Graphs

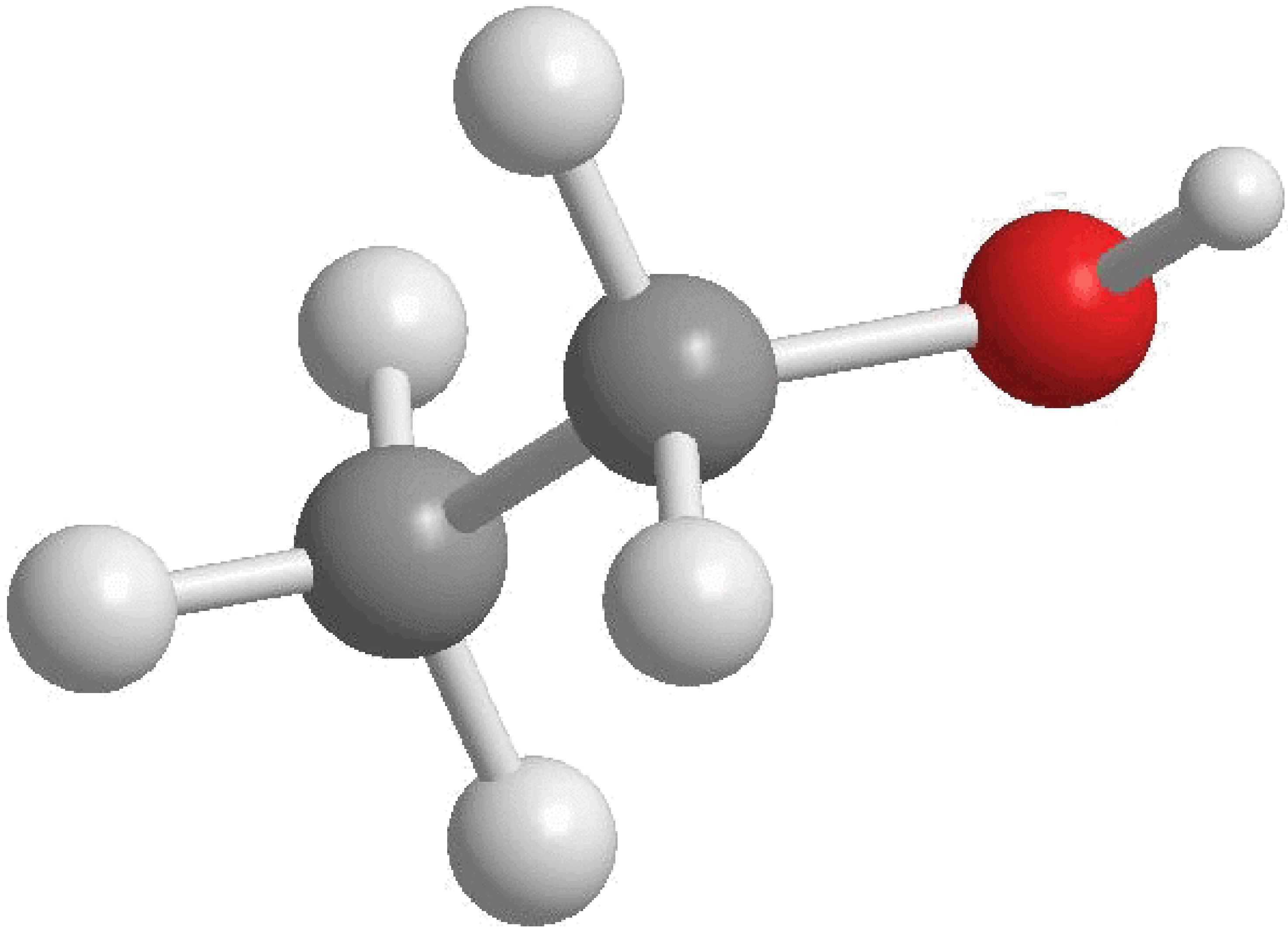
Problem Set One due right now in the box up front.

Mathematical Structures

- Just as there are common data structures in programming, there are common mathematical structures in discrete math.
- So far, we've seen simple structures like sets and natural numbers, but there are many other important structures out there.
- Over the next few weeks, we'll explore several of them.

Graphs

Chemical Bonds



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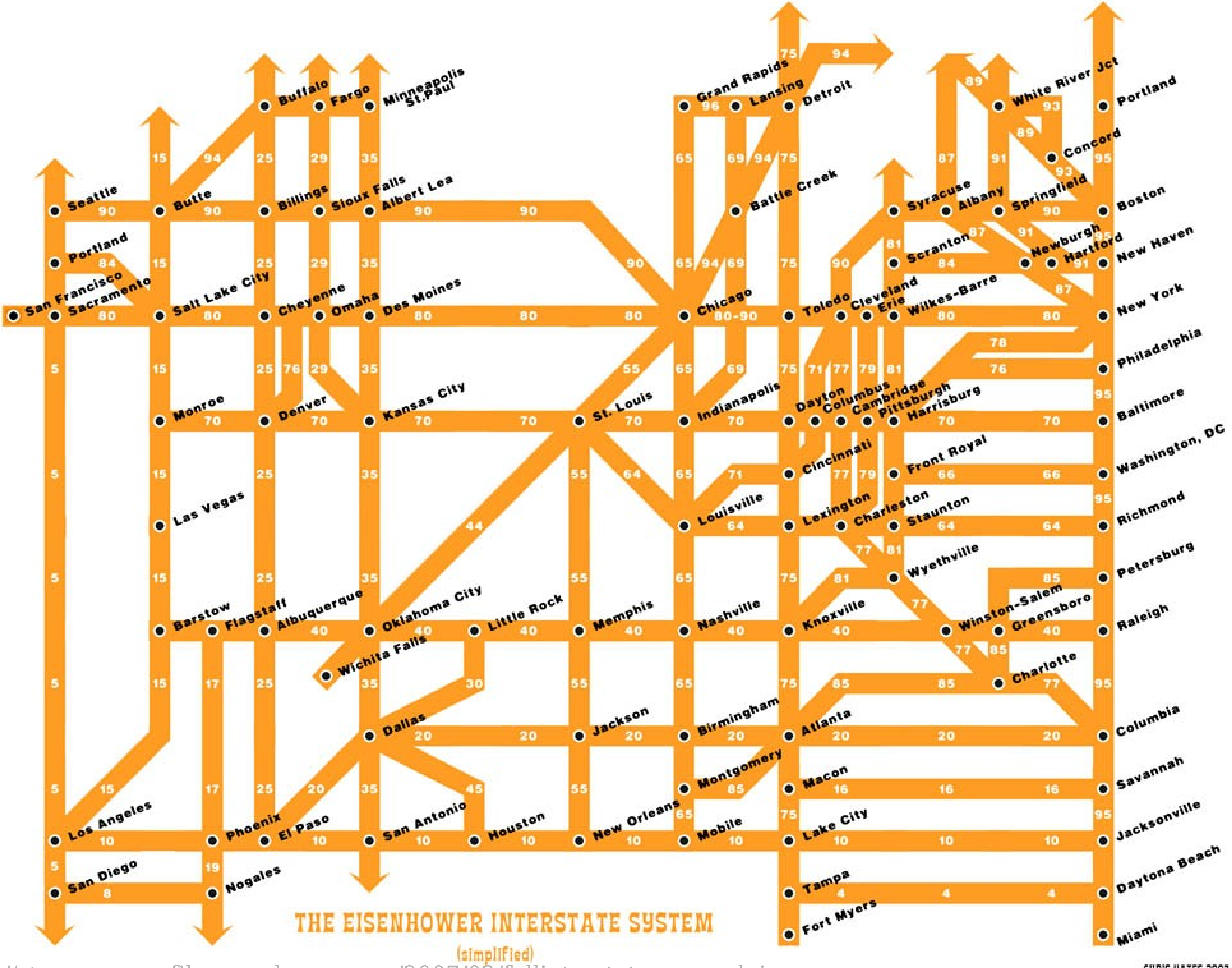
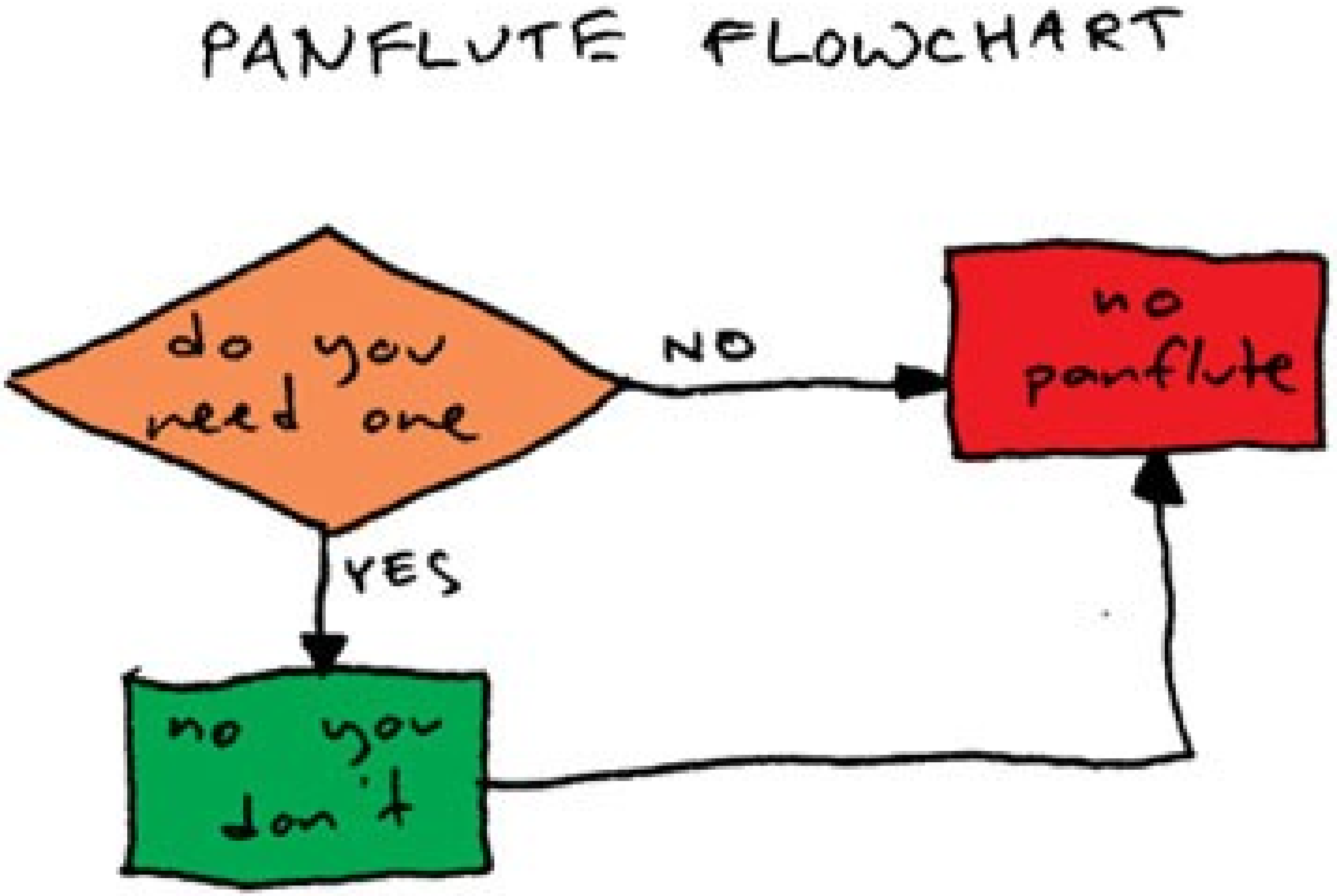


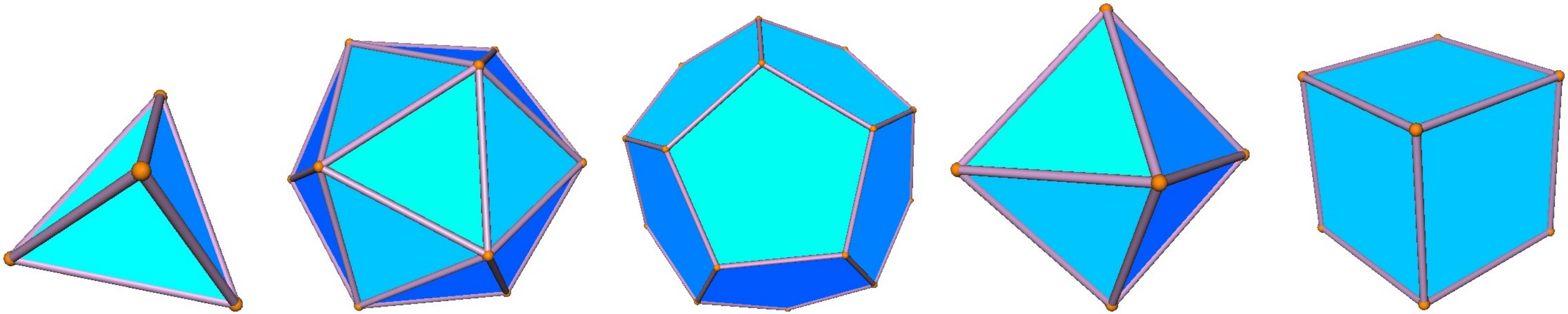
THE EISENHOWER INTERSTATE SYSTEM

(simplified)

http://strangemaps.files.wordpress.com/2007/02/fullinterstatemap-web.jpg

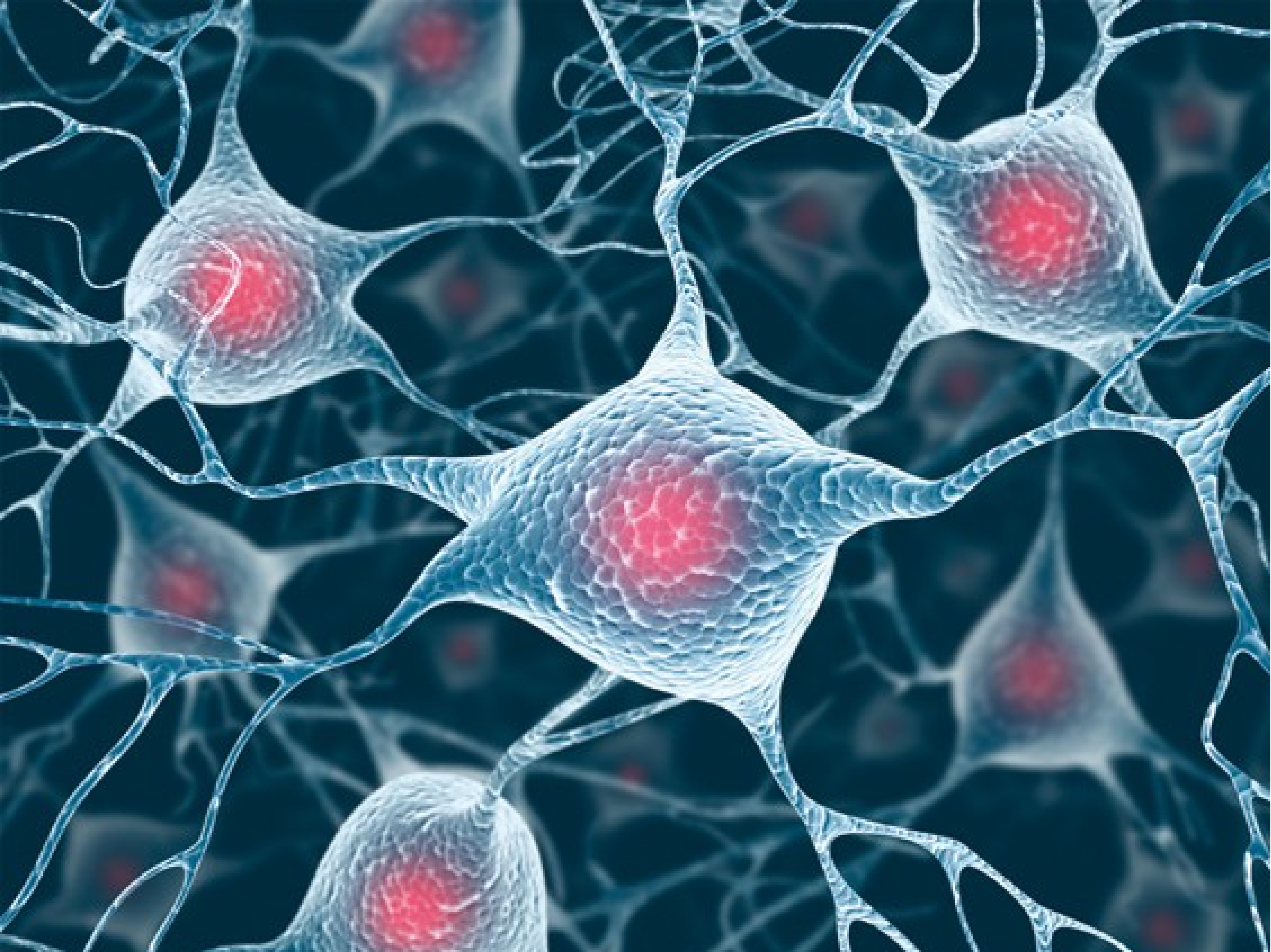


http://www.toothpastefordinner.com/

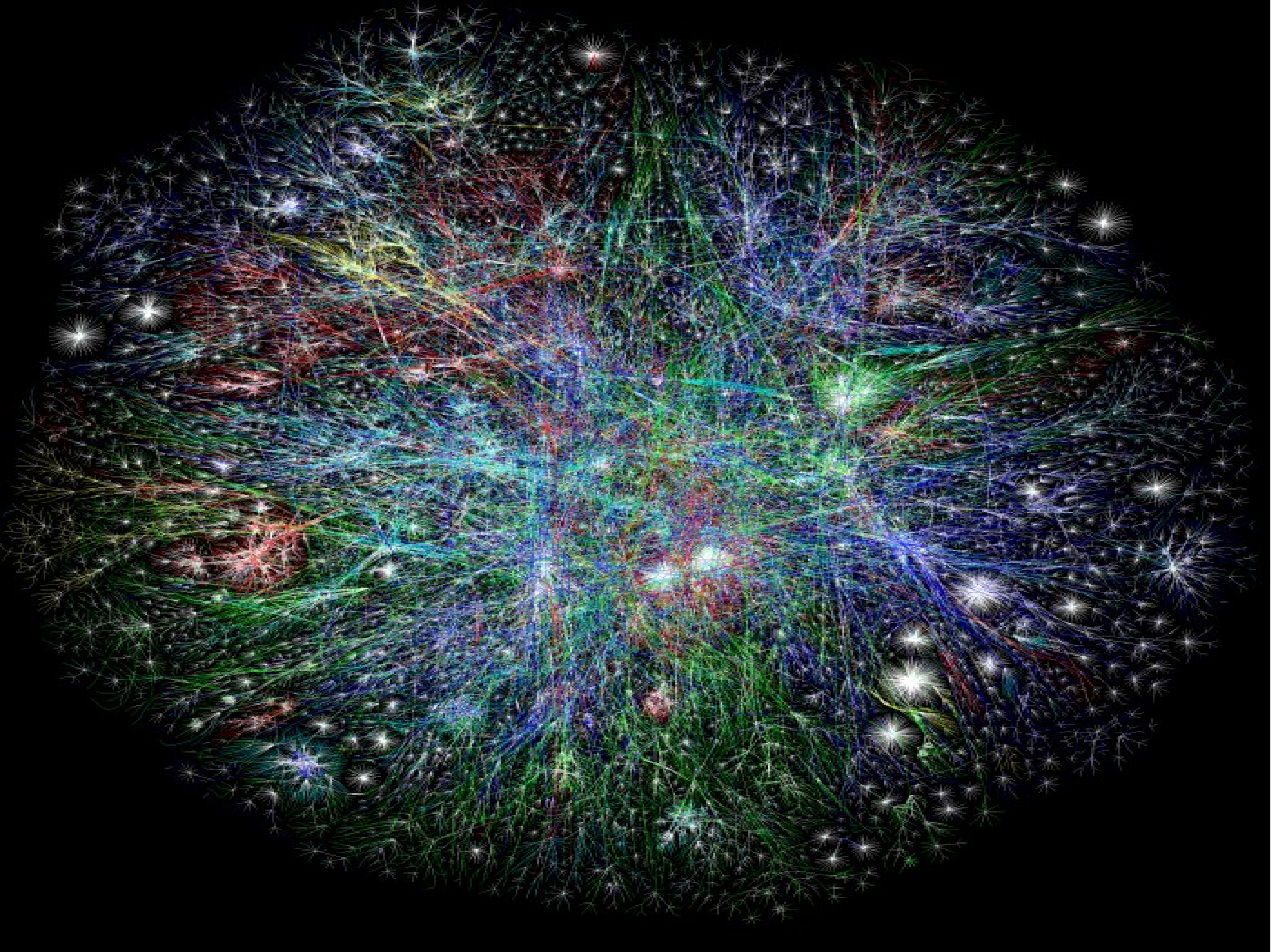


Tetrahedron Icosahedron Dodecahedron Octahedron Cube

http://www.princeton.edu/pr/pictures/l-r/packingproblem/pu-platonic-solids.jpg



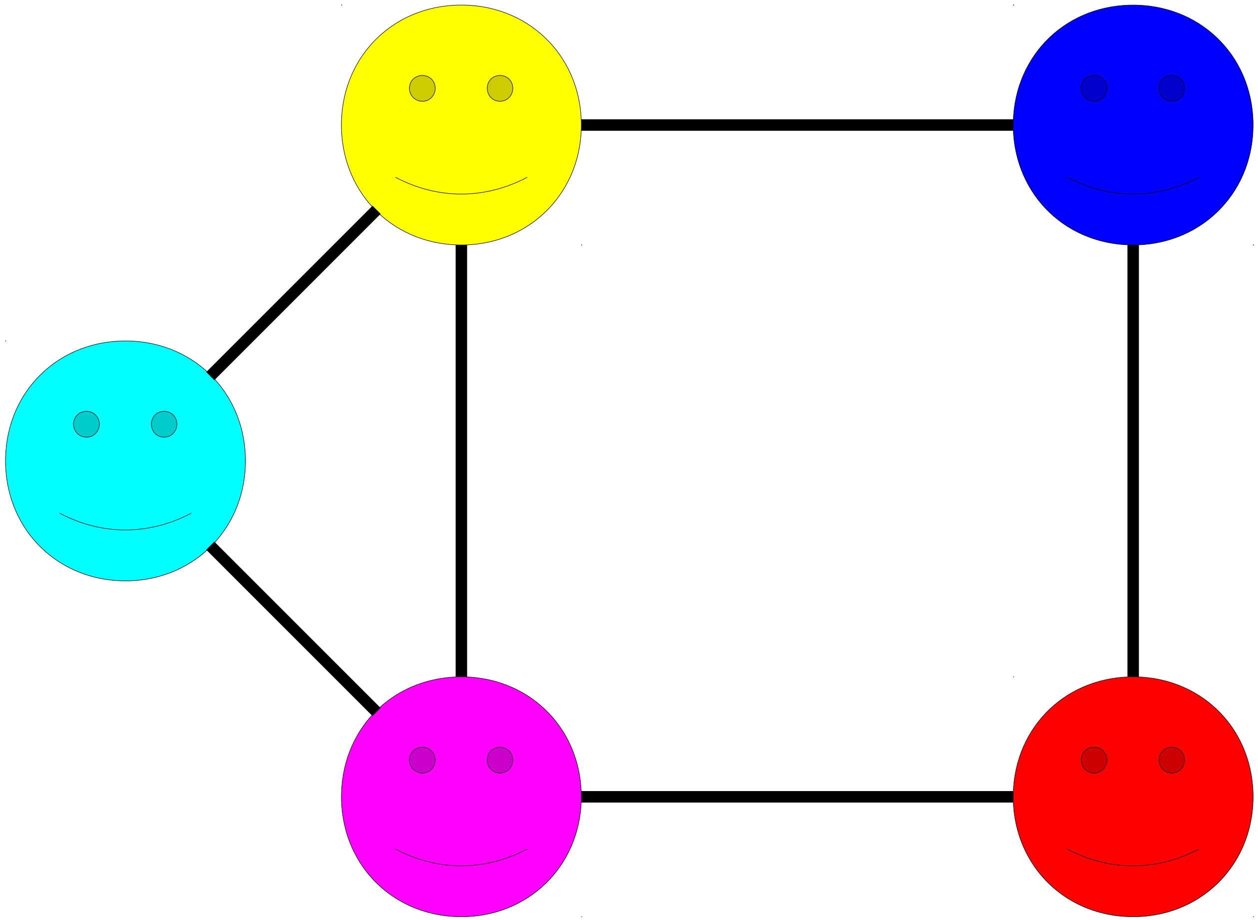
Me too!



What's in Common

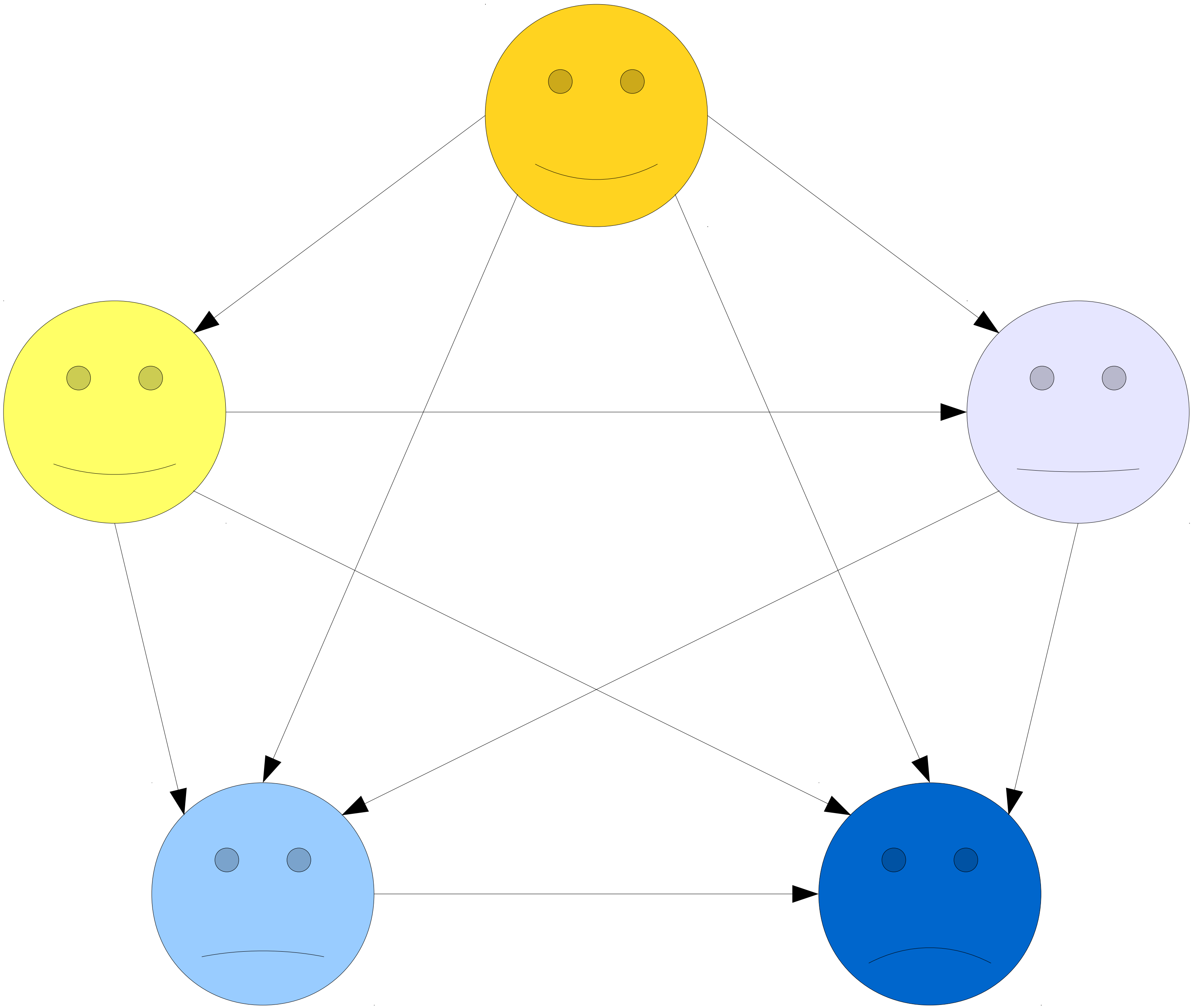
- Each of these structures consists of
  - Individual objects and
  - Links between those objects.
- Goal: find a general framework for describing these objects and their properties.

A **graph** is a mathematical structure for representing relationships.

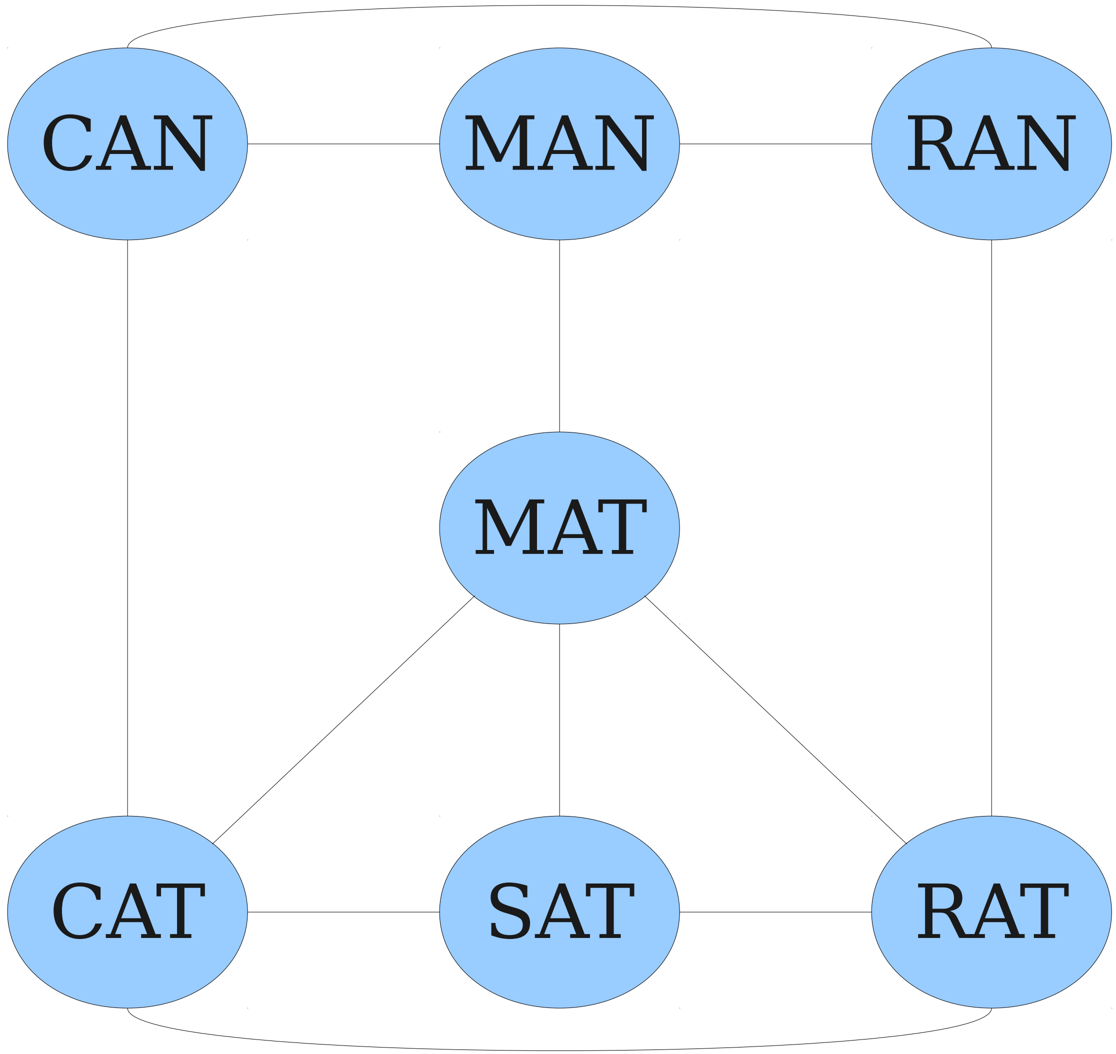


A graph consists of a set of **nodes** (or ***vertices***) connected by **edges** (or ***arcs***)

Some graphs are **directed**.



Some graphs are **undirected**.



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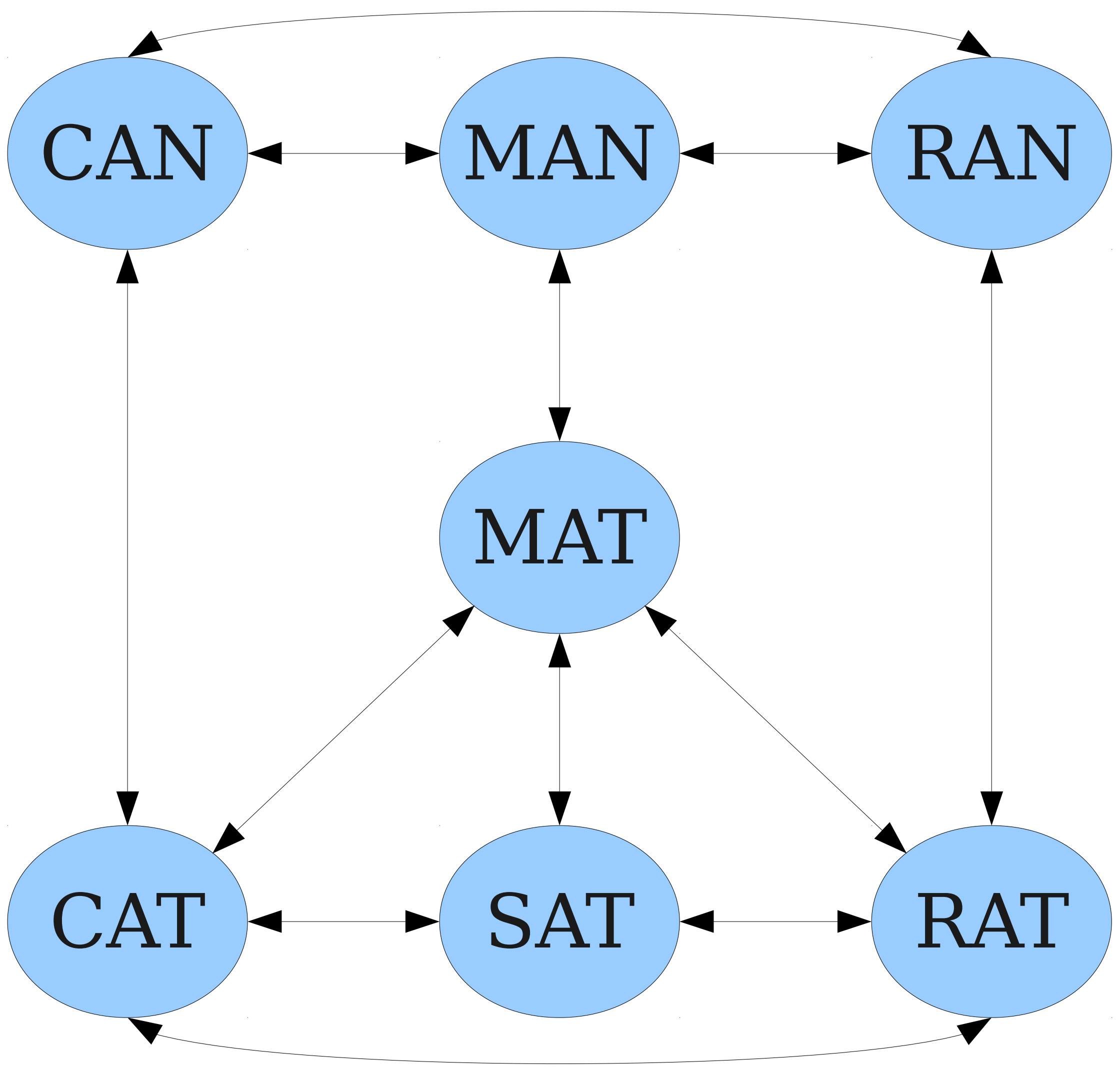


You can think of them as directed graphs with edges both ways.

Formalizing Graphs

- How might we define a graph mathematically?
- Need to specify
  - What the nodes in the graph are, and
  - What the edges are in the graph.
- The nodes can be pretty much anything.
- What about the edges?

Ordered and Unordered Pairs

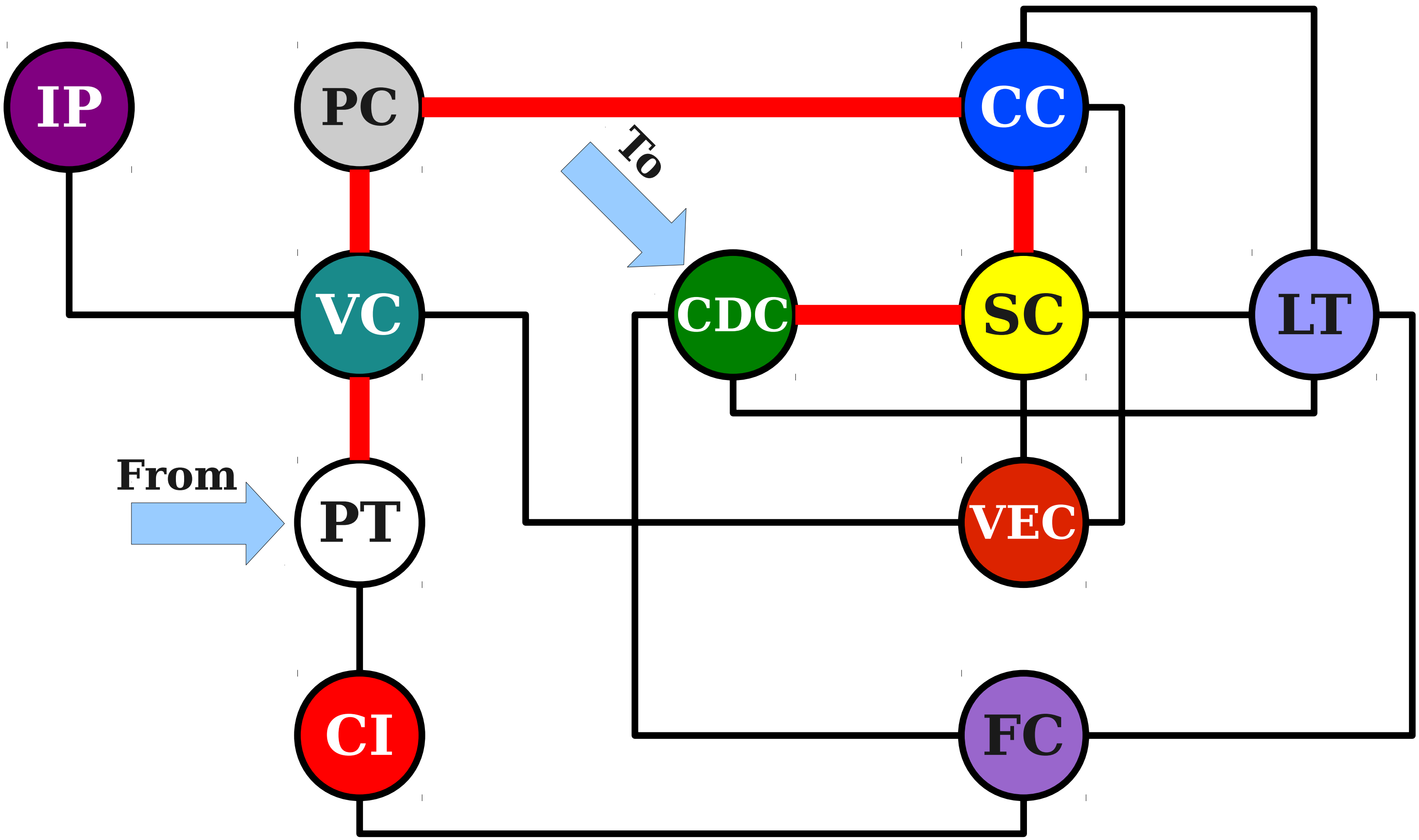
- An **unordered pair** is a set $\{a, b\}$ of two elements (remember that sets are unordered).
  - $\{0, 1\} = \{1, 0\}$
- An **ordered pair** (a, b) is a pair of elements in a specific order.
  - $(0, 1) \neq (1, 0)$.
  - Two ordered pairs are equal iff each of their components are equal.

Formalizing Graphs

- Formally, a **graph** is an ordered pair $G = (V, E)$, where
  - V is a set of nodes.
  - E is a set of edges.
- G is defined as an *ordered* pair so it's clear which set is the nodes and which is the edges.
- V can be any set whatsoever.
- E is one of two types of sets:
  - A set of *unordered* pairs of elements from V.
  - A set of *ordered* pairs of elements from V.

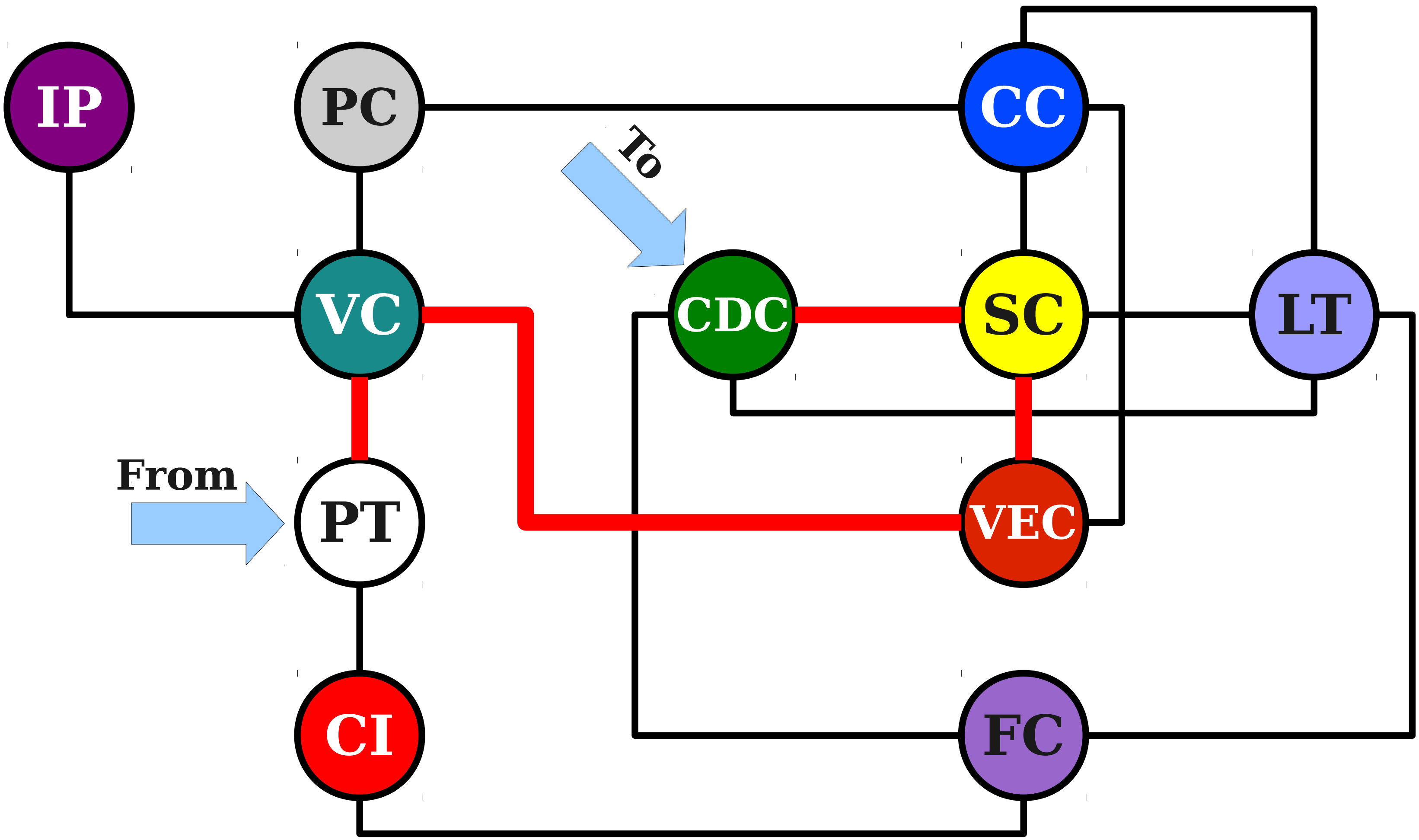
Undirected Connectivity

Navigating a Graph



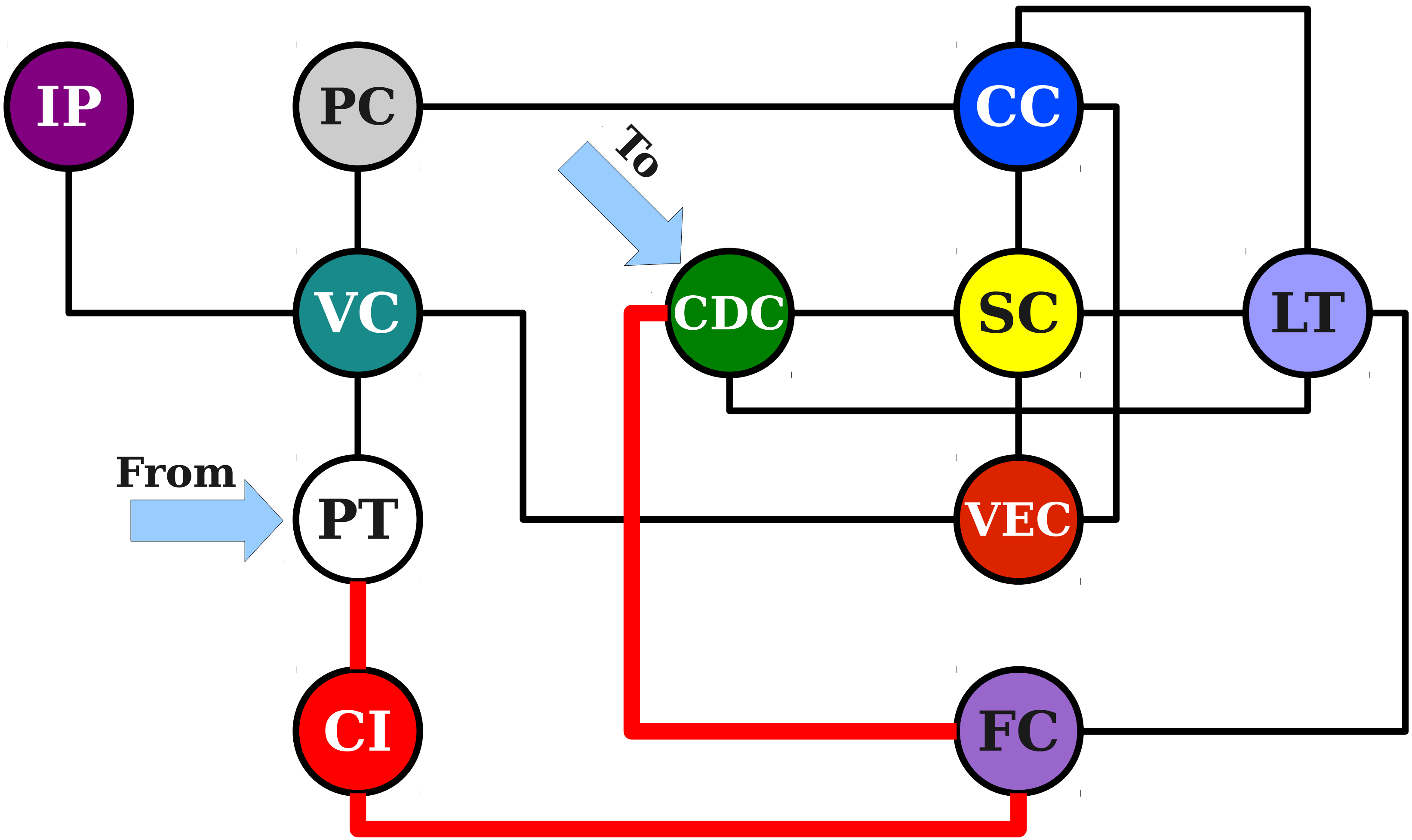
PT → VC → PC → CC → SC → CDC

Navigating a Graph



PT → VC → VEC → SC → CDC

Navigating a Graph

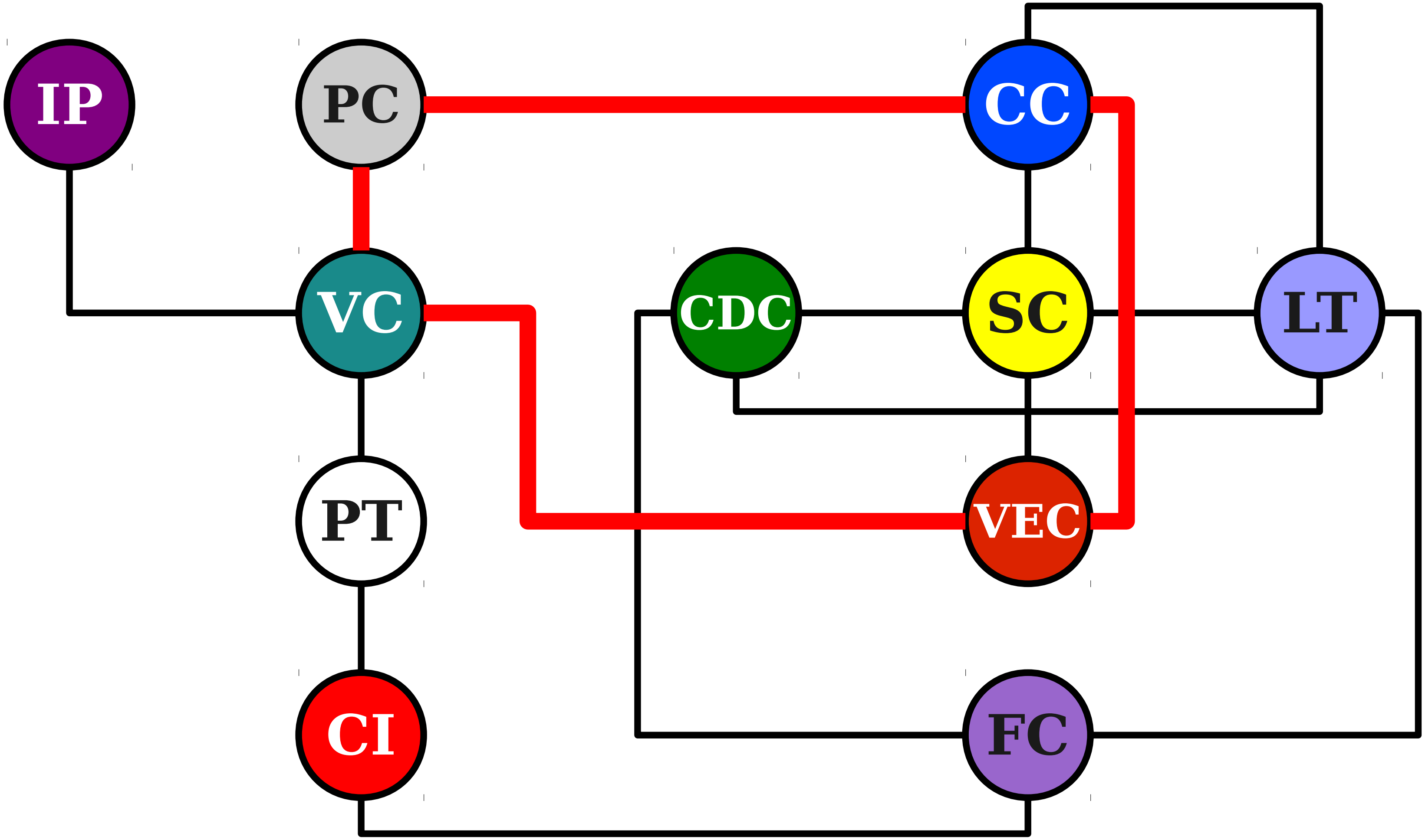


PT → CI → FC → CDC

A **path** from v_1 to v_n is a sequence of nodes $v_1, v_2, \ldots, v_n$ where $(v_k, v_{k+1}) \in E$ for all natural numbers in the range $1 \leq k \leq n - 1$.

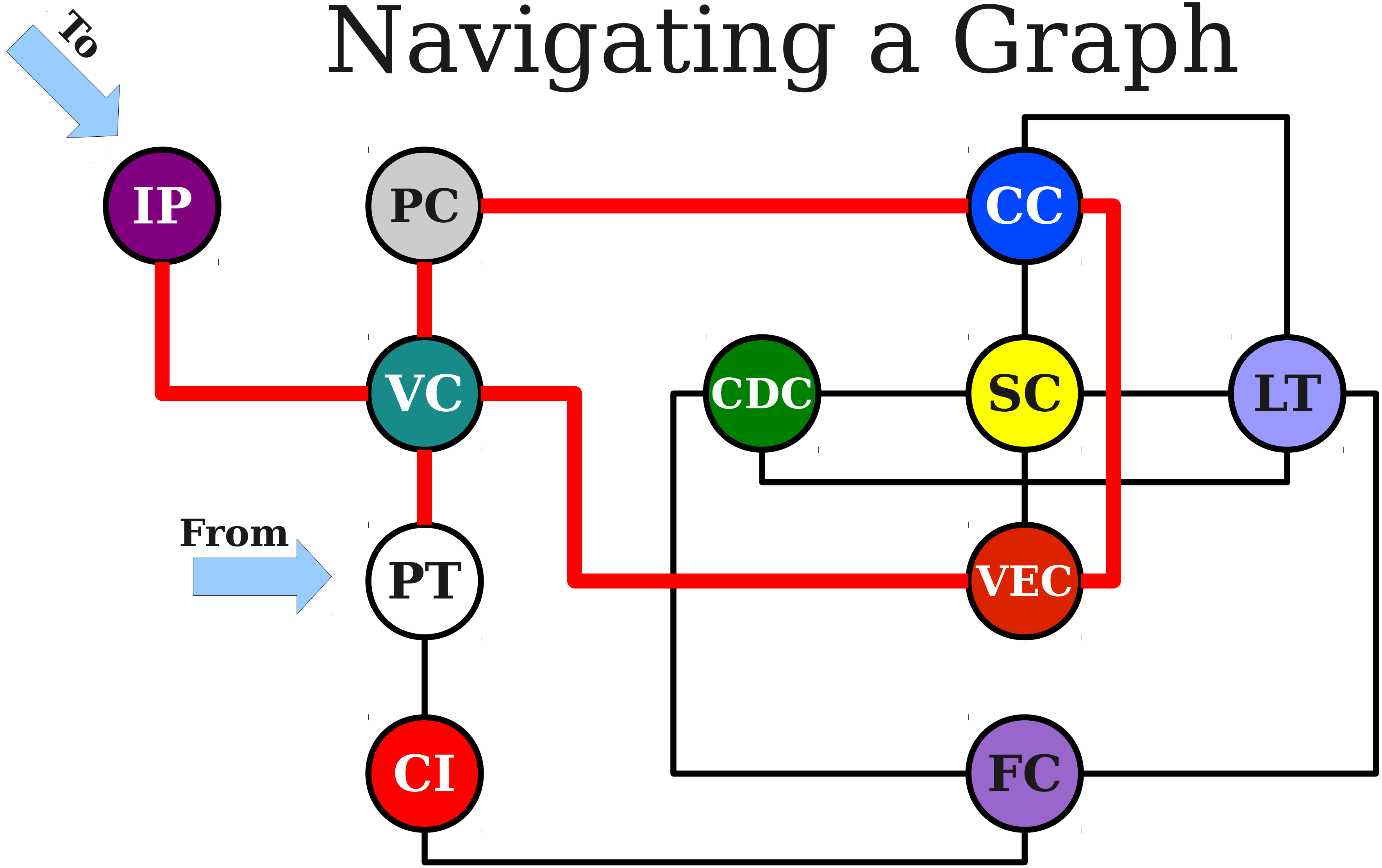
The **length** of a path is the number of edges it contains, which is one less than the number of nodes in the path.

Navigating a Graph



PC → CC → VEC → VC → PC

Navigating a Graph



PT → VC → PC → CC → VEC → VC → IP

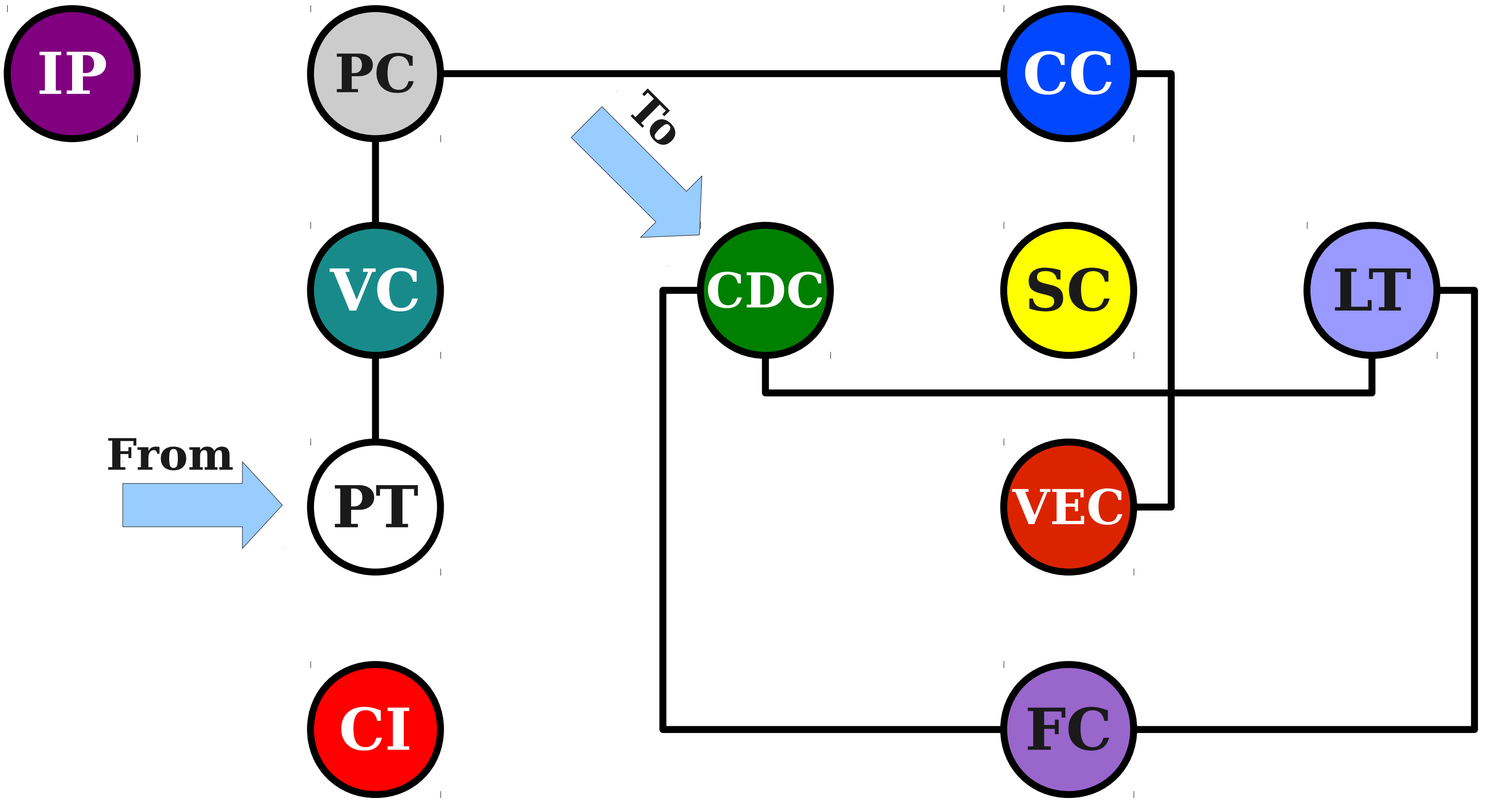
A **cycle** in a graph is a path from a node to itself.

The **length** of a cycle is the number of edges in that cycle.

A **simple path** in a graph is a path that does not revisit any nodes or edges.

A **simple cycle** in a graph is a cycle that does not revisit any nodes or edges (except the start/end node).

Navigating a Graph



In an undirected graph, two nodes u and v are called **connected** iff there is a path from u to v.

We denote this as $\boldsymbol{u \leftrightarrow v}$.

If u is not connected to v, we write $\boldsymbol{u \nleftrightarrow v}$.

Next Time

- **The Rest of The Lecture**
  - *Sorry about the fire alarm!*
  - Connected components.
  - Planar graphs.
- **Binary Relations**
  - Equivalence relations.
  - Partial orders (ITA).