Global Optimization

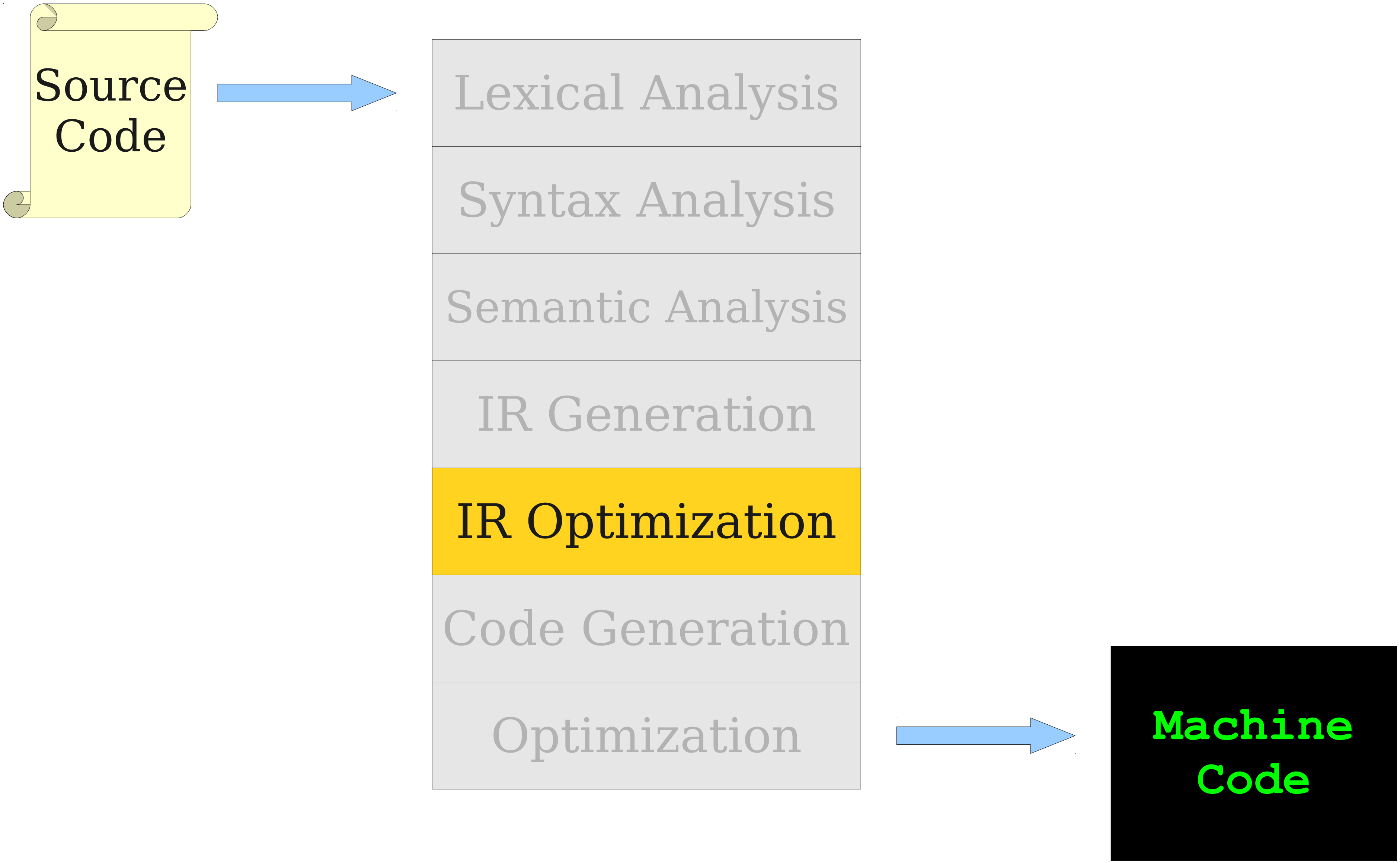
Part II

Announcements

- Programming Project 4 due Saturday, August 18 at 11:30AM.
    - OH today and tomorrow.
    - Ask questions via email!
    - Ask questions via Piazzza!
    - **No late submissions.**

Four Square!
5:30PM Thursday, Outside Gates

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

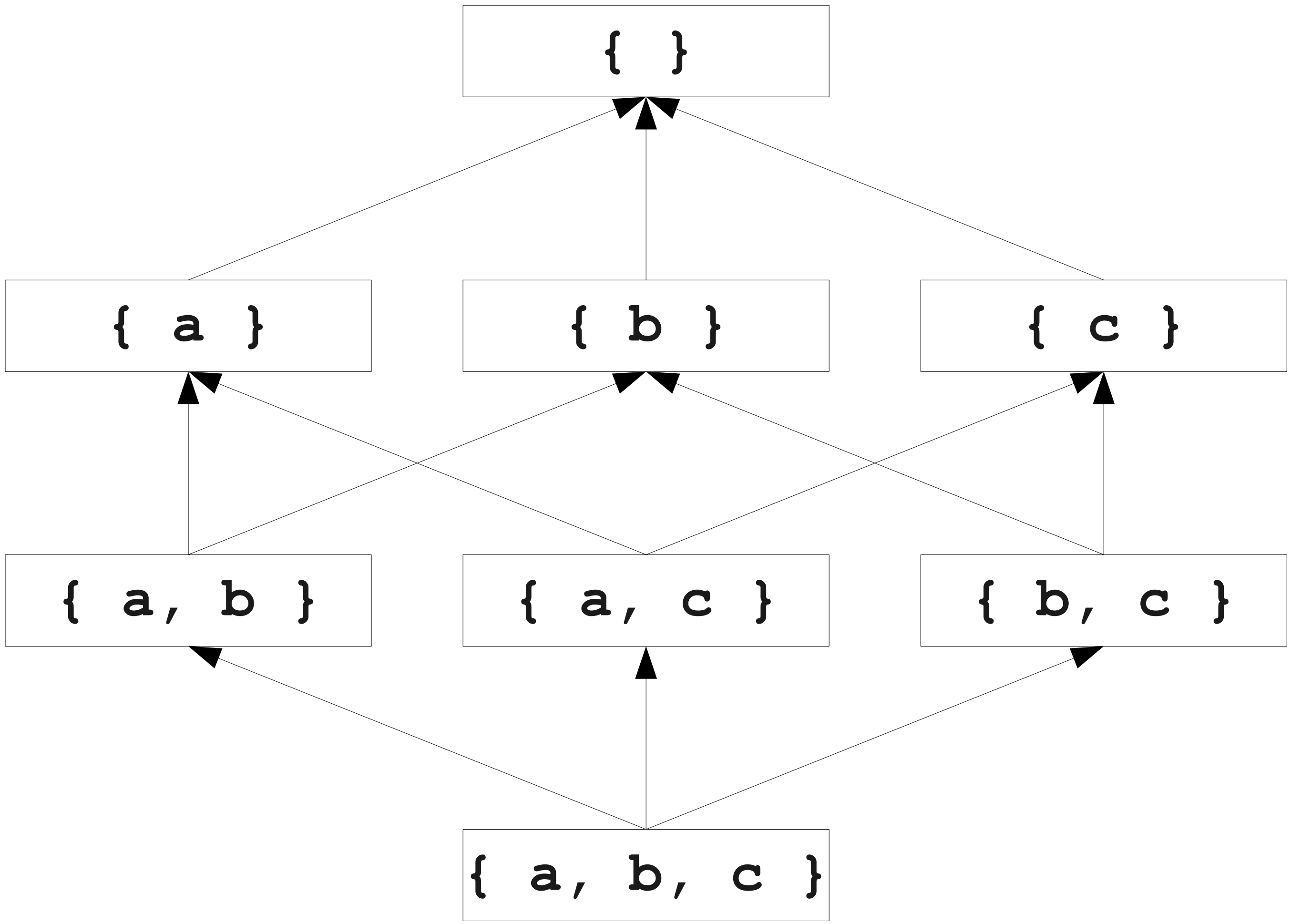
Review: Why Global Analysis is Hard

- Need to be able to handle multiple predecessors/successors for a basic block.
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value (but the analysis still needs to terminate!)
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it.

Review: Meet Semilattices

- A **meet semilattice** is a ordering defined on a set of elements.
- Any two elements have some **meet** that is the largest element smaller than both elements.
- There is a unique **top element**, which is at least as large as any other element.
- Intuitively:
  - The meet of two elements represents combining information from two elements.
  - The top element element represents “no information yet.”

Meet Semilattices for Liveness



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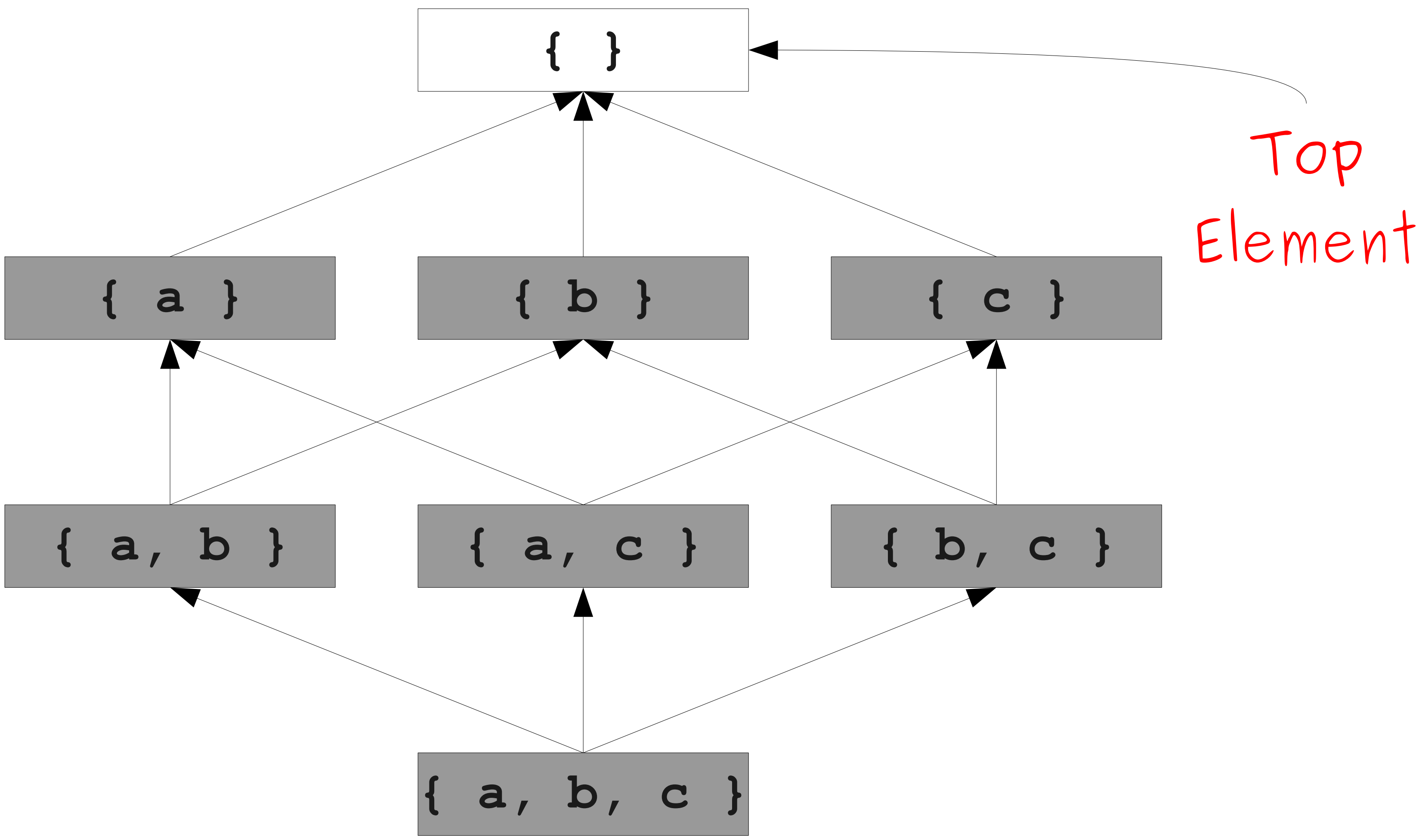

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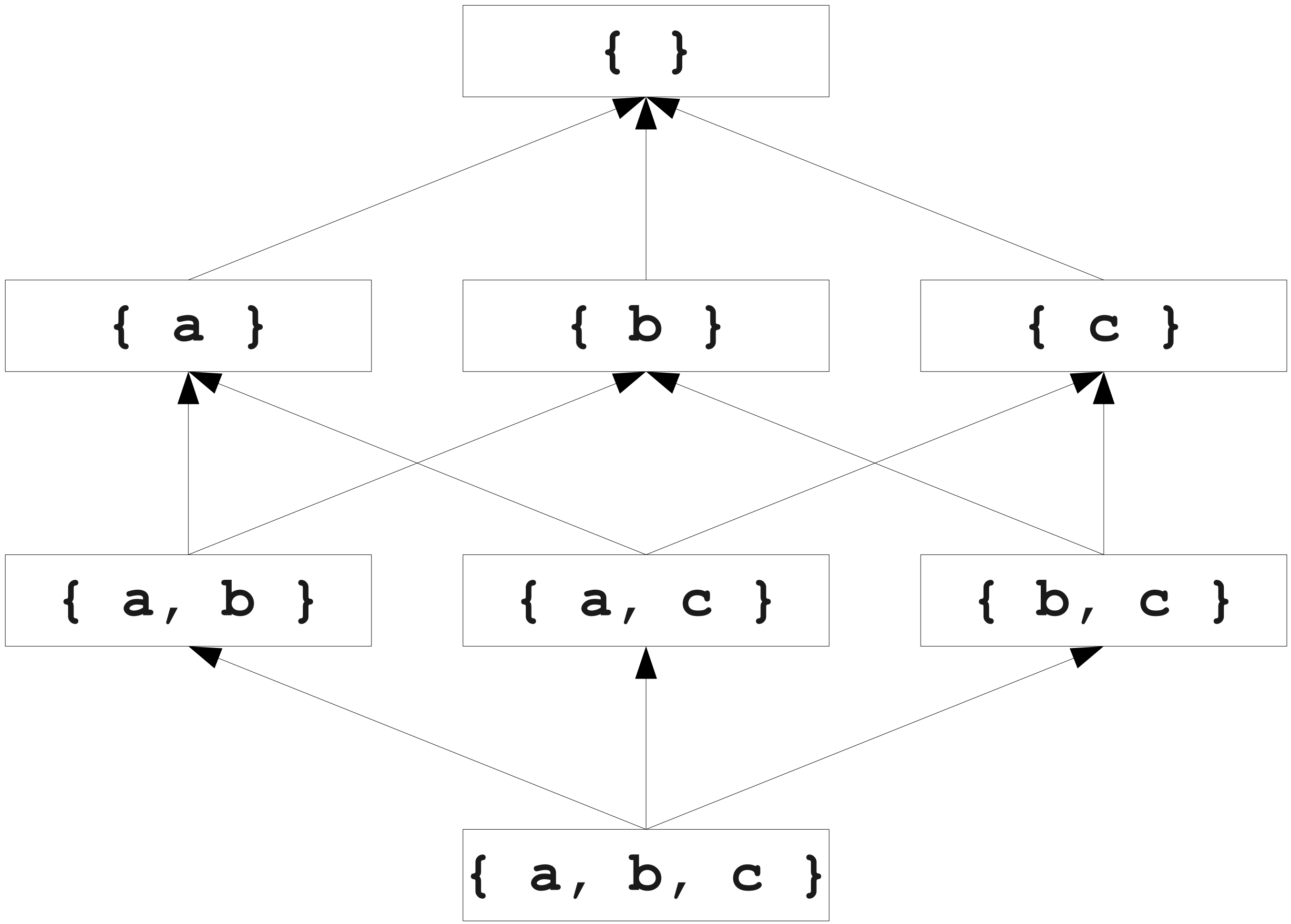
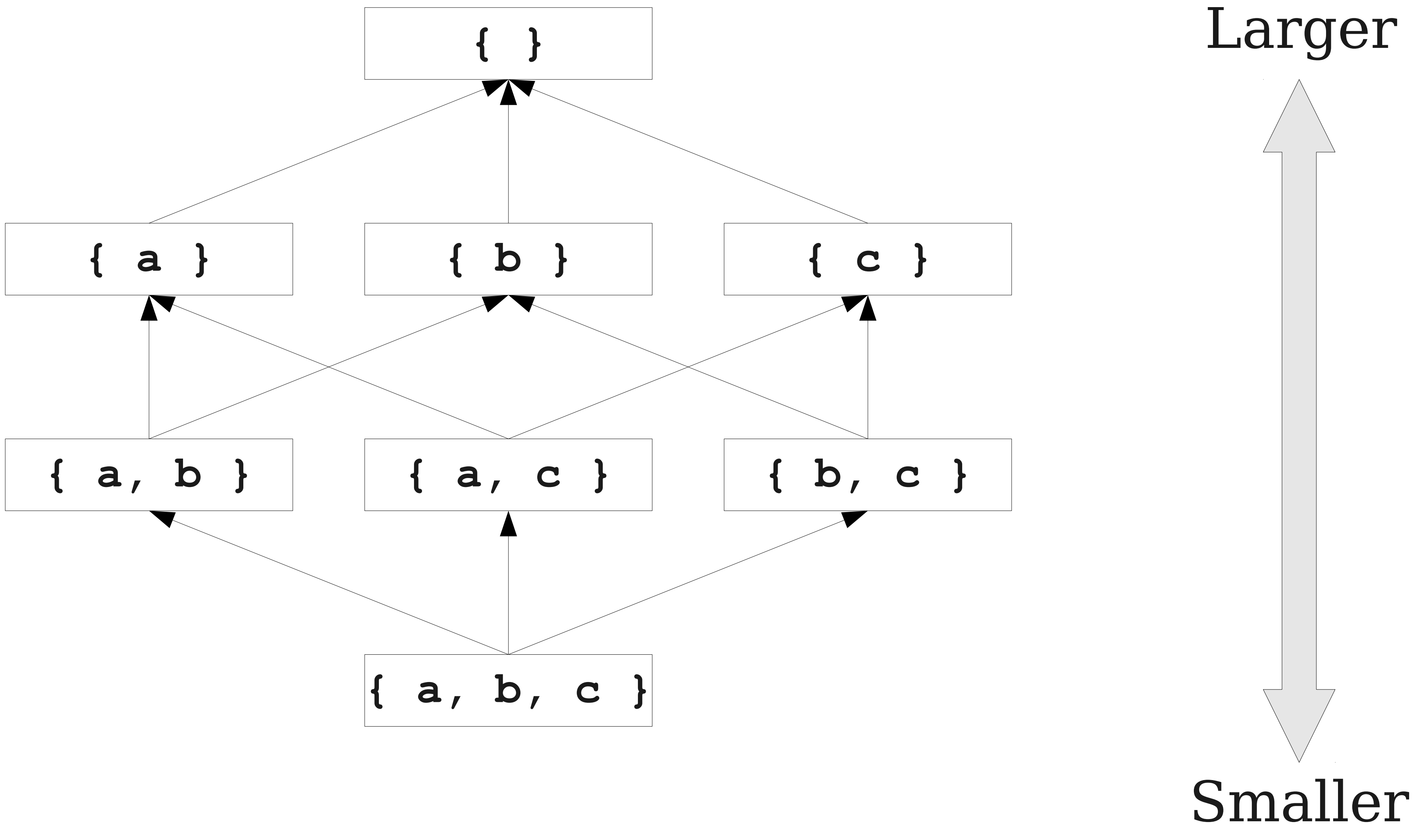


Review: Meet Semilattices

- A **meet semilattice** is a pair $(D, \wedge)$, where
  - D is a domain of elements.
  - $\wedge$ is a **meet operator** that is
    - **idempotent:** $x \wedge x = x$
    - **commutative:** $x \wedge y = y \wedge x$
    - **associative:** $(x \wedge y) \wedge z = x \wedge (y \wedge z)$
- If $x \wedge y = z$, we say that z is the **meet** or (**greatest lower bound**) of x and y.
- Every meet semilattice has a **top element** denoted $\top$ such that $\top \wedge x = x$ for all x.

Meet Semilattices and Orderings



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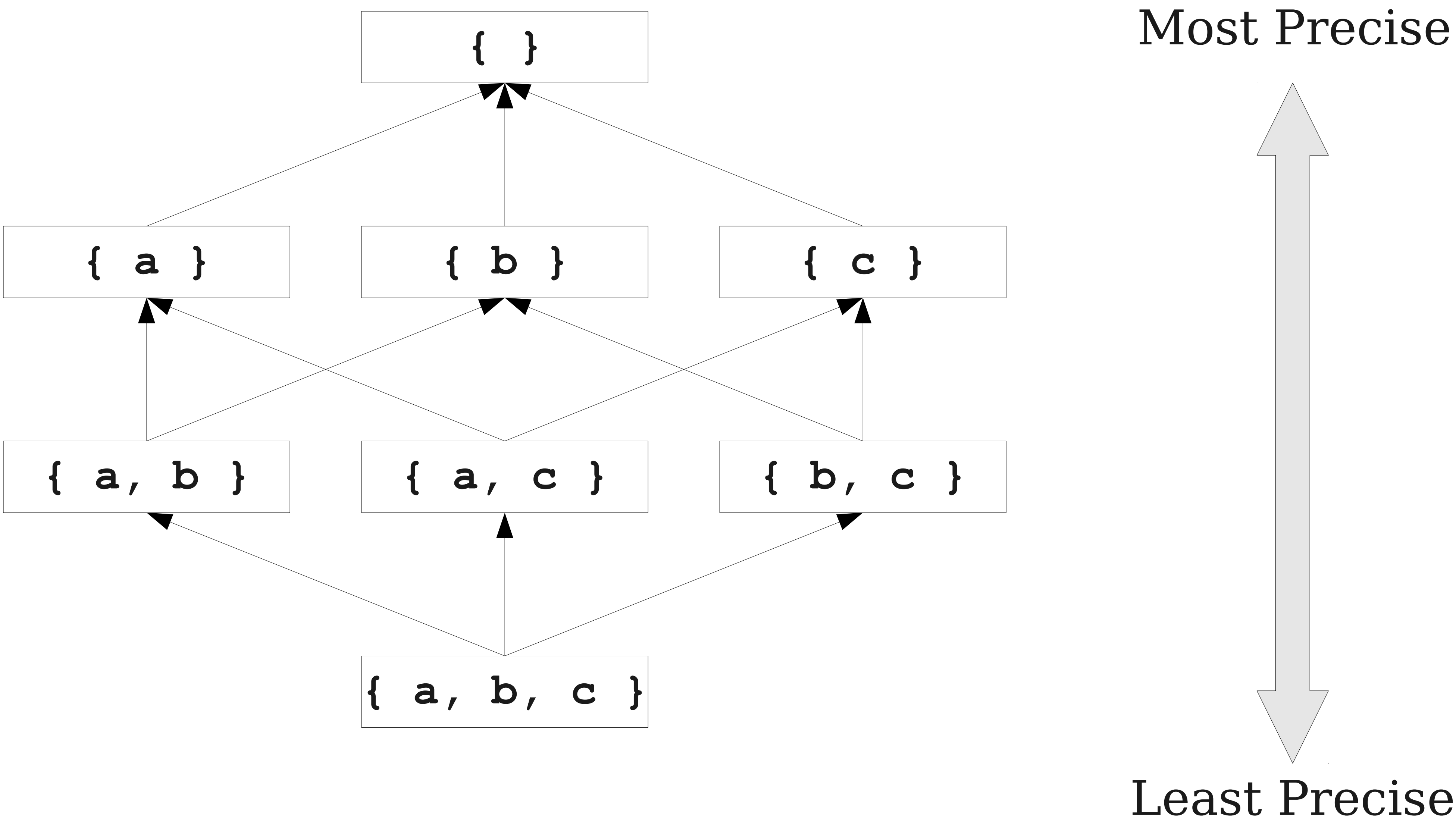


Meet Semilattices and Orderings

- Every meet semilattice $(D, \wedge)$ induces an ordering relationship $\leq$ over its elements.
- Define $\mathbf{x \leq y \quad iff \quad x \wedge y = x}$
- Need to prove
  - Reflexivity: $x \leq x$
  - Antisymmetry: If $x \leq y$ and $y \leq x$, then $x = y$.
  - Transitivity: If $x \leq y$ and $y \leq z$, then $x \leq z$.

An Example Semilattice

- The set of natural numbers and the **max** function.
- Idempotent
  - **max**{a, a} = a
- Commutative
  - **max**{a, b} = **max**{b, a}
- Associative
  - **max**{a, **max**{b, c}} = **max**{**max**{a, b}, c}
- Top element is 0:
  - **max**{0, a} = a
- What is the ordering over these elements?

A Semilattice for Liveness

- Sets of live variables and the set union operation.
- Idempotent:
  - $x \cup x = x$
- Commutative:
  - $x \cup y = y \cup x$
- Associative:
  - $(x \cup y) \cup z = x \cup (y \cup z)$
- Top element:
  - The empty set: $\emptyset \cup x = x$
- What is the ordering over these elements?

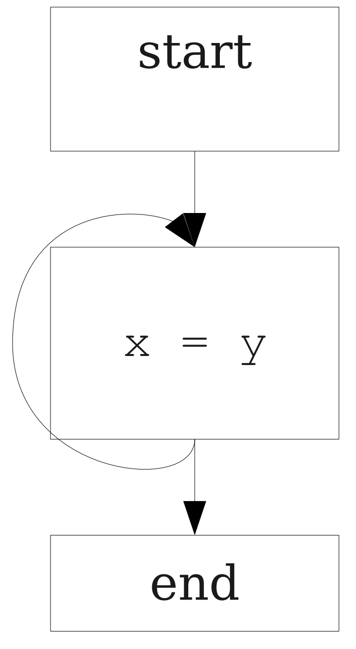
Proving Termination

- Our algorithm for running these analyses continuously loops until no changes are detected.
- Given this, how do we know the analyses will eventually terminate?
- In general, **we don't**.

A Nonterminating Analysis

- The following analysis will loop infinitely on any CFG containing a loop:
- **Direction:** Forward
- **Domain:** $\mathbb{N}$
- **Meet operator:** **max**
- **Transfer function:** $f(n) = n + 1$
- **Initial value:** 0

A Nonterminating Analysis



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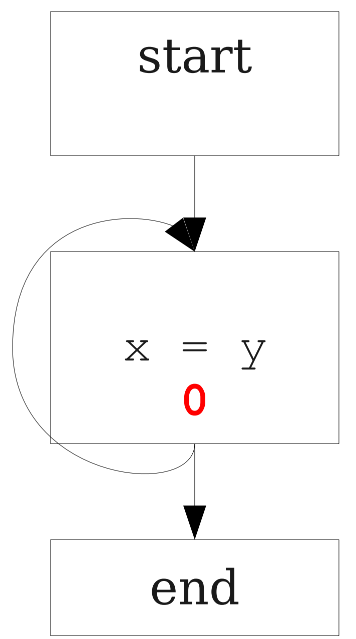

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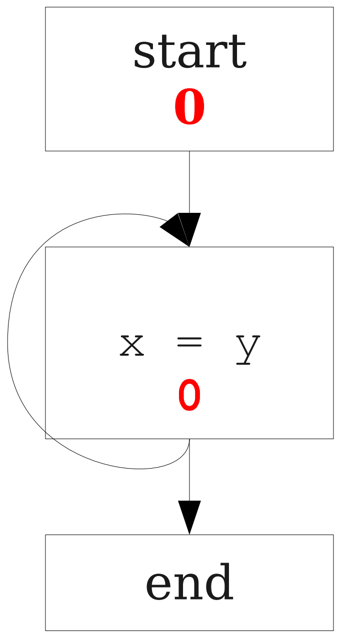

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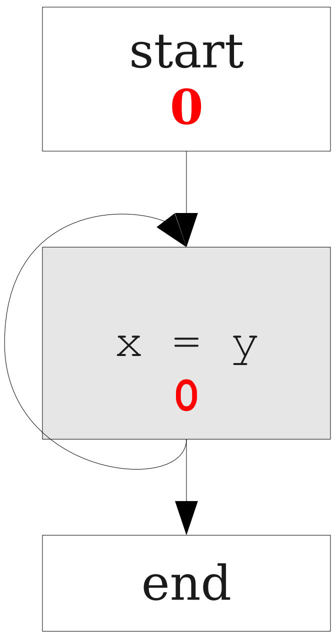

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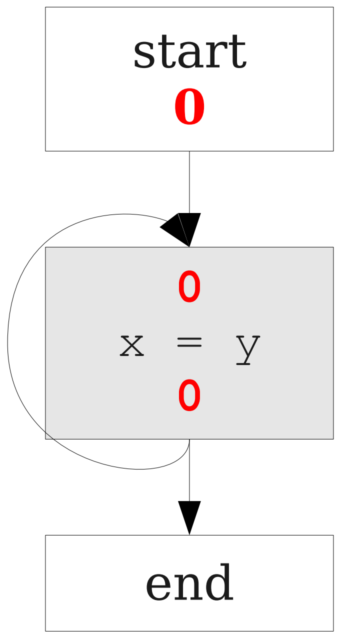

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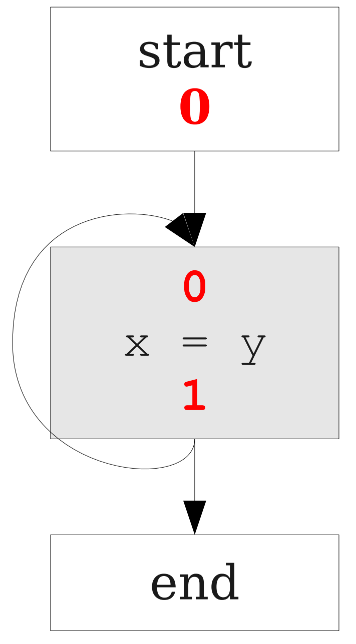

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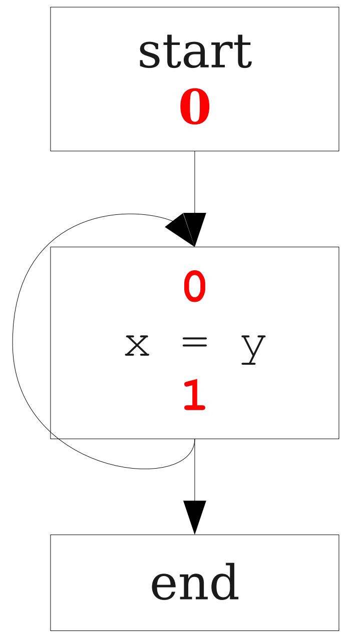

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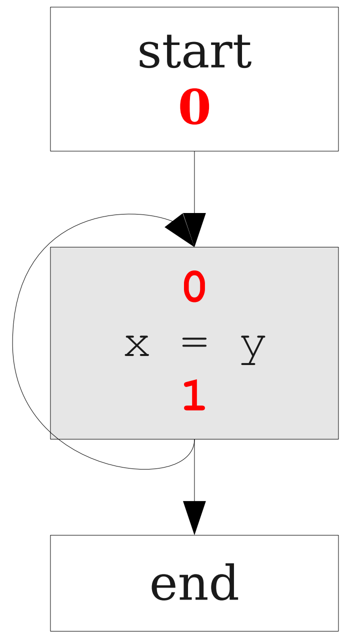

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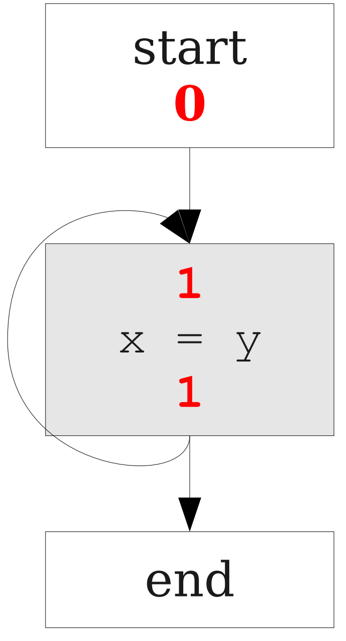

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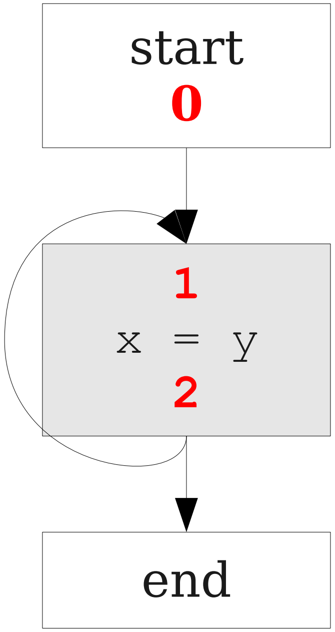

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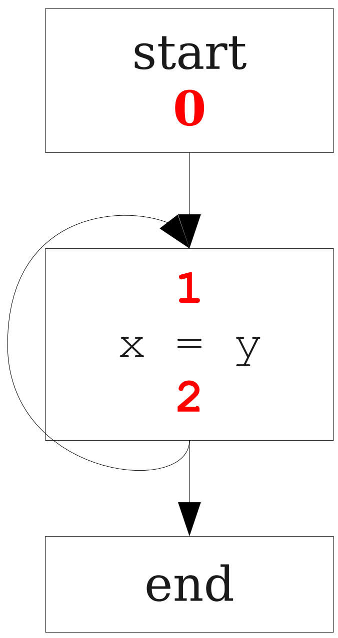

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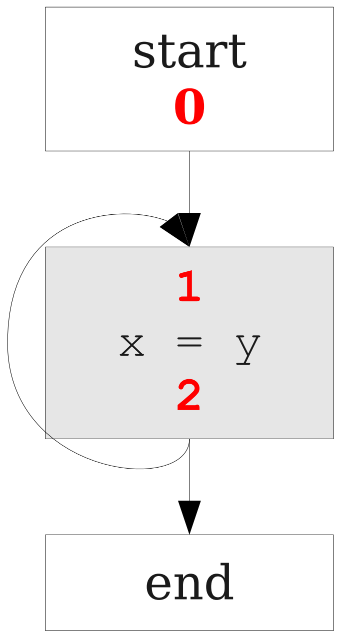

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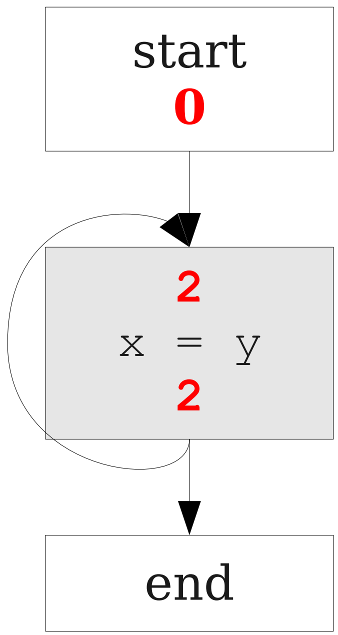

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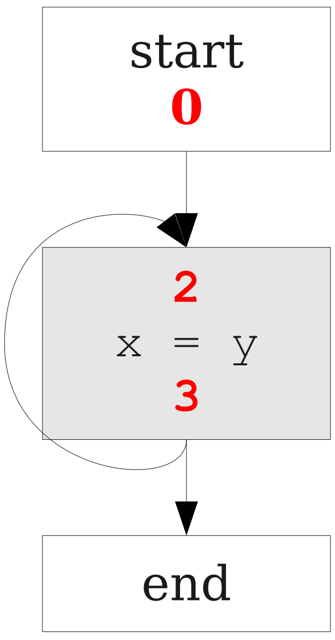

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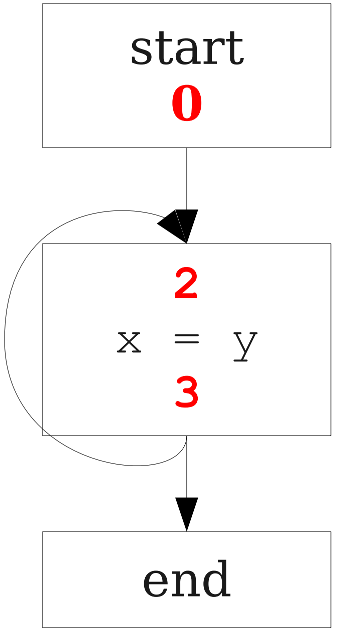
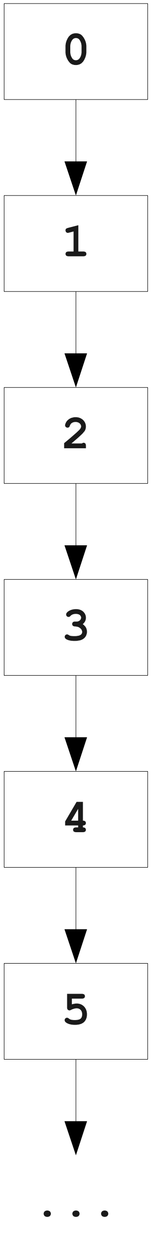


Why Doesn't This Terminate?

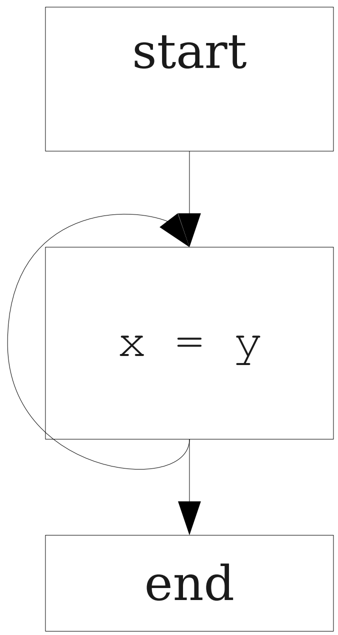


- **Values can decrease without bound.**
  - Note that "decrease" refers to the lattice ordering, not the ordering on the natural numbers.
- The **height** of a semilattice is the length of the longest decreasing sequence in that semilattice.
- The dataflow framework is not guaranteed to terminate for semilattices of infinite height.
- Note that a semilattice can be infinitely large but have finite height (e.g. constant propagation).

Another Nonterminating Analysis

- This analysis works on a finite-height semilattice, but will not terminate on certain CFGs:
- **Direction:** Forward
- **Domain:** Boolean values `true` and `false`
- **Meet operator:** Logical AND
- **Transfer function:** Logical NOT
- **Initial value:** `true`

Another Nonterminating Analysis



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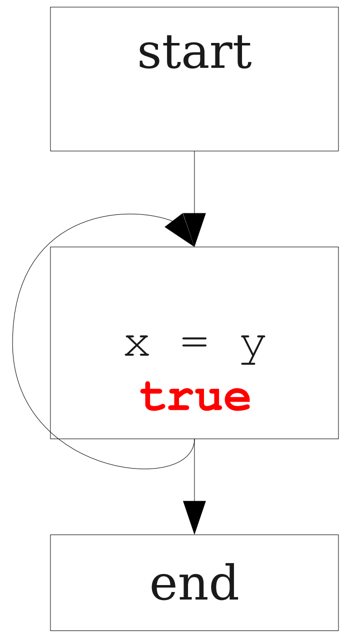

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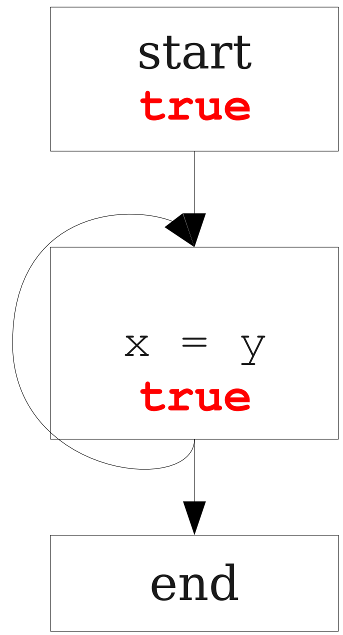

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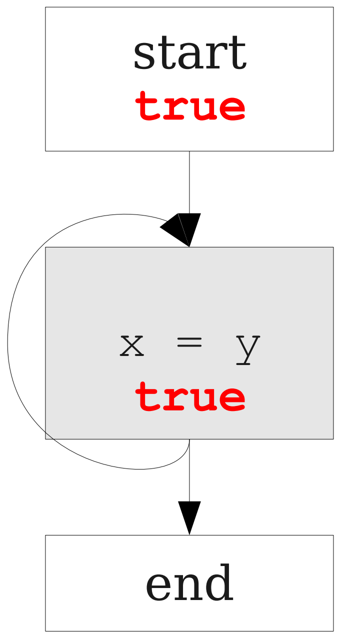

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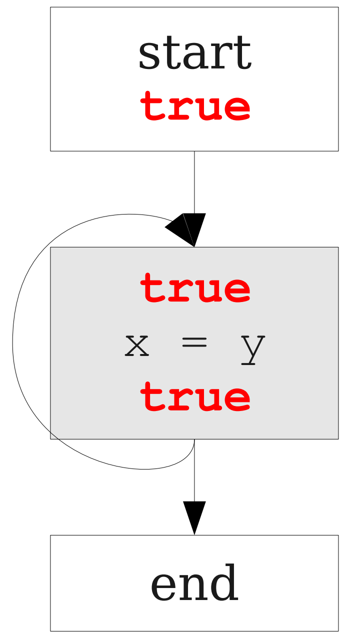

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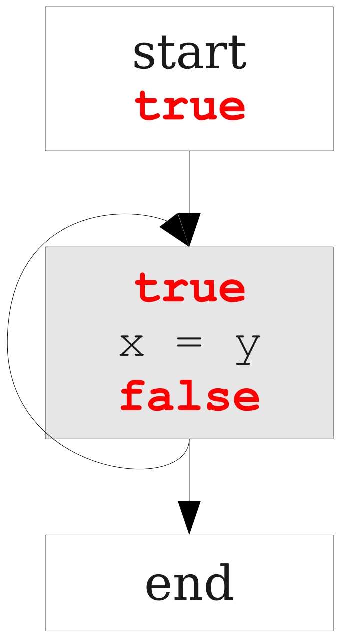

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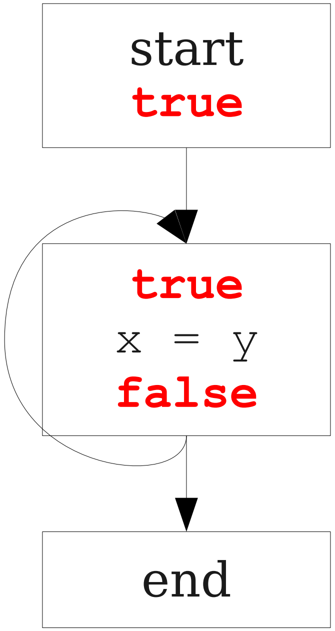

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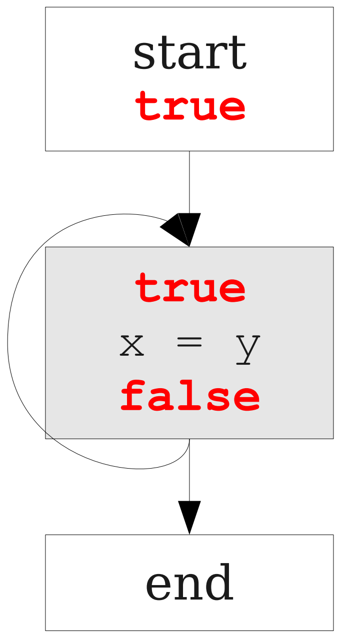

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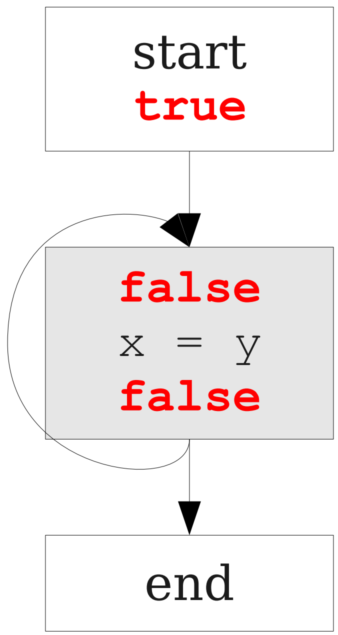

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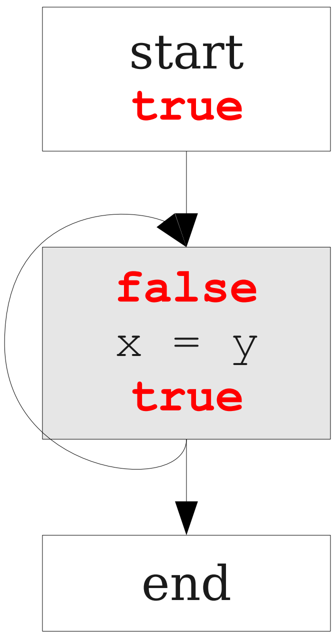

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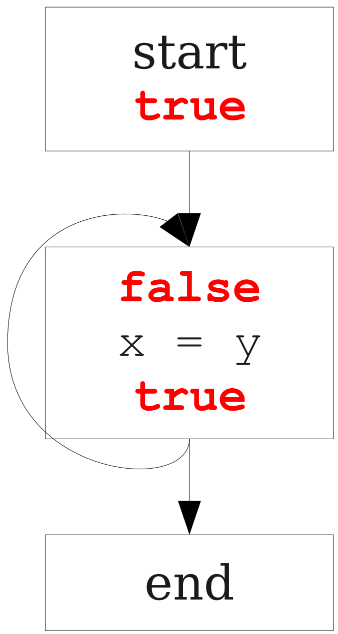

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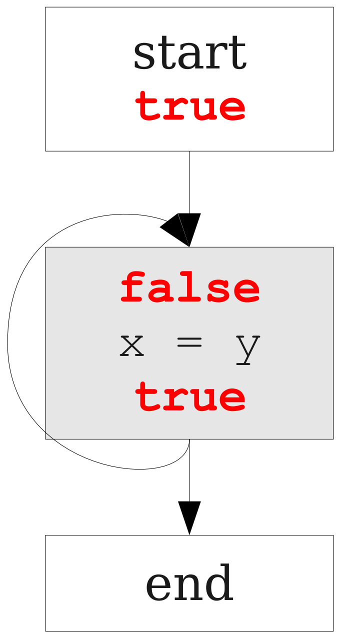

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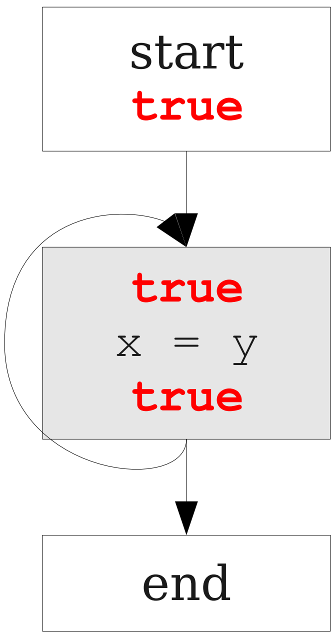

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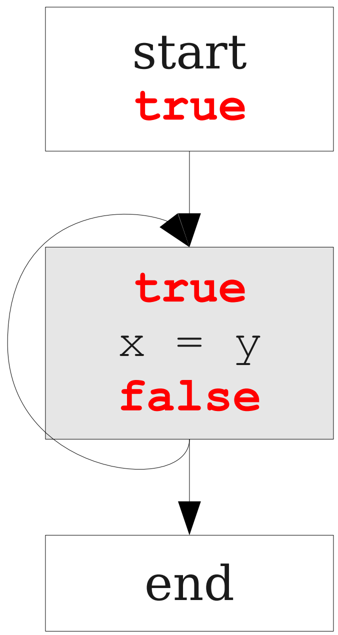

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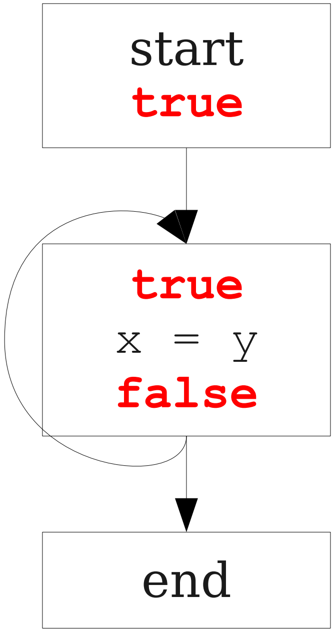

What Went Wrong (This Time)?

- **Values can loop indefinitely.**
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What's wrong
with cycling forever?

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- If the transfer function can keep pushing values back up again, then the values might cycle forever.
- How can we fix this?

Monotone Transfer Functions

- A transfer function is **monotone** iff

$$\textbf{if } x \le y\textbf{, then } f(x) \le f(y)$$

- Intuitively, if you know less information about a program point, you can't "gain back" more information about that program point.
- Many transfer functions are monotone, including those for liveness and constant propagation.
- Note: Monotonicity does **not** mean that $f(x) \le x$; we'll see an example.

Liveness and Monotonicity

- A transfer function is **monotone** iff

  if $x \leq y$, then $f(x) \leq f(y)$

- Recall our transfer function for **a = b + c** is
  - $f_{a\,=\,b\,+\,c}(V) = (V - \mathbf{a}) \cup \{\mathbf{b}, \mathbf{c}\}$
- Recall that our meet semilattice has set union as a transfer function and induces an ordering relationship $X \leq Y$ iff $X \supseteq Y$.
- Is this monotone?

Constant Propagation is Monotone

- A transfer function is **monotone** iff

  if $x \leq y$, then $f(x) \leq f(y)$

- Recall our transfer functions are
  - $f_{x=k}(V) = k$
  - $f_{x=a+b}(V) =$ Not a Constant
  - $f_{y=a+b}(V) = V$
- Is this monotonic?

The Grand Result

- **Theorem**: A dataflow analysis with a finite-height semilattice and family of monotone transfer functions always terminates.
- Proof sketch:
  - Run the data-flow iteration once to get some initial values.
  - From this point forward:
    - The meet operator can only bring values down.
    - The transfer function can never raise values back up above where they were in the past (monotonicity)
    - Values cannot decrease indefinitely (finite height)

Partial-Redundancy Elimination

Code Size is Not Execution Time

- All of the analyses we've seen so far have worked by simplifying or eliminating IR code.
- However, much of optimization results from *moving* code from one basic block to another.

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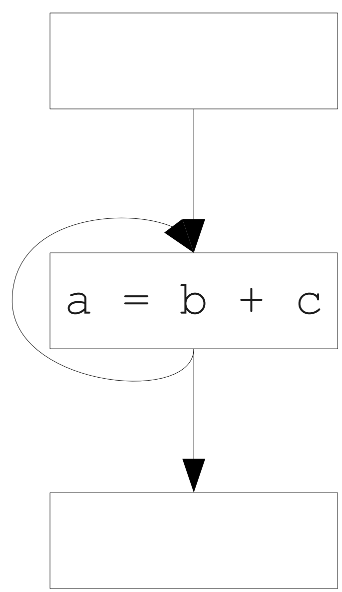

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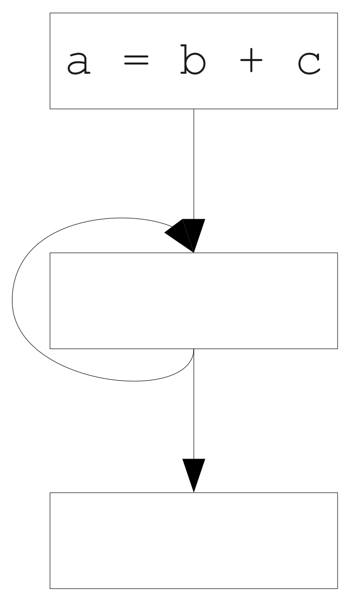

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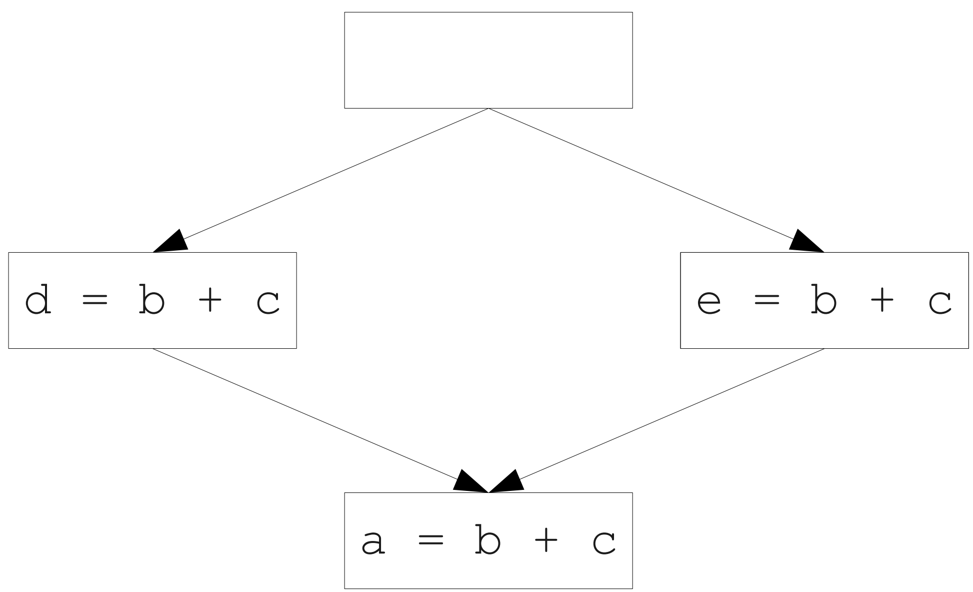

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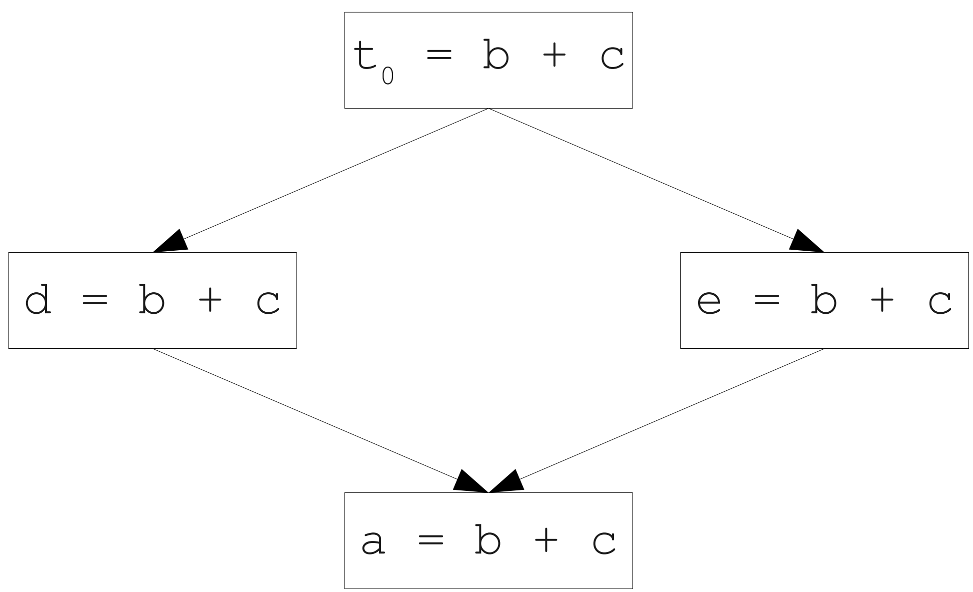

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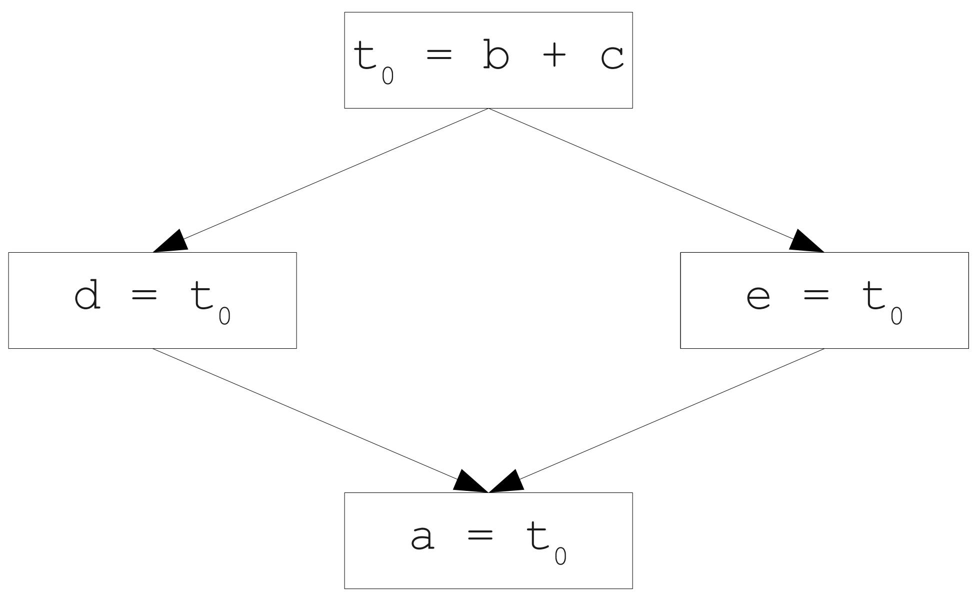
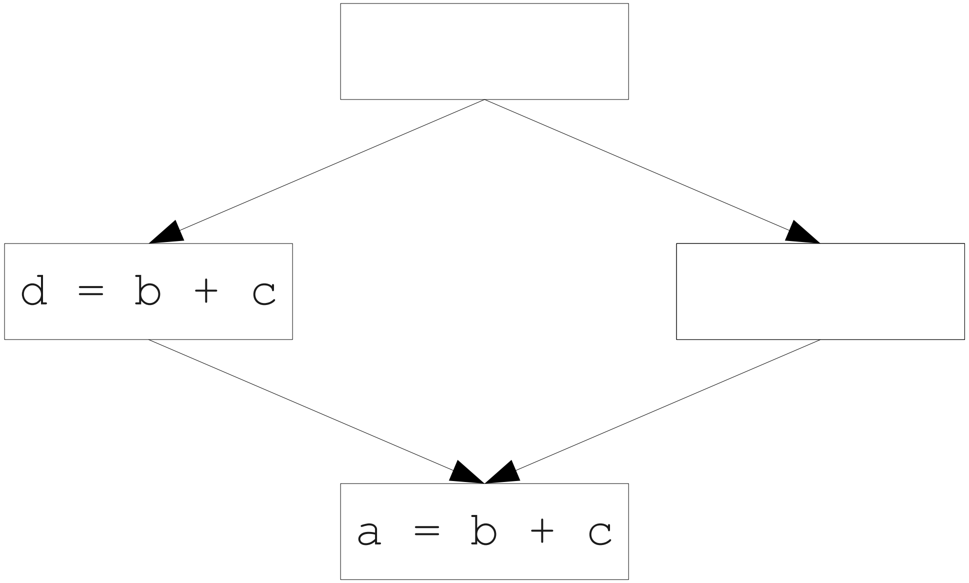


Eliminating Redundancy

- A computation in a program is said to be **redundant** if it computes a value that is already known.
- Common subexpressions are one example of redundancy.
- Loop-invariant code is another example.
- Virtually all optimizing compilers have some logic to try to eliminate redundancy.

Partial Redundancy

- One of the trickiest cases of redundancy to eliminate is **partial redundancy**.
- A computation is **partially redundant** if its value is known on only some of the paths that reach it.



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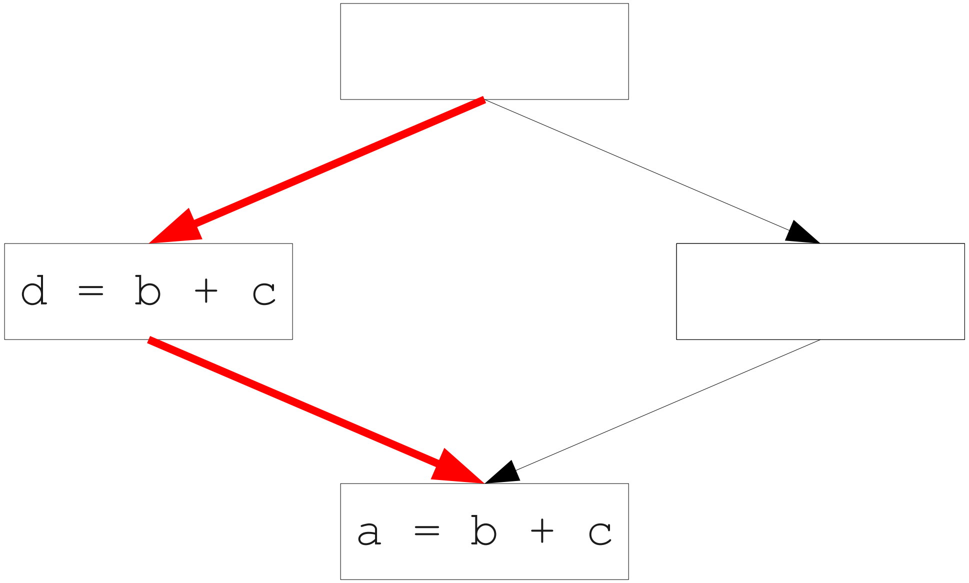

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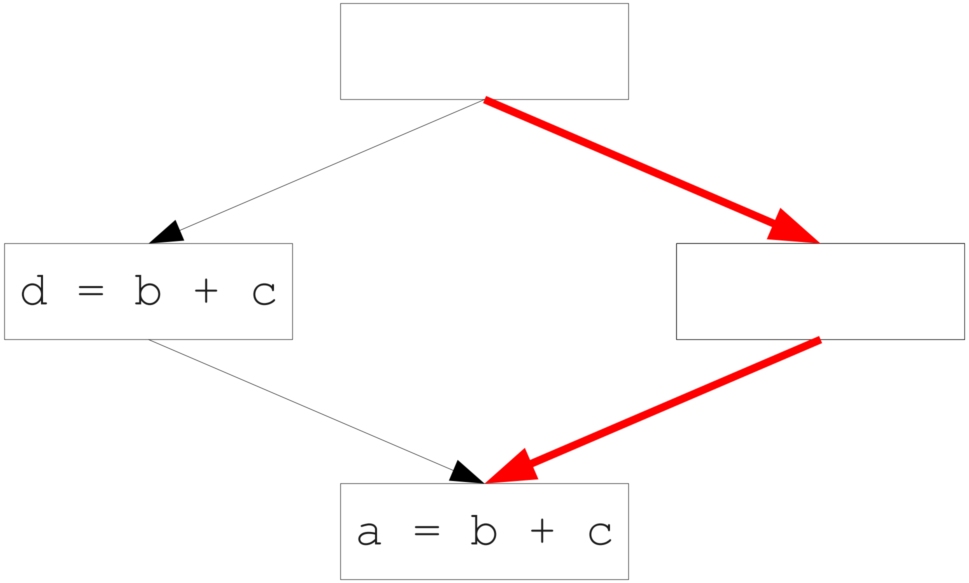
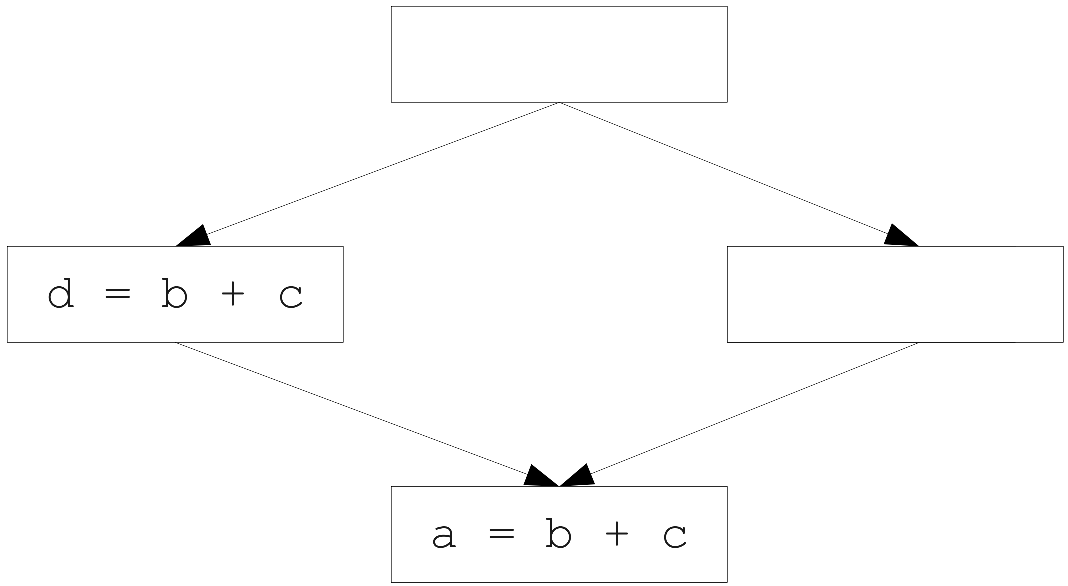


Eliminating Partial Redundancy

- Goal: Eliminate partial redundancy without making any execution of the program do more work than before.
- Optimized code should **always** be at least as good as the original.

The Key Observation



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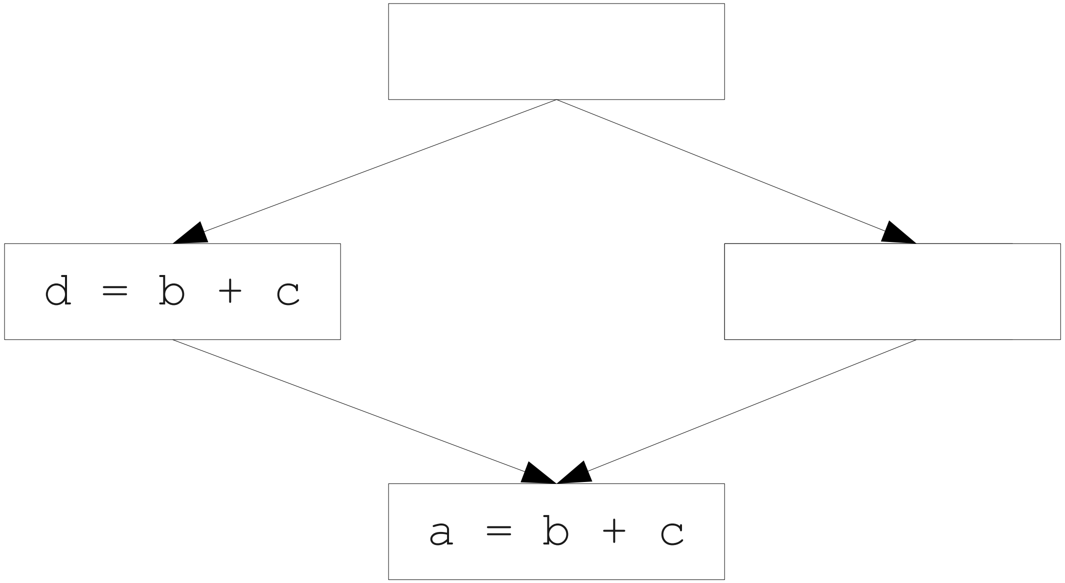

Where in the program is it <u>guaranteed</u> that we will eventually need the value of **b + c**?

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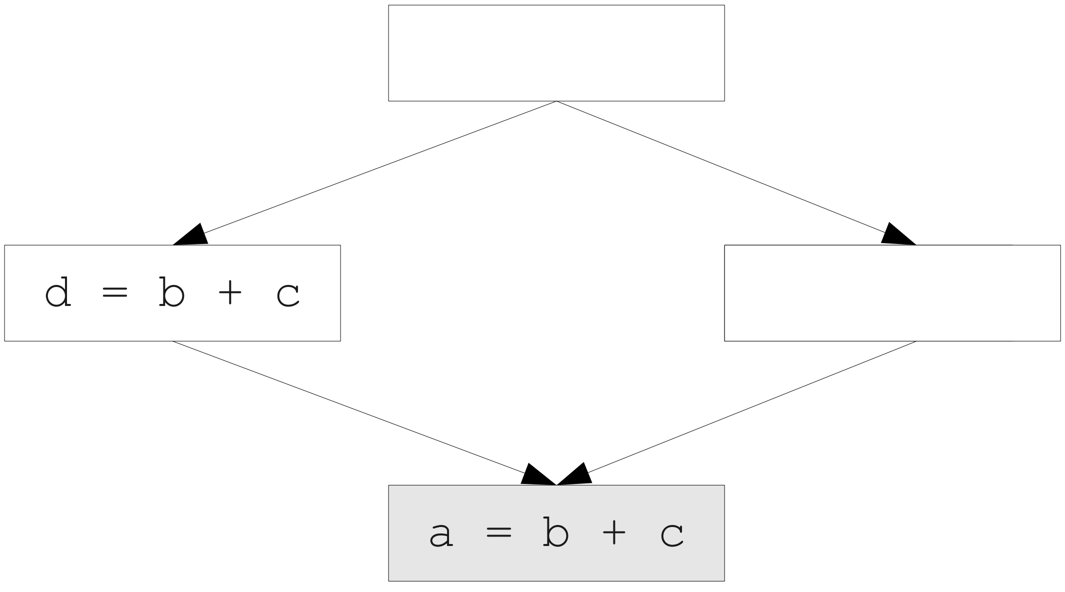

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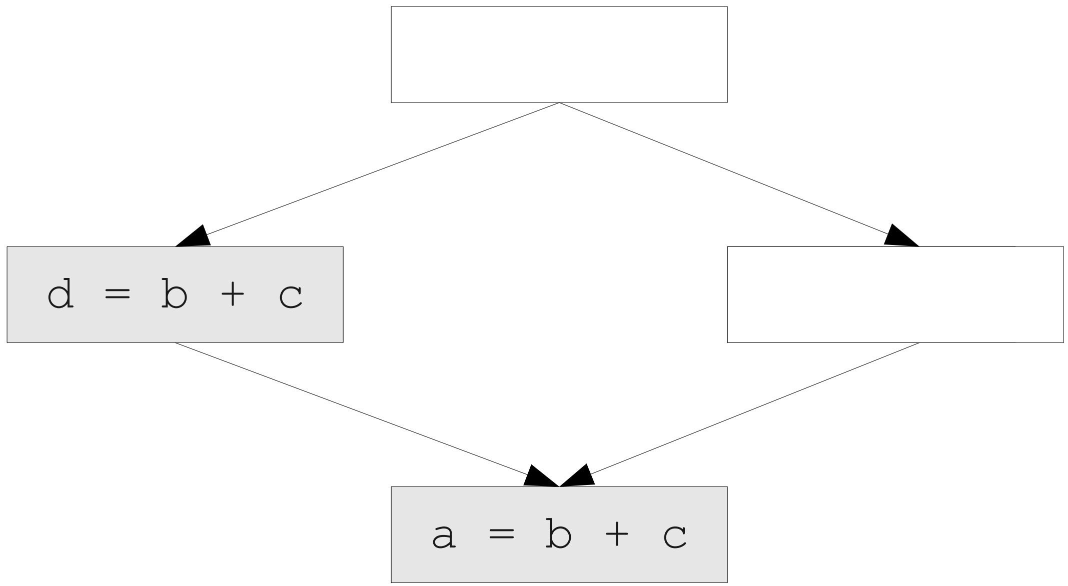

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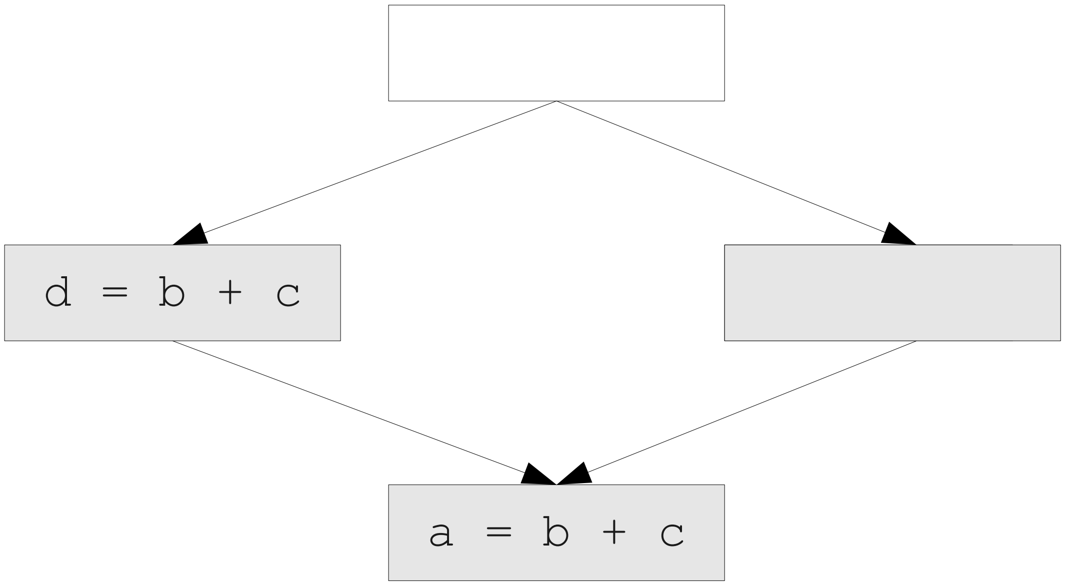

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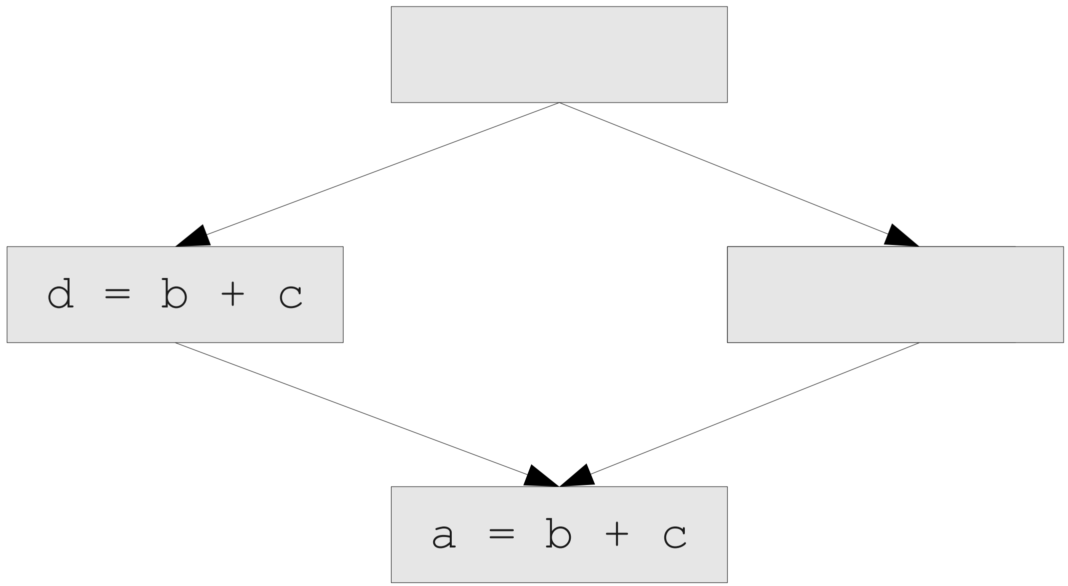
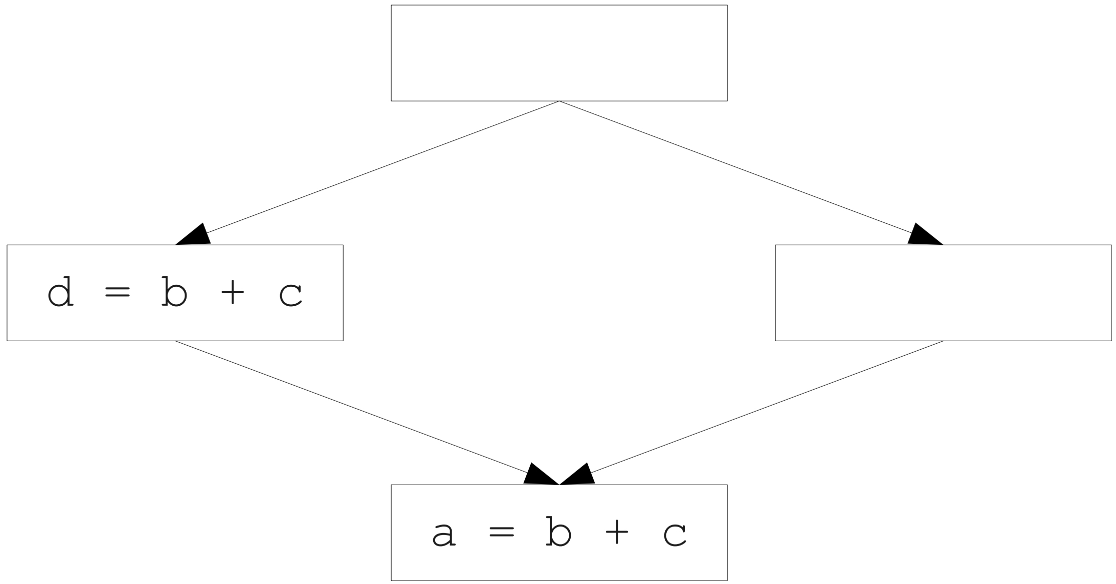


Where in the program is it <u>guaranteed</u> that we will eventually need the value of **b + c**?

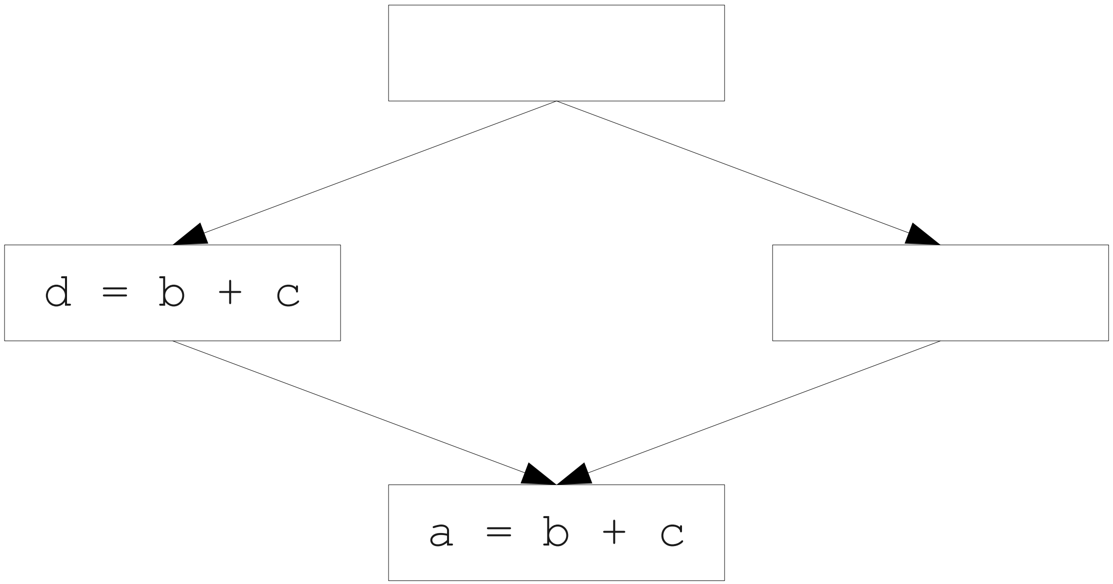
An expression is called **anticipated** at a program point if the expression is guaranteed to be used after that point.

Although not all paths through the program might directly need an expression, they may **anticipate** the expression.

The Second Key Observation

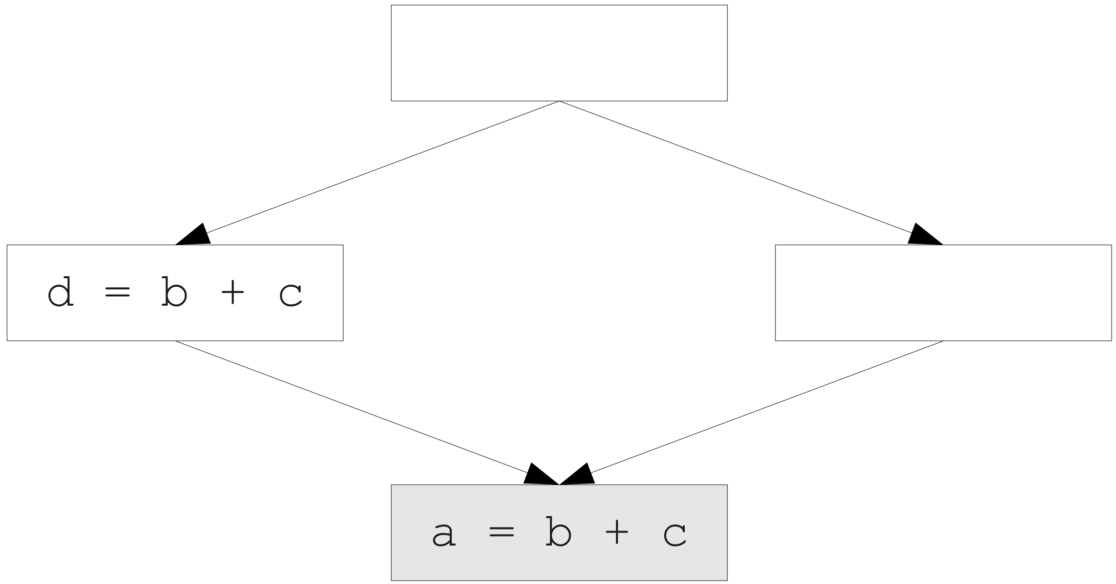


The Second Key Observation



Where in the program
is the value of **`b + c`**
already computed?

The Second Key Observation



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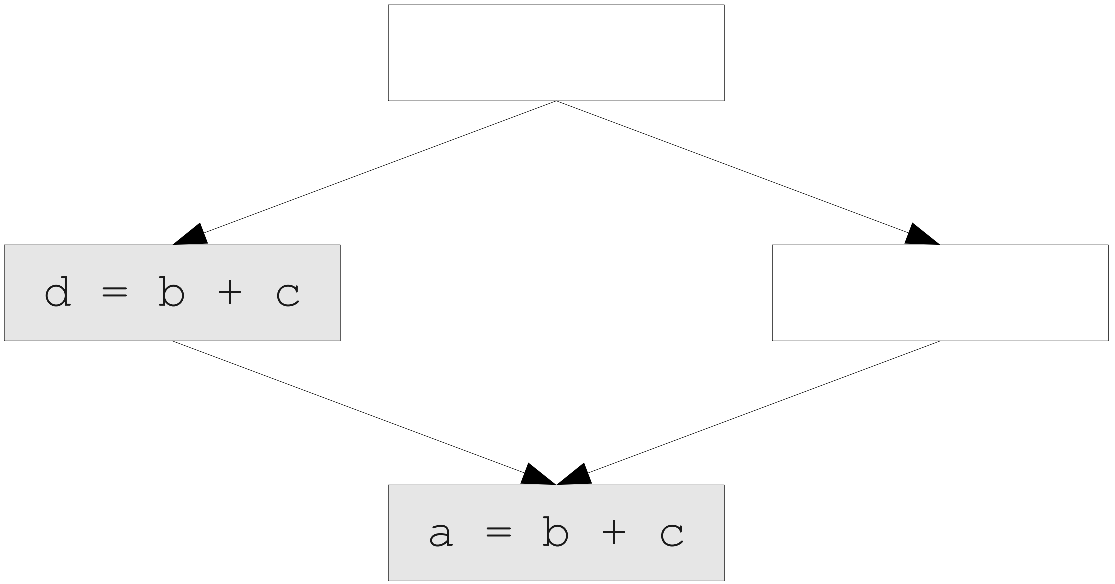

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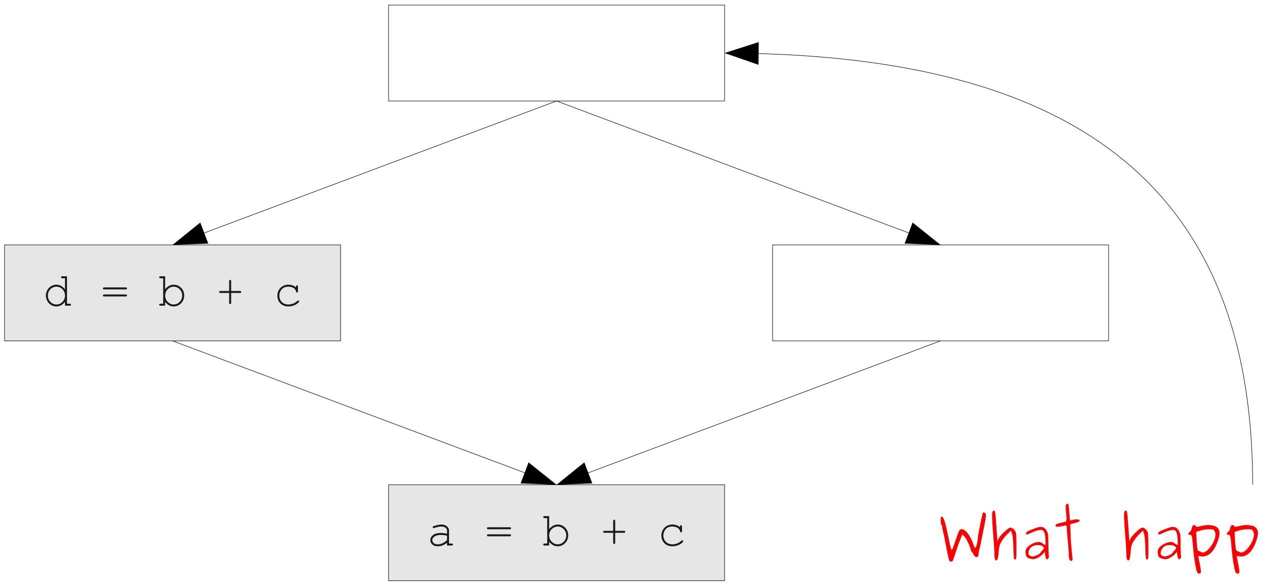
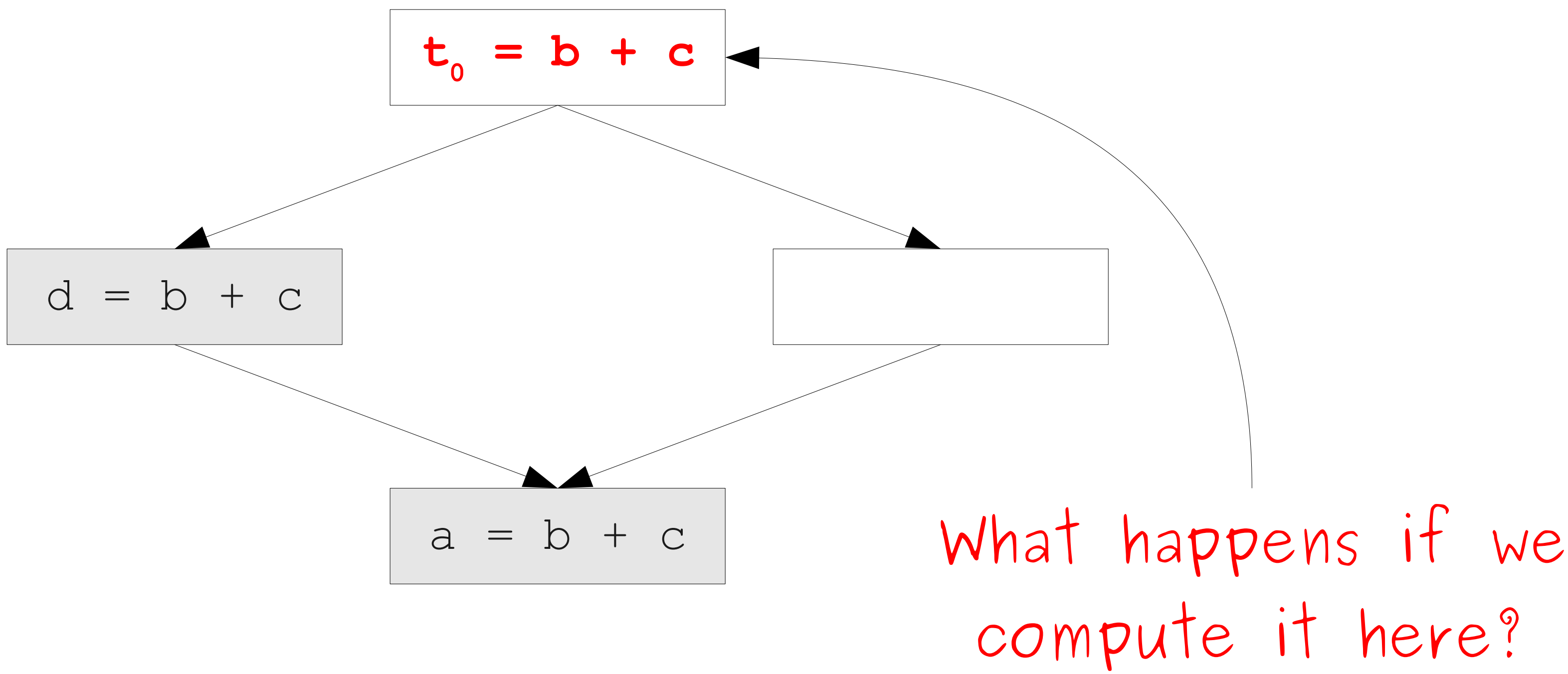


What happens if we compute it here?

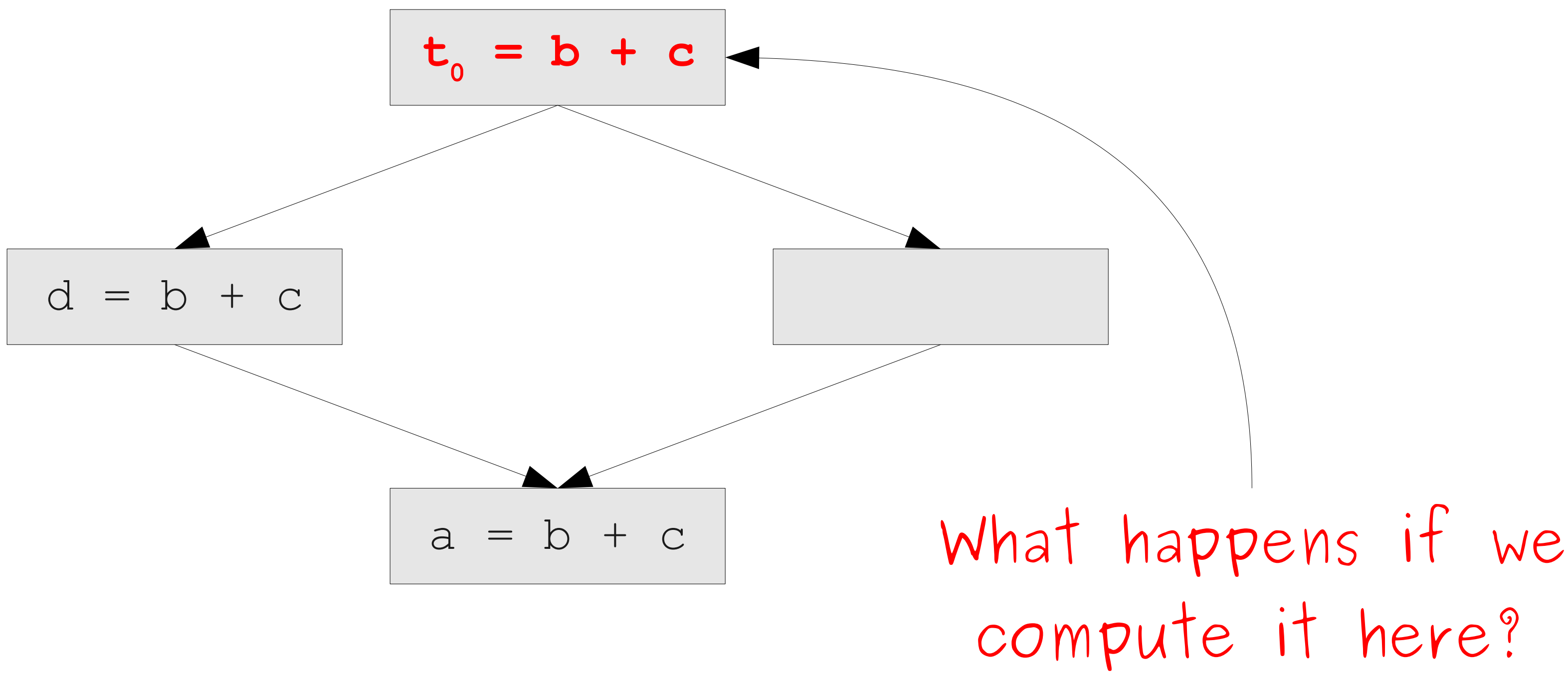
Where in the program is the value of **b + c** already computed?

The Second Key Observation



Where in the program is the value of **b + c** already computed?

The Second Key Observation



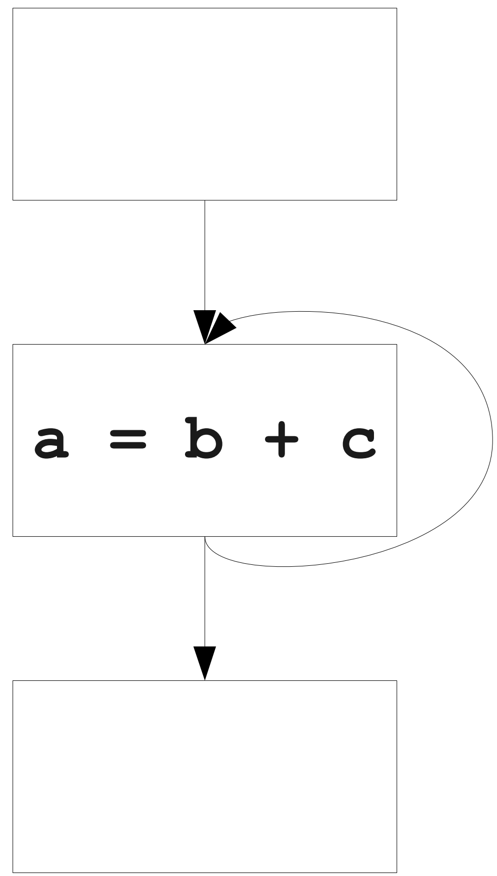
Where in the program is the value of **b + c** already computed?

Partial-Redundancy Elimination

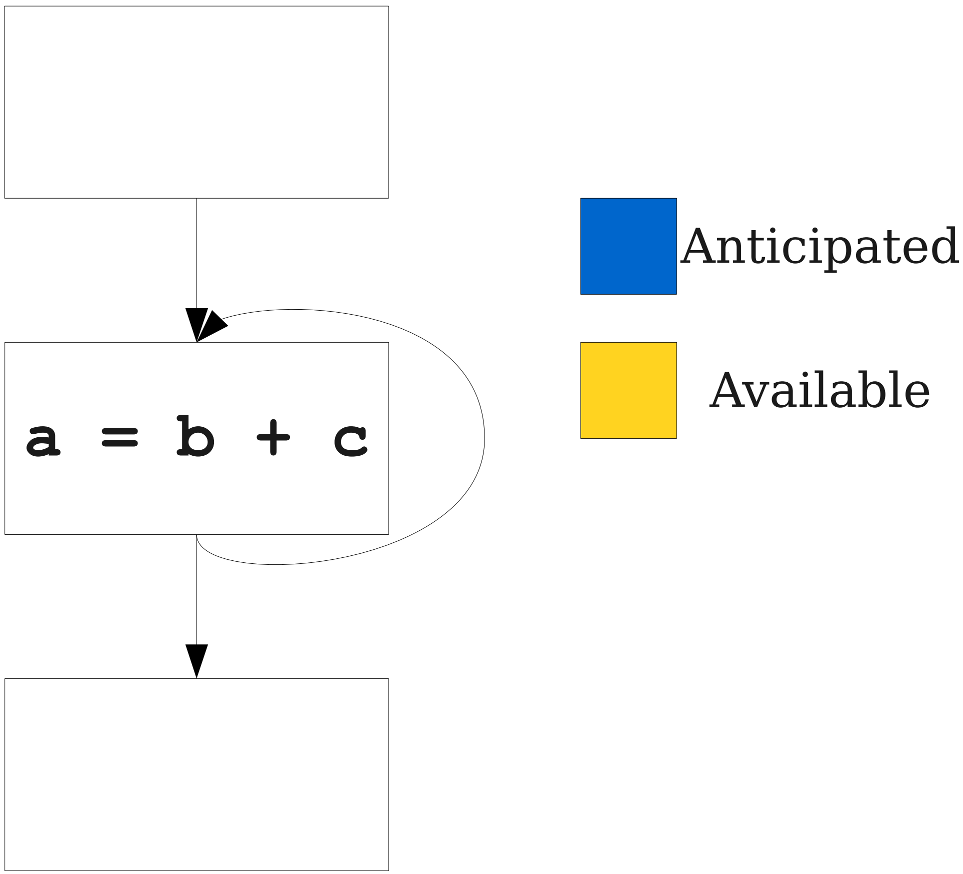
- **Idea:** Make the expression *available* everywhere that it's *anticipated*.
- Run an analysis to locate where the expression is anticipated.
- Run a second analysis to locate where the expression is available.
- Place the expression at the earliest locations where the expression is **anticipated** but not **available**.

Eliminating Redundancy

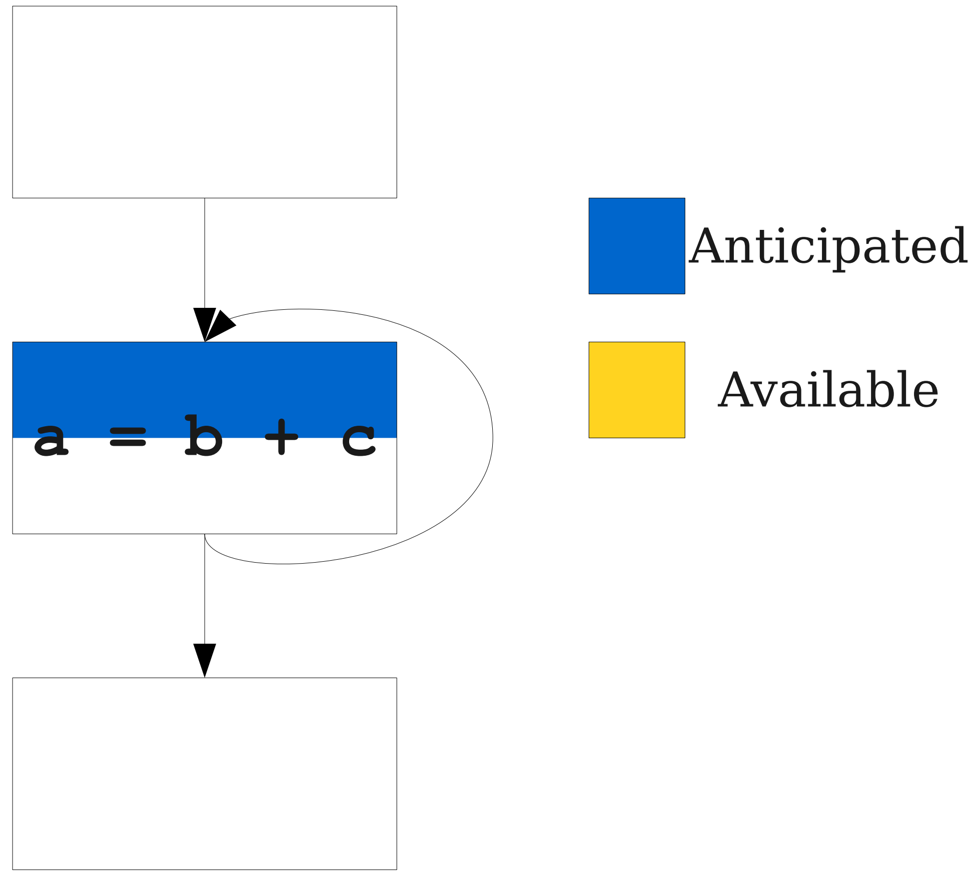
Eliminating Redundancy



Eliminating Redundancy



Eliminating Redundancy



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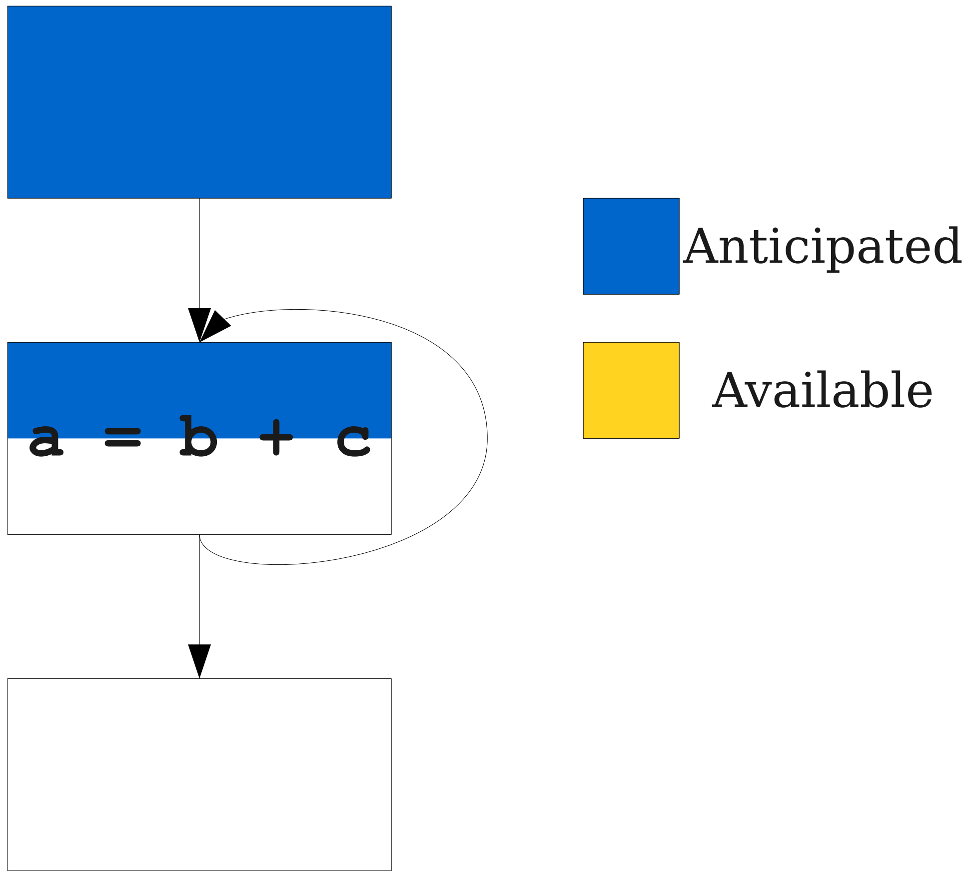

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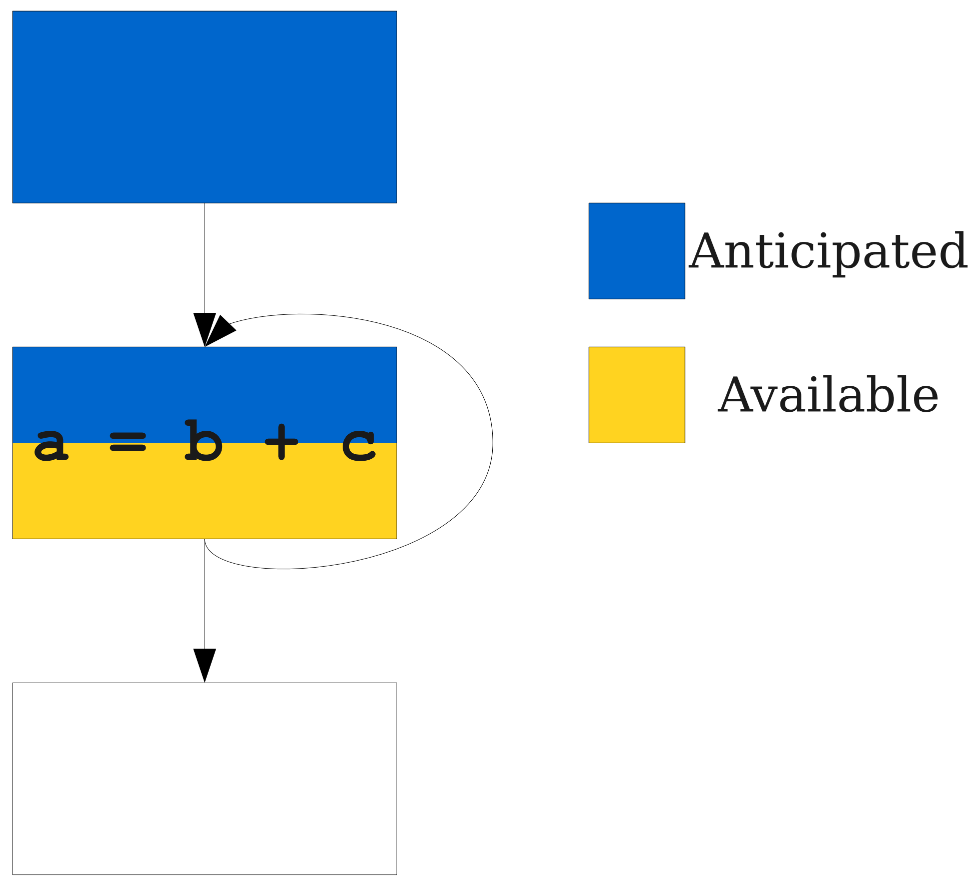

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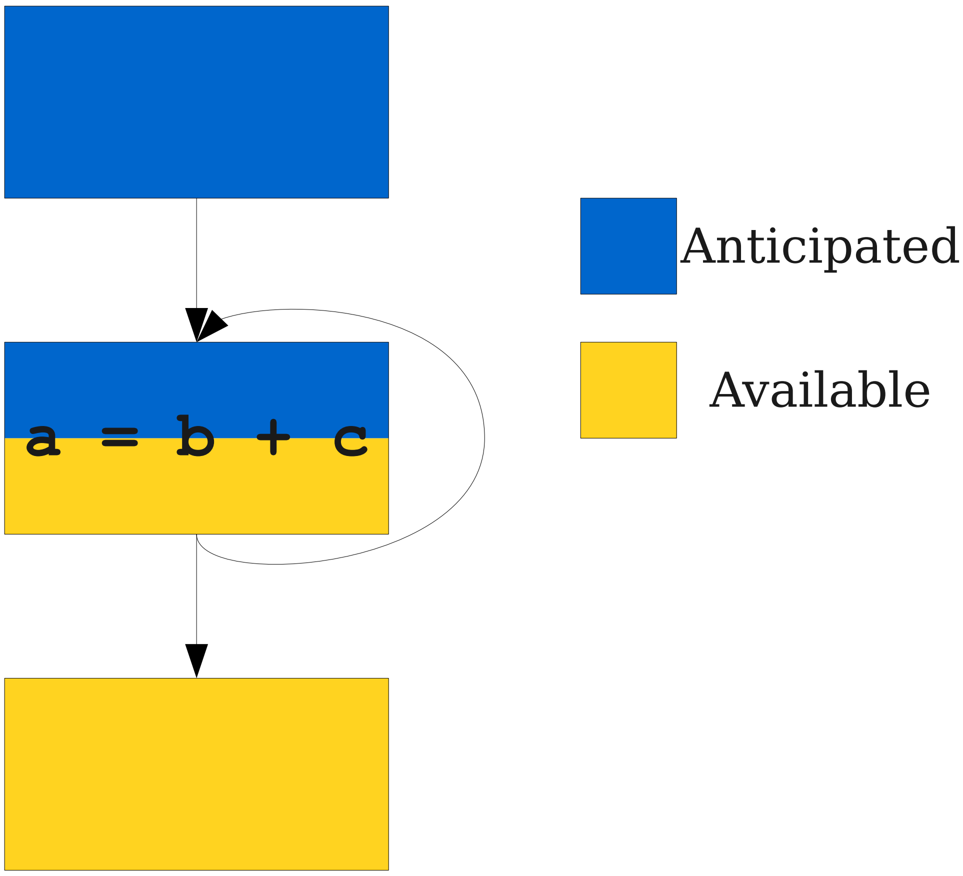

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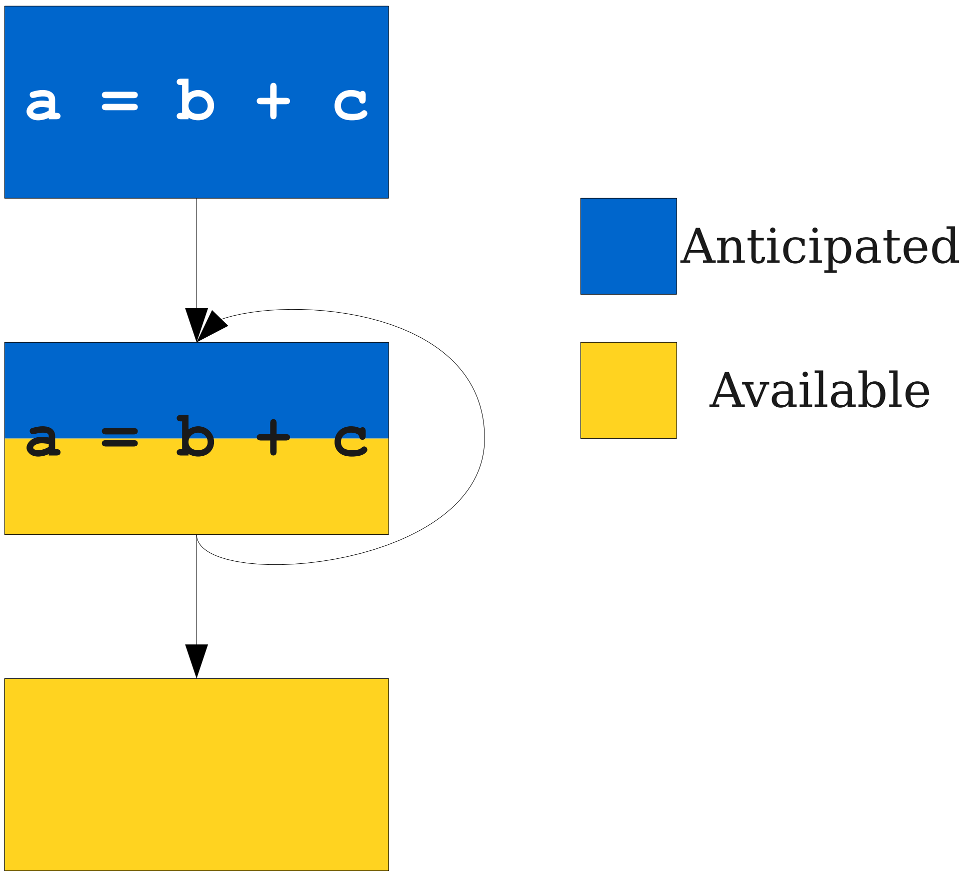

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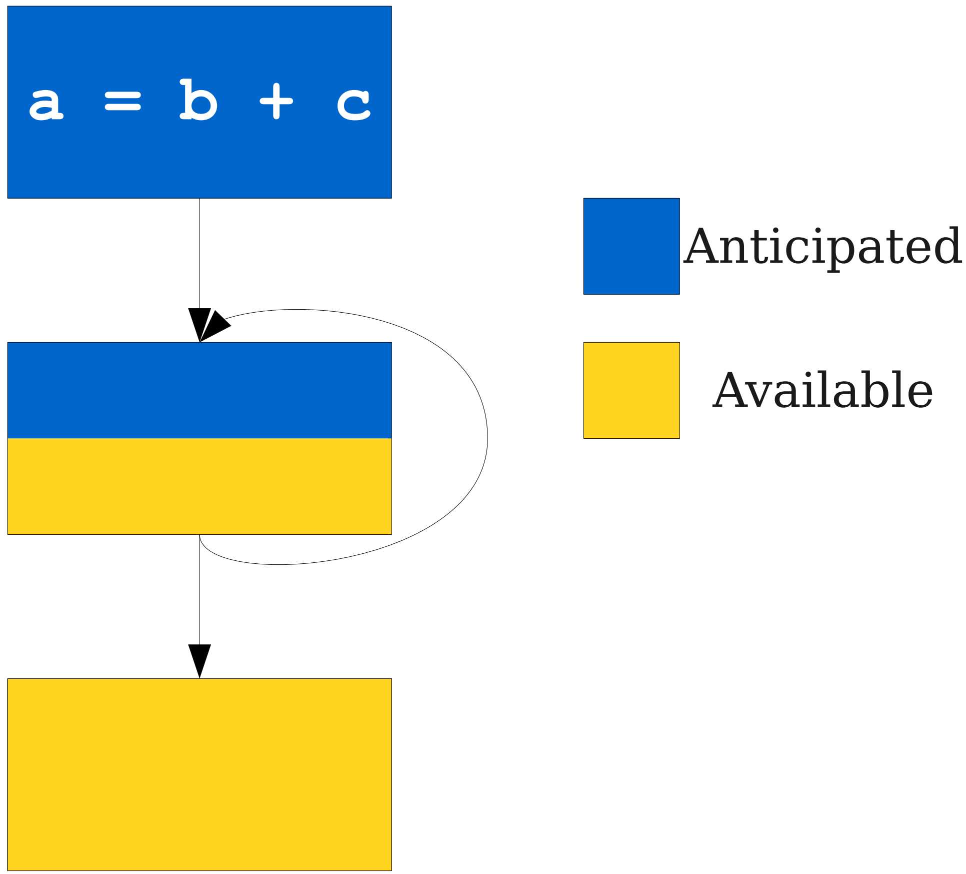

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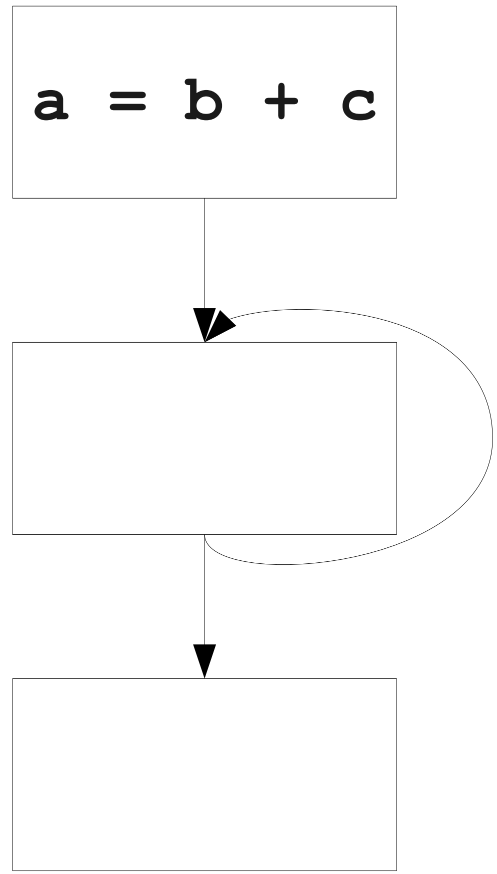

Eliminating Redundancy II

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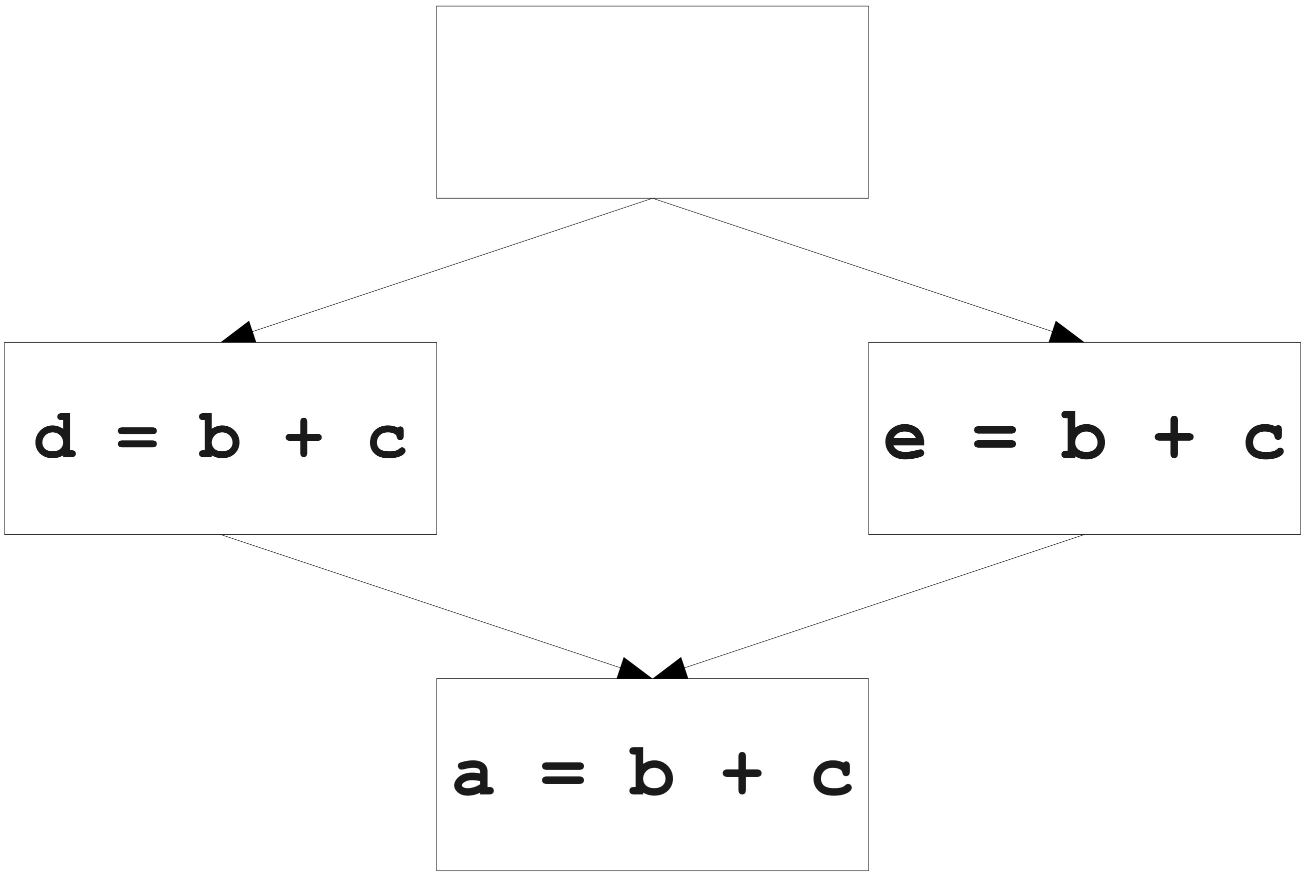

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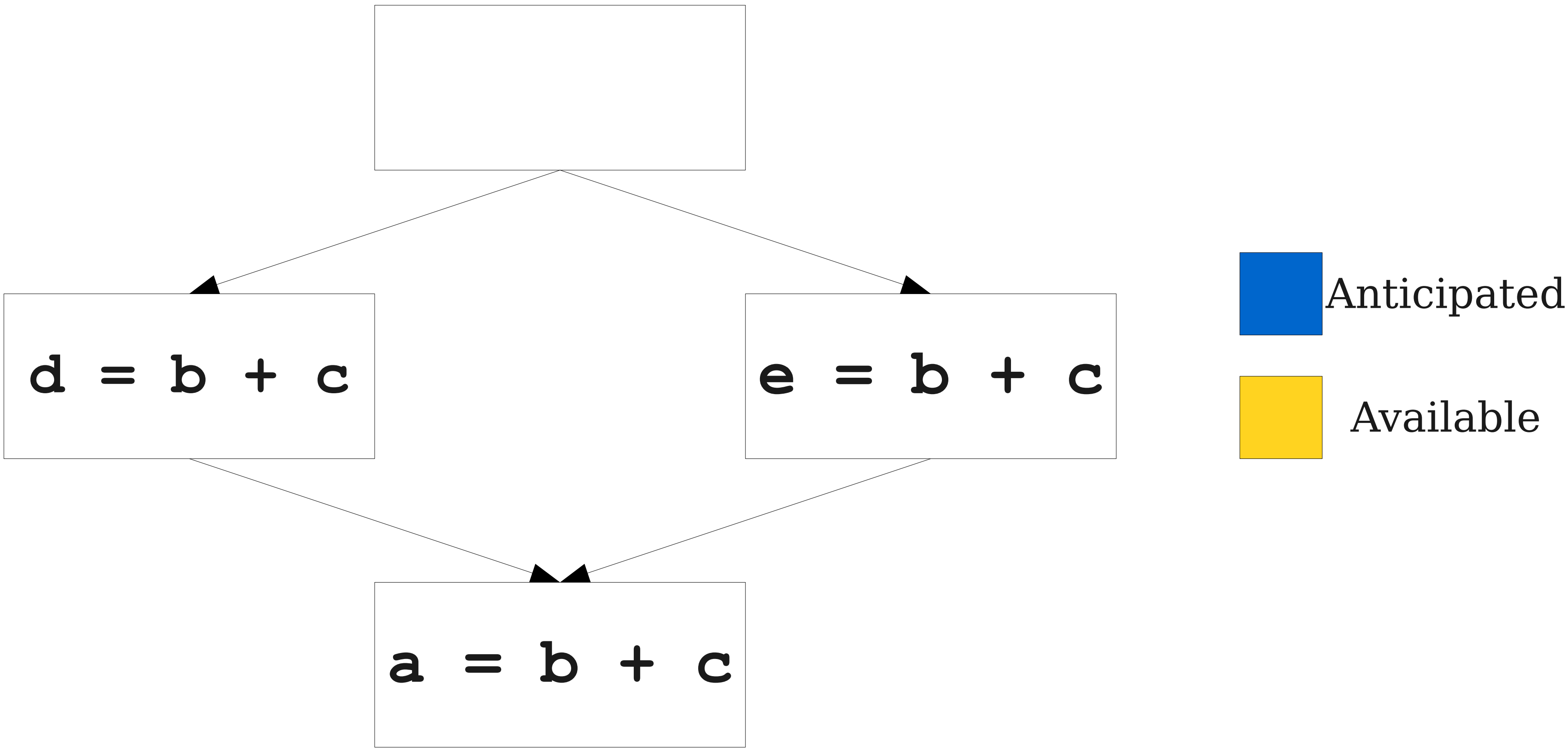

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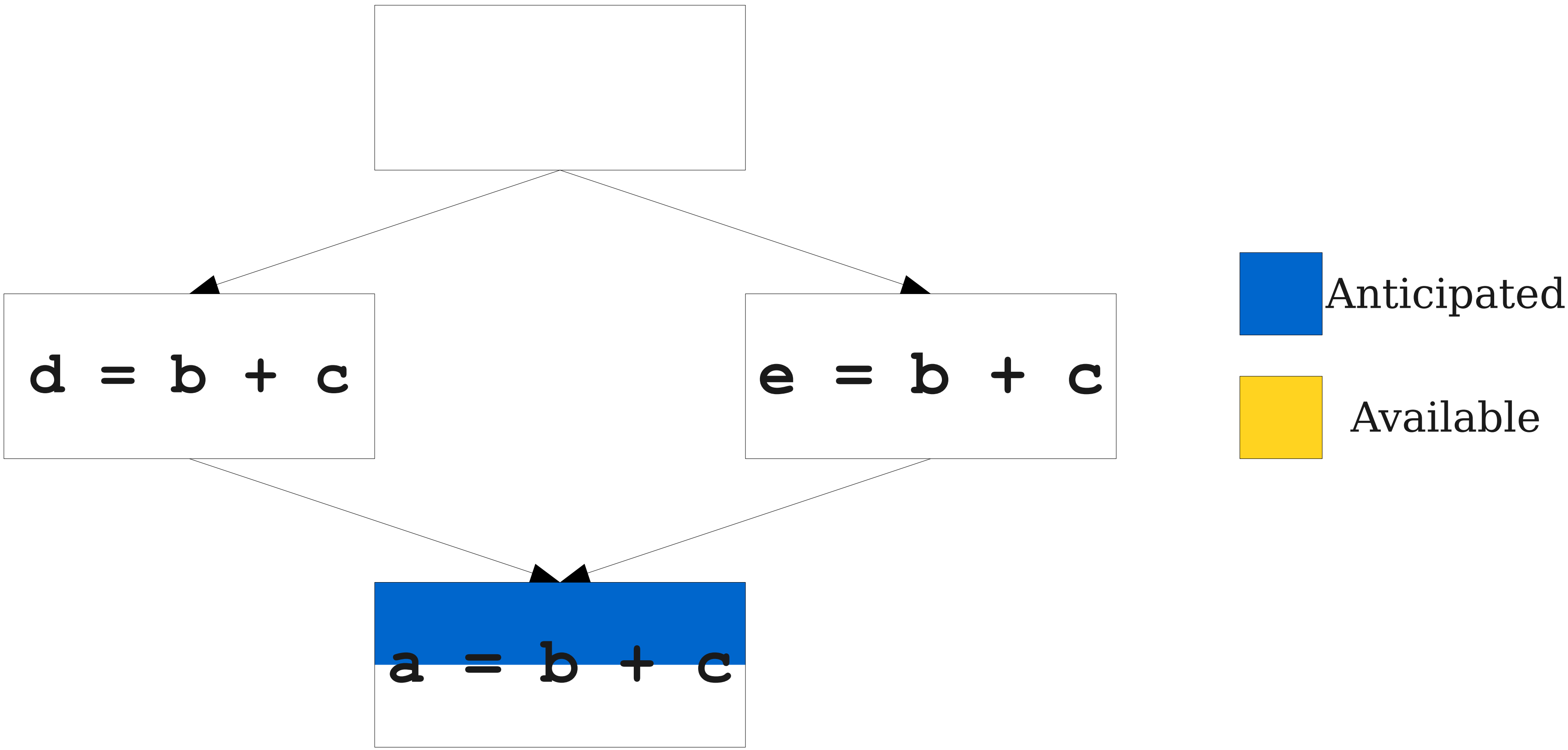

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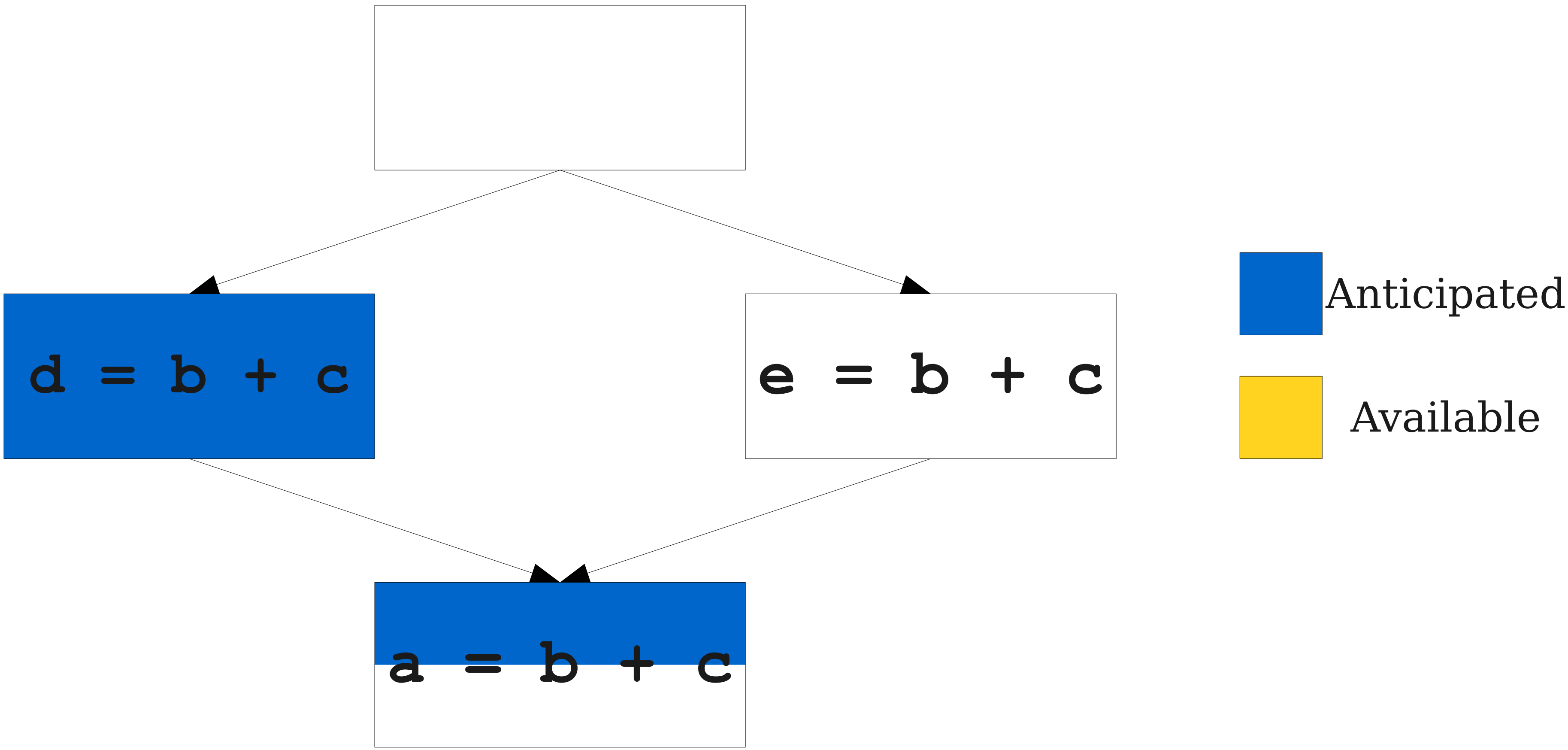

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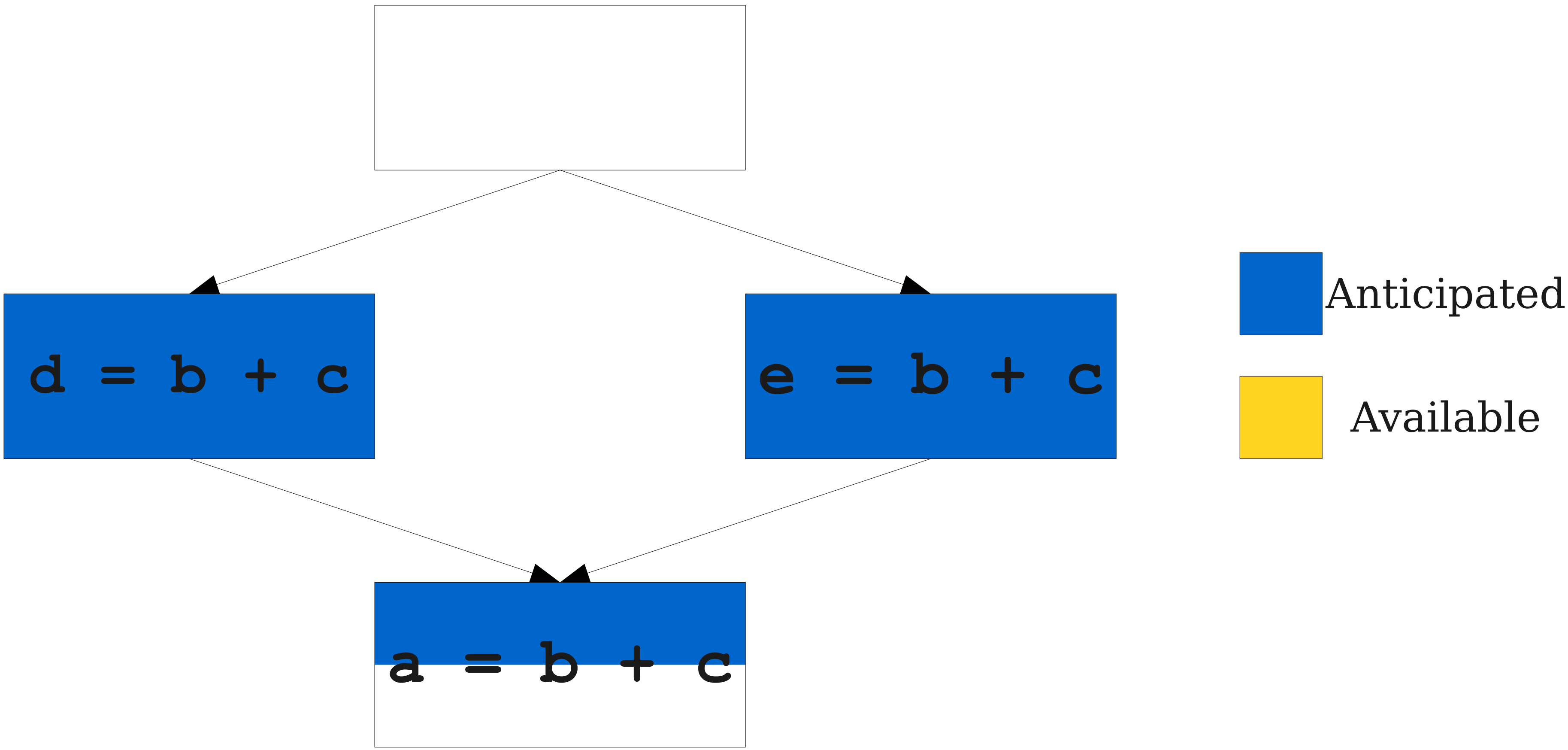

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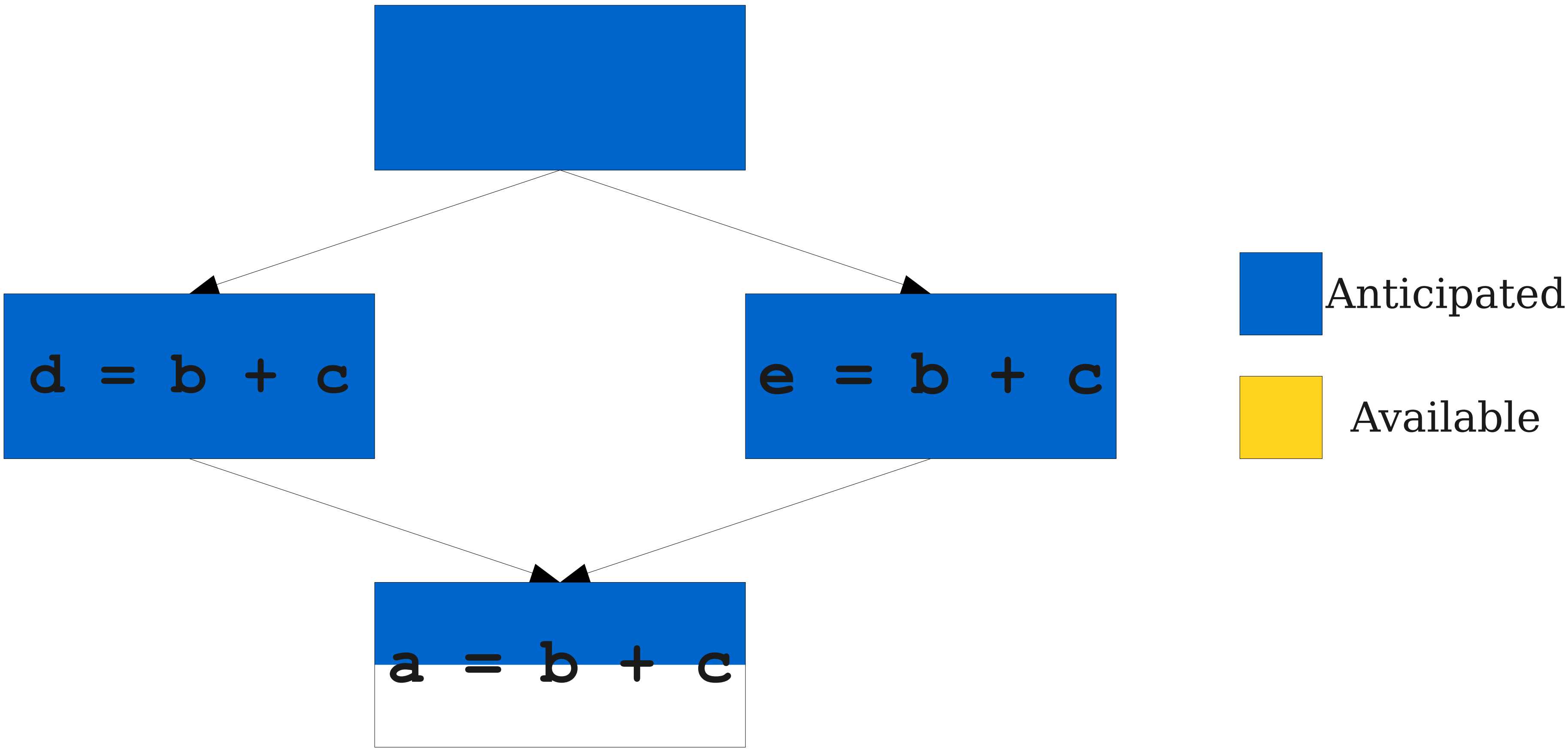

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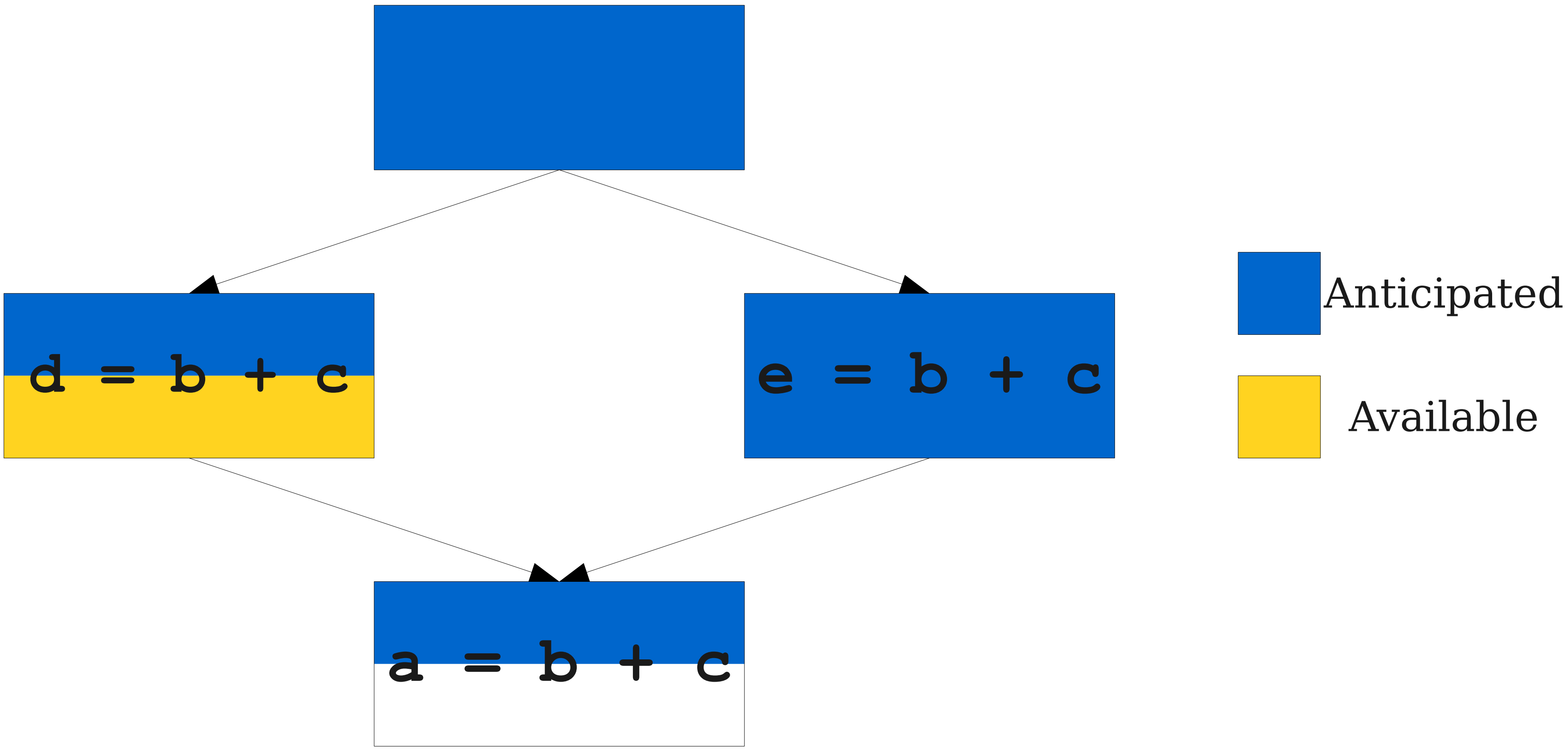

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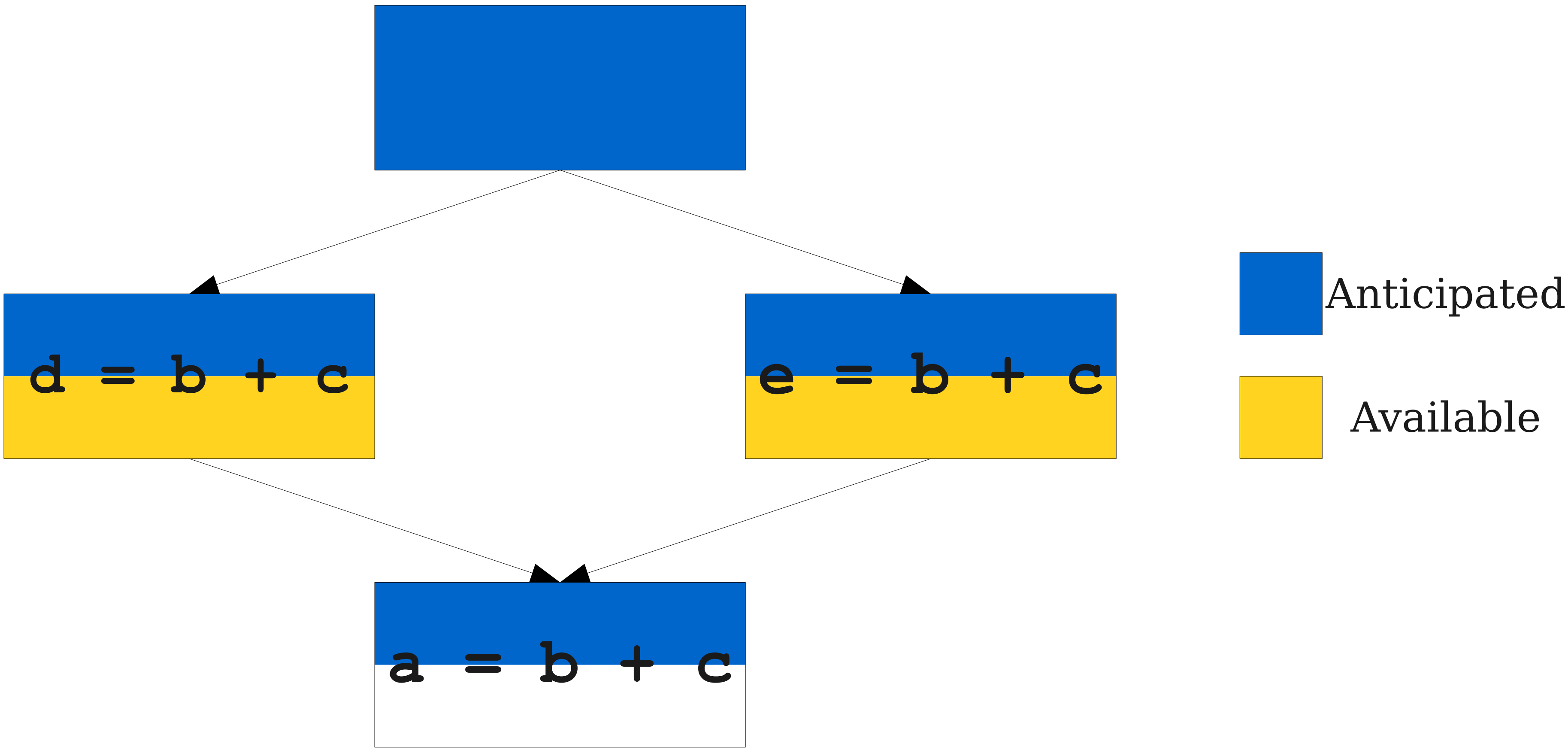

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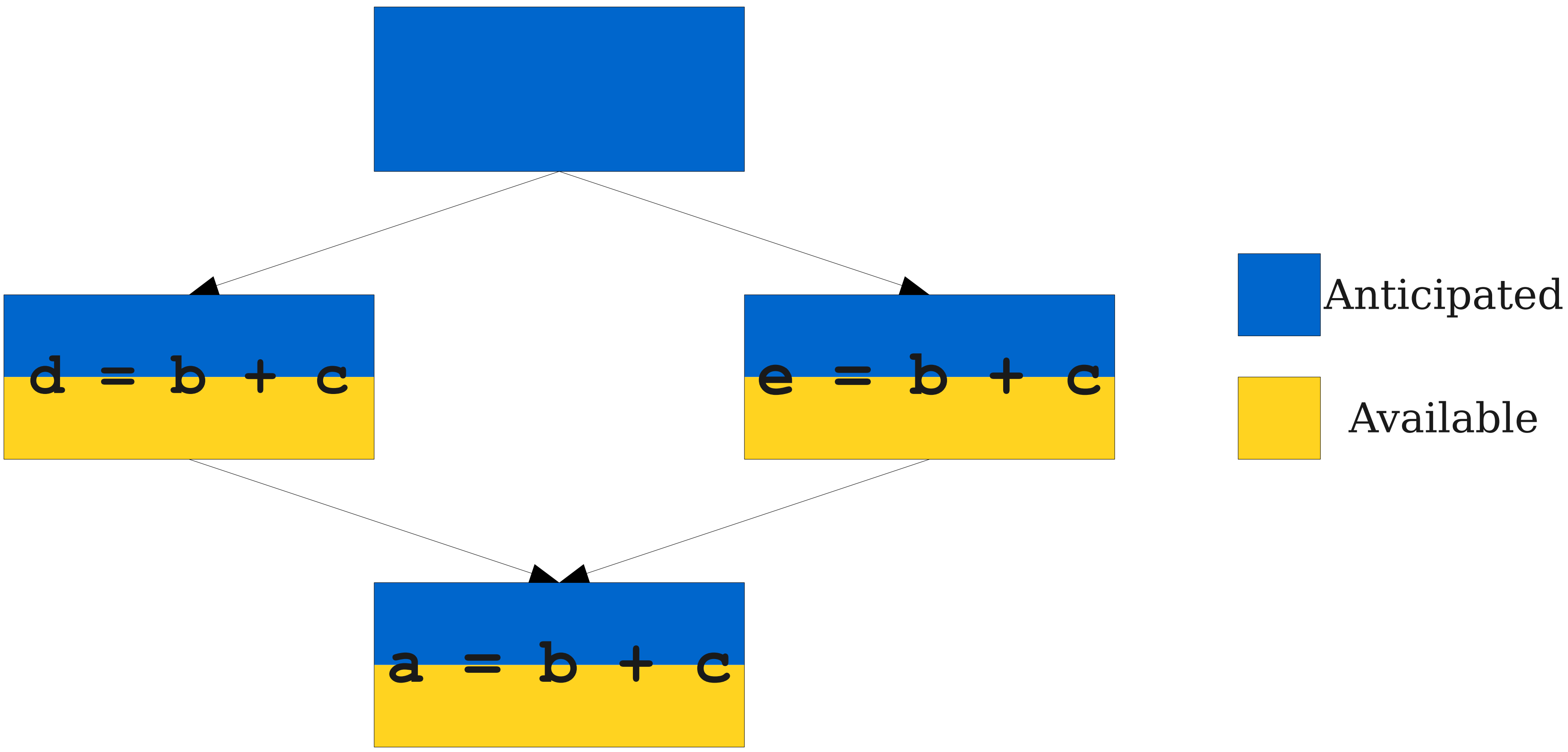

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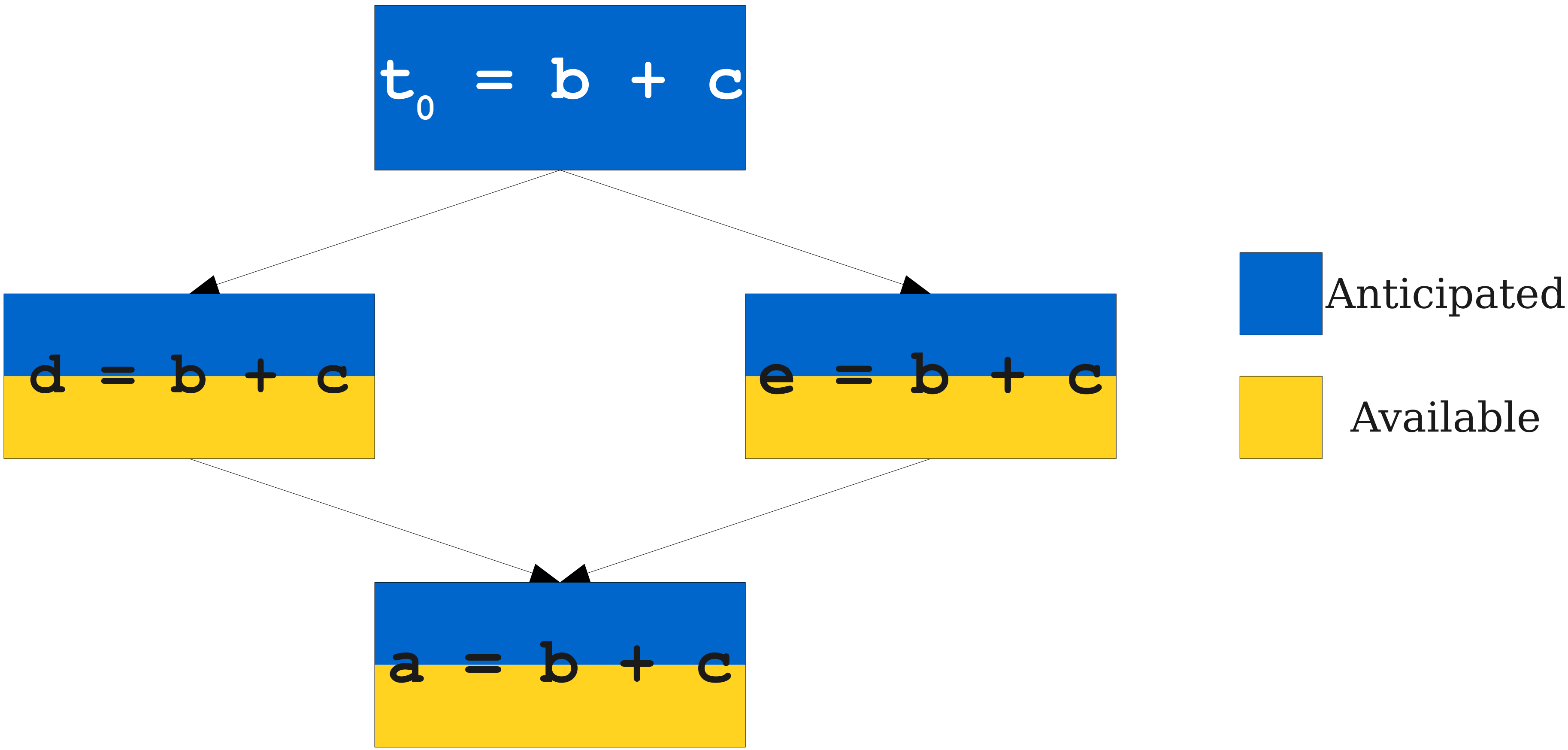

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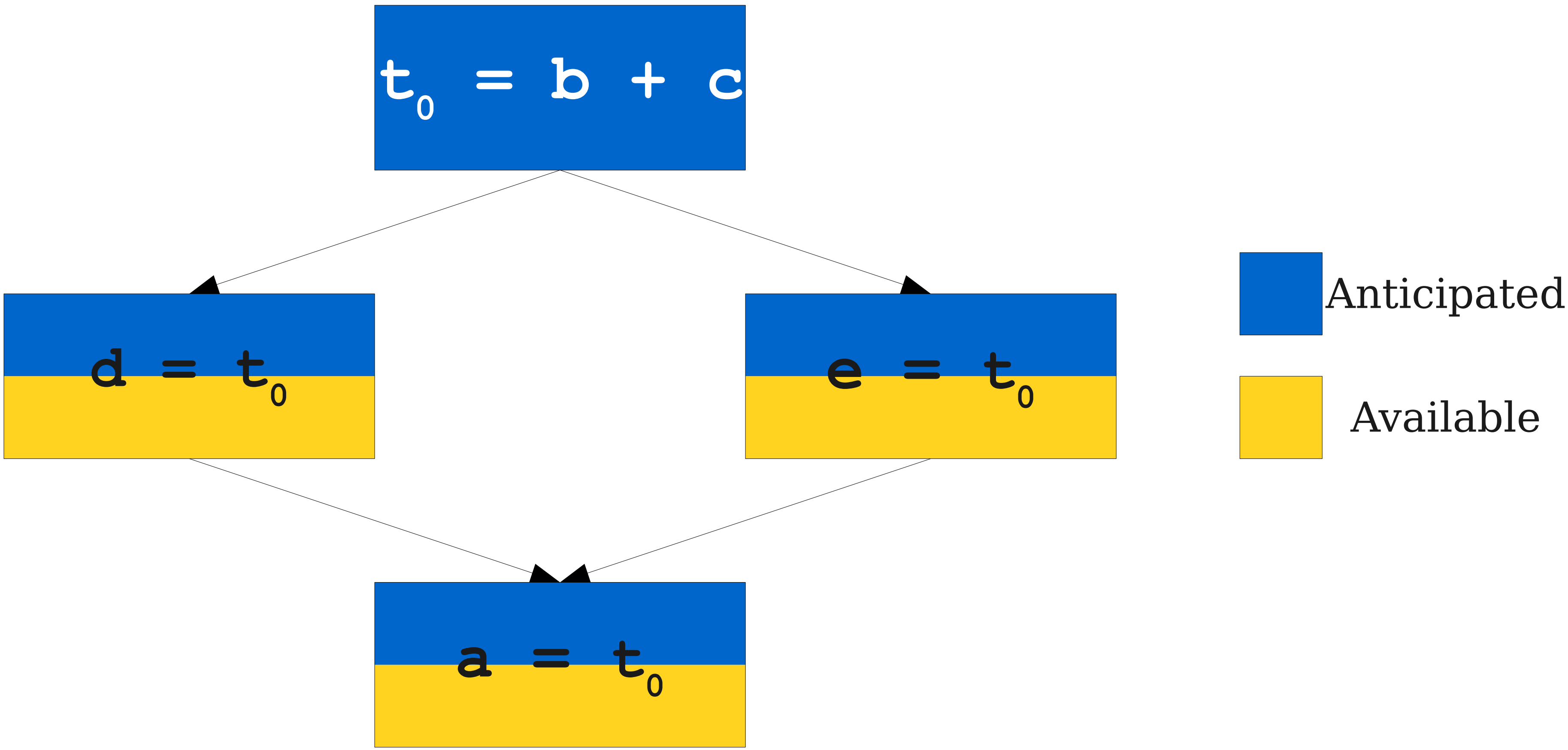

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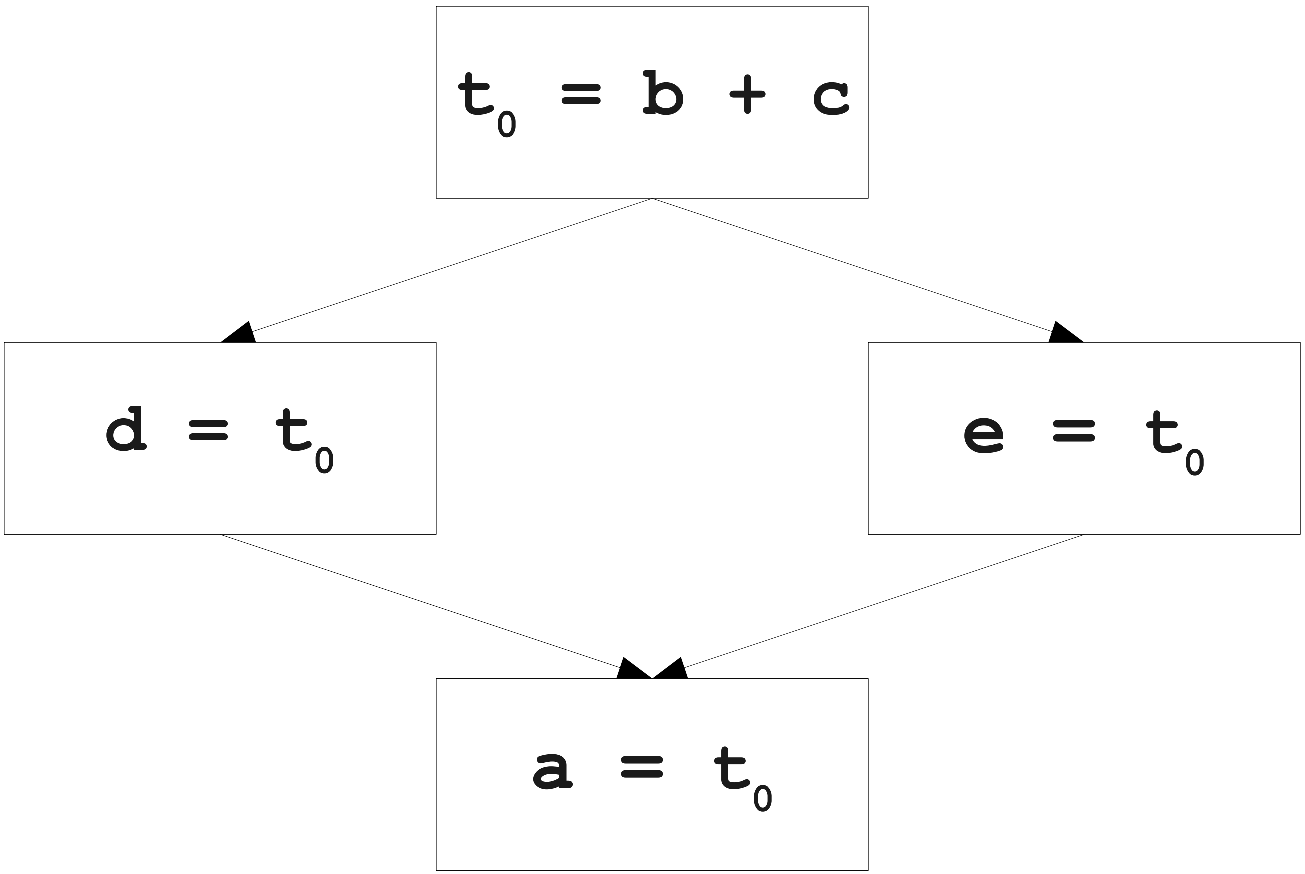

Eliminating Redundancy III

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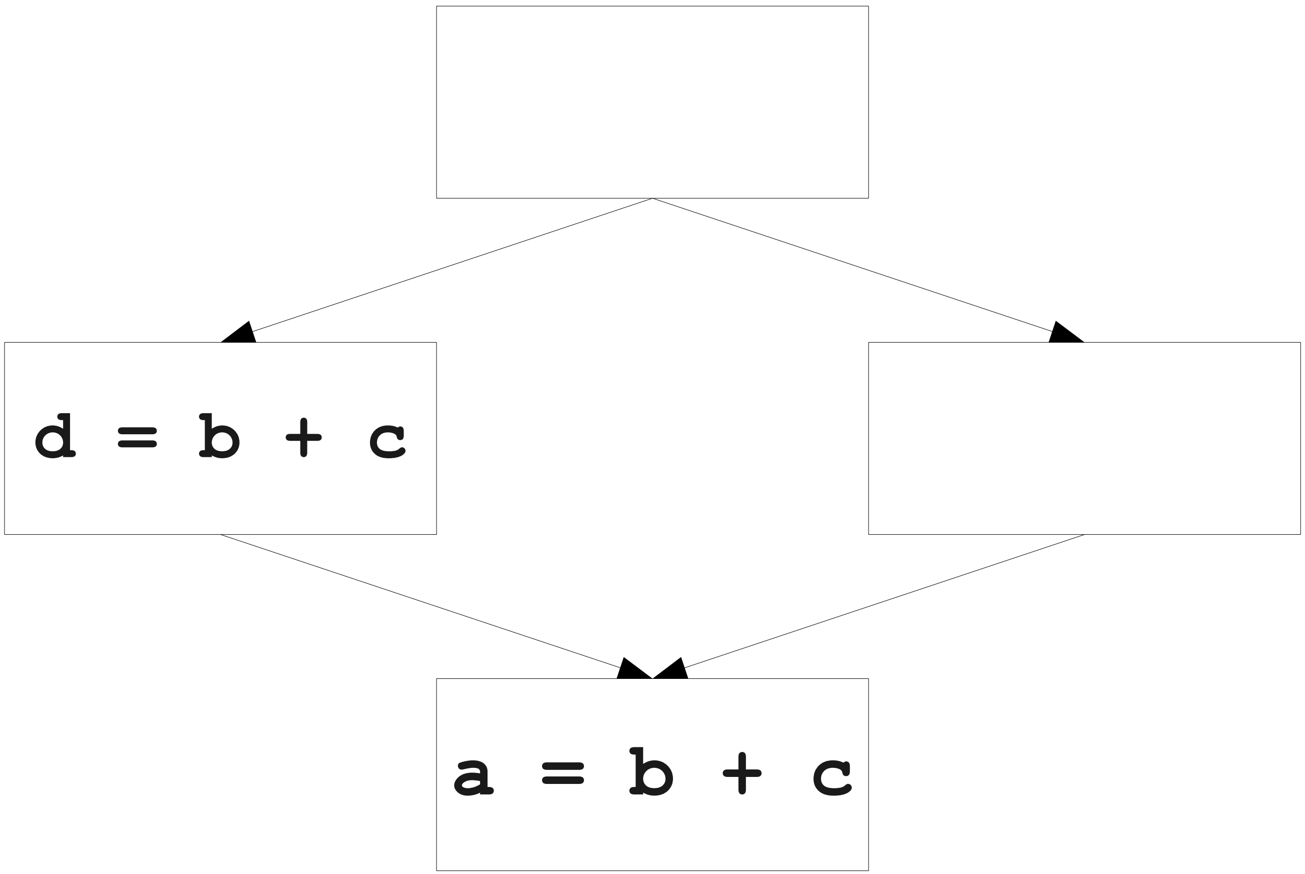

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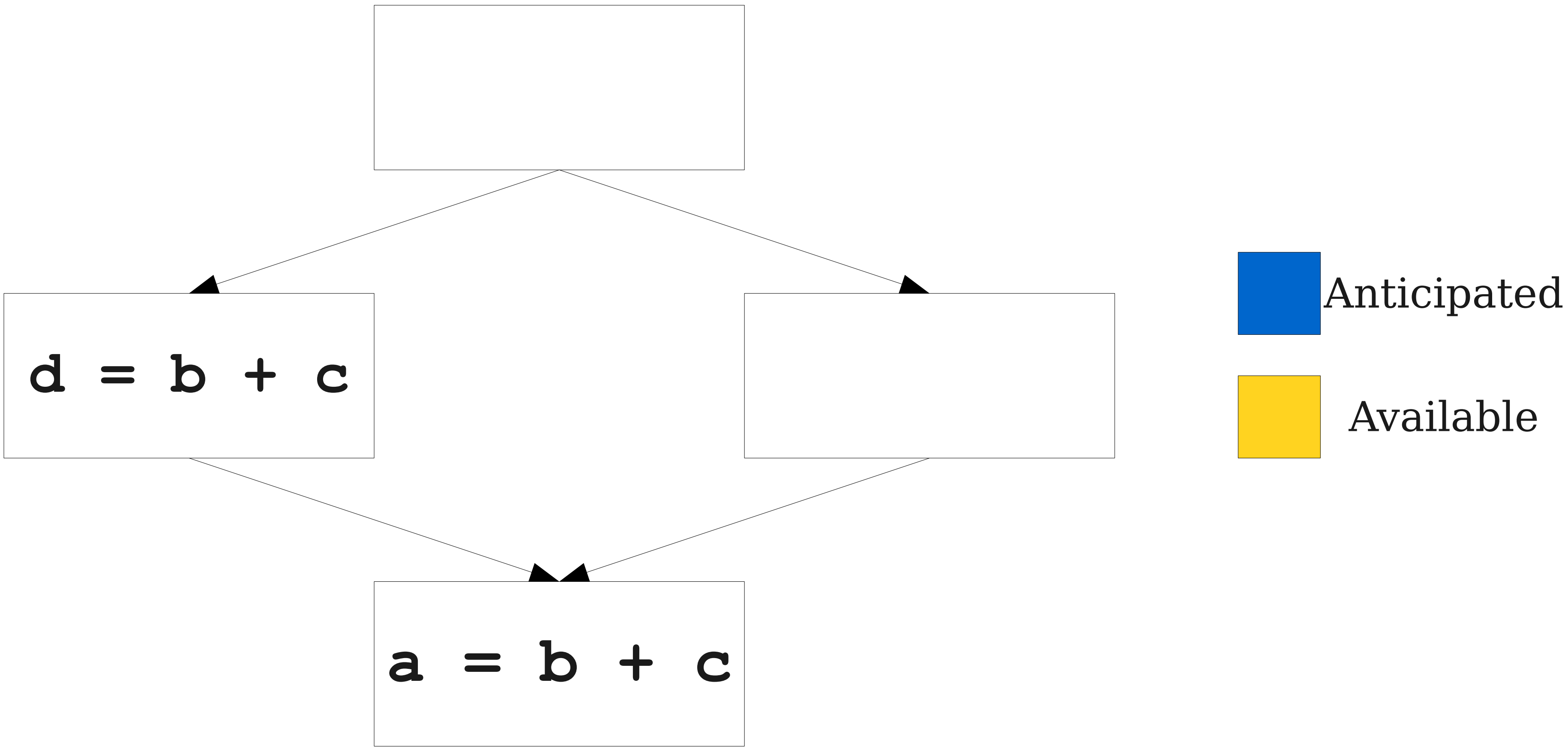

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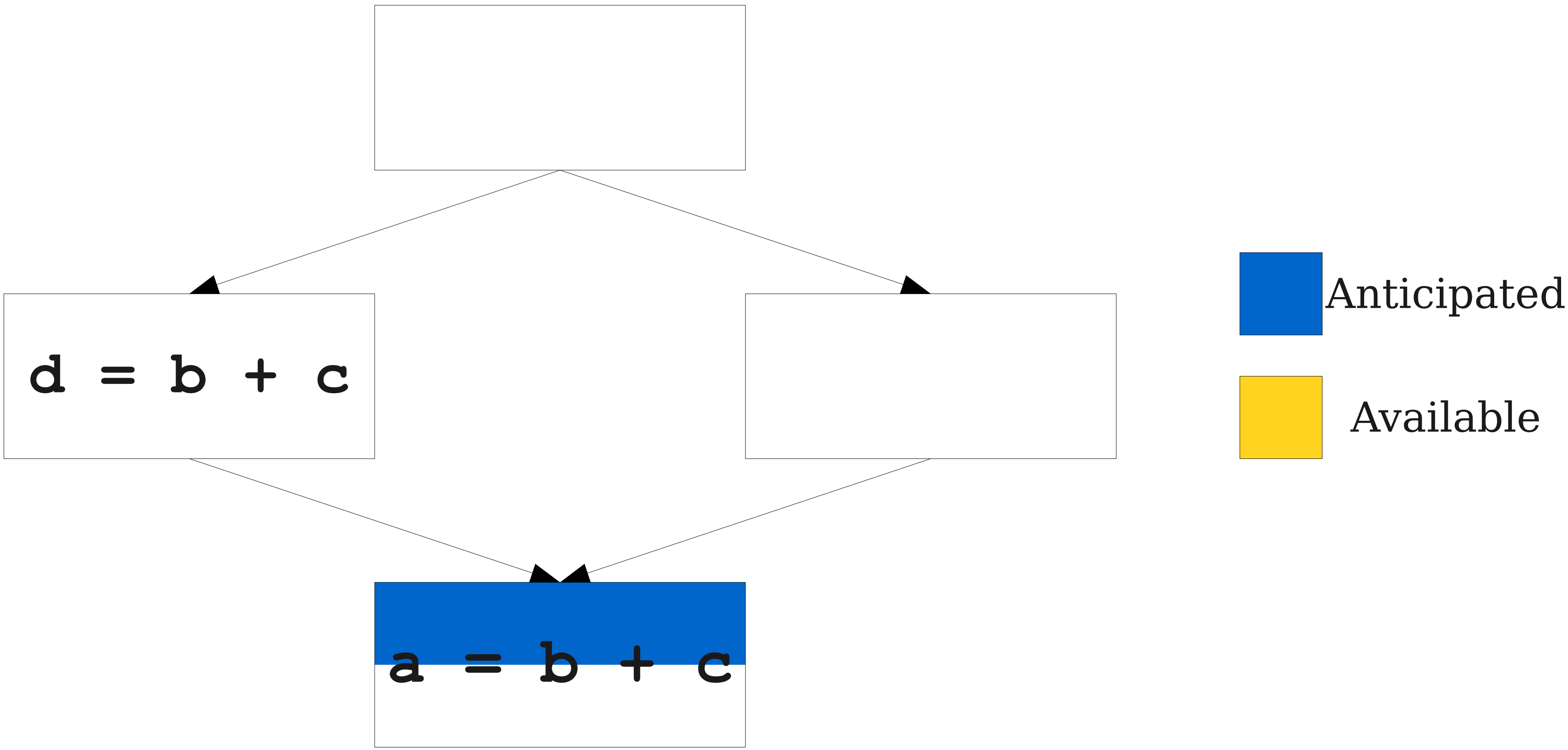

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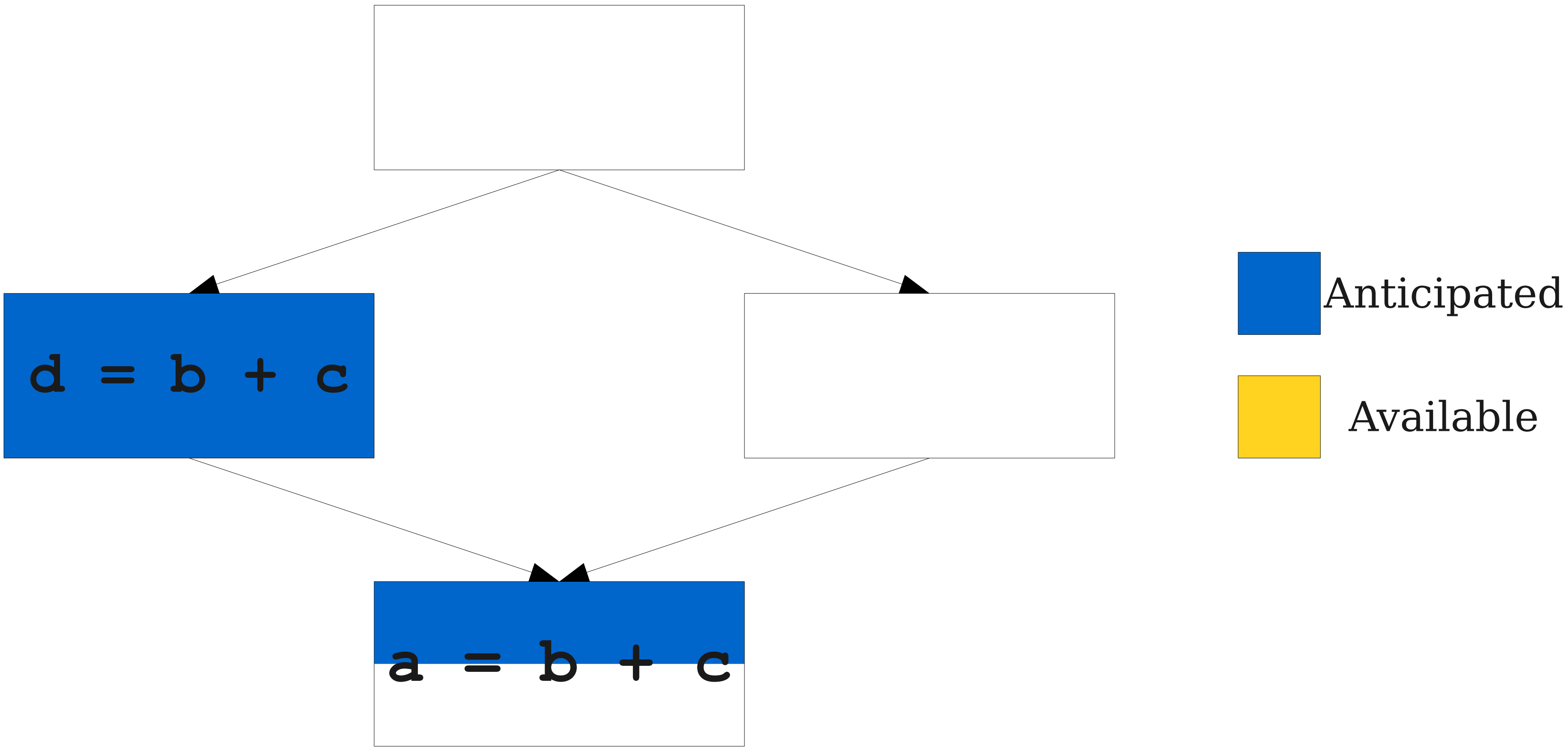

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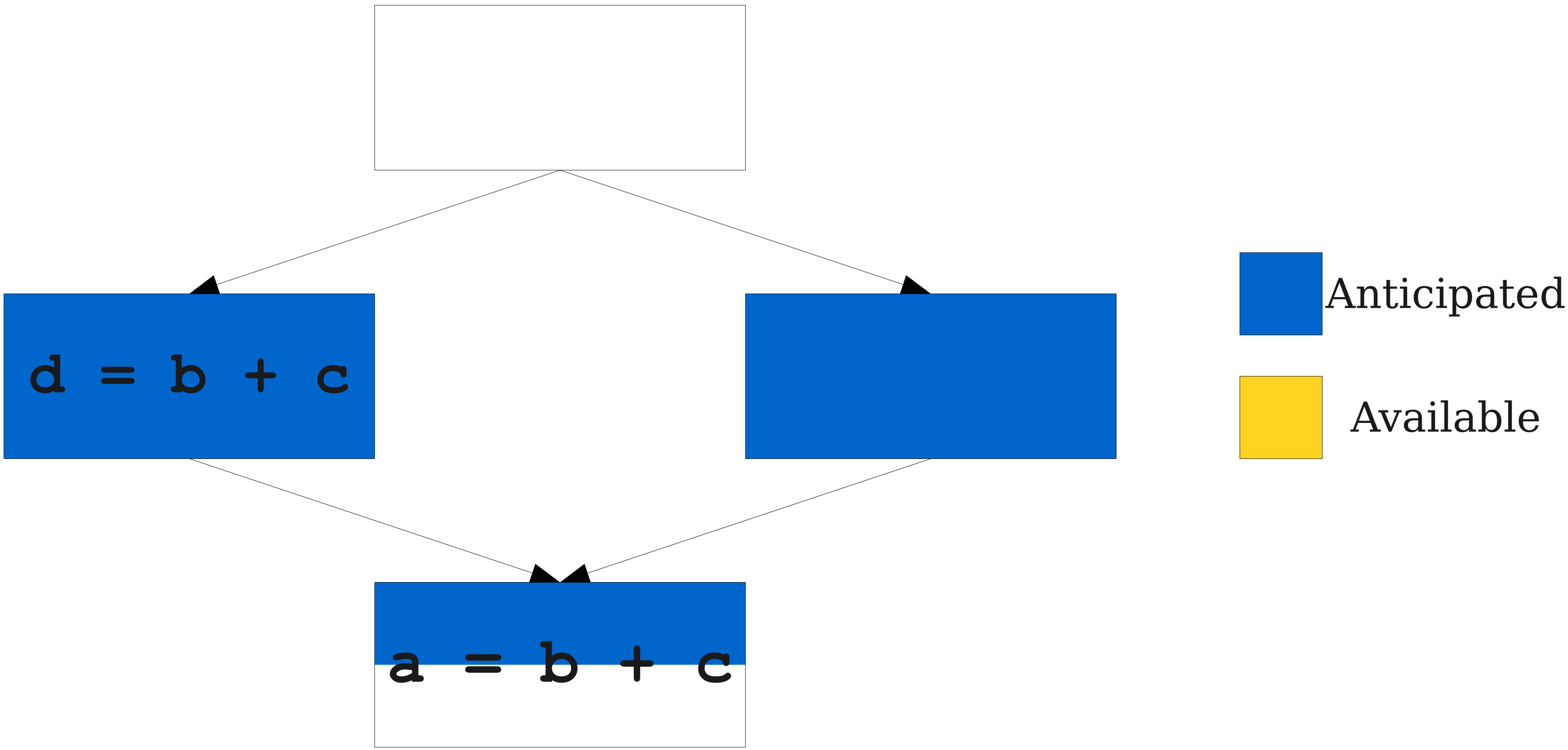

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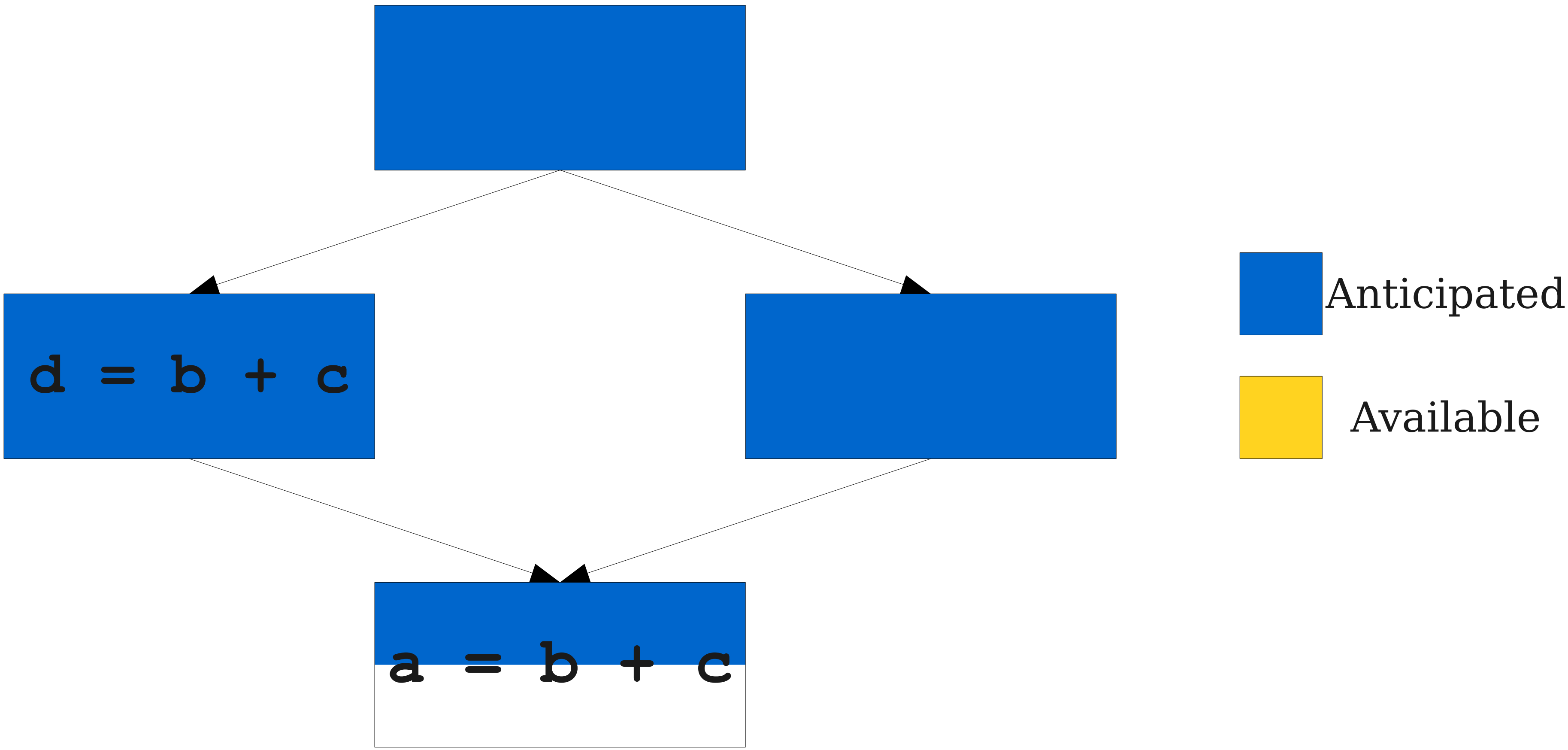

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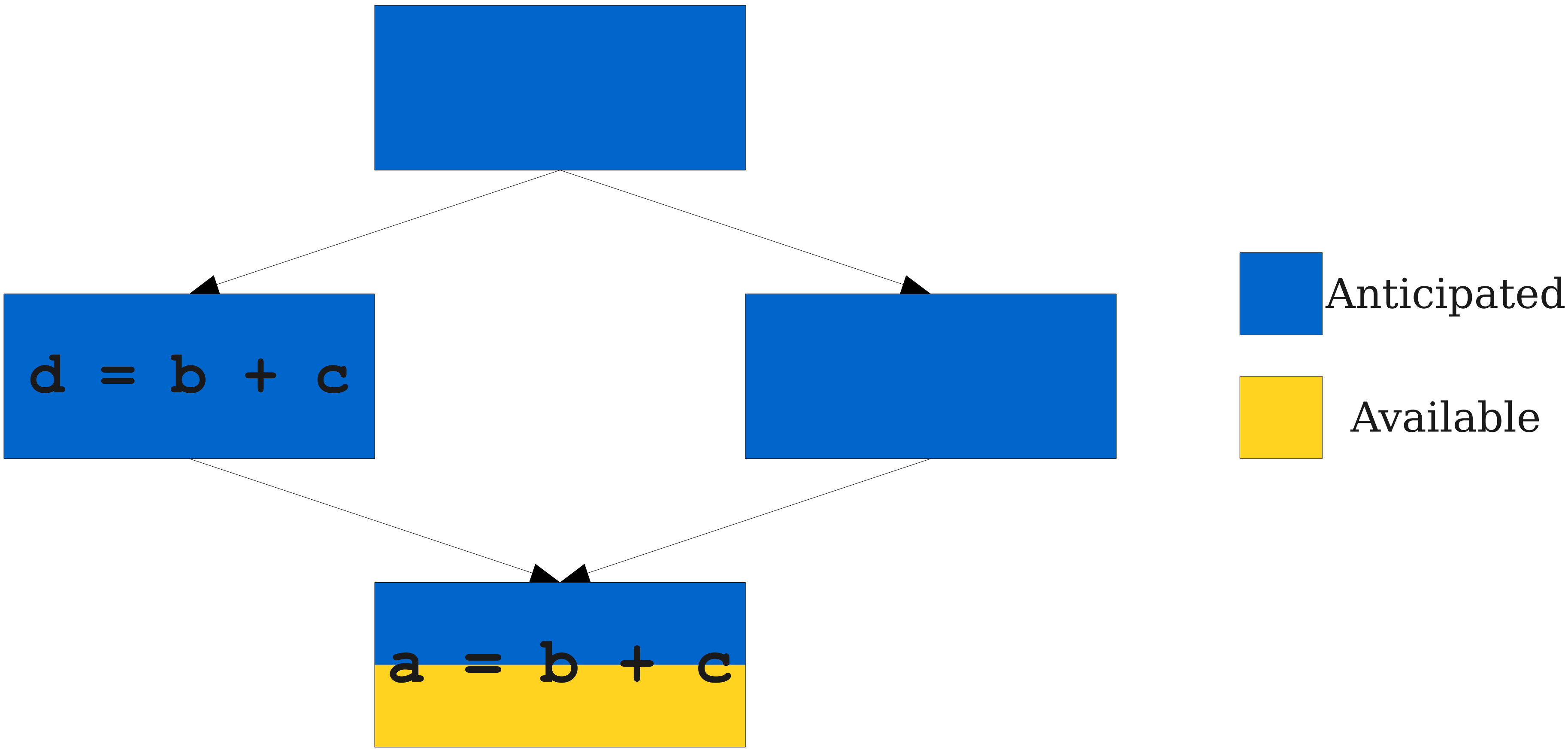

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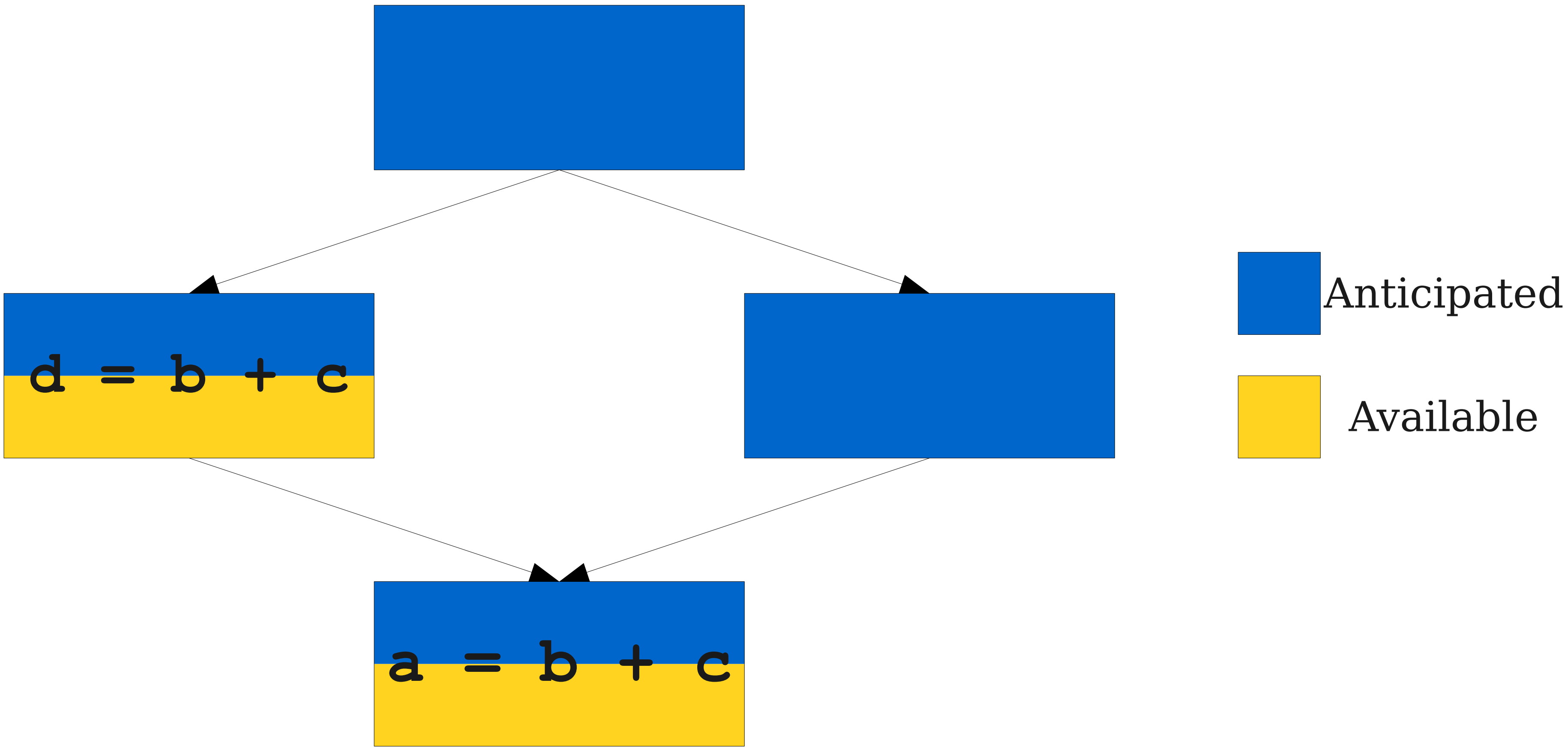

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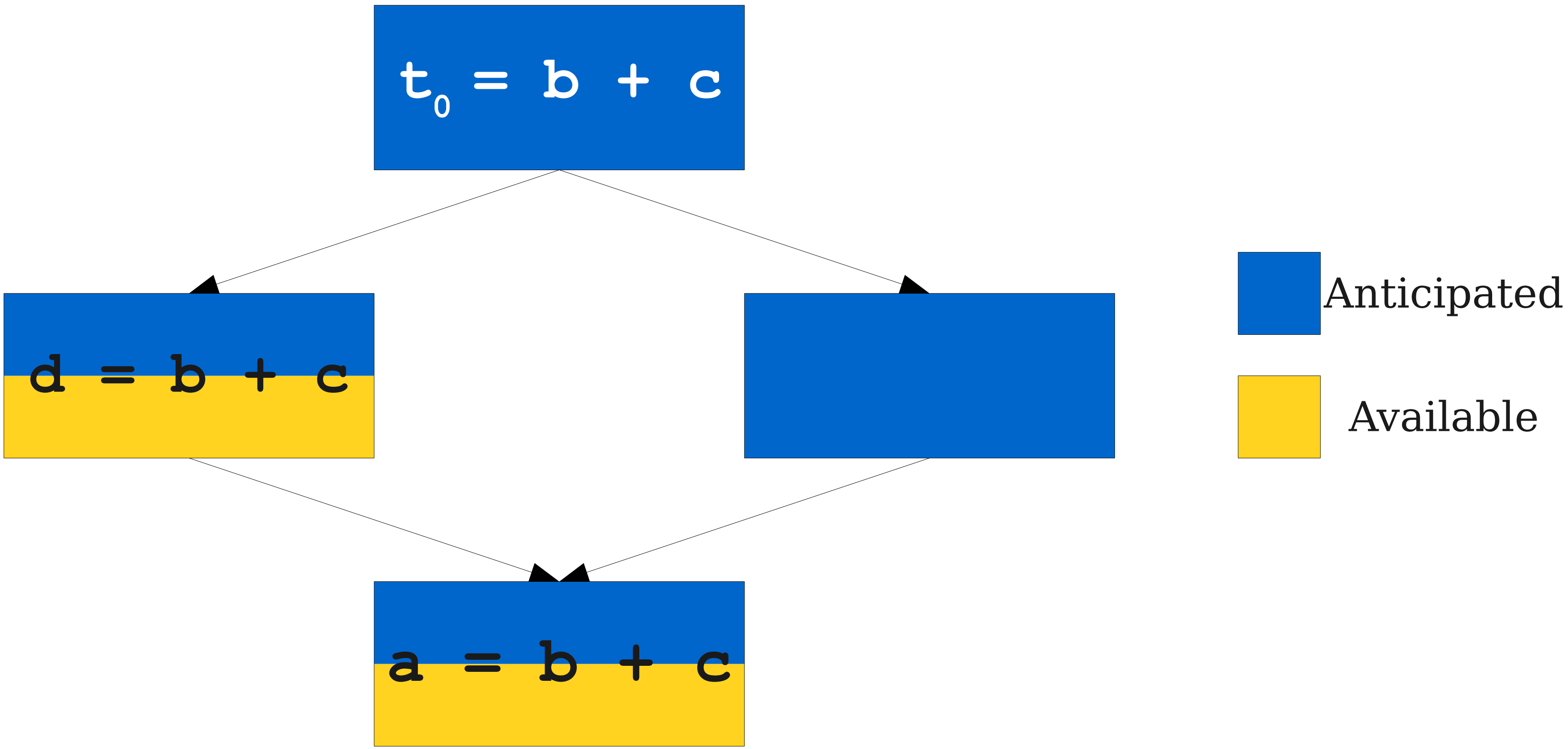

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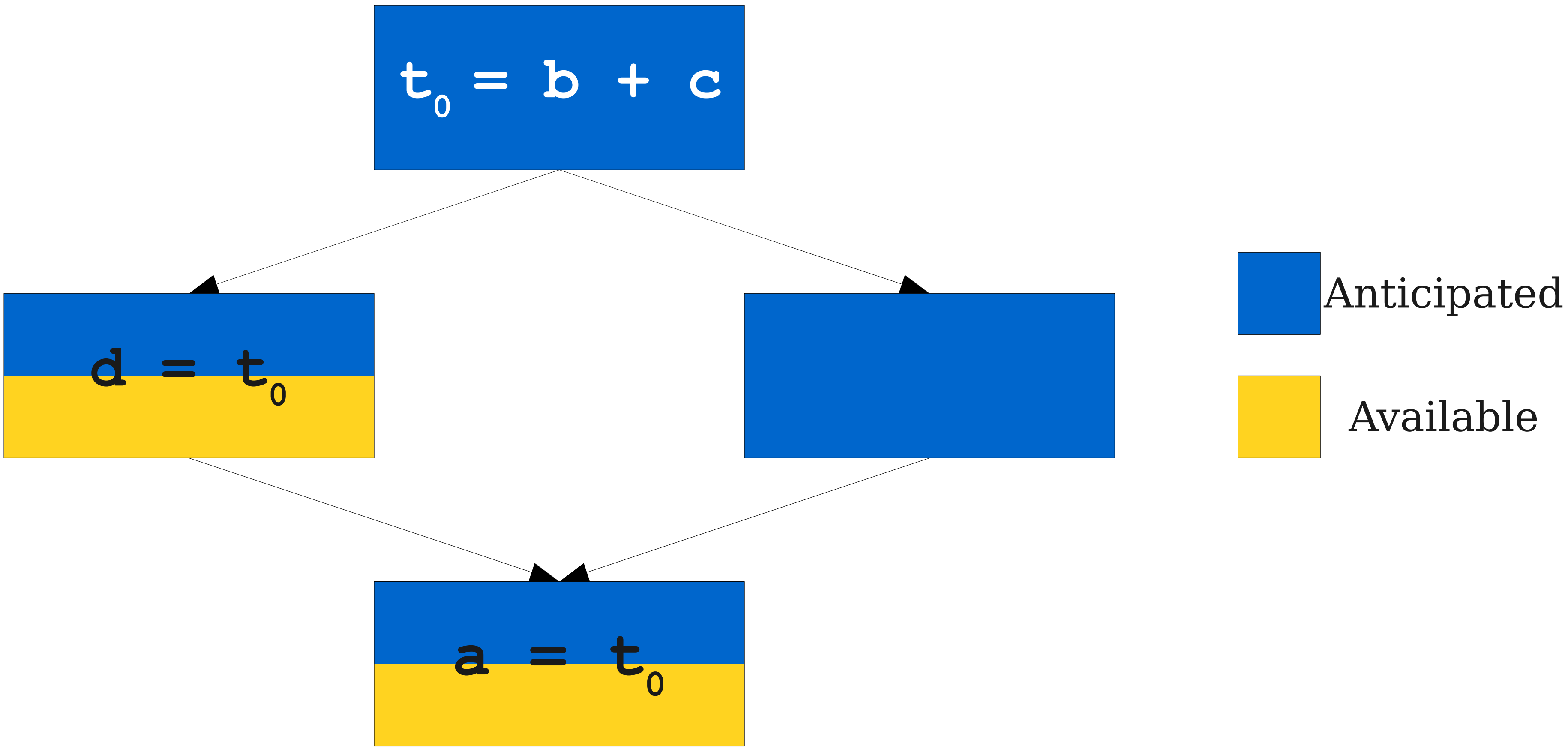

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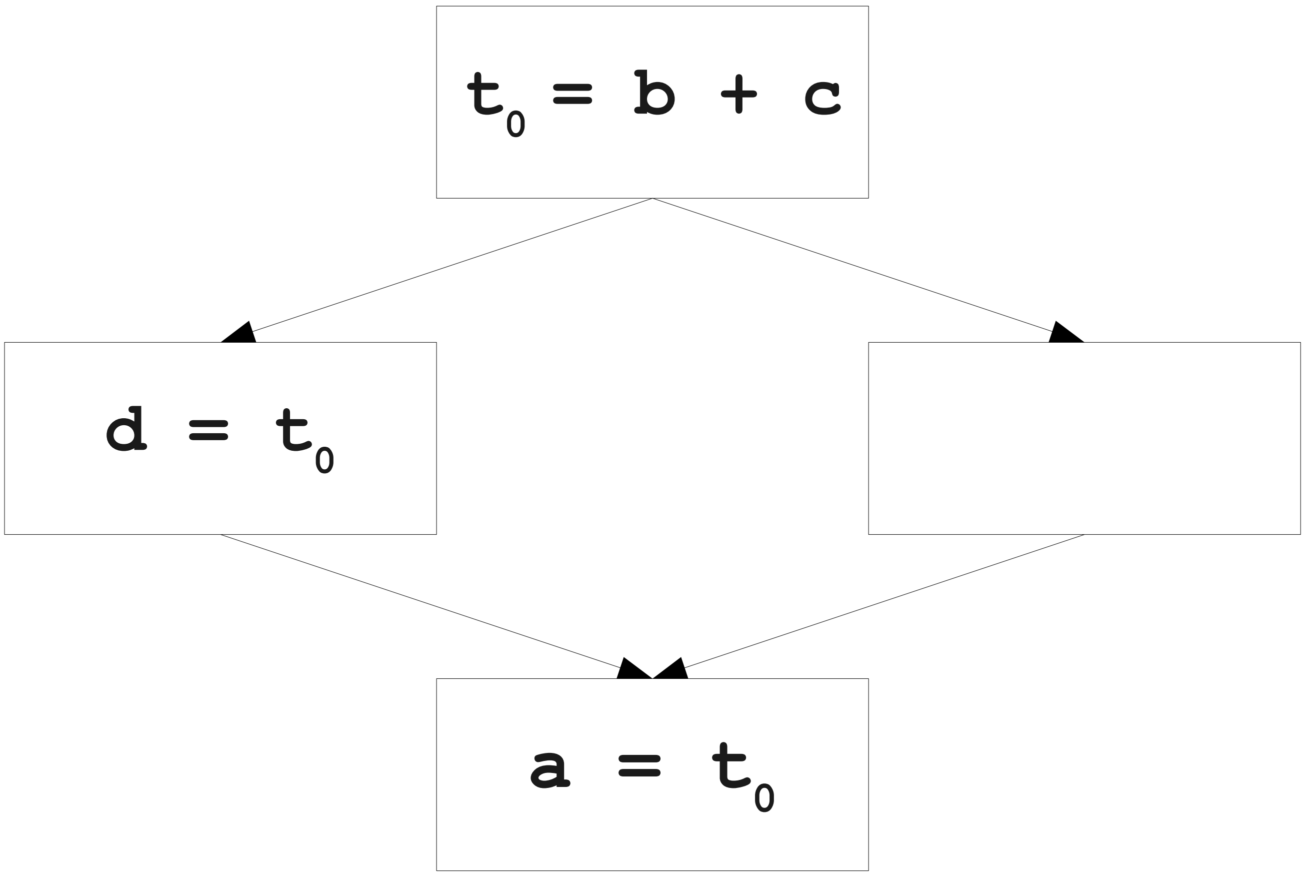

Partial Redundancy Elimination

- Powerful optimization; handles a huge number of disparate cases.
- Subsumes common subexpression elimination, loop invariant code motion, full redundancy elimination, and copy propagation.
- Almost all compilers do this.

In Practice

- Partial-redundancy elimination is typically implemented using *four* dataflow analyses.
  - Pass one: Determine where anticipated.
  - Pass two: Determine where available.
  - Pass three: Find best placement.
  - Pass four: Cleanup unnecessary temporaries.
- A bit more complex than what we covered:
  - Have to add basic blocks at some points.
  - For very complex CFGs, might miss some redundancy.
- See Dragon Book, Ch. 9.5 for more details.

Summary

- The **dataflow framework** gives a unified framework for defining global analyses.
- All of the following analyses can be formulated in the dataflow framework:
  - Global dead code elimination.
  - Global constant propagation.
  - Partial redundancy elimination.
- **Meet semilattices** give a way of describing how to initialize the analysis and combine intermediate results.
- **Monotone transfer functions**, combined with **finite-height lattices**, are necessary to guarantee termination.

Next Time

- **Register Allocation**
  - The memory hierarchy.
  - Naive register allocation.
  - Linear scan allocation.
  - Graph-coloring allocation.