Graphs

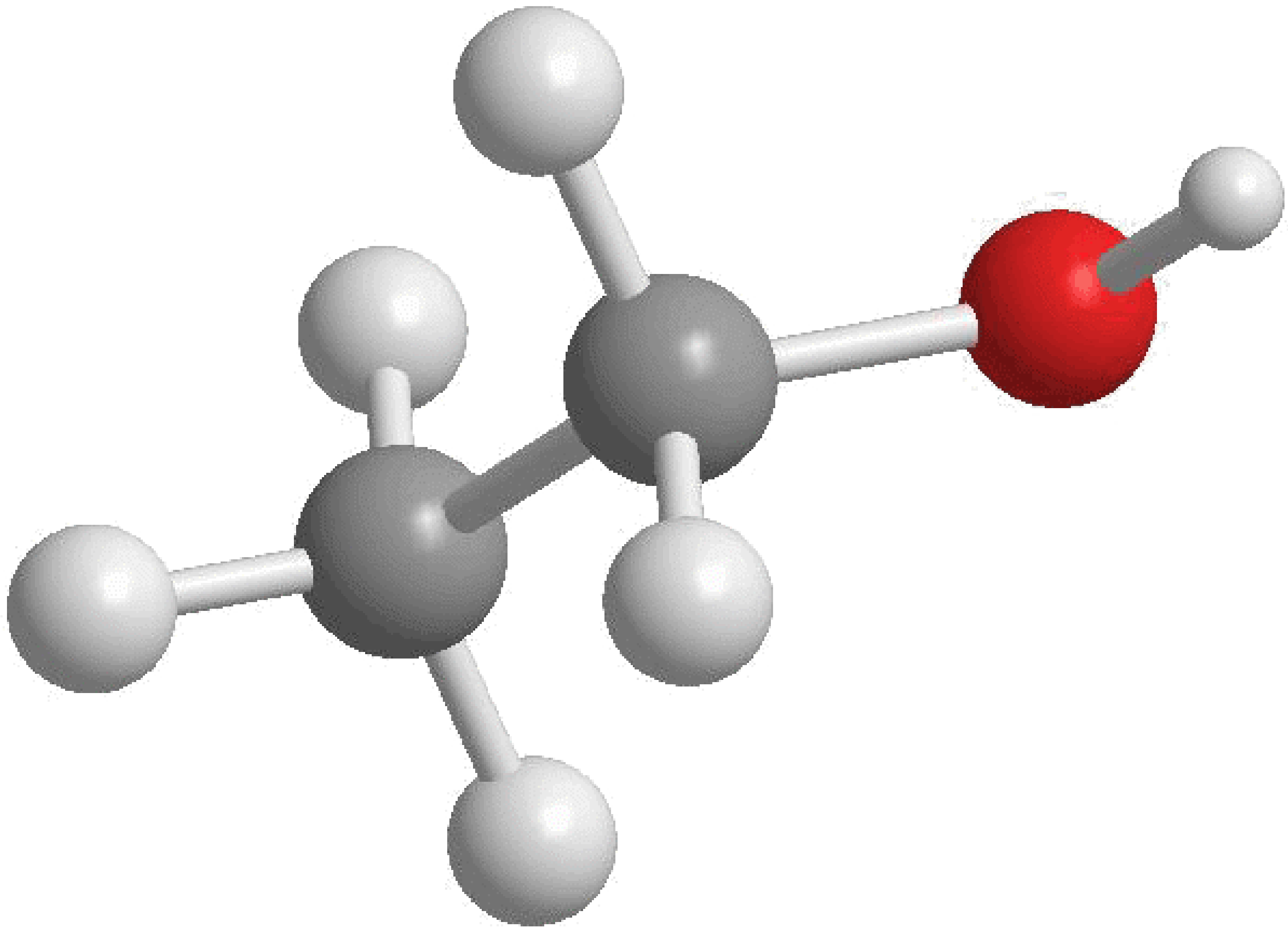
Problem Set One
due right now in
the box up front.

Mathematical Structures

- Just as there are common data structures in programming, there are common mathematical structures in discrete math.
- So far, we've seen simple structures like sets and natural numbers, but there are many other important structures out there.
- Over the next few weeks, we'll explore several of them.

Graphs

Chemical Bonds



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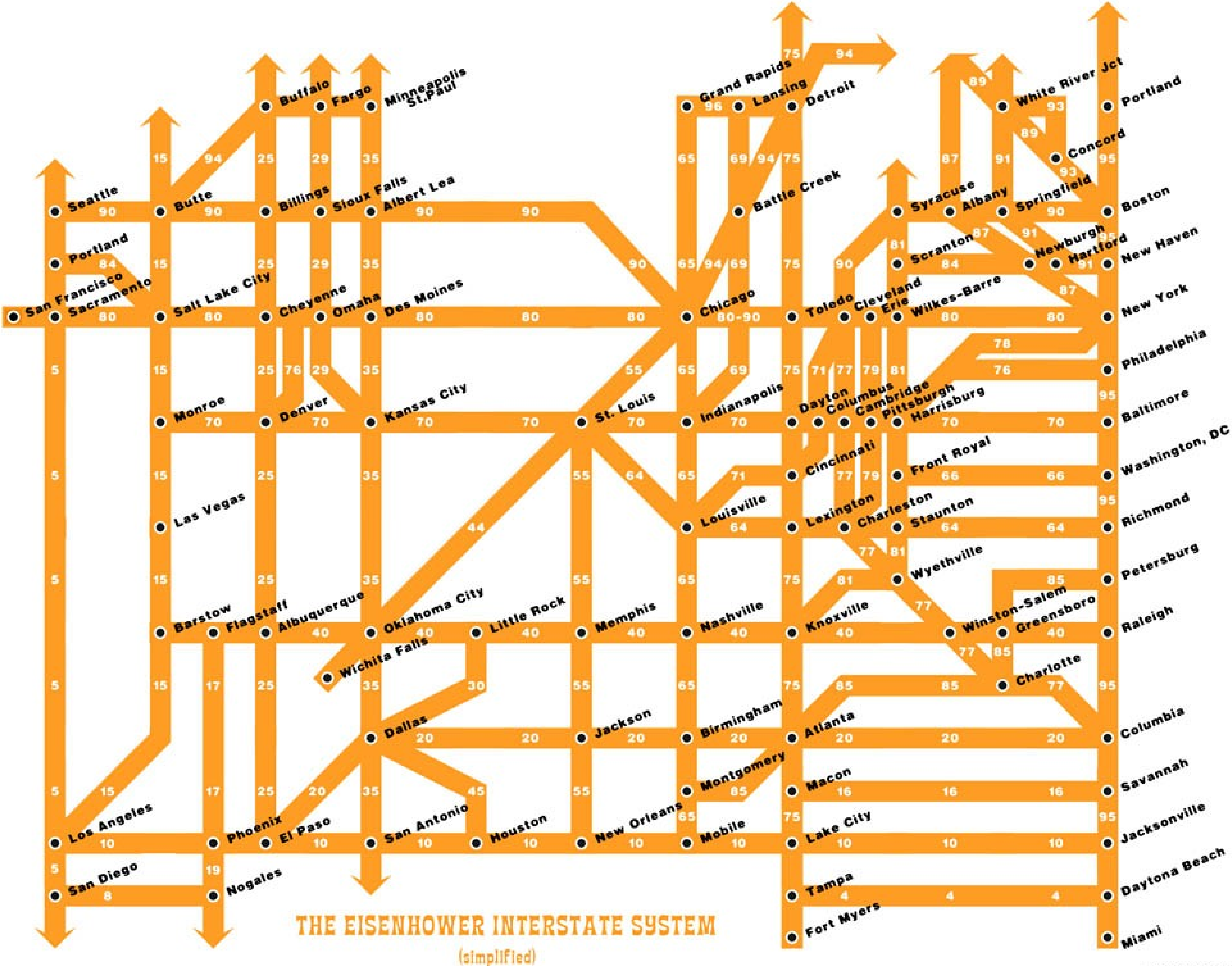
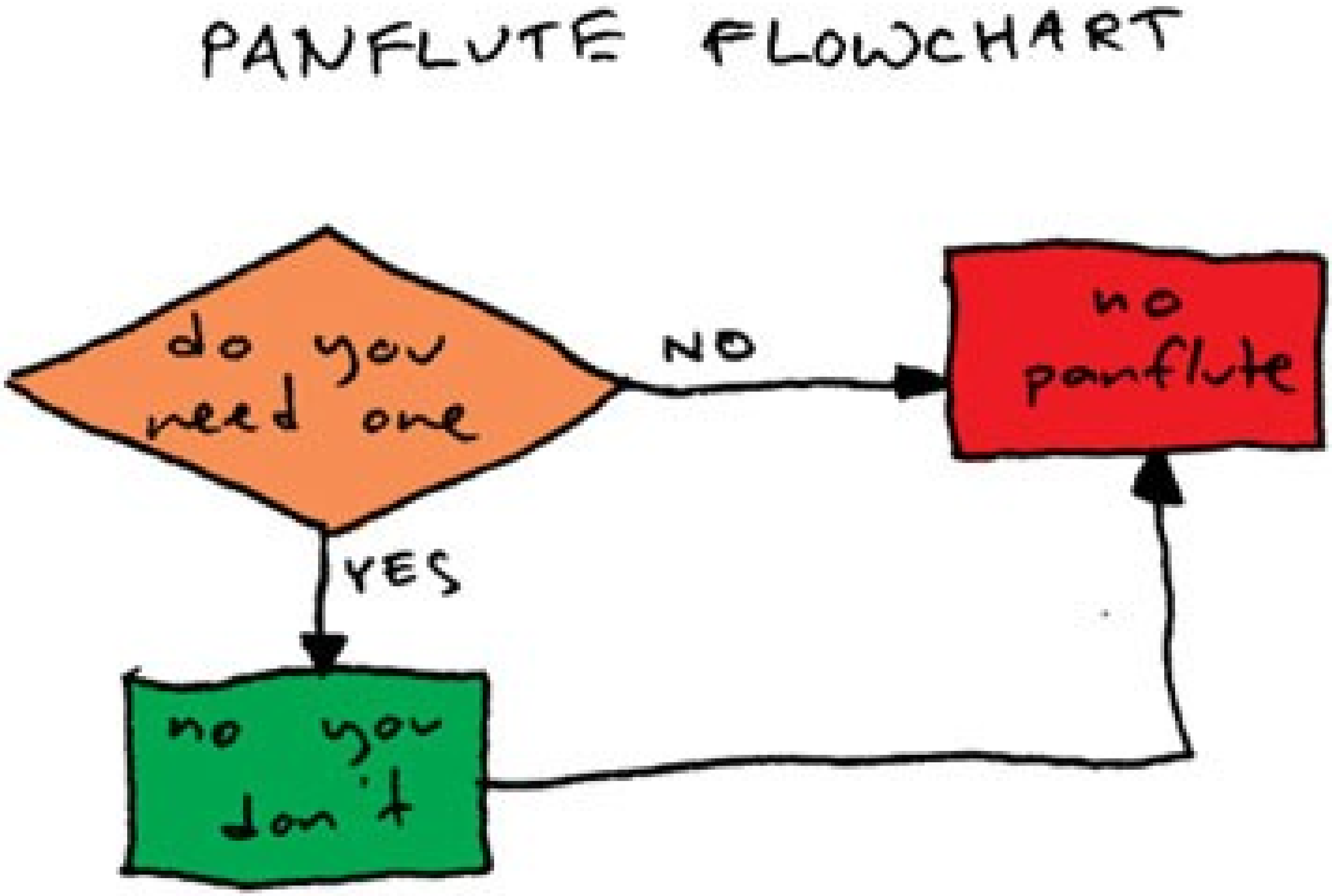
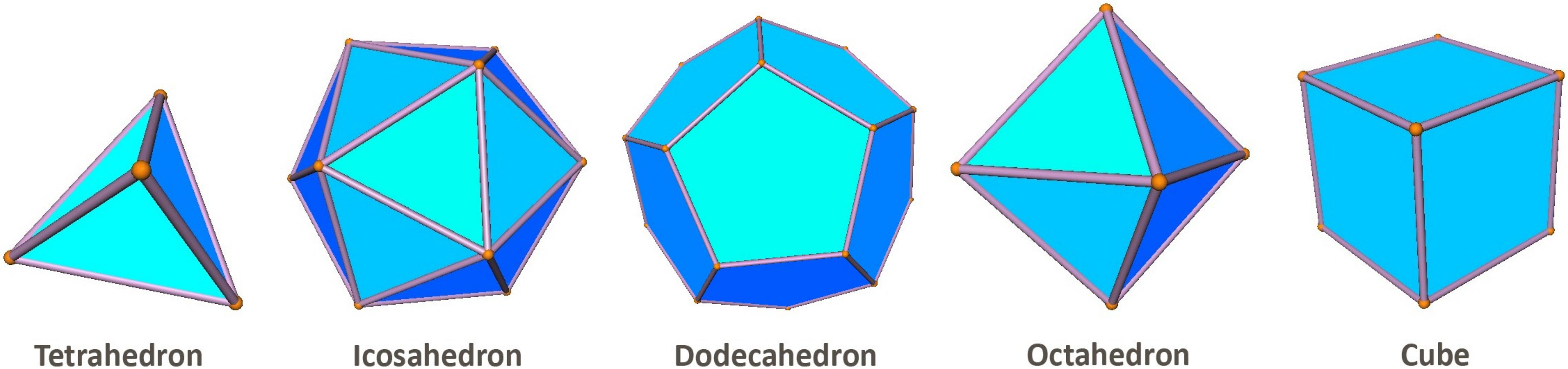

THE EISENHOWER INTERSTATE SYSTEM
(simplified)
CHRIS YATES 2007
Seattle
Portland
San Francisco
Sacramento
Los Angeles
San Diego
Butte
Salt Lake City
Monroe
Las Vegas
Barstow
Flagstaff
Phoenix
Nogales
Buffalo
Billings
Cheyenne
Denver
Albuquerque
El Paso
Fargo
Sioux Falls
Omaha
Minneapolis
St.Paul
Albert Lea
Des Moines
Kansas City
Oklahoma City
Wichita Falls
Dallas
San Antonio
Houston
Little Rock
St. Louis
Memphis
Jackson
New Orleans
Grand Rapids
Lansing
Detroit
Battle Creek
Chicago
Indianapolis
Louisville
Nashville
Birmingham
Montgomery
Mobile
Toledo
Dayton
Columbus
Cincinnati
Cleveland
Erie
Cambridge
Pittsburgh
Lexington
Charleston
Knoxville
Atlanta
Macon
Lake City
Tampa
Fort Myers
Syracuse
Albany
Scranton
Wilkes-Barre
Harrisburg
Front Royal
Staunton
Wyethville
Winston-Salem
Greensboro
Charlotte
White River Jct
Concord
Springfield
Newburgh
Hartford
Portland
Boston
New Haven
New York
Philadelphia
Baltimore
Washington, DC
Richmond
Petersburg
Raleigh
Columbia
Savannah
Jacksonville
Daytona Beach
Miami


PANFLUTE FLOWCHART
do you need one
NO
no panflute
YES
no you don't



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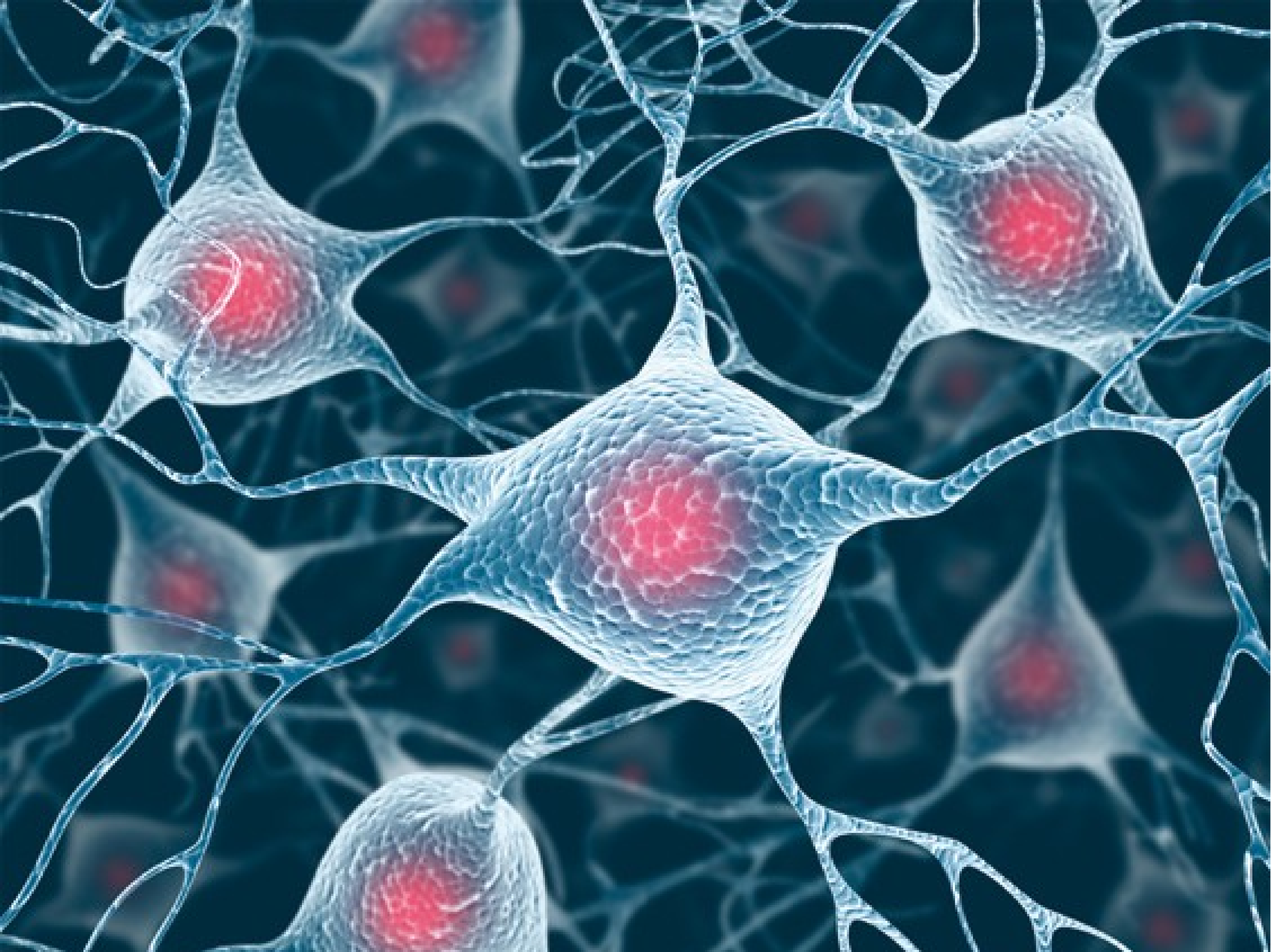

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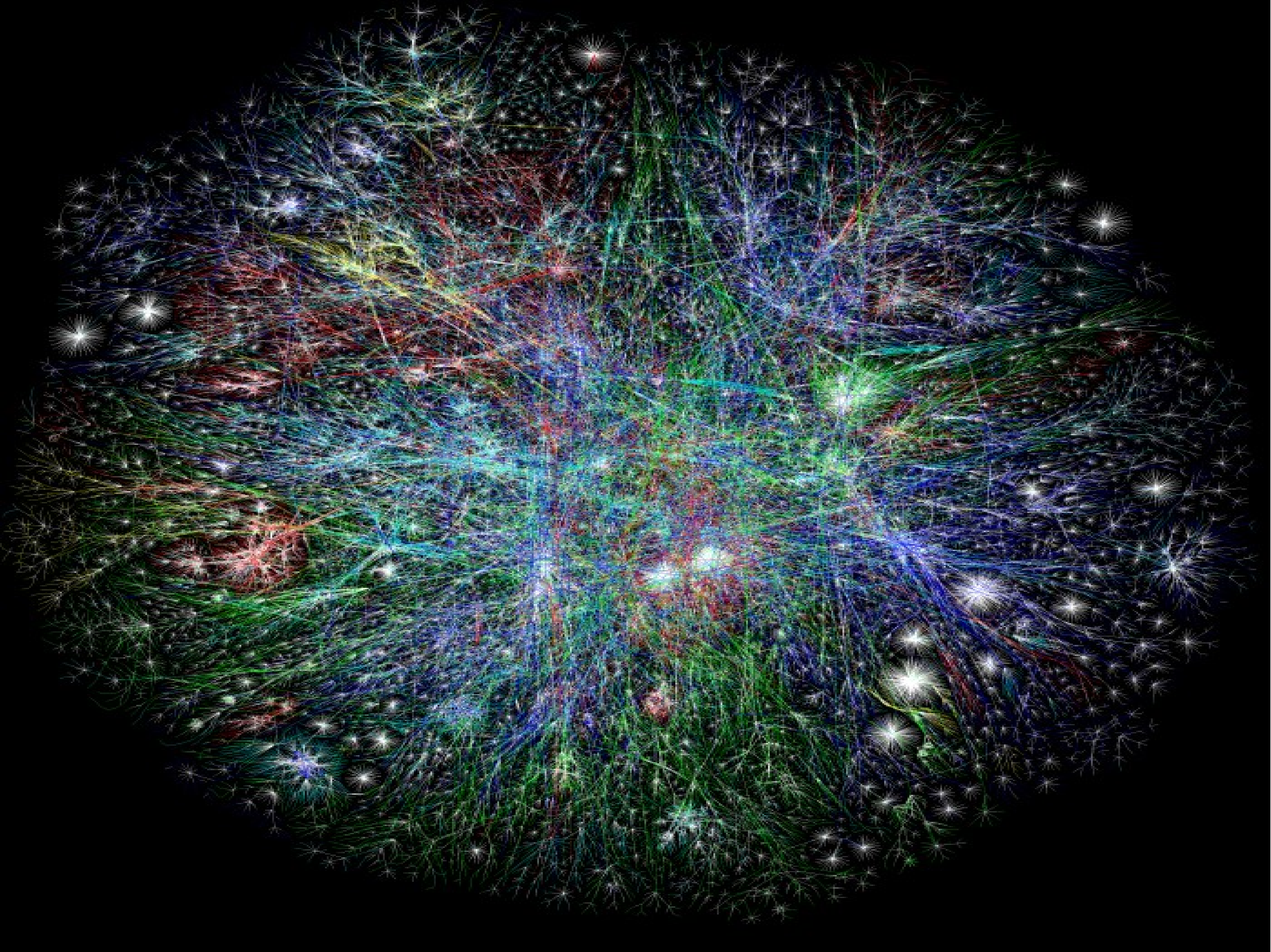
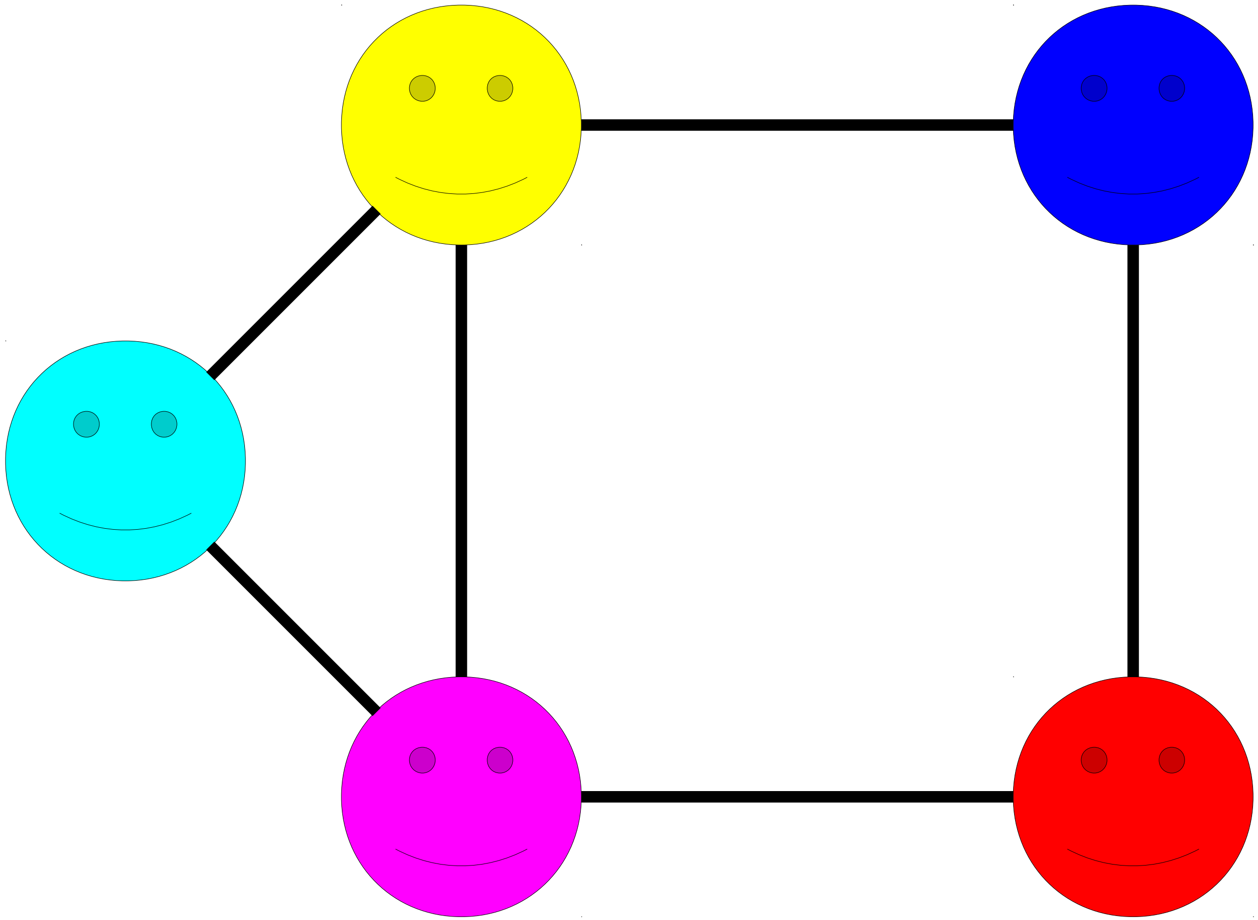
Me too!



What's in Common

- Each of these structures consists of
  - Individual objects and
  - Links between those objects.
- Goal: find a general framework for describing these objects and their properties.

A **graph** is a mathematical structure for representing relationships.



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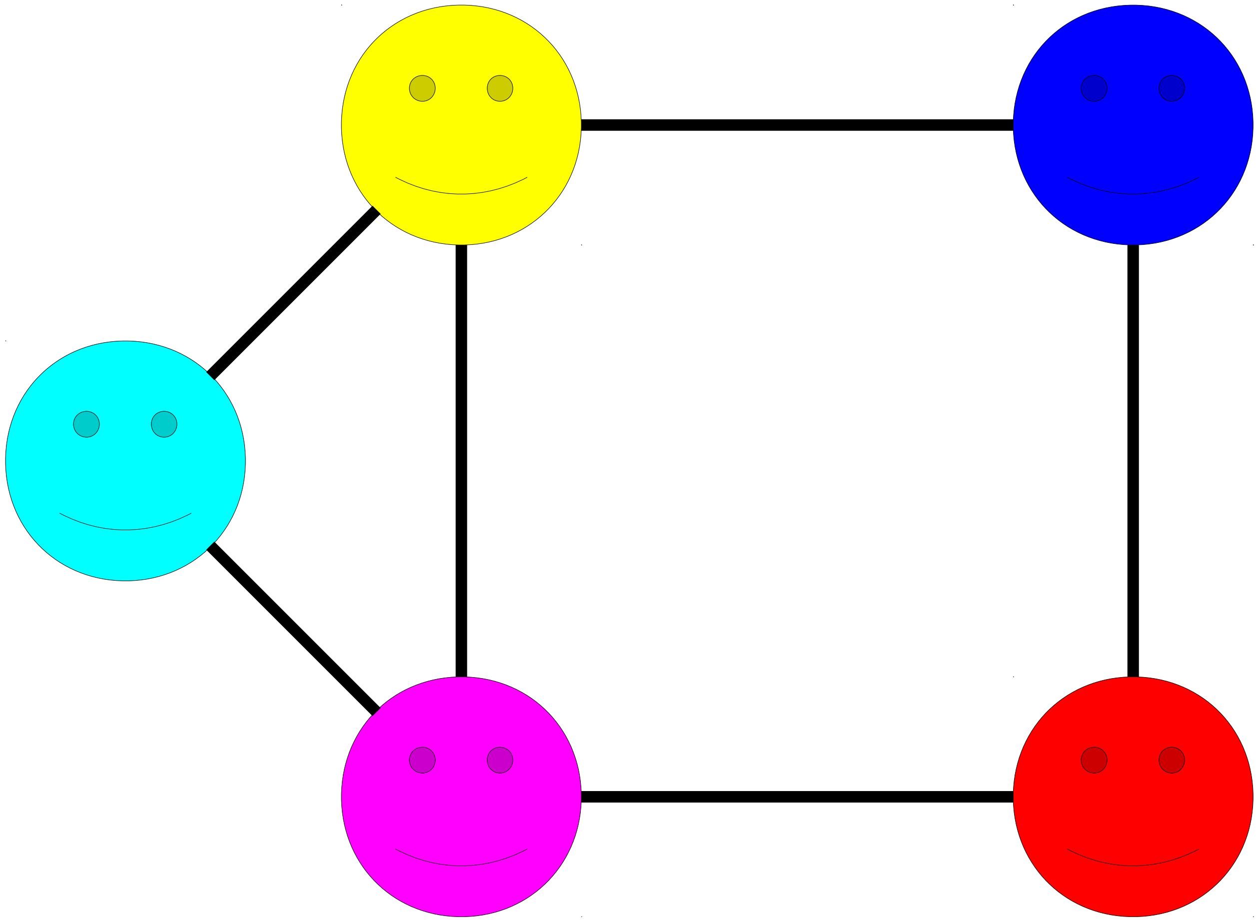

A graph consists of a set of **nodes** (or ***vertices***) connected by **edges** (or ***arcs***)

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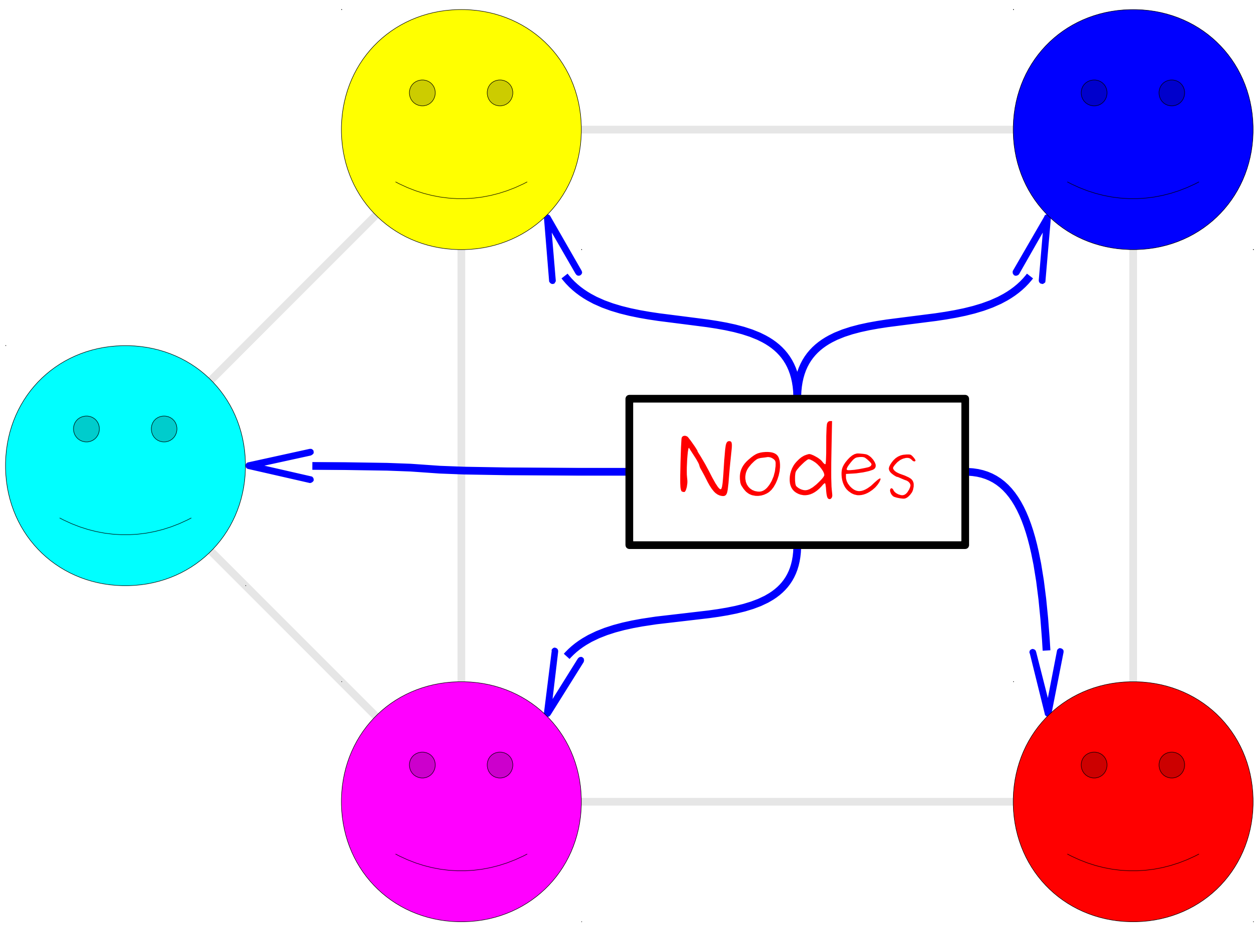

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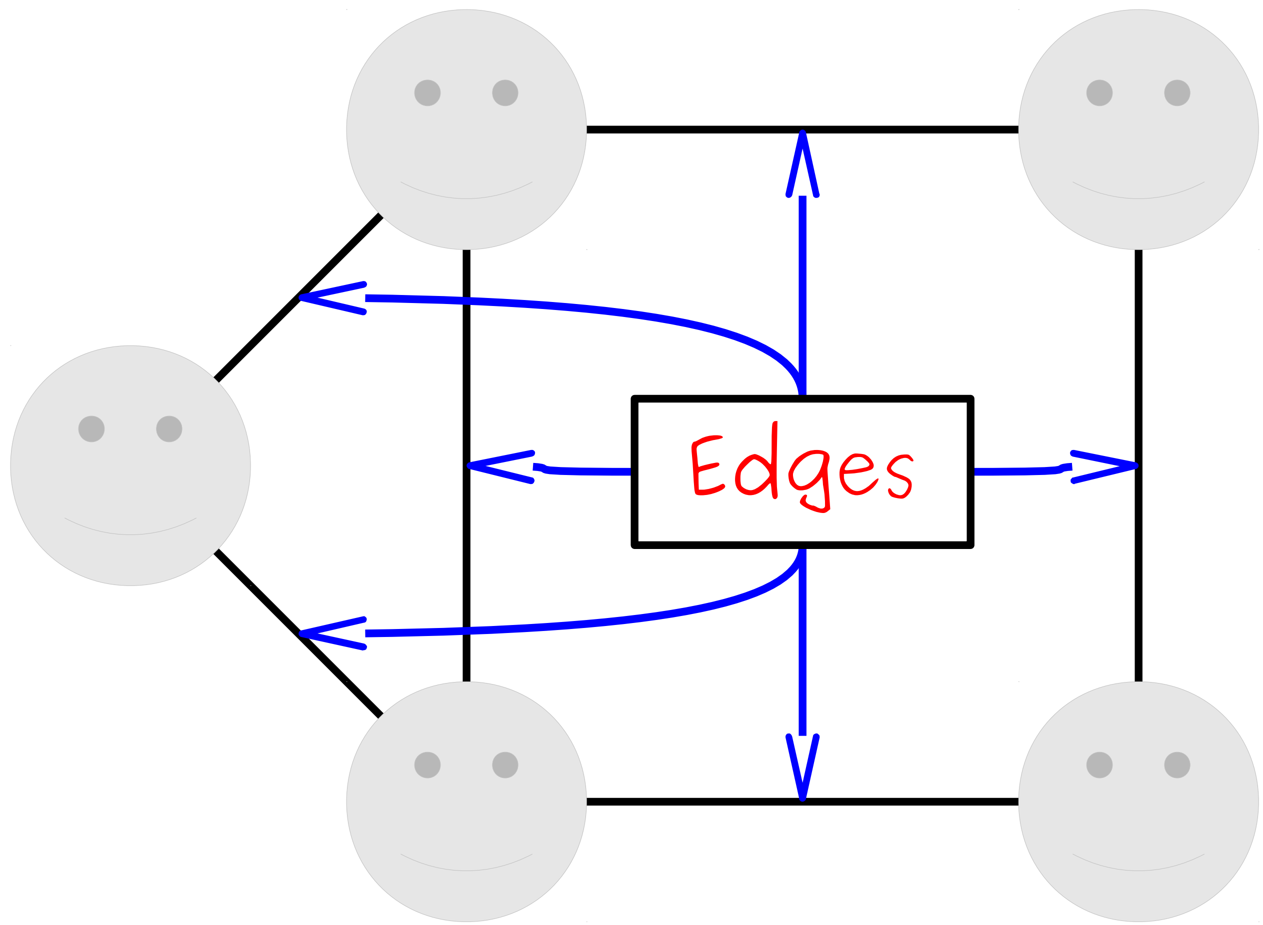

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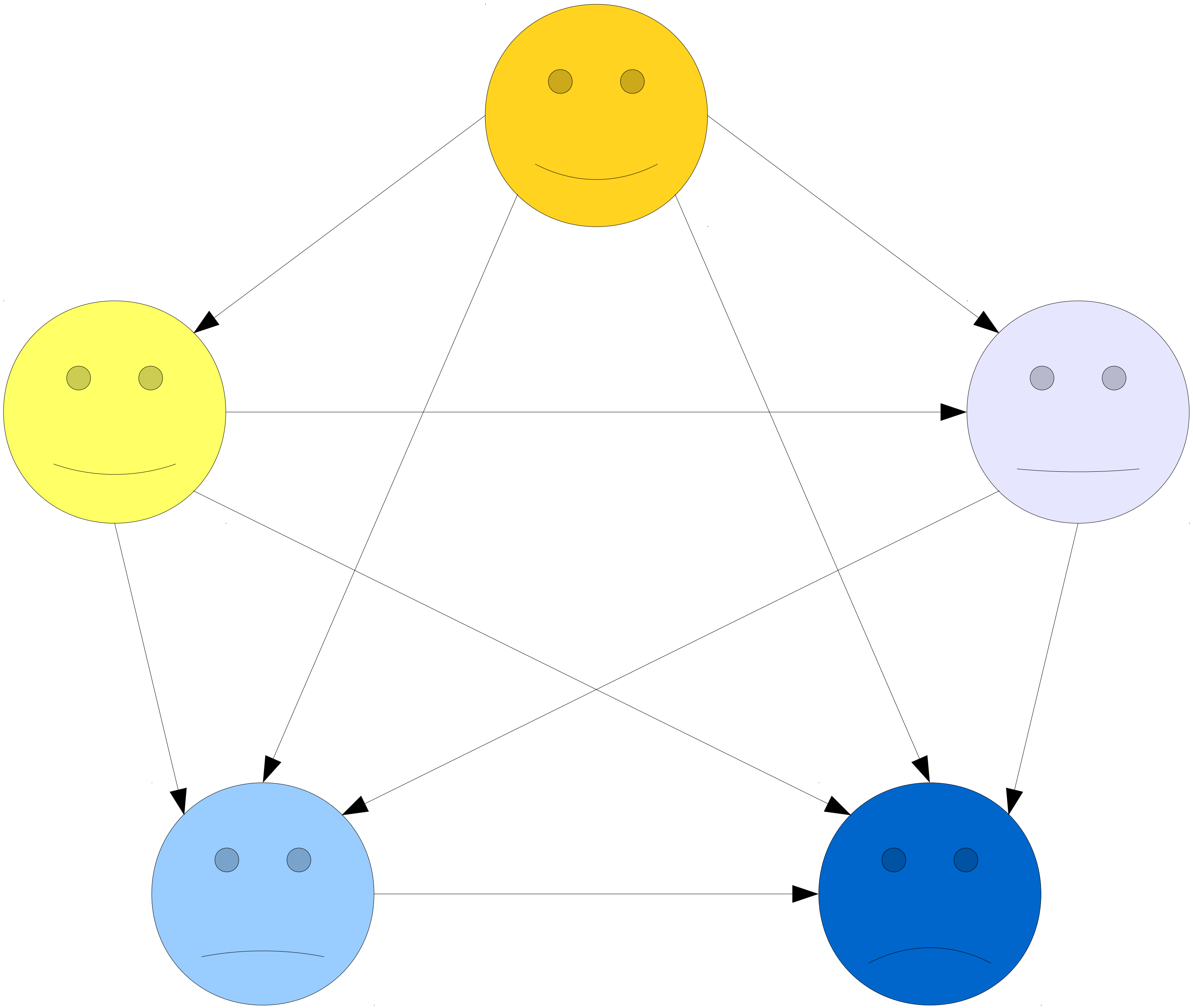
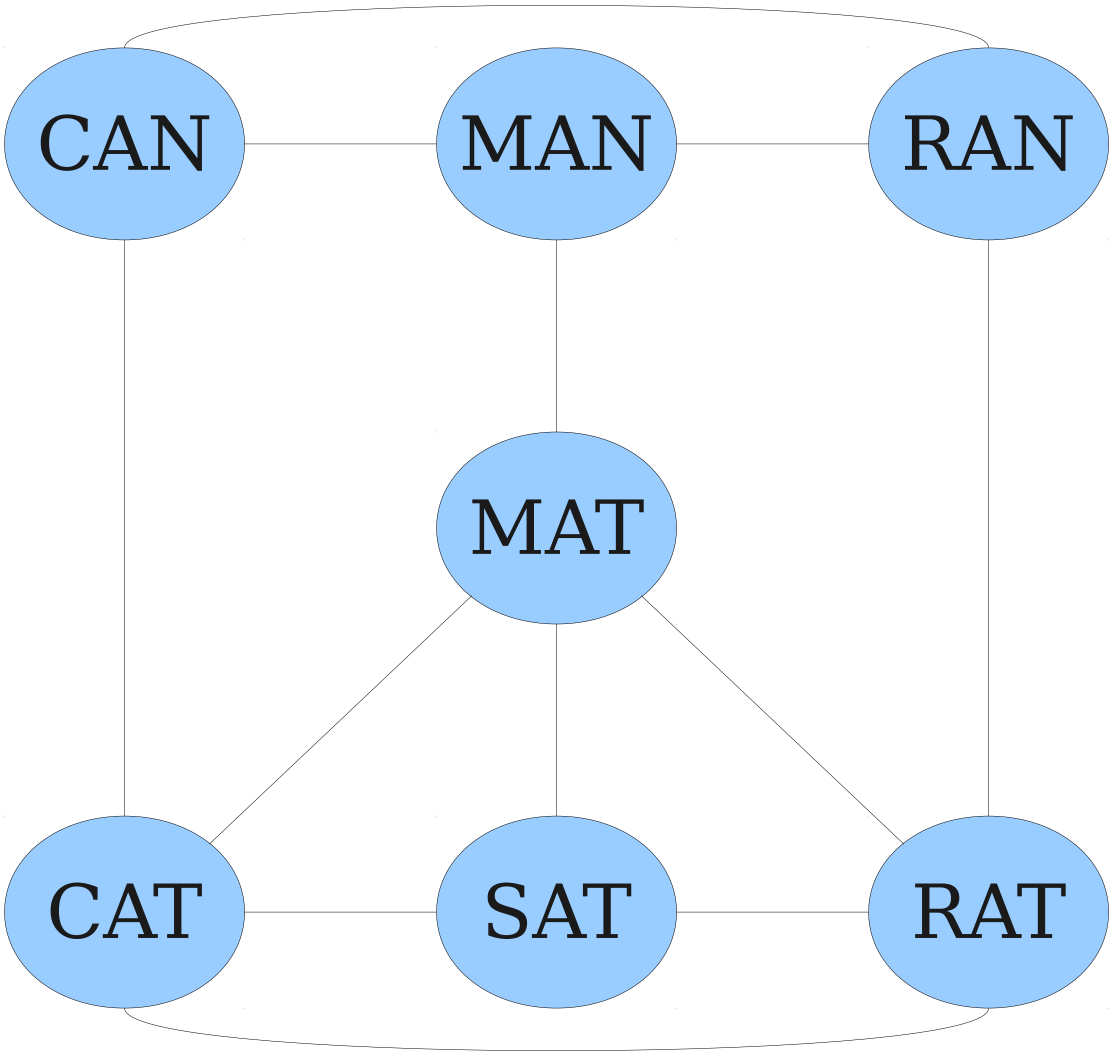
Some graphs are **directed**.



Some graphs are **undirected**.



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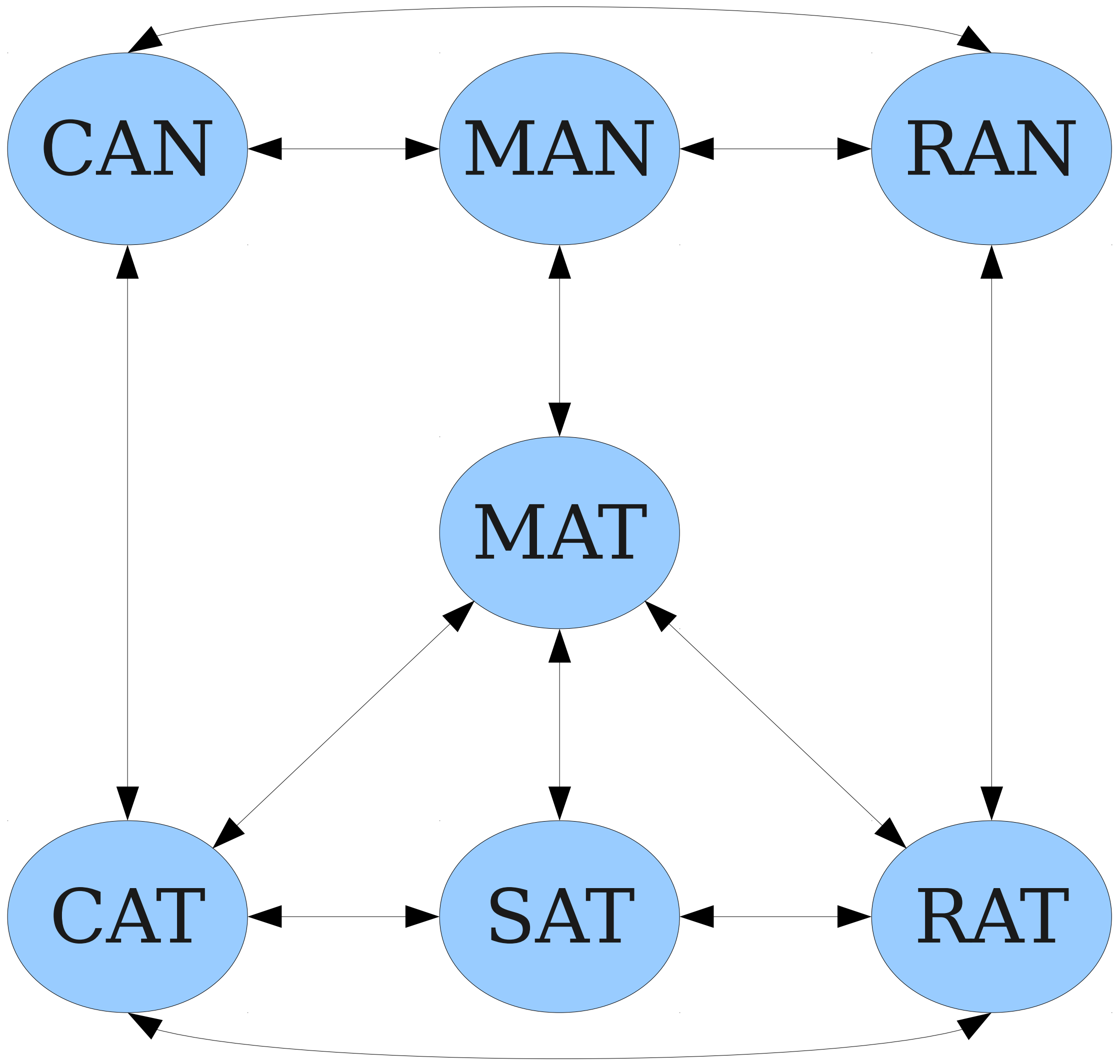


You can think of them as directed graphs with edges both ways.

Formalizing Graphs

- How might we define a graph mathematically?
- Need to specify
  - What the nodes in the graph are, and
  - What the edges are in the graph.
- The nodes can be pretty much anything.
- What about the edges?

Ordered and Unordered Pairs

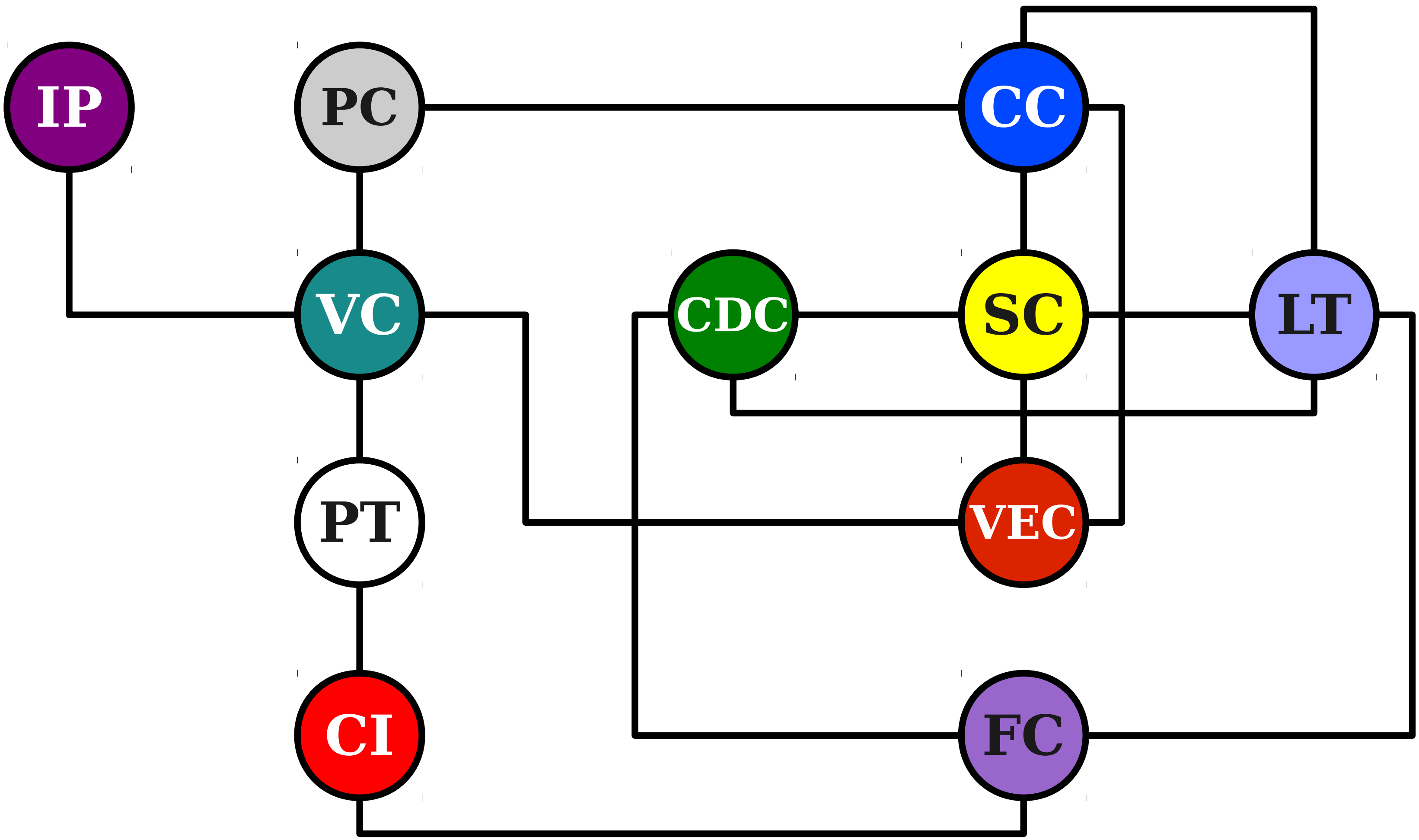
- An **unordered pair** is a set $\{a, b\}$ of two elements (remember that sets are unordered).
  - $\{0, 1\} = \{1, 0\}$
- An **ordered pair** (a, b) is a pair of elements in a specific order.
  - $(0, 1) \neq (1, 0)$.
  - Two ordered pairs are equal iff each of their components are equal.

Formalizing Graphs

- Formally, a **graph** is an ordered pair $G = (V, E)$, where
  - V is a set of nodes.
  - E is a set of edges.
- G is defined as an *ordered* pair so it's clear which set is the nodes and which is the edges.
- V can be any set whatsoever.
- E is one of two types of sets:
  - A set of *unordered* pairs of elements from V.
  - A set of *ordered* pairs of elements from V.

Undirected Connectivity

Navigating a Graph



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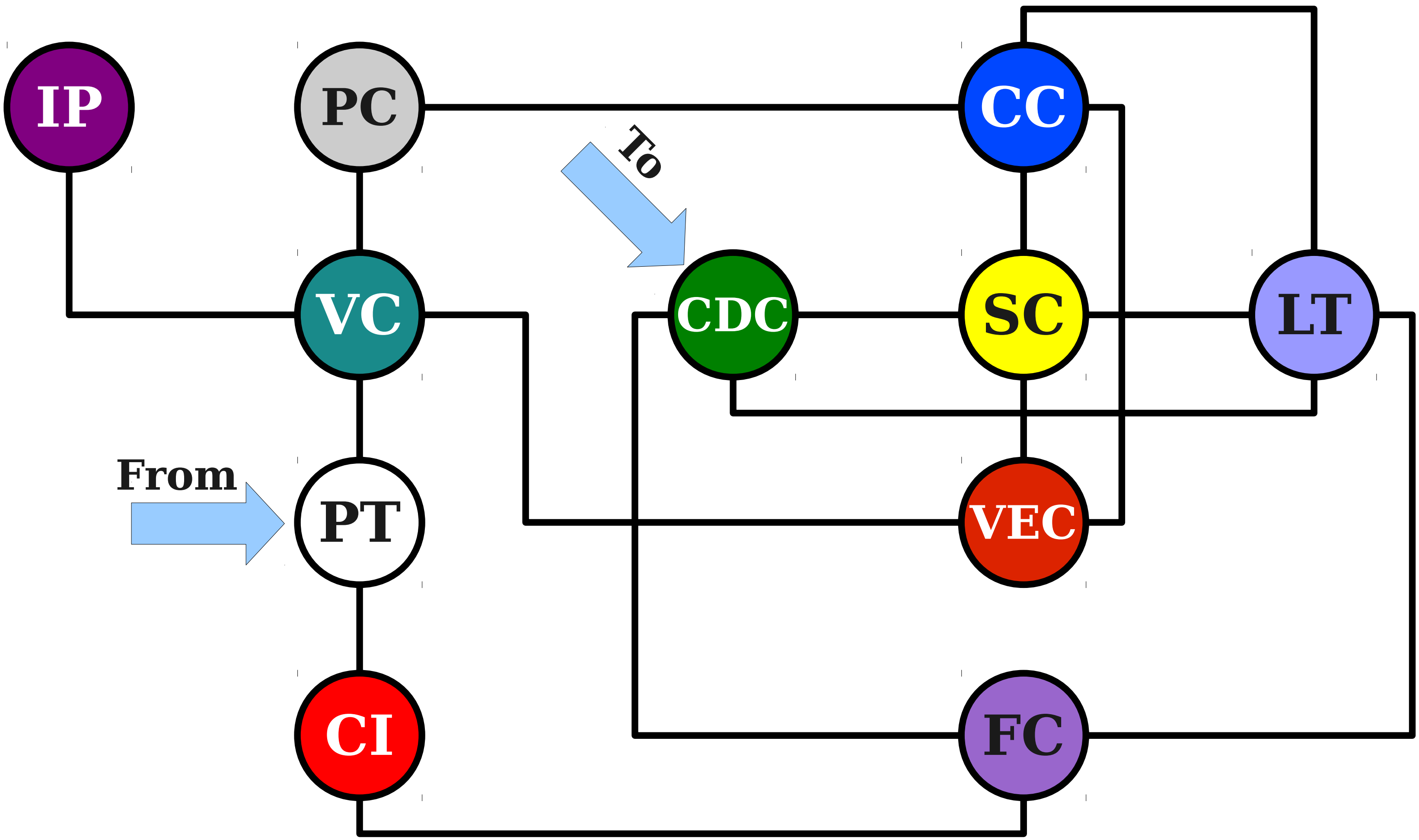

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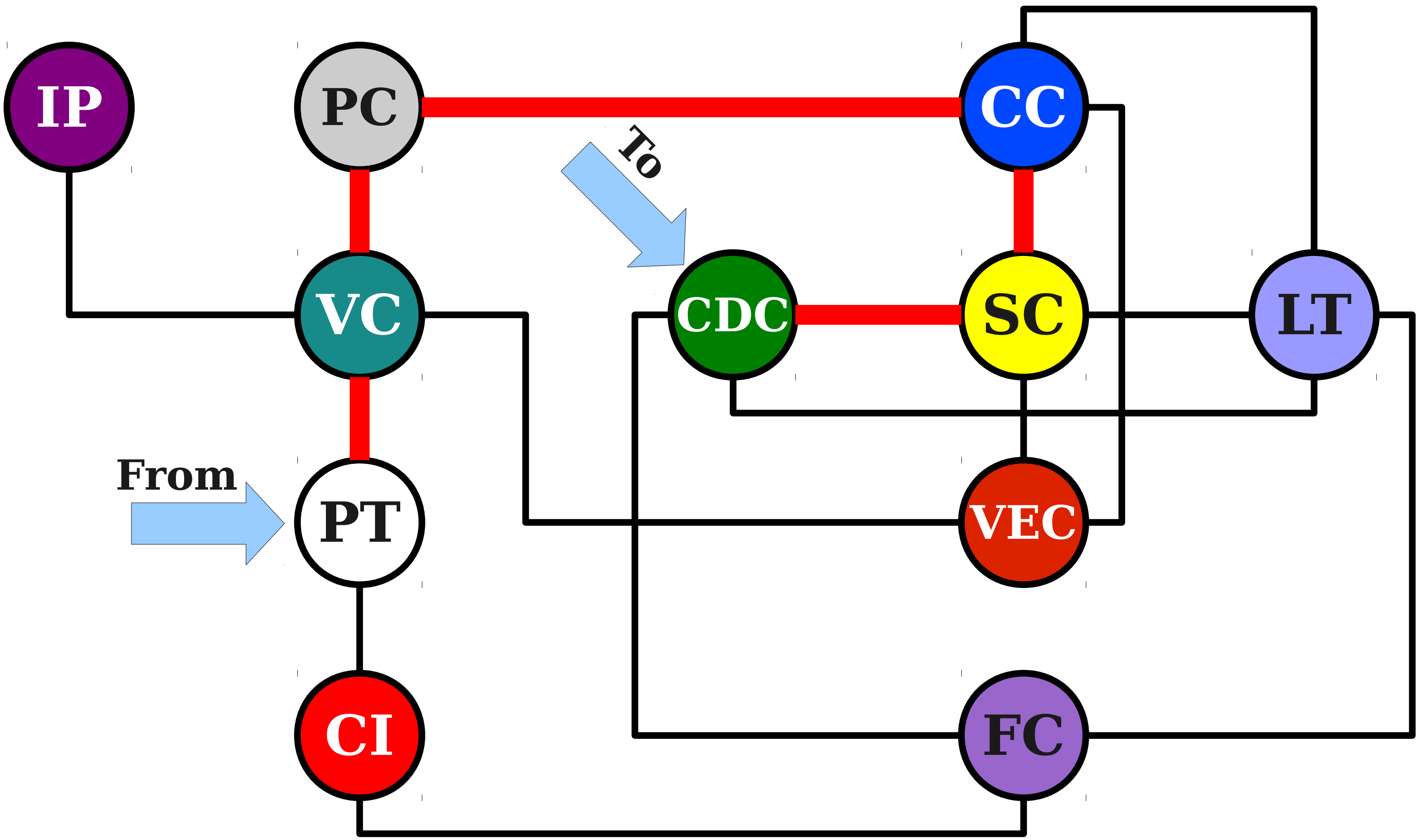

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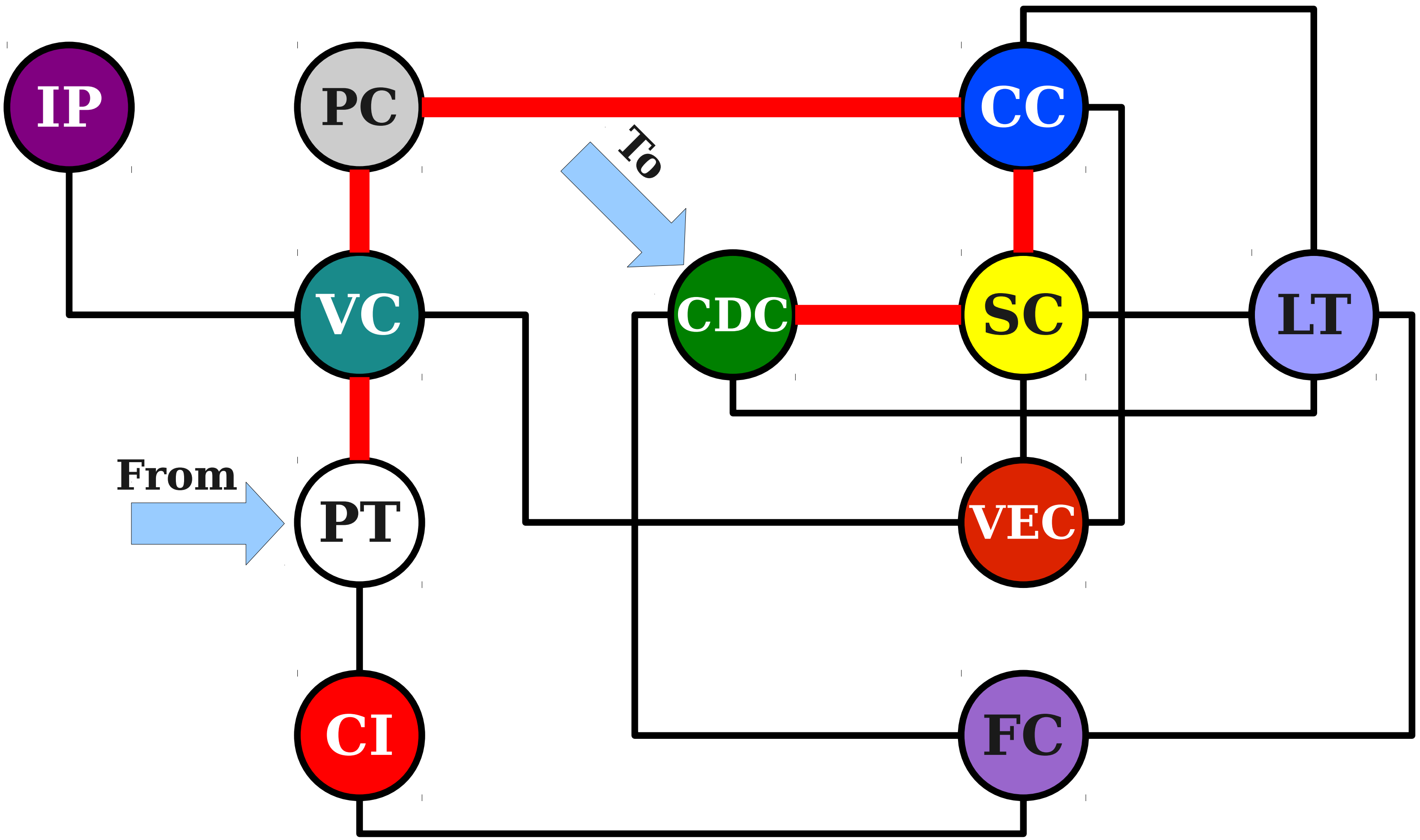
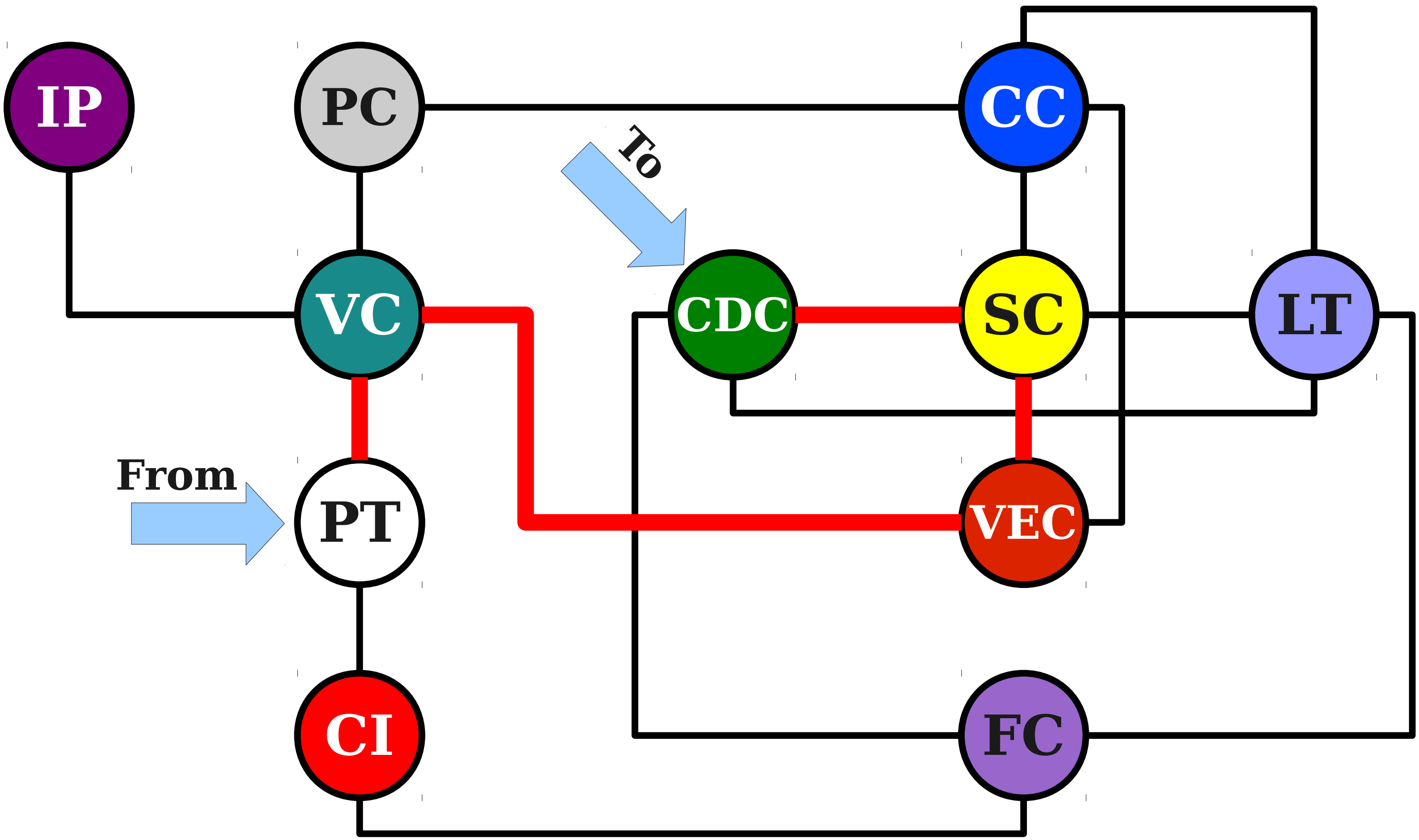


PT → VC → PC → CC → SC → CDC

Navigating a Graph



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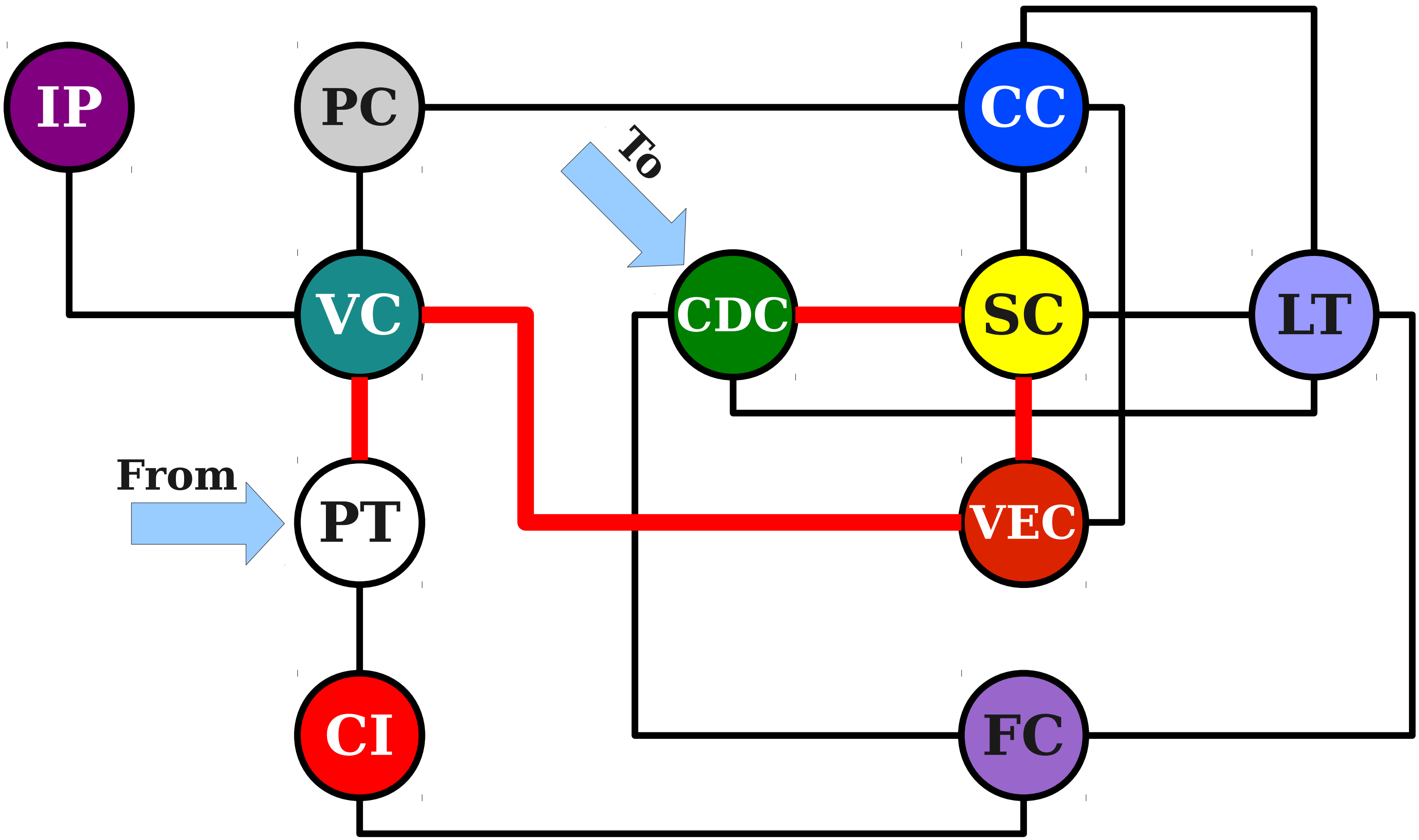

PT → VC → VEC → SC → CDC

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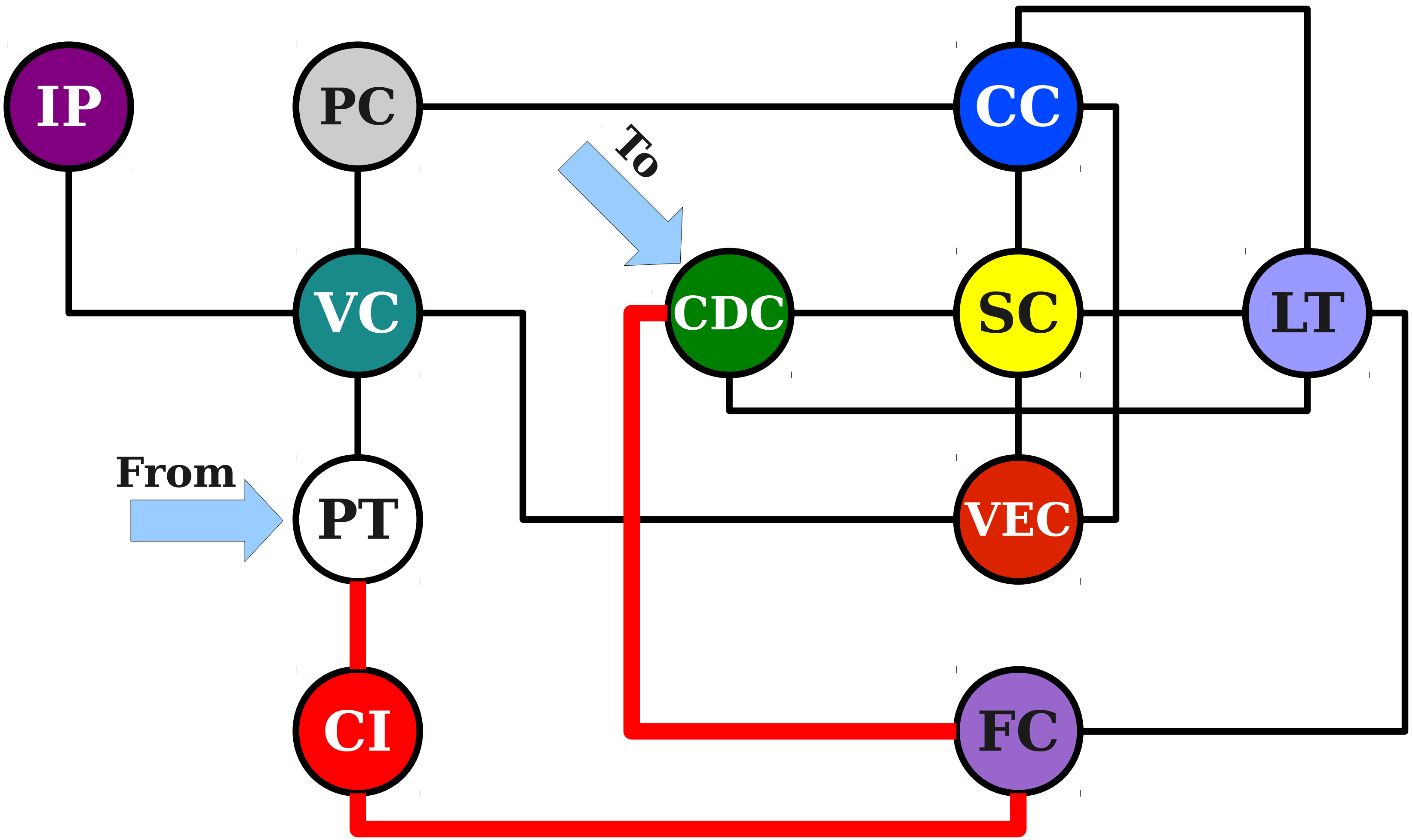

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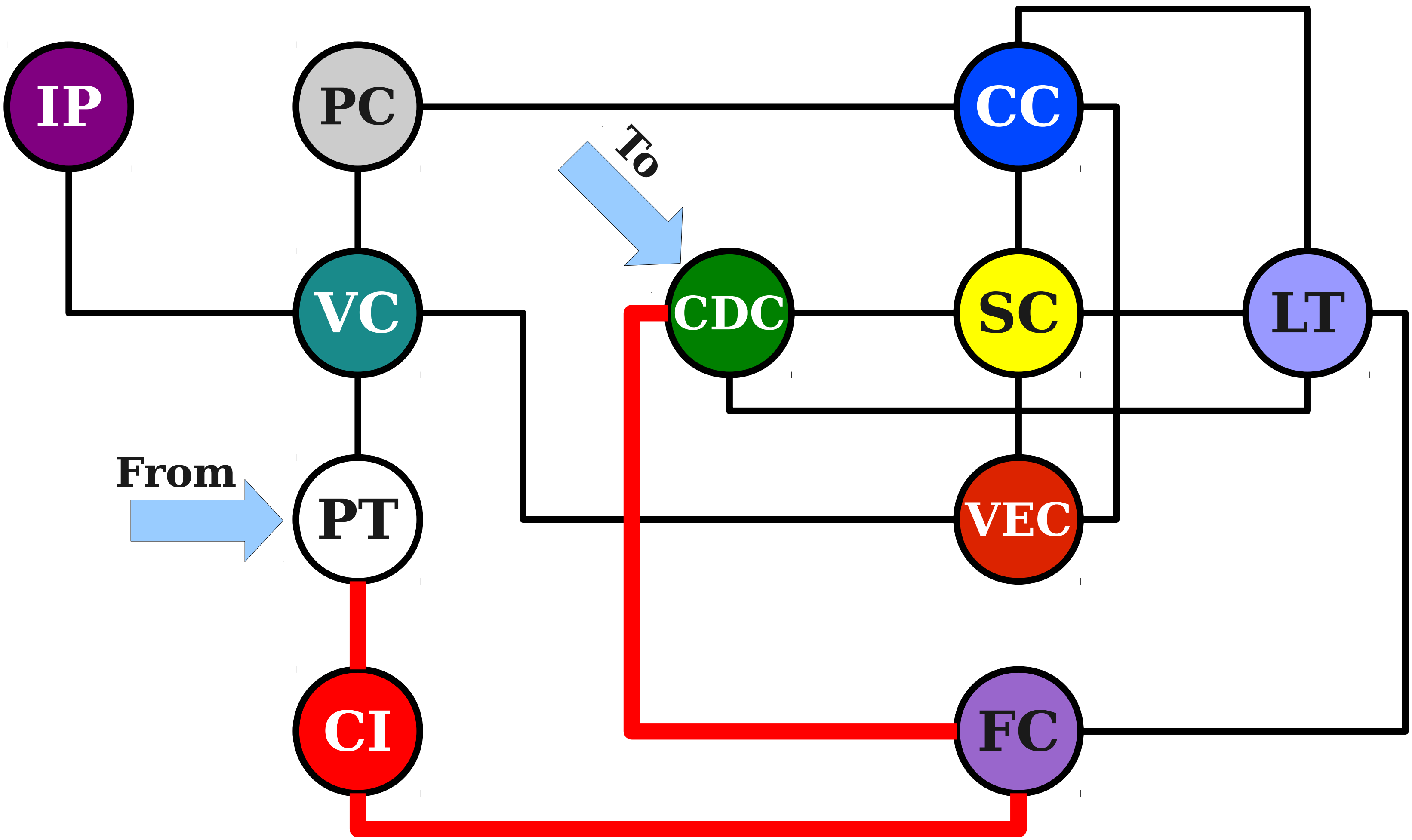

PT → CI → FC → CDC

A **path** from v_1 to v_n is a sequence of nodes $v_1, v_2, \ldots, v_n$ where $(v_k, v_{k+1}) \in E$ for all natural numbers in the range $1 \leq k \leq n - 1$.

The **length** of a path is the number of edges it contains, which is one less than the number of nodes in the path.

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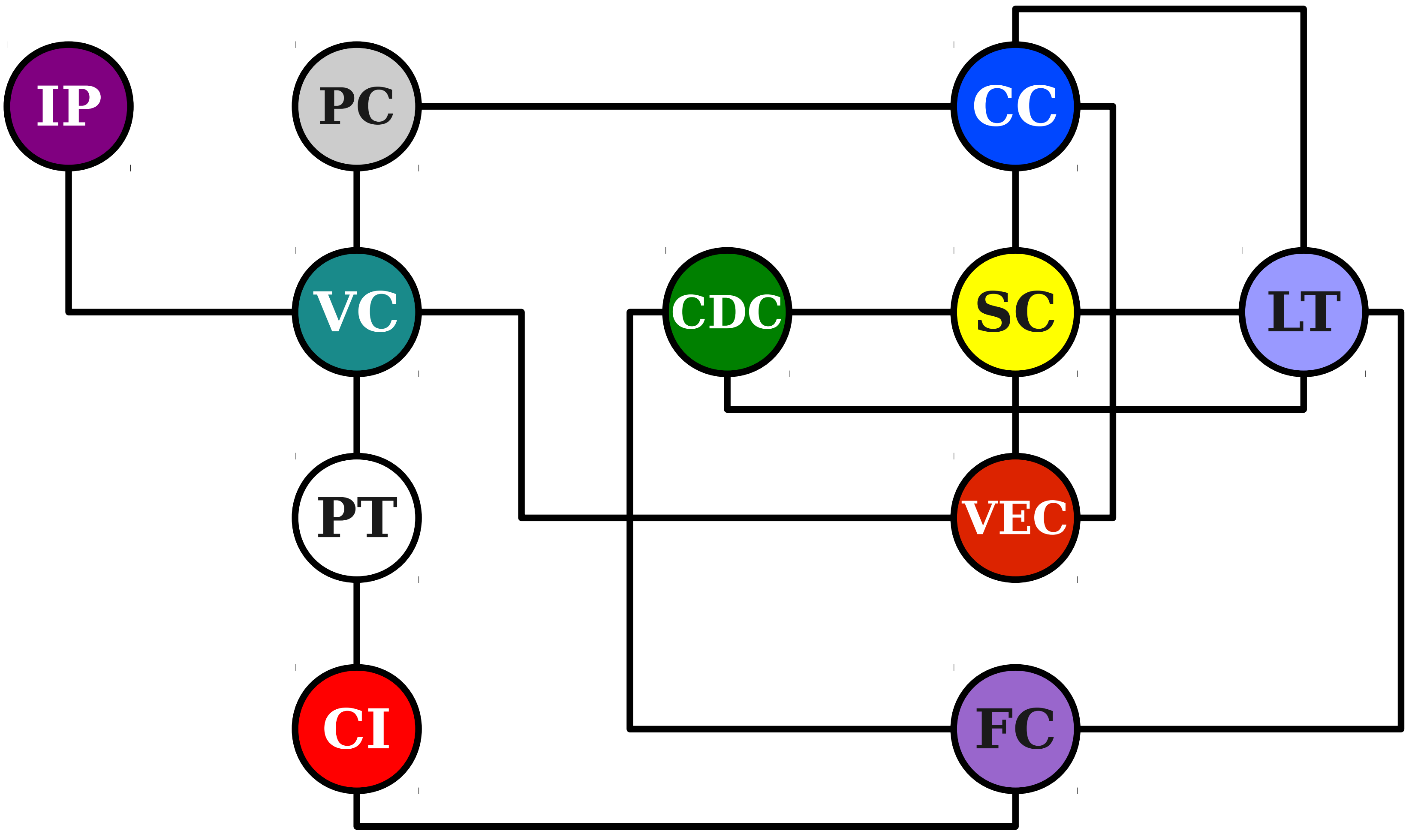

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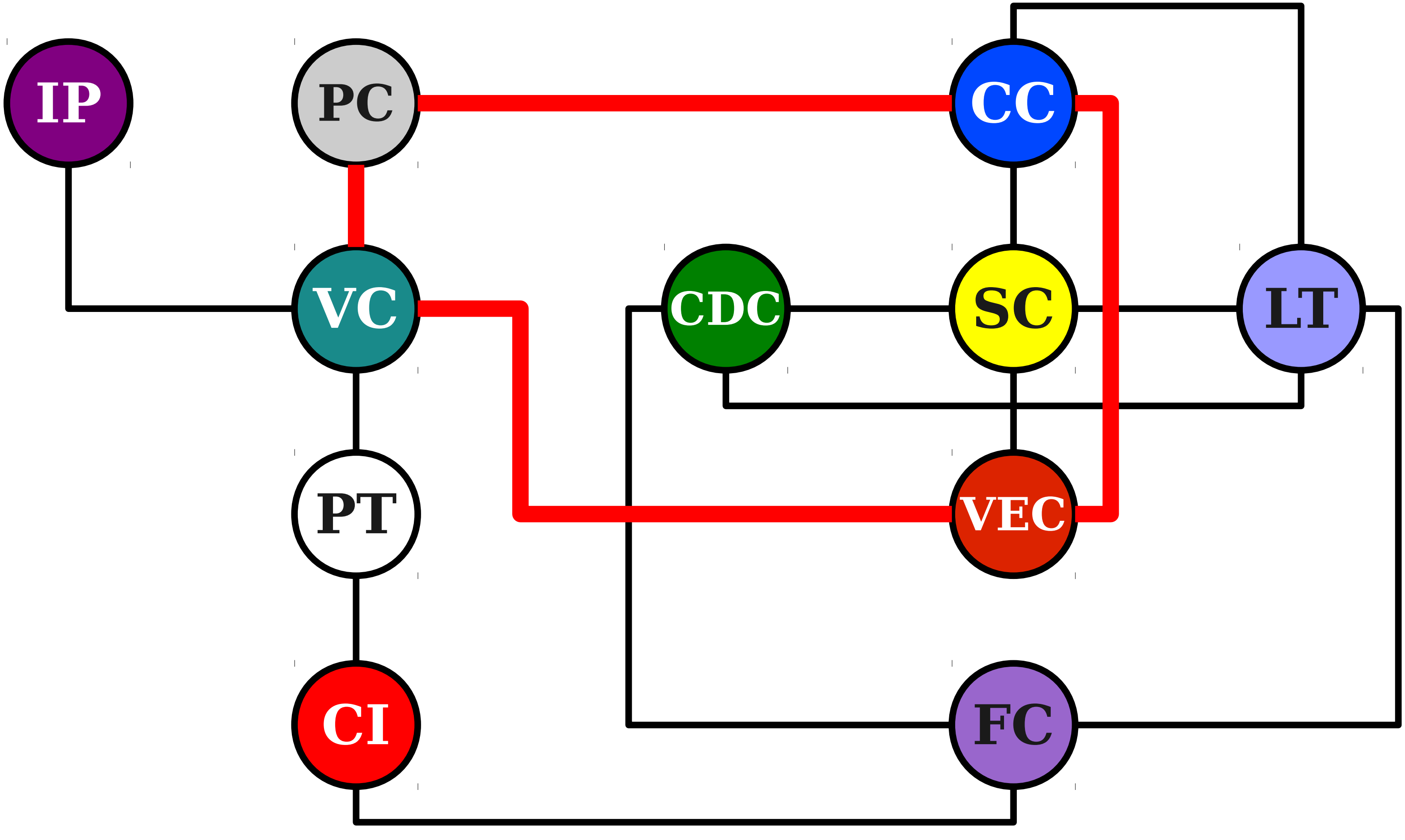

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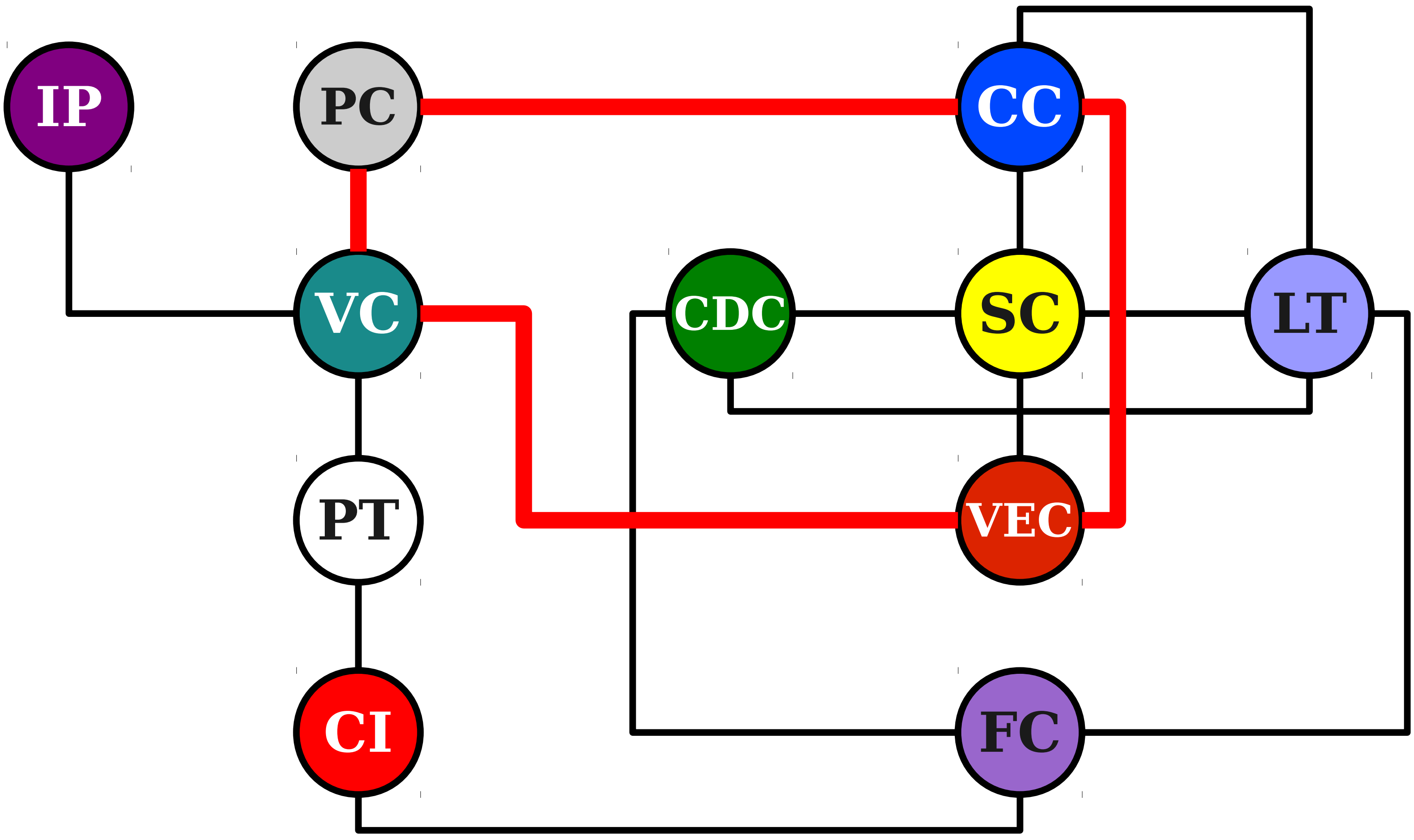
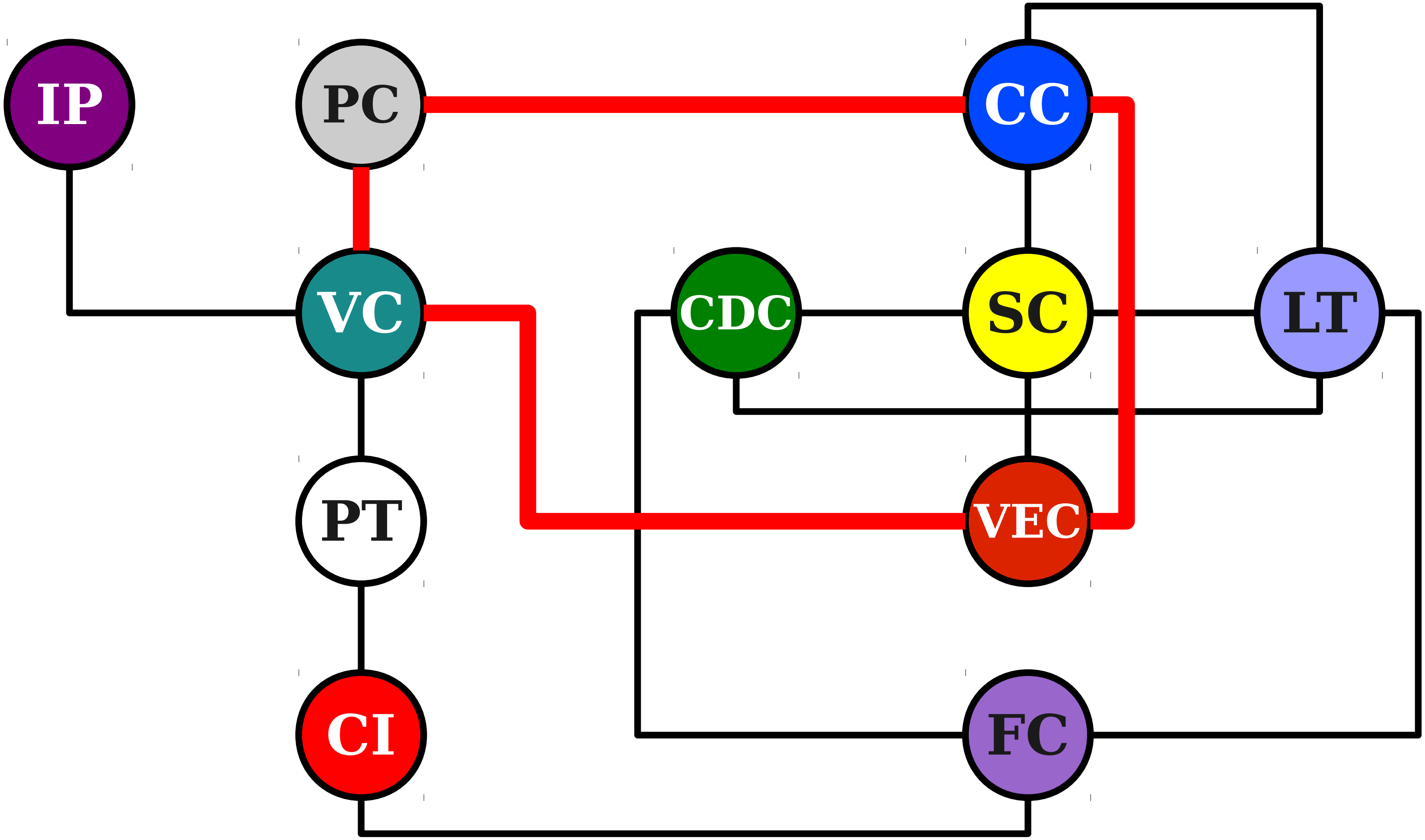


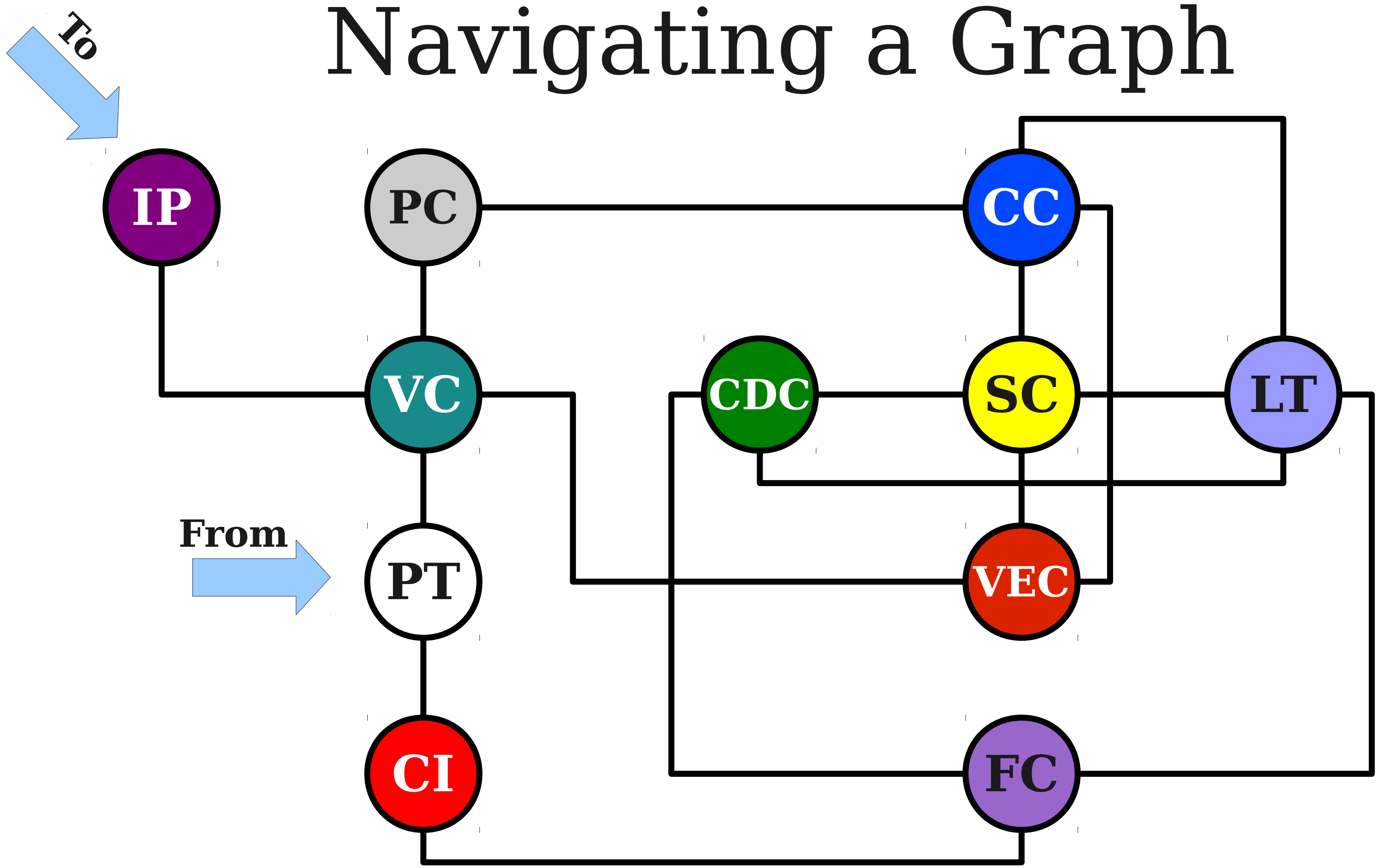
PC → CC → VEC → VC → PC

Navigating a Graph



PC → CC → VEC → VC → PC → CC → VEC → VC → PC

Navigating a Graph



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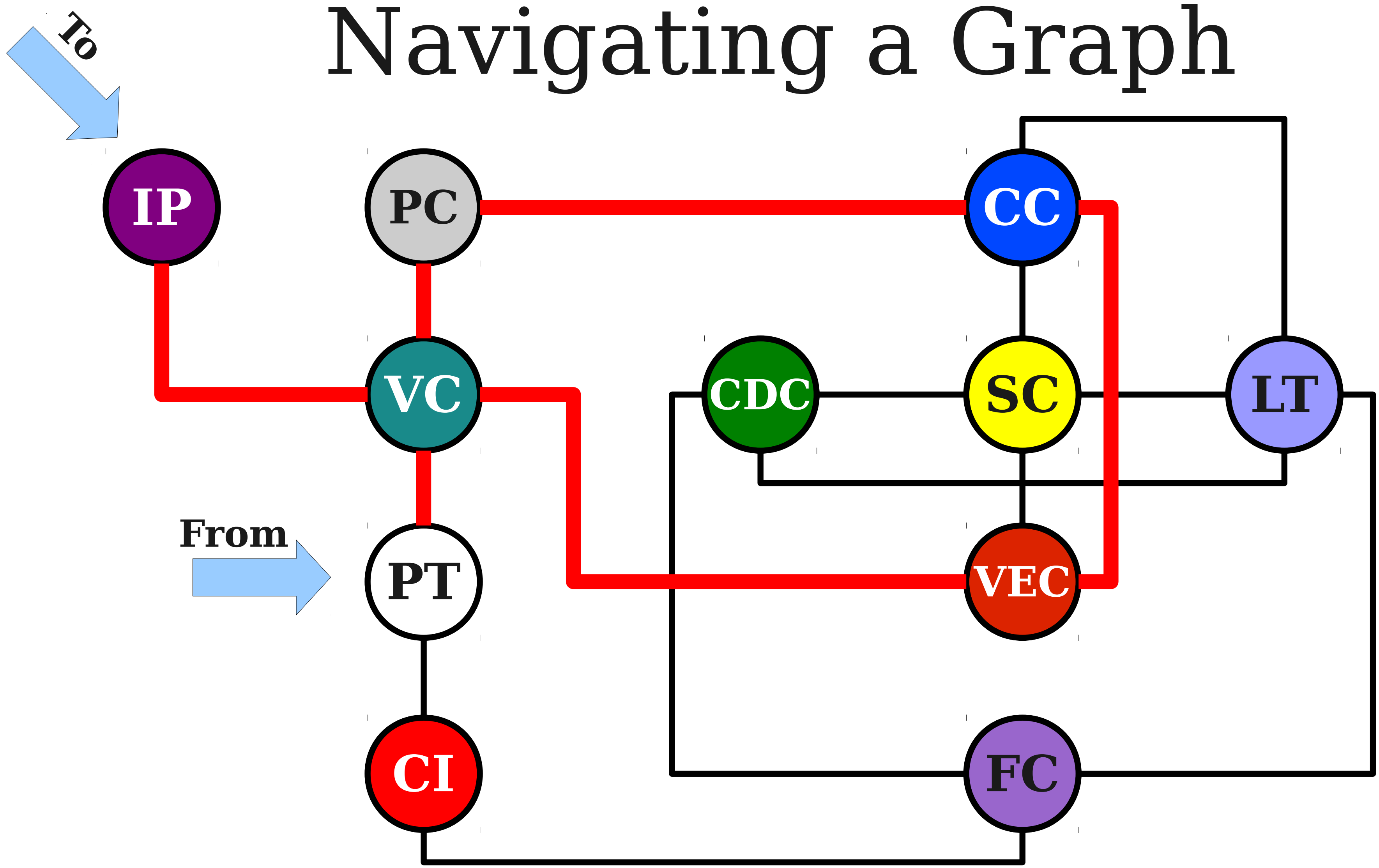

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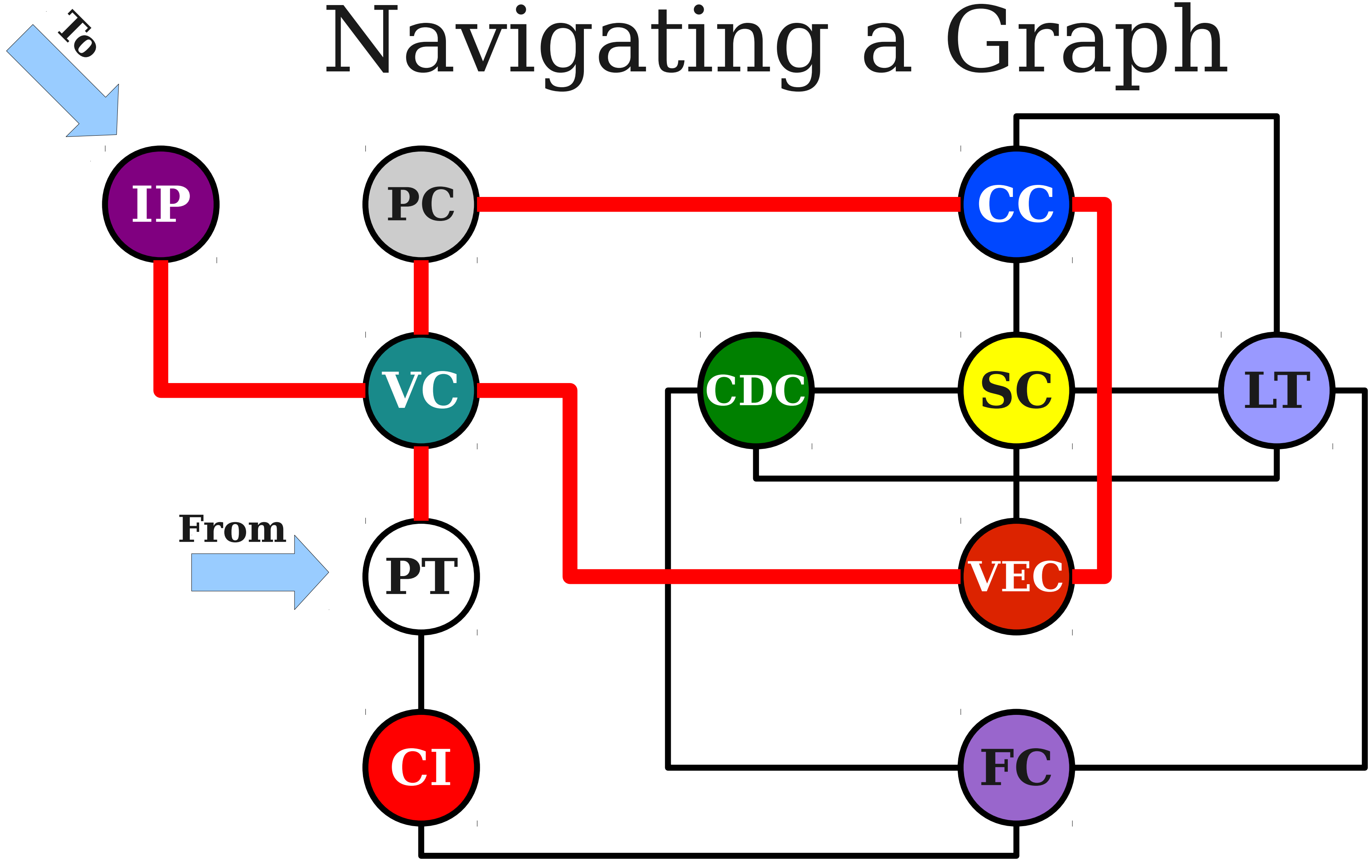


PT → VC → PC → CC → VEC → VC → IP

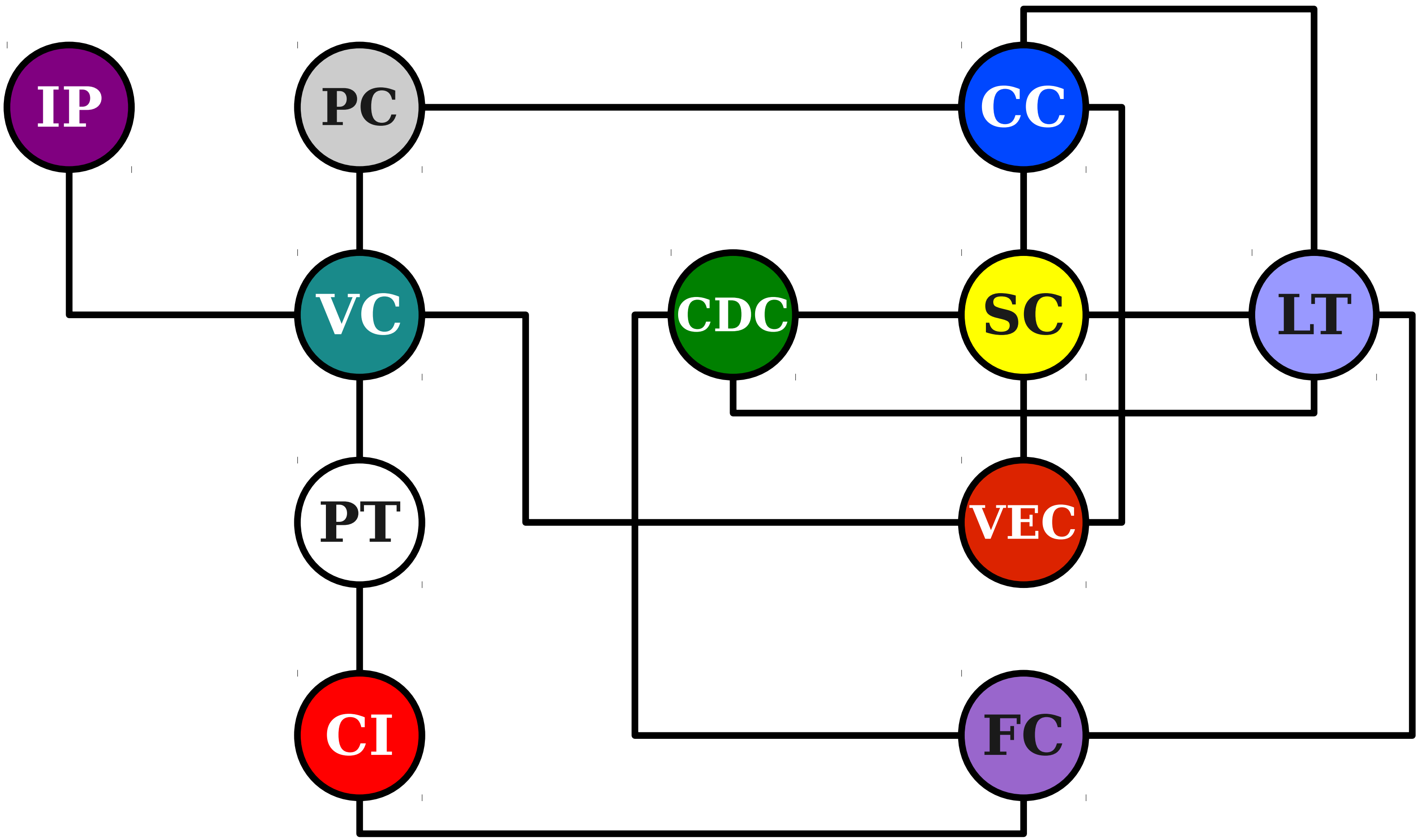
A **cycle** in a graph is a path from a node to itself.

The **length** of a cycle is the number of edges in that cycle.

A **simple path** in a graph is a path that does not revisit any nodes or edges.

A **simple cycle** in a graph is a cycle that does not revisit any nodes or edges (except the start/end node).

Navigating a Graph



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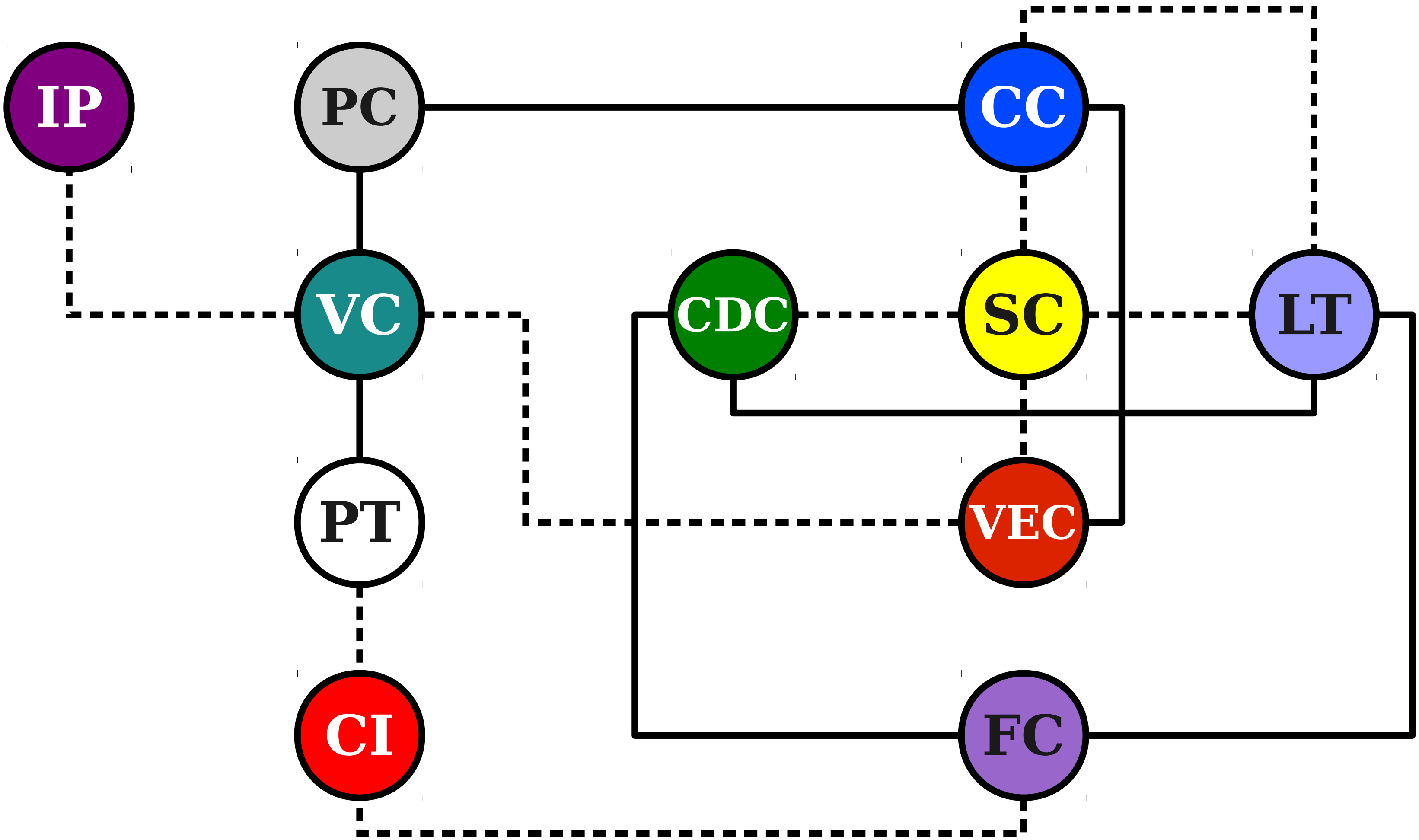

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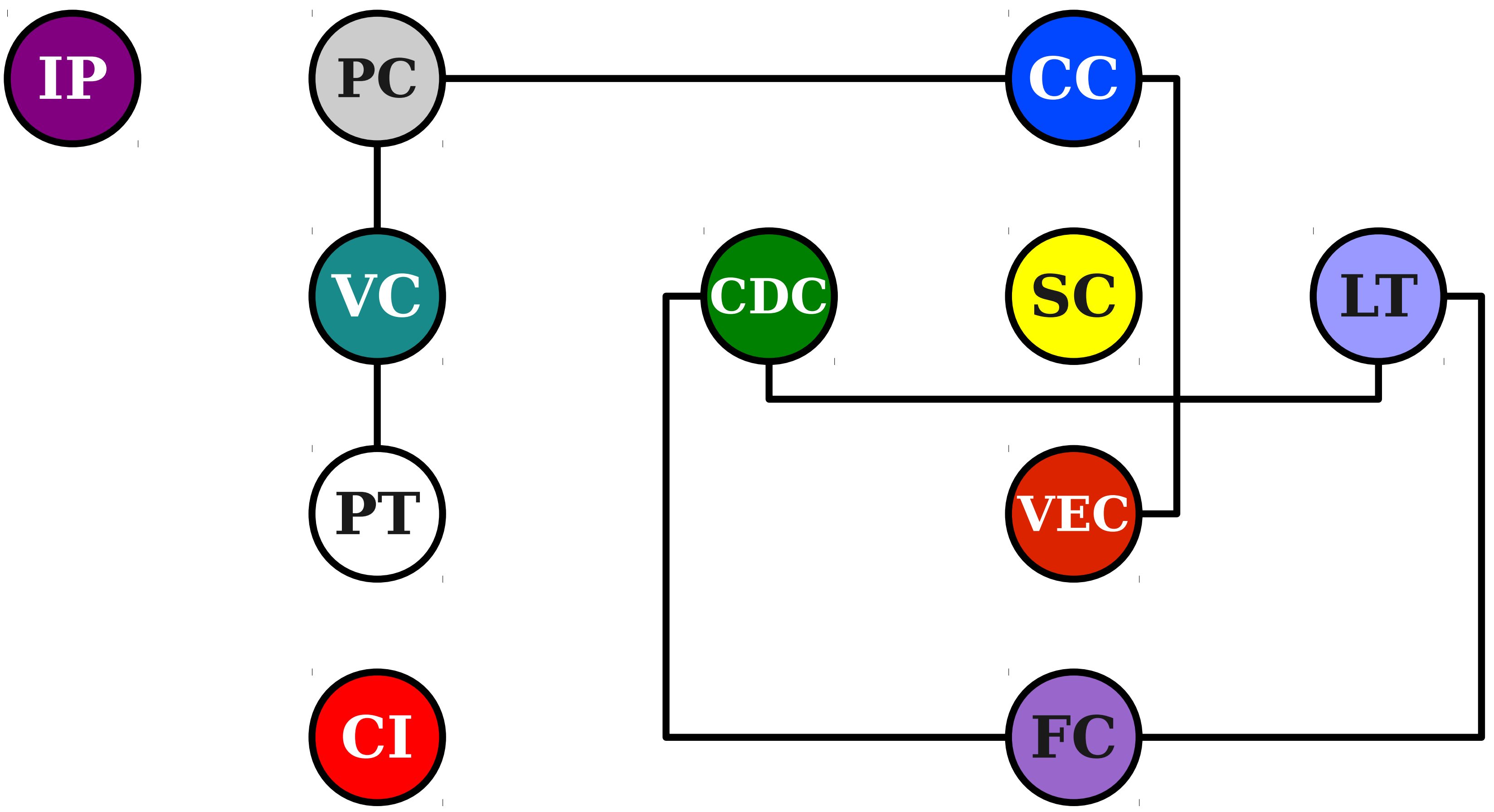

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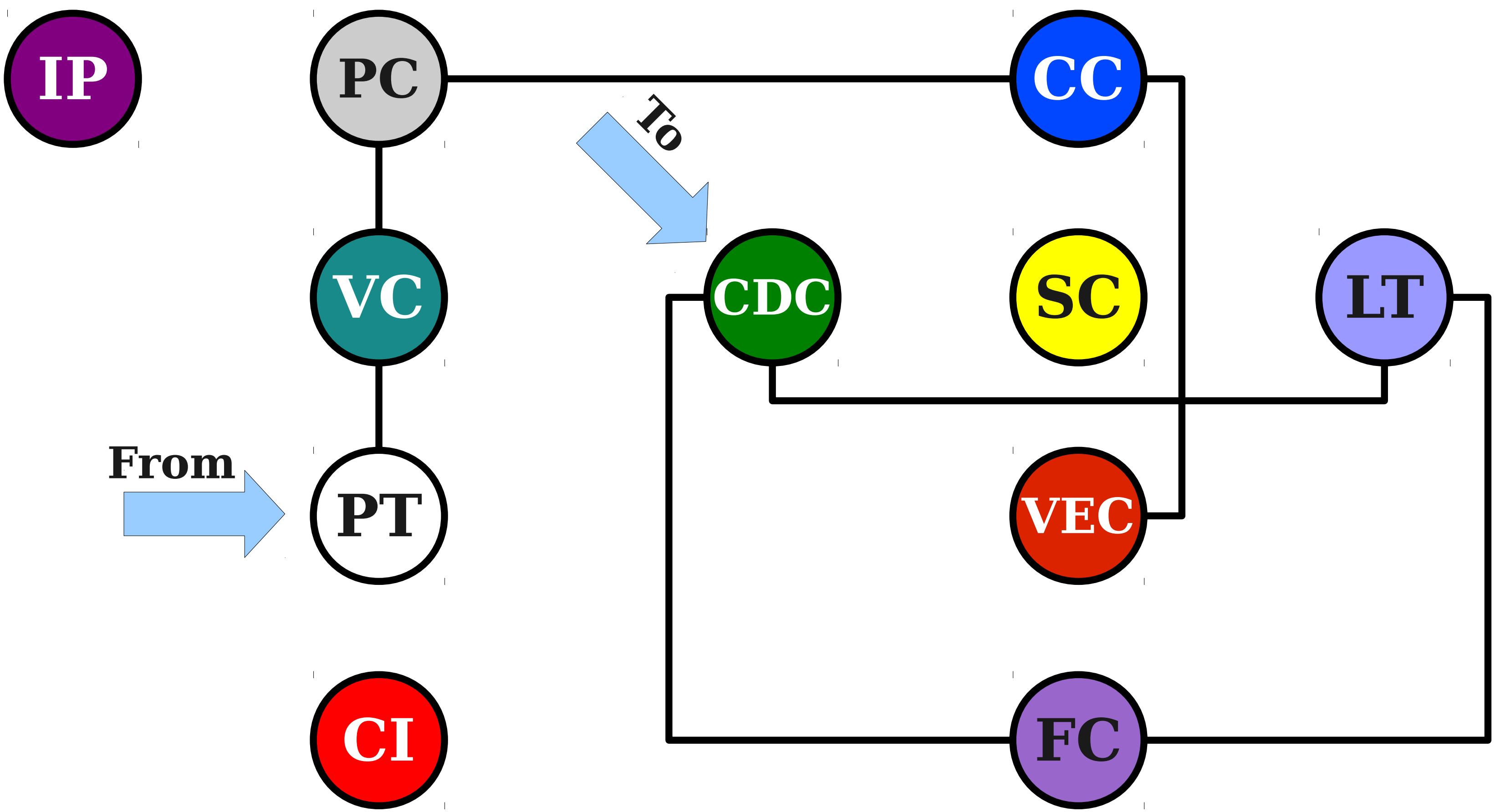

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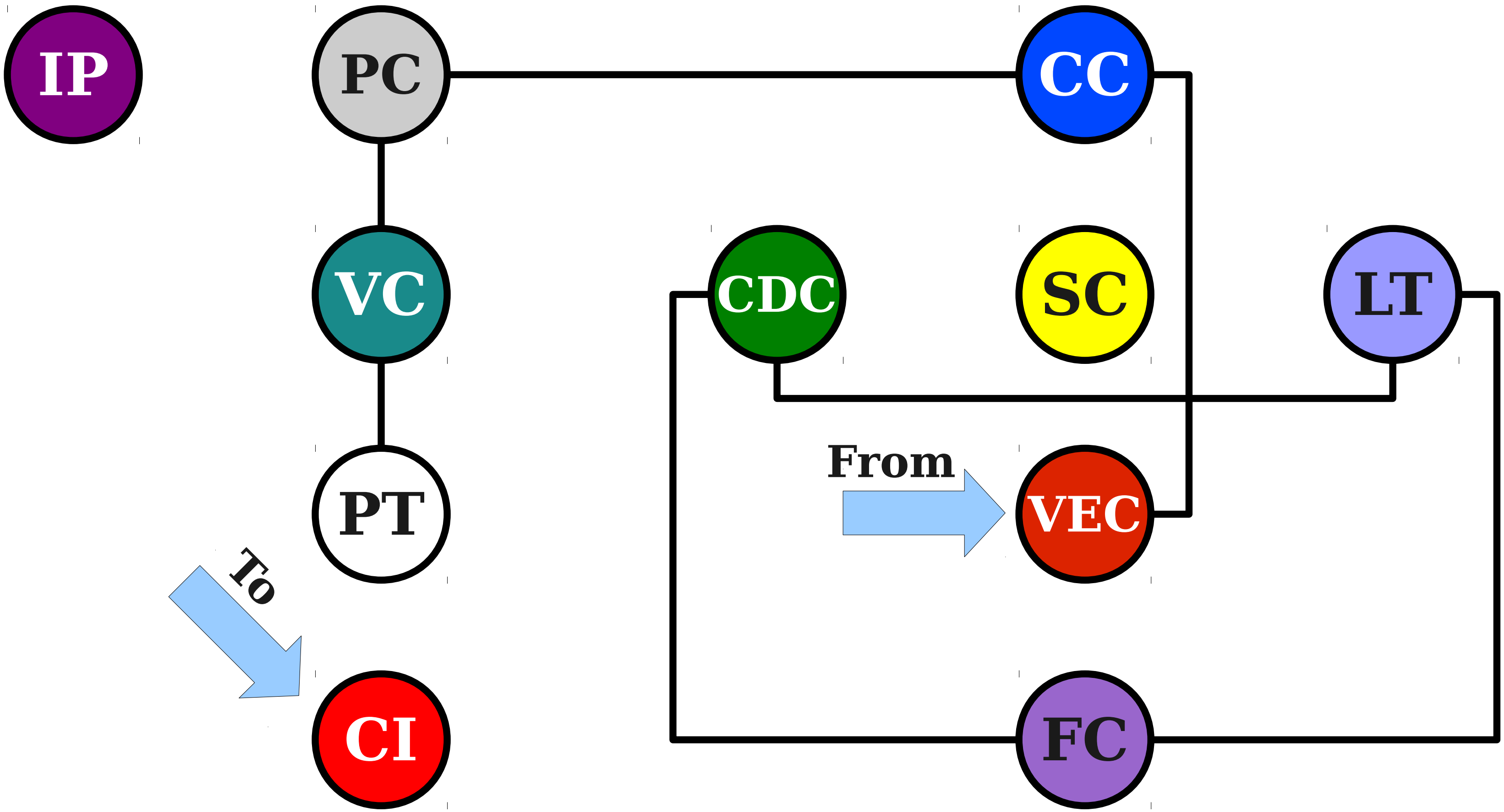

In an undirected graph, two nodes u and v are called **connected** iff there is a path from u to v.

We denote this as $\boldsymbol{u \leftrightarrow v}$.

If u is not connected to v, we write $\boldsymbol{u \nleftrightarrow v}$.

Next Time

- **The Rest of The Lecture**
  - *Sorry about the fire alarm!*
  - Connected components.
  - Planar graphs.
- **Binary Relations**
  - Equivalence relations.
  - Partial orders (ITA).