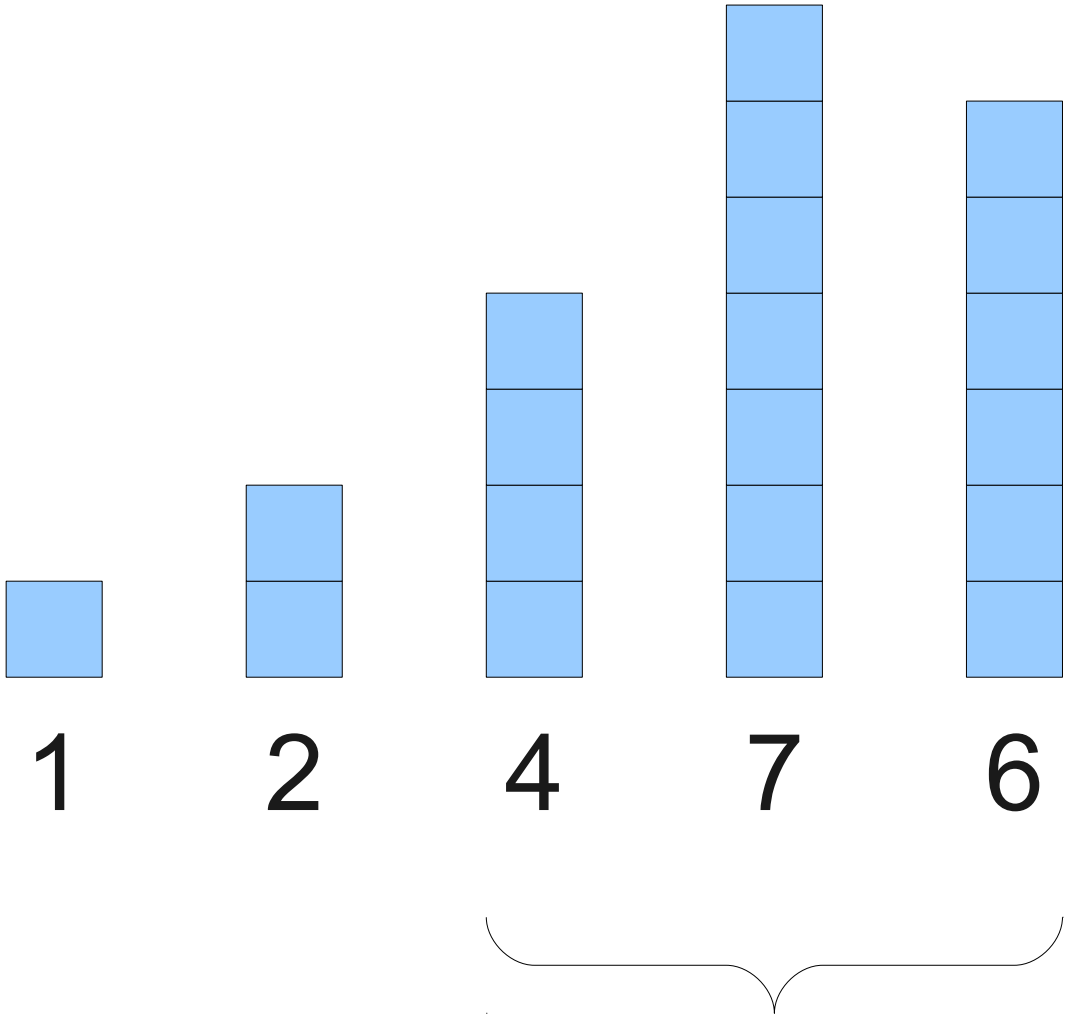

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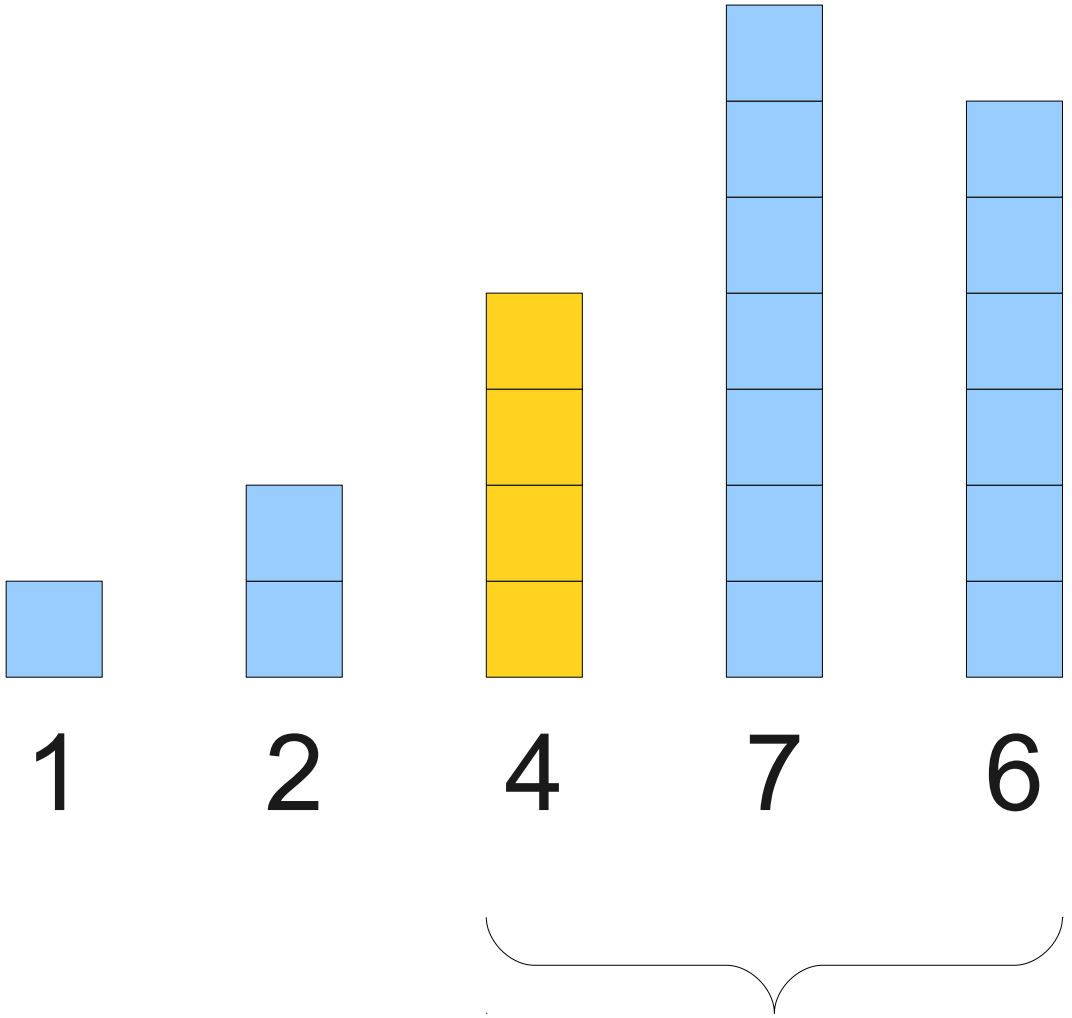

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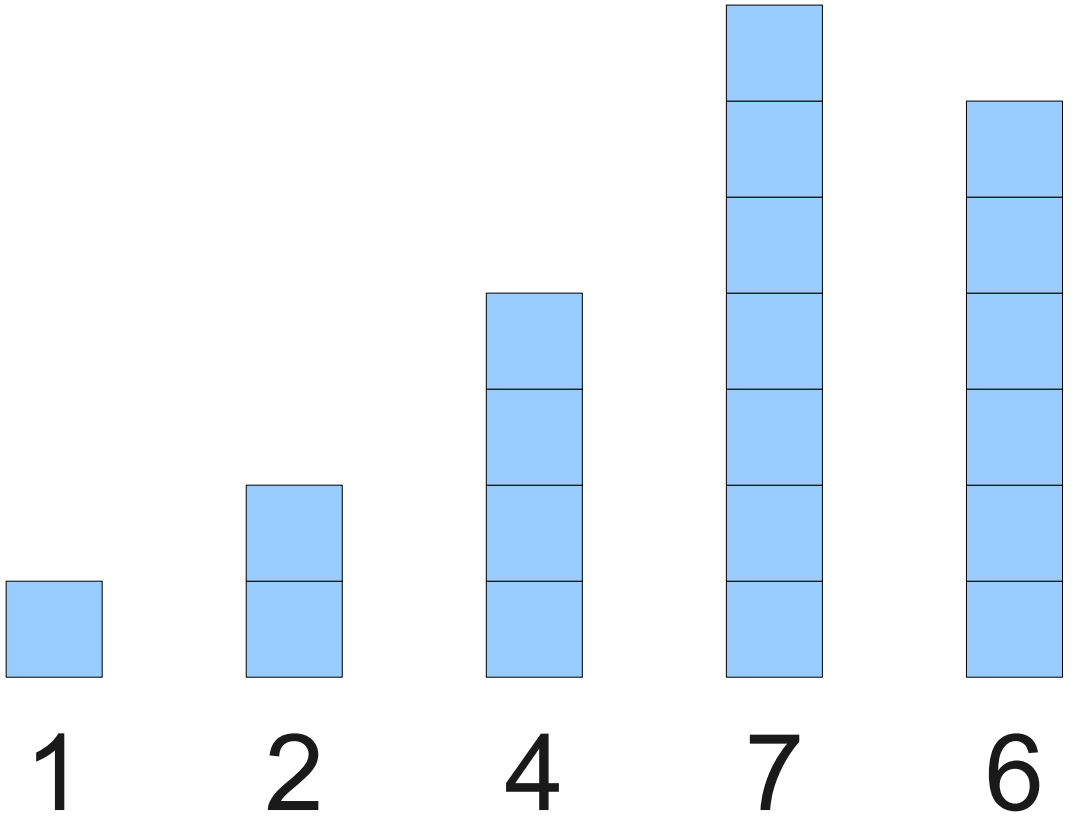

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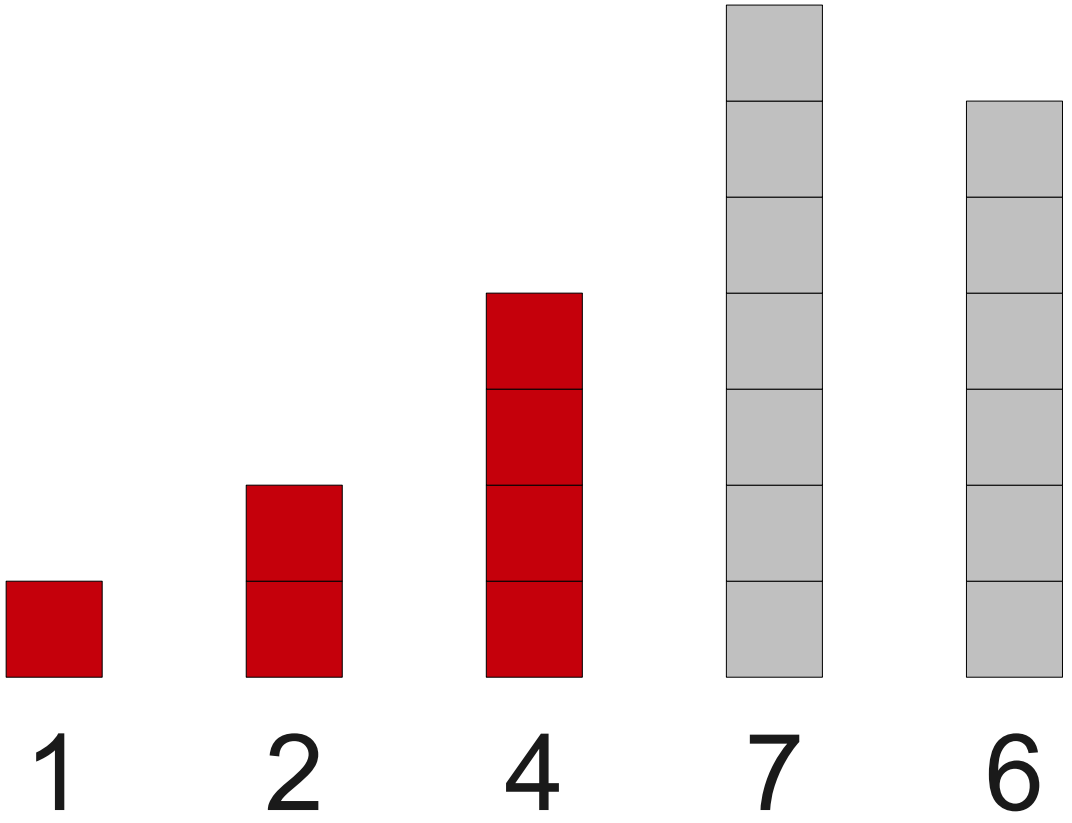
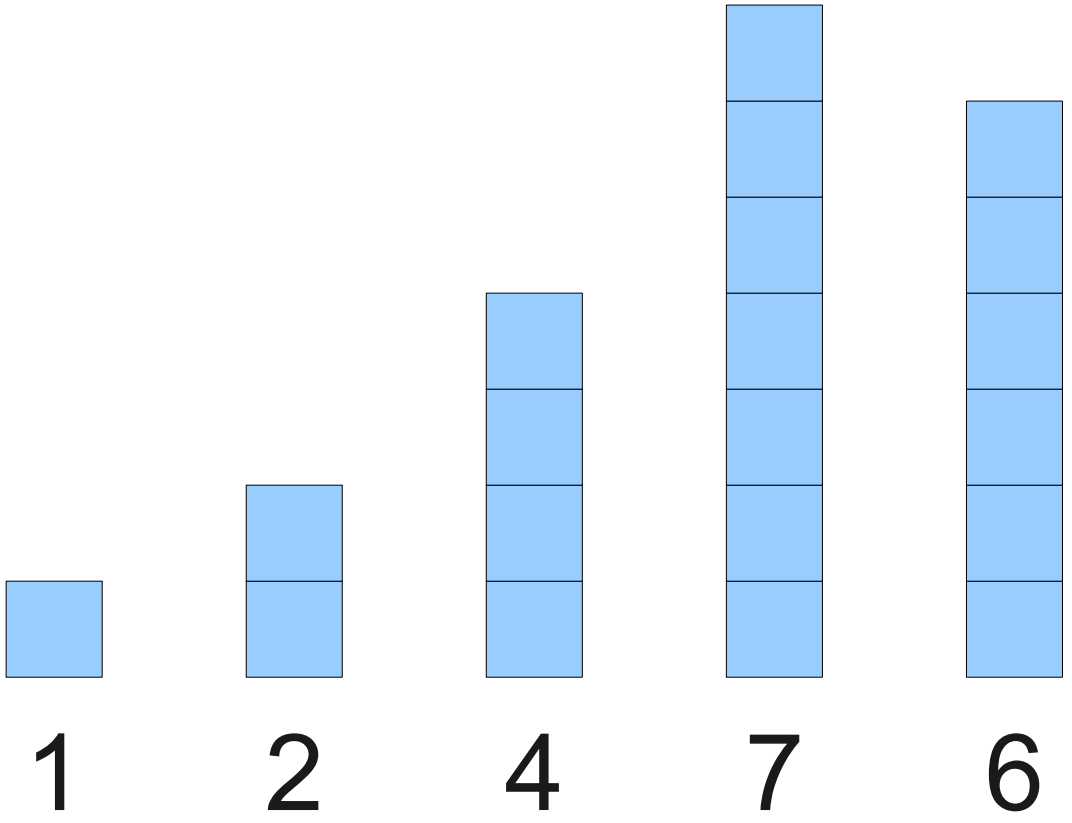


An Second Idea: **Selection Sort**



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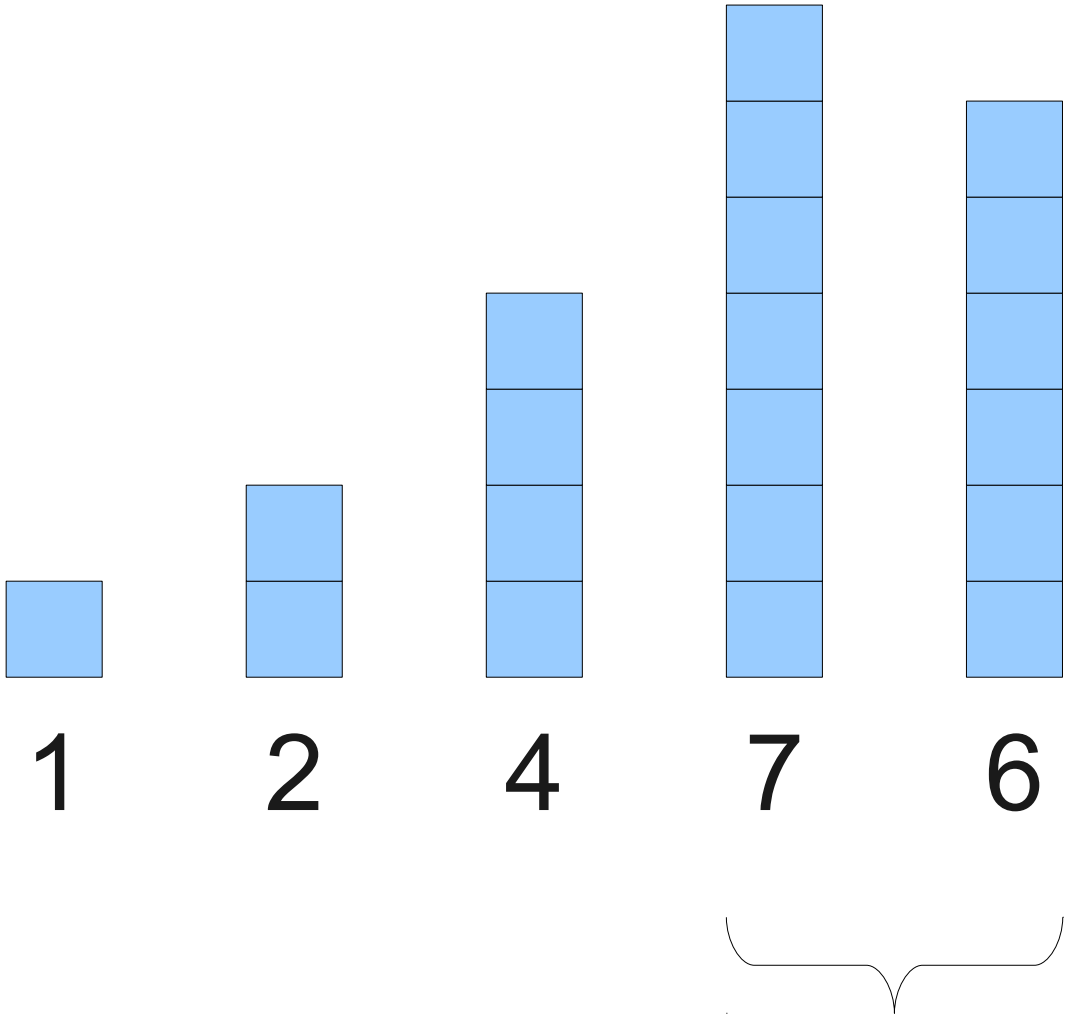

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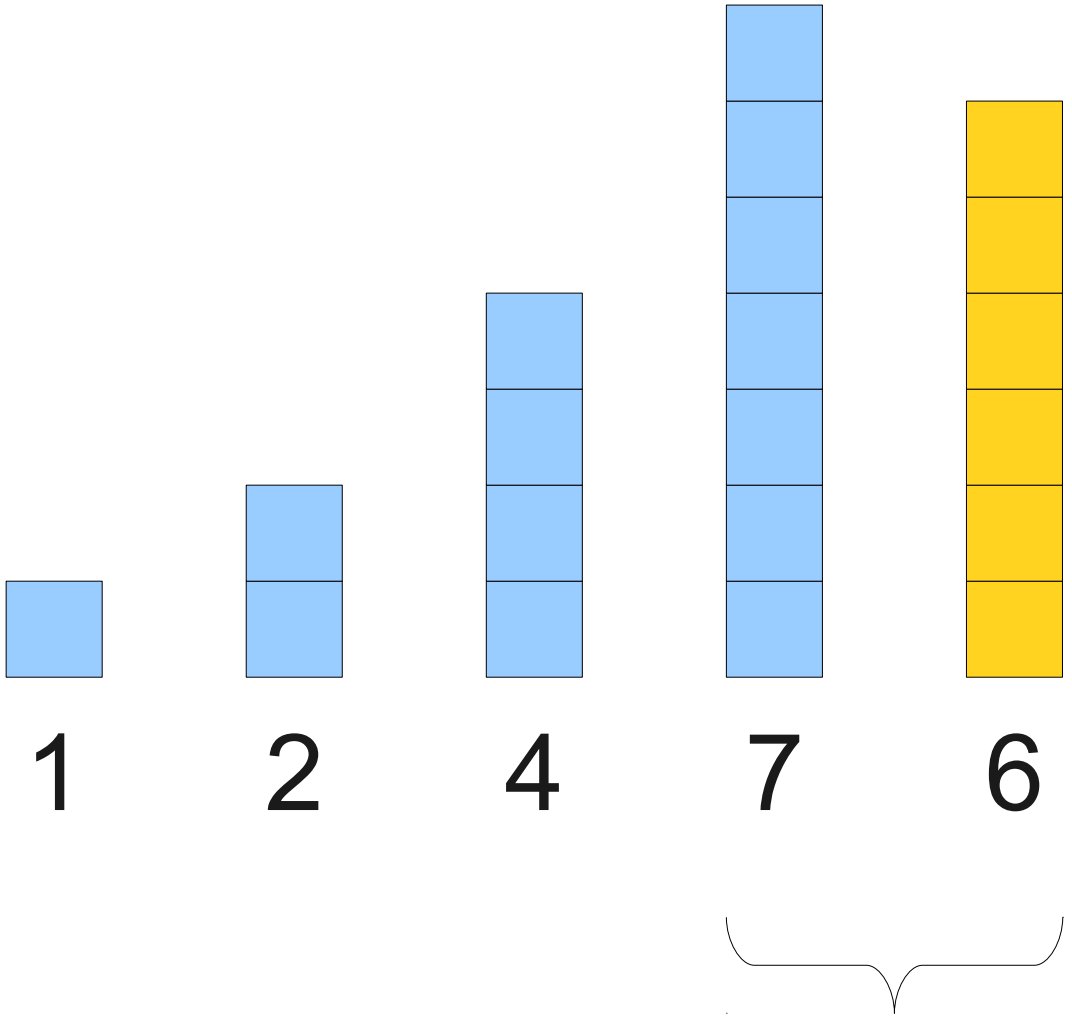

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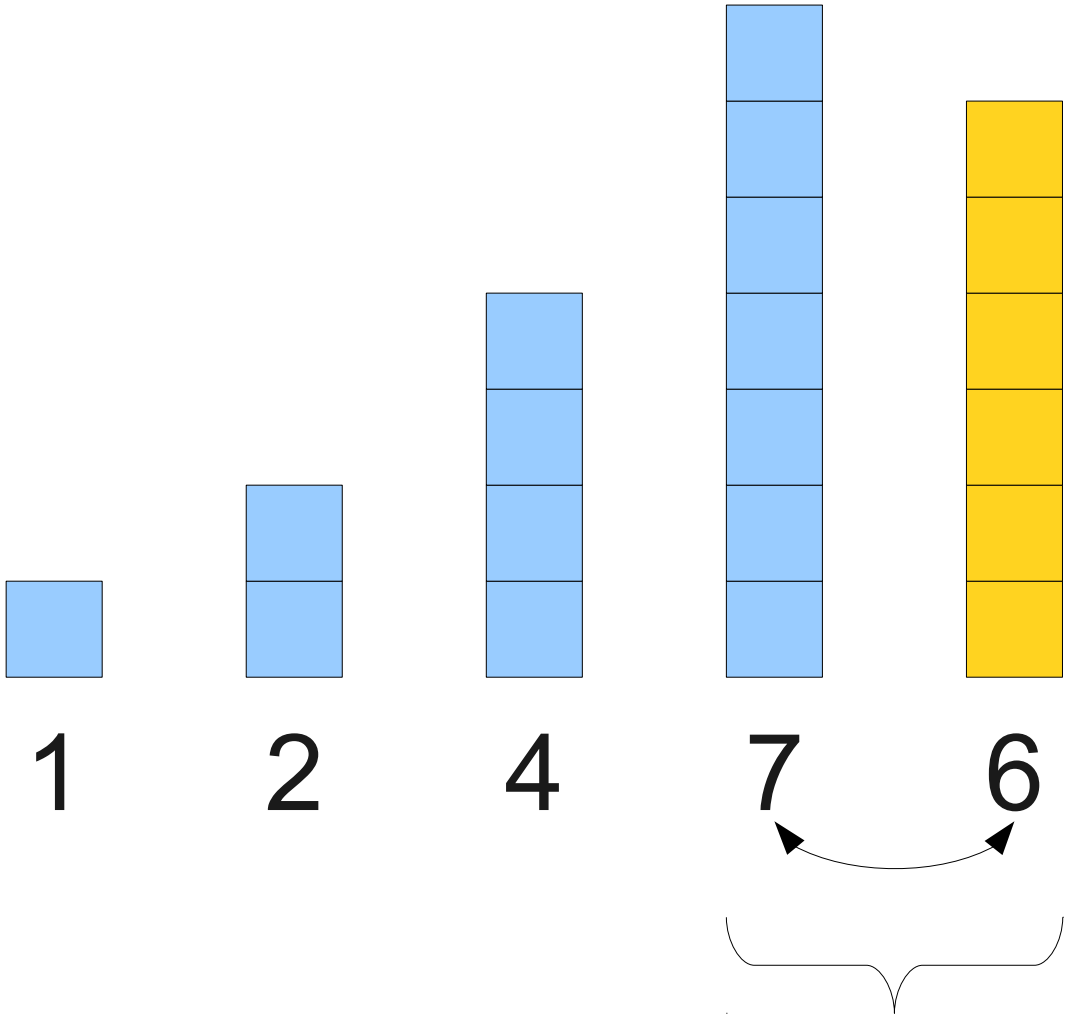

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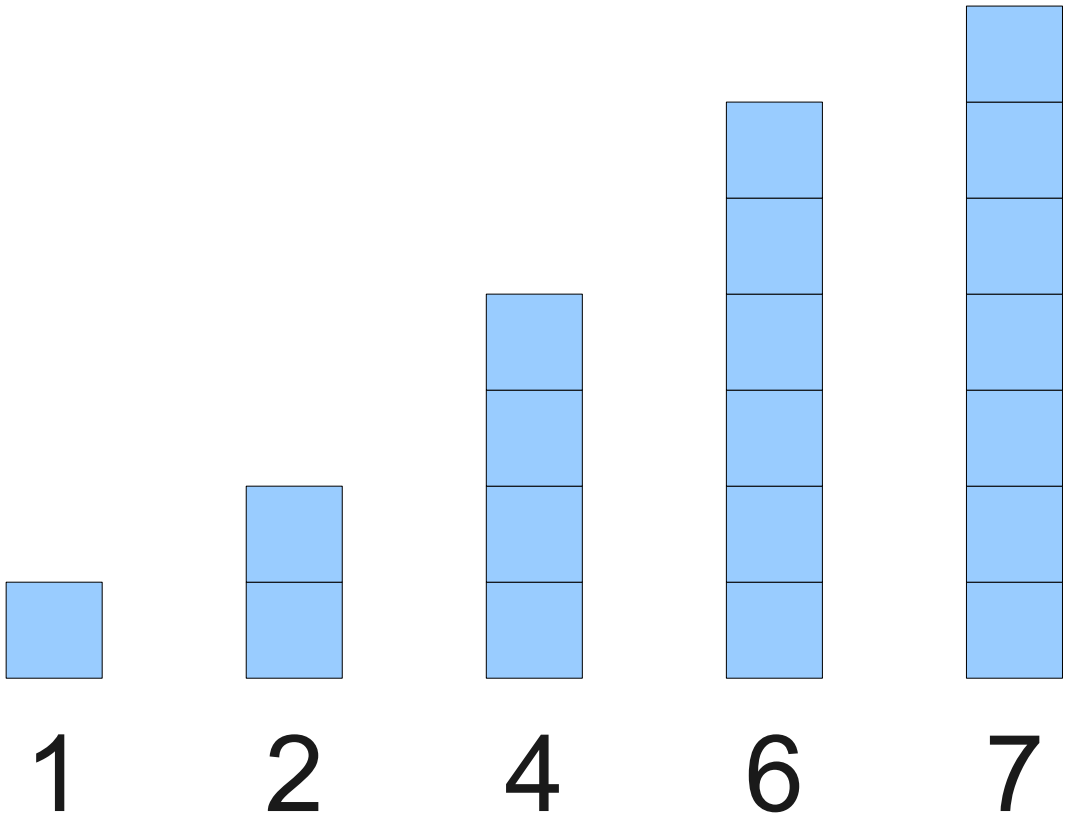

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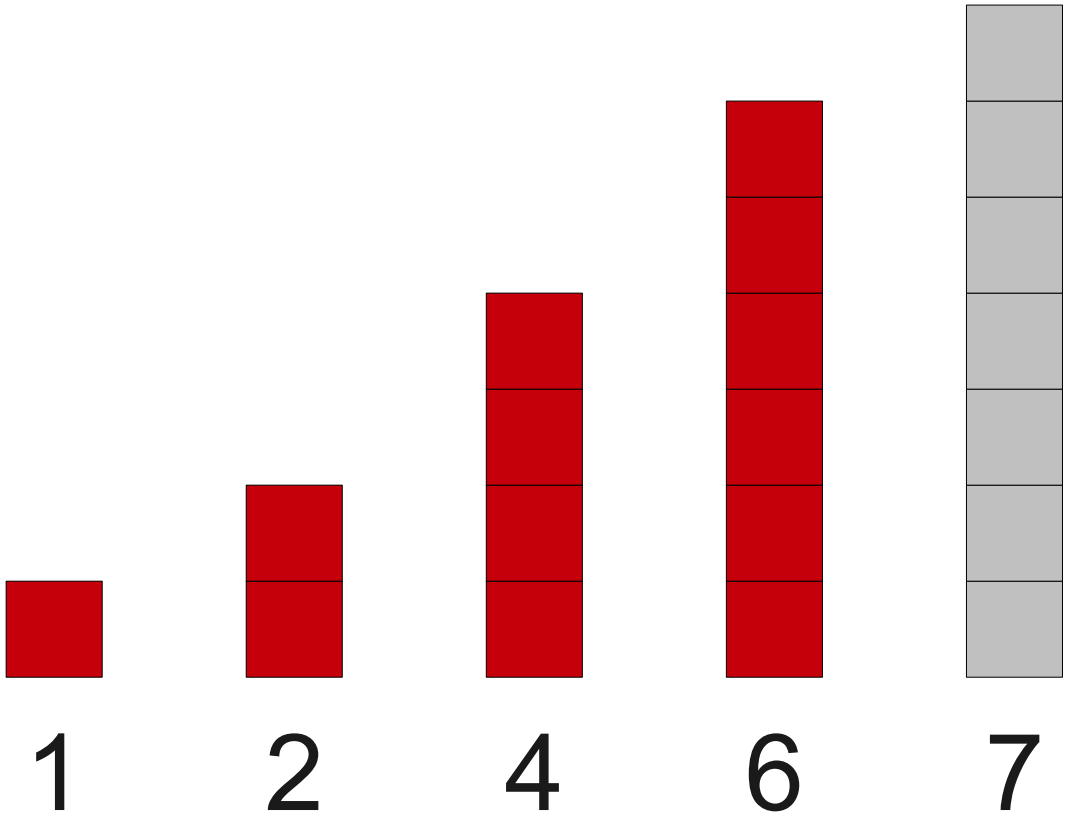

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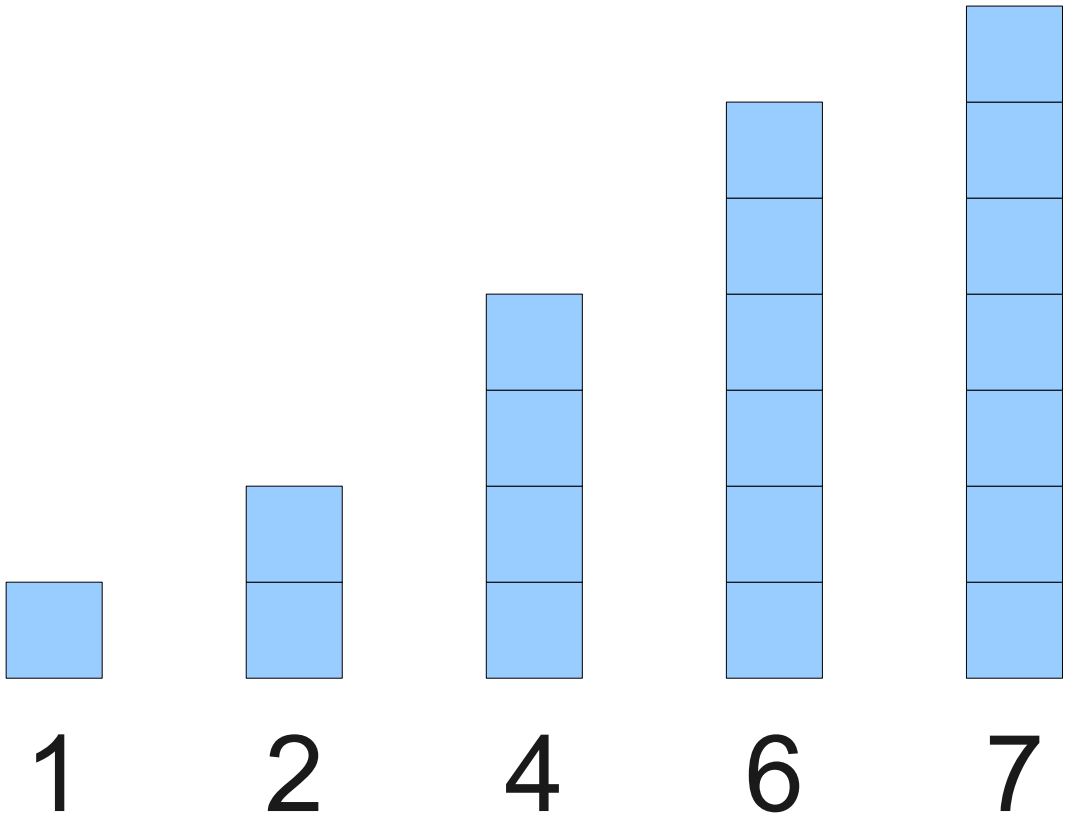

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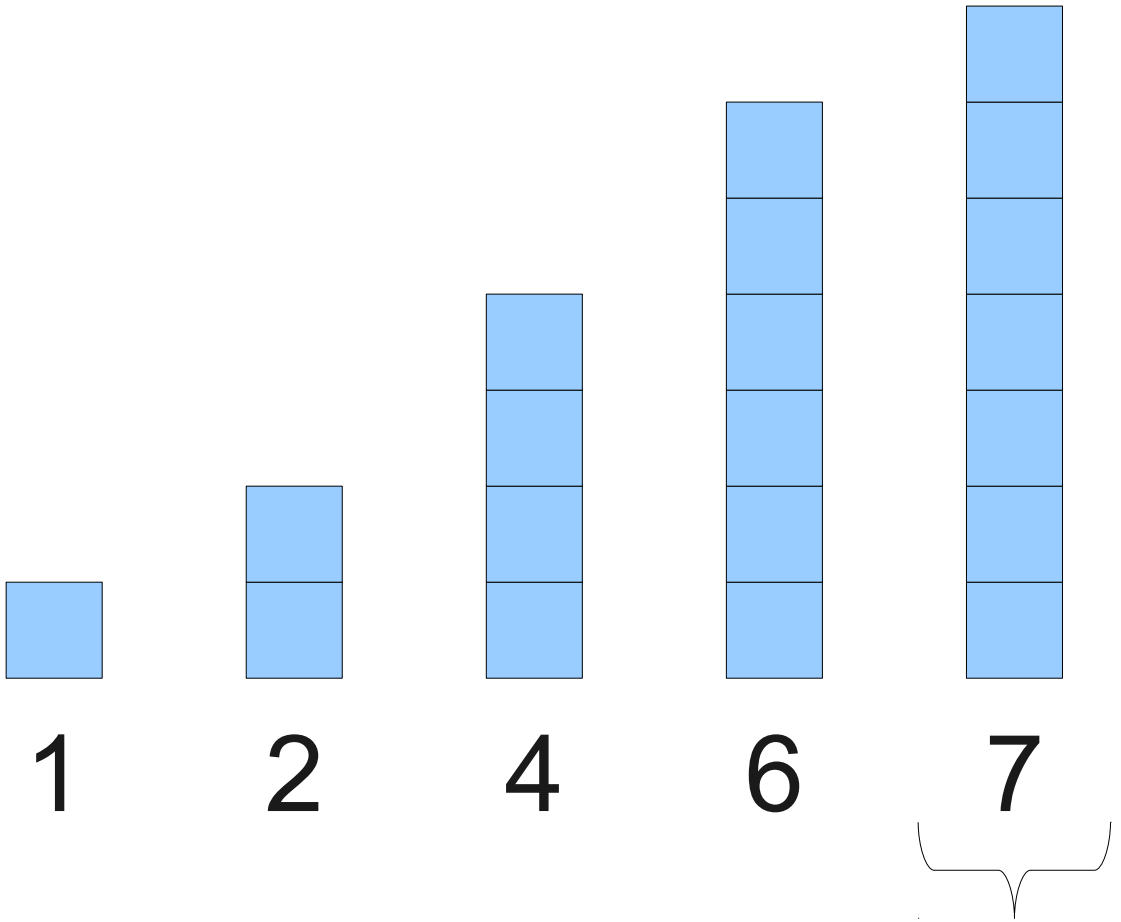

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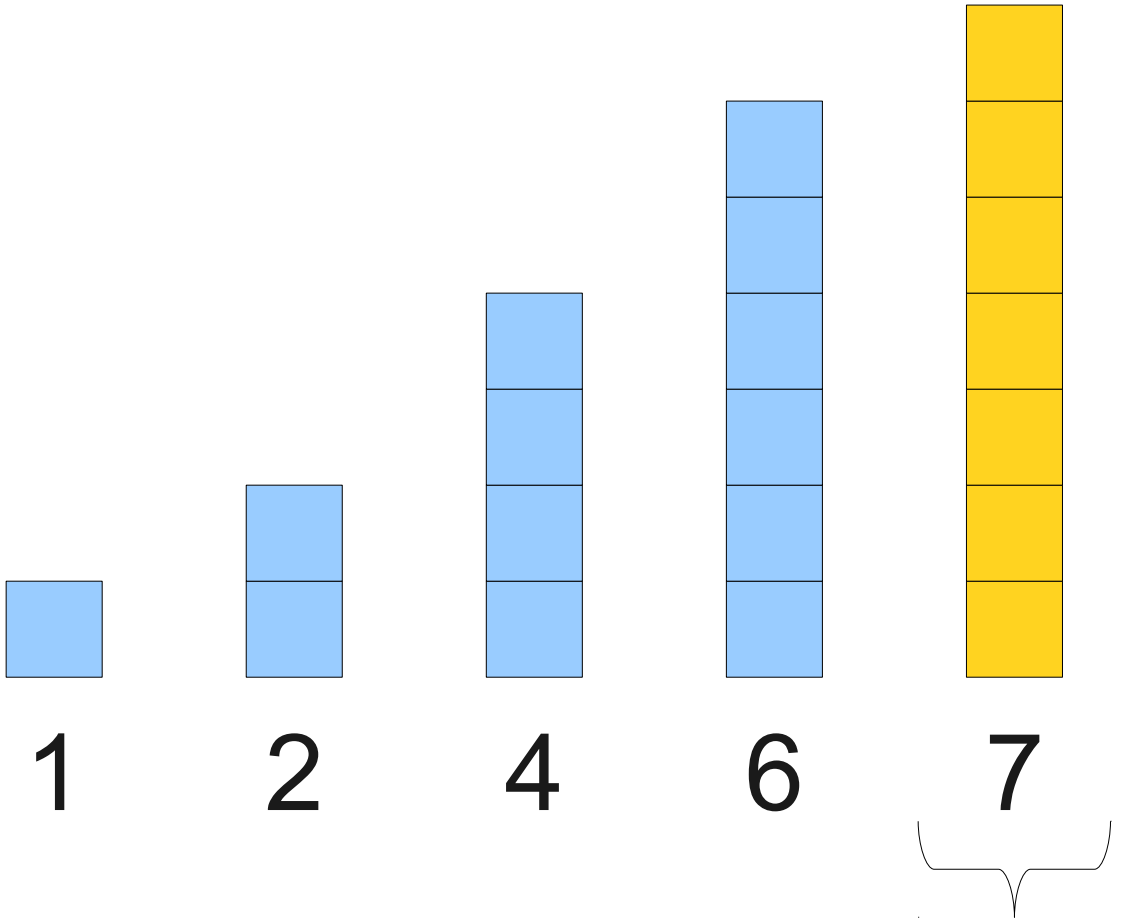

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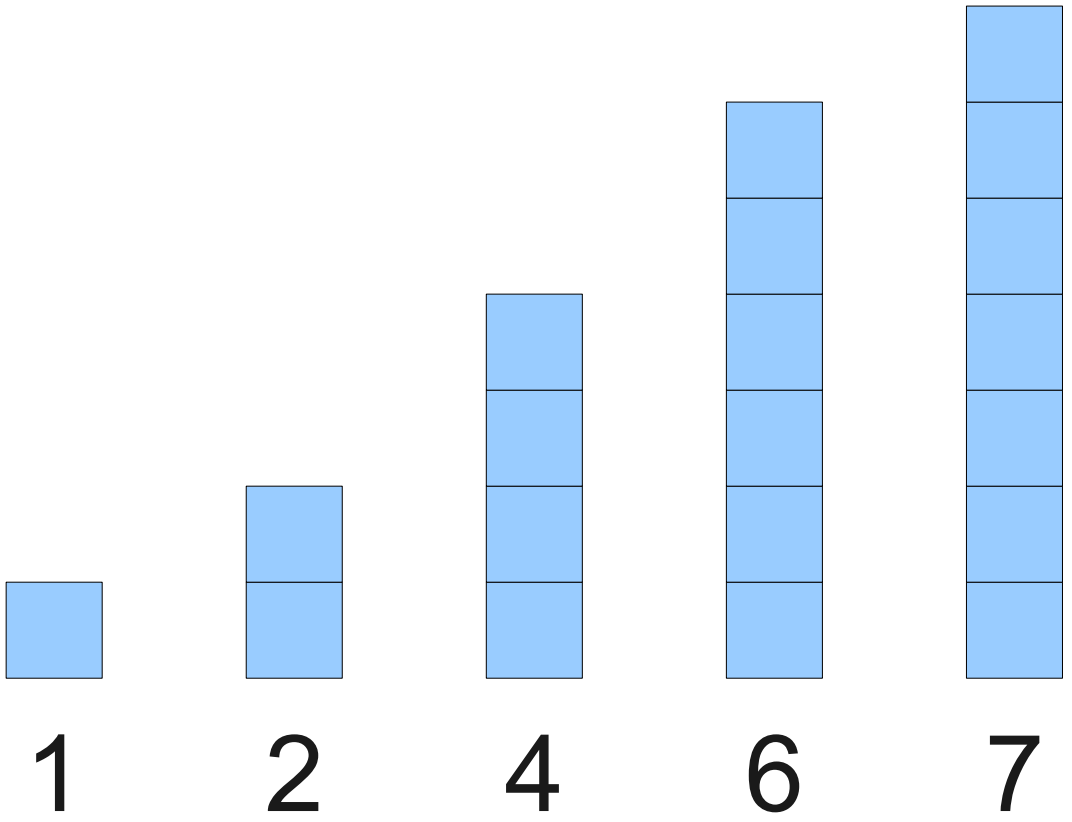

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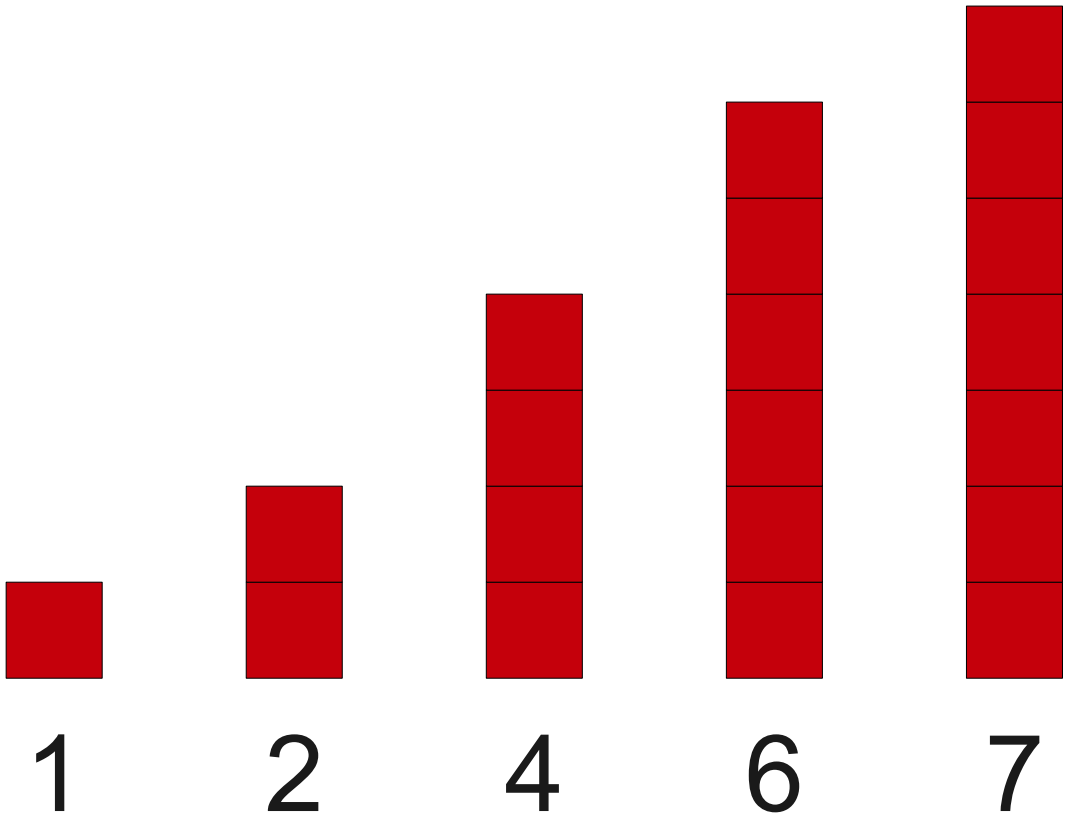


Selection Sort

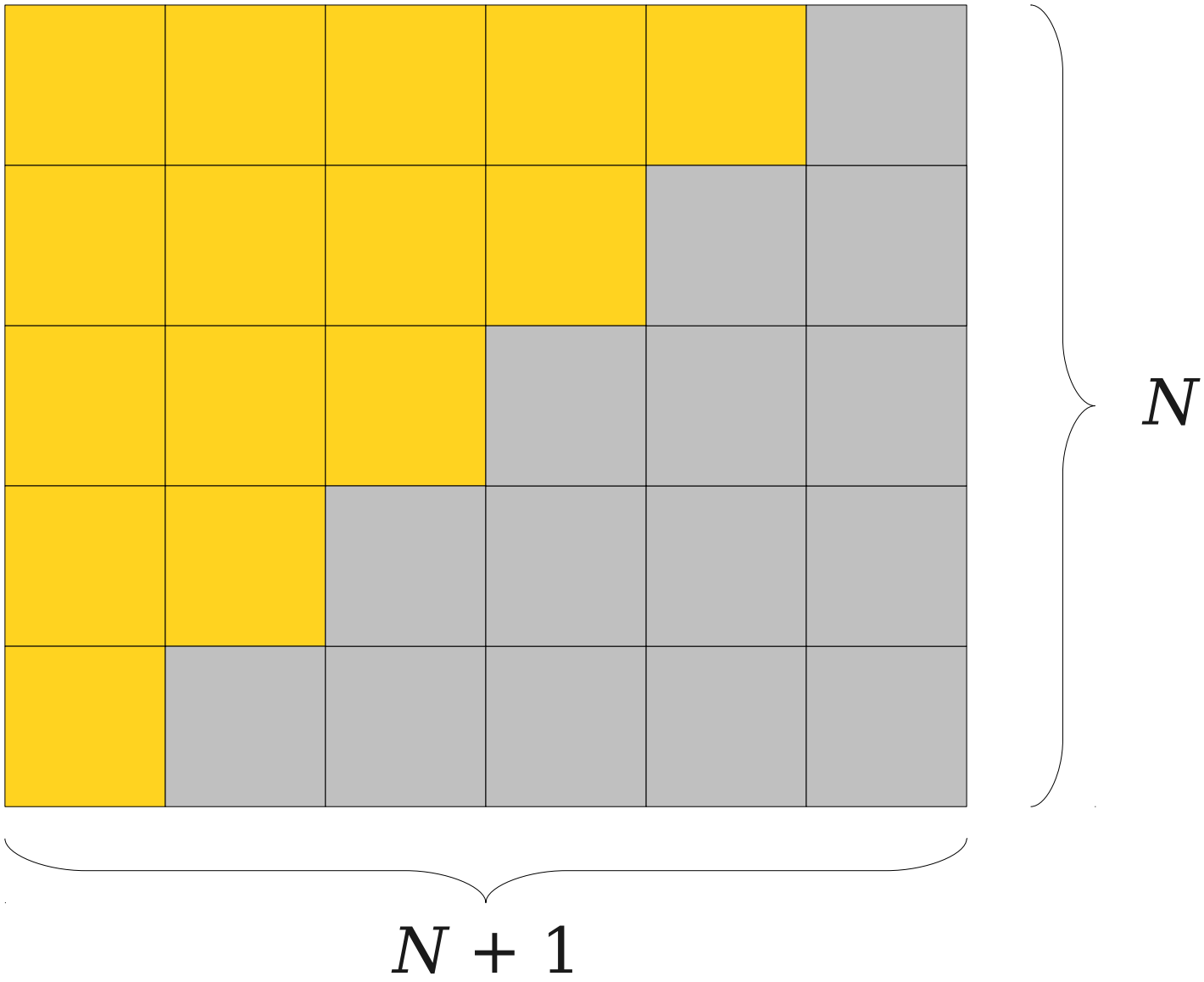
- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position.
- (etc.)

```
private void selectionSort(int[] elems) {
    for (int index = 0; index < elems.length; index++) {
        int smallestIndex = indexOfSmallest(elems, index);
        swap(elems, index, smallestIndex);
    }
}

private int indexOfSmallest(int[] elems, int startPoint) {
    int smallestIndex = startPoint;
    for (int i = startPoint + 1; i < elems.size(); i++) {
        if (elems[i] < elems[smallestIndex])
            smallestIndex = i;
    }
    return smallestIndex;
}

private void swap(int[] arr, int a, int b) {
    int temp = arr[a];
    arr[a] = arr[b];
    arr[b] = temp;
}
```

Analyzing Selection Sort

- How much work do we do for selection sort?
- To find the smallest value, we need to look at all N array elements.
- To find the second-smallest value, we need to look at N – 1 array elements.
- To find the third-smallest value, we need to look at N – 2 array elements.
- Work is $N + (N - 1) + (N - 2) + \ldots + 1$.

$$1 + 2 + \ldots + (N - 1) + N = N(N+1) / 2$$



An Interesting Observation

- Selection sort does roughly N^2 / 2 array lookups.
- Suppose we double the number of elements in the array we want to sort.
- How much longer will it take to sort the new array?

$$\begin{aligned} & \textit{newTime} \,/\, \textit{oldTime} \\ & \approx ((2N)^2 \,/\, 2) \,/\, (N^2 \,/\, 2) \\ & \approx (2N)^2 \,/\, N^2 \\ & \approx 4N^2 \,/\, N^2 \\ & \approx 4 \end{aligned}$$

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- So we should expect it to take about four times longer.

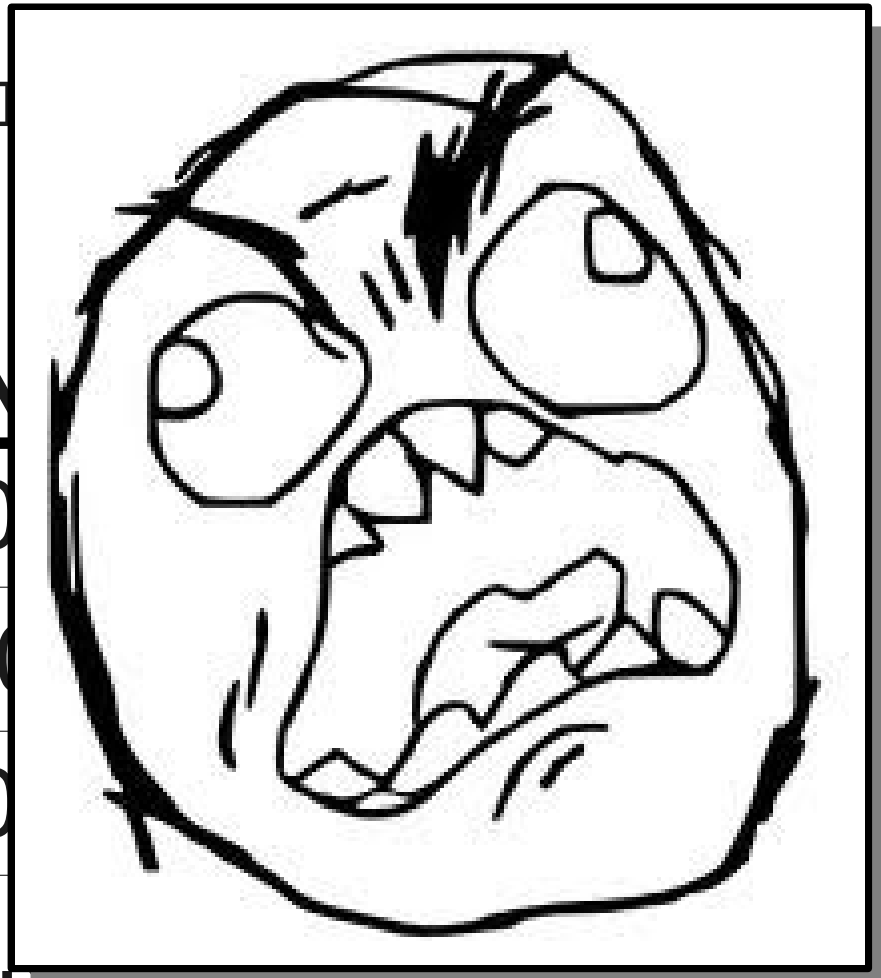
Analyzing Selection Sort

- Work done is roughly $N^2 / 2$.

N	$N^2 / 2$
10	50
100	5,000
1000	500,000
1,000,000	500,000,000,000
1,000,000,000	500,000,000,000,000,000

Analyzing Selection Sort

- Work do



A Few Other Sorting Algorithms

Yes, This Exists

Sorting algorithms done as
Hungarian folk dances:

http://www.youtube.com/user/AlgoRythmics