AA278A Homework 3: Control of Hybrid Systems.

Assigned May 20; Due June 7

We consider the motivational example given in Lecture Notes 7.

Problem 1: Two-aircraft collision avoidance, no mode switching.

Consider the case in which the aircraft follow straight paths only (mode 1 of the motivational example), and collision avoidance is achieved using linear velocity control only. Thus, the continuous inputs are the airspeeds of the aircraft ($u = v_1, d = v_2$) and assume that the airspeeds are known to vary over specified ranges: $u \in U = [\underline{v}_1, \overline{v}_1] \subset \mathbb{R}^+$, $d \in D = [\underline{v}_2, \overline{v}_2] \subset \mathbb{R}^+$, and model reduces to

$$\begin{aligned} \dot{x}_r &= -u + d\cos\psi_r \\ \dot{y}_r &= d\sin\psi_r \\ \dot{\psi}_r &= 0 \end{aligned} \tag{1}$$

Design a MATLAB program which plots the subset of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone in T seconds. You can choose T to be anything you like; what happens as $T \to -\infty$?

For your code, use $[\underline{v}_1, \overline{v}_1] = [2, 4]$, $[\underline{v}_2, \overline{v}_2] = [1, 5]$, and consider four different values of ψ_r: $\pi/2$, 0, $-\pi/4$, and $-\pi/2$.

Problem 2: Two-aircraft collision avoidance, mode switching

Now consider the three mode example of Lectures Notes 7.

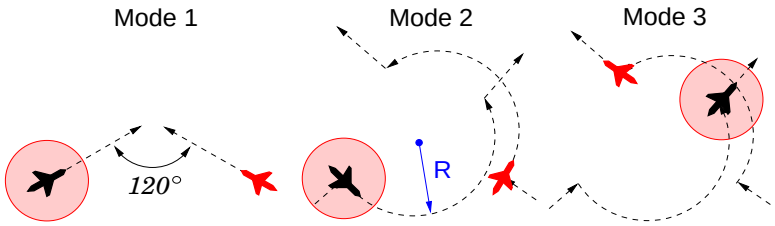

Figure 1: Two aircraft in three modes of operation: in modes 1 and 3 the aircraft follow a straight course and in mode 2 the aircraft follow a half circle. The initial relative heading (120°) is preserved throughout.

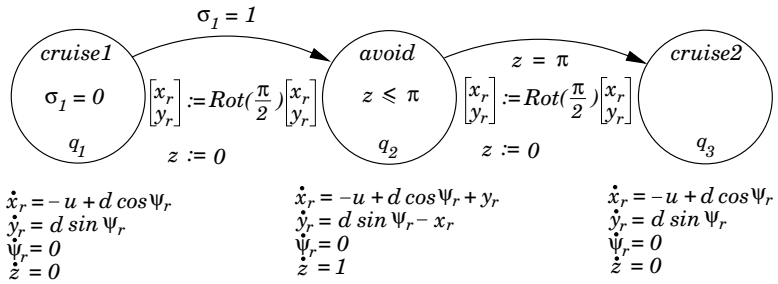

Figure 2: In q_1 both aircraft follow a straight course, in q_2 a half circle, and in q_3 both aircraft return to a straight course.

Assume that, in the straight modes, $\omega_1 = \omega_2 = 0$, and in the circular arc mode, $\omega_1 = \omega_2 = 1$; and assume that, in all modes, $v_1 = v_2 = 5$. Assume that in all modes, $\psi_r = 2\pi/3$.

Show that by increasing the radius of the circular arc in the "avoid" mode, the set of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone decreases in size. You can use the code that you wrote for Problem 1 and the "overlapping set" argument presented in class, and answer this question using a set of illustrations.