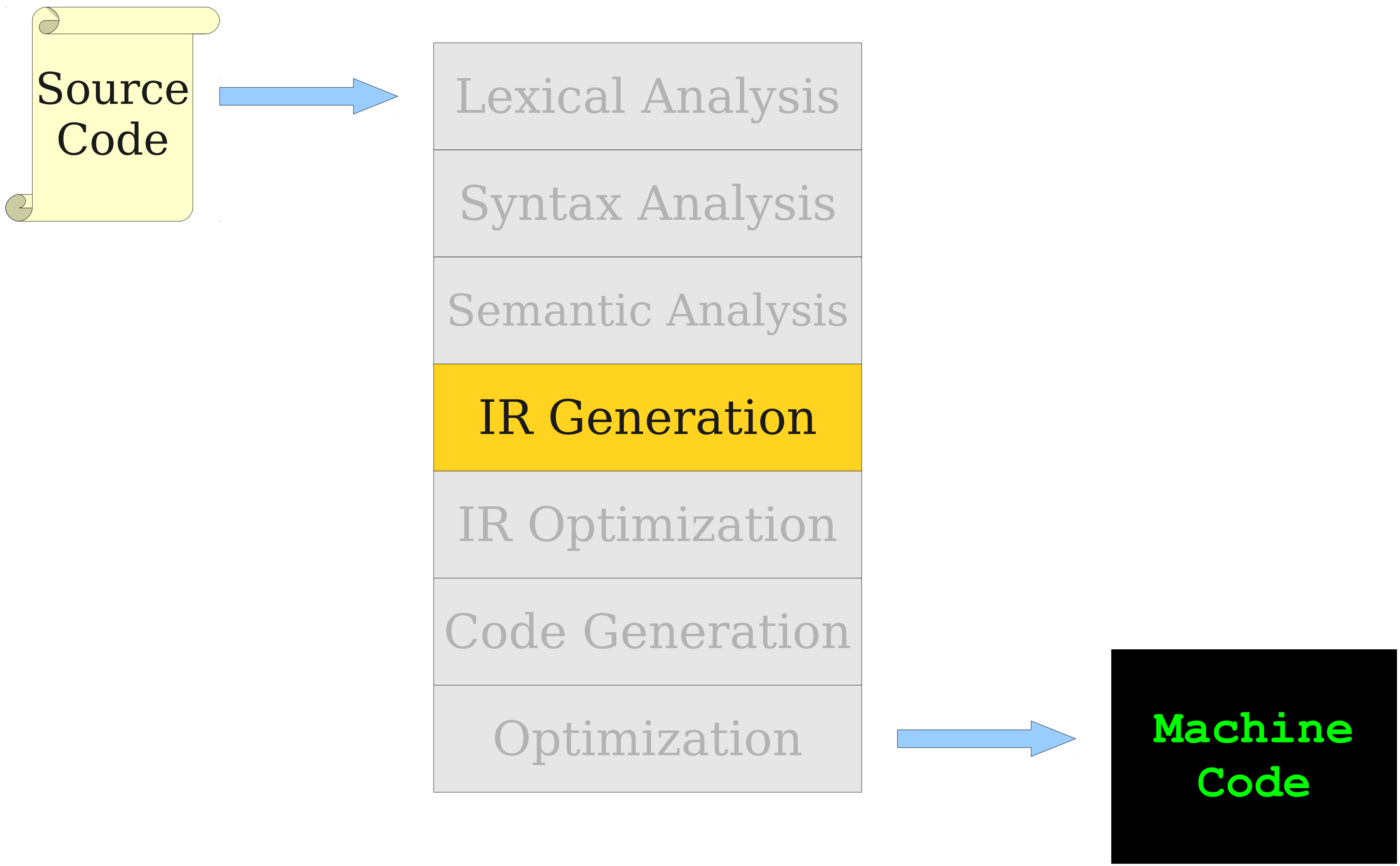
IR Optimization

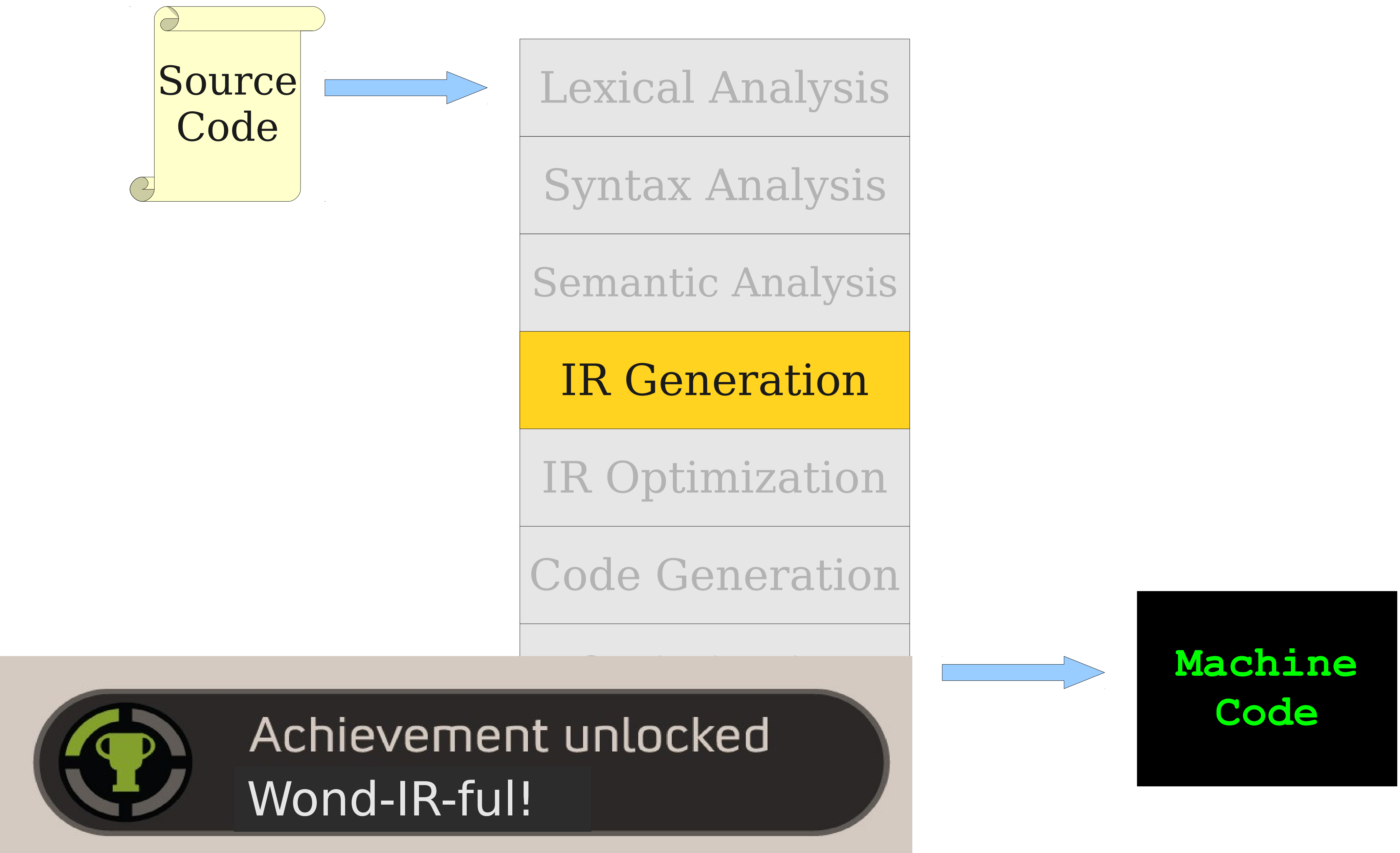
Announcements

- Programming Project 3 due Monday at 11:59PM.
- Programming Project 3 checkpoint graded; feedback emailed out.
  - Please let us know ASAP if you haven't heard back yet!

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

Where We Are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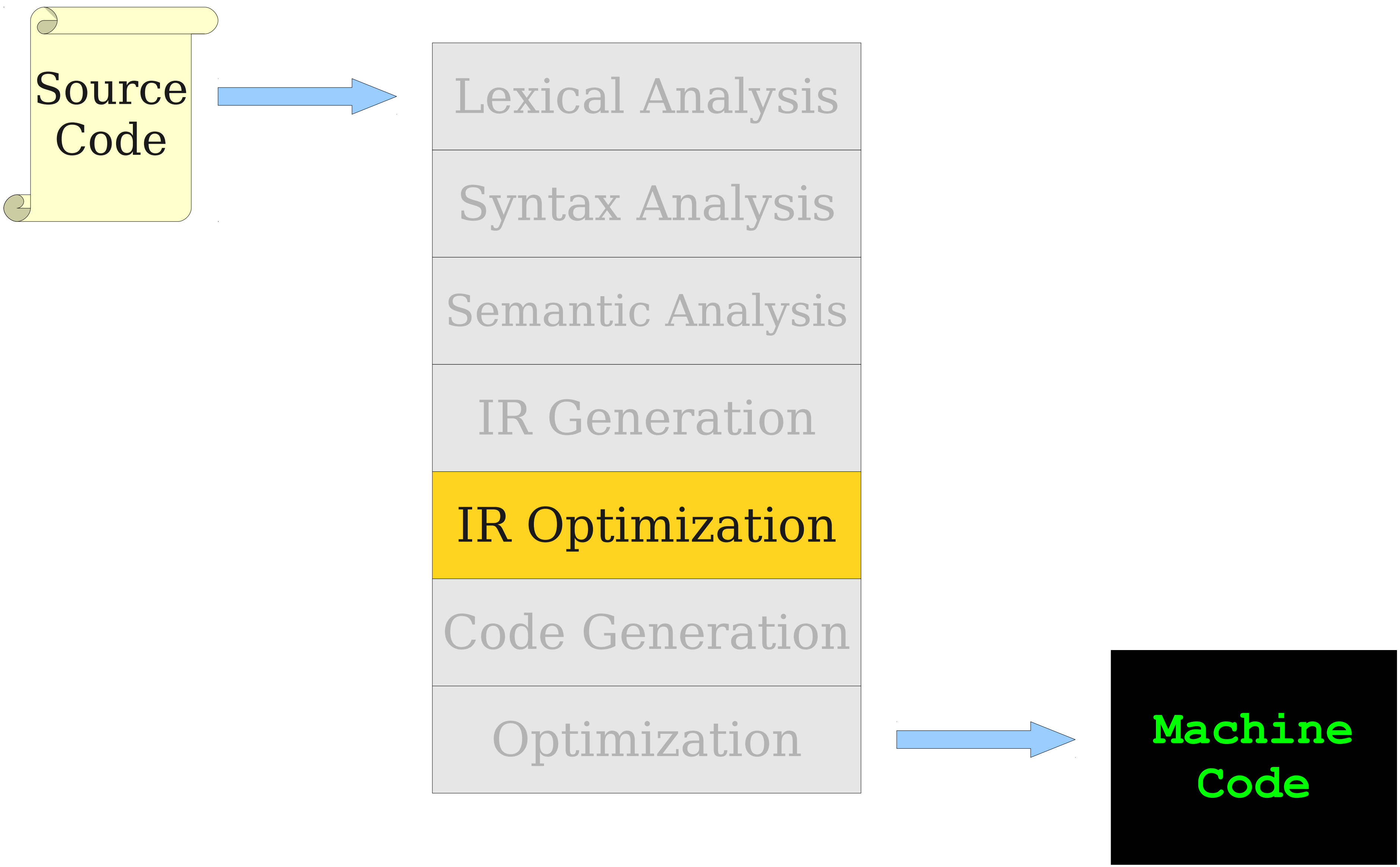
Achievement unlocked
Wond-IR-ful!

Where We Are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IR Optimization

- **Goal**: Improve the IR generated by the previous step to take better advantage of resources.
- One of the most important and complex parts of any modern compiler.
- A very active area of research.
- There is a whole class (CS243) dedicated to this material.

Sources of Optimization

- In order to optimize our IR, we need to understand why it can be improved in the first place.
- **Reason one:** IR generation introduces redundancy.
  - A naïve translation of high-level language features into IR often introduces subcomputations.
  - Those subcomputations can often be sped up, shared, or eliminated.
- **Reason two:** Programmers are lazy.
  - Code executed inside of a loop can often be factored out of the loop.
  - Language features with side effects often used for purposes other than those side effects.

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
bool b3;

b1 = x + x < y
b2 = x + x == y
b3 = x + x > y
```

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
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b1 = x + x < y
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```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

_t2 = x + x;
_t3 = y;
b2 = _t2 == _t3;

_t4 = x + x;
_t5 = y;
b3 = _t5 < _t4;
```

Optimizations from IR Generation

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b1 = x + x < y
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```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

b2 = _t0 == _t1;

b3 = _t0 < _t1;
```

```
while (x < y + z) {
    x = x - y;
}
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
```

```
_L0:
    _t0 = y + z;
    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
    x = x - y;
    Goto _L0;
_L1:
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
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_L0:
    _t0 = y + z;
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while (x < y + z) {
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    _t0 = y + z;
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    x = x - y;
    Goto _L0;
_L1:
```

A Note on Terminology

- The term “optimization” implies looking for an “optimal” piece of code for a program.
- This is, in general, undecidable.
  - e.g. create a program that can be simplified iff some other program halts.
- Our goal will be IR *improvement* rather than IR *optimization*.

The Challenge of Optimization

- A good optimizer
  - Should never change the observable behavior of a program.
  - Should produce IR that is as efficient as possible.
  - Should not take too long to process inputs.
- Unfortunately:
  - Even good optimizers sometimes introduce bugs into code.
  - Optimizers often miss “easy” optimizations due to limitations of their algorithms.
  - Almost all interesting optimizations are **NP**-hard or undecidable.

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?
- **Runtime** (make the program as fast as possible at the expense of time and power)
- **Memory usage** (generate the smallest possible executable at the expense of time and power)
- **Power consumption** (choose simple instructions at the expense of speed and memory usage)
- Plus a lot more (minimize function calls, reduce use of floating-point hardware, etc.)

IR Optimization vs Code Optimization

- There is not always a clear distinction between what belongs to "IR optimization" versus "code optimization."
- Typically:
  - IR optimizations try to perform simplifications that are valid across all machines.
  - Code optimizations try to improve performance based on the specifics of the machine.
- Some optimizations are somewhere in-between:
  - Replacing **`x * 0.5`** with **`x / 2`**

Overview of IR Optimization

- **Formalisms and Terminology** (Today)
  - Control-flow graphs.
  - Basic blocks.
- **Local optimizations** (Today)
  - Speeding up small pieces of a function.
- **Global optimizations** (Monday)
  - Speeding up functions as a whole.
- **The dataflow framework** (Monday/Wednesday)
  - Defining and implementing a wide class of optimizations.

Formalisms and Terminology

Analyzing a Program

- In order to optimize a program, the compiler has to be able to reason about the properties of that program.
- An analysis is called **sound** if it never asserts an incorrect fact about a program.
- All the analyses we will discuss in this class are sound.
  - *(Why?)*

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

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int x;
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if (y < 5)
    x = 137;
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Print(x);
```

"At this point in the program, **x** holds some integer value."

Soundness

```
int x;
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if (y < 5)
    x = 137;
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Print(x);
```

"At this point in the program, **x** is either 137 or 42"

Soundness

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int x;
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if (y < 5)
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else
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Print(x);
```

"At this point in the program, **x** is 137"

Soundness

```
int x;
int y;

if (y < 5)
    x = 137;
else
    x = 42;

Print(x);
```

"At this point in the program, **x** is either 137, 42, or 271"

Semantics-Preserving Optimizations

- An optimization is **semantics-preserving** if it does not alter the semantics of the original program.
- Examples:
  - Eliminating unnecessary temporary variables.
  - Computing values that are known statically at compile-time instead of runtime.
  - Evaluating constant expressions outside of a loop instead of inside.
- Non-examples:
  - Replacing bubble sort with quicksort.
- The optimizations we will consider in this class are all semantics-preserving.

A Formalism for IR Optimization

- Every phase of the compiler uses some new abstraction:
  - Scanning uses regular expressions.
  - Parsing uses CFGs.
  - Semantic analysis uses proof systems and symbol tables.
  - IR generation uses ASTs.
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization.

Visualizing IR

```
main:
    BeginFunc 40;
    _tmp0 = LCall _ReadInteger;
    a = _tmp0;
    _tmp1 = LCall _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    PushParam a;
    LCall _PrintInt;
    PopParams 4;
    EndFunc;
```

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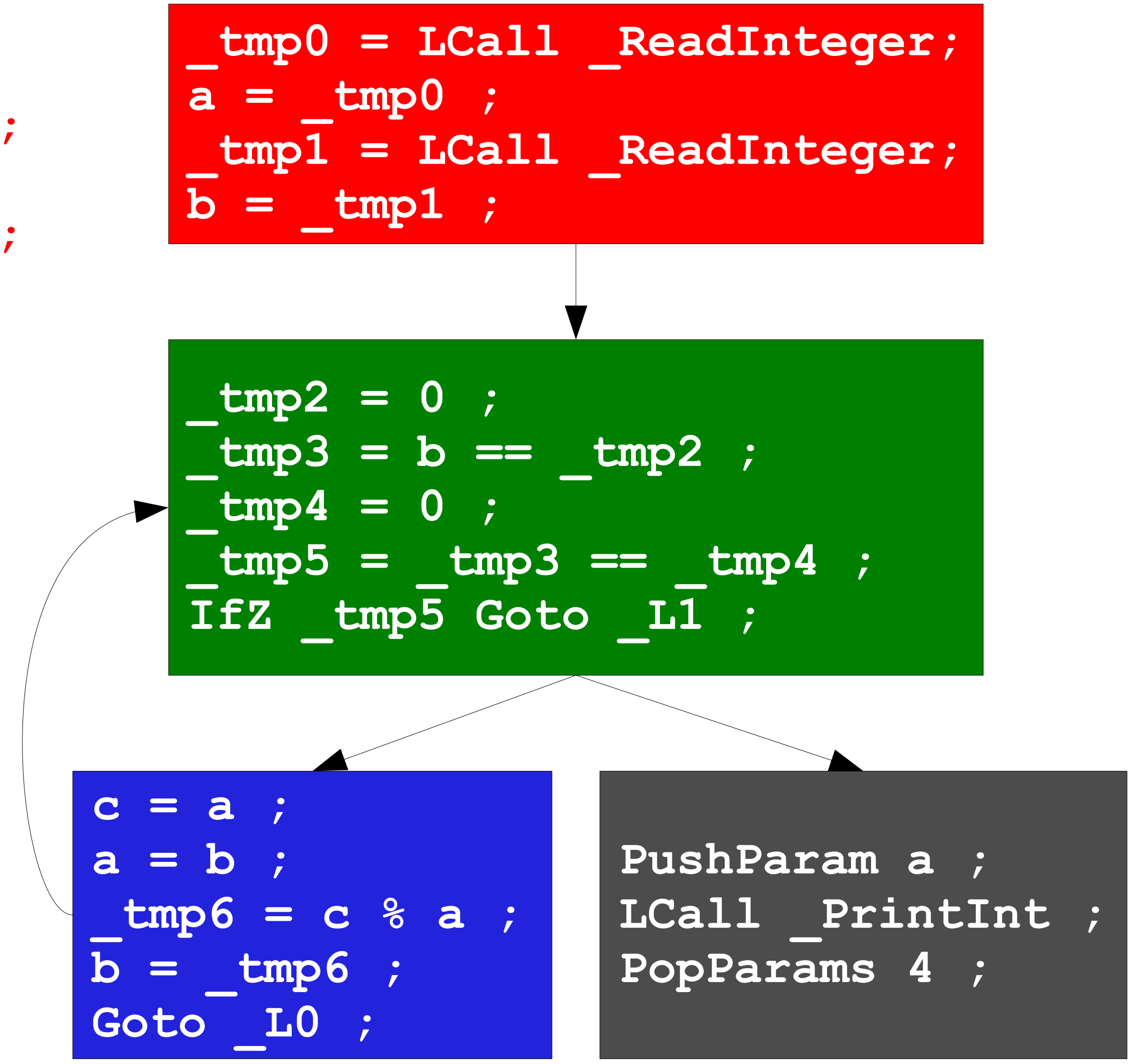

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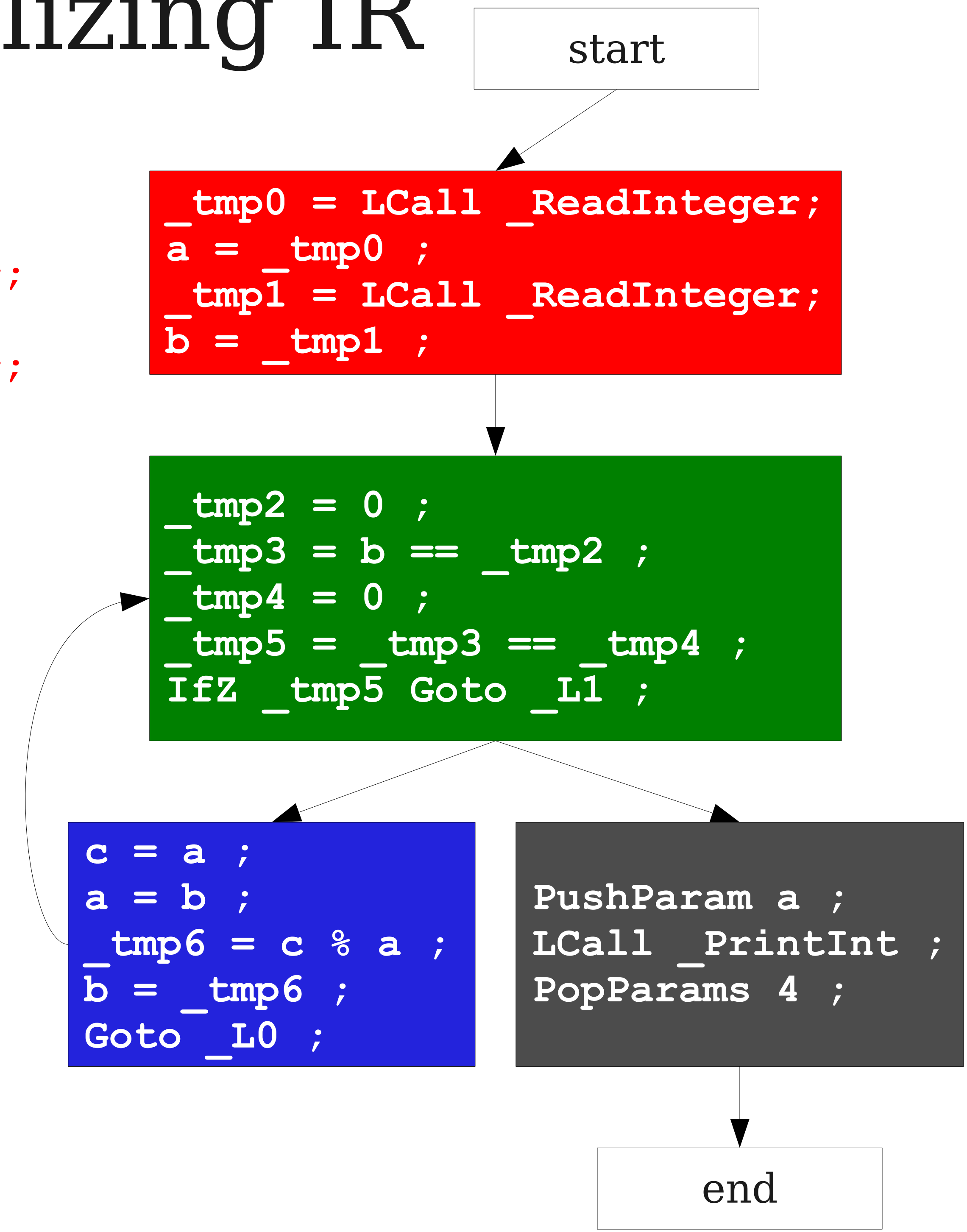


Basic Blocks

- A **basic block** is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence.
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence.
- Informally, a sequence of instructions that always execute as a group.

Control-Flow Graphs

- A **control-flow graph** (CFG) is a graph of the basic blocks in a function.
  - The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context-free grammar.”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block.
- There is a dedicated node for the start and end of a function.

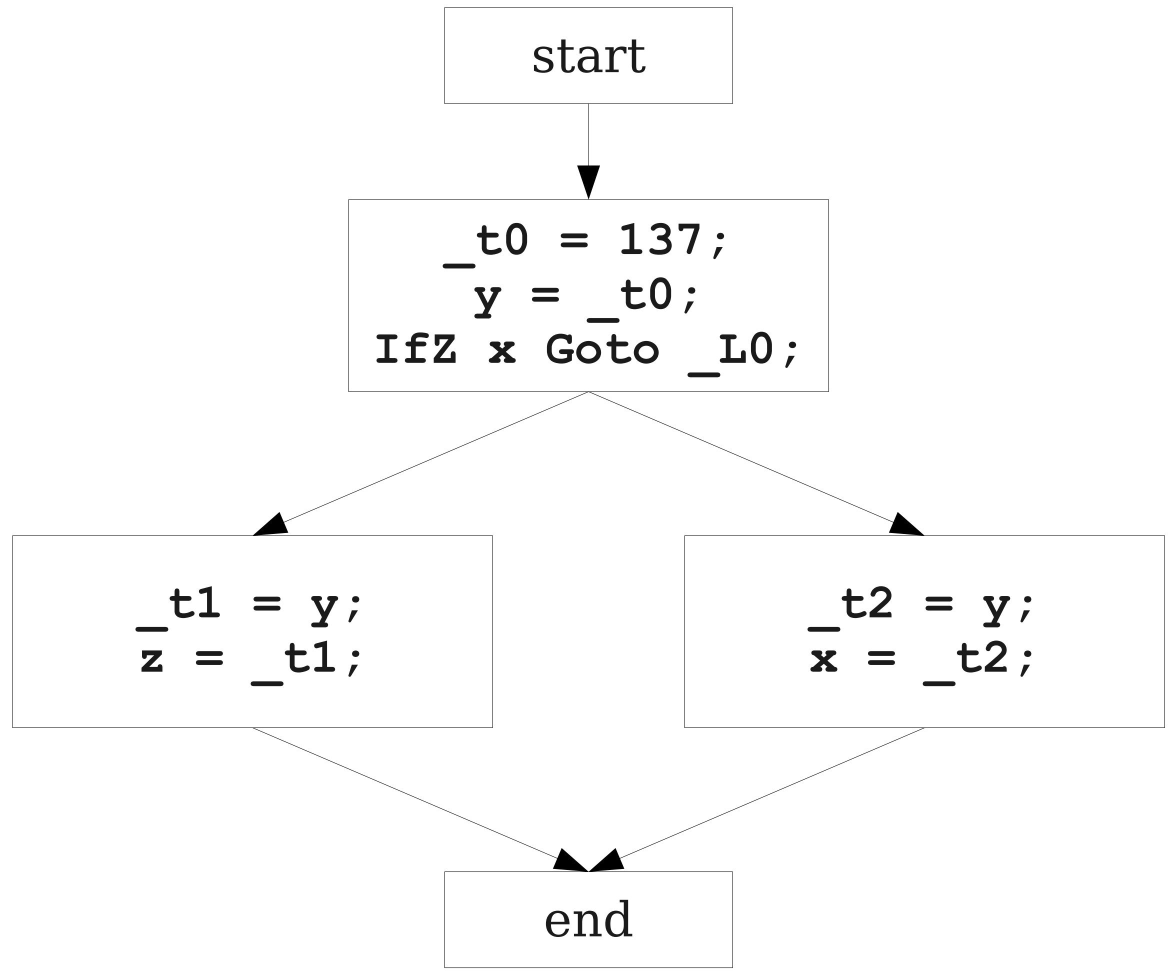
Types of Optimizations

- An optimization is **local** if it works on just a single basic block.
- An optimization is **global** if it works on an entire control-flow graph.
- An optimization is **interprocedural** if it works across the control-flow graphs of multiple functions.
  - We won't talk about this in this course.

Local Optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



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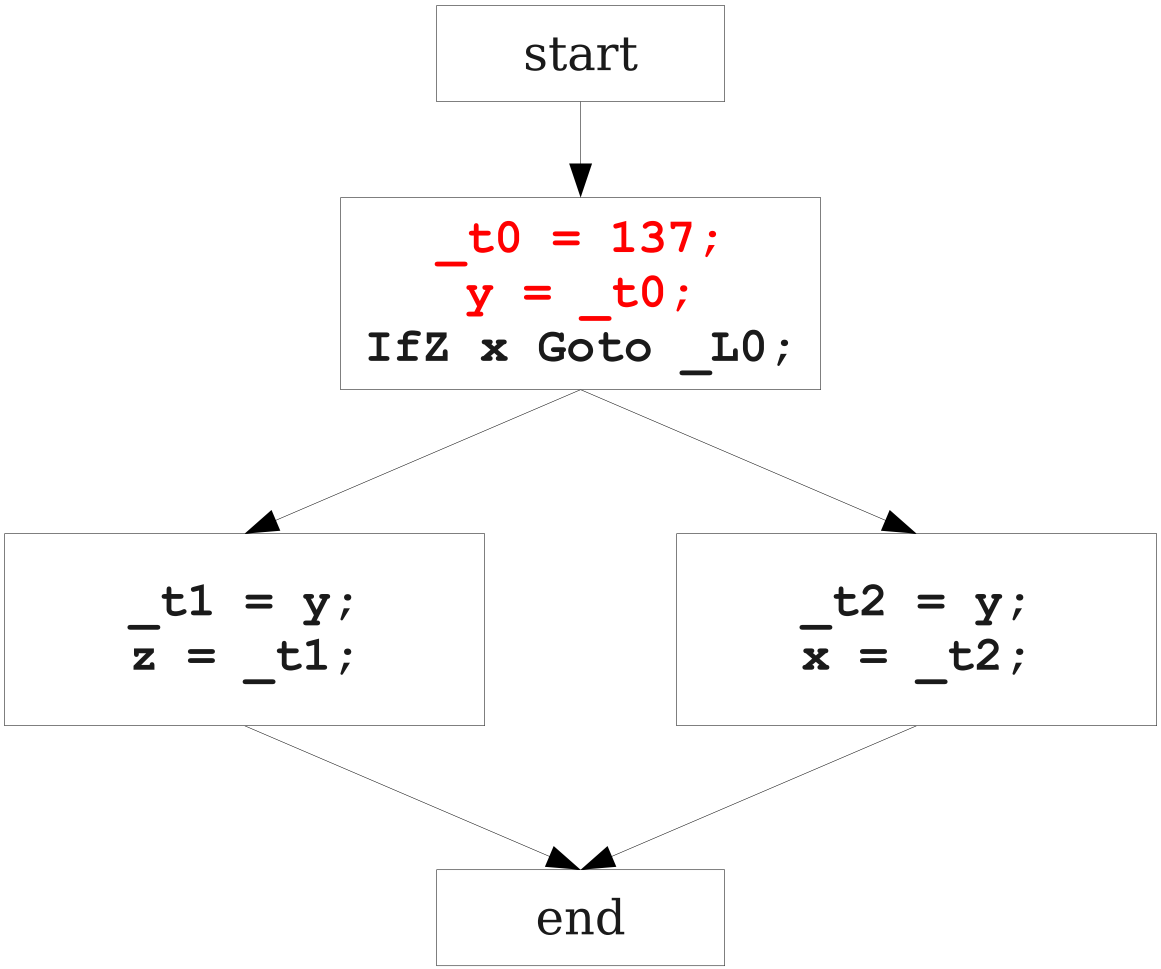

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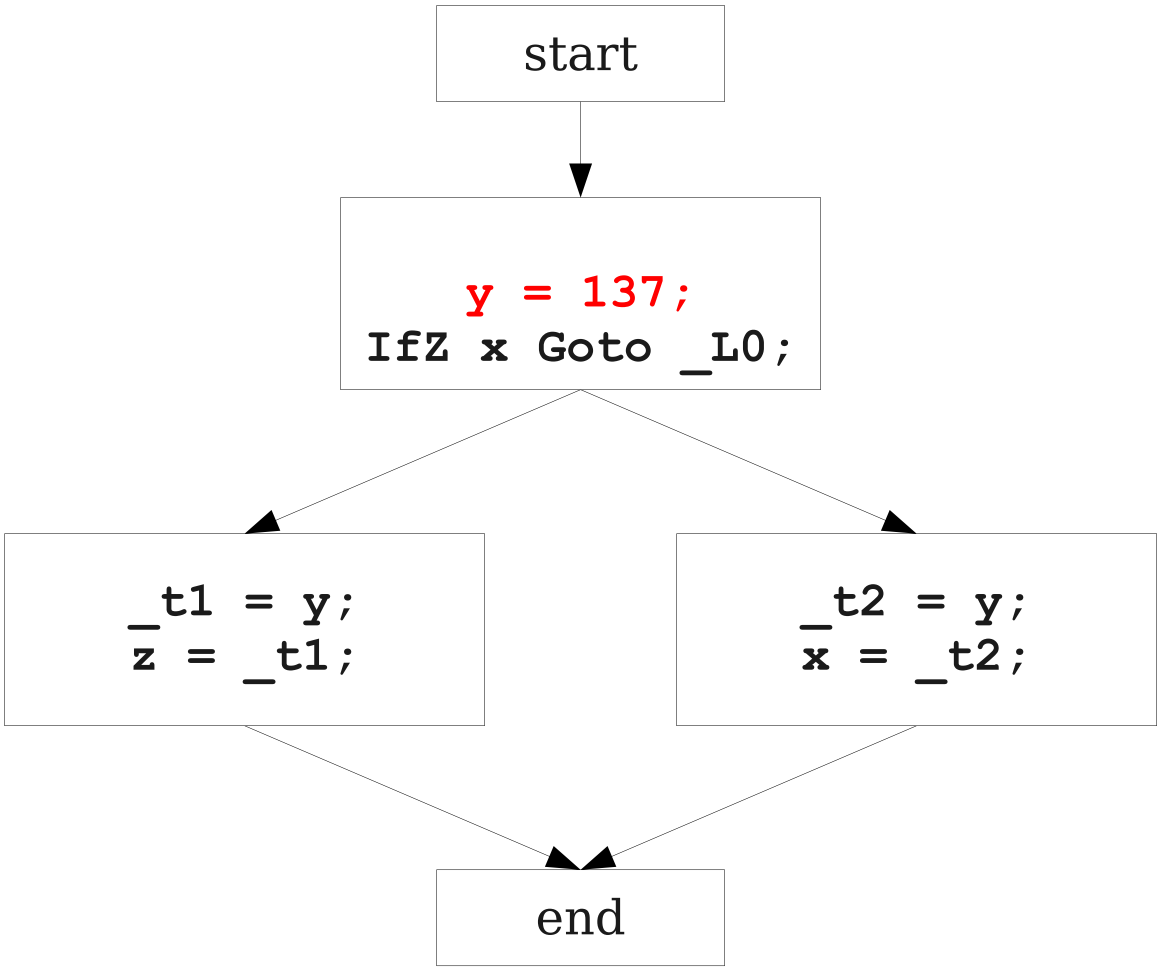

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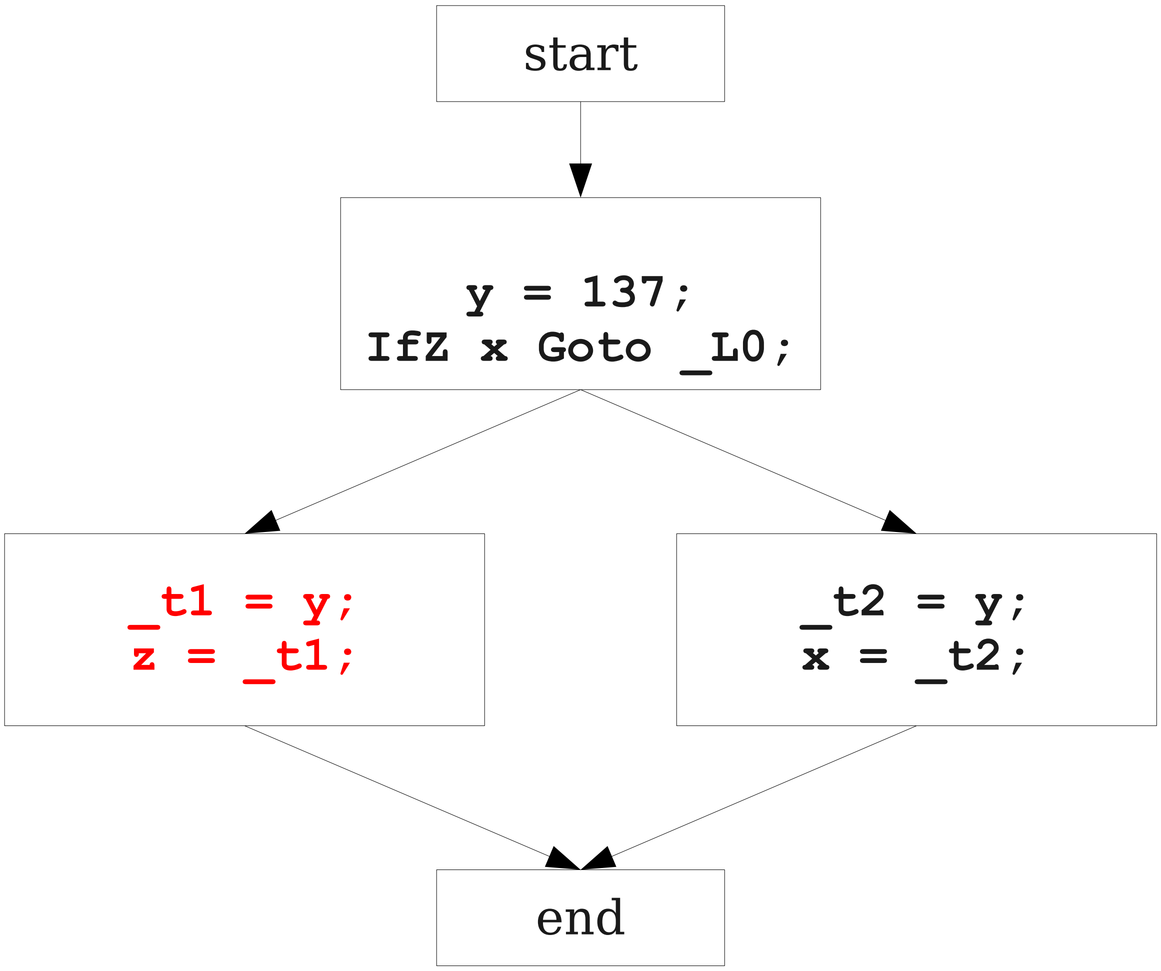

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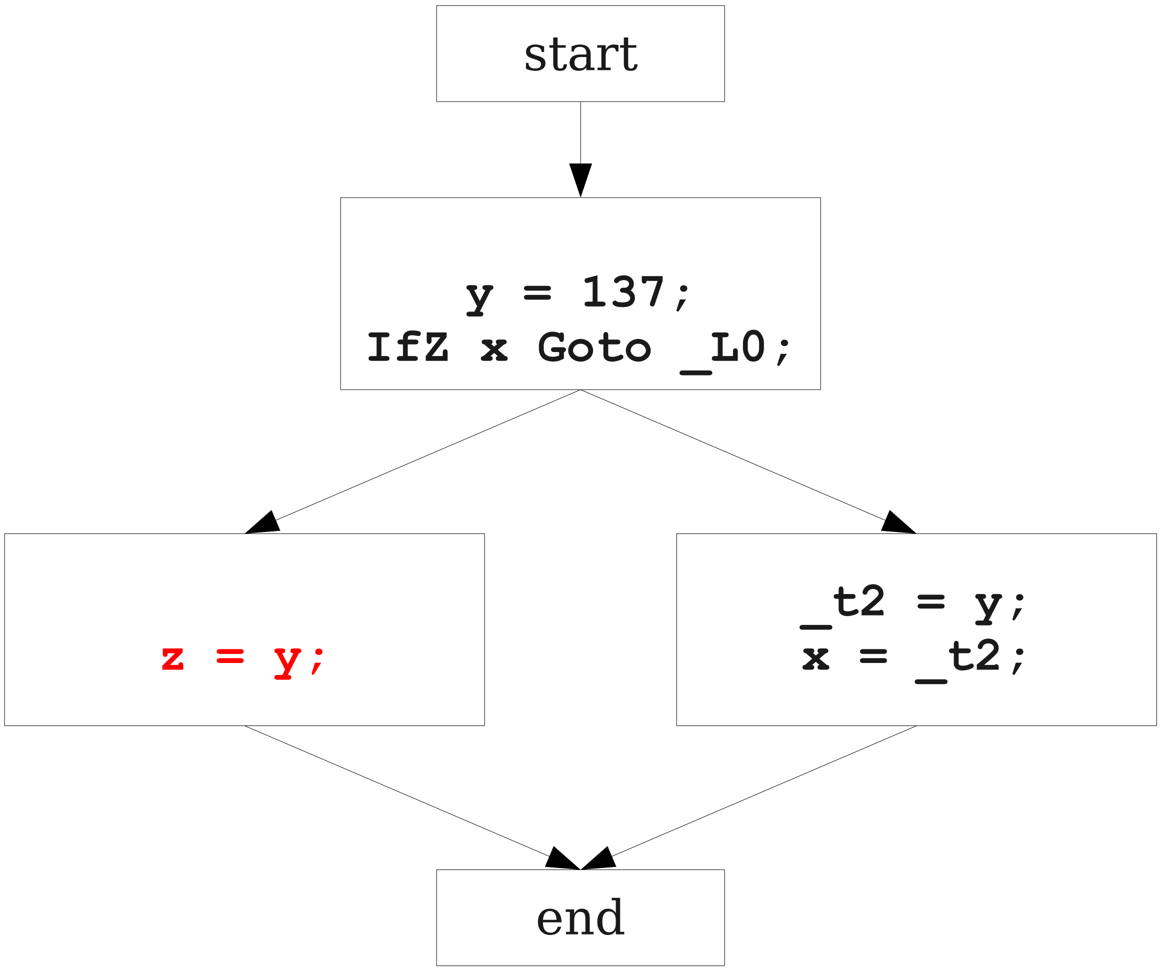

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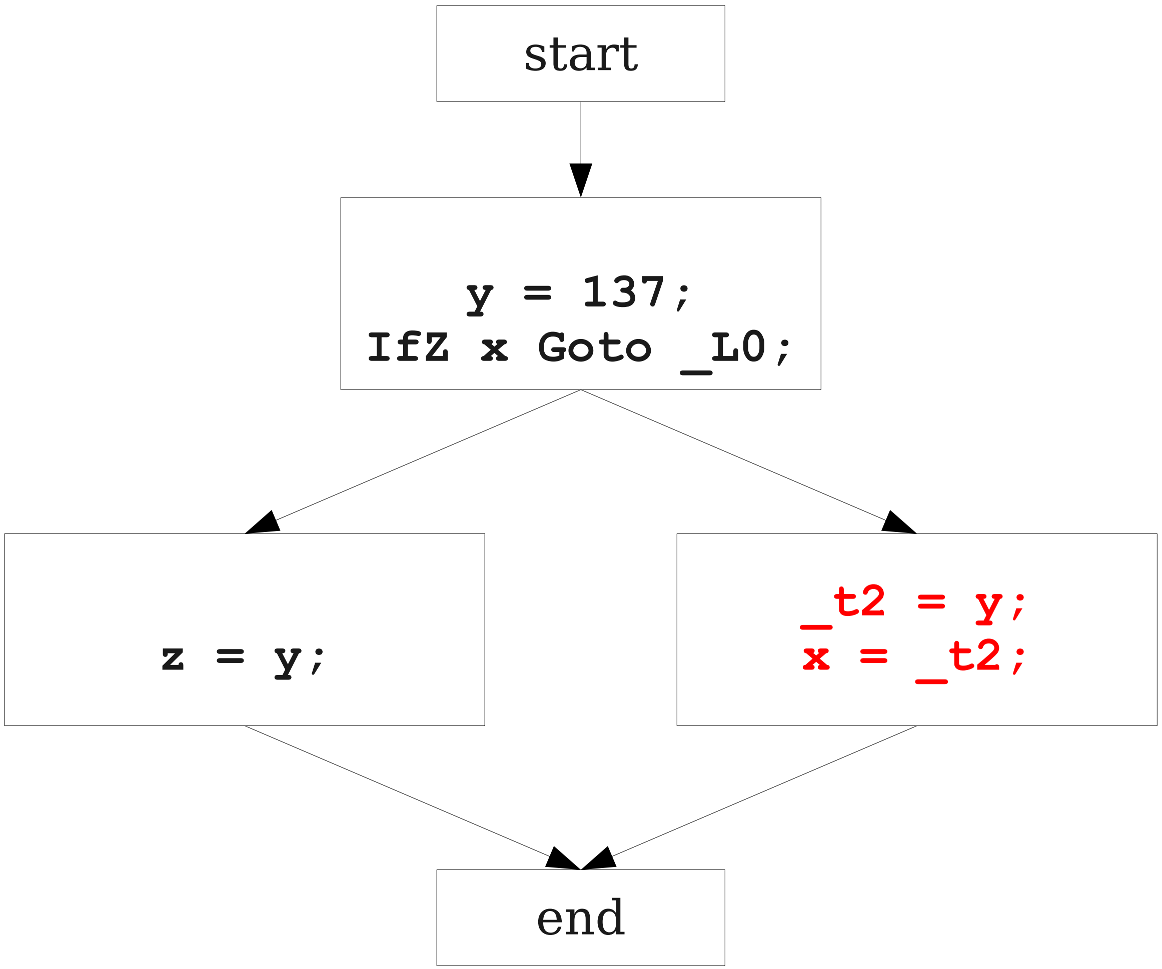

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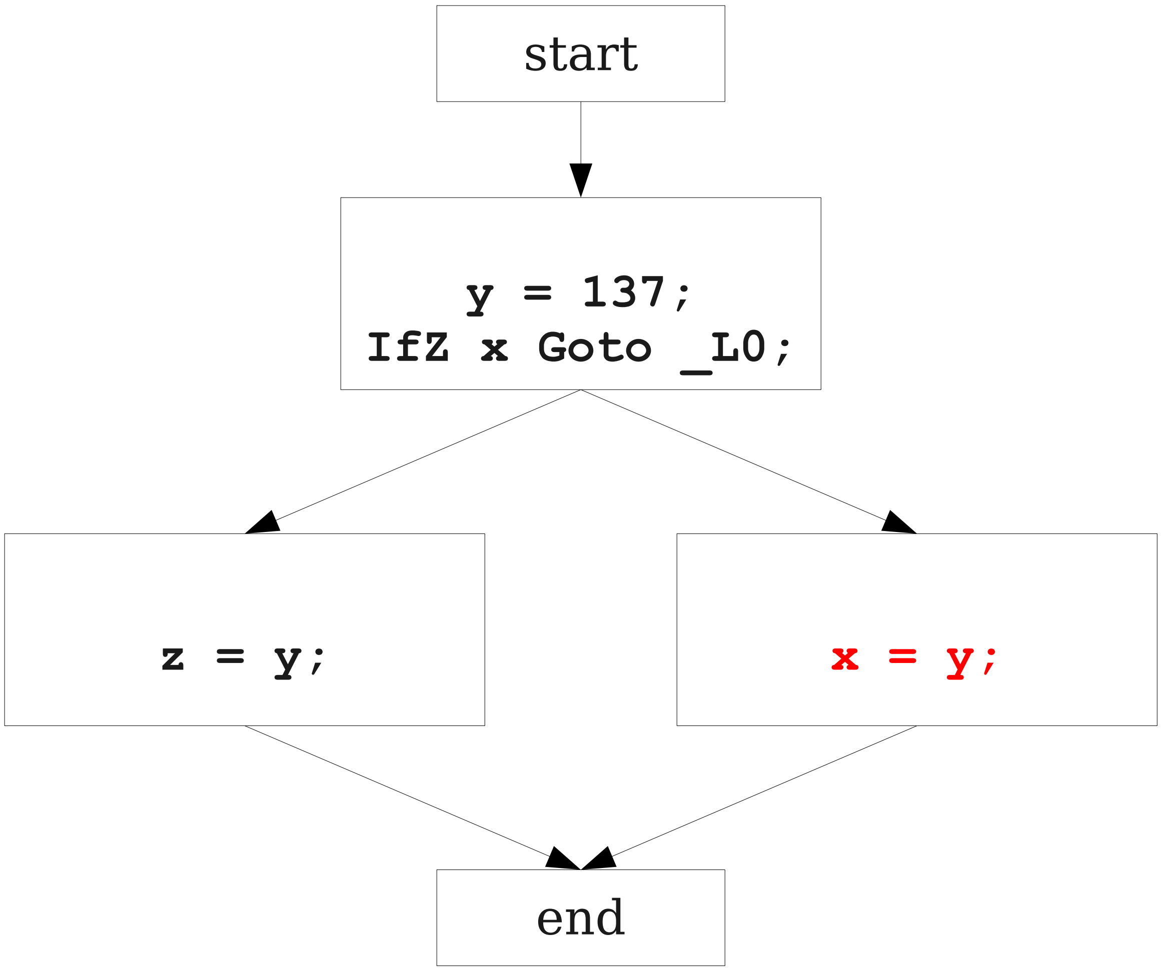

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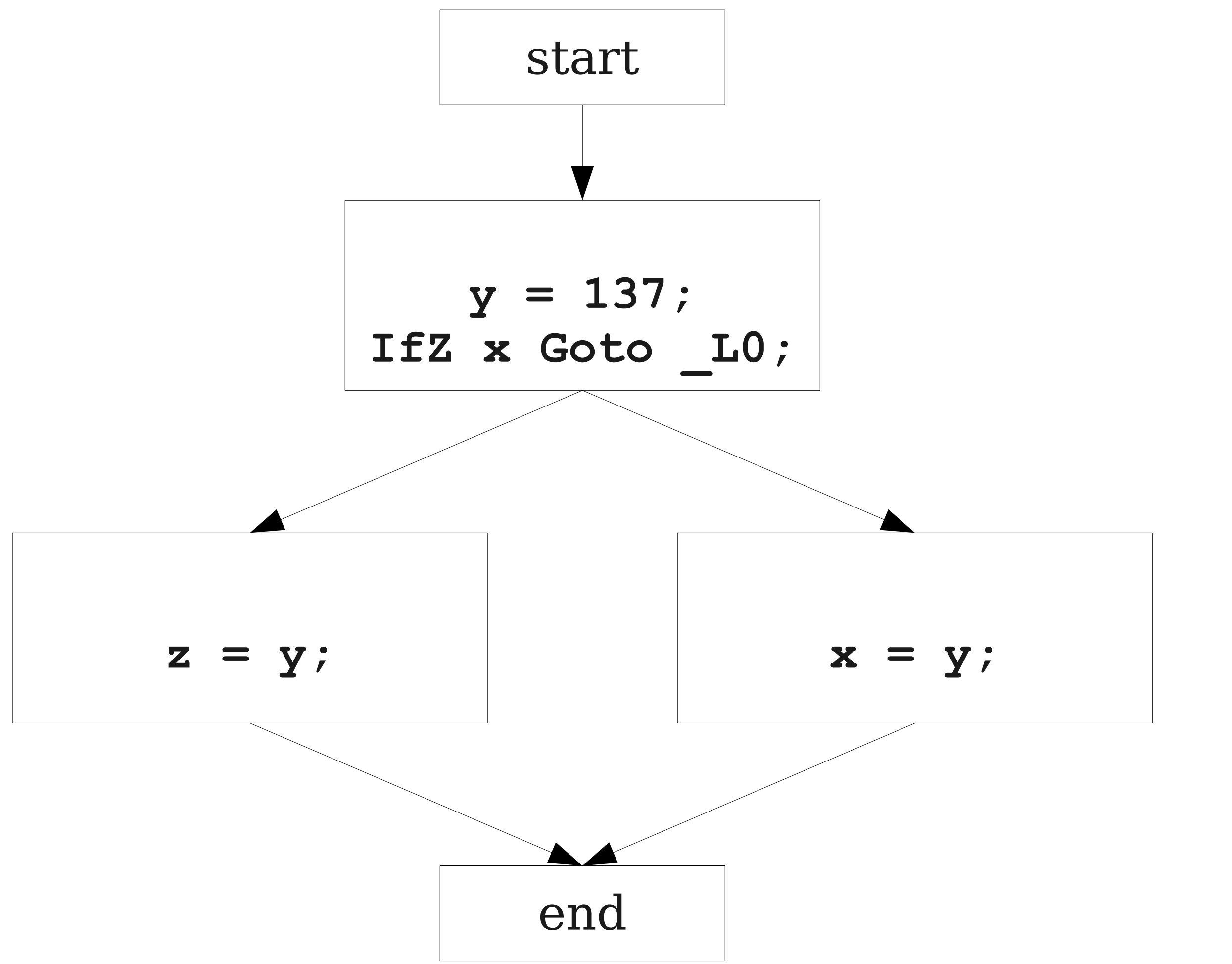

Global Optimizations

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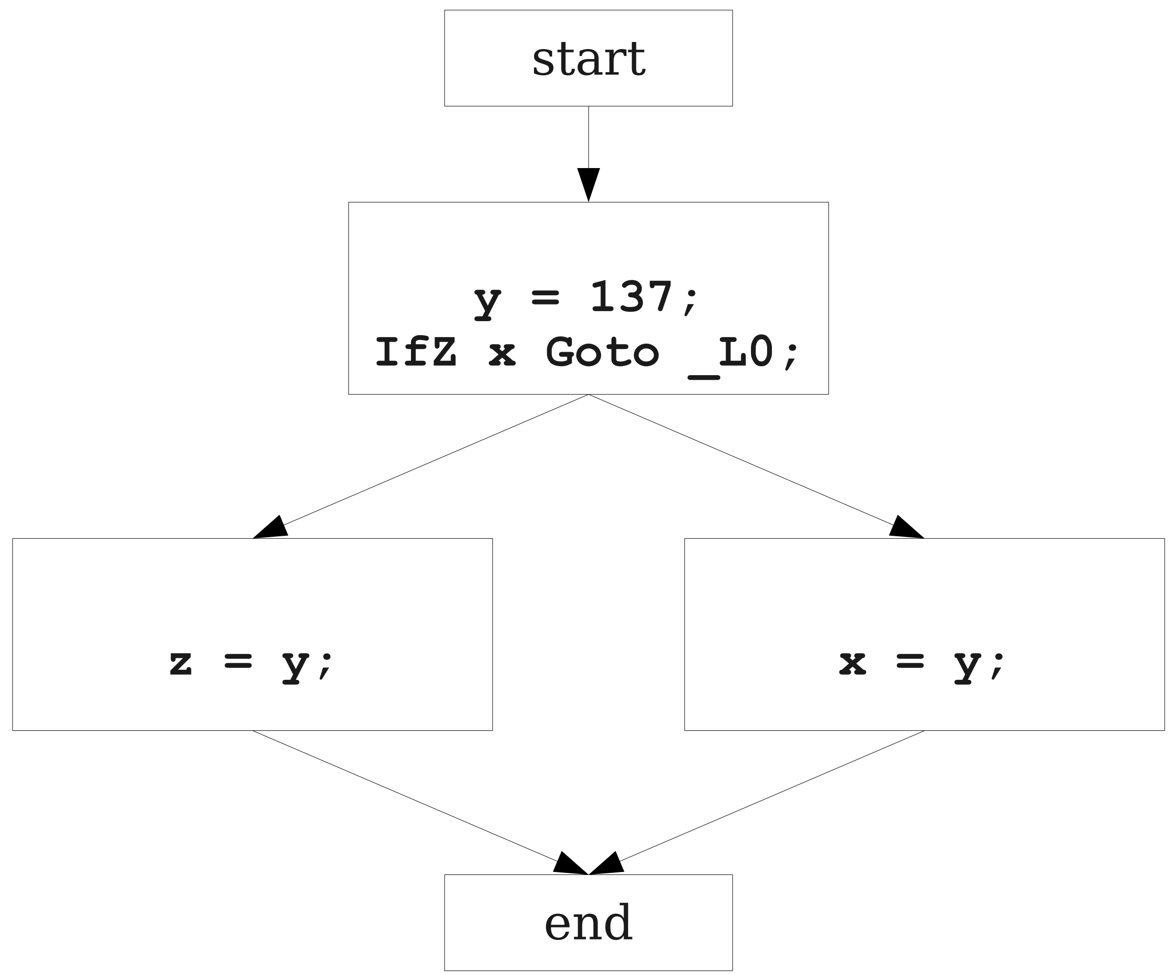

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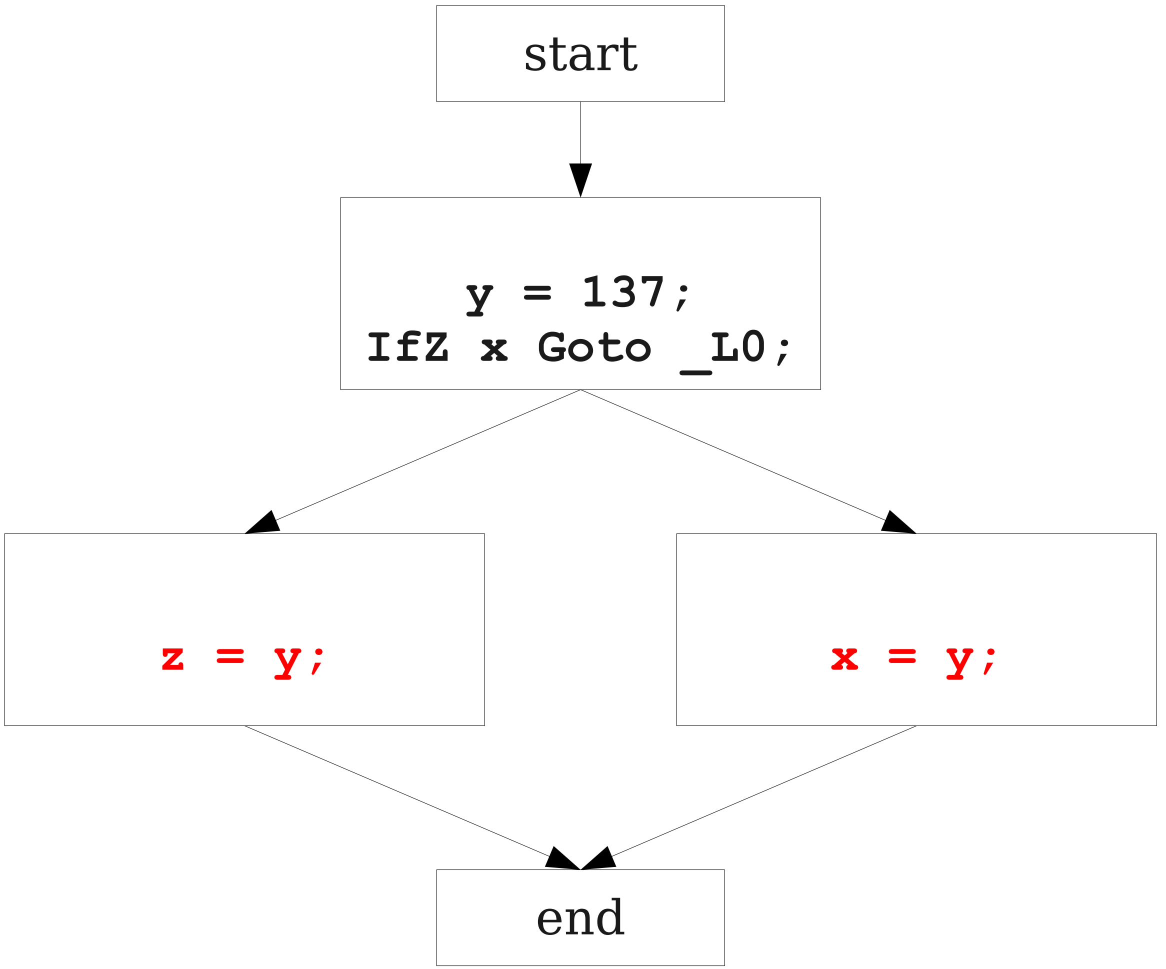

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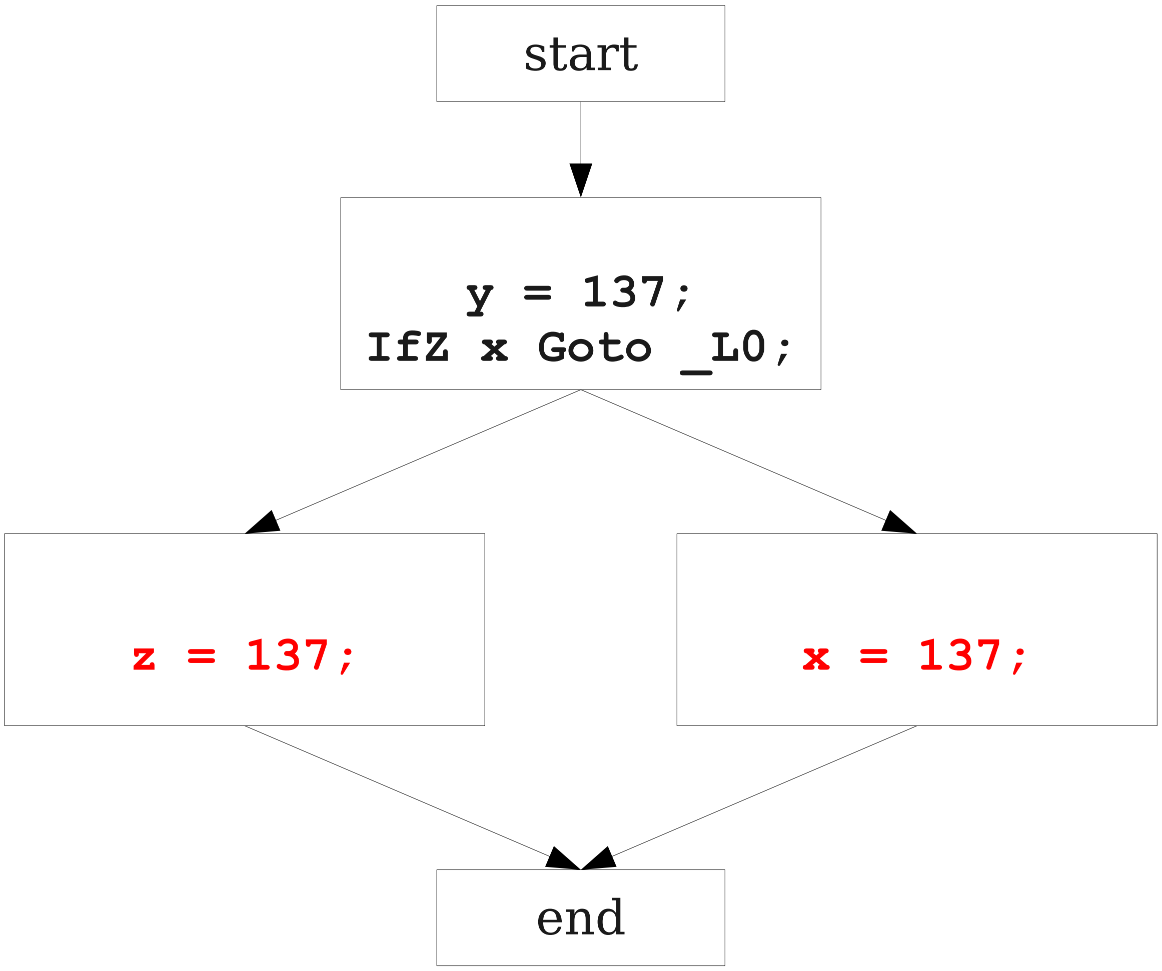

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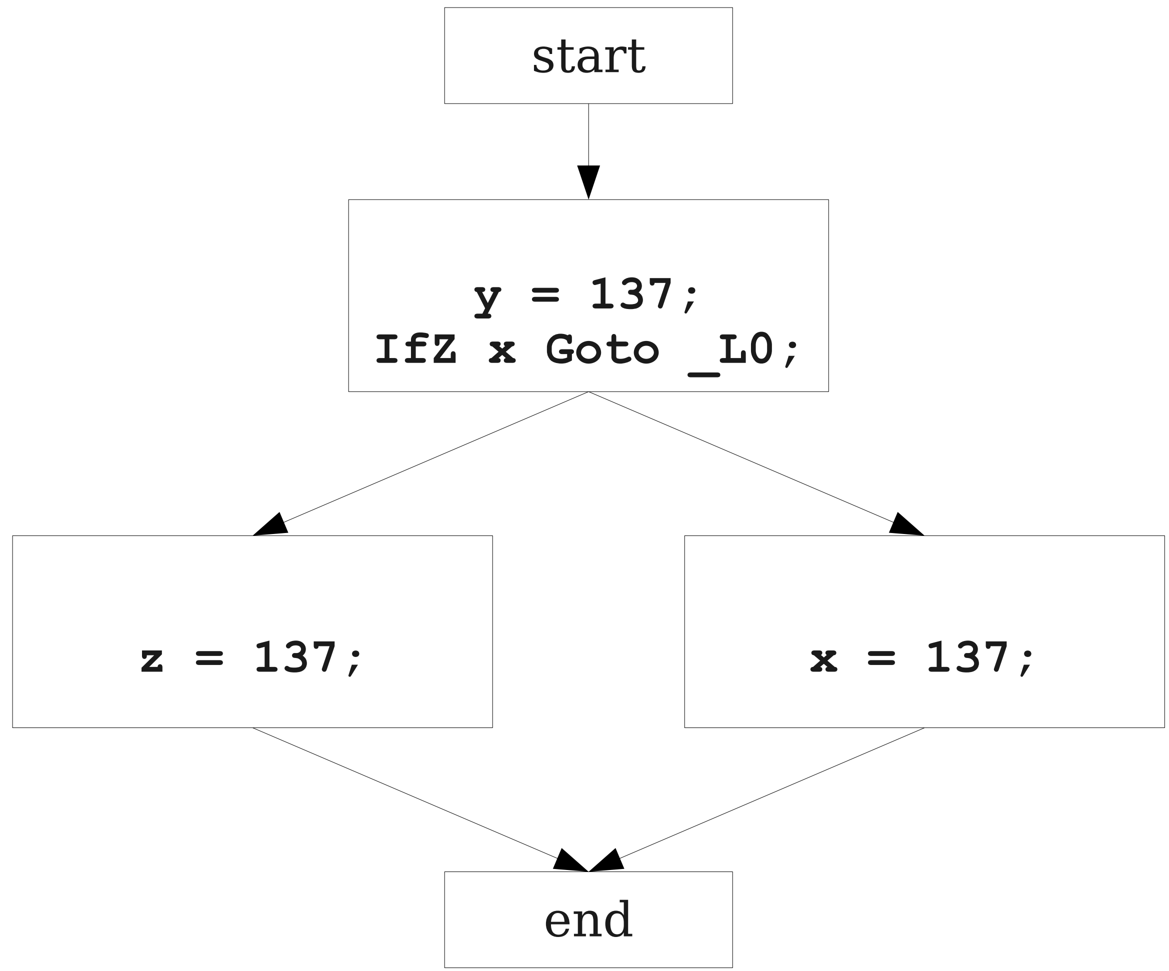

Local Optimizations

Common Subexpression Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

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PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
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x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

Common Subexpression Elimination

- If we have two variable assignments

```
v1 = a op b
    …
v2 = a op b
```

  and the values of $\mathbf{v_1}$, **a**, and **b** have not changed between the assignments, rewrite the code as

```
v1 = a op b
    …
v2 = v1
```

- Eliminates useless recalculation.
- Paves the way for later optimizations.

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
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x = new Object;
a = 4;
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_tmp0 = 4 ;
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*(_tmp1) = _tmp2 ;
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_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
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_tmp0 = 4 ;
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_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
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x.fn(a + b);
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_tmp0 = 4 ;
PushParam _tmp0 ;
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a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
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_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = *(_tmp1) ;
_tmp7 = *(_tmp6) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
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_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp6) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp6) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
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_tmp3 = _tmp0 ;
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_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp0 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp0 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

- If we have a variable assignment

  $v_1 = v_2$

  then as long as $\mathbf{v_1}$ and $\mathbf{v}_2$ are not reassigned, we can rewrite expressions of the form

  $a = \ldots v_1 \ldots$

  as

  $a = \ldots v_2 \ldots$

  provided that such a rewrite is legal.
- This will help immensely later on, as you'll see.

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
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```

Dead Code Elimination

```
Object x;
int a;
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x = new Object;
a = 4;
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x.fn(a + b);
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
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ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
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a = 4 ;
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Dead Code Elimination

```
Object x;
int a;
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x = new Object;
a = 4;
c = a + b;
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
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Dead Code Elimination

```
Object x;
int a;
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x = new Object;
a = 4;
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
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_tmp2 = Object ;
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_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
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PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
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```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;


_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;



_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

- An assignment to a variable **v** is called **dead** if the value of that assignment is never read anywhere.
- **Dead code elimination** removes dead assignments from IR.
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned.

For Comparison

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
_tmp4 = _tmp0 + b ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Applying Local Optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization.
  - Common subexpression elimination eliminates unnecessary statements.
  - Copy propagation helps identify dead code.
  - Dead code elimination removes statements that are no longer needed.
- To get maximum effect, we may have to apply these optimizations numerous times.

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Other Types of Local Optimization

- **Arithmetic Simplification**
  - Replace “hard” operations with easier ones.
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- **Constant Folding**
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite **x = 4 * 5;** as **x = 20;**.

Implementing Local Optimization

Optimizations and Analyses

- Most optimizations are only possible given some analysis of the program's behavior.
- In order to implement an optimization, we will talk about the corresponding program analyses.

Available Expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the **available expressions** in a program.
- An expression is called **available** if some variable in the program holds the value of that expression.
- In common subexpression elimination, we replace an available expression by the variable holding its value.
- In copy propagation, we replace the use of a variable by the available expression it holds.

Finding Available Expressions

- Initially, no expressions are available.
- Whenever we execute a statement **a = b + c**:
  - Any expression holding **a** is invalidated.
  - The expression **a = b + c** becomes available.
- **Idea**: Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable.

Available Expressions

```
  a = b;
  c = b;
d = a + b;
e = a + b;
  d = b;
f = a + b;
```

Available Expressions

```
   { }
  a = b;

  c = b;

d = a + b;

e = a + b;

  d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

{ }

```
a = b;
```

{ a = b }

```
c = b;
```

{ a = b, c = b }

```
d = a + b;
```

{ a = b, c = b, d = a + b }

```
e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;

f = a + b;
```

Available Expressions

{ }

```
a = b;
```

{ a = b }

```
c = b;
```

{ a = b, c = b }

```
d = a + b;
```

{ a = b, c = b, d = a + b }

```
e = a + b;
```

{ a = b, c = b, d = a + b, e = a + b }

```
d = b;
```

{ a = b, c = b, d = b, e = a + b }

```
f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

{ }

```
a = b;
```

{ a = b }

```
c = a;
```

{ a = b, c = b }

```
d = a + b;
```

{ a = b, c = b, d = a + b }

```
e = a + b;
```

{ a = b, c = b, d = a + b, e = a + b }

```
d = b;
```

{ a = b, c = b, d = b, e = a + b }

```
f = a + b;
```

{ a = b, c = b, d = b, e = a + b, f = a + b }

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = e;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

Live Variables

- The analysis corresponding to dead code elimination is called **liveness analysis**.
- A variable is **live** at a point in a program if later in the program its value will be read before it is written to again.
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a = b + c**:
  - Just before the statement, **a** is not alive, since its value is about to be overwritten.
  - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
  - *(what if we have* **a = a + b***?)*

Liveness Analysis

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

Liveness Analysis

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;
```

{ a, b, e }

```
d = a;
```

{ b, d, e }

```
f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
    { b }
    a = b;
   { a, b }
    c = a;
   { a, b }
  d = a + b;
 { a, b, d }
    e = d;
 { a, b, e }
    d = a;
 { b, d, e }

   { b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }

{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;


d = a + b;

e = d;

d = a;
```

{ b, d }

Liveness Analysis II

```
a = b;



d = a + b;

e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
a = b;



d = a + b;
```
{ a, b, d }
```
e = d;
```
{ a, b }
```
d = a;
```
{ b, d }

Liveness Analysis II

```
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }

{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;


d = a + b;


d = a;
{b, d}
```

Liveness Analysis III

```
a = b;



d = a + b;

{a, b}

d = a;
{b, d}
```

Liveness Analysis III

```
a = b;
```

{a, b}

```
d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

Liveness Analysis III

{b}

```
a = b;
```

{a, b}

```
d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}



{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
a = b;
```

```
d = a;
```

A Combined Algorithm

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

{b, d}

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
{b, d}
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
 a = b;

 c = a;

d = a + b;

 e = d;
 {a, b}
 d = a;


 {b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;


{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;

c = a;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;
c = a;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;
```

```
{a, b}
d = a;
```

```
{b, d}
```

A Combined Algorithm

```
{b}
a = b;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;
```

```
d = a;
```

Next Time

- **Formalisms for Local Optimizations**
  - Transfer functions and semantics
- **Global optimization**
  - Optimizing across basic blocks.
  - Meet operators and the dataflow framework.