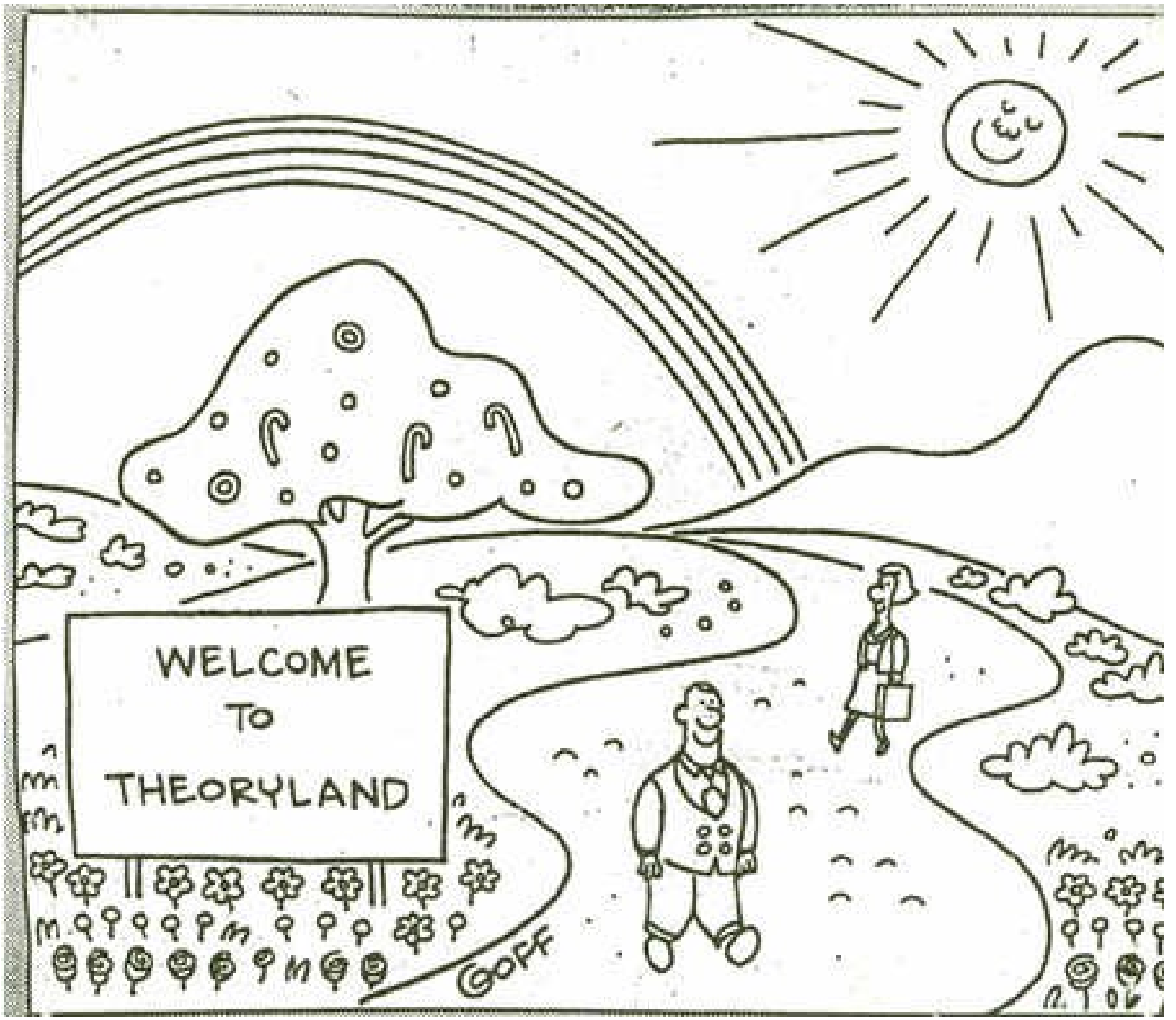
Advanced Parsing Techniques

Announcements

- Written Set 1 graded.
  - Hard copies available for pickup right now.
  - Electronic submissions: feedback returned later today.

Where We Are

Where We Are



Parsing so Far

- We've explored five deterministic parsing algorithms:
  - LL(1)
  - LR(0)
  - SLR(1)
  - LALR(1)
  - LR(1)
- These algorithms all have their limitations.
- Can we parse arbitrary context-free grammars?

Why Parse Arbitrary Grammars?

- **They're easier to write**.
  - Can leave operator precedence and associativity out of the grammar.
  - No worries about shift/reduce or FIRST/FOLLOW conflicts.
- If ambiguous, can filter out invalid trees at the end.
  - Generate candidate parse trees, then eliminate them when not needed.
- Practical concern for some languages.
  - We need to have C and C++ compilers!

Questions for Today

- How do you go about parsing ambiguous grammars *efficiently*?
- How do you produce *all possible parse trees*?
- What else can we do with a general parser?

The Earley Parser

Motivation: The Limits of LR

- LR parsers use shift and reduce actions to reduce the input to the start symbol.
- LR parsers cannot deterministically handle shift/reduce or reduce/reduce conflicts.
- However, they can nondeterministically handle these conflicts by guessing which option to choose.
- What if we try all options and see if any of them work?

The Earley Parser

- Maintain a collection of Earley items, which are LR(0) items annotated with a start position.
  - The item **A** → ***α*·*ω*** **@n** means we are working on recognizing **A** → ***αω***, have seen ***α***, and the start position of the item was the **n**th token.
- Using techniques similar to LR parsing, try to scan across the input creating these items.
- We're done when we find an item **S** → **E**· **@1** at the very last position.

Earley in Action

S → **E**
E → **E** `+` **E**
E → `int`

Earley in Action

`int`	`+`	`int`	`+`	`int`

S → **E**
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Earley in Action

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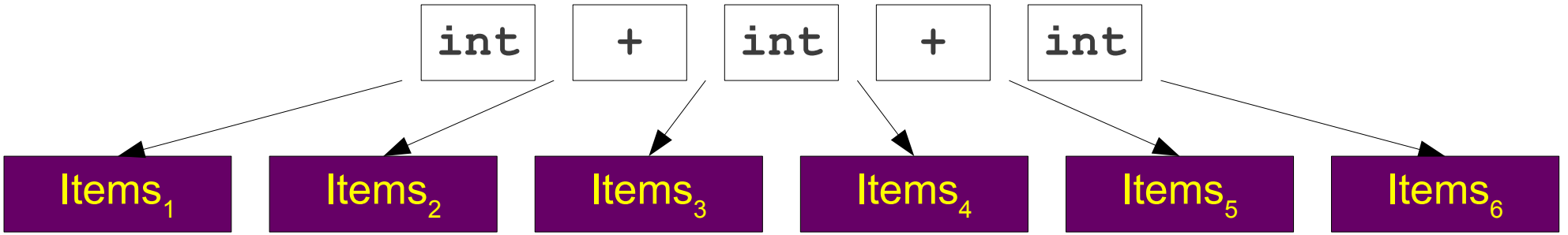
Earley in Action



S → E
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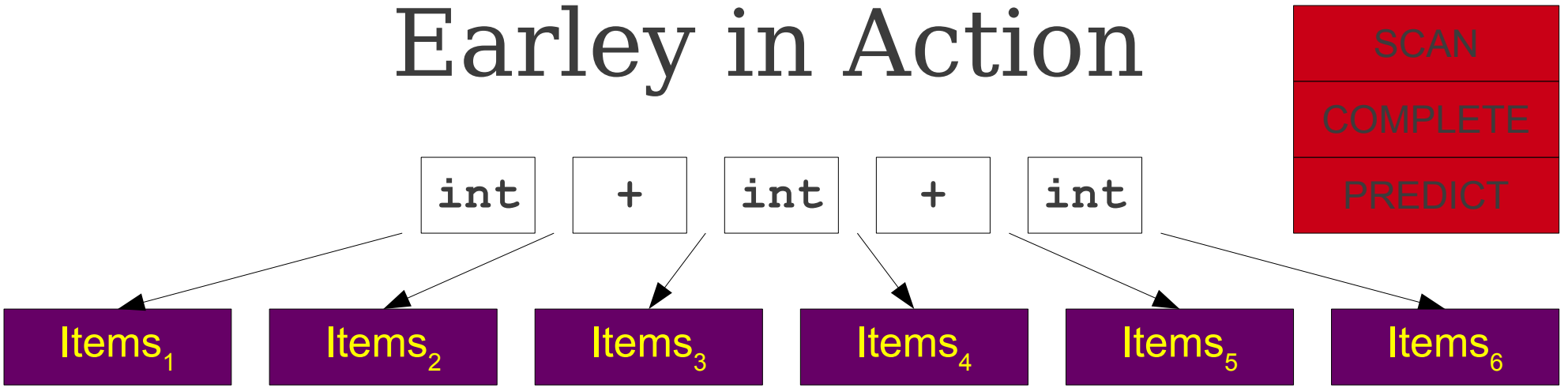
Earley in Action



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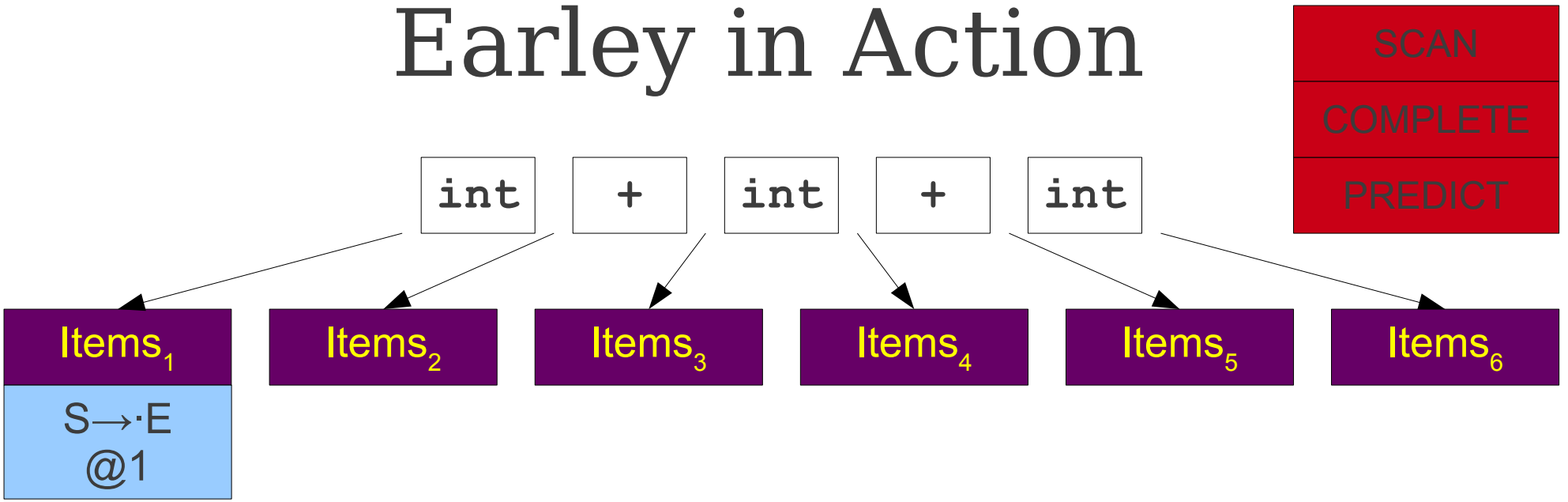

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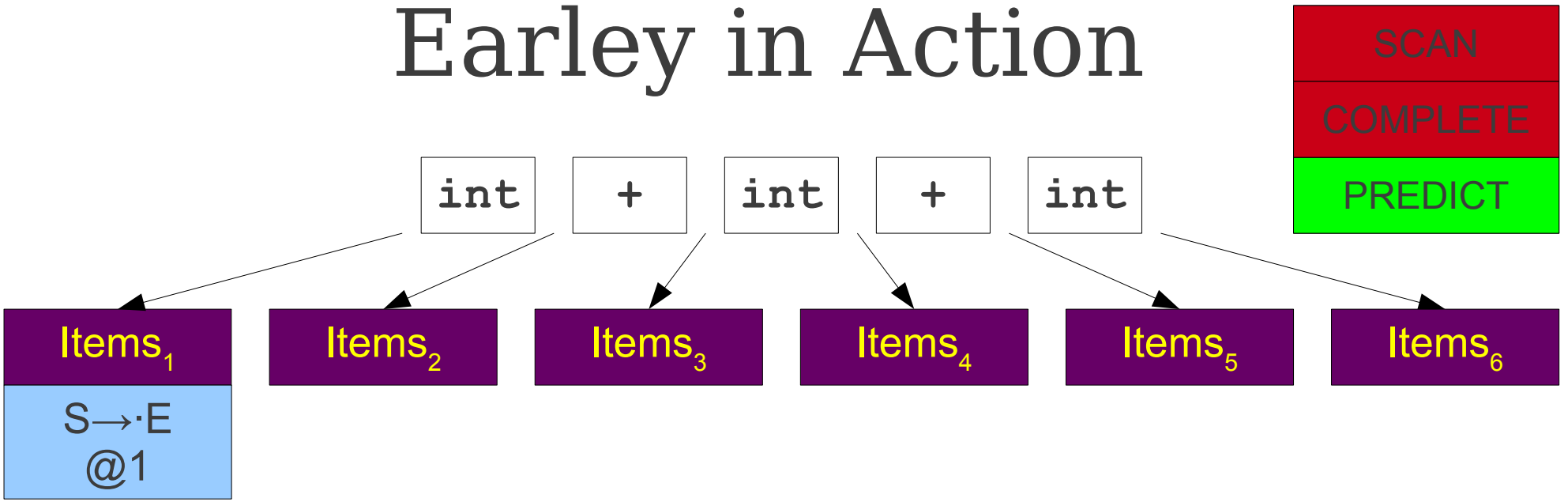

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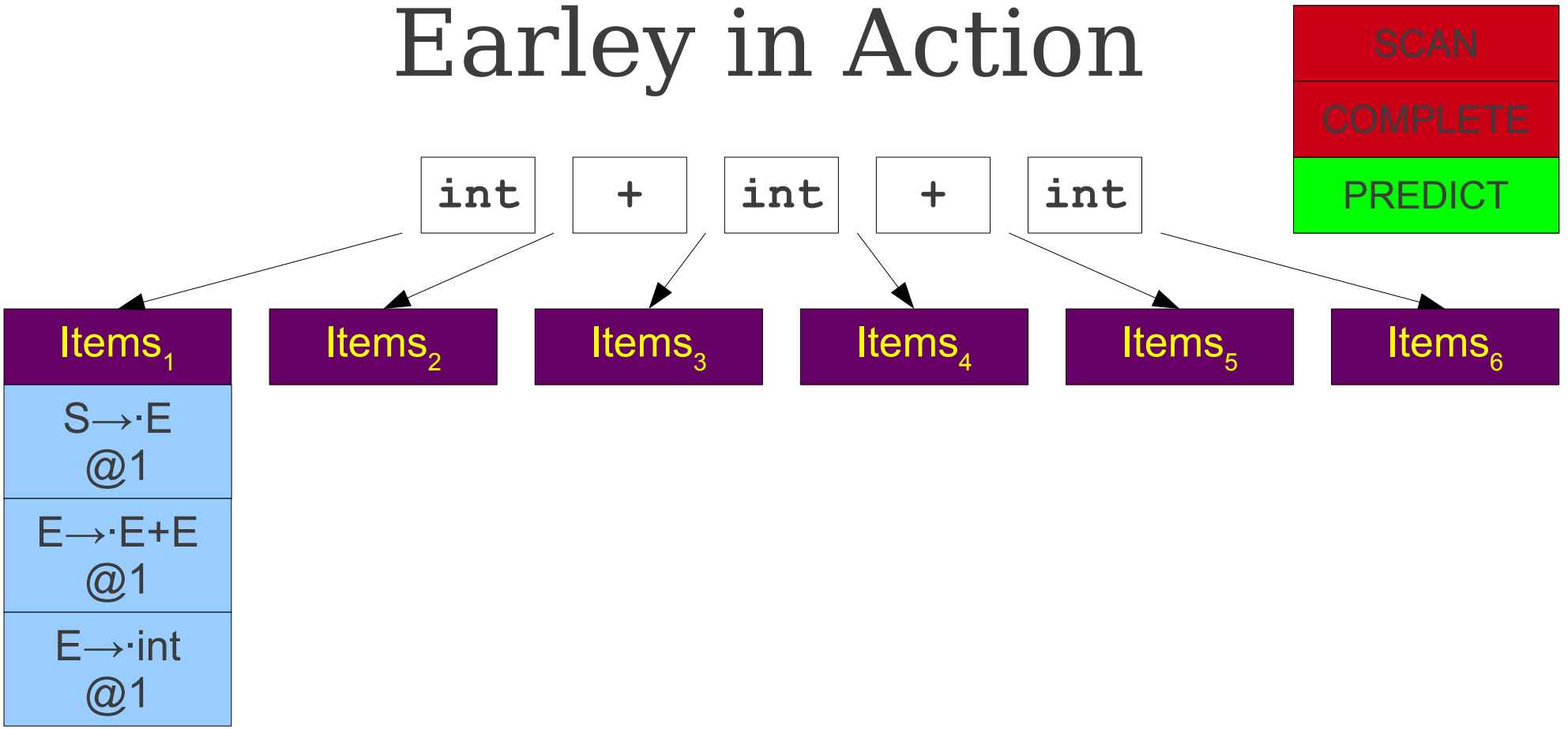

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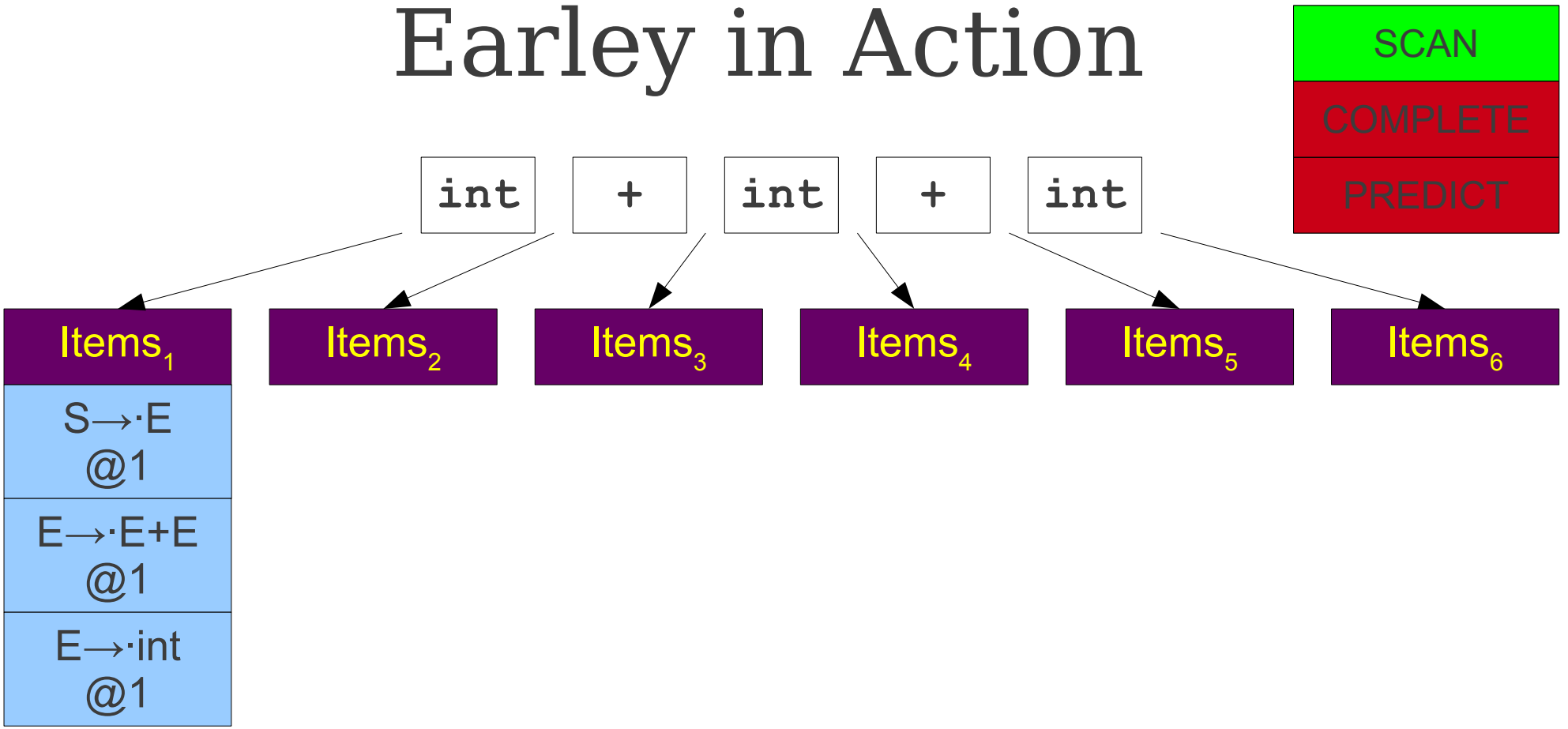

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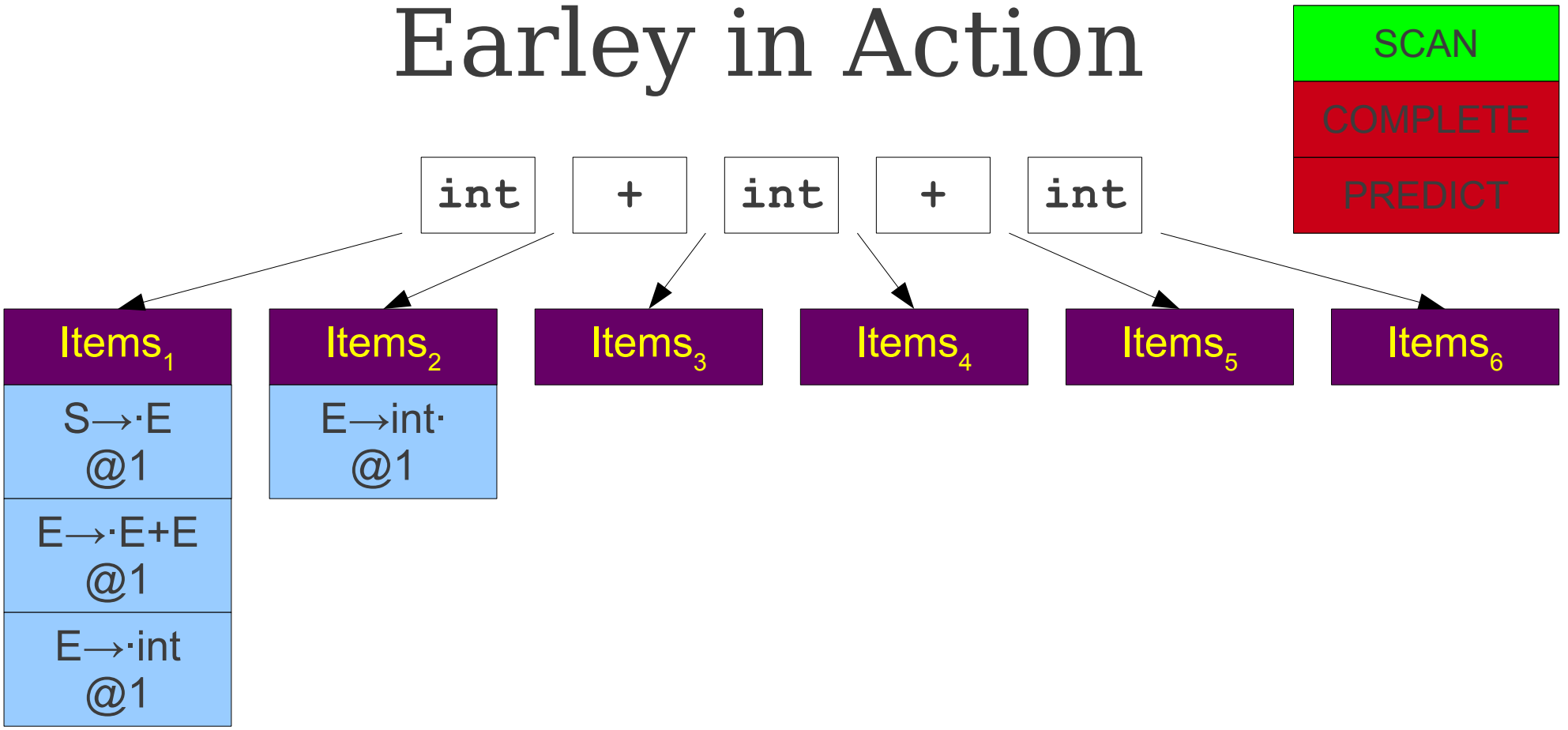

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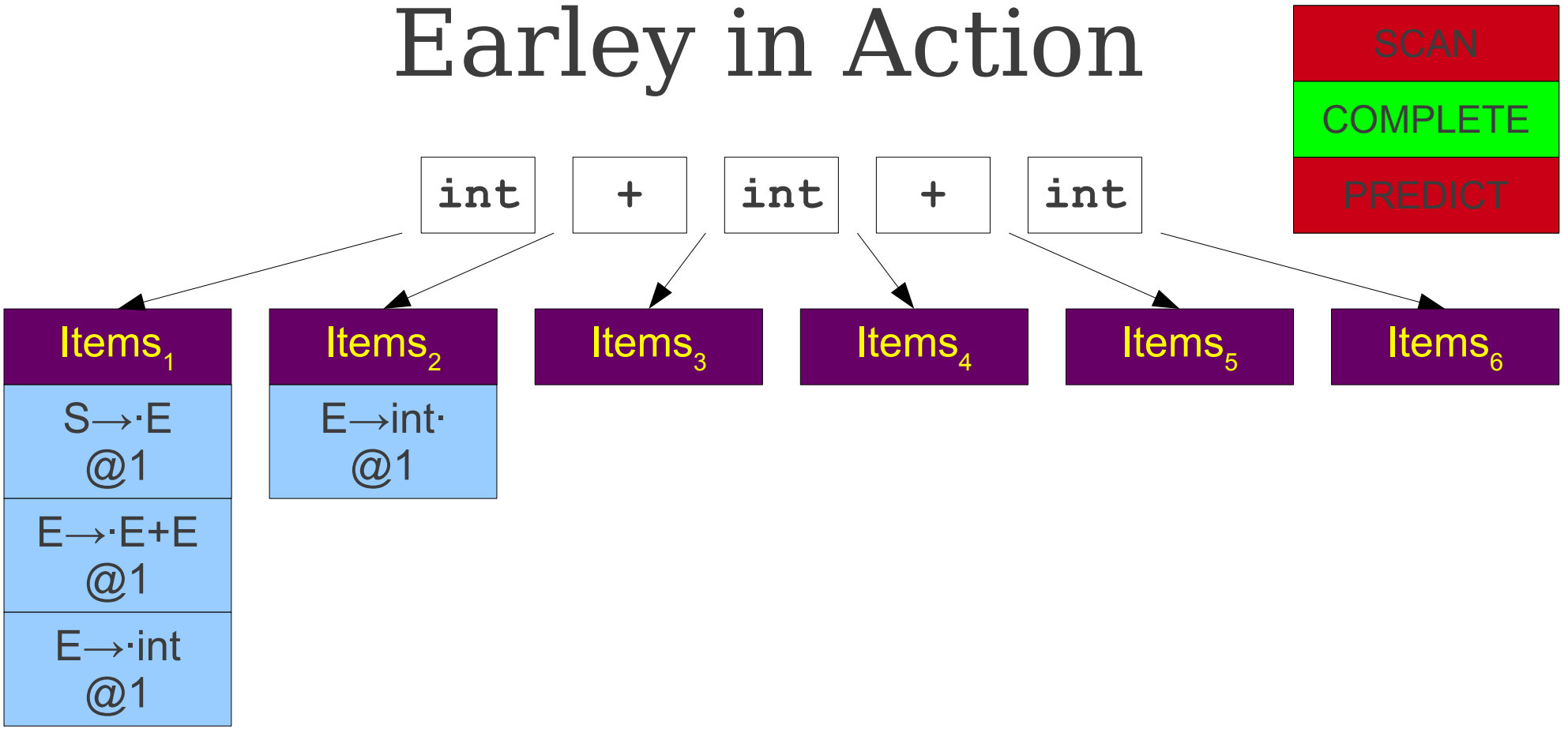

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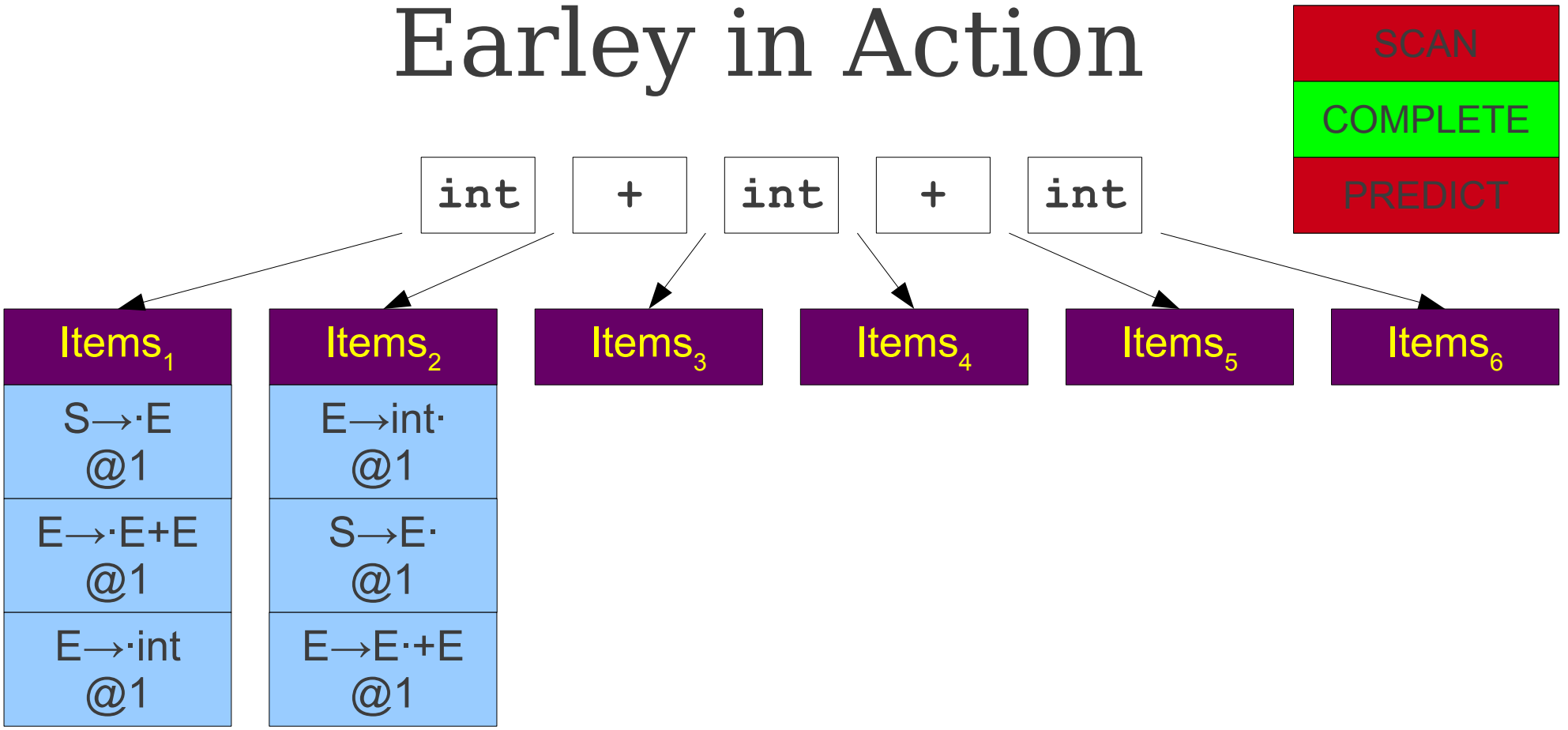

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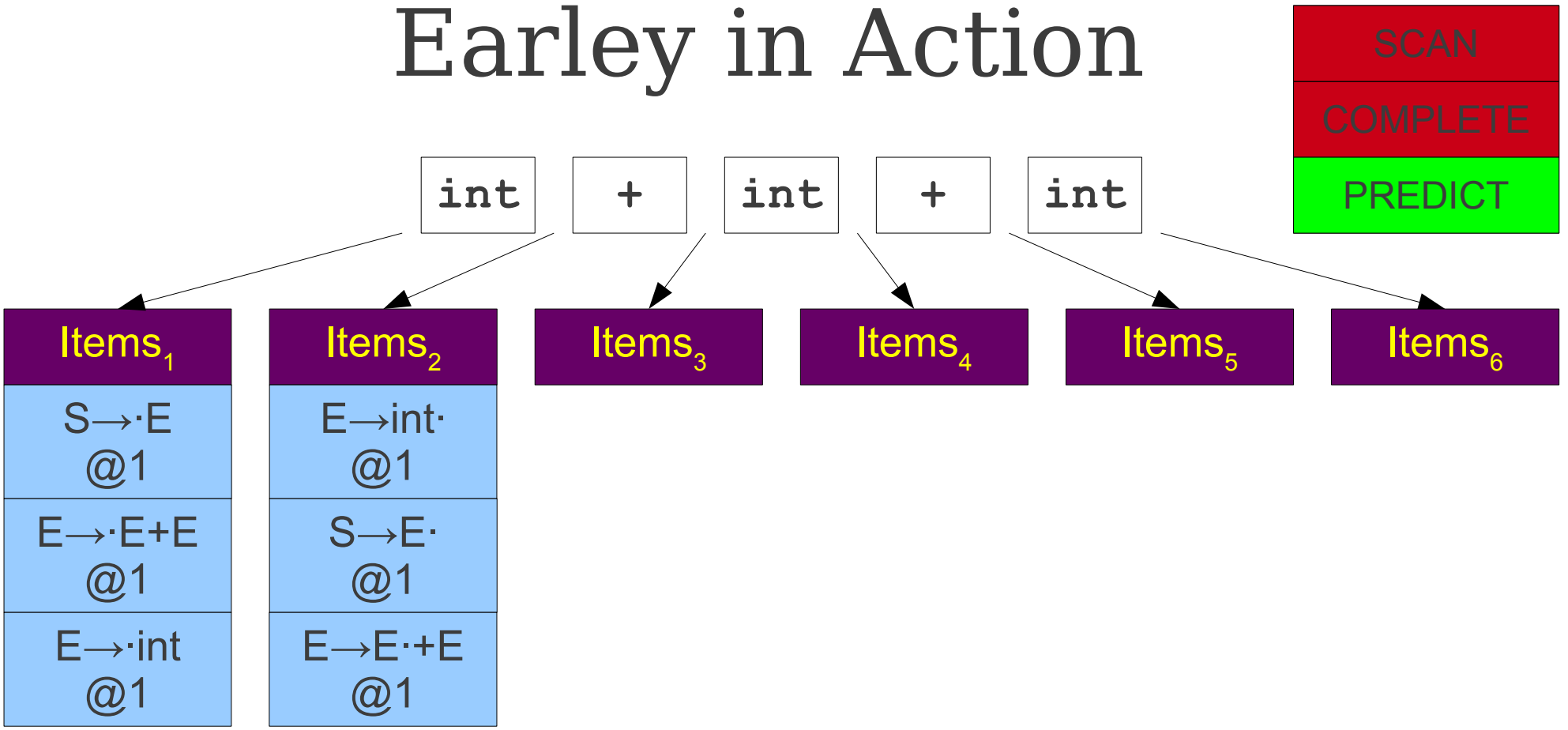

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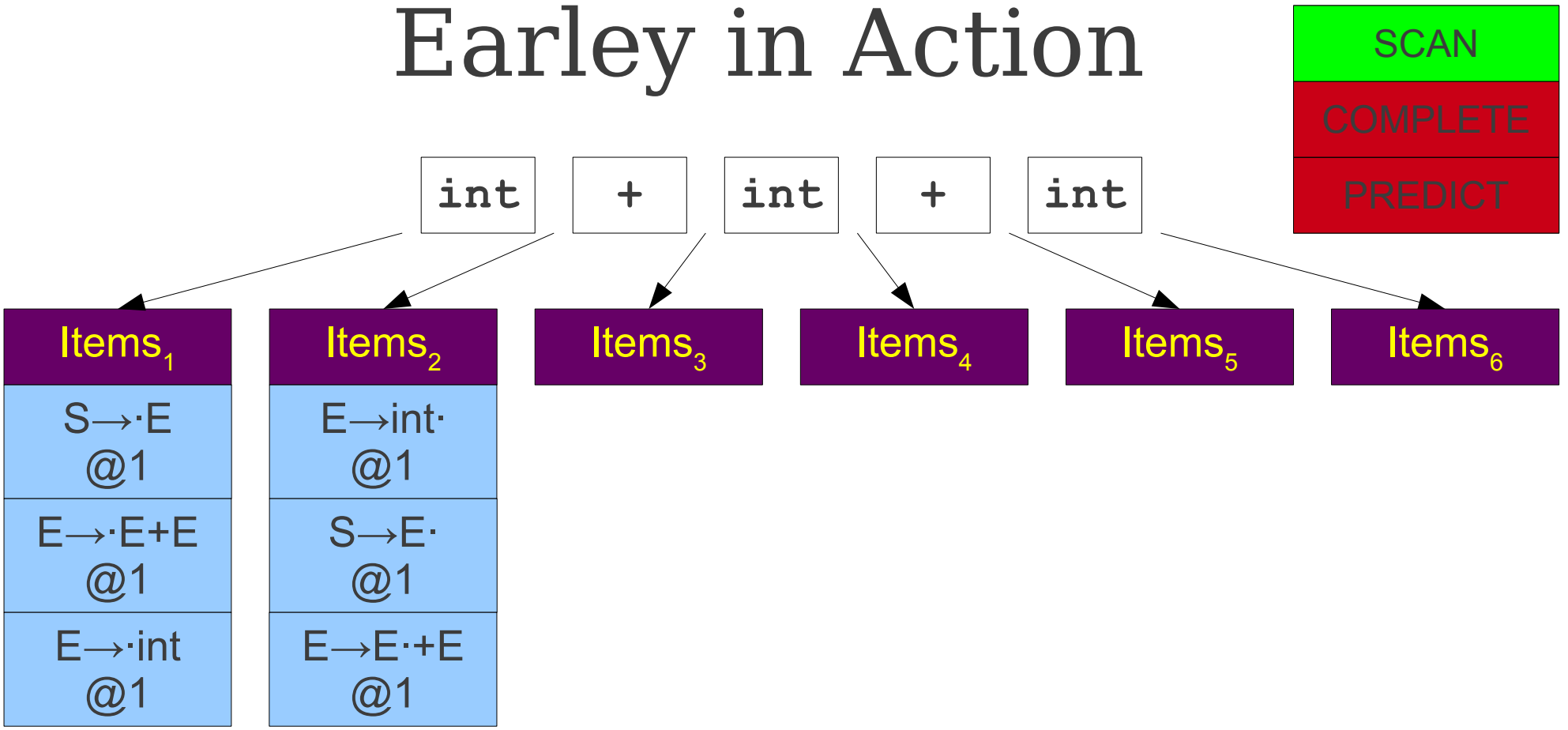

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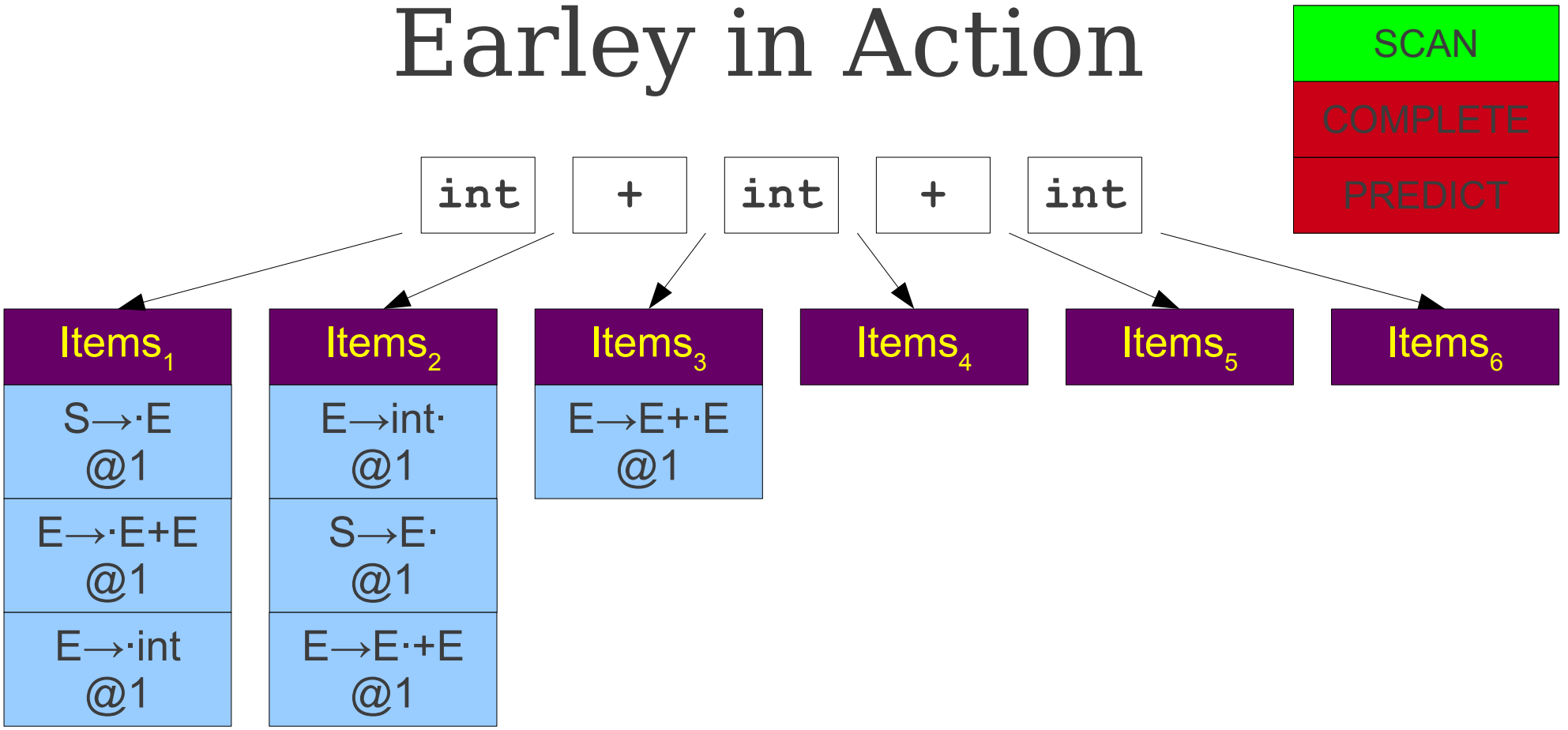

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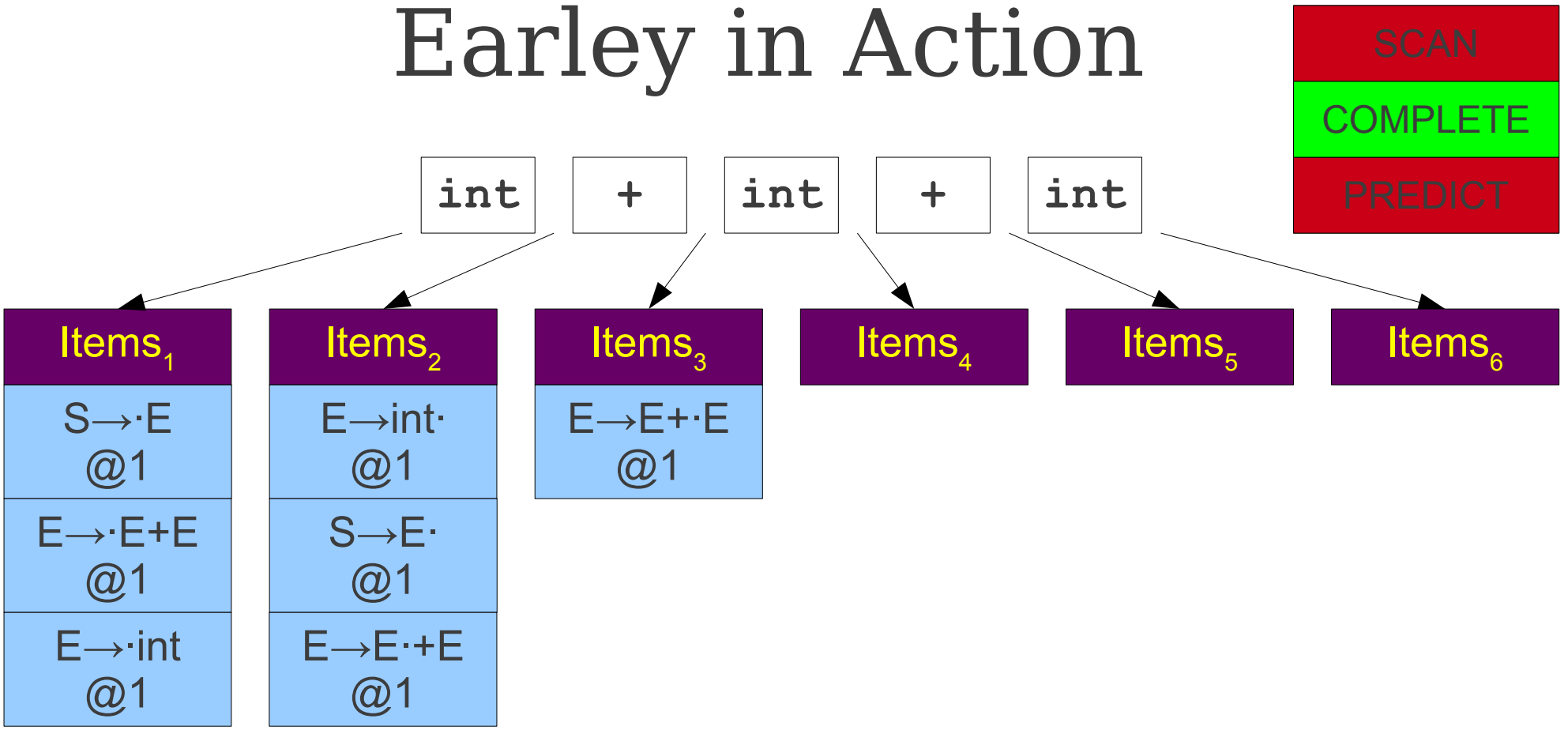

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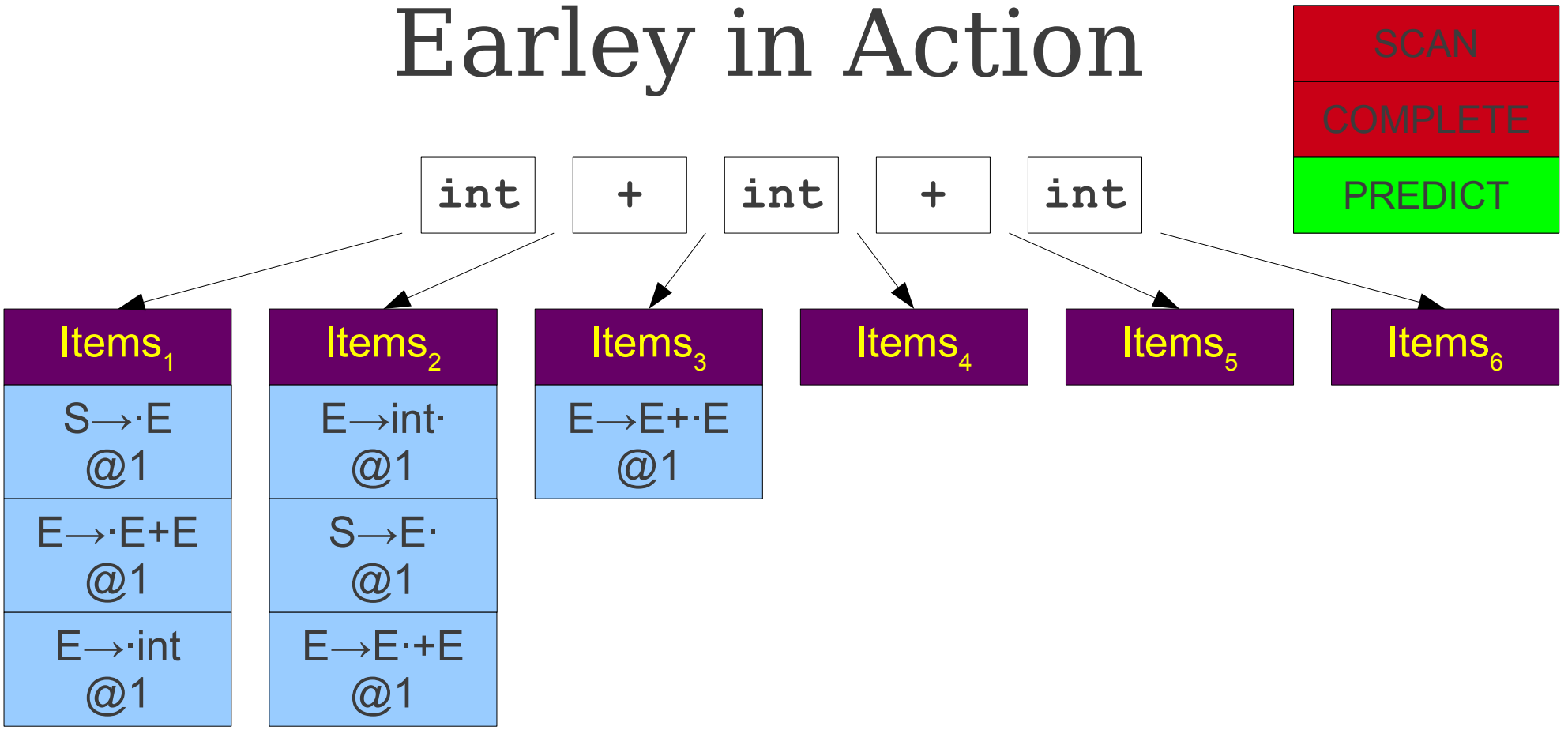

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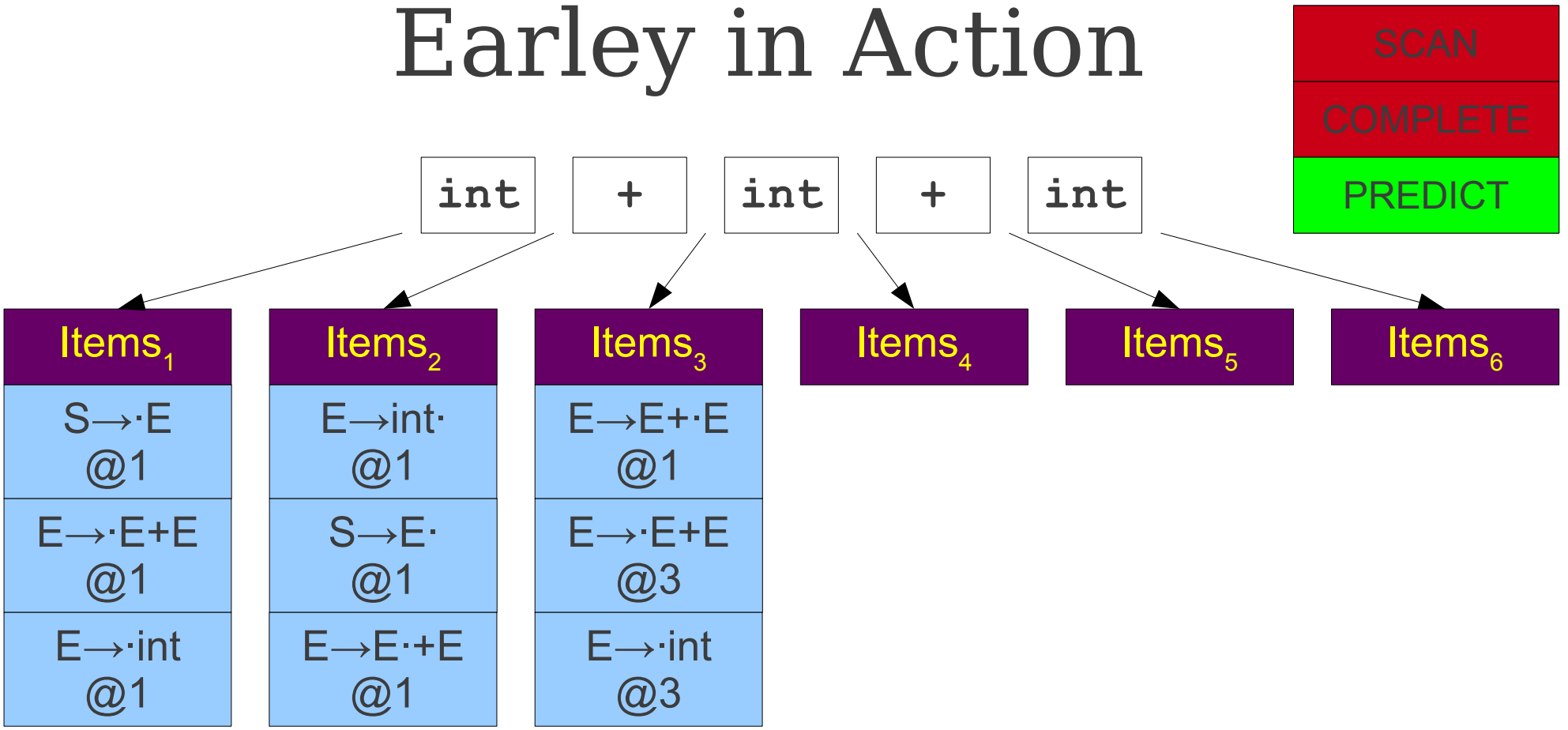

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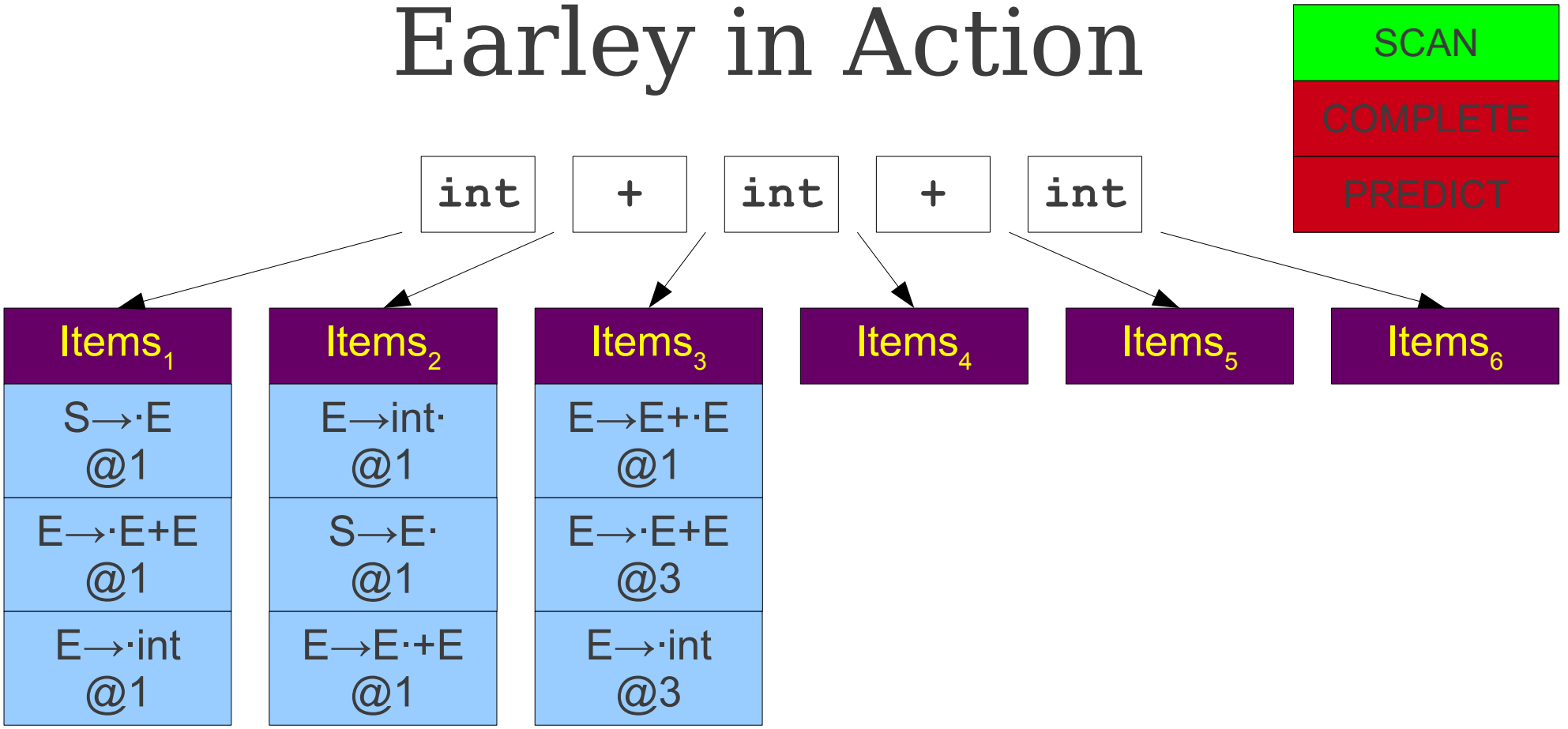

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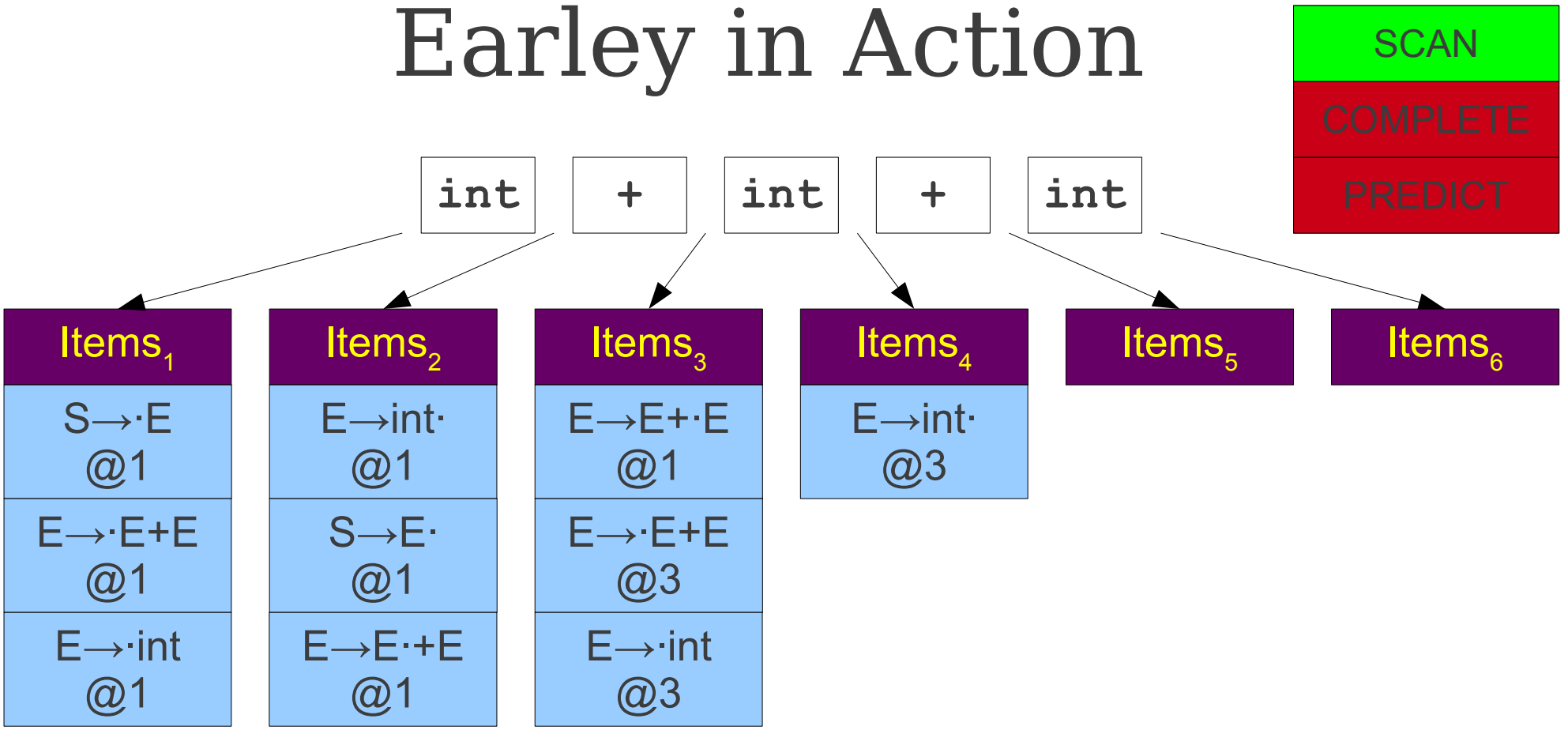

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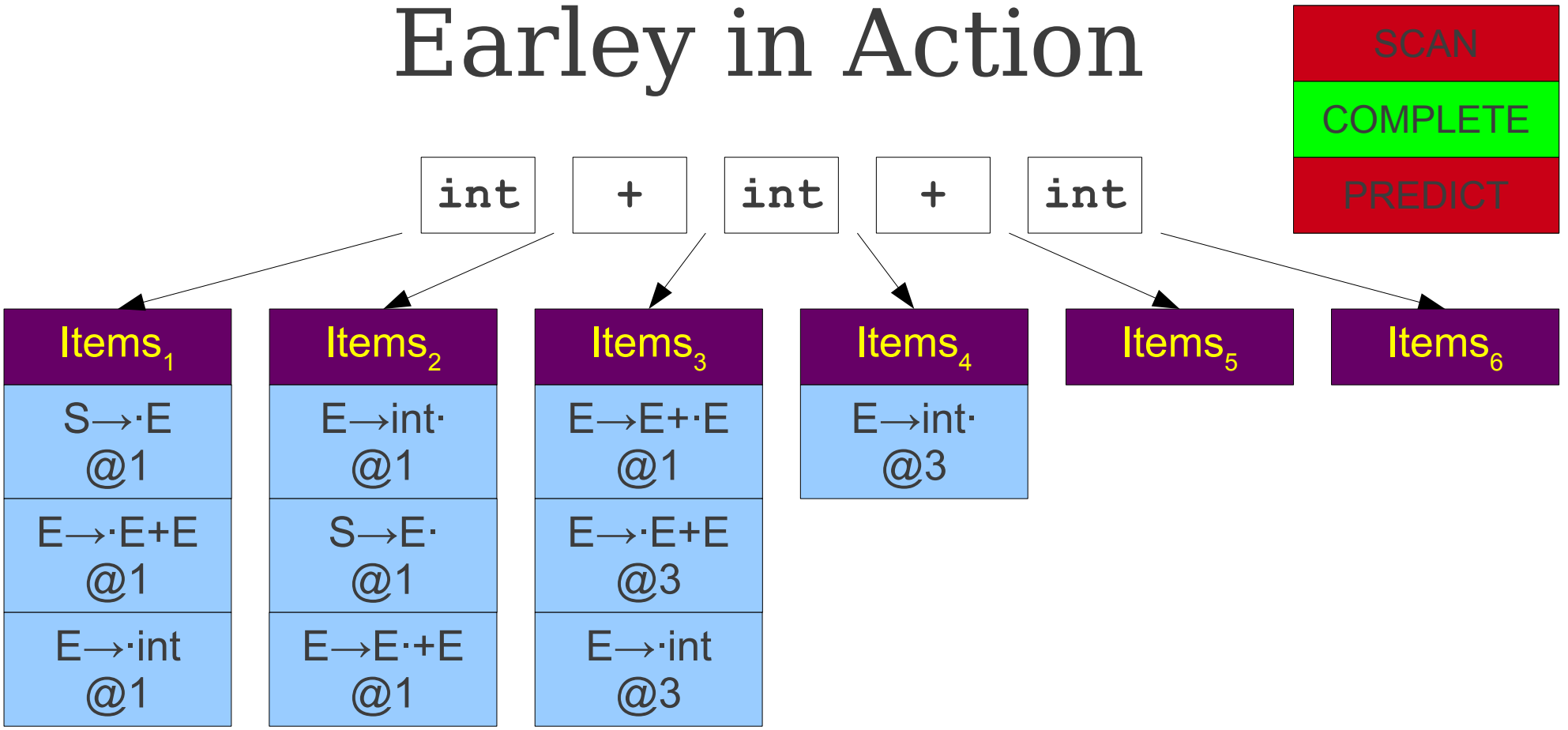

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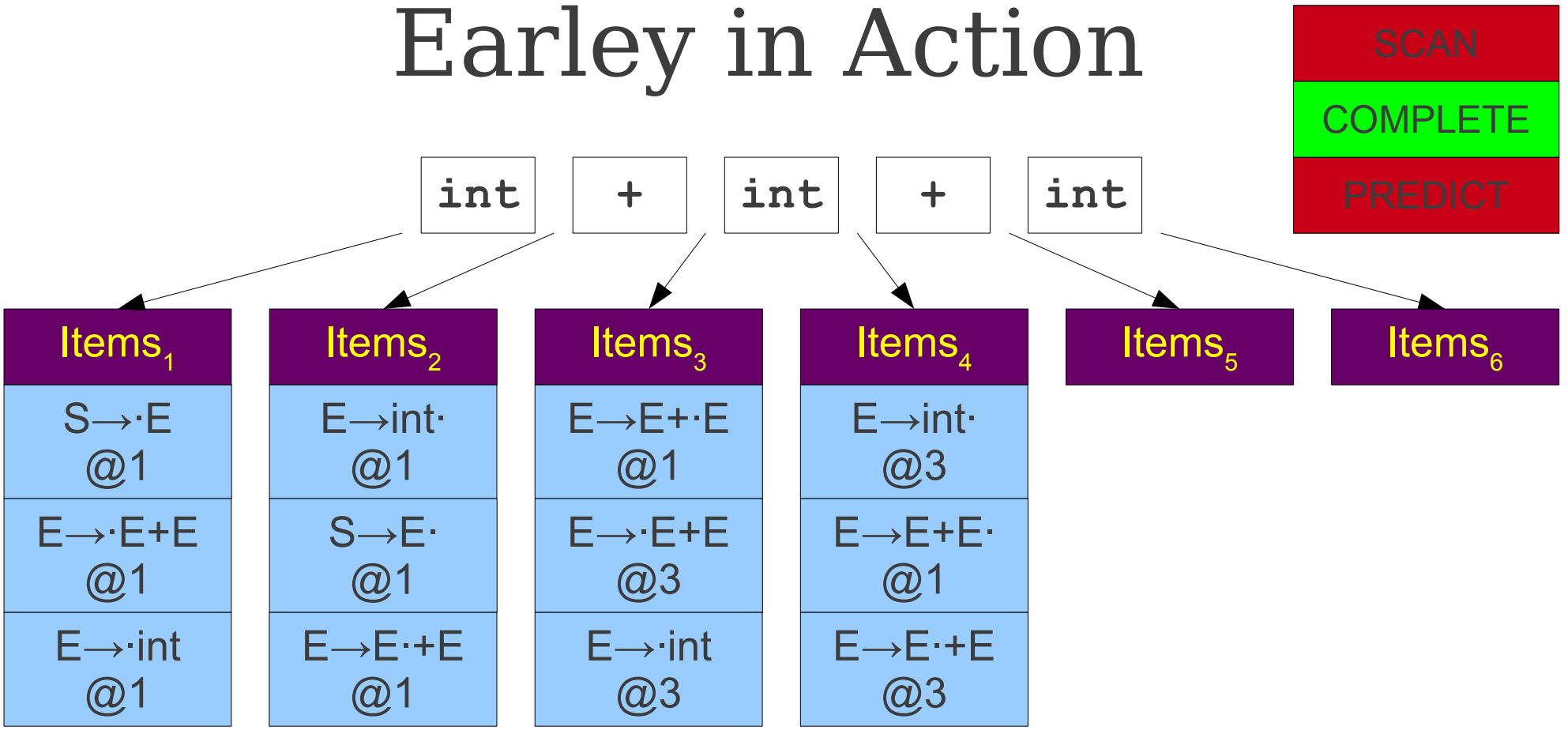

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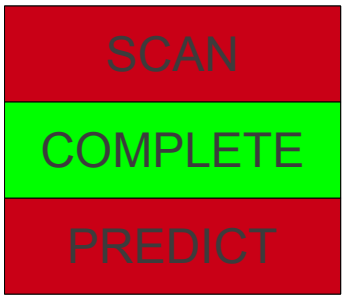

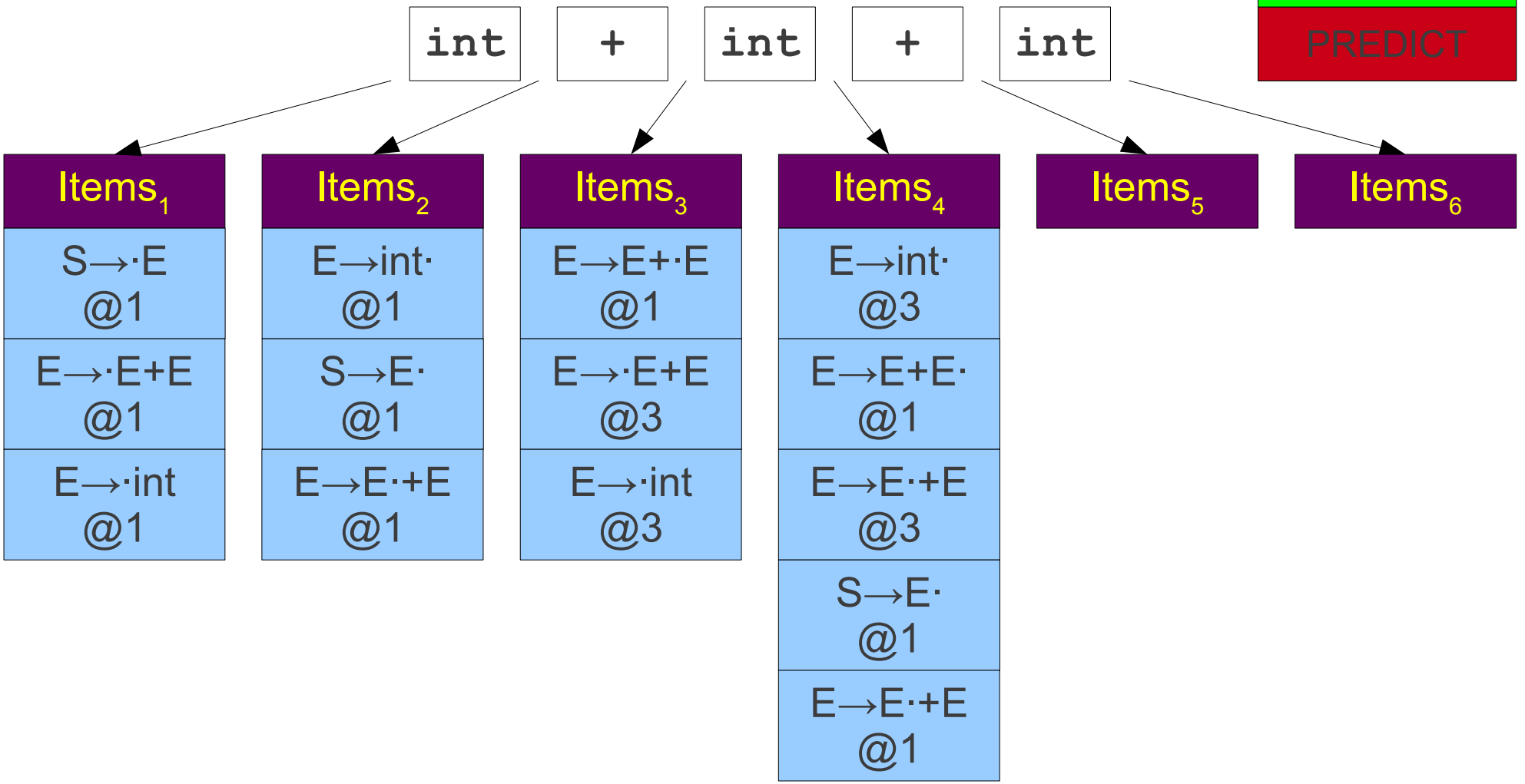

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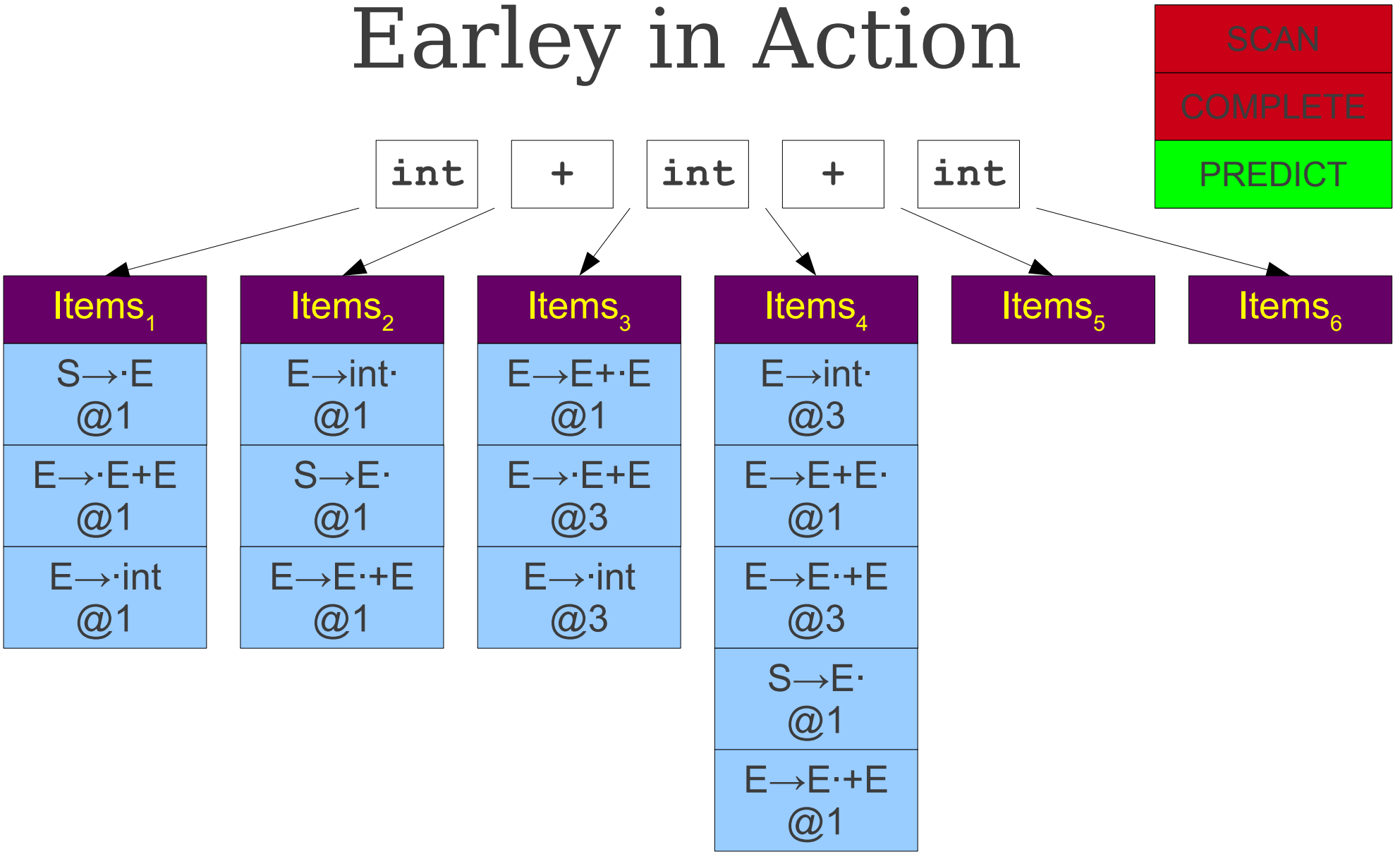

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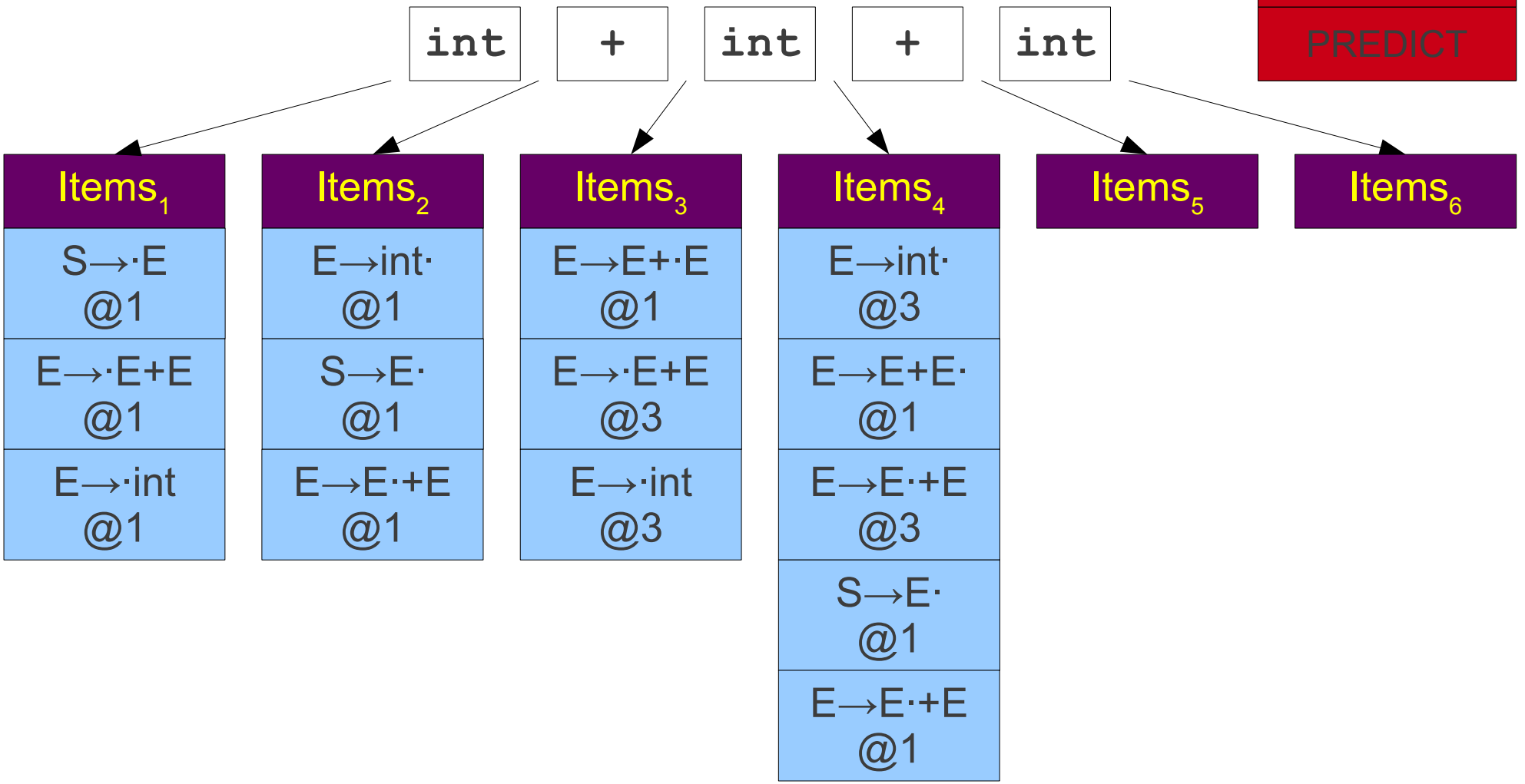

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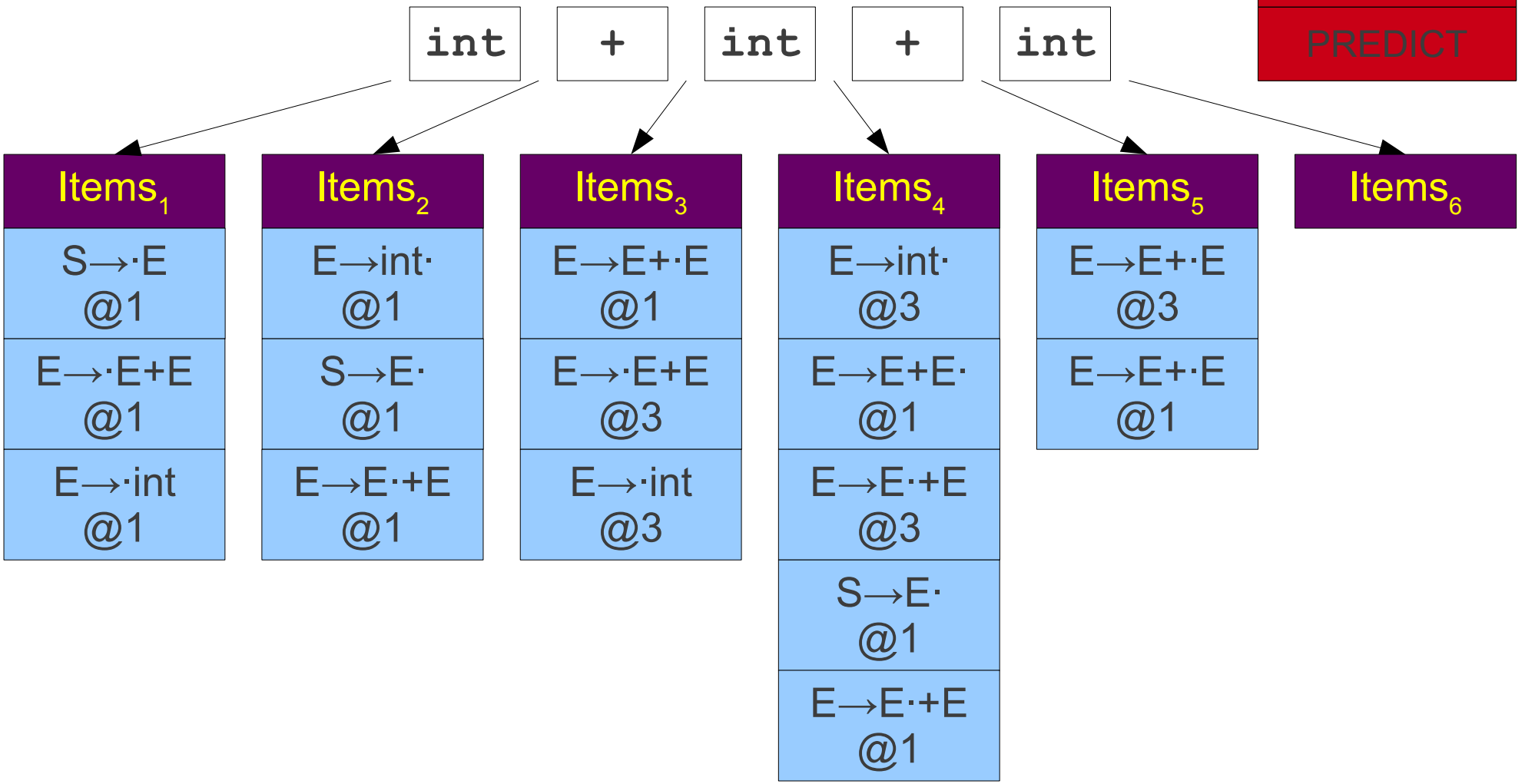

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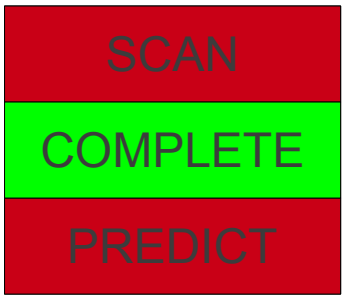

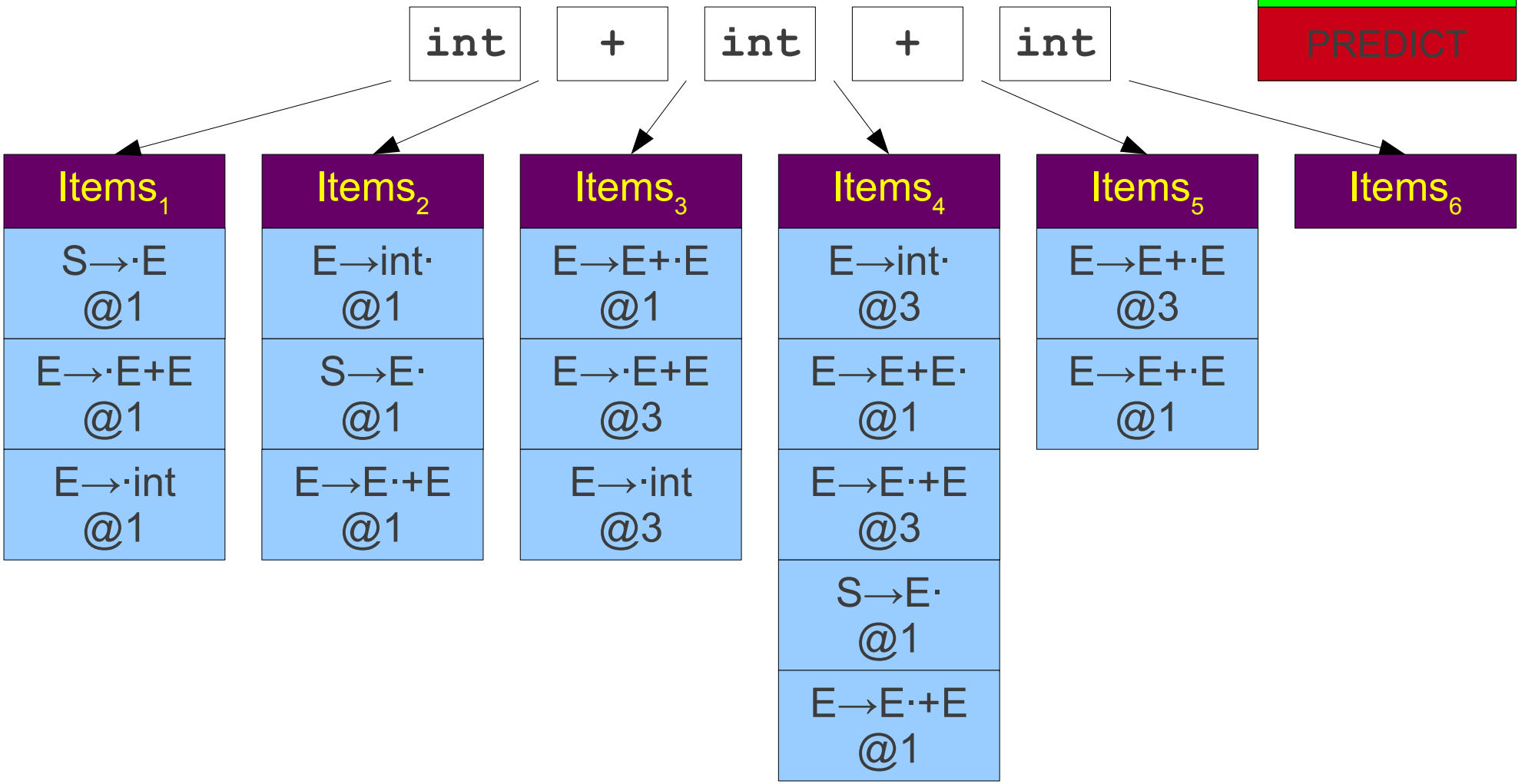

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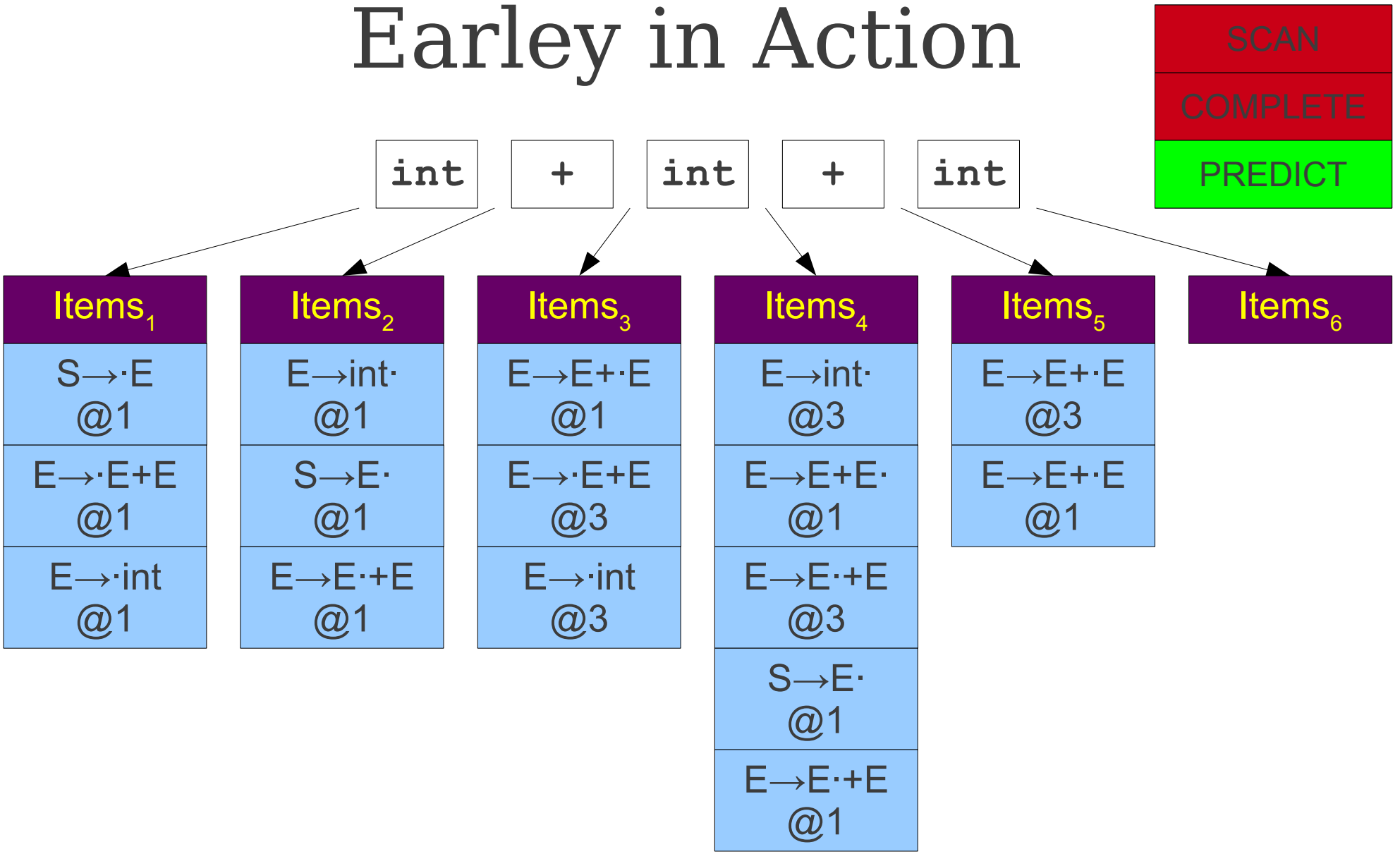

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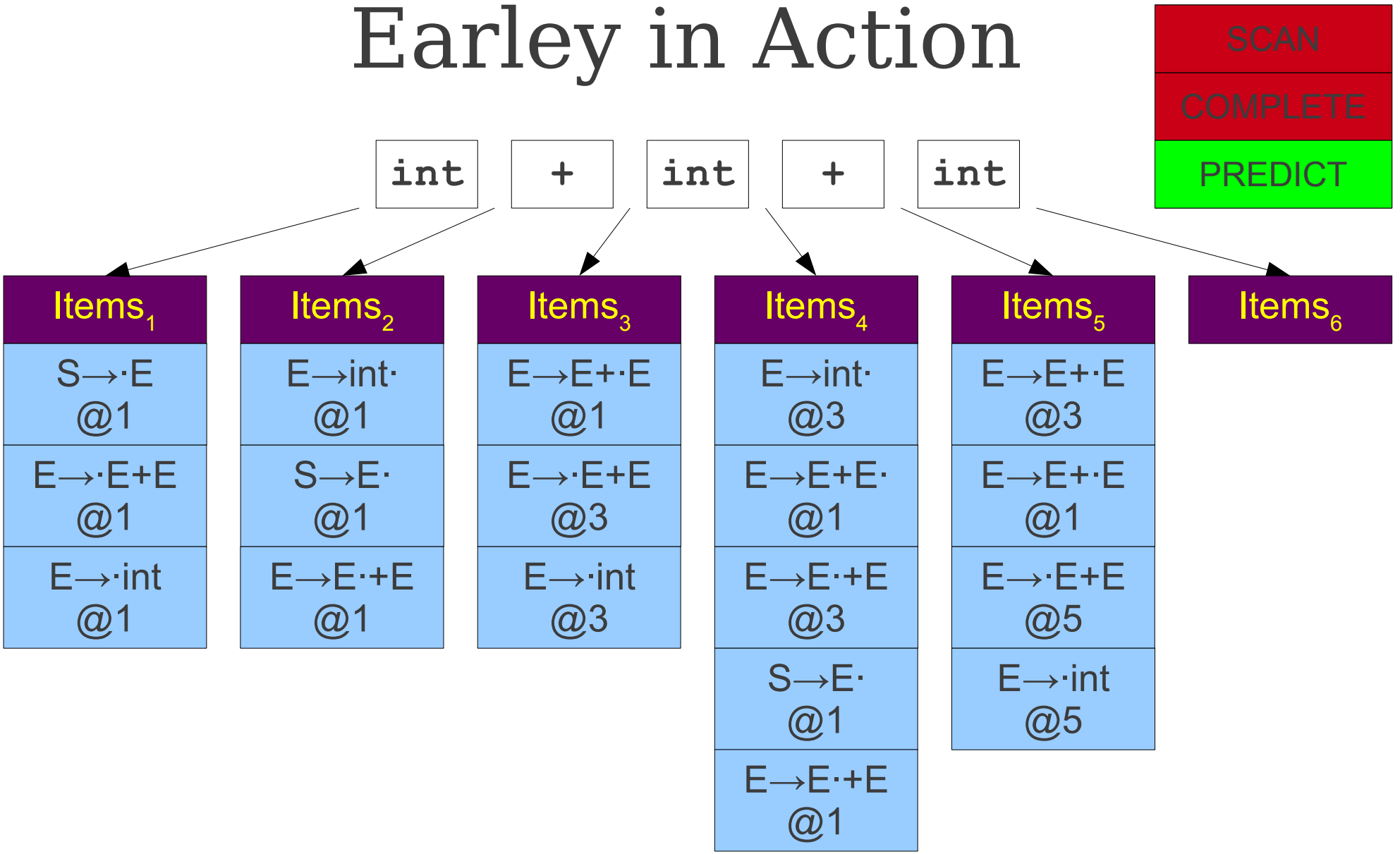

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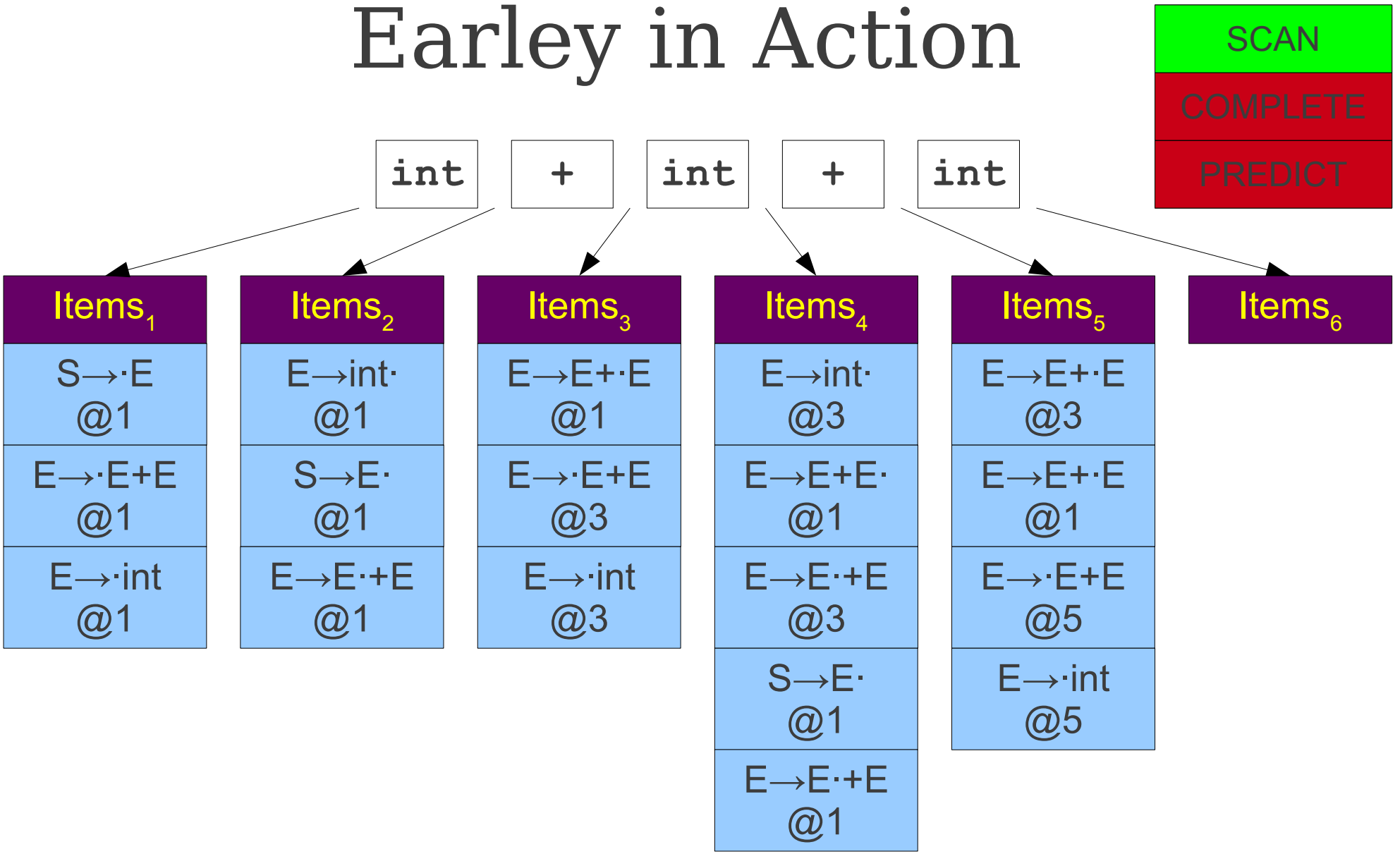

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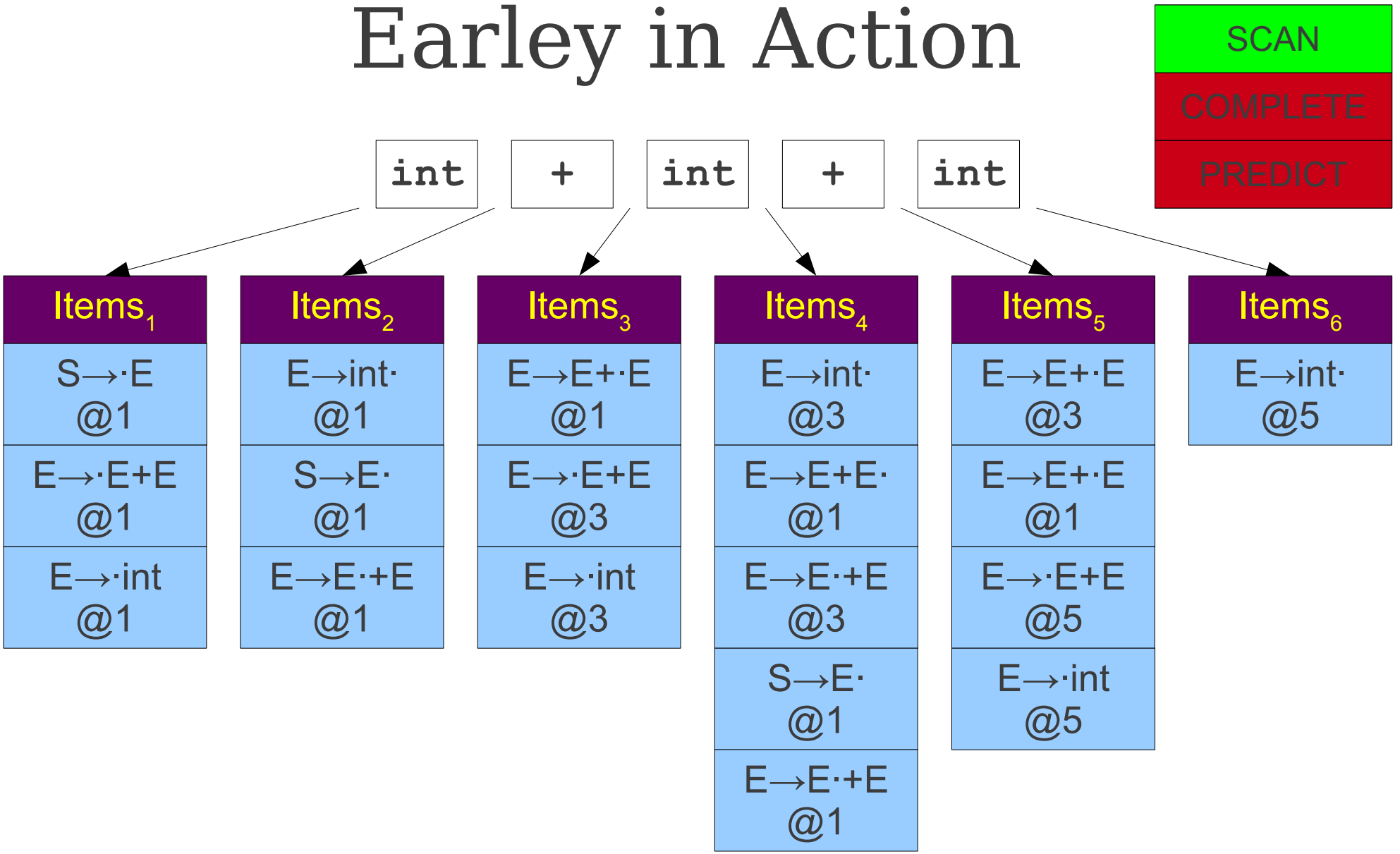

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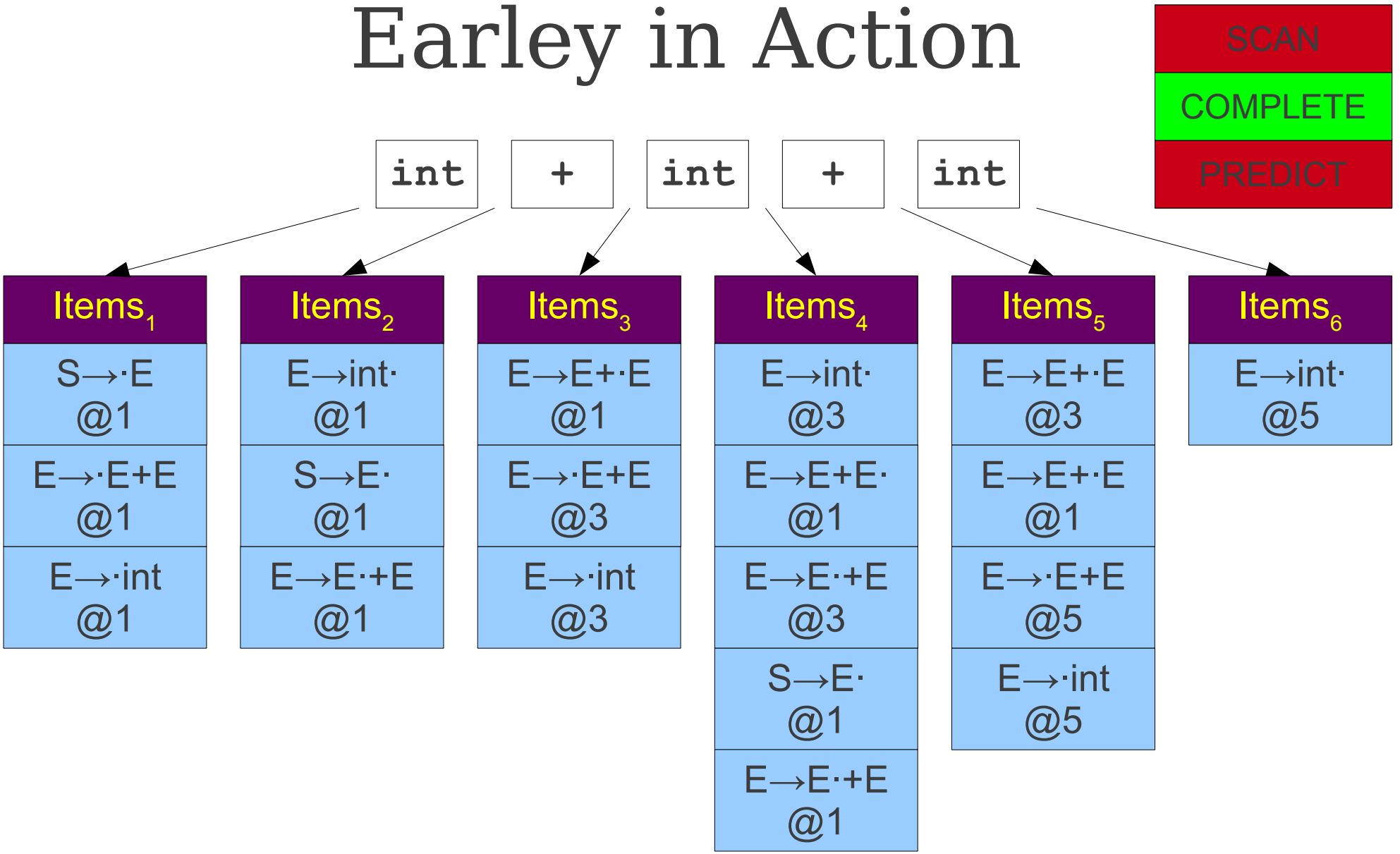

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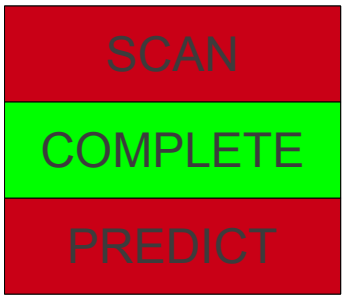

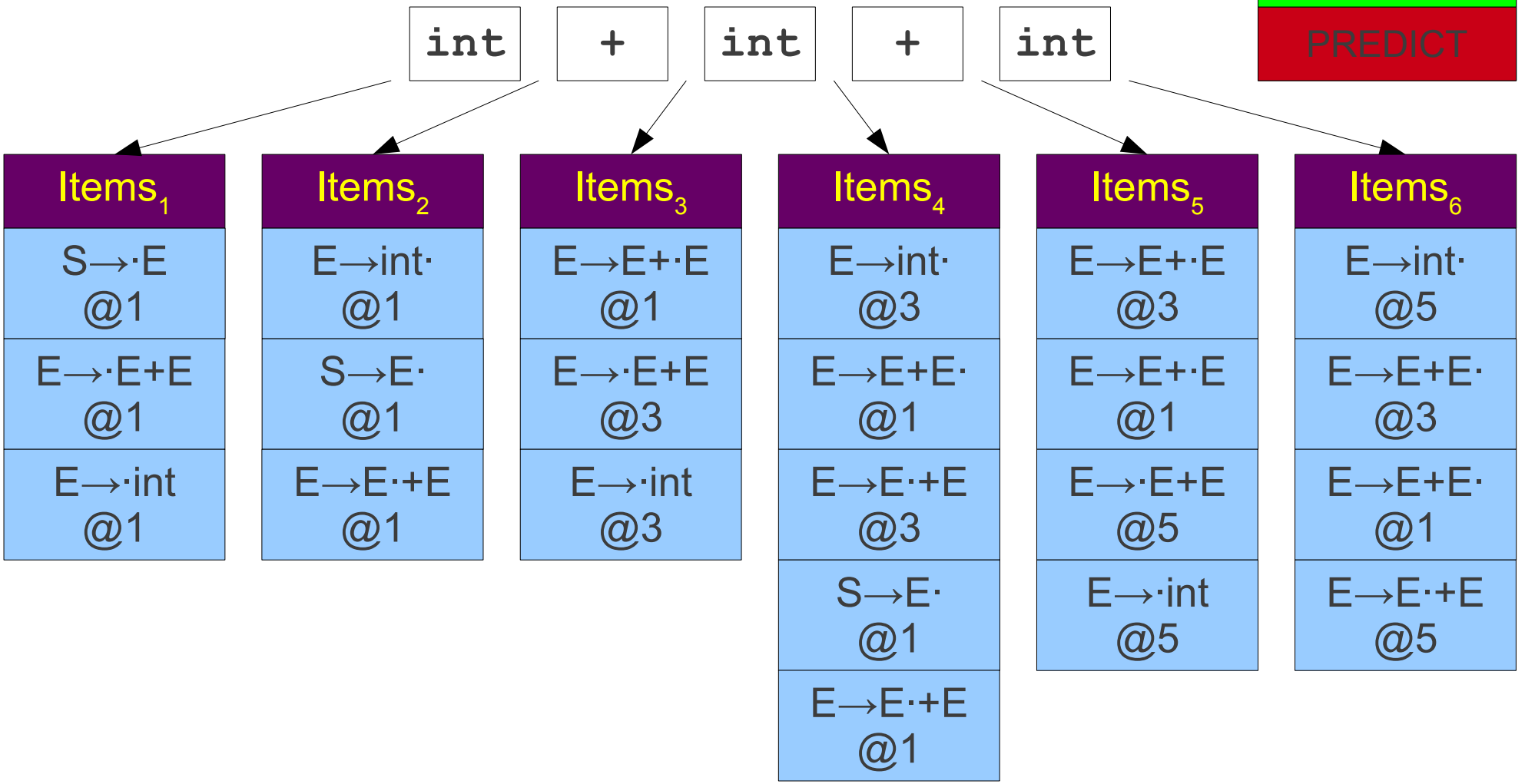

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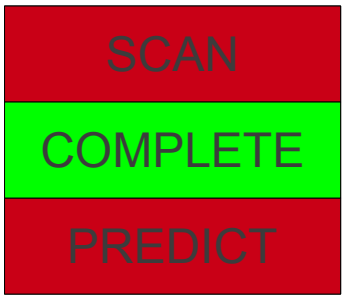

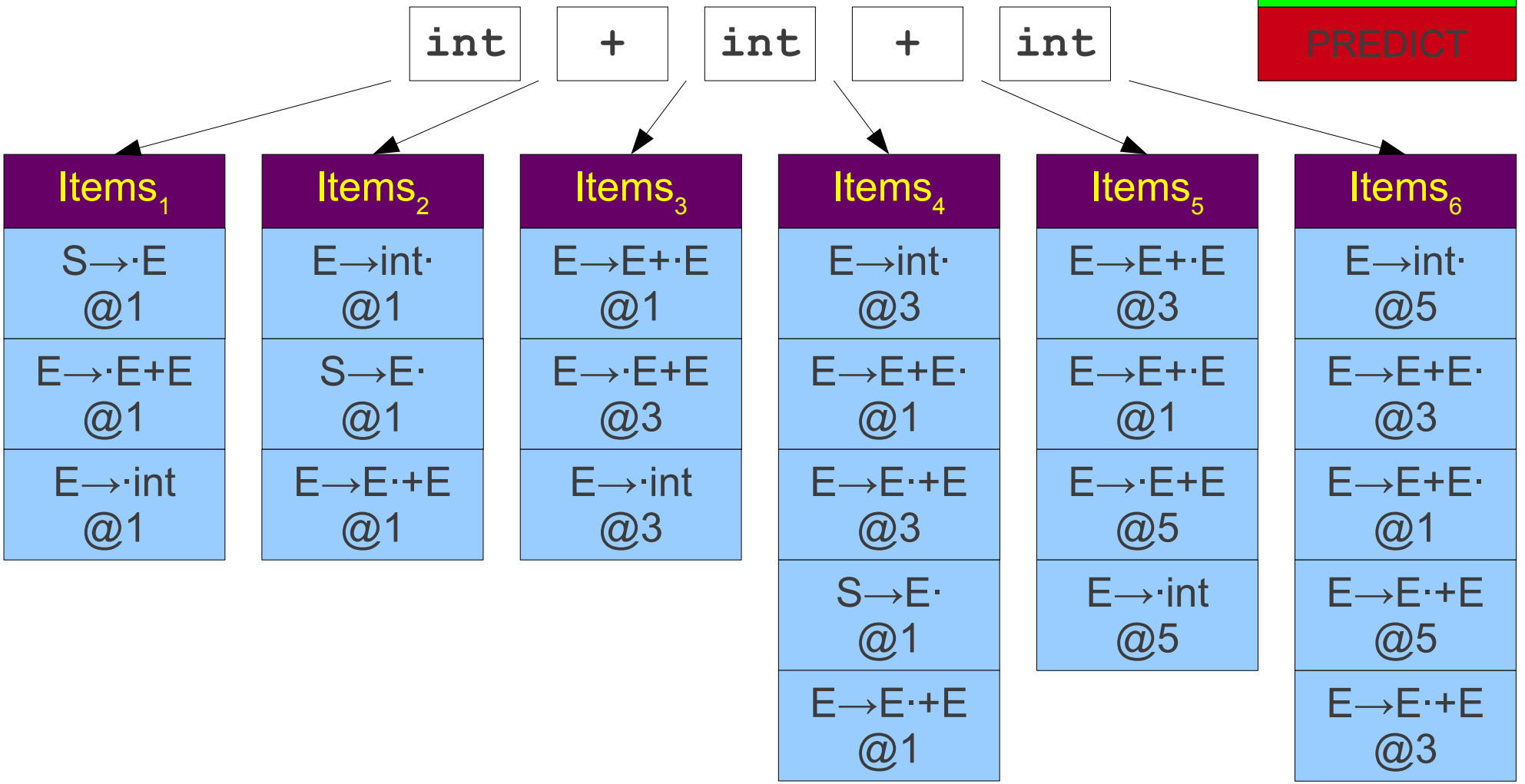

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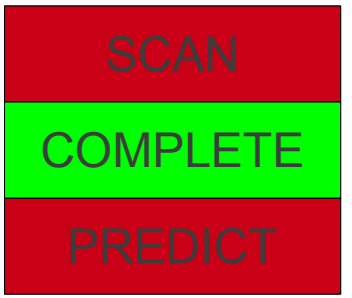

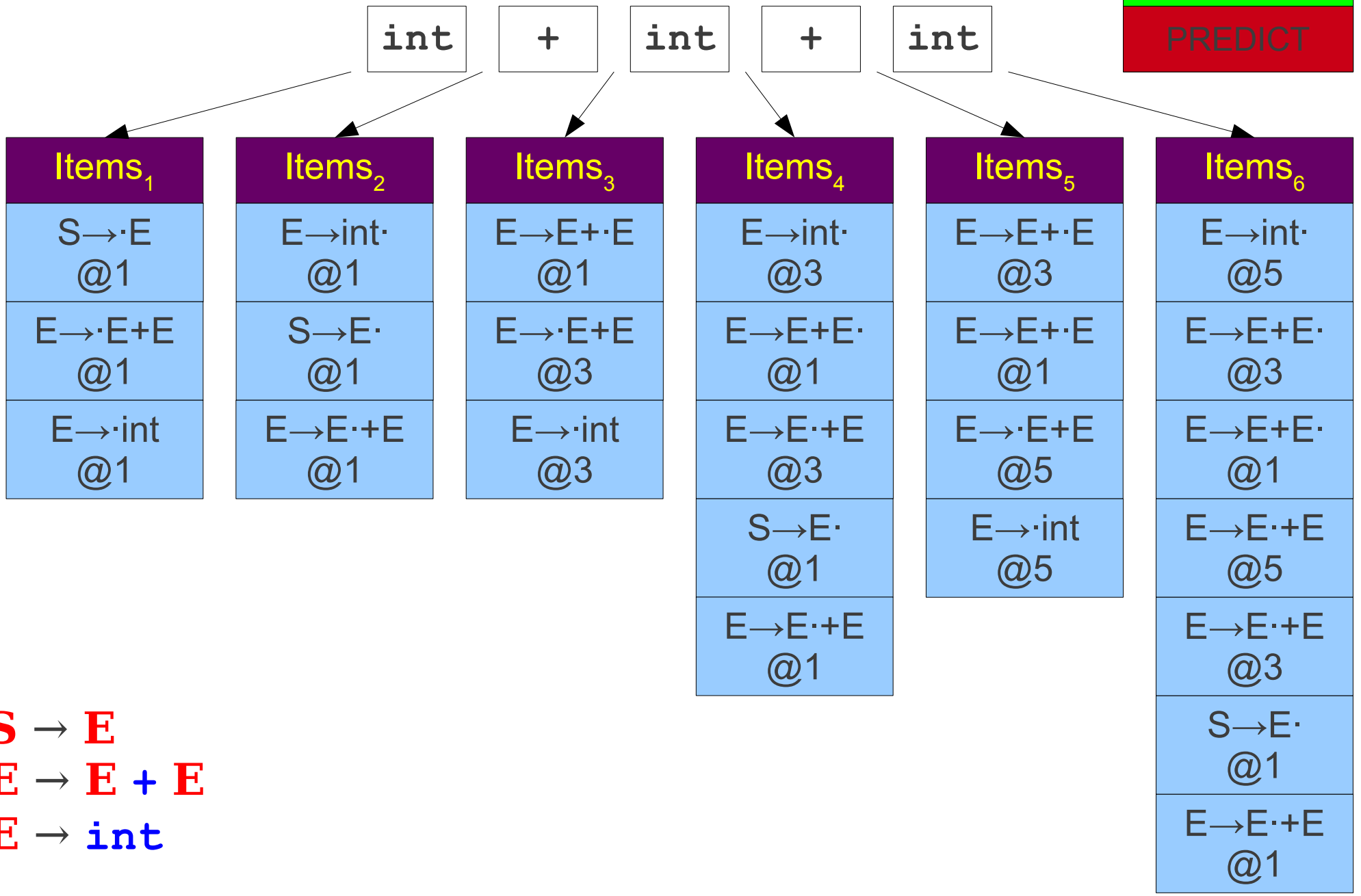

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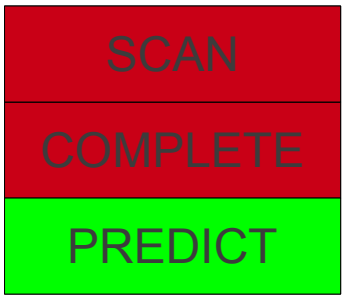

Items$_1$	Items$_2$	Items$_3$	Items$_4$	Items$_5$	Items$_6$
S→·E @1	E→int· @1	E→E+·E @1	E→int· @3	E→E+·E @3	E→int· @5
E→·E+E @1	S→E· @1	E→·E+E @3	E→E+E· @1	E→E+·E @1	E→E+E· @3
E→·int @1	E→E·+E @1	E→·int @3	E→E·+E @3	E→·E+E @5	E→E+E· @1
			S→E· @1	E→·int @5	E→E·+E @5
			E→E·+E @1		E→E·+E @3
					S→E· @1
					E→E·+E @1

S → E
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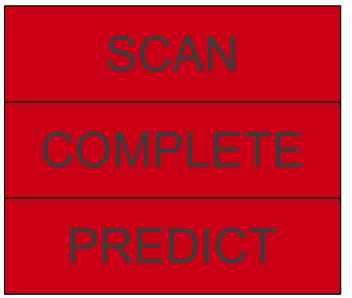
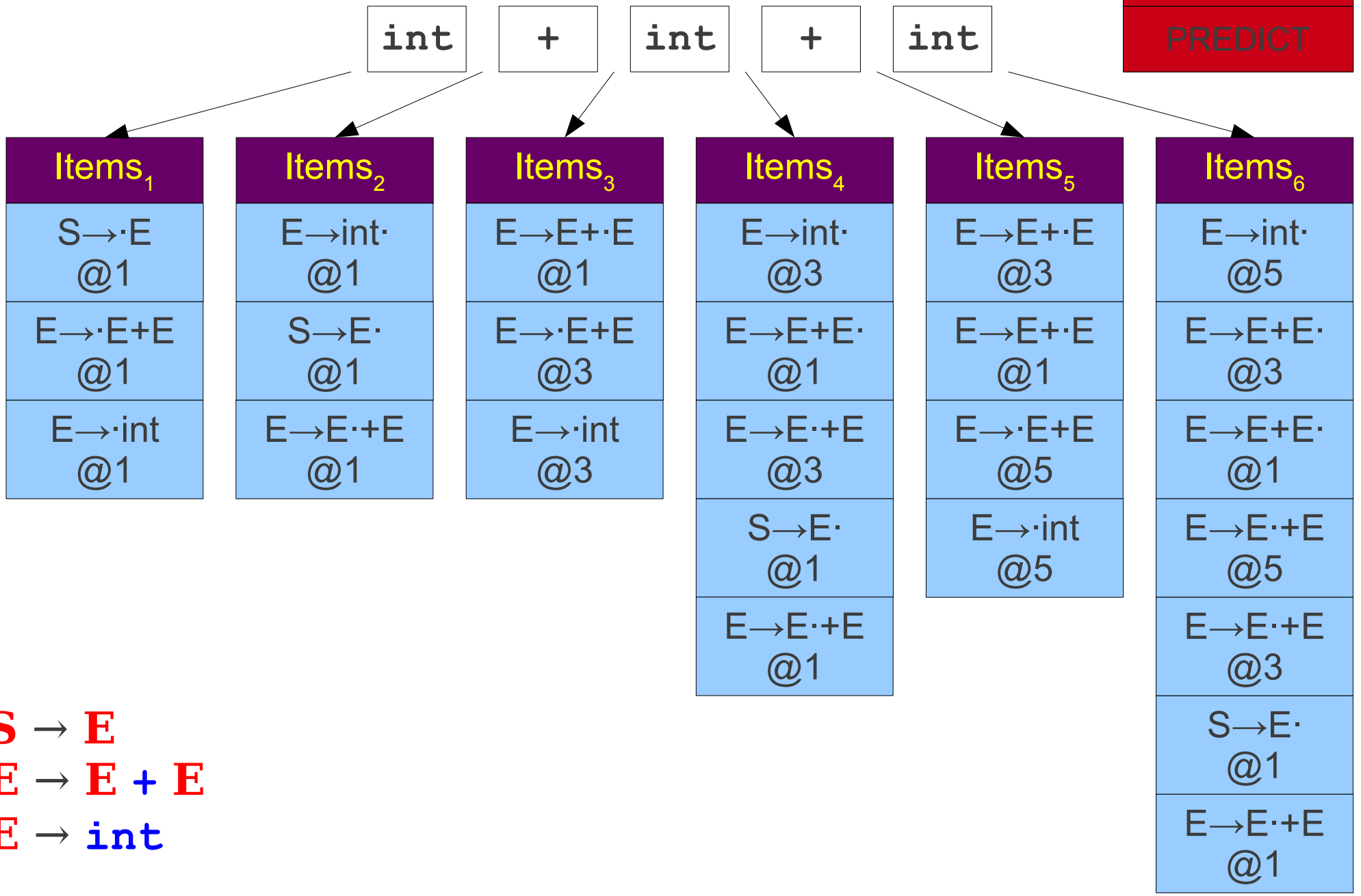
E → int

Earley in Action





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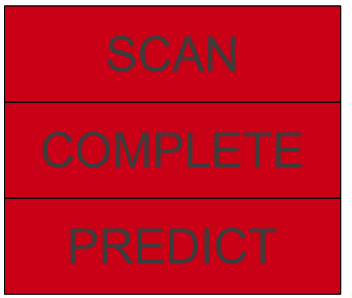

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E→·E+E @1	S→E· @1	E→·E+E @3	E→E+E· @1	E→E+·E @1	E→E+E· @3
E→·int @1	E→E·+E @1	E→·int @3	E→E·+E @3	E→·E+E @5	E→E+E· @1
			S→E· @1	E→·int @5	E→E·+E @5
			E→E·+E @1		E→E·+E @3
					S→E· @1
					E→E·+E @1

S → E
E → E + E
E → int

The Earley Algorithm

- **For now, assume no ε-rules**.
- Begin with the item **S** → ·**E** @1 in the first item set.
- Apply a **predict** step:
  - For each item **A** → ***α*·B*ω*** **@n** in the *k*th item set, add the item **B** → ·***γ*** @**k** to the ***k***th item set for each production **B** → ***γ***.
- Apply a **scan** step:
  - For each item **A** → ***α*·`t`*ω*** **@n** in the *k*th item set, if the *k*th token is t, add **A** → ***α*`t`·*ω*** **@n** to the (*k* + 1)st item set.
- Apply a **complete** step:
  - For each item **A** → ***γ*·** **@n** in the *kth* item set, for each item **B** → ***α*·A*ω*** **@m** in the nth item set, add **B** → ***α*A·*ω*** **@m** to the *k*th item set.

Supporting ε-Rules

- Simple modification to the **predict** step:

  If there is an item **A** → ***α*·B*ω*** **@n** in the *k*th item set, where **ε** ∈ FIRST(**B**), add the item **A** → ***α*B·*ω*** **@n** to the *k*th item set.

- Intuition: One prediction is that **B** ends up expanding out to nothing.

The Efficiency of Earley

- How many LR(0) items are possible in a grammar G?

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  - O(|G|)

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- How many Earley items can there be?
  - One for each triple of (LR(0) item, start position, end position).
  - $O(|G|n^2)$
- How many times can each item be generated?
  - Item at position k could be added from positions 1, 2, …, k – 1
  - O(n)

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  - O(|G|)
- How many Earley items can there be?
  - One for each triple of (LR(0) item, start position, end position).
  - $O(|G|n^2)$
- How many times can each item be generated?
  - Item at position k could be added from positions 1, 2, …, k – 1
  - O(n)
- Overall complexity is **$O(|G|n^3)$**

The Efficiency of Earley

- How many LR(0) items are possible in a grammar G?
  - One for each possible position in any production.
  - O(|G|)
- How many Earley items can there be?
  - One for each triple of (LR(0) item, start position, end position).
  - $O(|G|n^2)$
- How many times can each item be generated?
  - Item at position k could be added from positions 1, 2, …, k – 1
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- Overall complexity is **$O(|G|n^3)$**
- For a fixed grammar, parse time is **$O(n^3)$**.

The Efficiency of Earley

- How many LR(0) items are possible in a grammar G?
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  - O(|G|)
- How many Earley items can there be?
  - One for each triple of (LR(0) item, start position, end position).
  - $O(|G|n^2)$
- How many times can each item be generated?
  - Item at position k could be added from positions 1, 2, …, k – 1
  - O(n)
- Overall complexity is **$O(|G|n^3)$**
- For a fixed grammar, parse time is **$O(n^3)$**.
- For a fixed grammar, memory usage always **$O(n^2)$**.

Interesting Results

- The Earley parser always runs in $O(n^2)$ on unambiguous grammars.
  - Intuition: Never generates the same item twice.
- If we add k tokens of lookahead before applying productions or reductions, the Earley parser can parse any LR(k) grammar in time $O(n)$.
  - Intuition: We never need to backtrack, so each item generated ends up being used.

Recognizers and Parsers

- We have just discussed the Earley *recognizer*, not the Earley *parser*.
- Right now, we can only detect whether a string is valid by seeing if **S** → **E**· @1 is in the last item set.
- We need to discuss how to upgrade our *recognizer* into a *parser*.
- If the grammar is ambiguous, how do we hand back multiple parse trees?

A Highly Ambiguous Grammar

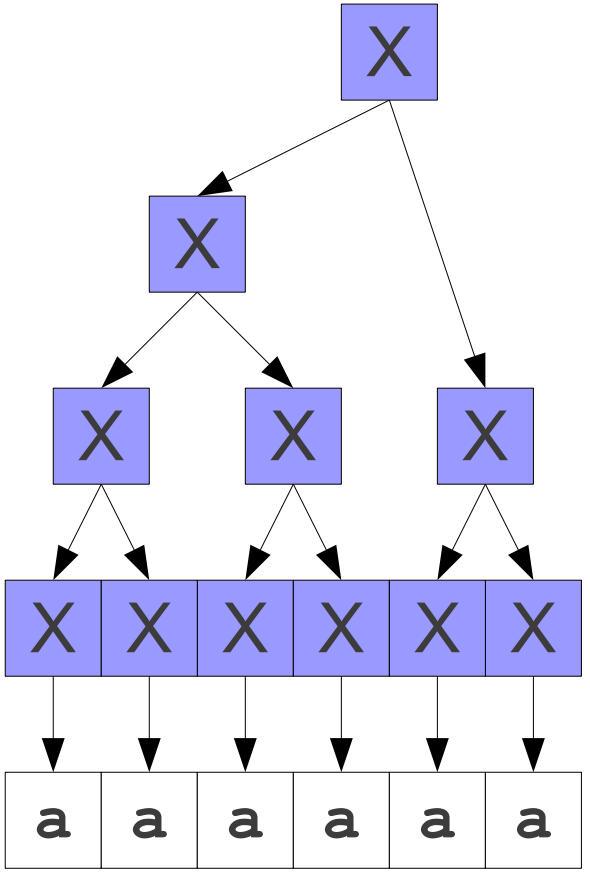
$X \rightarrow XX$
$X \rightarrow a$

A Highly Ambiguous Grammar

X → XX
X → a

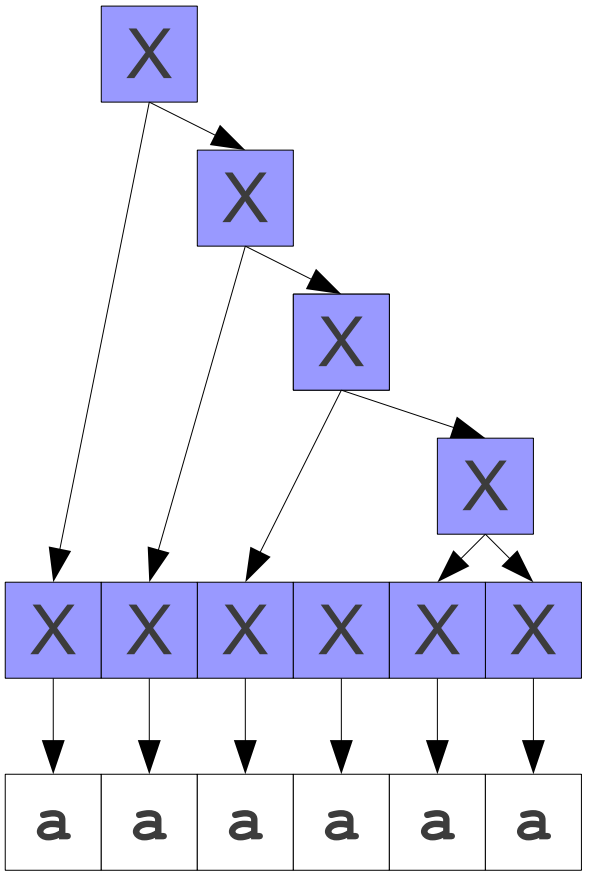
A Highly Ambiguous Grammar

X → XX
X → a



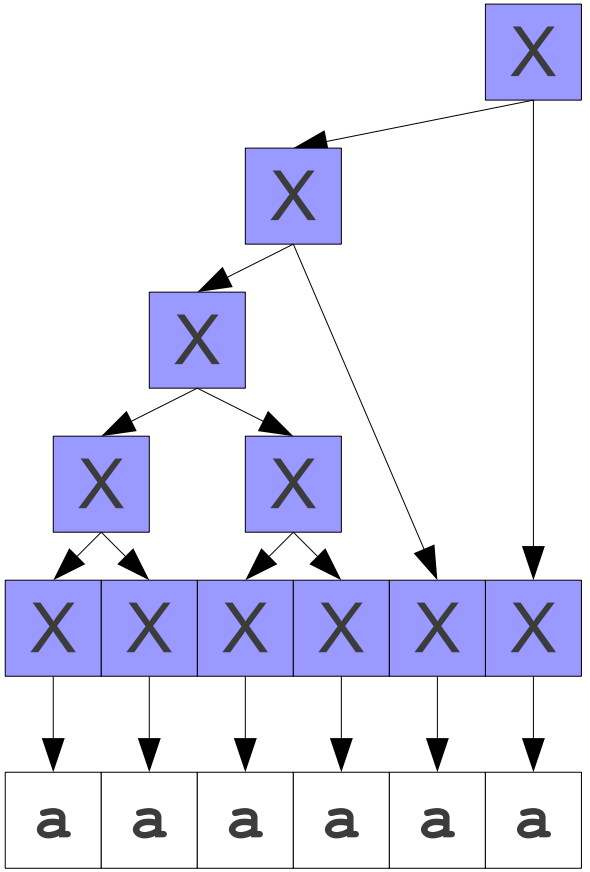
A Highly Ambiguous Grammar

$X \rightarrow XX$
$X \rightarrow a$



A Highly Ambiguous Grammar

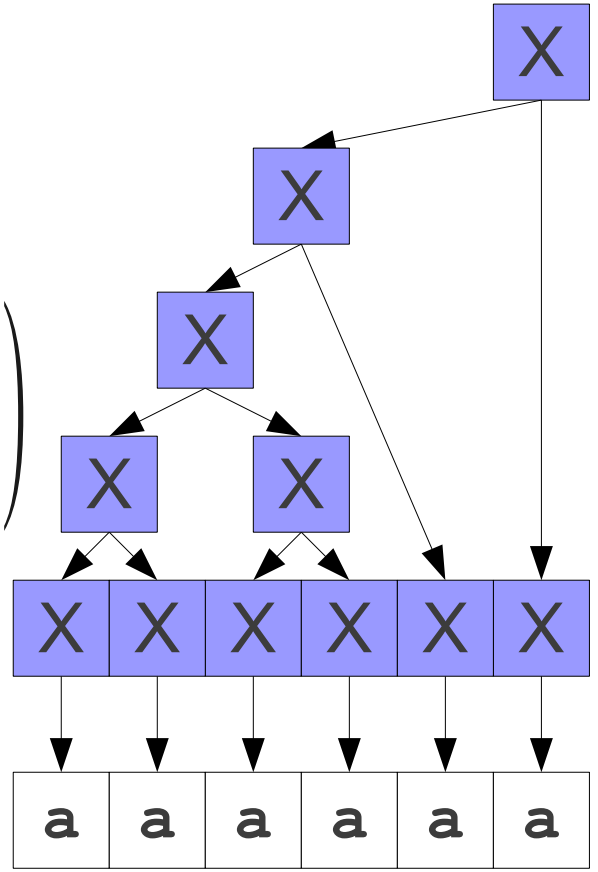
$X \rightarrow XX$
$X \rightarrow a$



A Highly Ambiguous Grammar

$X \rightarrow XX$
$X \rightarrow a$

$$PT(n) = \frac{1}{n}\binom{2n-2}{n-1}$$



An Infinitely Ambiguous Grammar

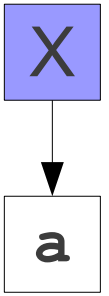
$X \rightarrow X$
$X \rightarrow a$

An Infinitely Ambiguous Grammar

$X \rightarrow X$
$X \rightarrow a$

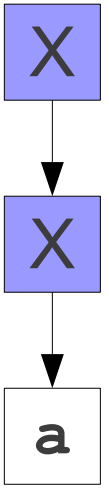
An Infinitely Ambiguous Grammar

$X \rightarrow X$
$X \rightarrow a$



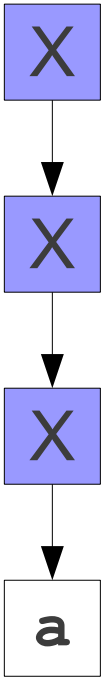
An Infinitely Ambiguous Grammar

$X \rightarrow X$
$X \rightarrow a$



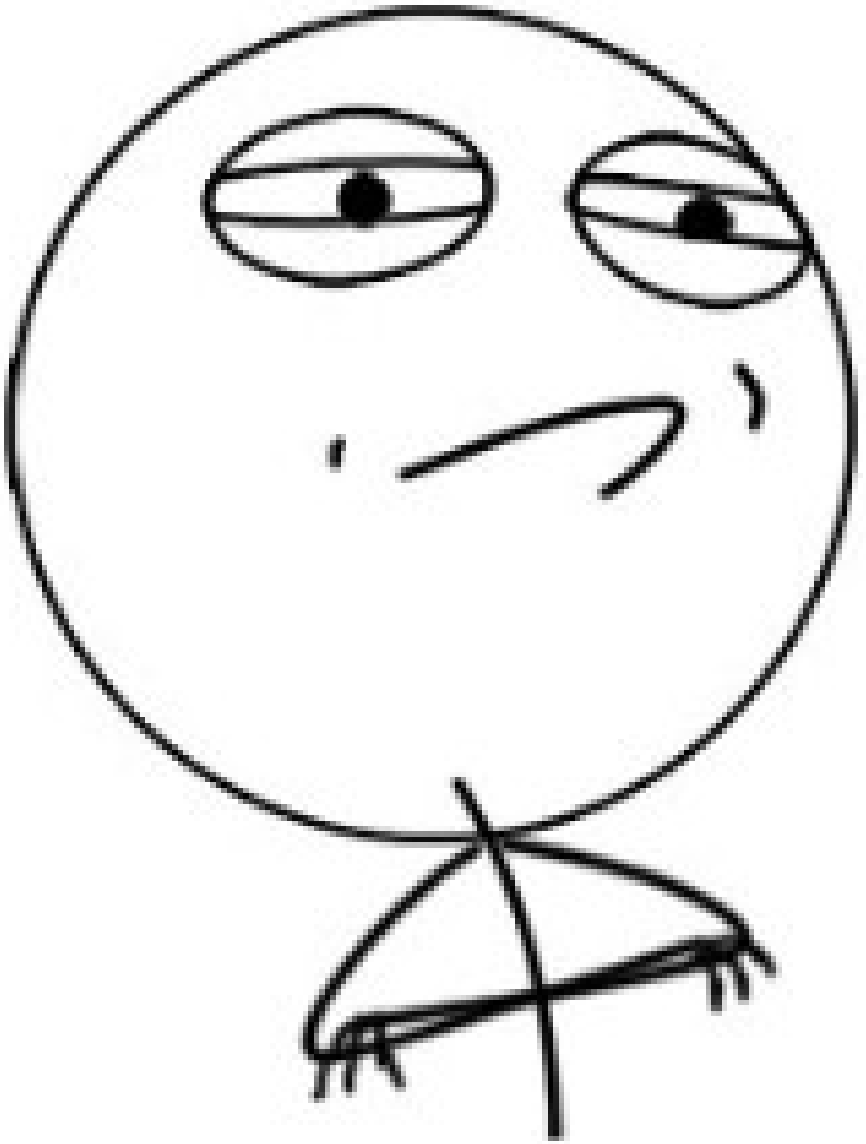
An Infinitely Ambiguous Grammar

$X \rightarrow X$
$X \rightarrow a$



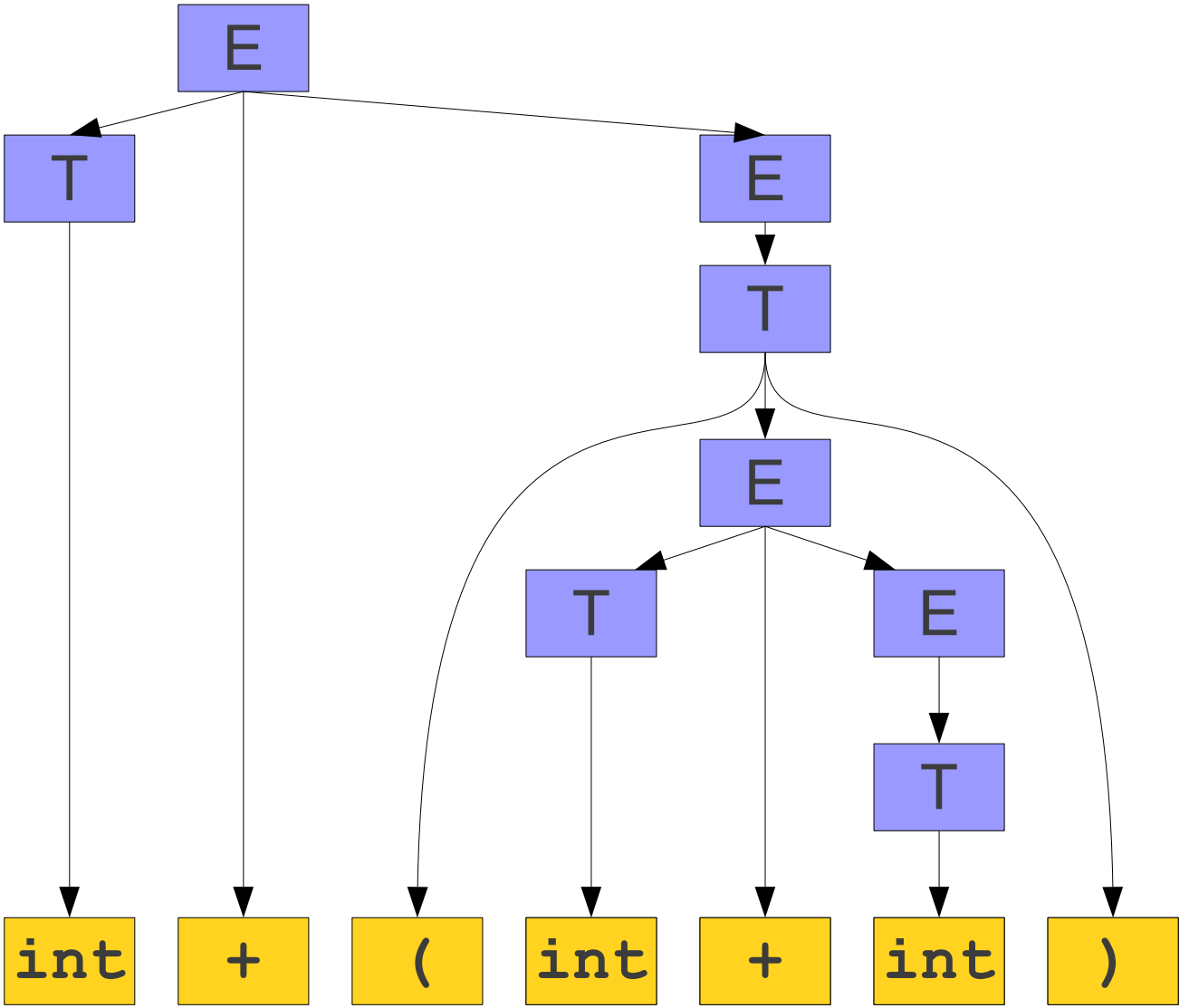
Given that there can be **infinitely many parse trees**, how could we possibly list all of them?

CHALLENGE ACCEPTED



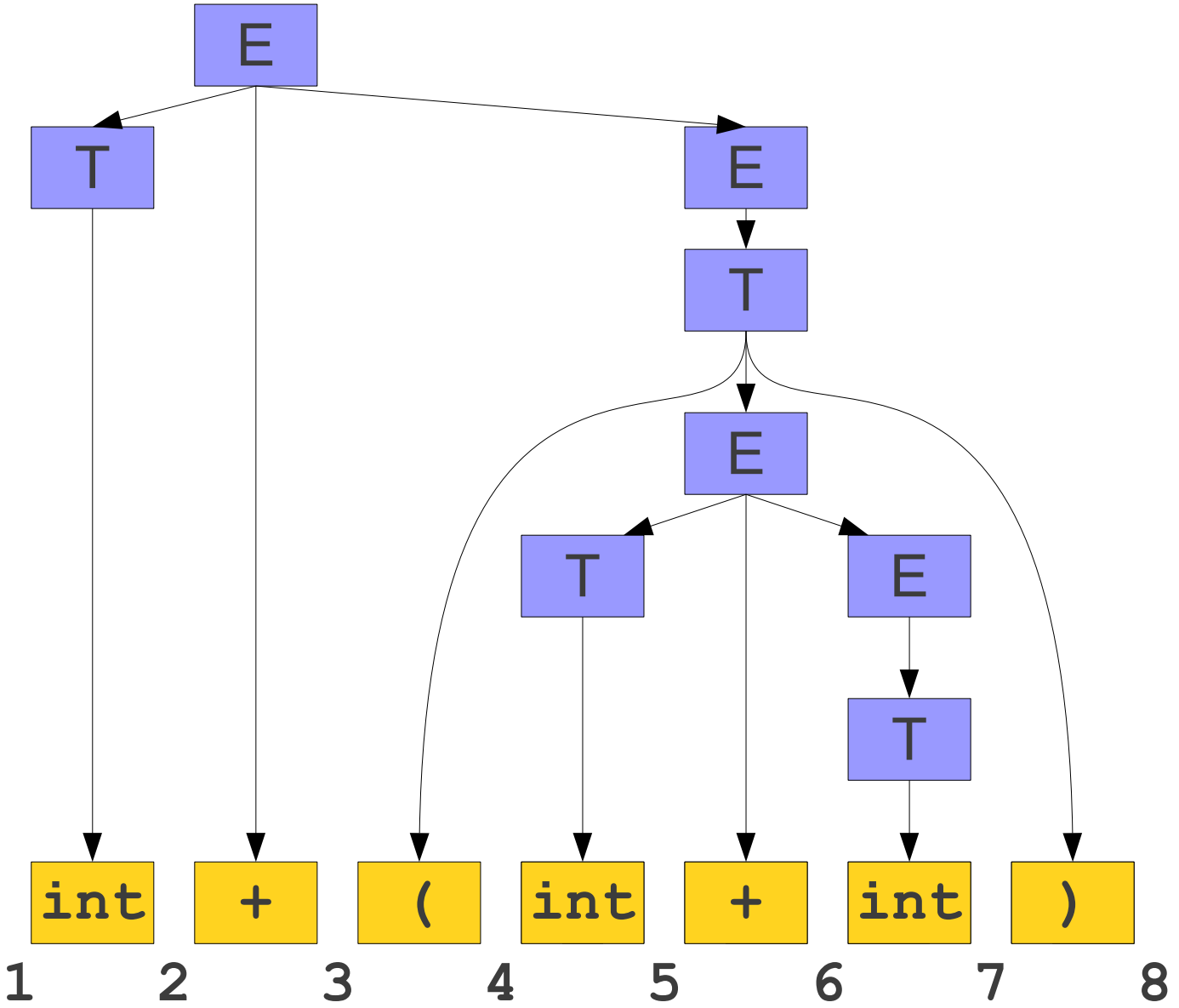
$E \rightarrow T$
$E \rightarrow T$ `+` E
$T \rightarrow$ `int`
$T \rightarrow$ `(`E`)`

A Simple Parse Tree



$E \rightarrow T$
$E \rightarrow T$ + E
$T \rightarrow$ **int**
$T \rightarrow$ (E)

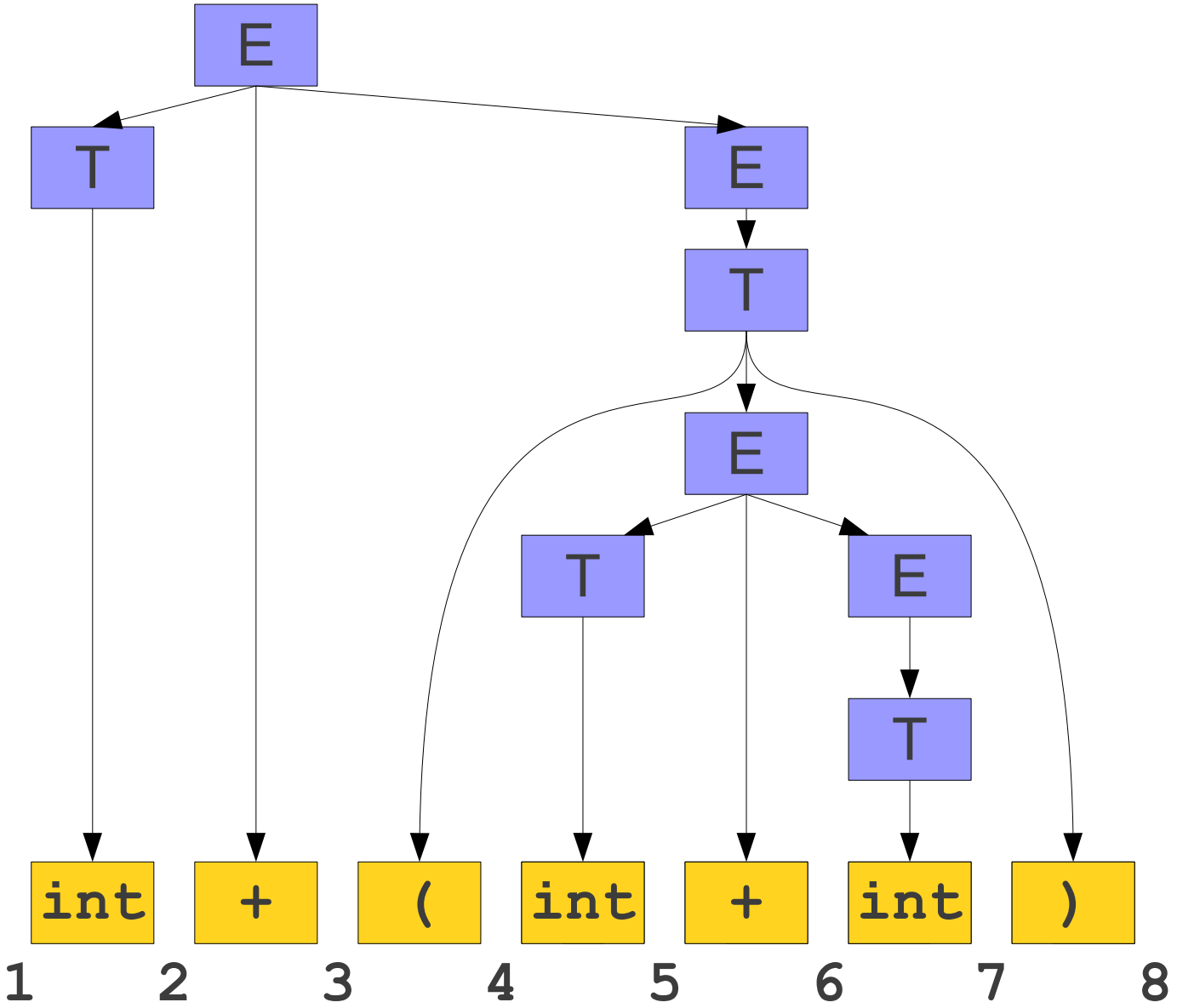
A Simple Parse Tree



$E \rightarrow T$
$E \rightarrow T$ `+` E
$T \rightarrow$ `int`
$T \rightarrow$ `(`E`)`

A Simple Parse Tree

$S \rightarrow E_{1\text{-}8}$

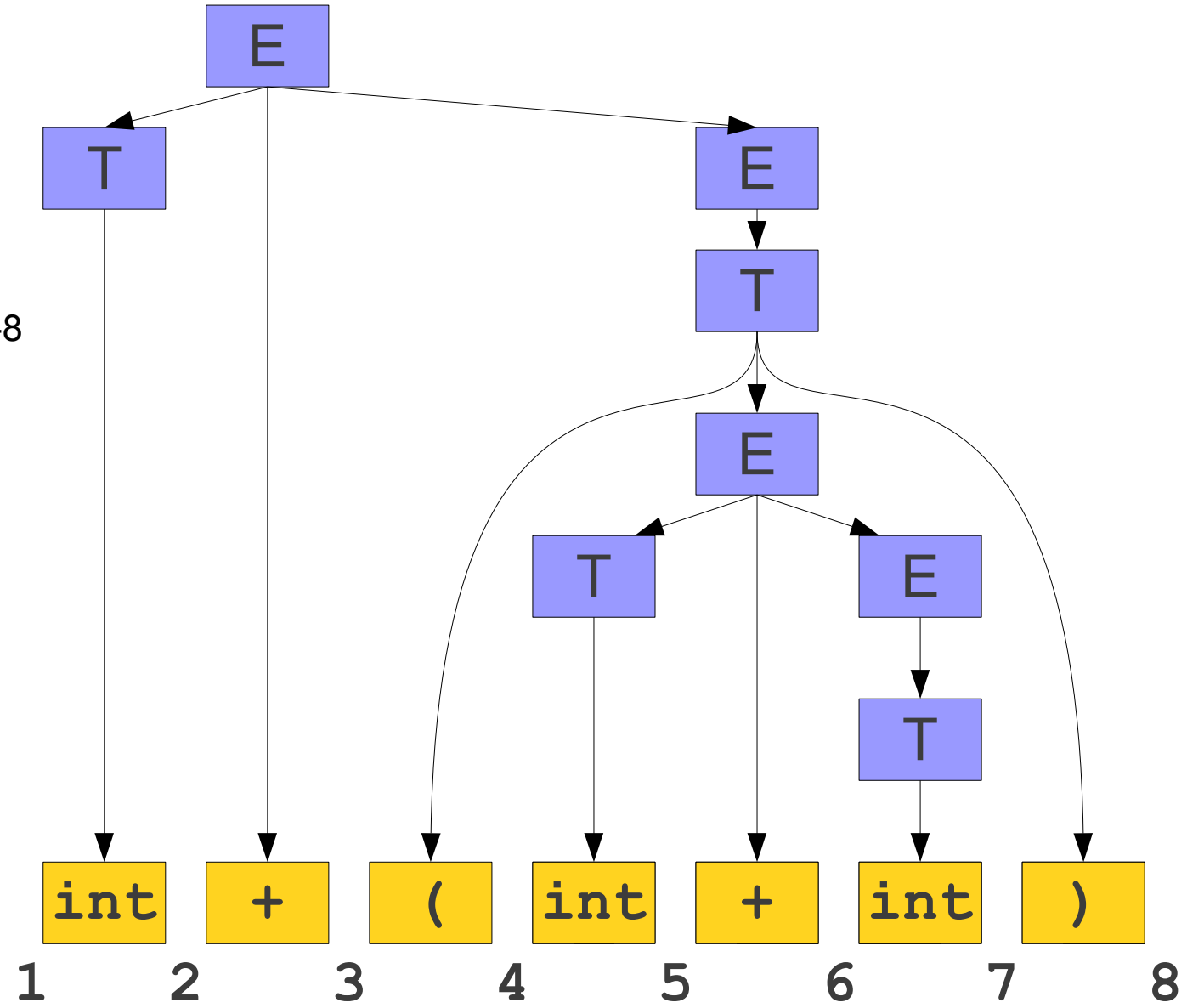


$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

$S \rightarrow E_{1\text{-}8}$

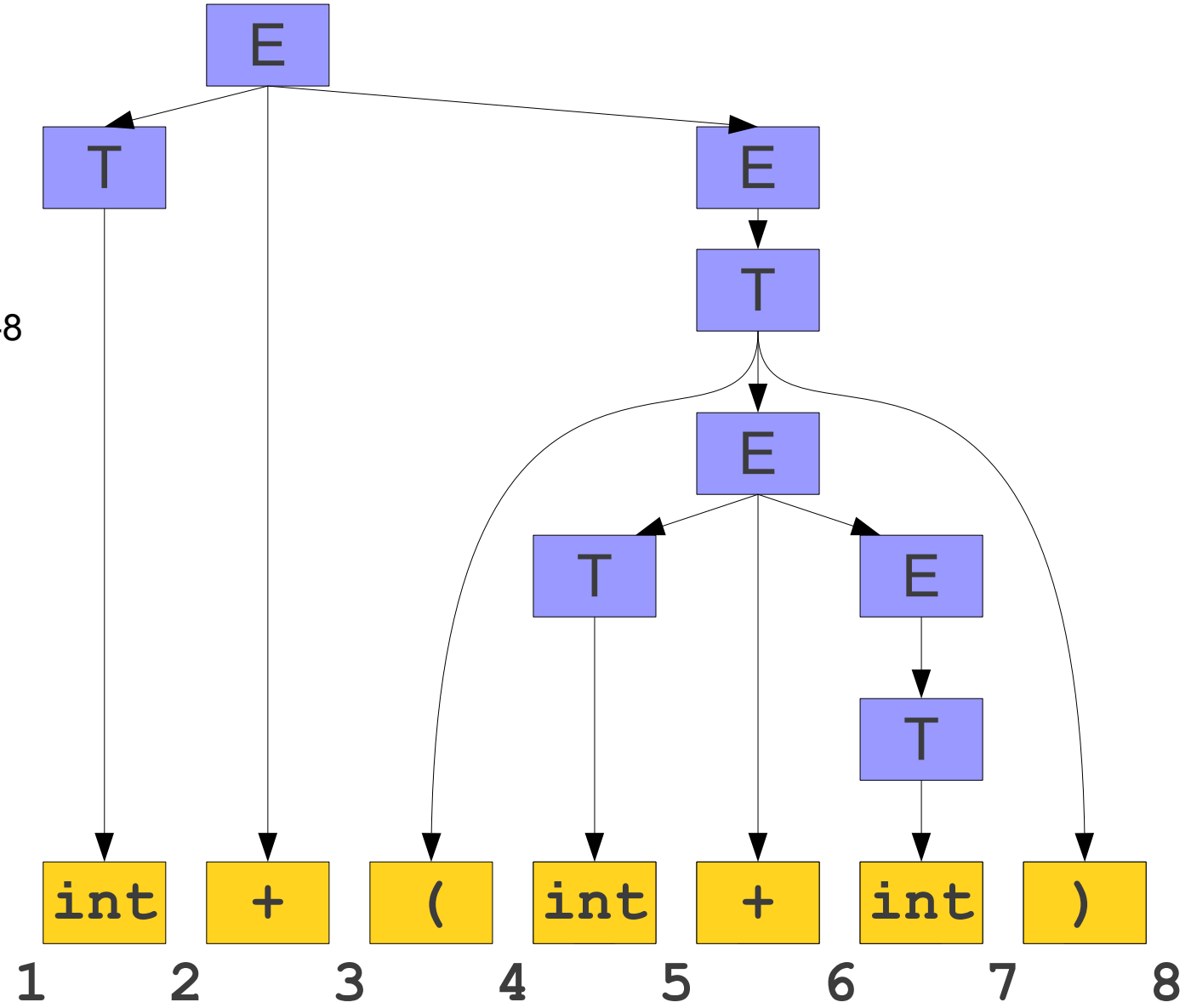
$E_{1\text{-}8} \rightarrow T_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}8}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

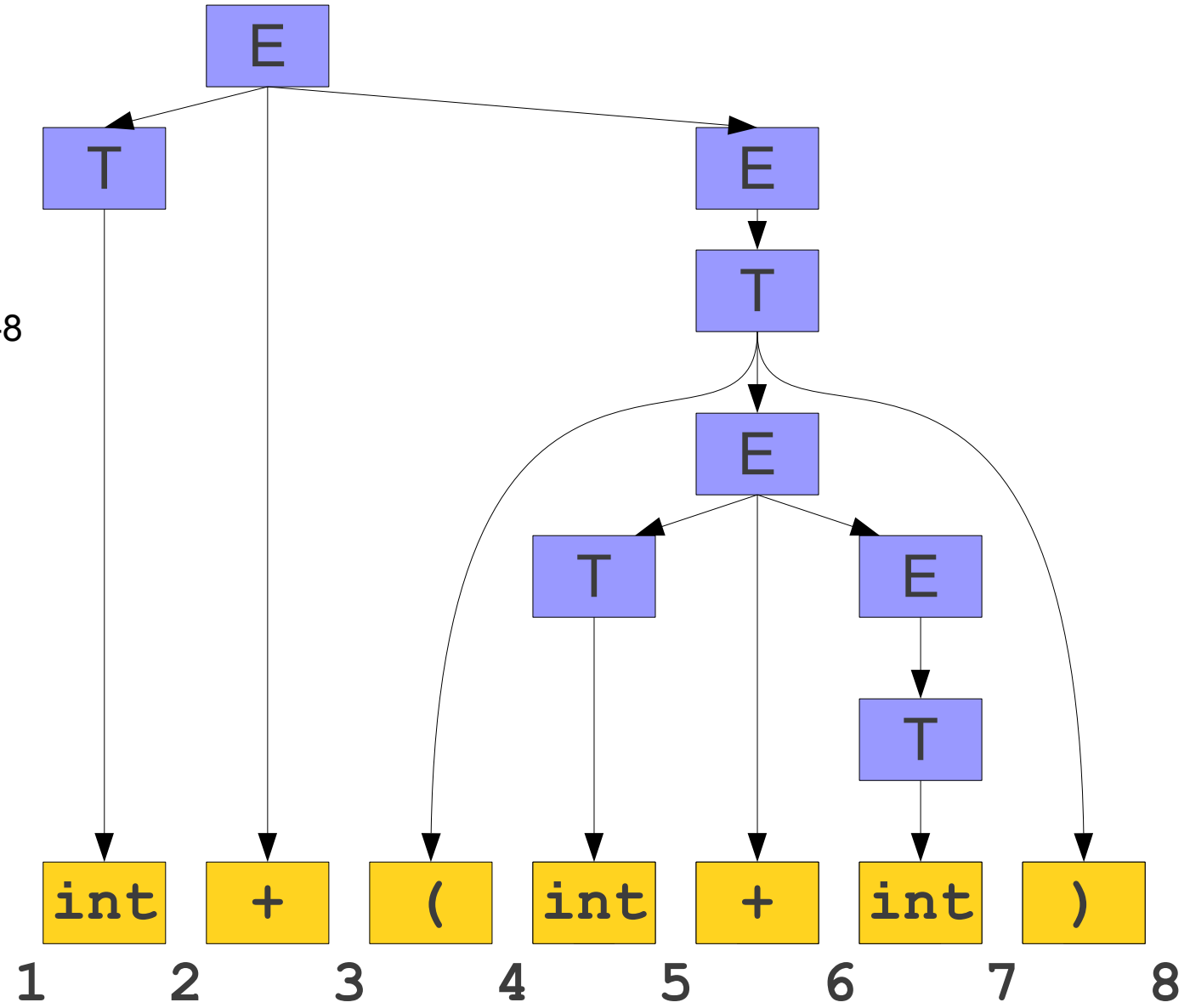
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

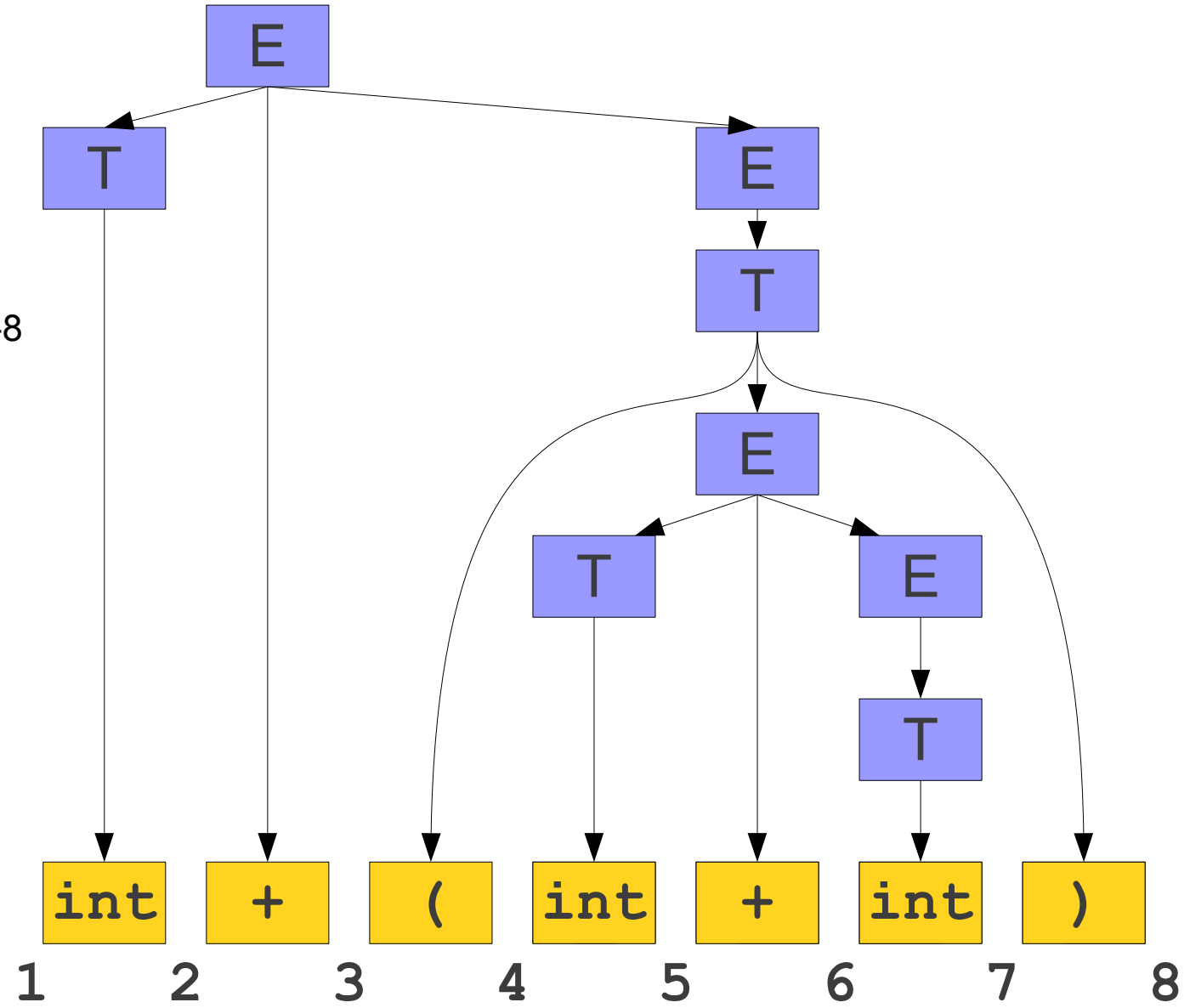
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$
$E_{3\text{-}8} \rightarrow T_{3\text{-}8}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

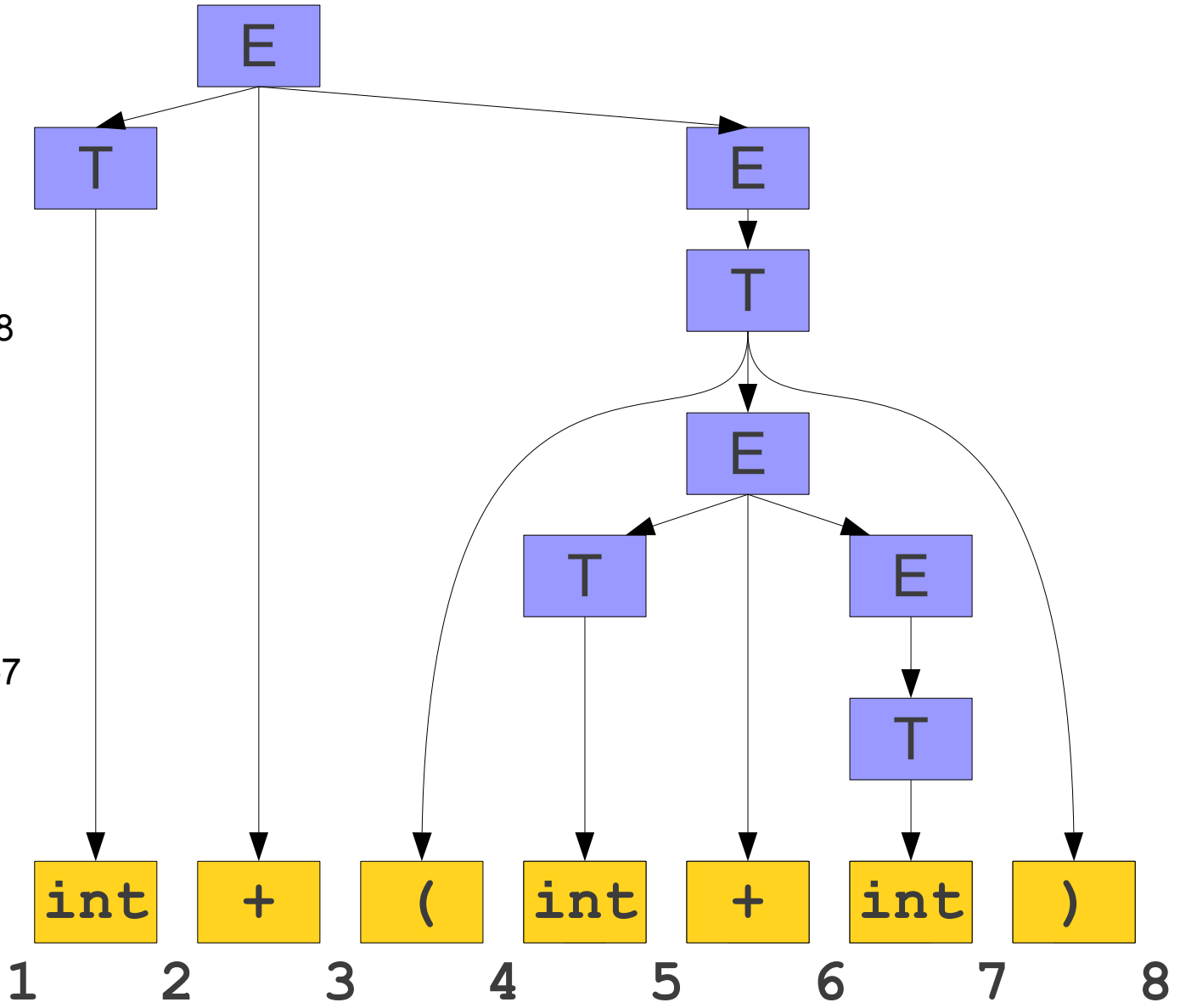
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$
$E_{3\text{-}8} \rightarrow T_{3\text{-}8}$
$T_{3\text{-}8} \rightarrow$ **(**$_{3\text{-}4}$ $E_{4\text{-}7}$ **)**$_{7\text{-}8}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

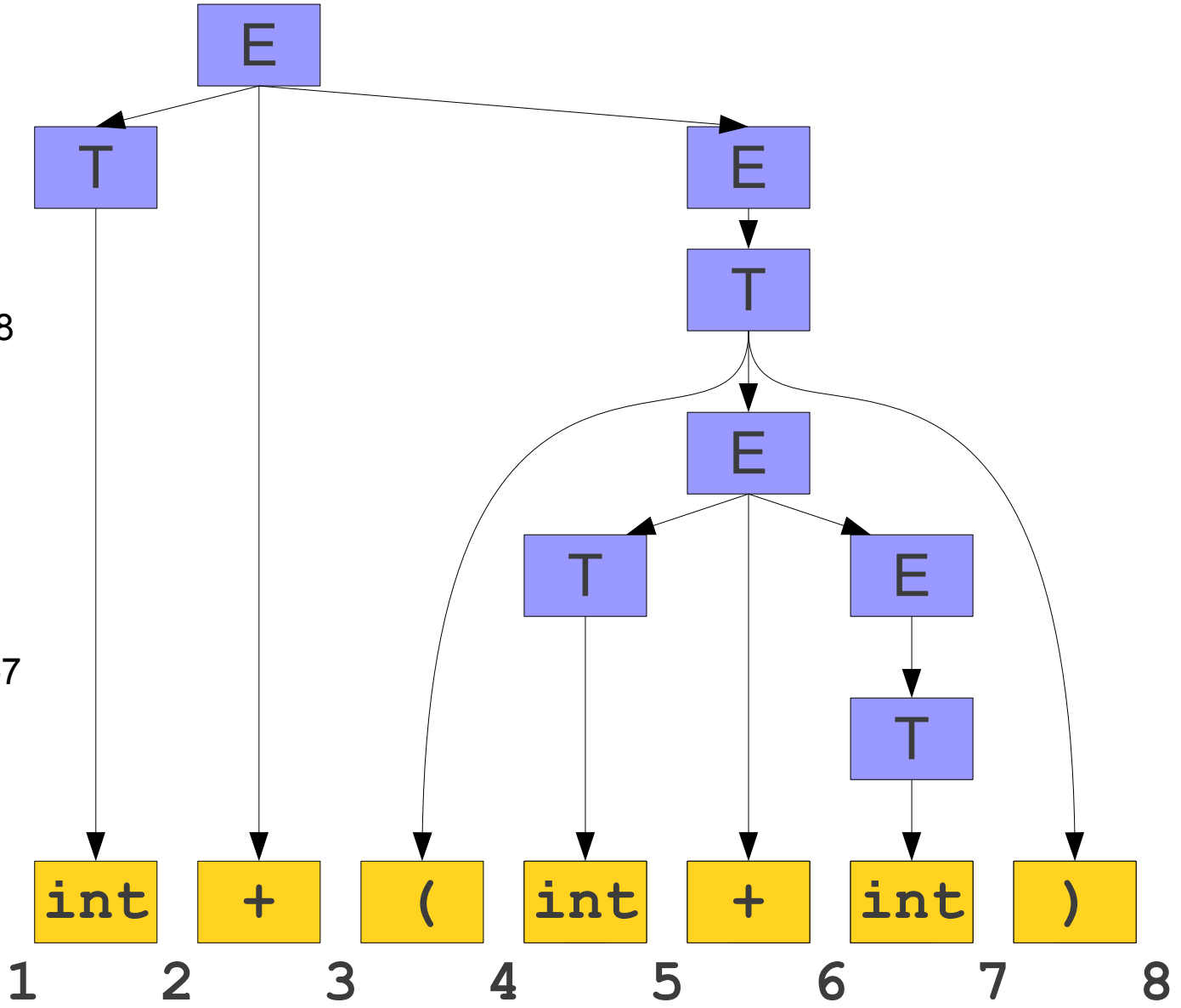
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$
$E_{3\text{-}8} \rightarrow T_{3\text{-}8}$
$T_{3\text{-}8} \rightarrow$ **(**$_{3\text{-}4}$ $E_{4\text{-}7}$ **)**$_{7\text{-}8}$
$E_{4\text{-}7} \rightarrow T_{4\text{-}5}$ **+**$_{5\text{-}6}$ $E_{6\text{-}7}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

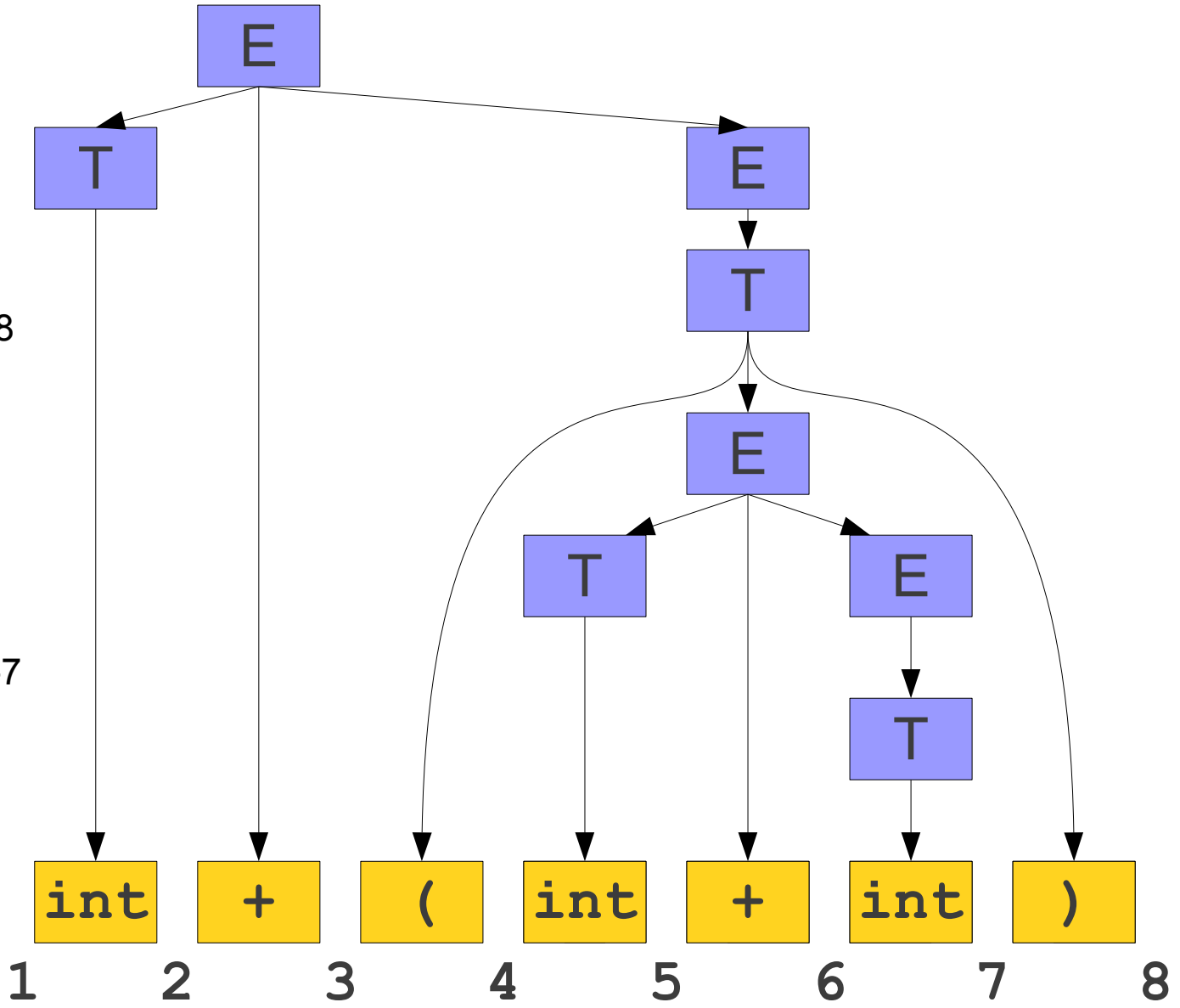
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$
$E_{3\text{-}8} \rightarrow T_{3\text{-}8}$
$T_{3\text{-}8} \rightarrow$ **(**$_{3\text{-}4}$ $E_{4\text{-}7}$ **)**$_{7\text{-}8}$
$E_{4\text{-}7} \rightarrow T_{4\text{-}5}$ **+**$_{5\text{-}6}$ $E_{6\text{-}7}$
$T_{4\text{-}5} \rightarrow$ **int**$_{4\text{-}5}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

A Simple Parse Tree

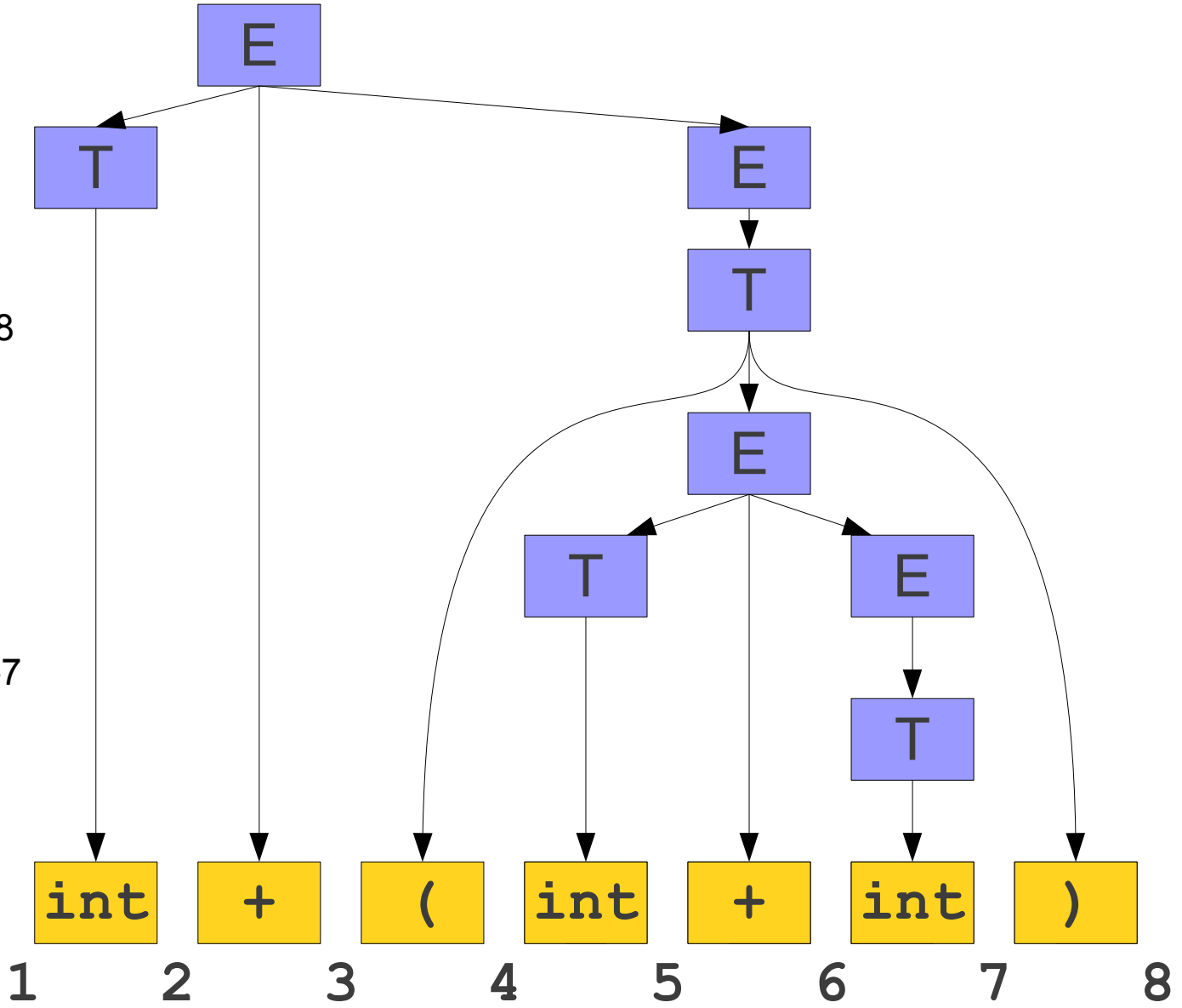
$S \rightarrow E_{1\text{-}8}$
$E_{1\text{-}8} \rightarrow T_{1\text{-}2}$ **+**$_{2\text{-}3}$ $E_{3\text{-}8}$
$T_{1\text{-}2} \rightarrow$ **int**$_{1\text{-}2}$
$E_{3\text{-}8} \rightarrow T_{3\text{-}8}$
$T_{3\text{-}8} \rightarrow$ **(**$_{3\text{-}4}$ $E_{4\text{-}7}$ **)**$_{7\text{-}8}$
$E_{4\text{-}7} \rightarrow T_{4\text{-}5}$ **+**$_{5\text{-}6}$ $E_{6\text{-}7}$
$T_{4\text{-}5} \rightarrow$ **int**$_{4\text{-}5}$
$E_{6\text{-}7} \rightarrow T_{6\text{-}7}$



$E \rightarrow T$
$E \rightarrow T$ **+** E
$T \rightarrow$ **int**
$T \rightarrow$ **(**E**)**

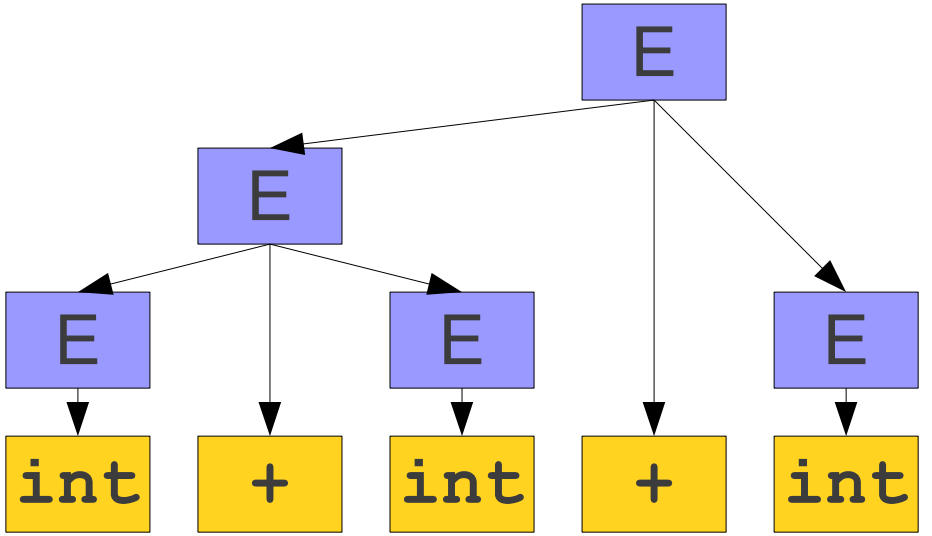
A Simple Parse Tree

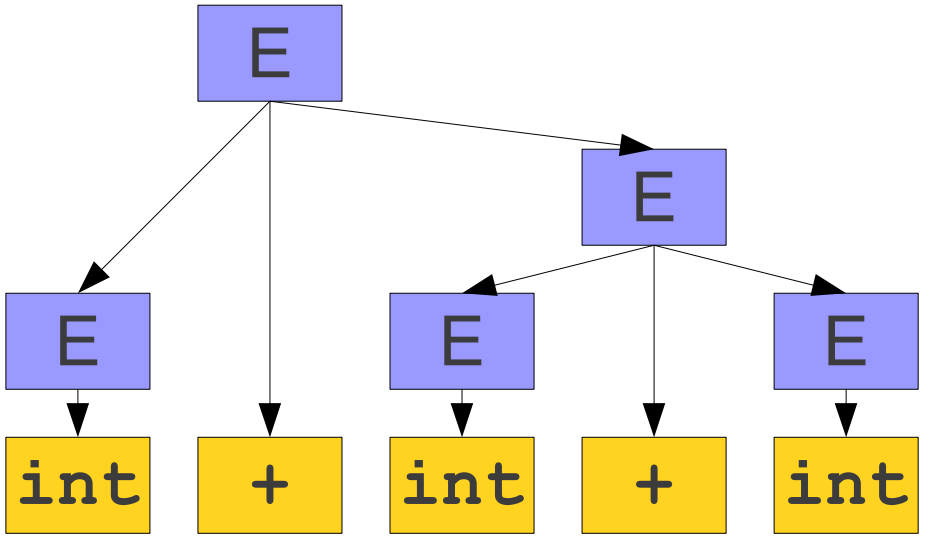
$S \rightarrow E_{1-8}$
$E_{1-8} \rightarrow T_{1-2}$ **+**$_{2-3}$ E_{3-8}
$T_{1-2} \rightarrow$ **int**$_{1-2}$
$E_{3-8} \rightarrow T_{3-8}$
$T_{3-8} \rightarrow$ **(**$_{3-4}$ E_{4-7} **)**$_{7-8}$
$E_{4-7} \rightarrow T_{4-5}$ **+**$_{5-6}$ E_{6-7}
$T_{4-5} \rightarrow$ **int**$_{4-5}$
$E_{6-7} \rightarrow T_{6-7}$
$T_{6-7} \rightarrow$ **int**$_{6-7}$



E → E + E
E → int

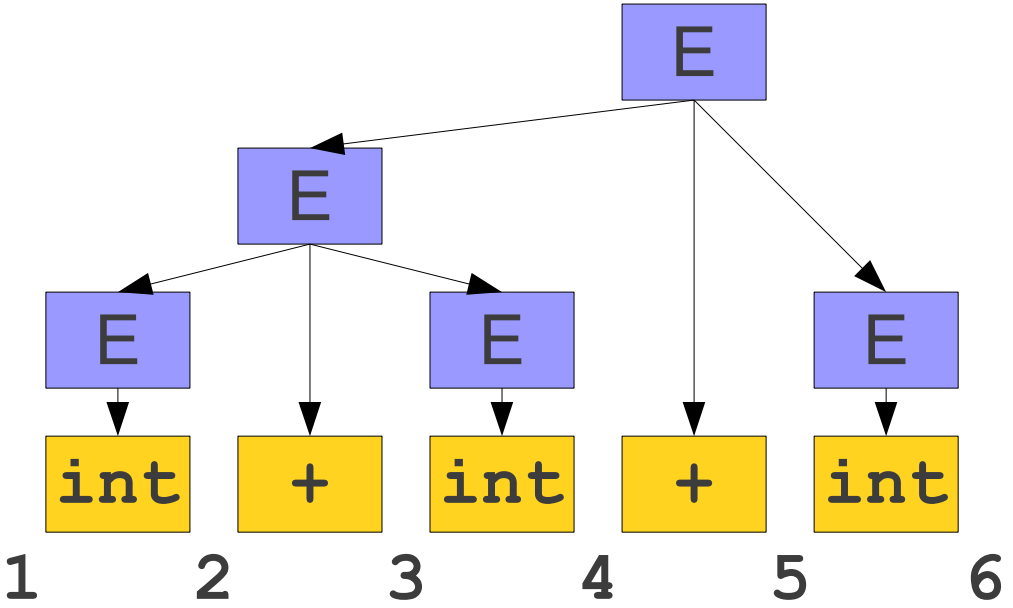
Several Parse Trees

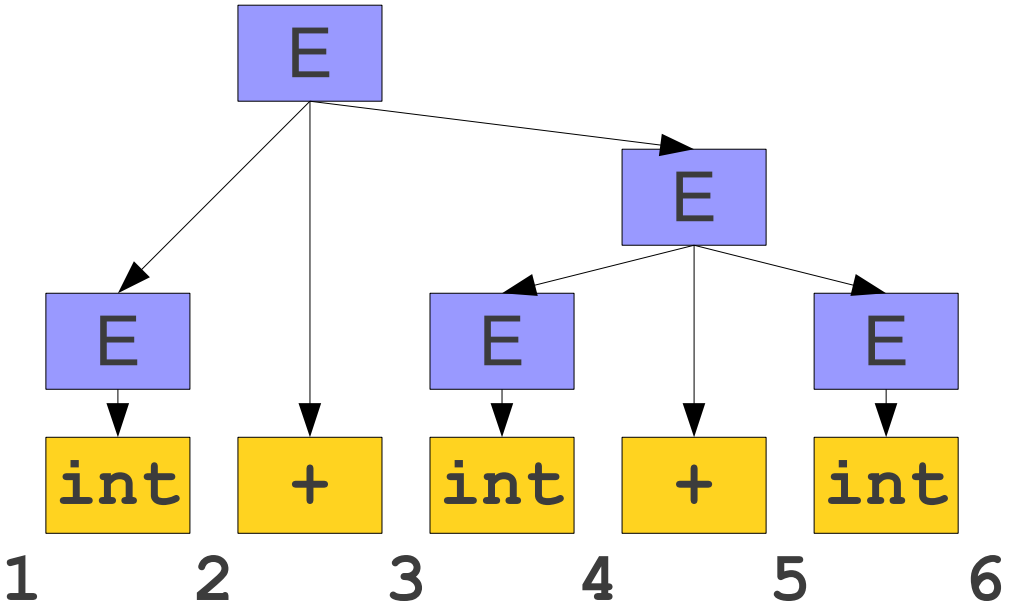




$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

Several Parse Trees

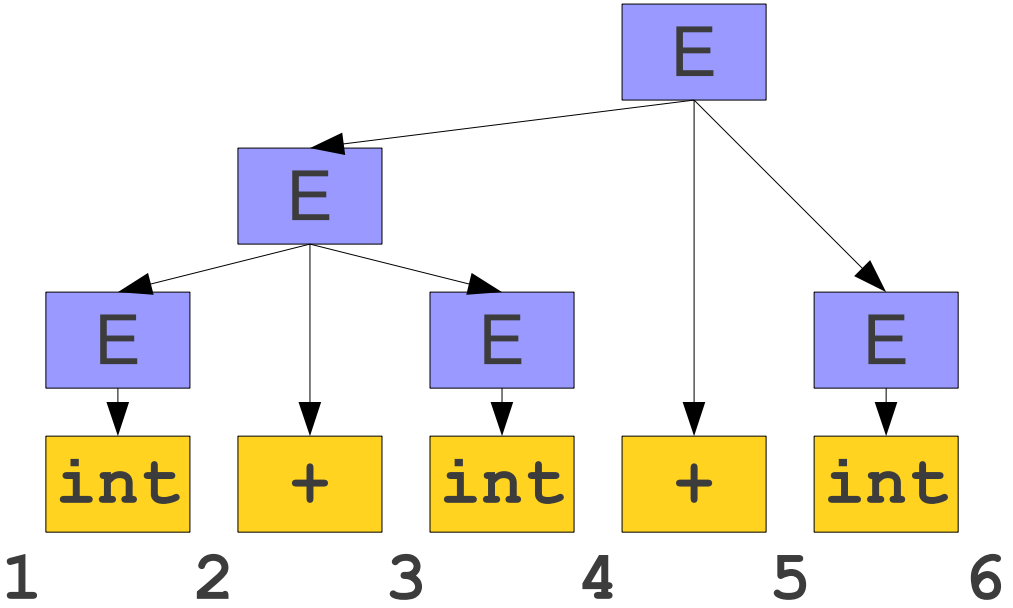


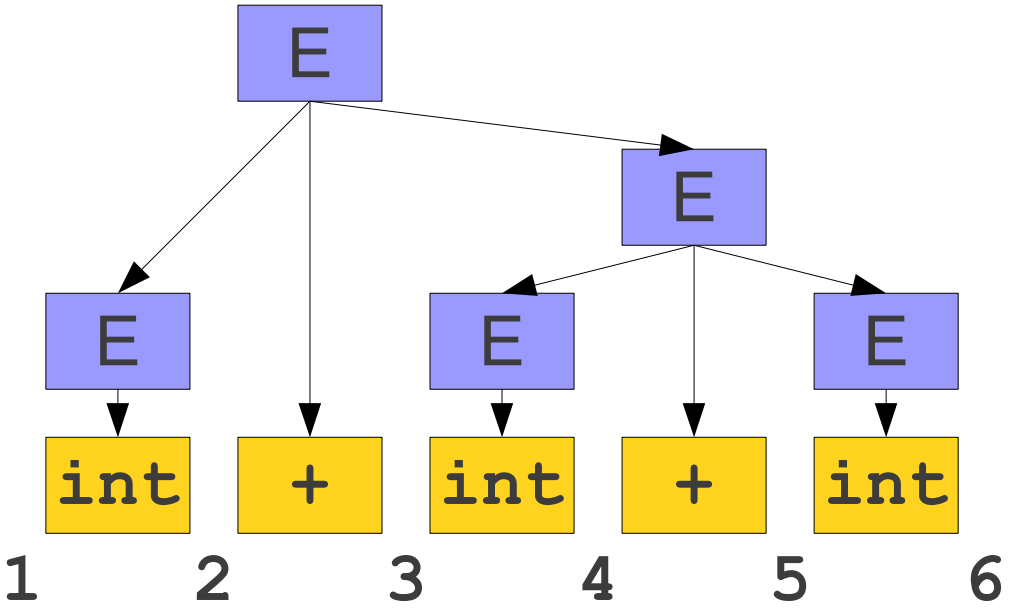


$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

Several Parse Trees

$S \rightarrow E_{1\text{-}6}$





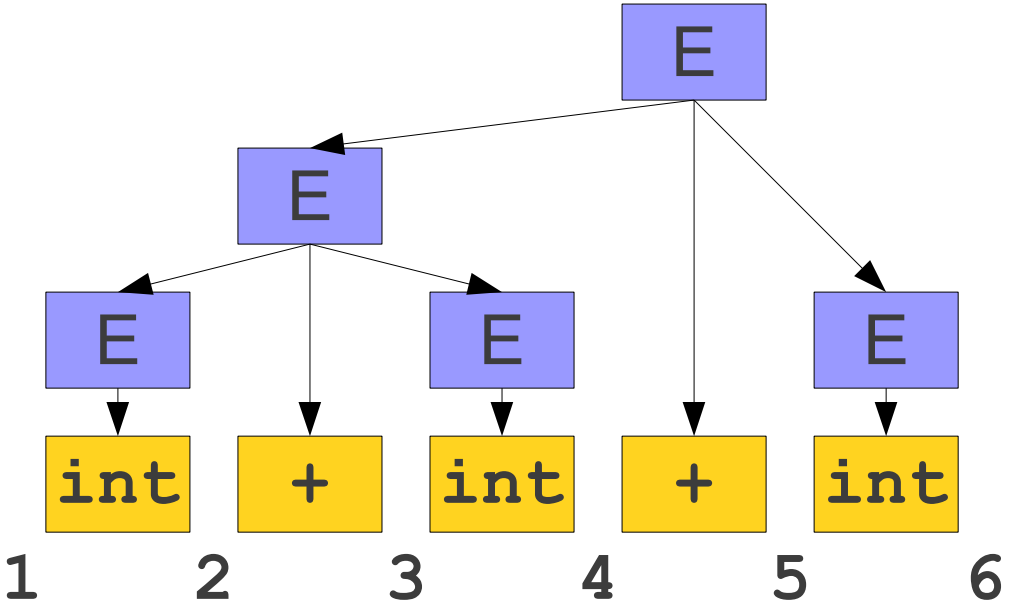
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

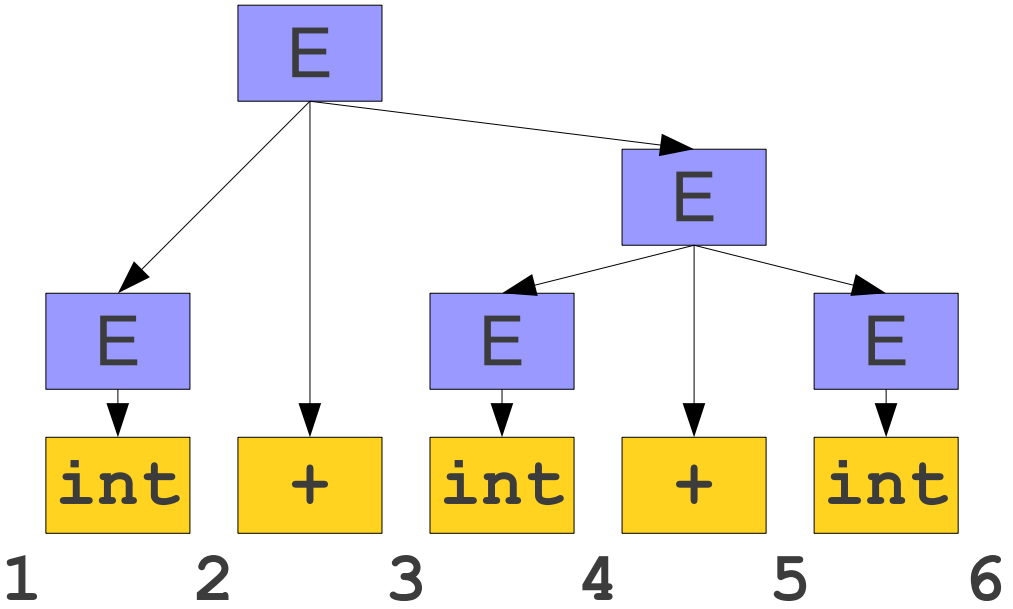
Several Parse Trees

$S \rightarrow E_{1\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}6}$





$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

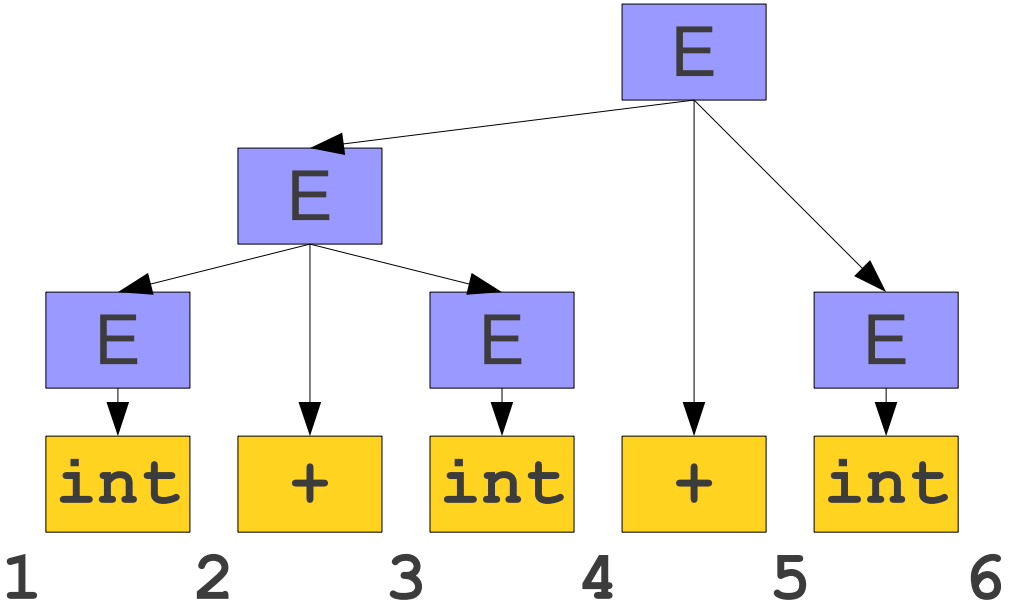
Several Parse Trees

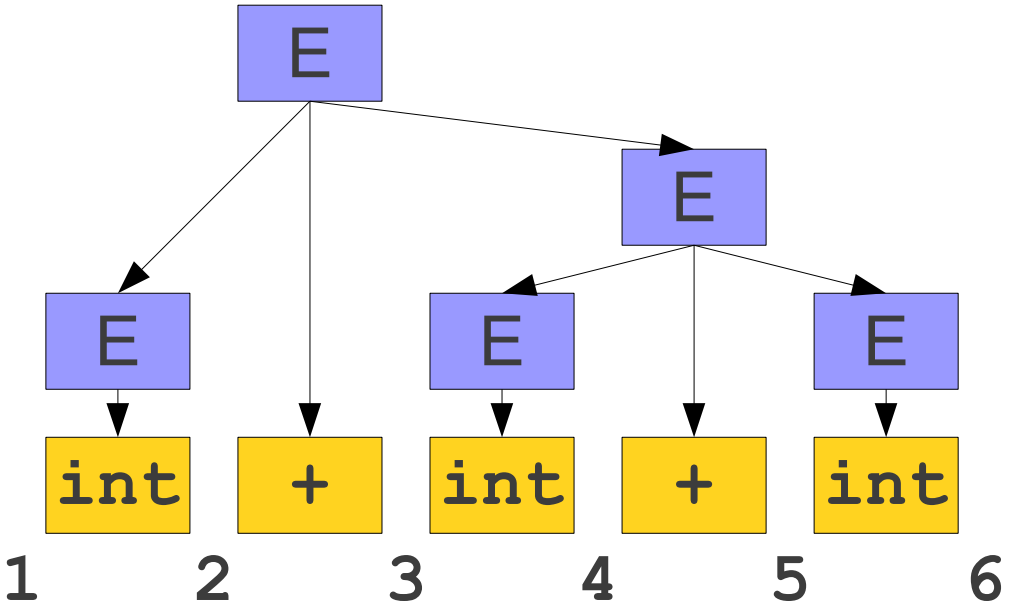
$S \rightarrow E_{1\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}6}$

$E_{1\text{-}4} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}4}$





$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

Several Parse Trees

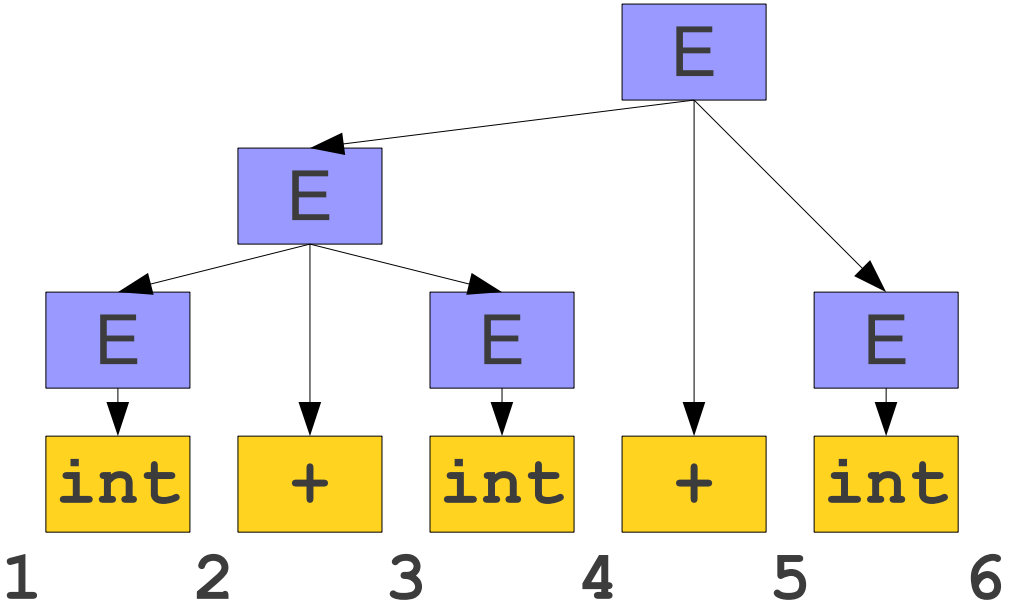
$S \rightarrow E_{1\text{-}6}$

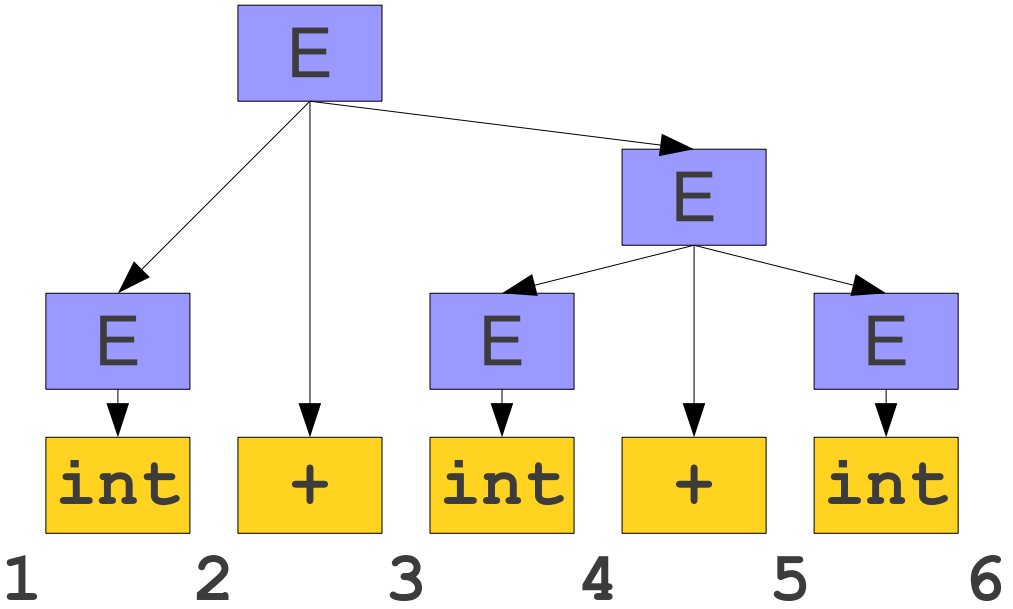
$E_{1\text{-}6} \rightarrow E_{1\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}6}$

$E_{1\text{-}4} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}4}$

$E_{3\text{-}6} \rightarrow E_{3\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$





$E \rightarrow E + E$
$E \rightarrow \texttt{int}$

Several Parse Trees

$S \rightarrow E_{1\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$

$E_{1\text{-}6} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}6}$

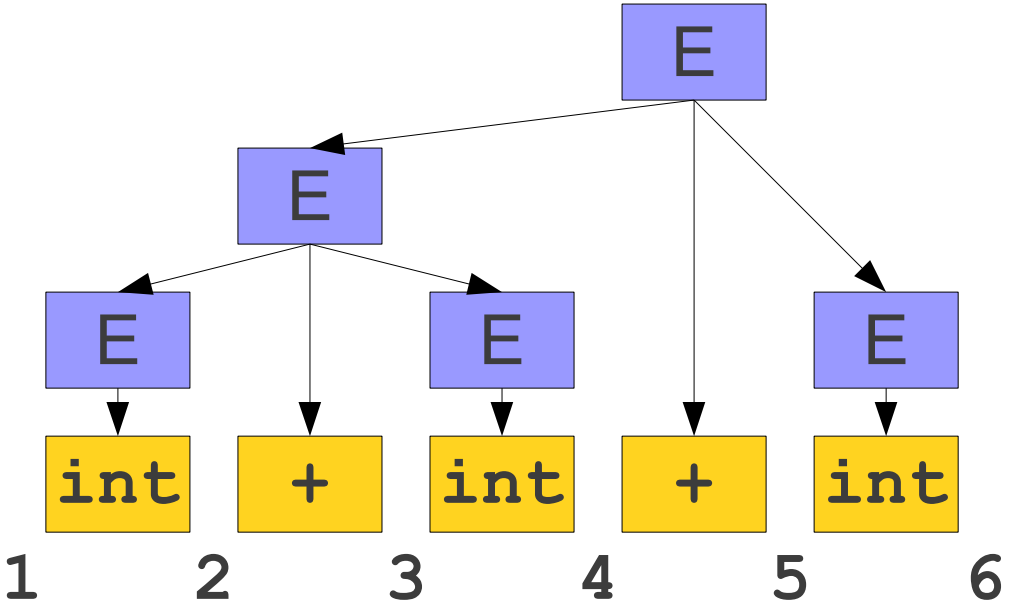
$E_{1\text{-}4} \rightarrow E_{1\text{-}2} +_{2\text{-}3} E_{3\text{-}4}$

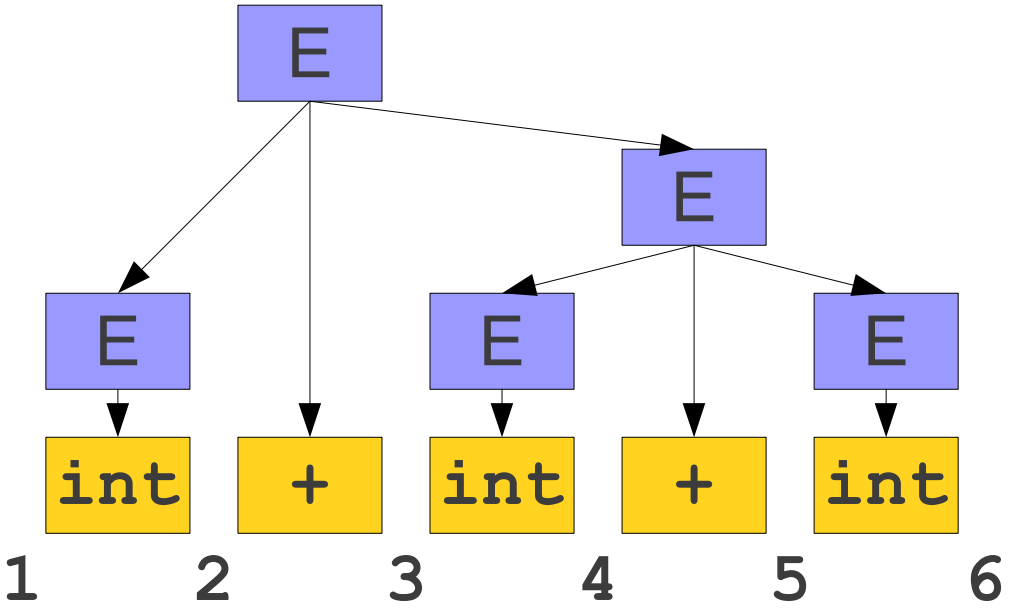
$E_{3\text{-}6} \rightarrow E_{3\text{-}4} +_{4\text{-}5} E_{5\text{-}6}$

$E_{1\text{-}2} \rightarrow \texttt{int}_{1\text{-}2}$

$E_{3\text{-}4} \rightarrow \texttt{int}_{3\text{-}4}$

$E_{5\text{-}6} \rightarrow \texttt{int}_{5\text{-}6}$

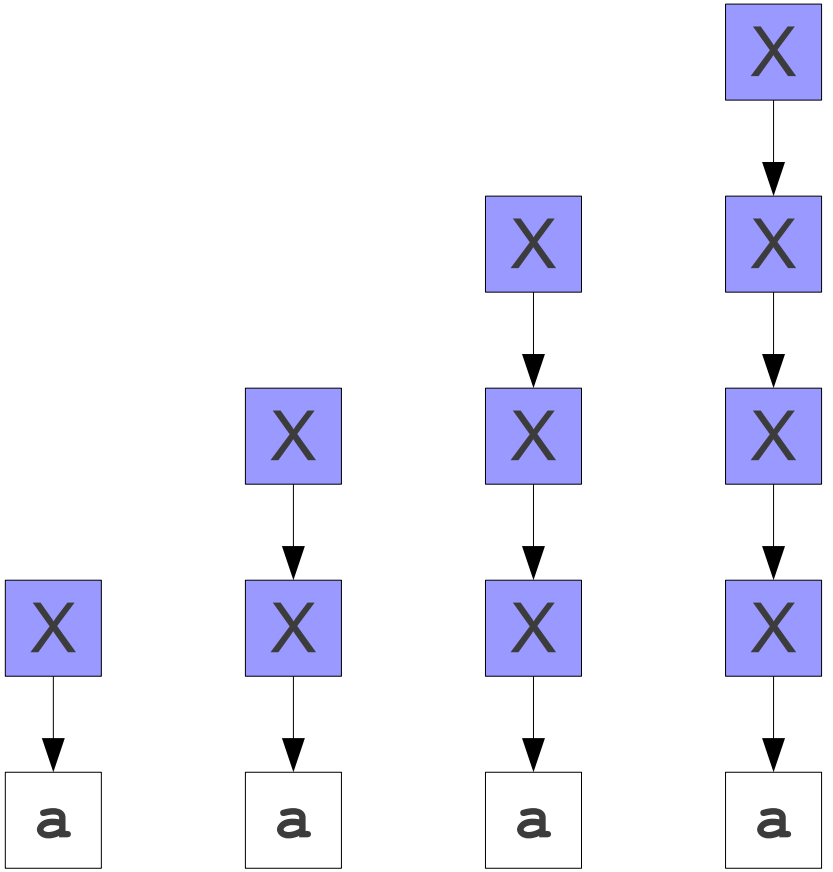




A **parse-forest grammar** is a context-free grammar for parse trees.

An Infinitely Ambiguous Grammar

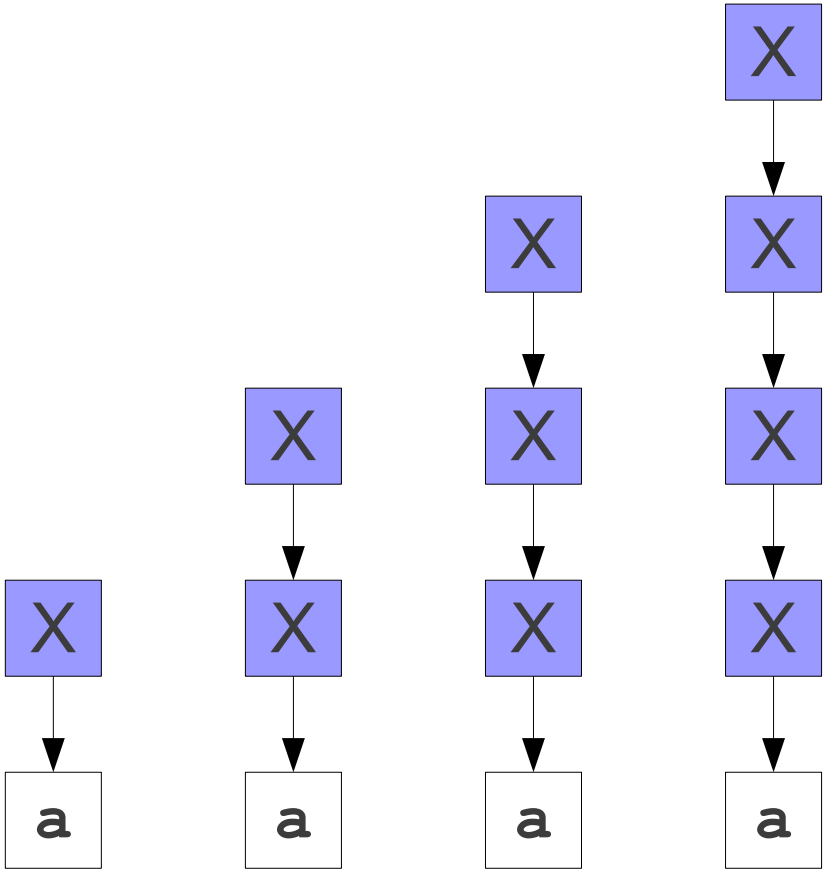
$X \rightarrow X$
$X \rightarrow a$



An Infinitely Ambiguous Grammar

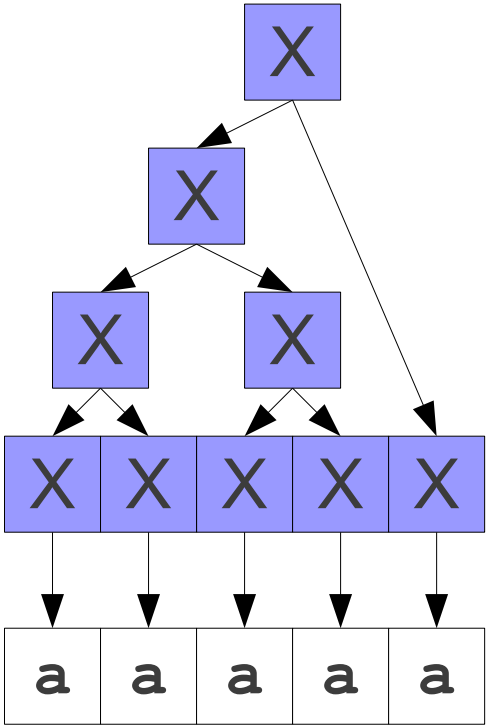
$X \rightarrow X$
$X \rightarrow a$

$S \rightarrow X_{1\text{-}2}$
$X_{1\text{-}2} \rightarrow X_{1\text{-}2}$
$X_{1\text{-}2} \rightarrow a_{1\text{-}2}$



A Highly Ambiguous Grammar

$X \rightarrow XX$
$X \rightarrow a$



A Highly Ambiguous Grammar

$X \rightarrow XX$

$X \rightarrow a$

$S \rightarrow X_{1\text{-}2}\ X_{2\text{-}6}$

$S \rightarrow X_{1\text{-}3}\ X_{3\text{-}6}$

$S \rightarrow X_{1\text{-}4}\ X_{4\text{-}6}$

$S \rightarrow X_{1\text{-}5}\ X_{5\text{-}6}$

$X_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$X_{2\text{-}3} \rightarrow a_{2\text{-}3}$

$X_{3\text{-}4} \rightarrow a_{3\text{-}4}$

$X_{4\text{-}5} \rightarrow a_{4\text{-}5}$

$X_{5\text{-}6} \rightarrow a_{5\text{-}6}$

$X_{1\text{-}3} \rightarrow X_{1\text{-}2}\ X_{2\text{-}3}$

$X_{2\text{-}4} \rightarrow X_{2\text{-}3} X_{3\text{-}4}$

$X_{3\text{-}5} \rightarrow X_{3\text{-}4}\ X_{4\text{-}5}$

$X_{4\text{-}6} \rightarrow X_{4\text{-}5}\ X_{5\text{-}6}$

$X_{1\text{-}4} \rightarrow X_{1\text{-}2}\ X_{2\text{-}4}$

$X_{1\text{-}4} \rightarrow X_{1\text{-}3}\ X_{3\text{-}4}$

$X_{2\text{-}5} \rightarrow X_{2\text{-}3}\ X_{3\text{-}5}$

$X_{2\text{-}5} \rightarrow X_{2\text{-}4}\ X_{4\text{-}5}$

$X_{3\text{-}6} \rightarrow X_{3\text{-}4}\ X_{4\text{-}6}$

$X_{3\text{-}6} \rightarrow X_{3\text{-}5}\ X_{5\text{-}6}$

$X_{1\text{-}5} \rightarrow X_{1\text{-}2} X_{2\text{-}5}$

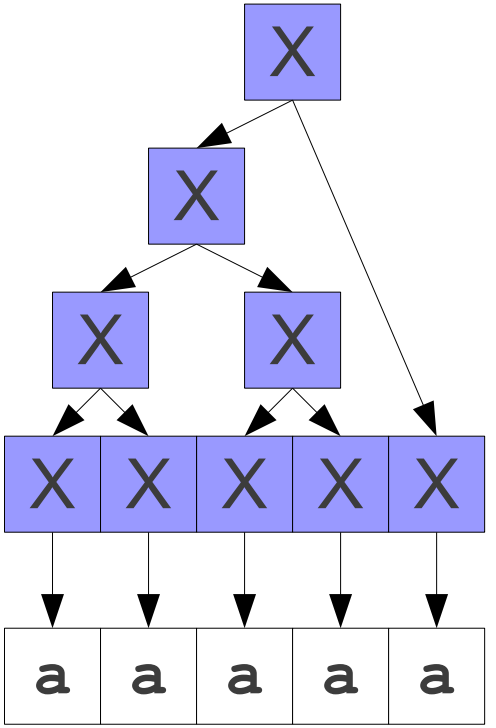
$X_{1\text{-}5} \rightarrow X_{1\text{-}3} X_{3\text{-}5}$

$X_{1\text{-}5} \rightarrow X_{1\text{-}4} X_{4\text{-}5}$

$X_{2\text{-}6} \rightarrow X_{2\text{-}3} X_{3\text{-}6}$

$X_{2\text{-}6} \rightarrow X_{2\text{-}4} X_{4\text{-}6}$

$X_{2\text{-}6} \rightarrow X_{2\text{-}5} X_{5\text{-}6}$



Parse Forest Grammars

- Compact framework for encoding (potentially infinitely many!) parse trees.
- Output of most (but not all) ambiguous grammar parsers.
- Size not guaranteed to be a polynomial in the size of the grammar.
  - May have every possible partition of every production in the parse tree.
- This is not a problem in practice.
  - Real grammars rarely trigger this behavior.
  - Techniques exist to obtain worst-case $O(n^3)$ size.

Building an Earley Parser

- **Idea**: Build up a parse-forest grammar as we compute item sets.
- Whenever we **complete** an item, add it to the resulting grammar.
- This will introduce unnecessary rules; we'll fix this later on.
- Some details are tricky; see Grune and Jacobs Ch. 13 for some of the finer points.

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

a	a	d

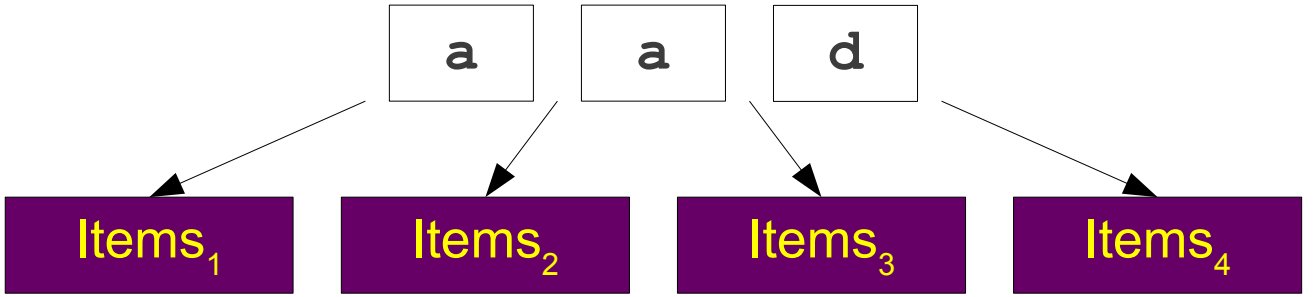
S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

a	a	d

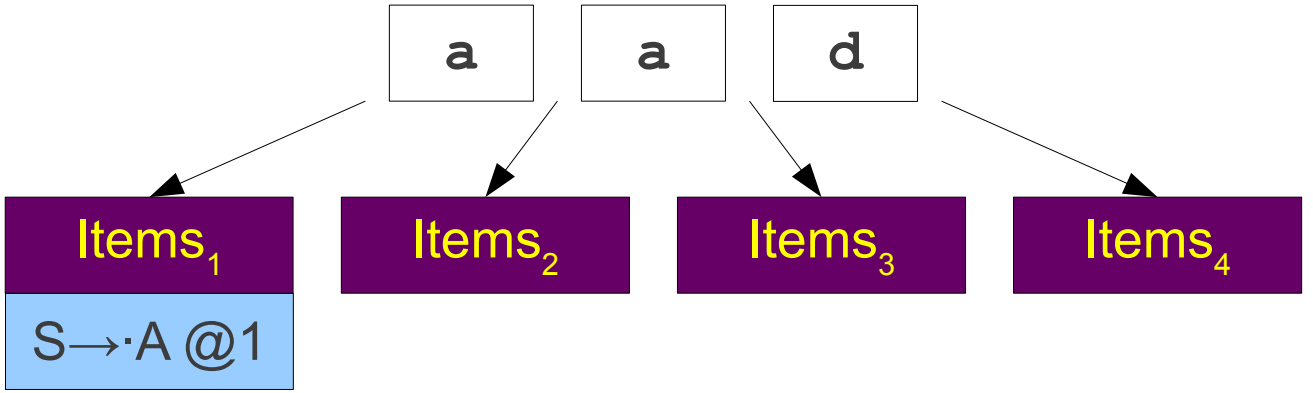
S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

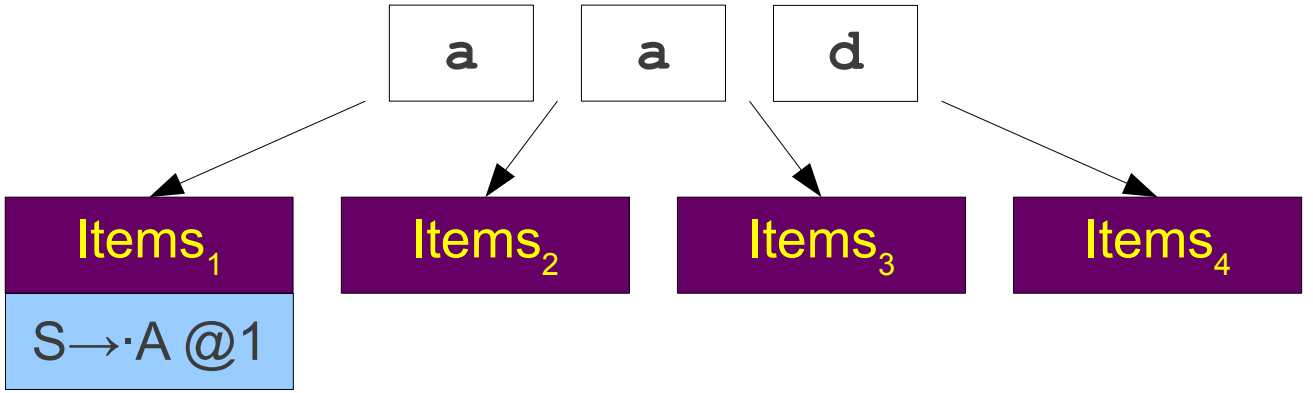
Earley Parsing



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

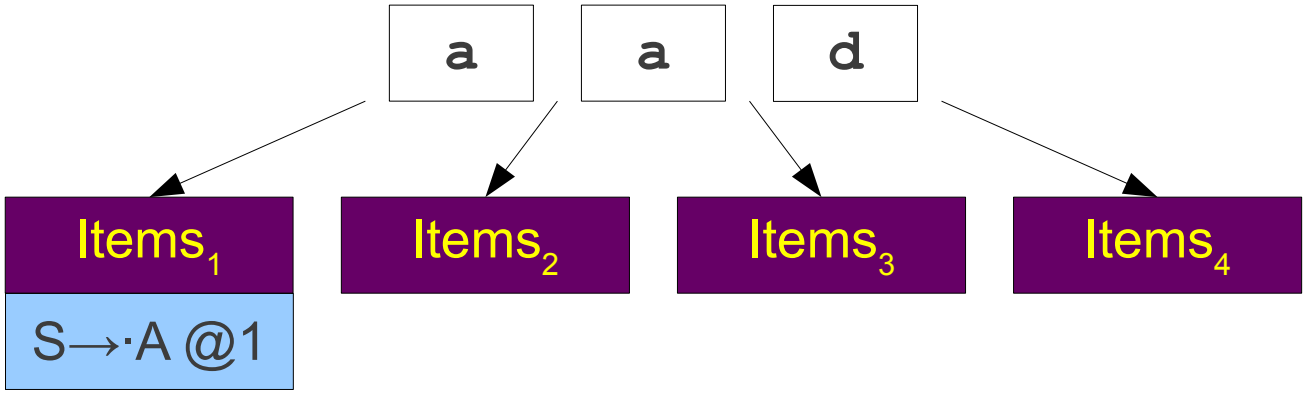
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

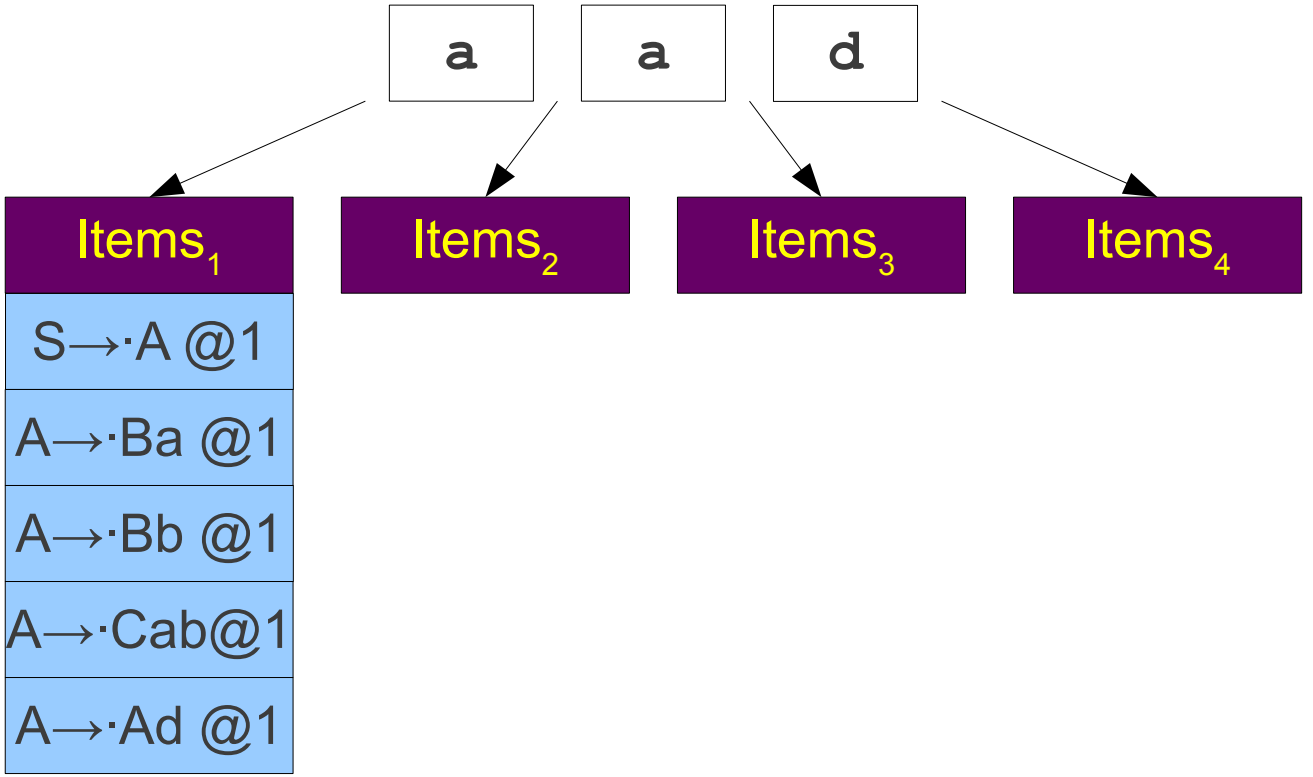
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

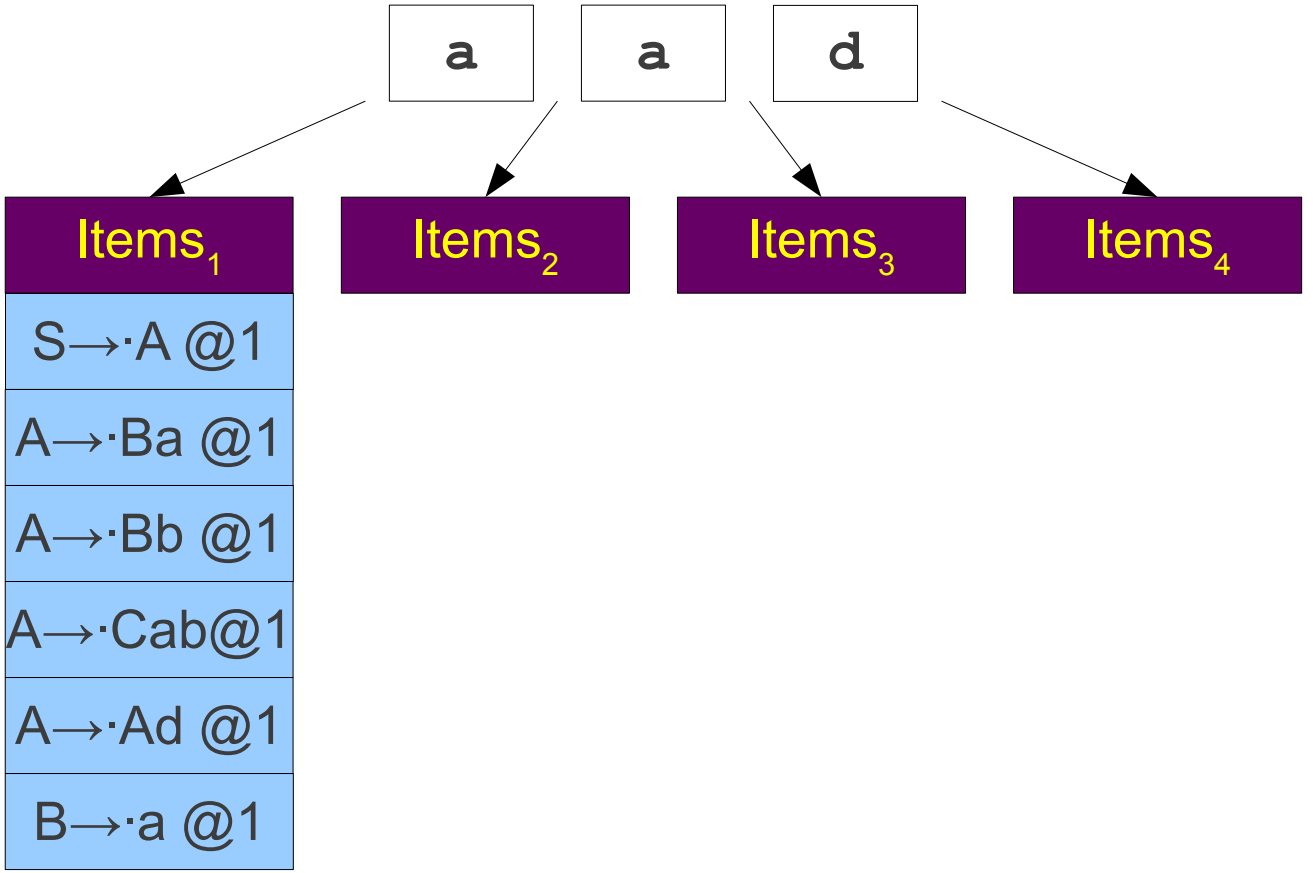
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

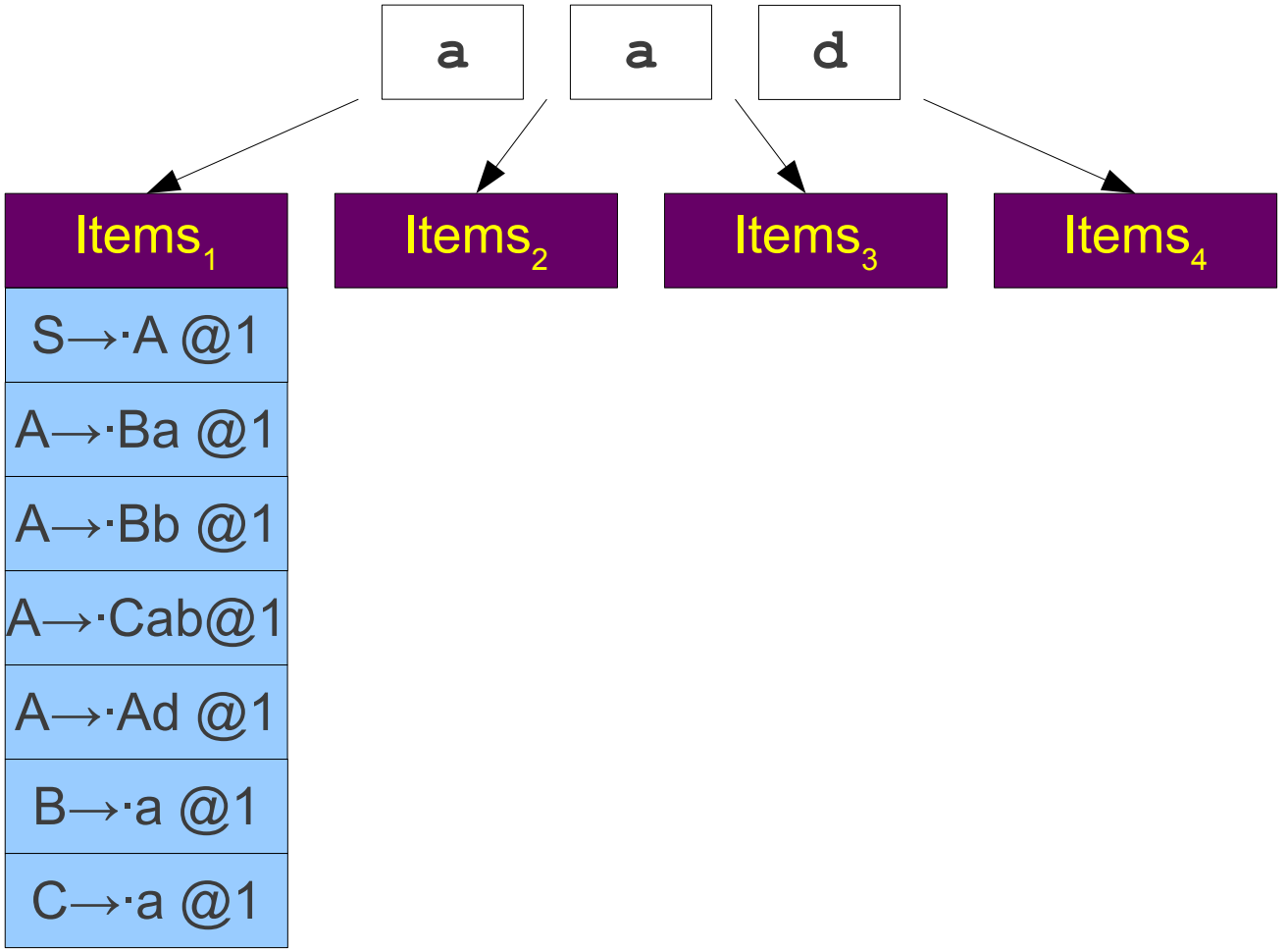
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

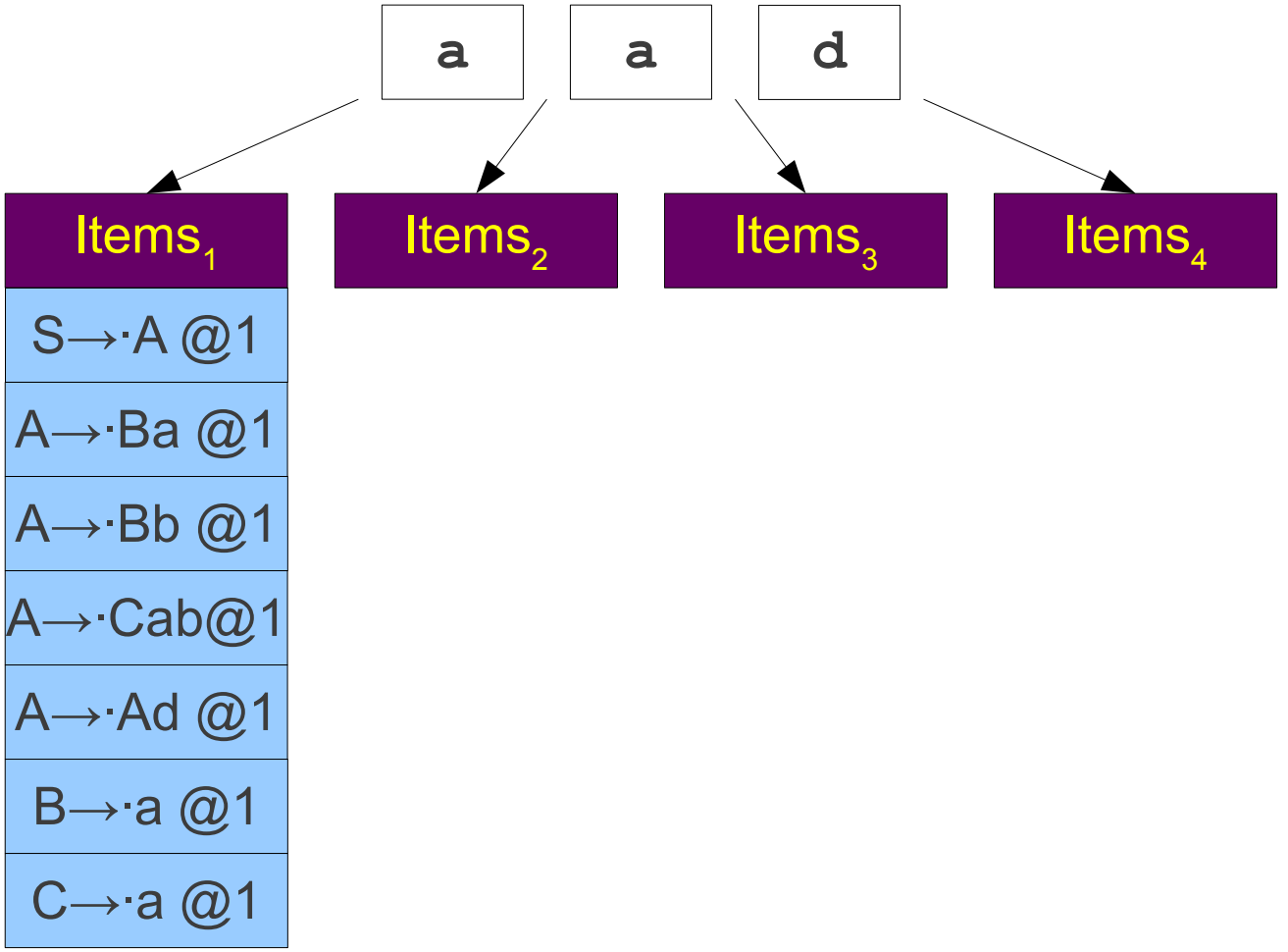
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

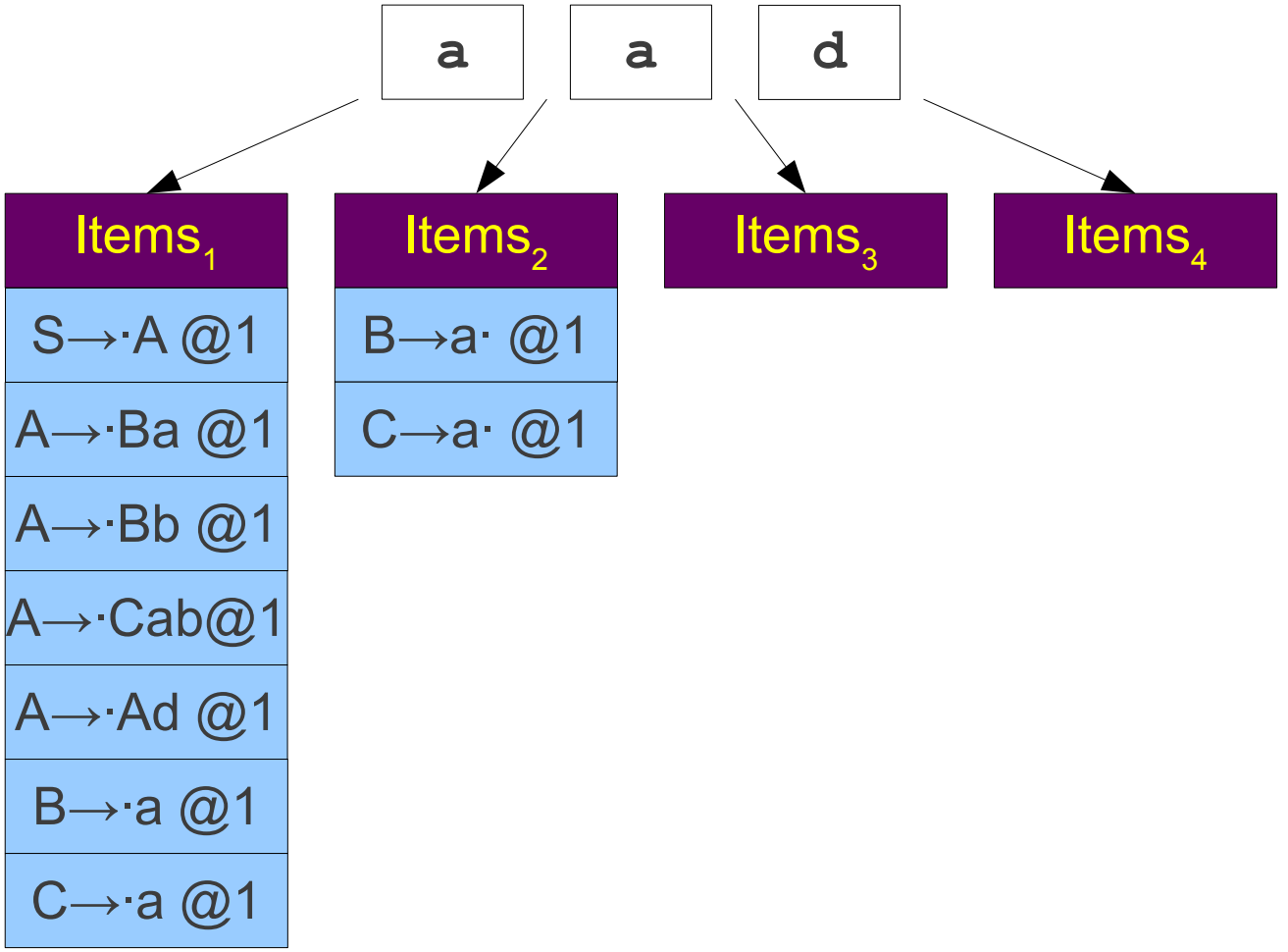
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

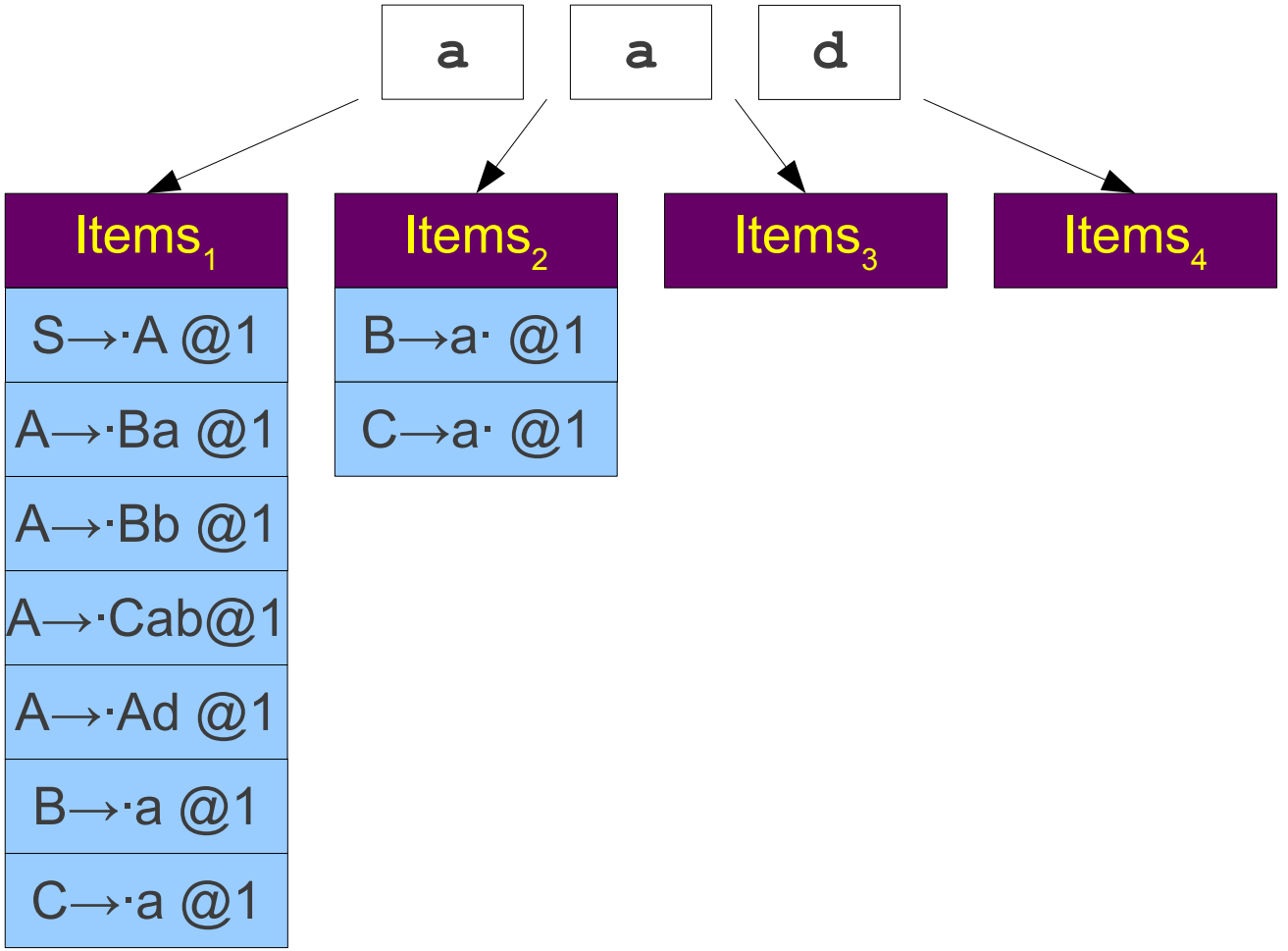
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

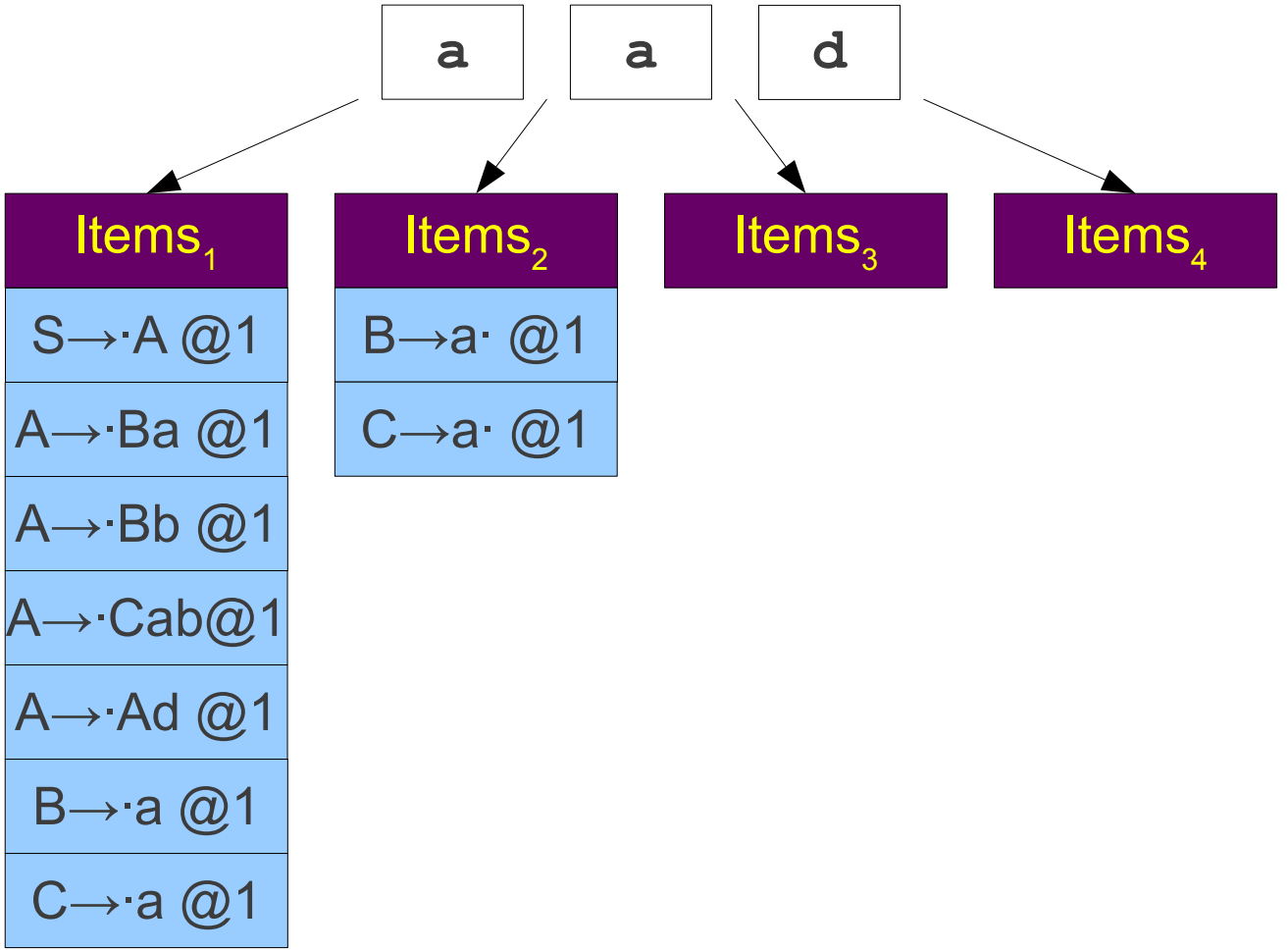


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

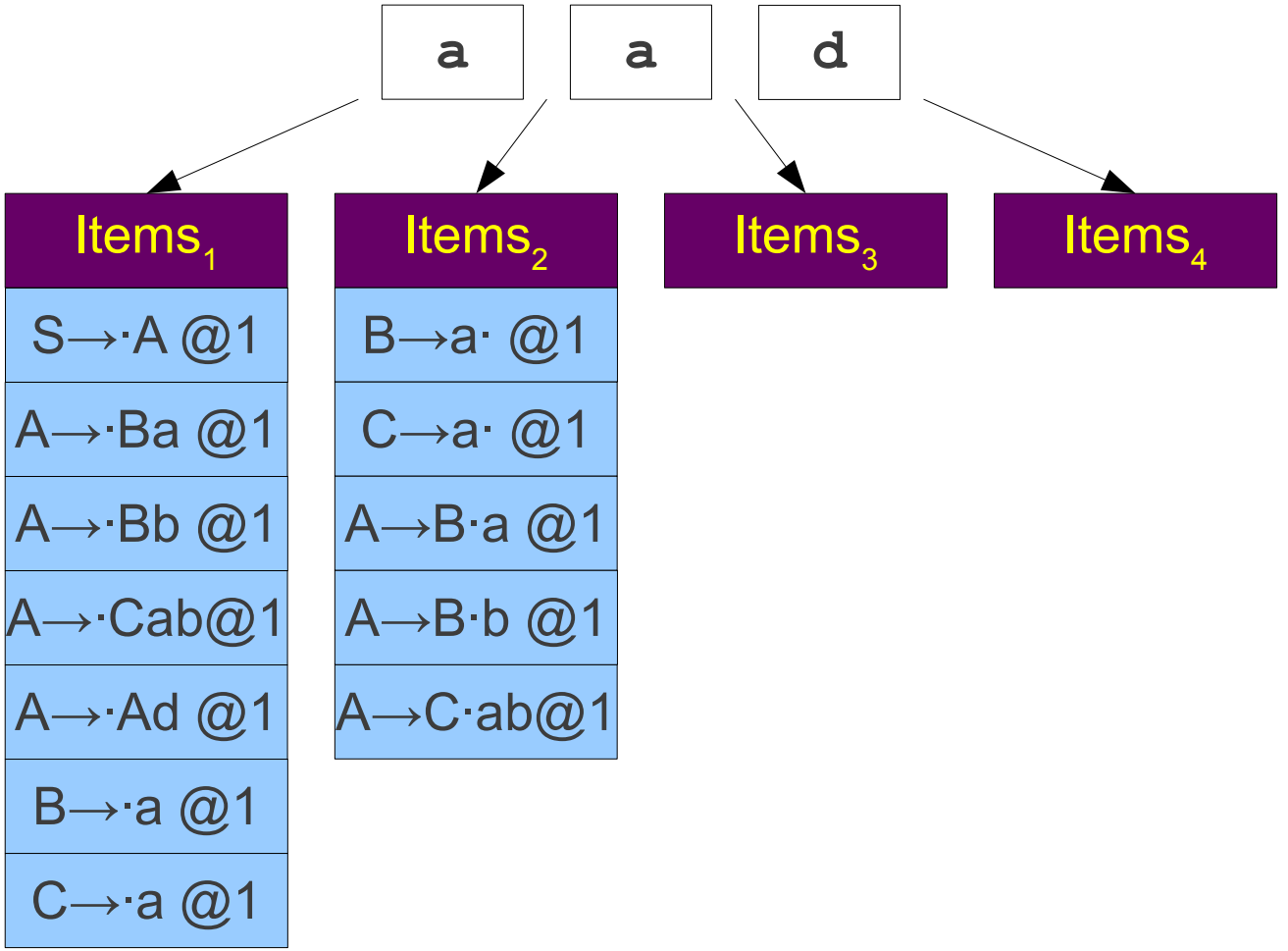


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$

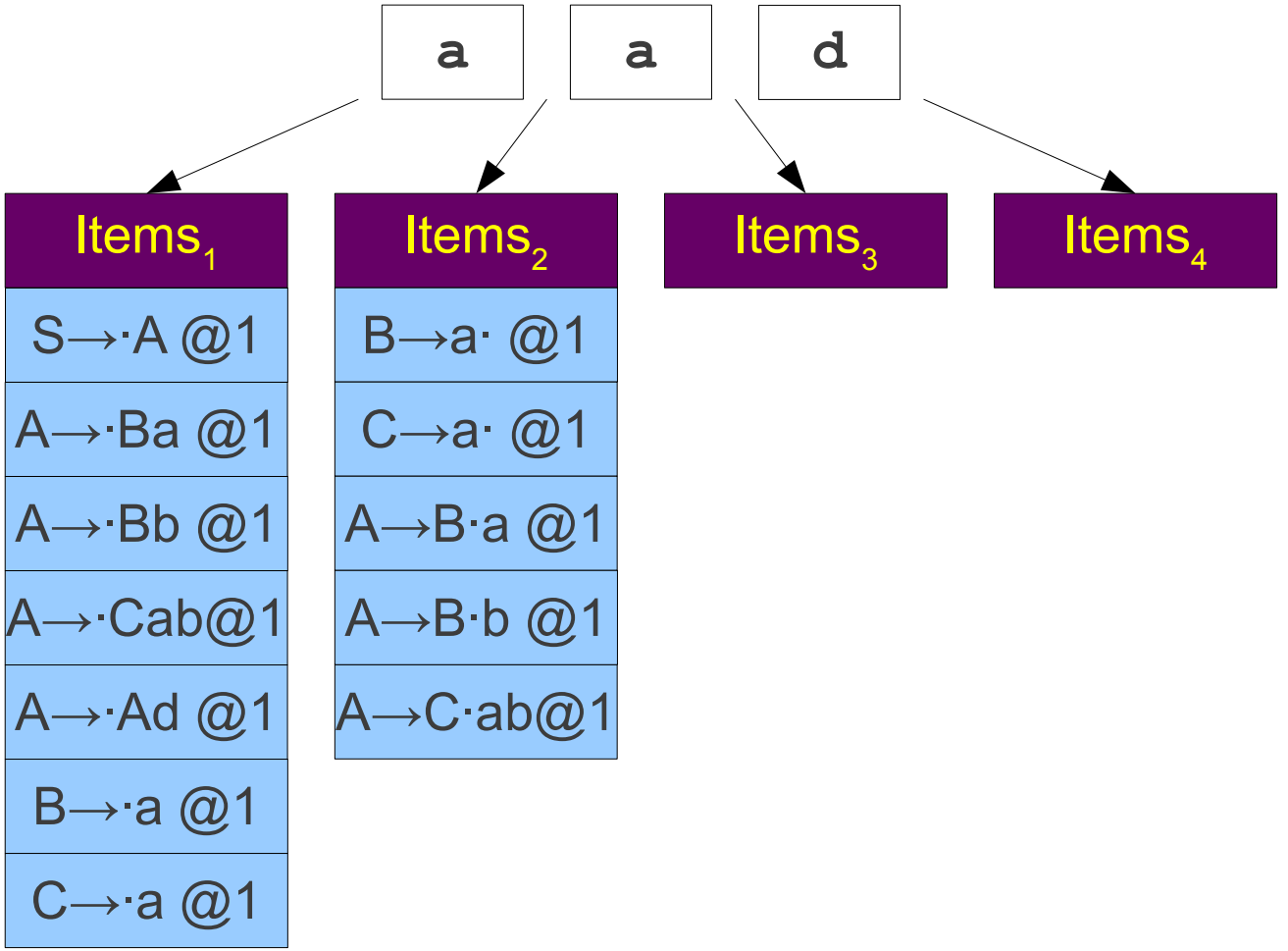


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

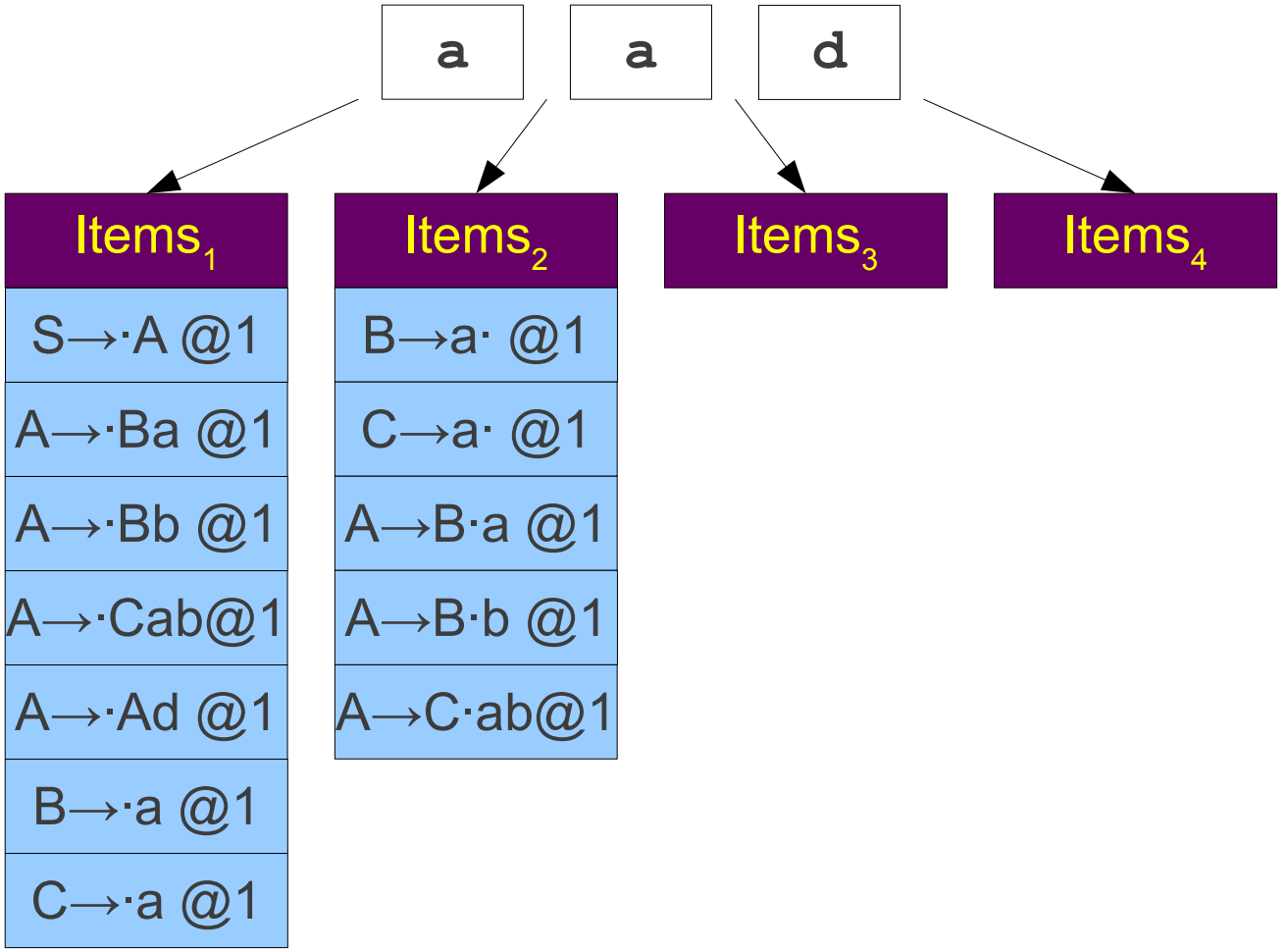


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

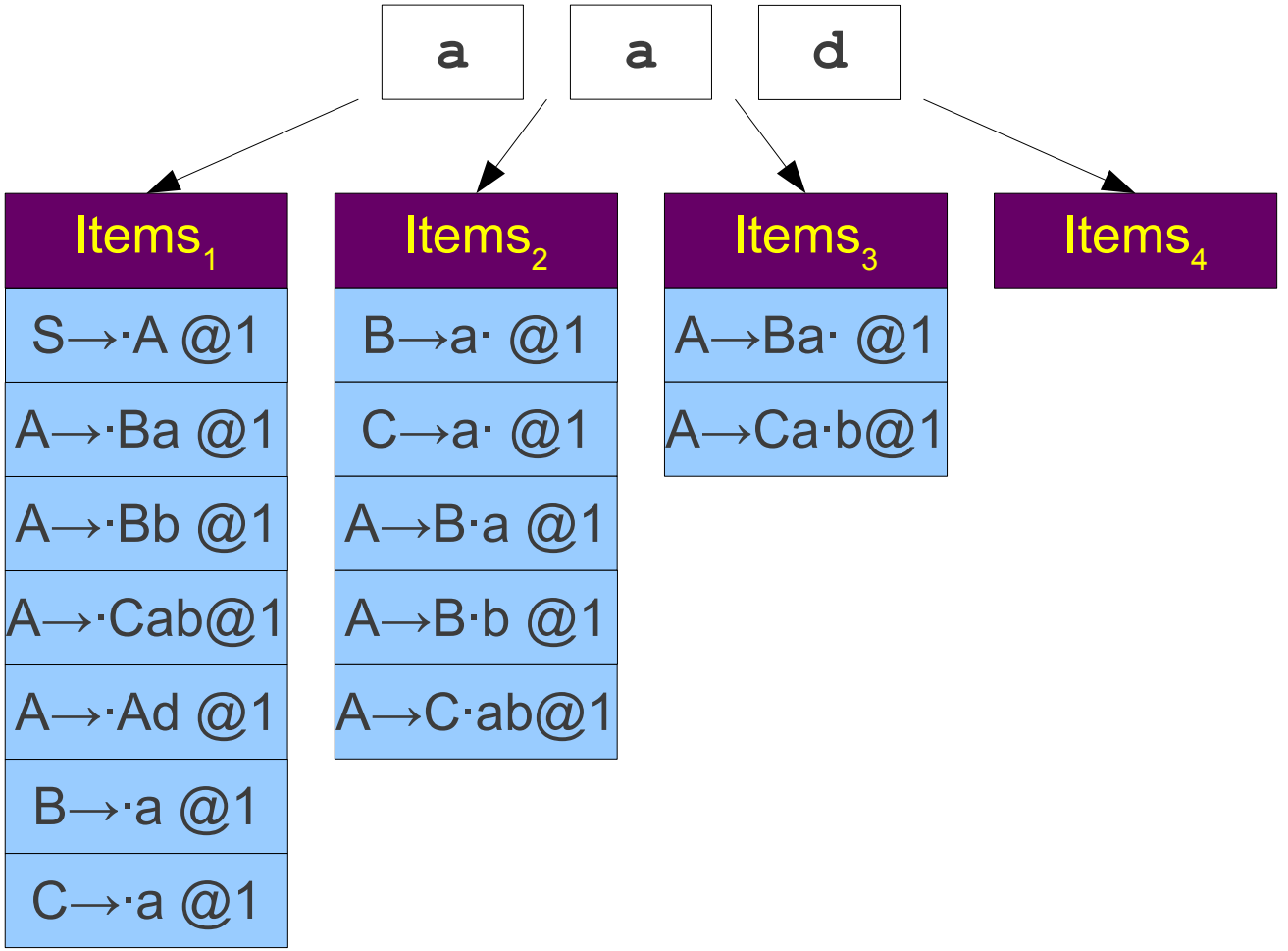


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

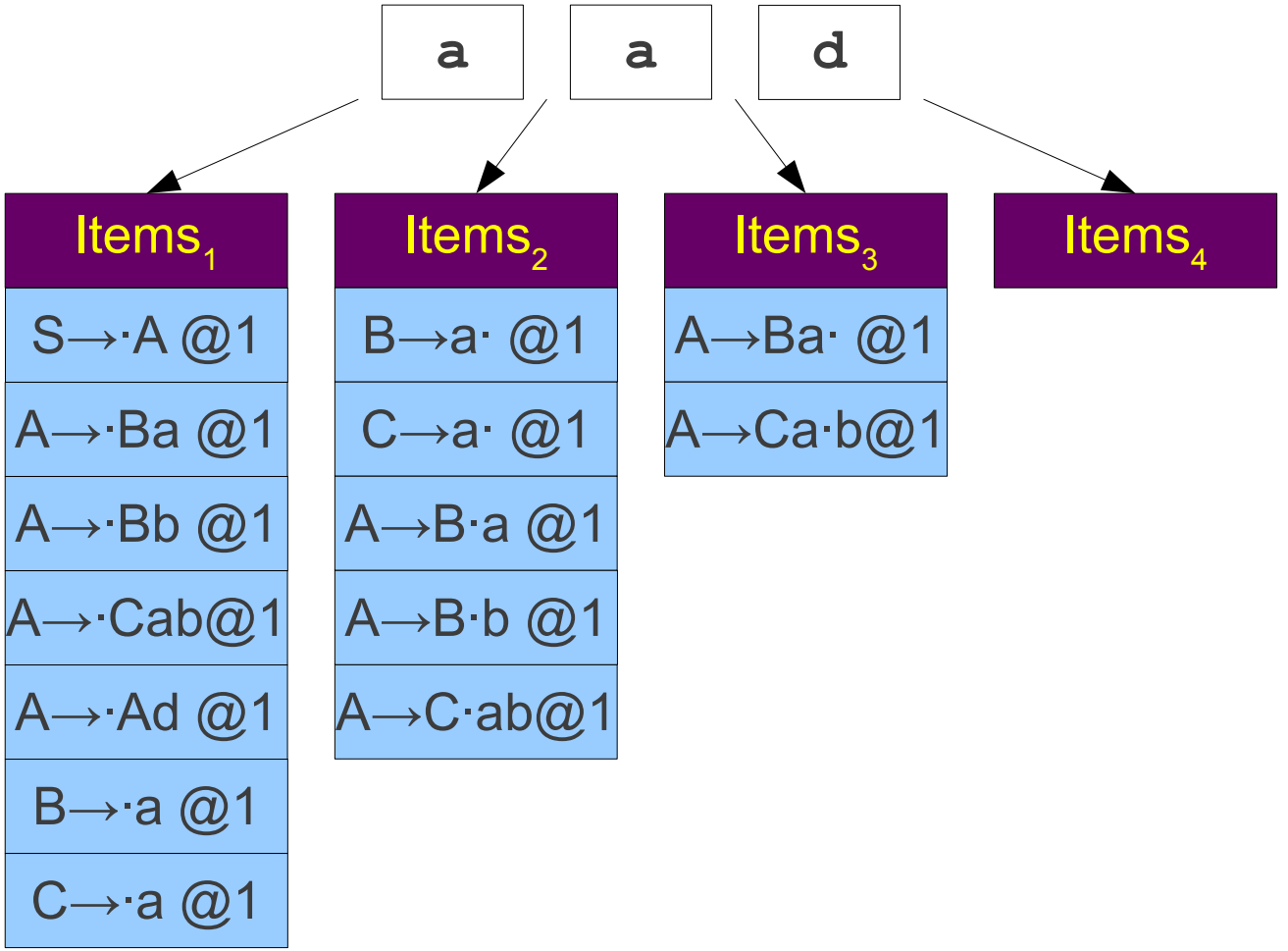


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

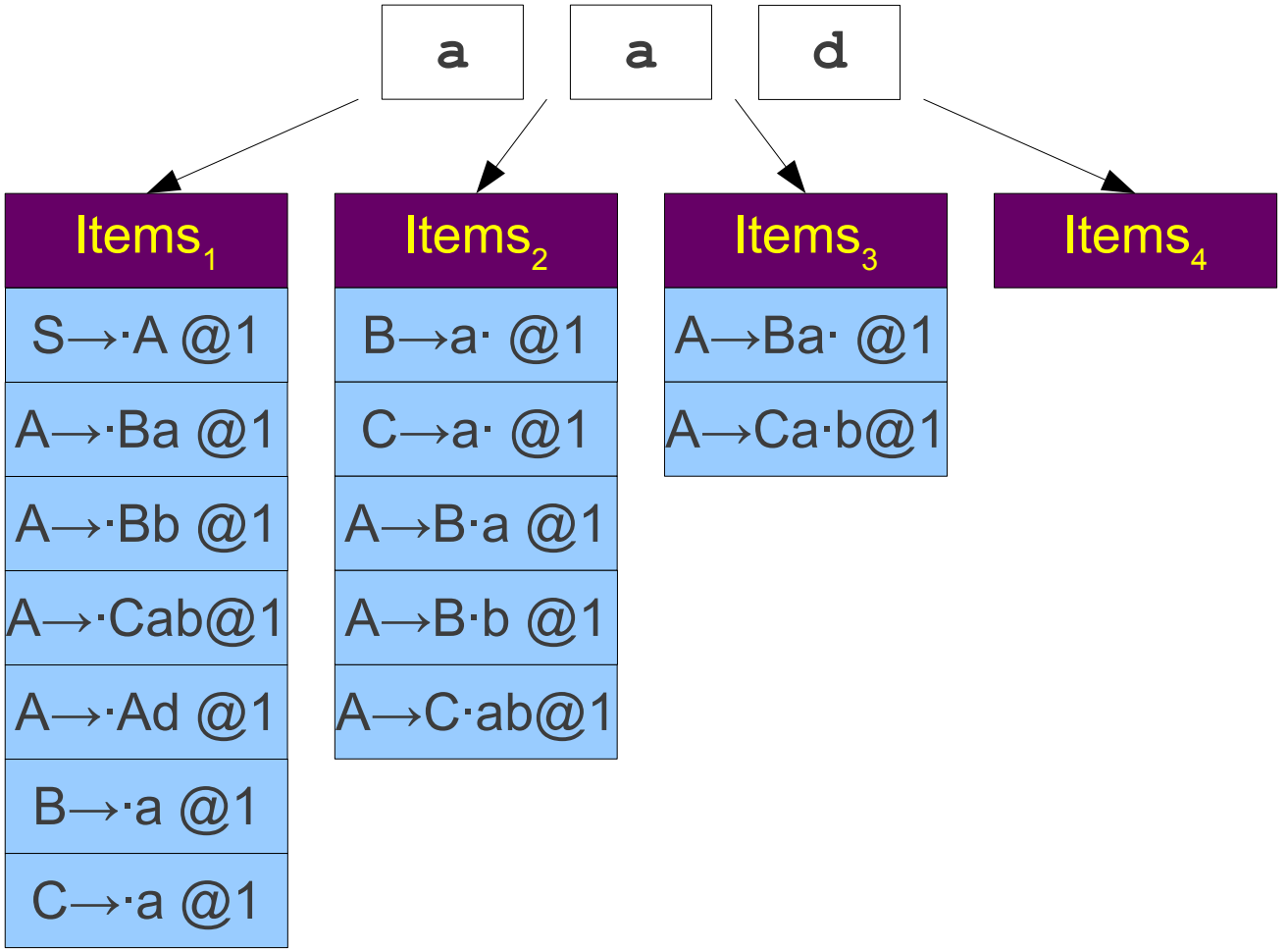


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$

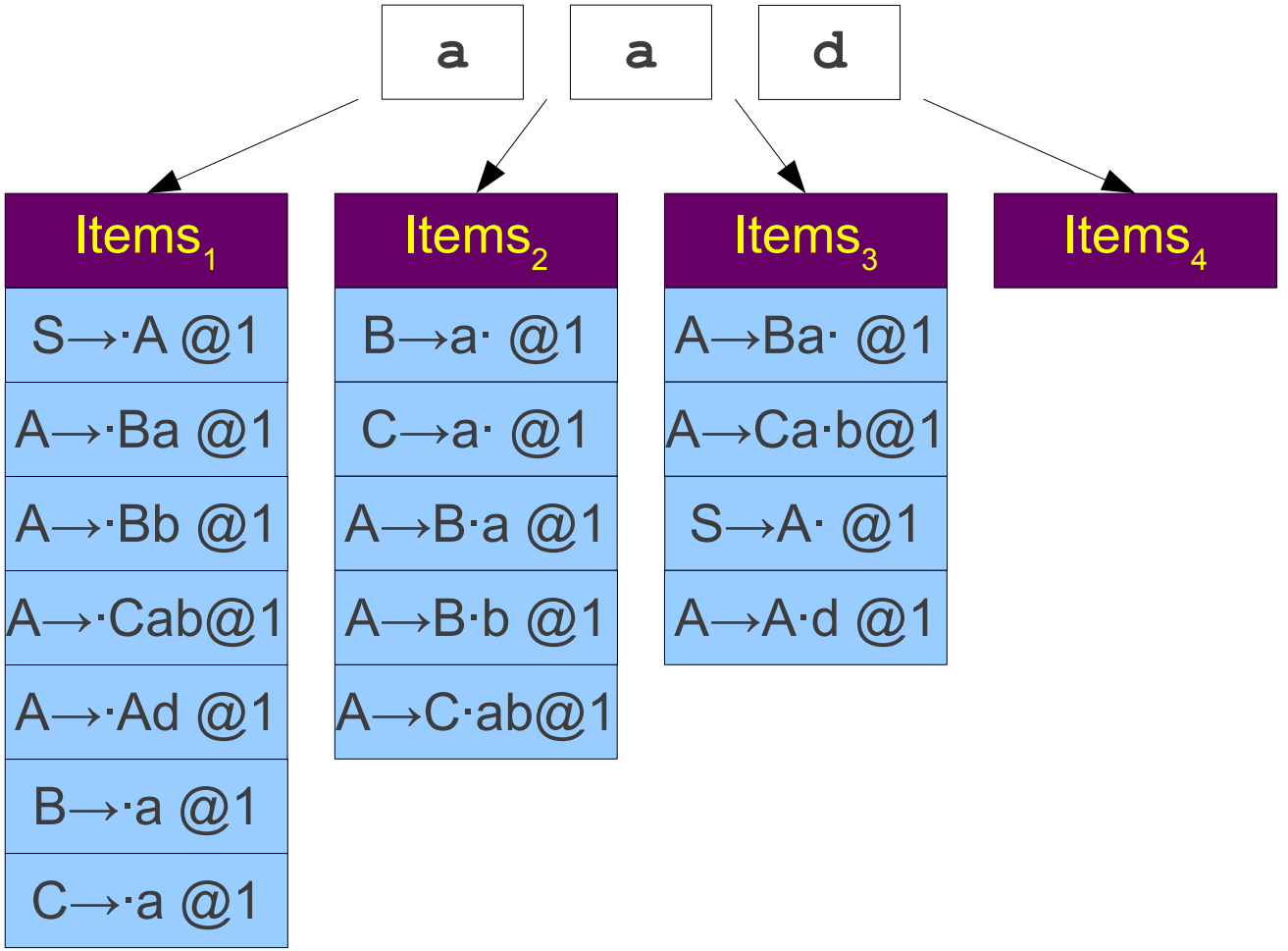


S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$

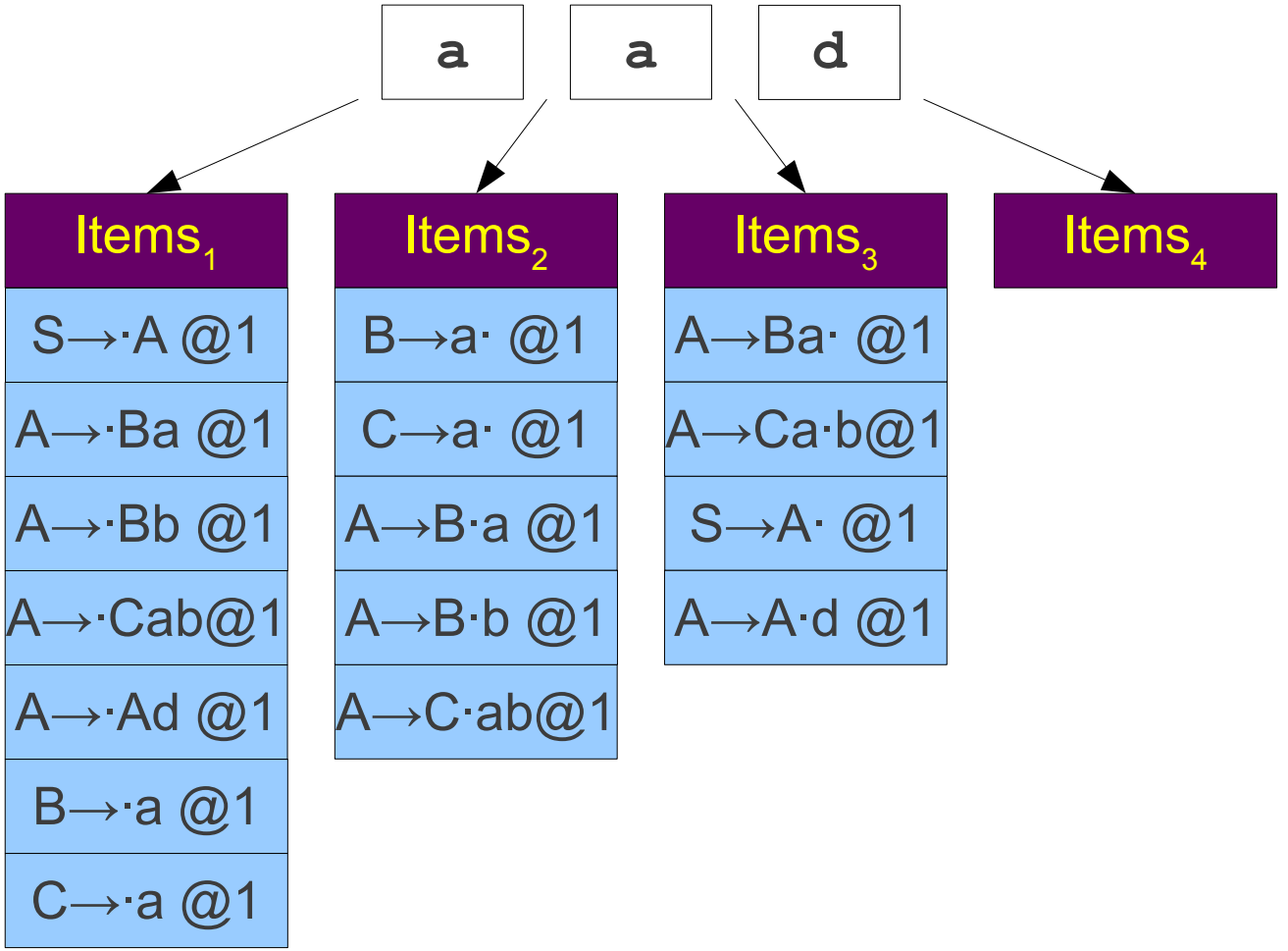


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

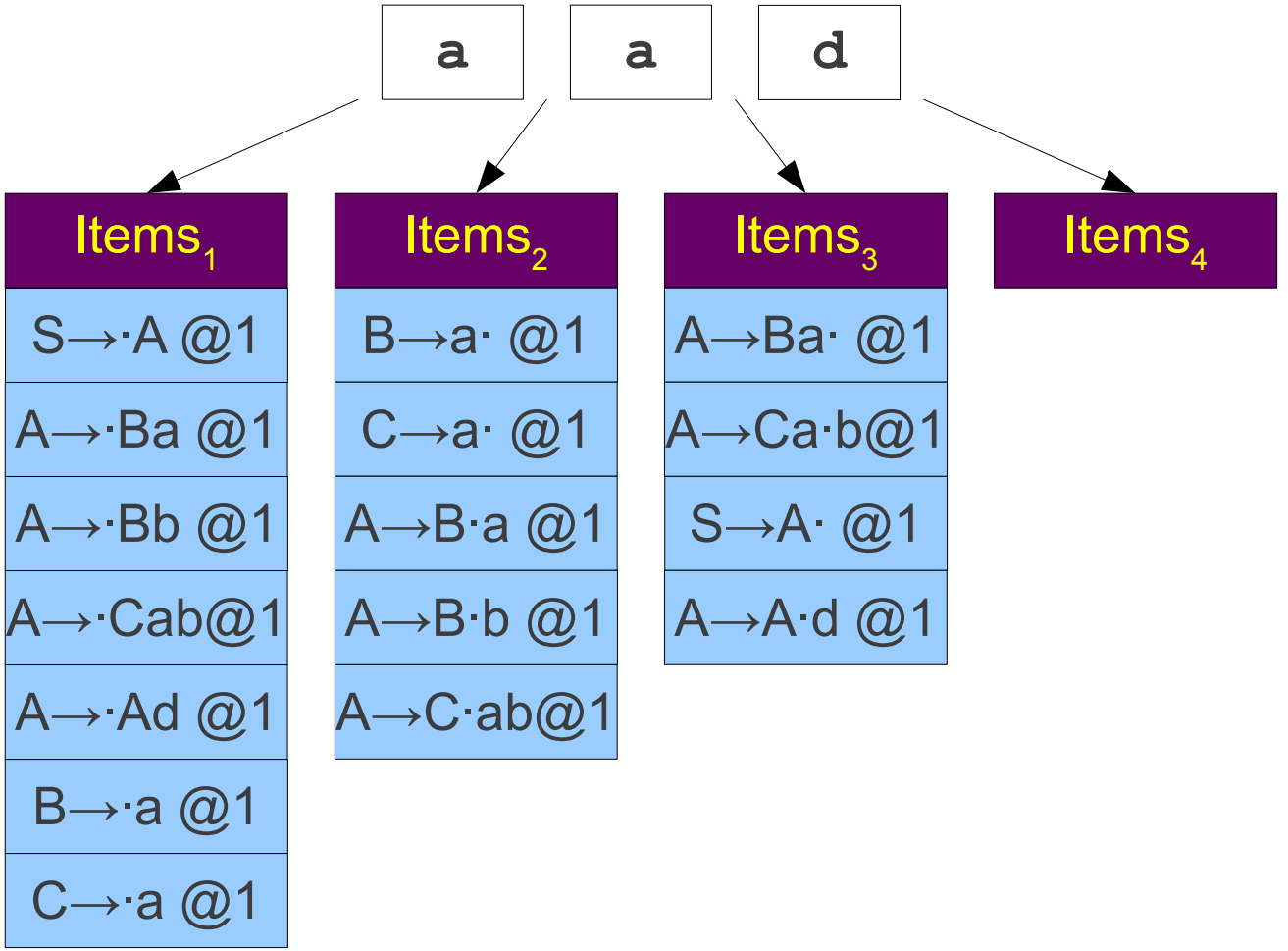


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

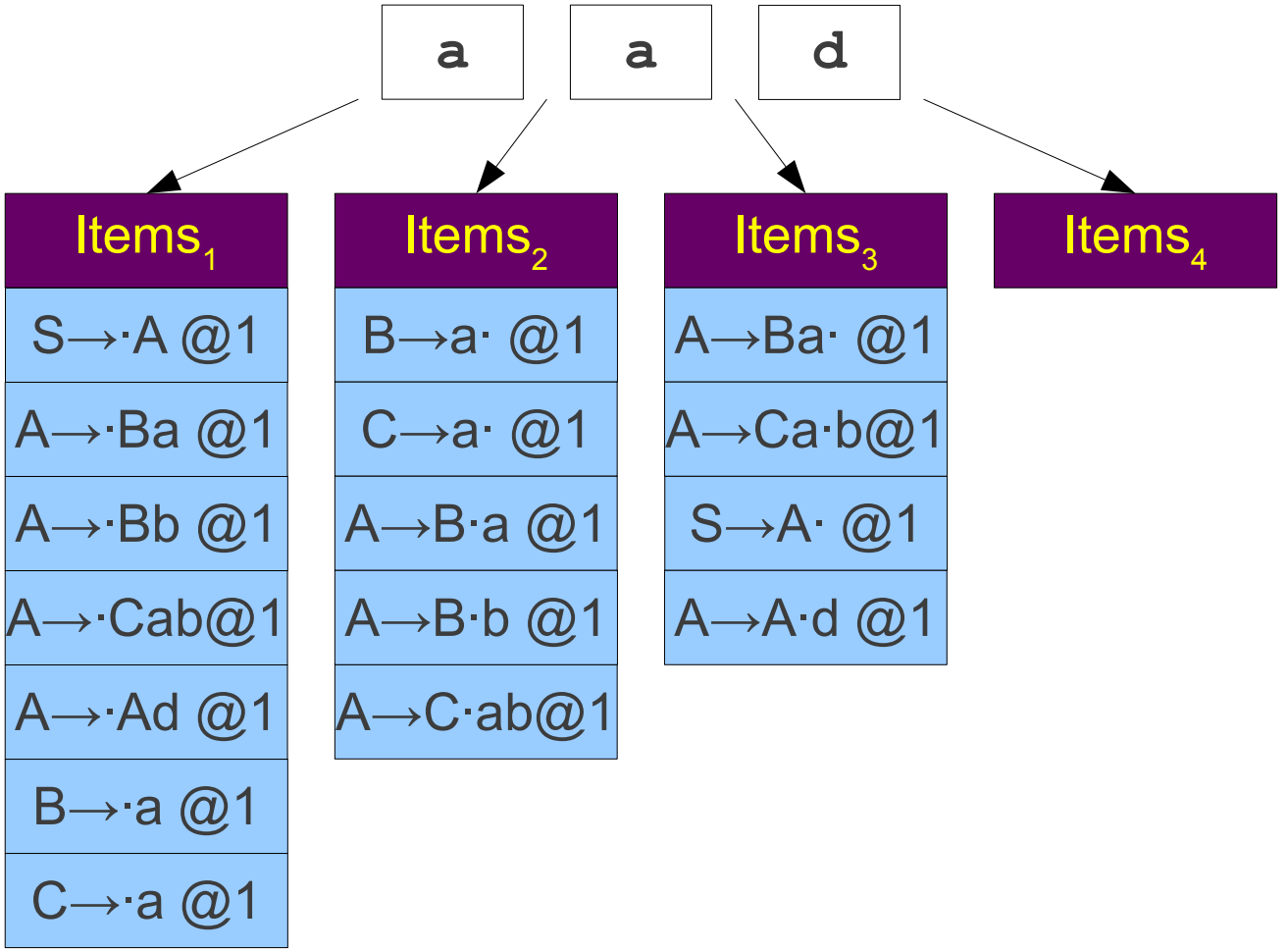


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

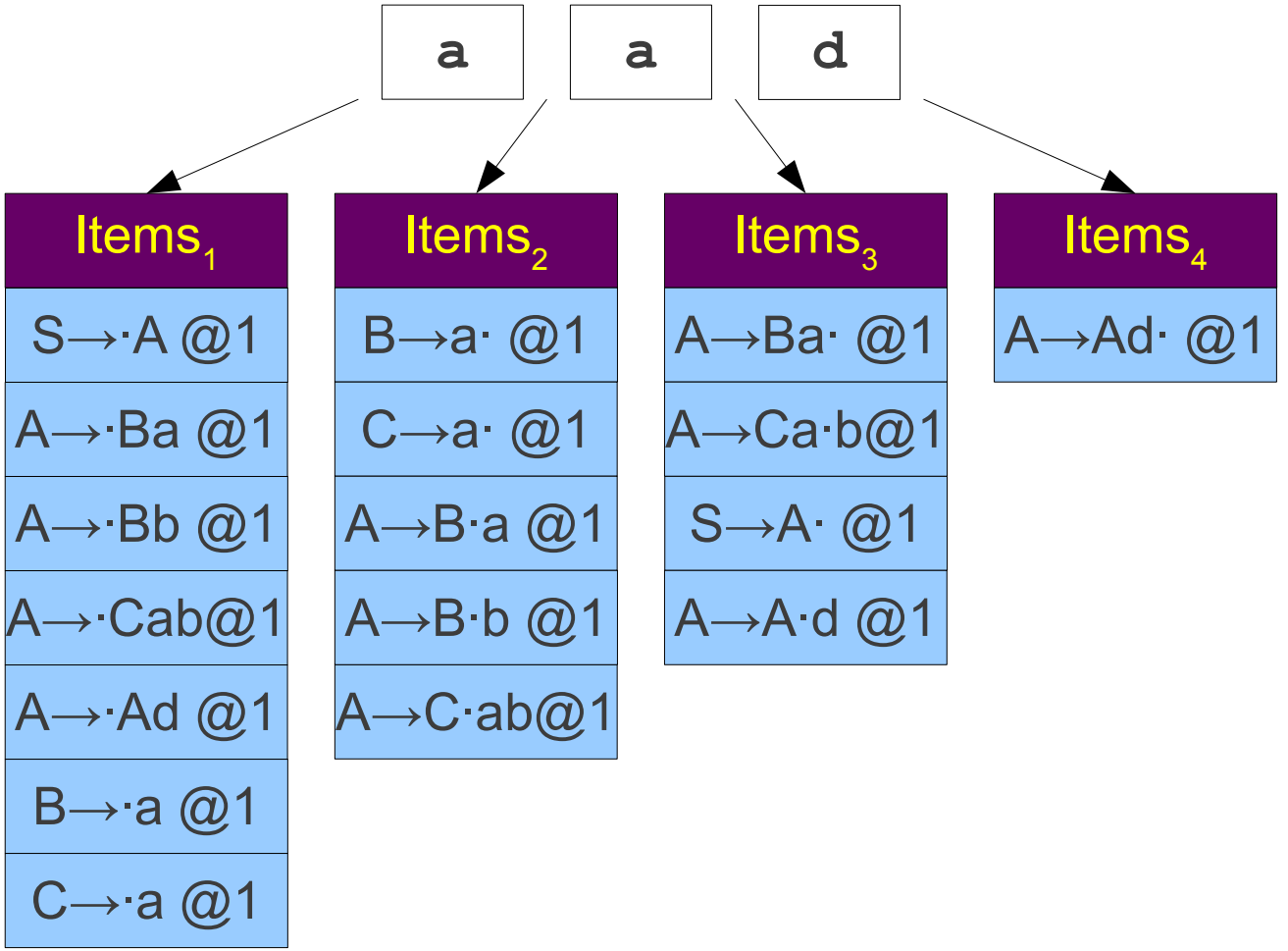


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

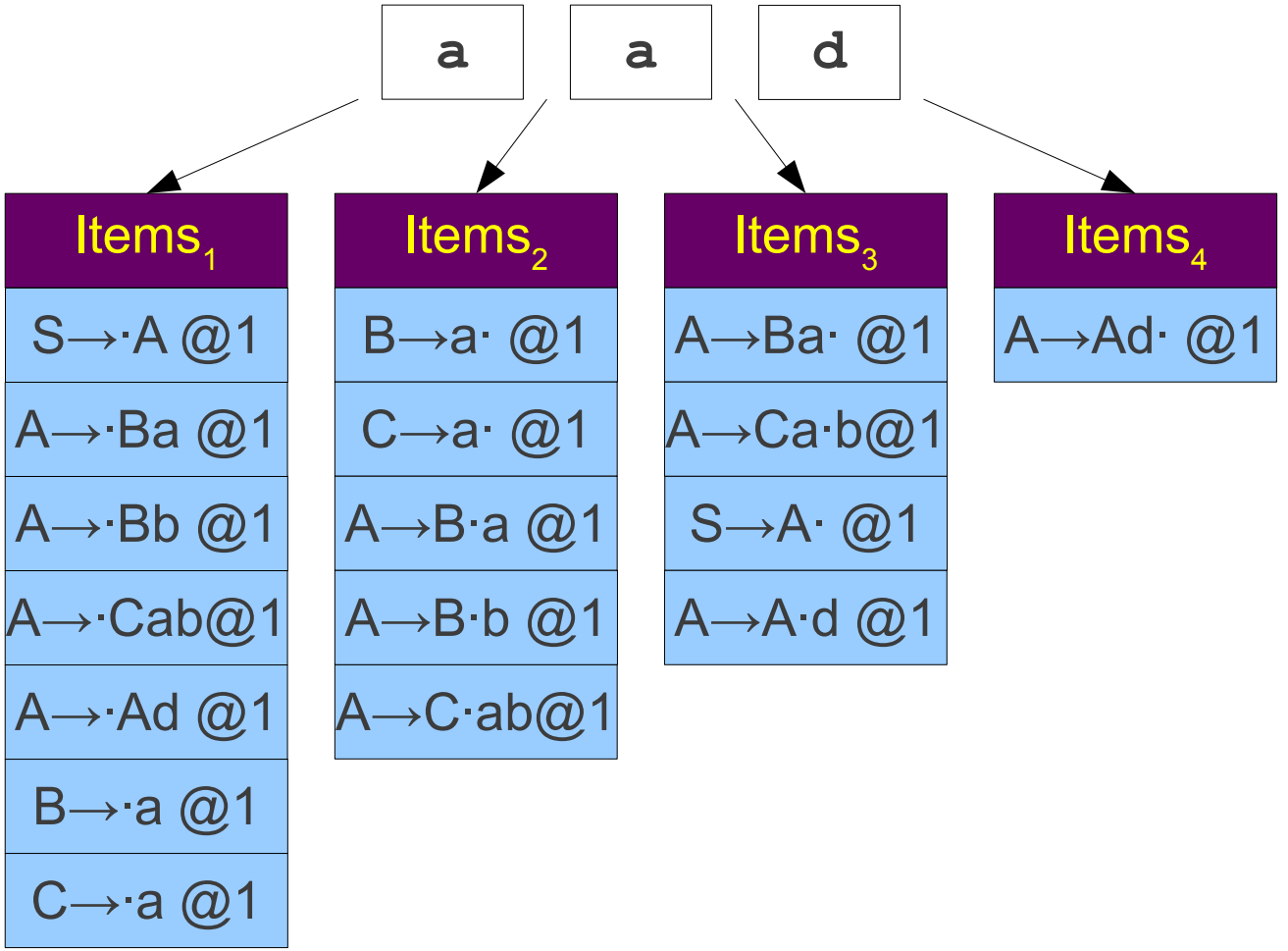


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

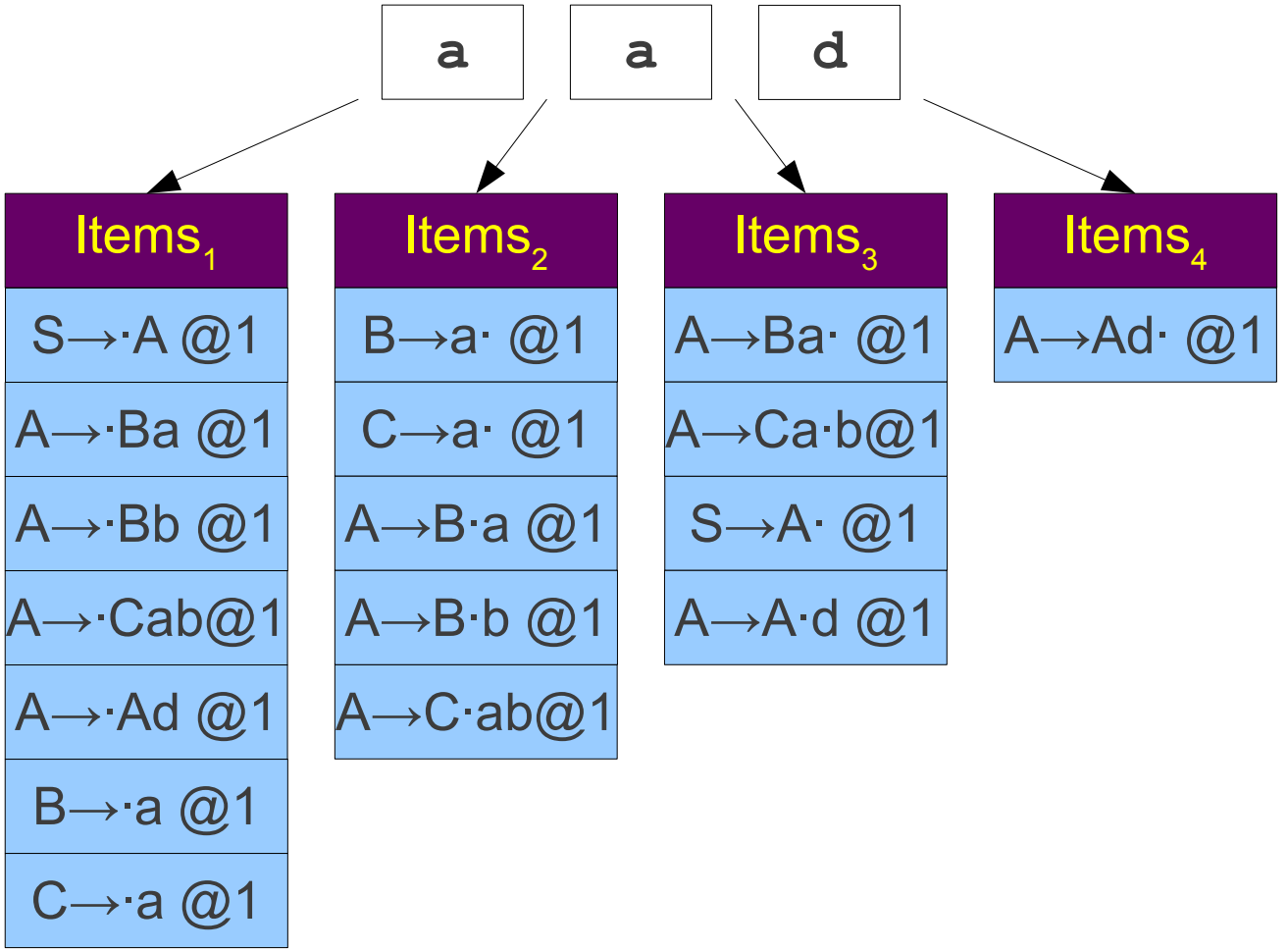


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$

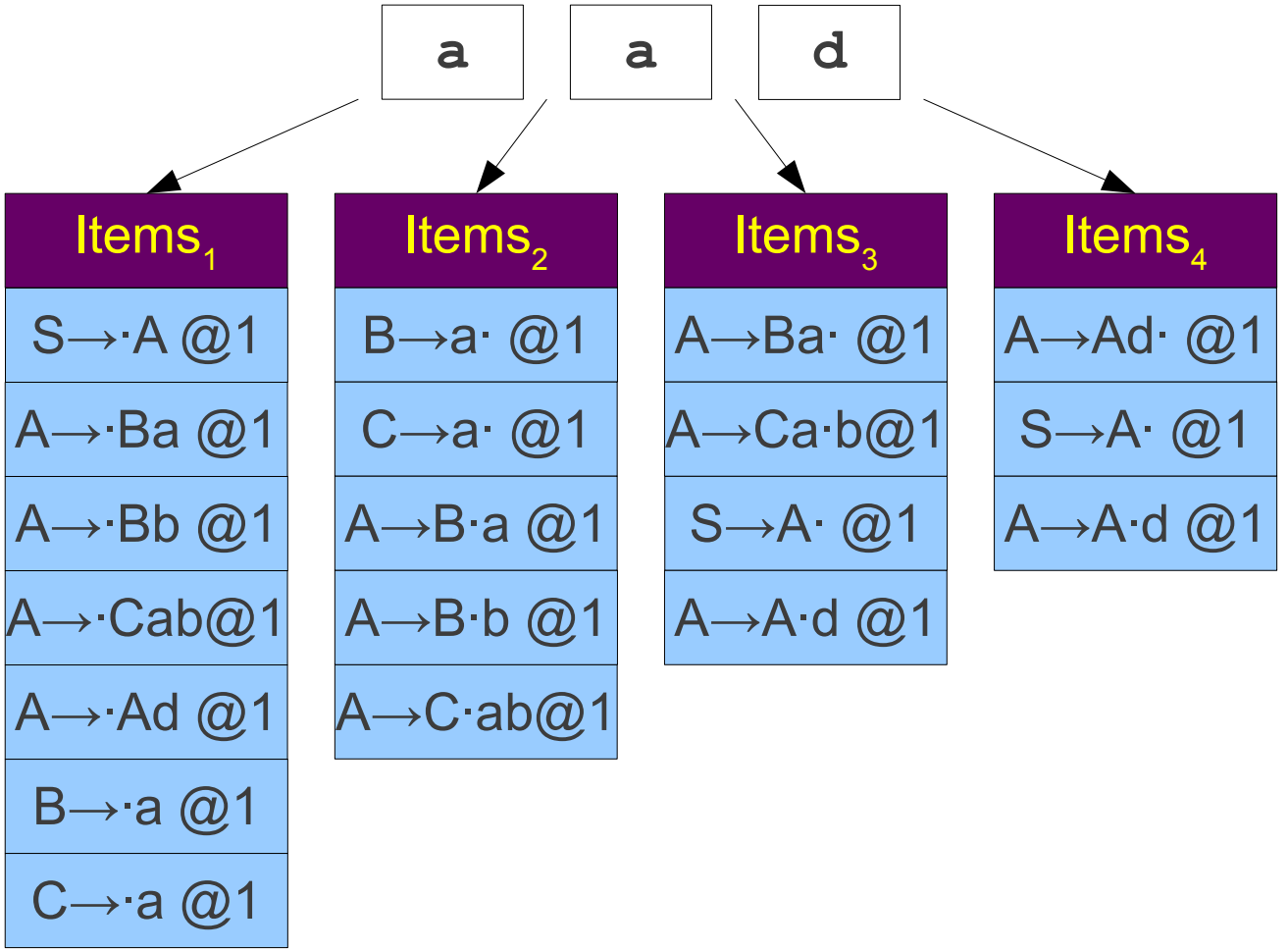


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$

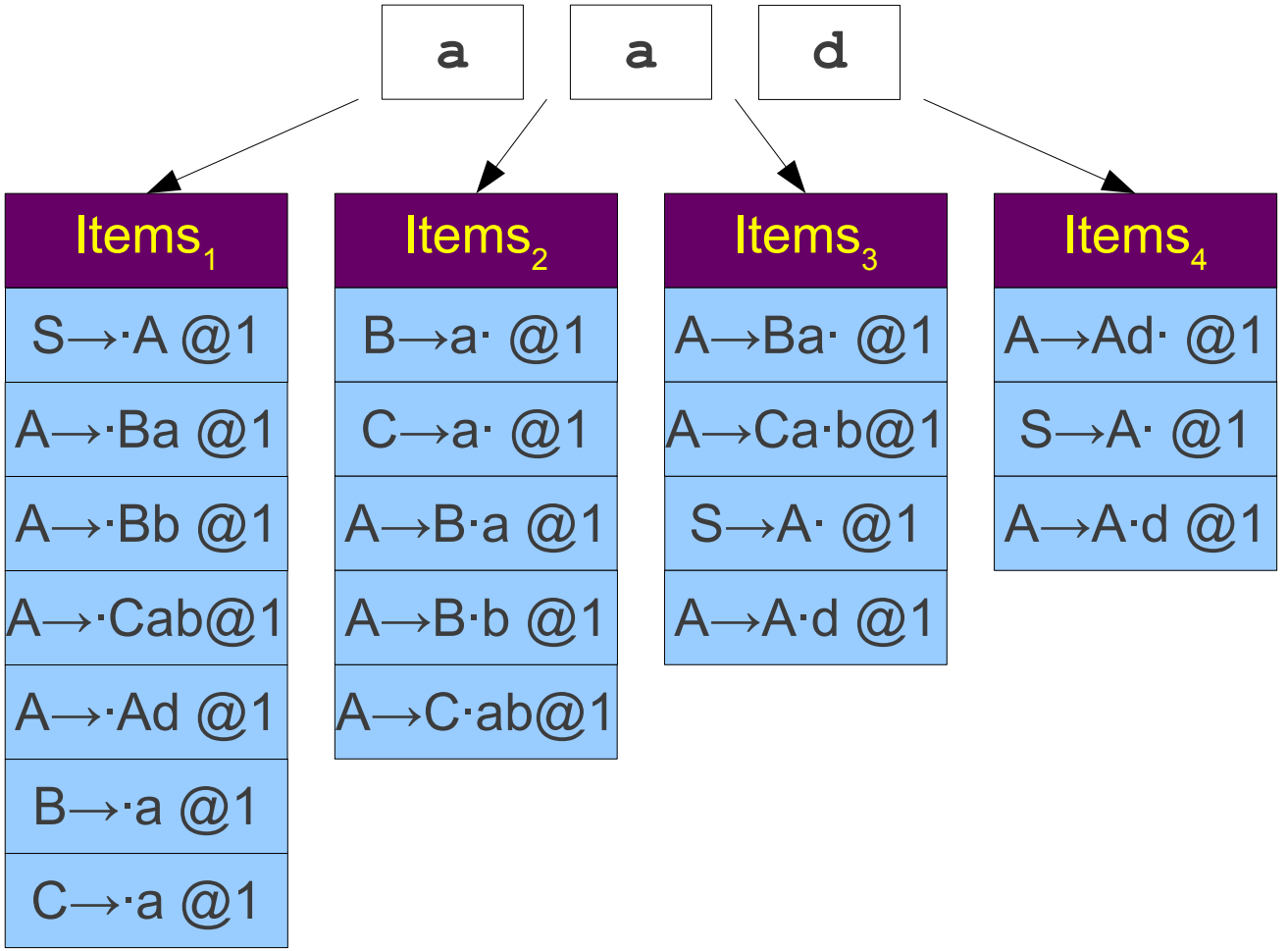


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$



$S \rightarrow A$
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$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
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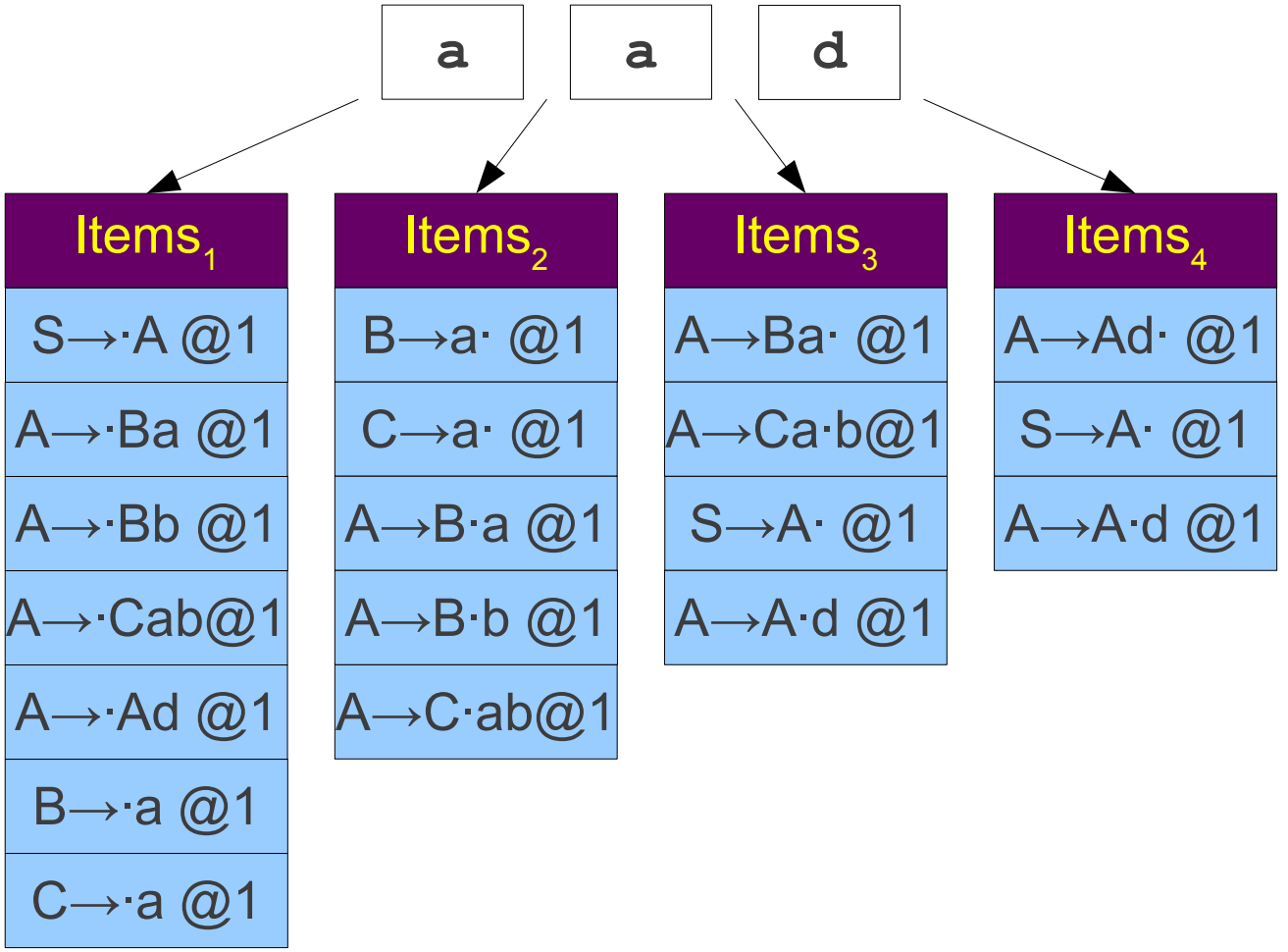
Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$

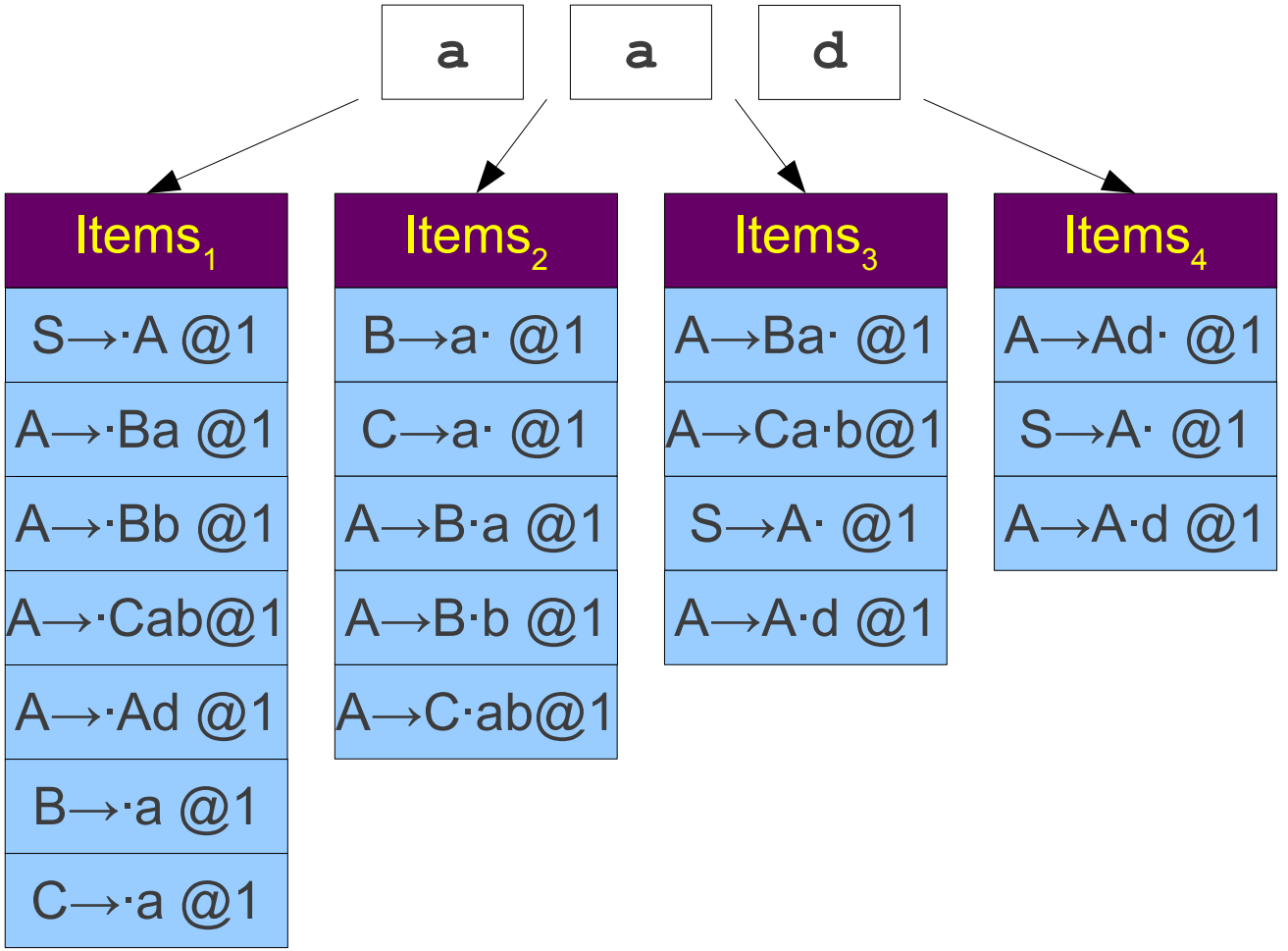


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$

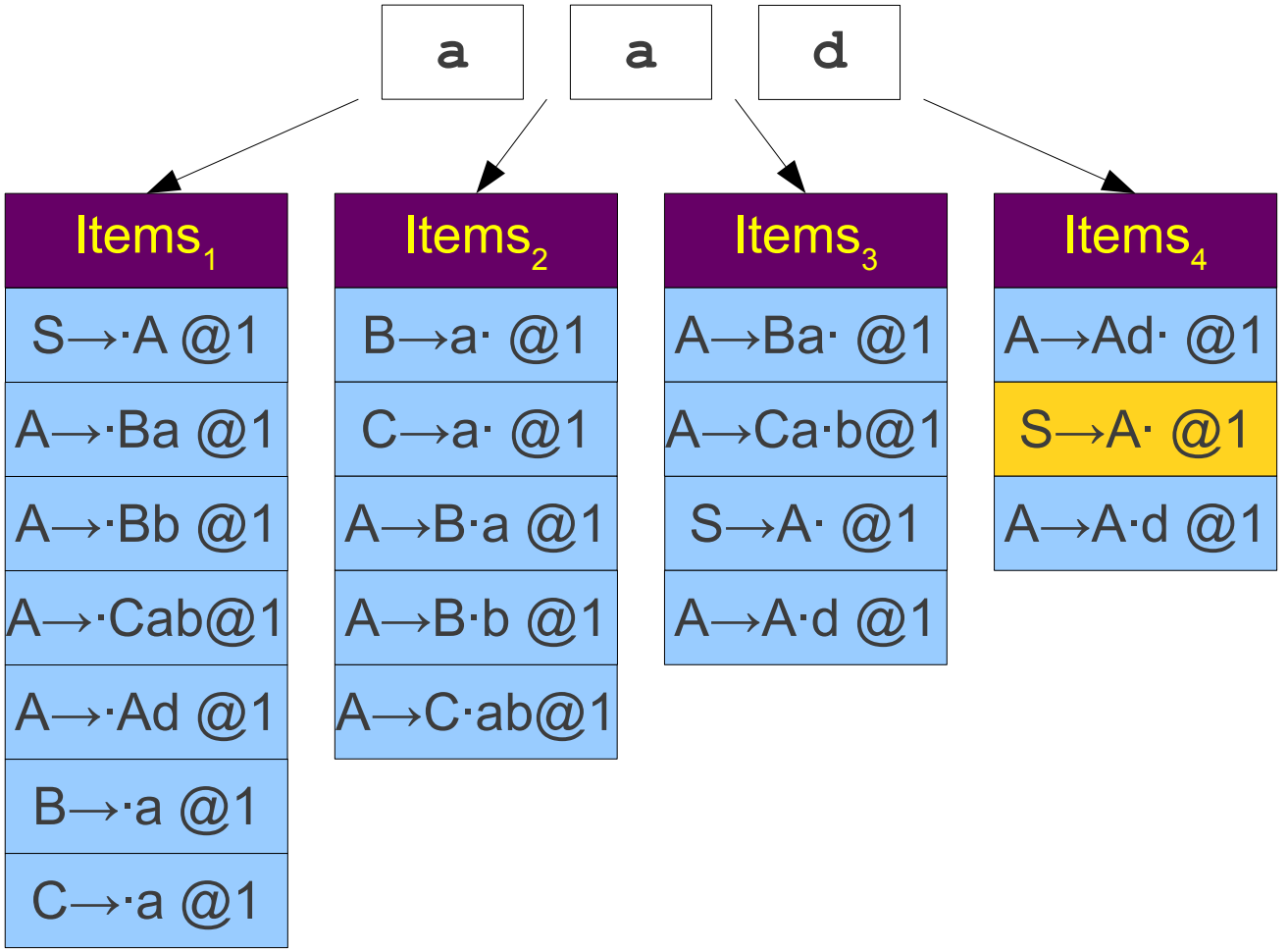


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$

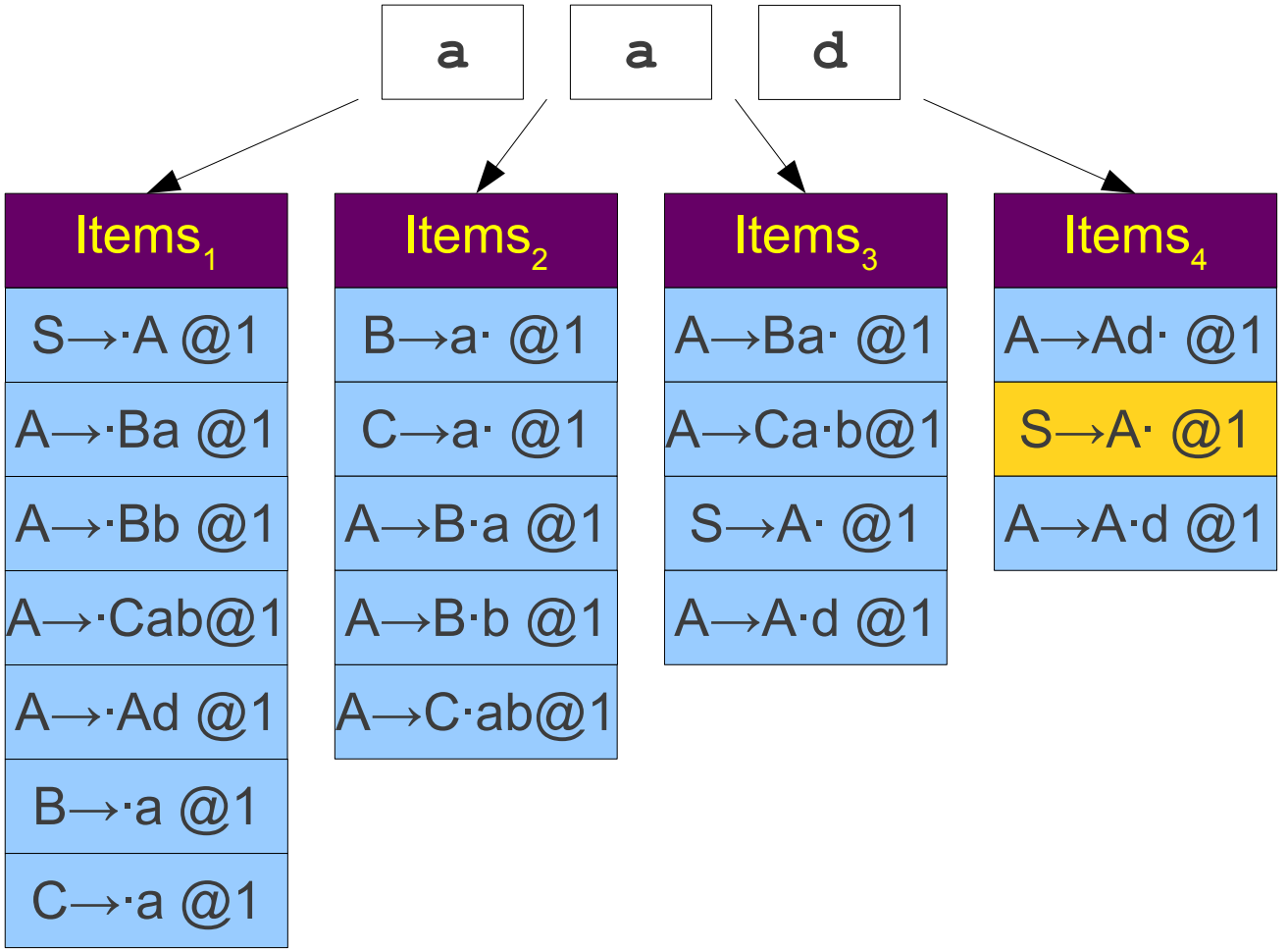


$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

Earley Parsing

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$
$S \rightarrow S_{1-4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$

Cleaning our Grammar

S

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$

Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

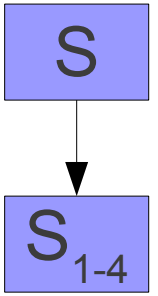
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

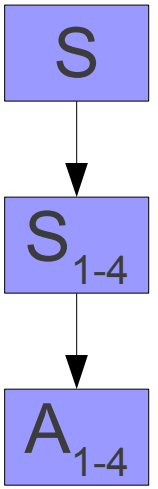
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

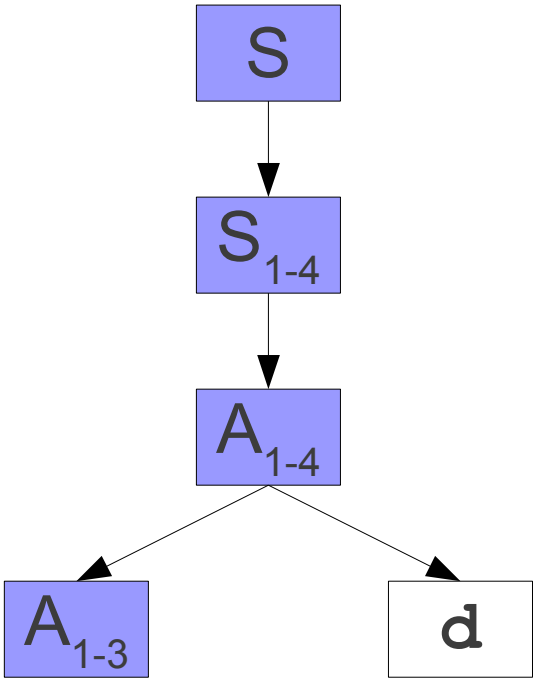
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

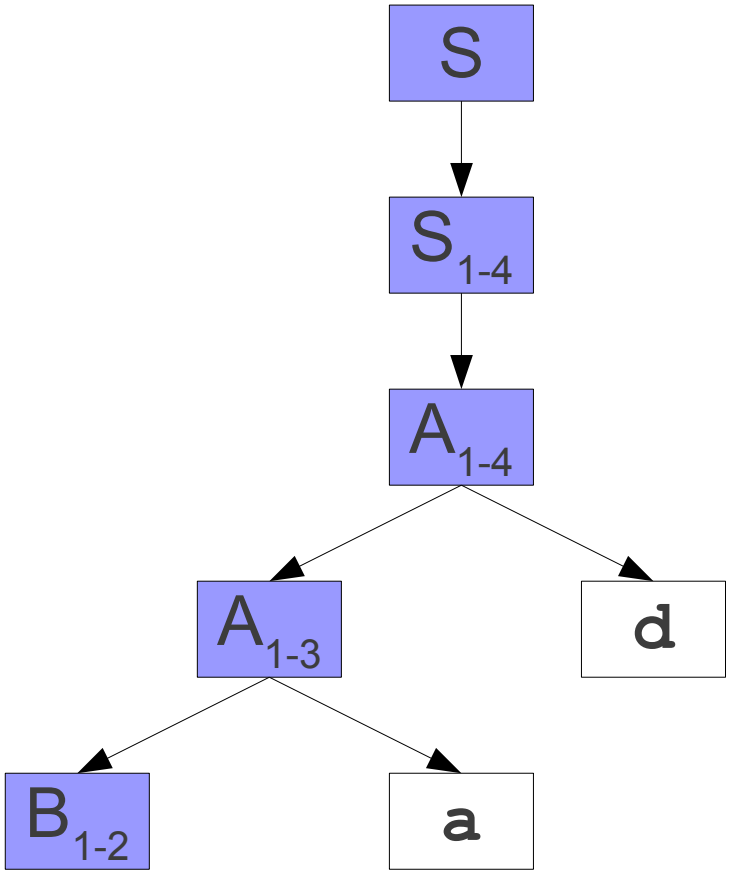
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

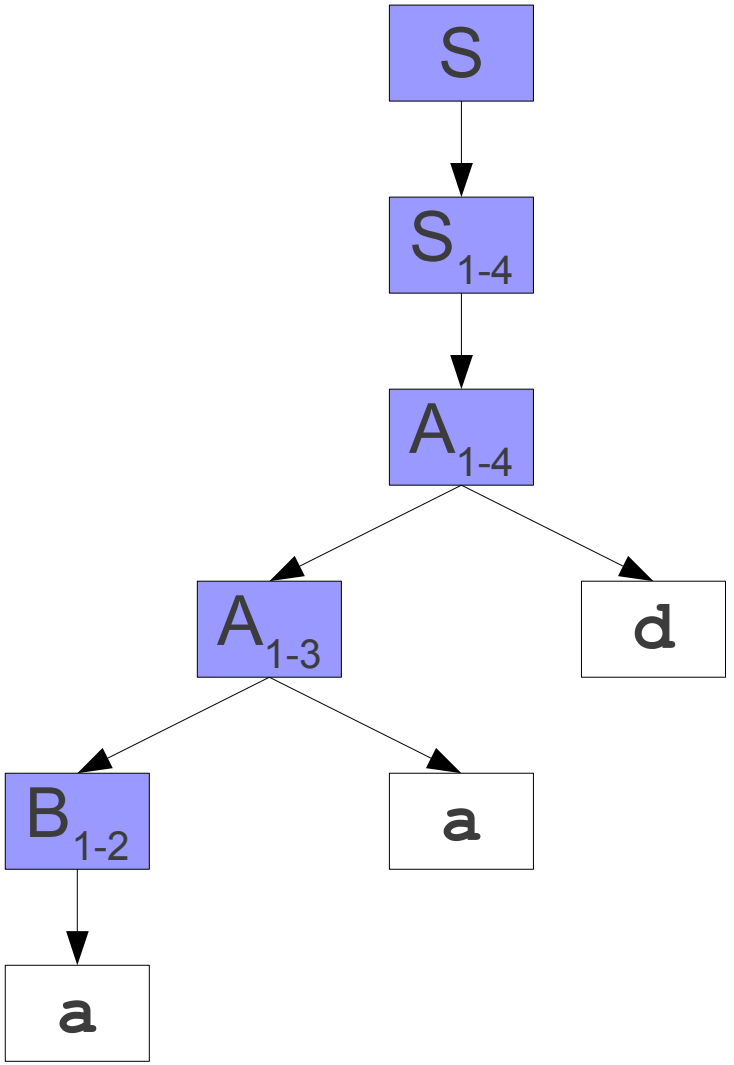
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

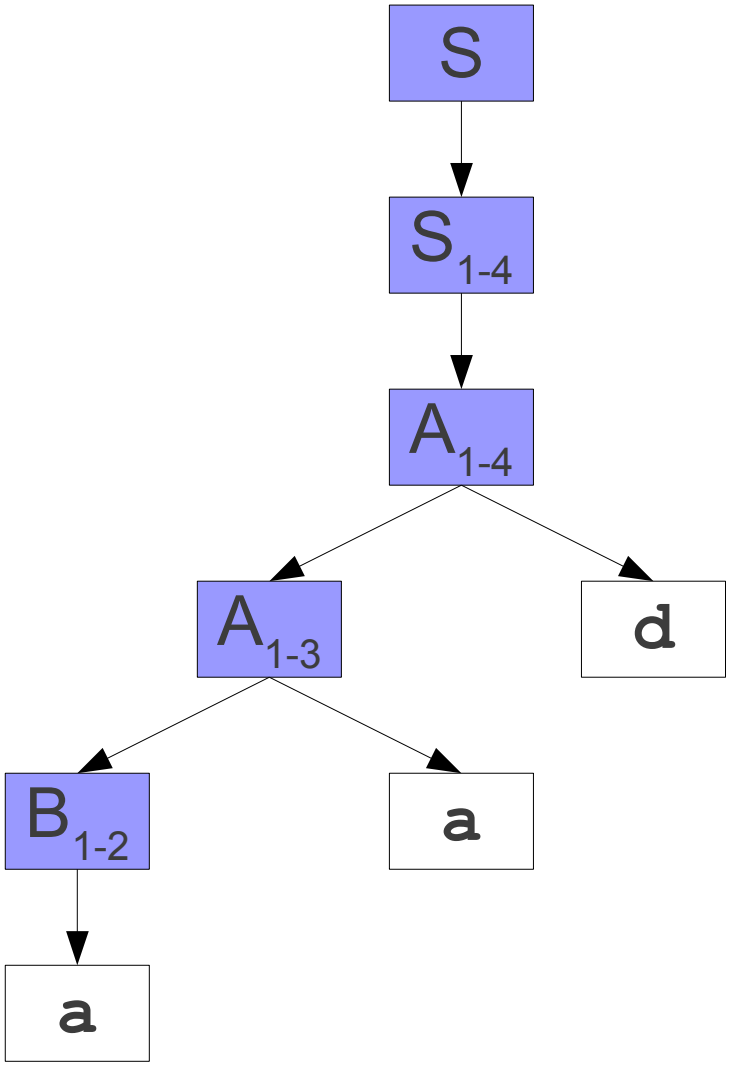
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

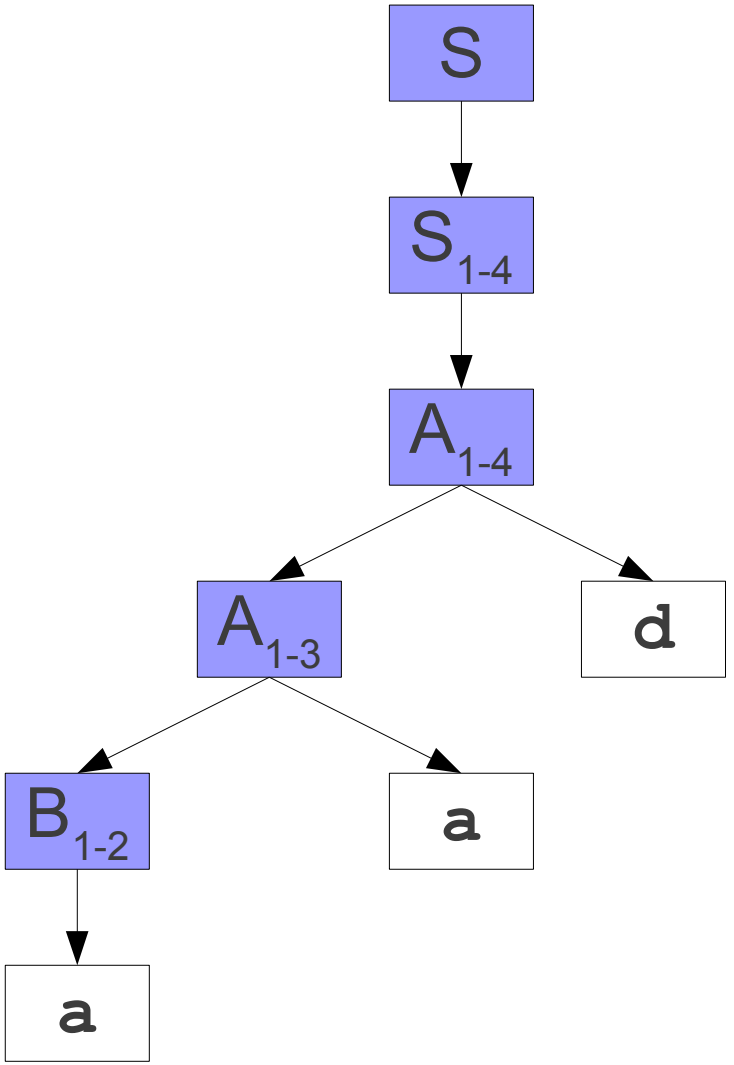
$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning our Grammar

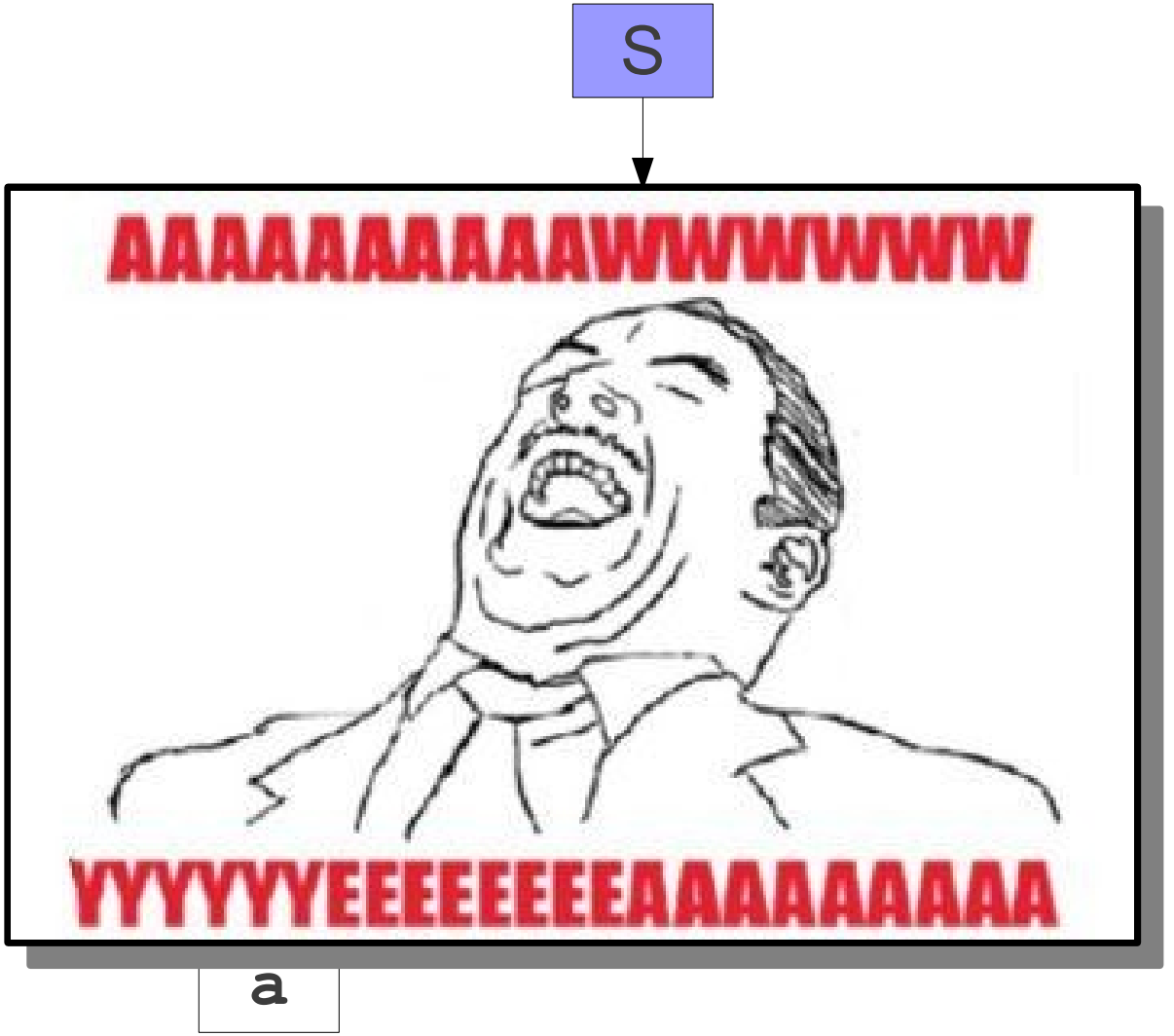
$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow S_{1\text{-}4}$



Cleaning a Parse Forest Grammar

- Remove all productions that can't be reached from the start symbol.
- Algorithm: (yet another) fixed-point iteration.
  - Set REACH = {**S**}
  - For each **A** ∈ REACH, and for each nonterminal **B** where **A** → ***α*B*ω***, add **B** to REACH.
- Remove all productions whose left-hand side is not in REACH.
- Can be made to run in time linear in the size of the grammar by encoding as a graph and doing a DFS.

Summary of Earley Parsing

- Parsing algorithm for arbitrary CFGs that, for any fixed grammar, runs quickly:
  - O(n) for LR(k) grammars (after adding lookahead)
  - O(n^2) for any unambiguous grammar.
  - O(n^3) for any ambiguous grammar.
- Outputs a parse forest grammar of all possible parse trees.

Intersection Parsing

$S \to A$
$A \to Ba$
$A \to Bb$
$A \to Cab$
$A \to Ad$
$B \to a$
$C \to a$

An Observation

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

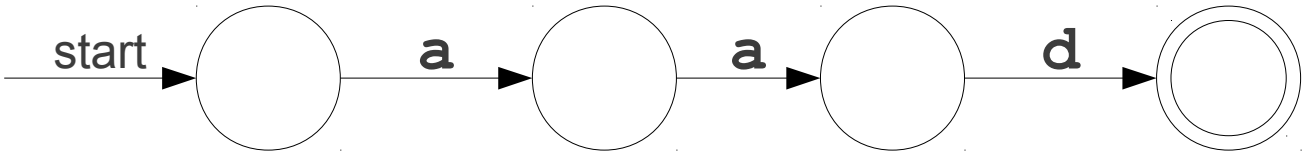
a	a	d

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

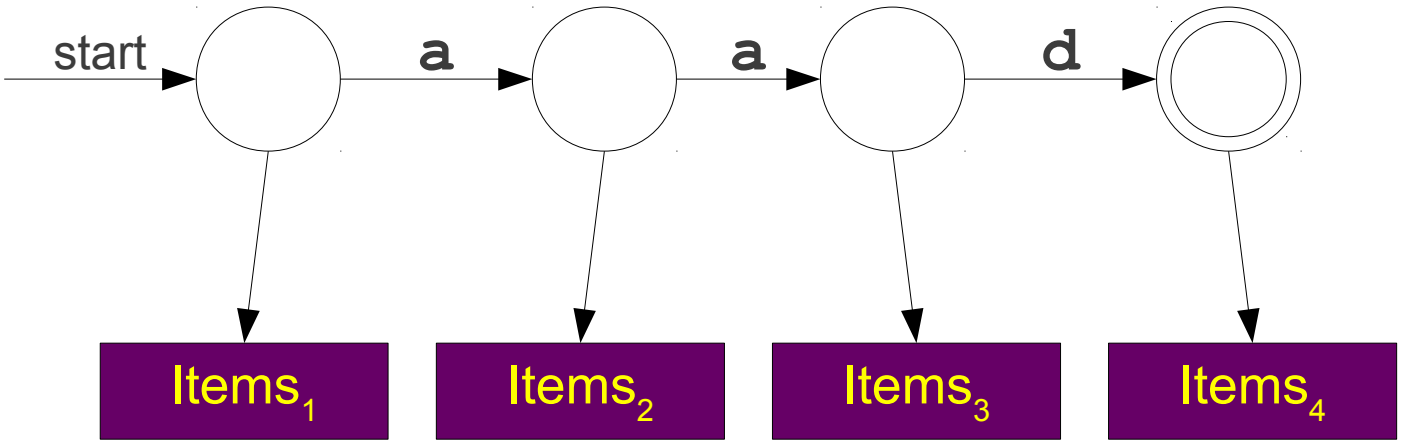
S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation



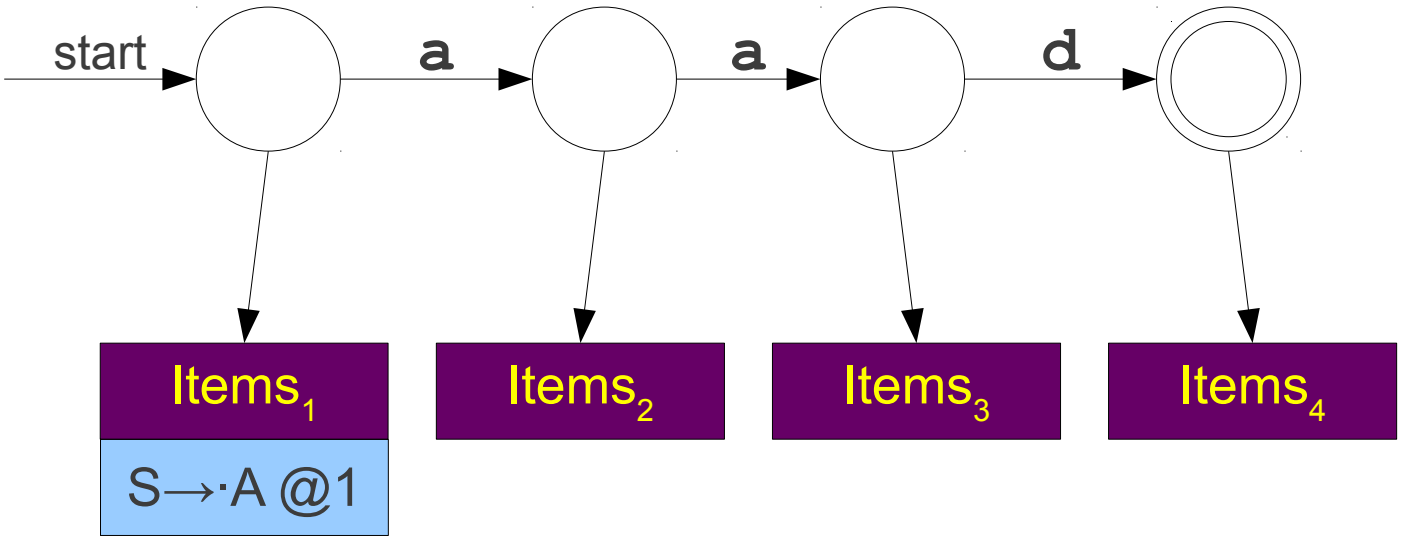
S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

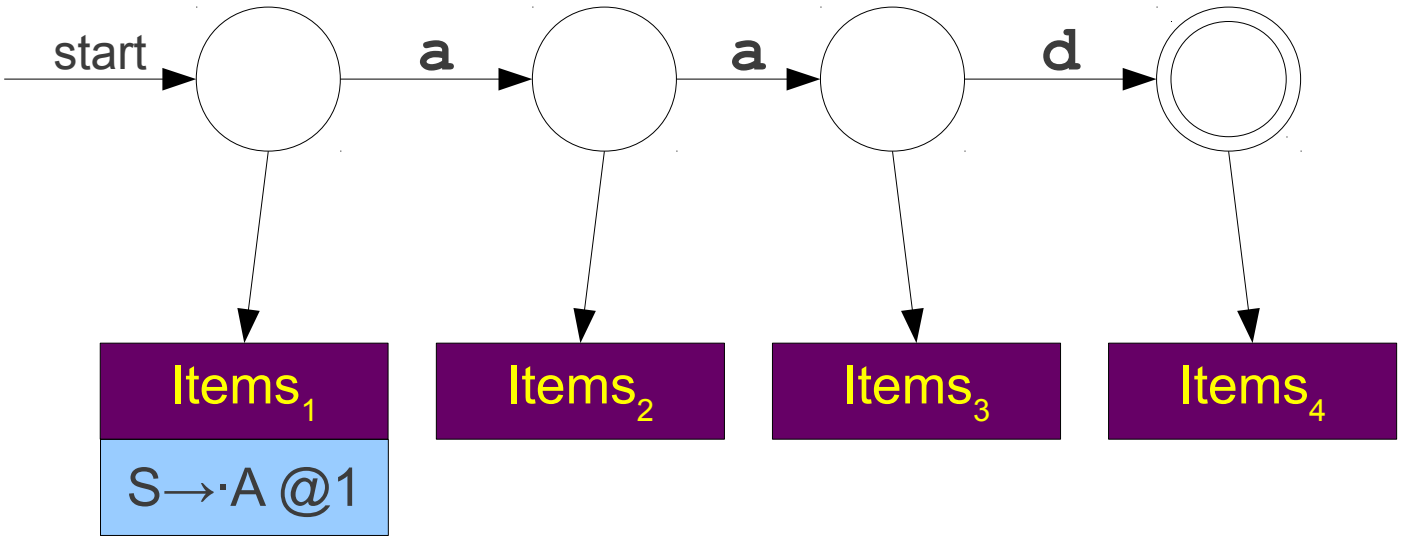
An Observation



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

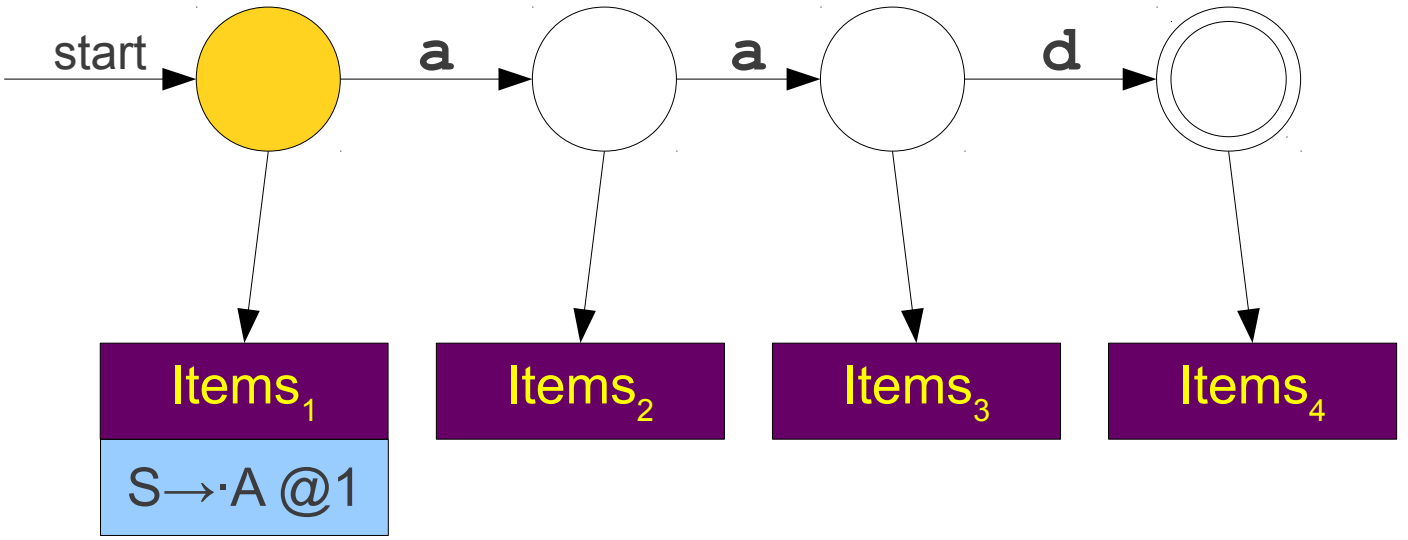
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

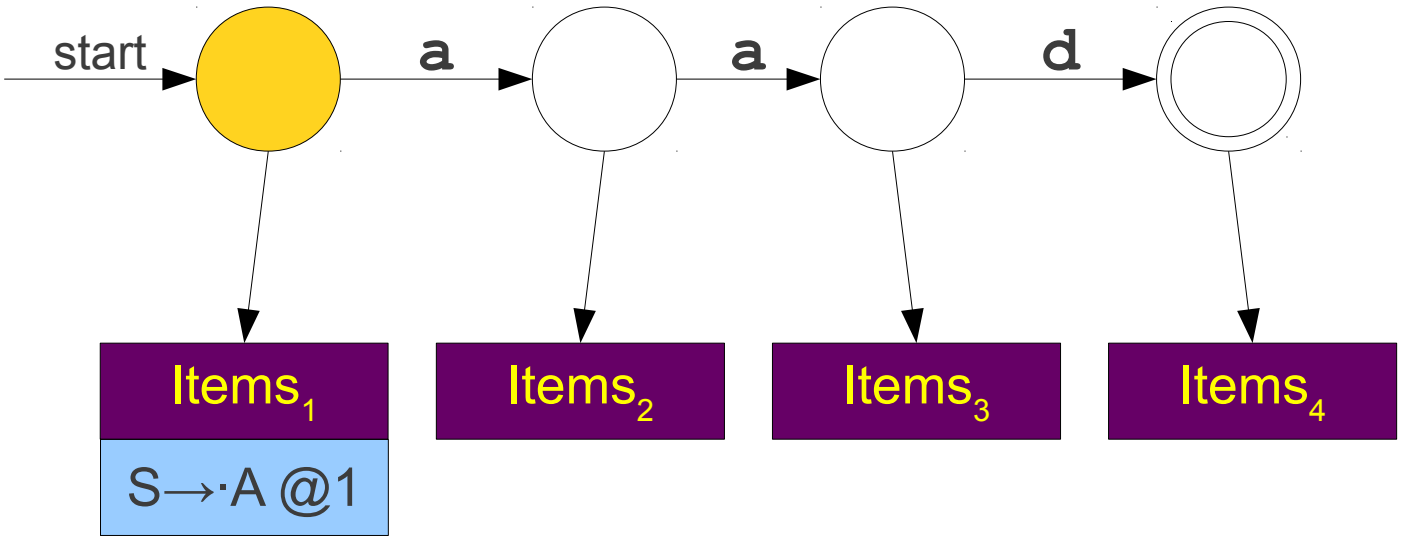
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

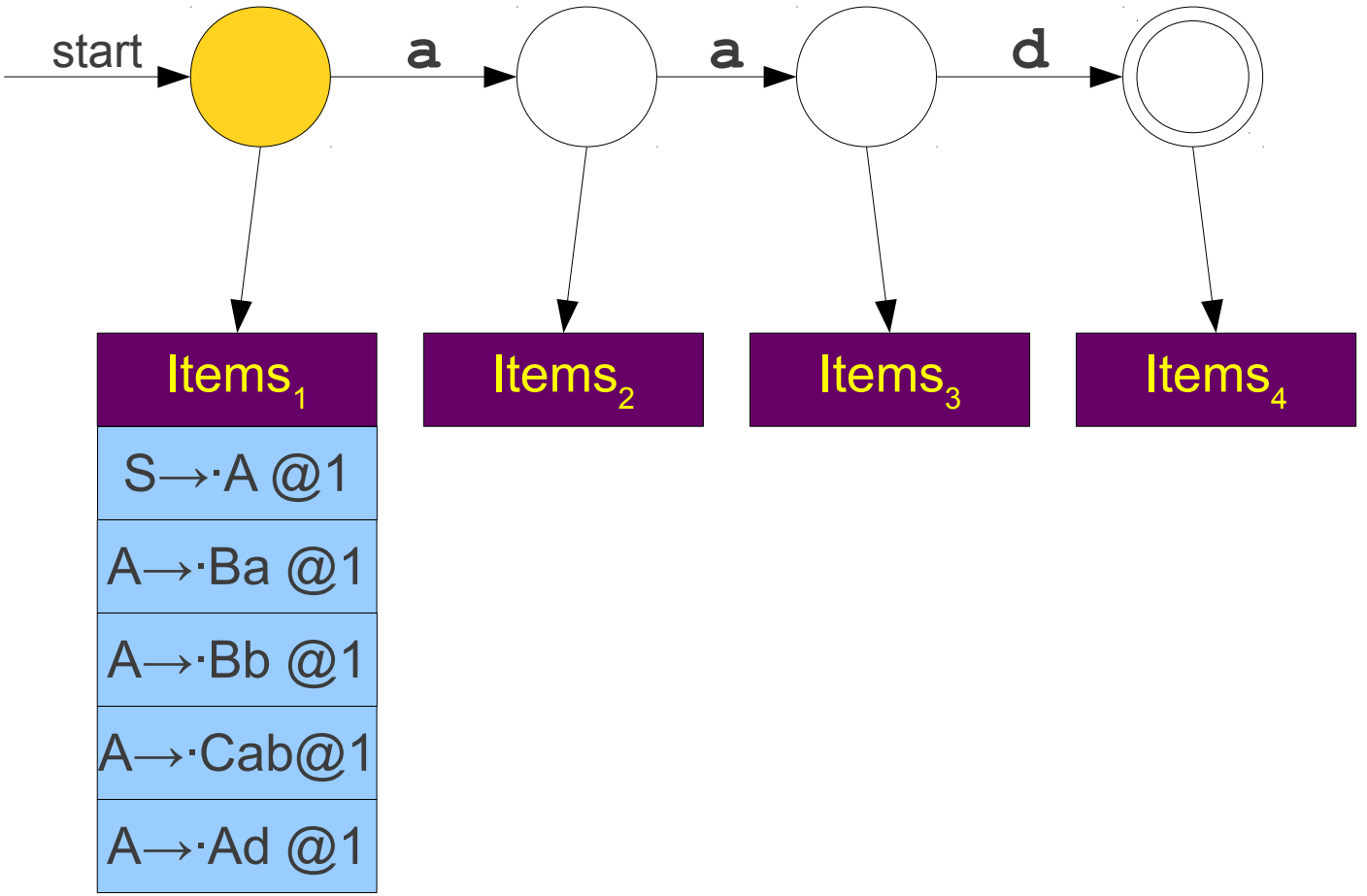
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

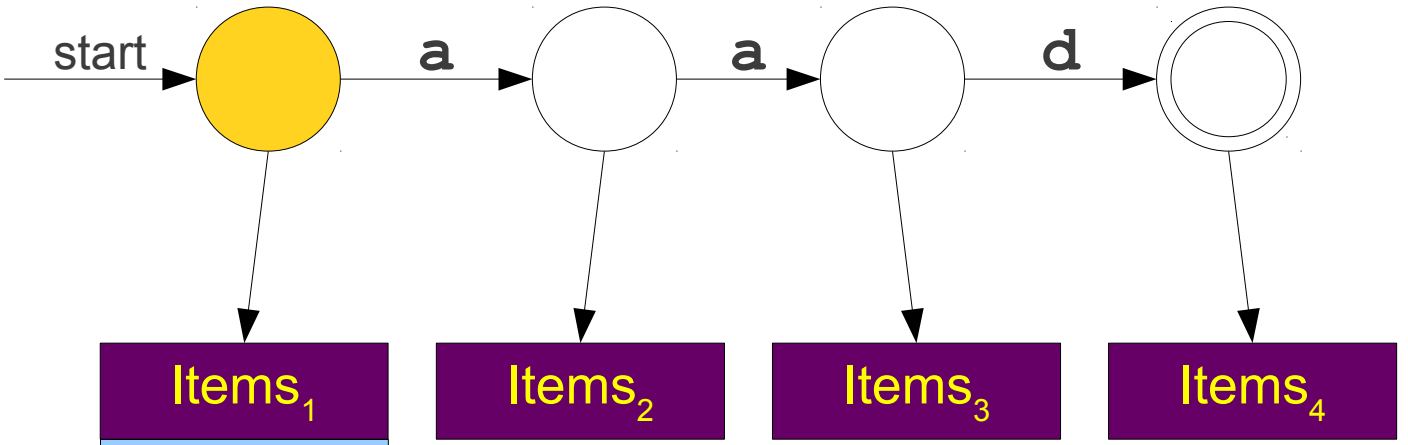
SCAN
COMPLETE
PREDICT



S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

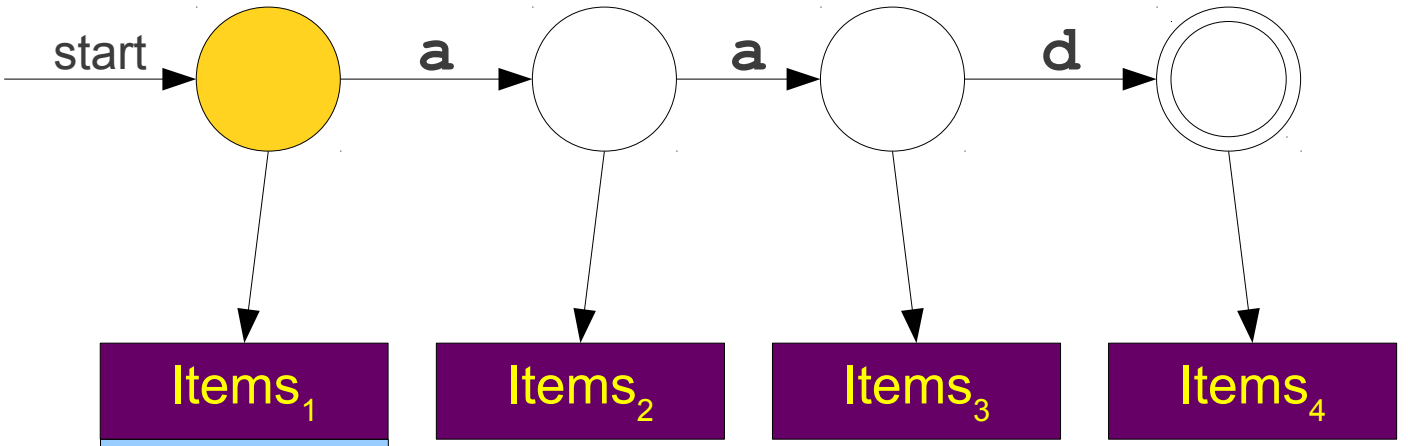


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1			
A→·Ba @1			
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

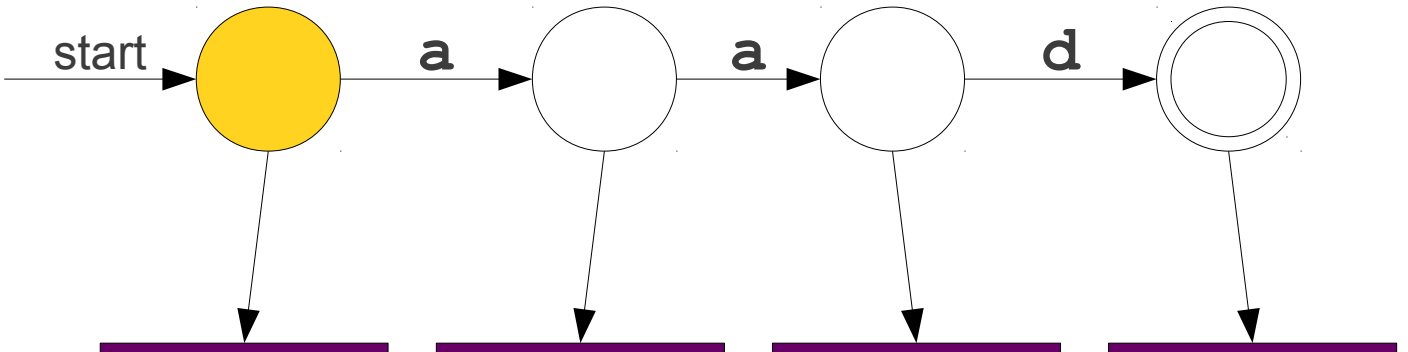


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1			
A→·Ba @1			
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

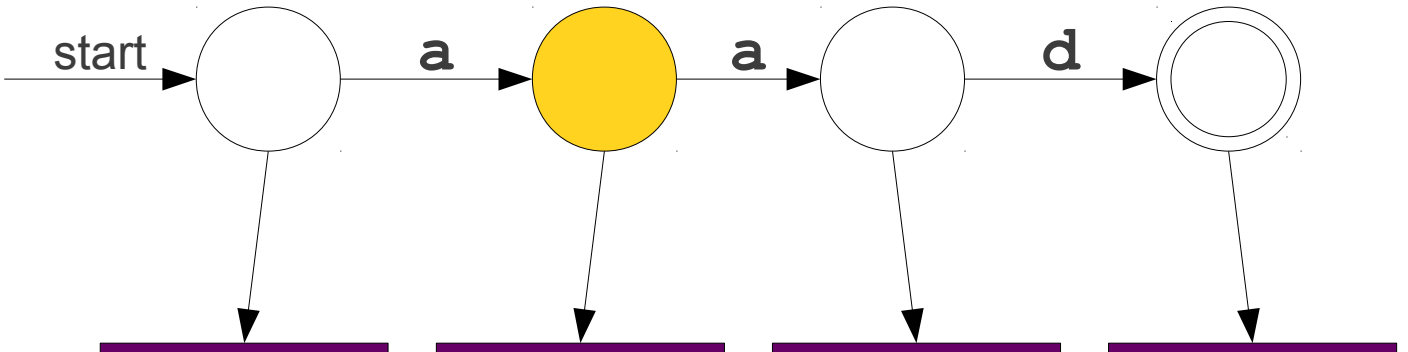


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1			
A→·Ba @1			
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

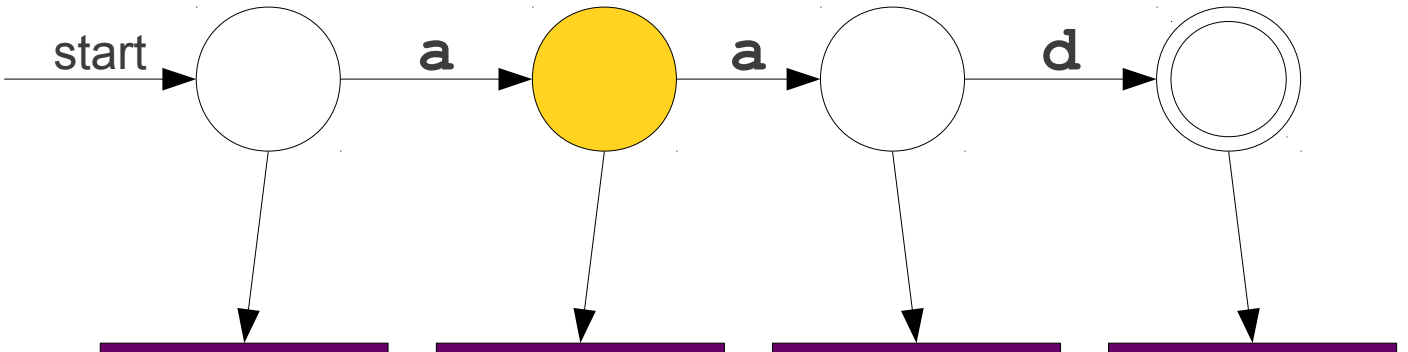


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



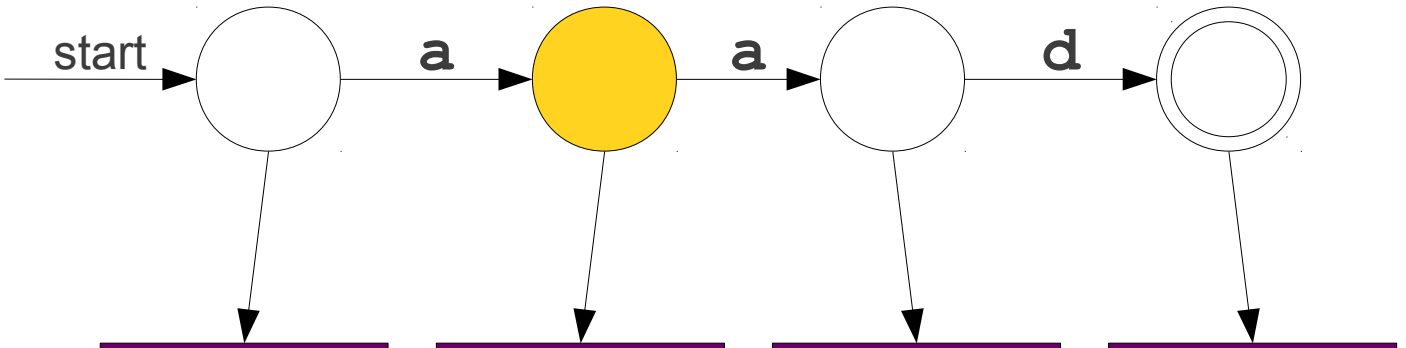
Items$_1$	Items$_2$	Items$_3$	Items$_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$



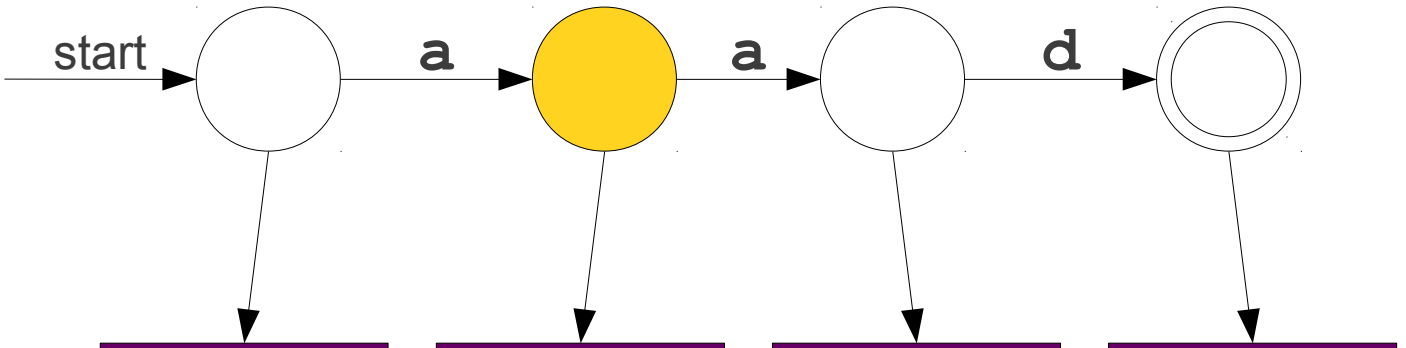
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1			
A→·Cab@1			
A→·Ad @1			
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

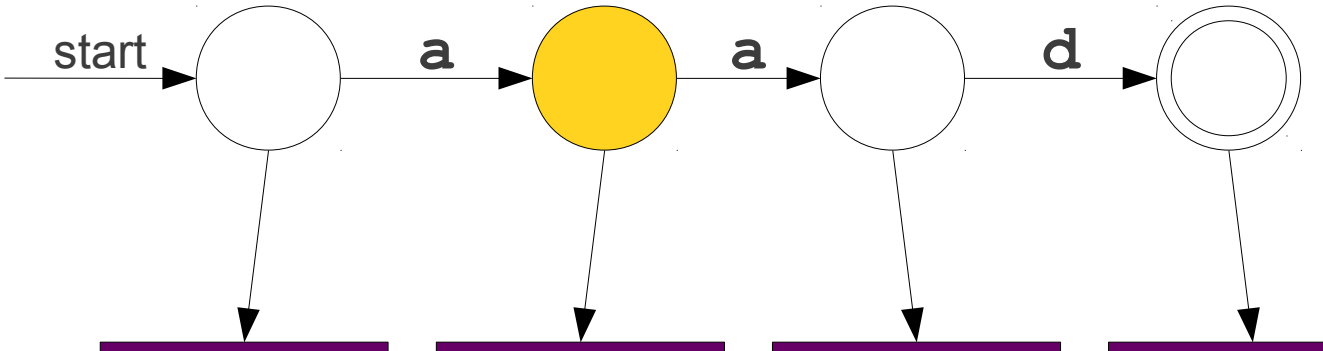


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



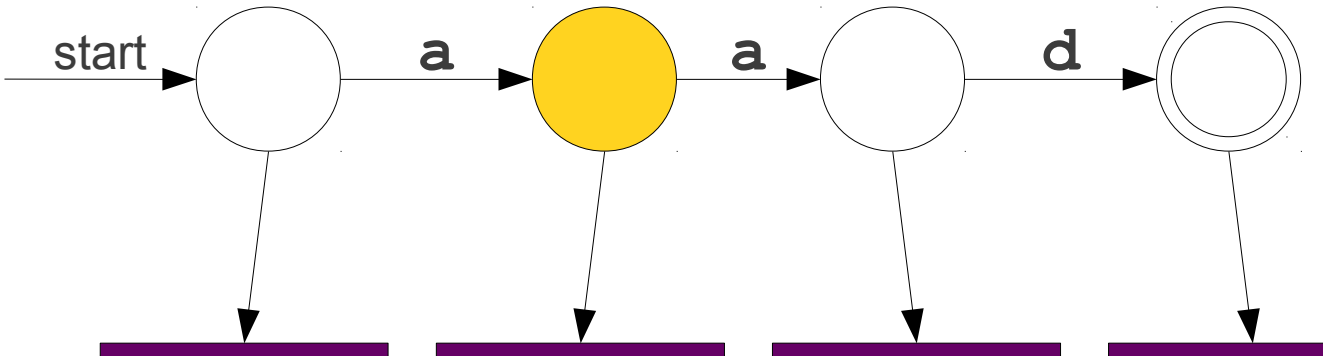
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



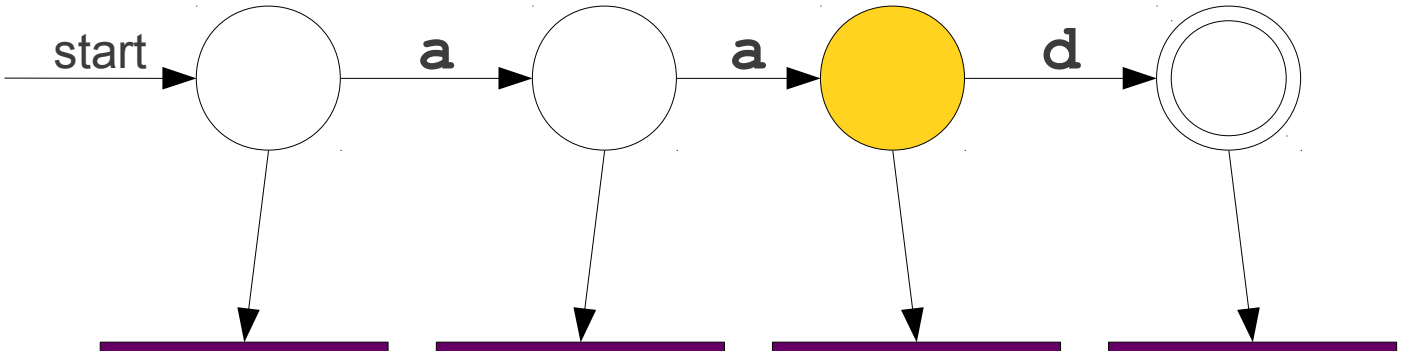
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1		
A→·Ba @1	C→a· @1		
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

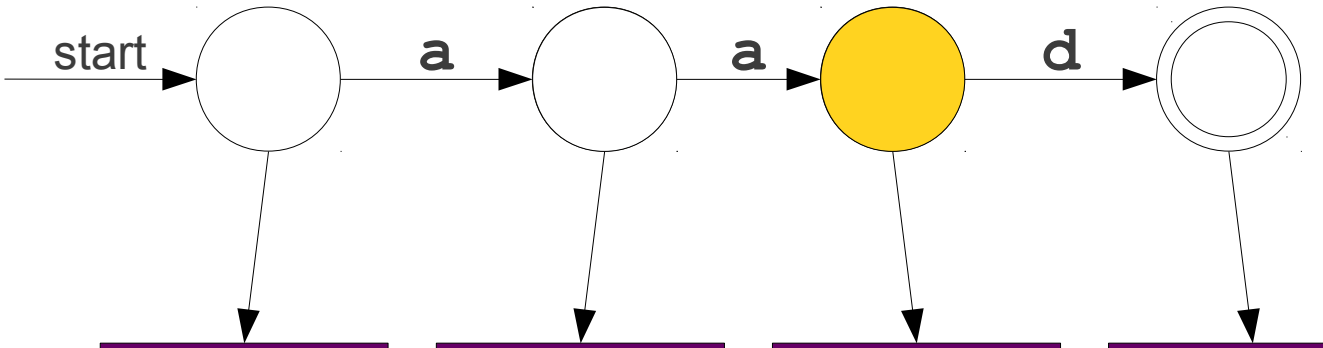
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$



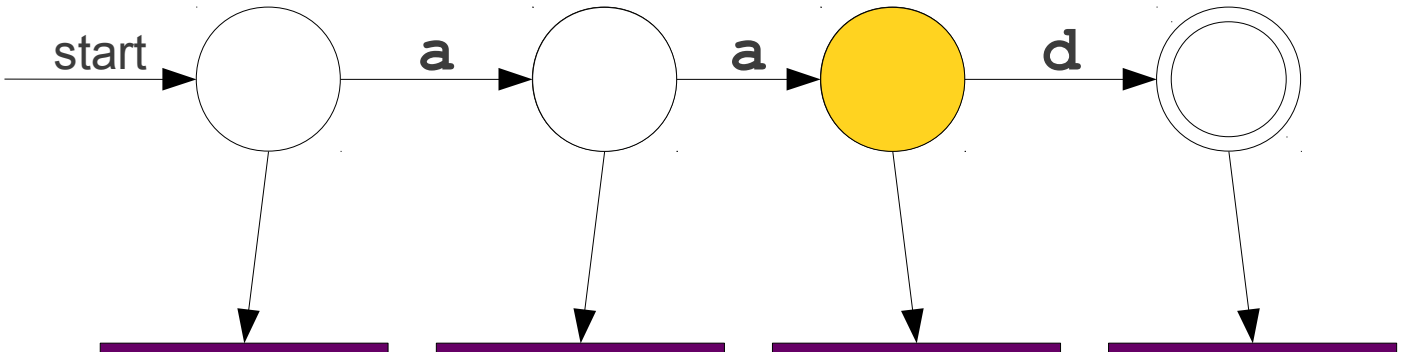
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$



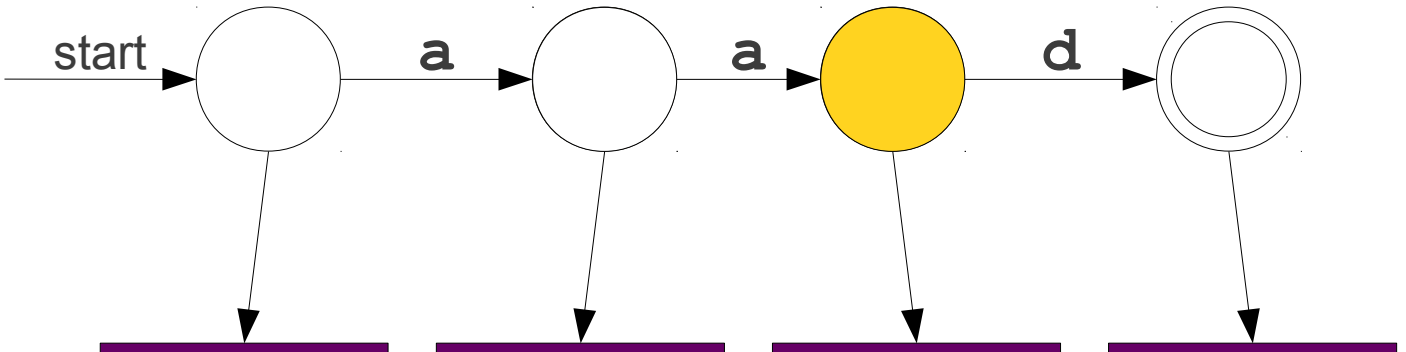
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1		
A→·Cab@1	A→B·b @1		
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$

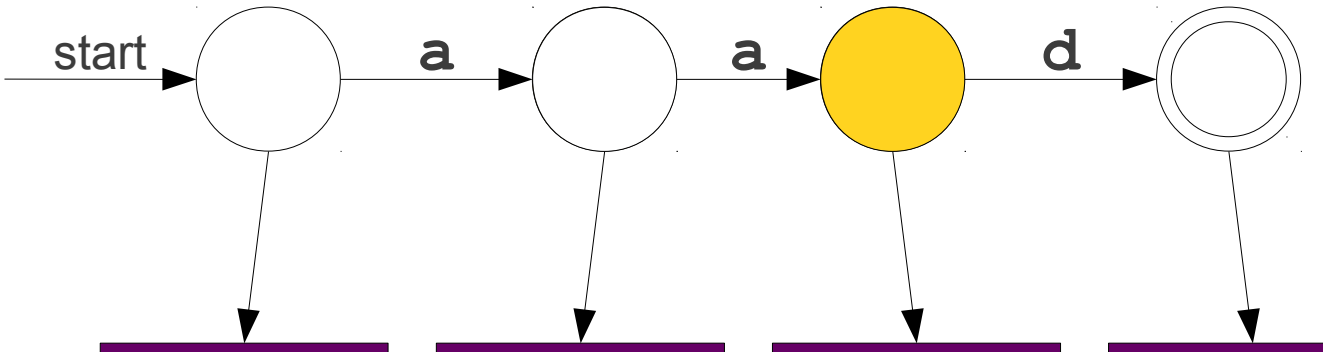


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



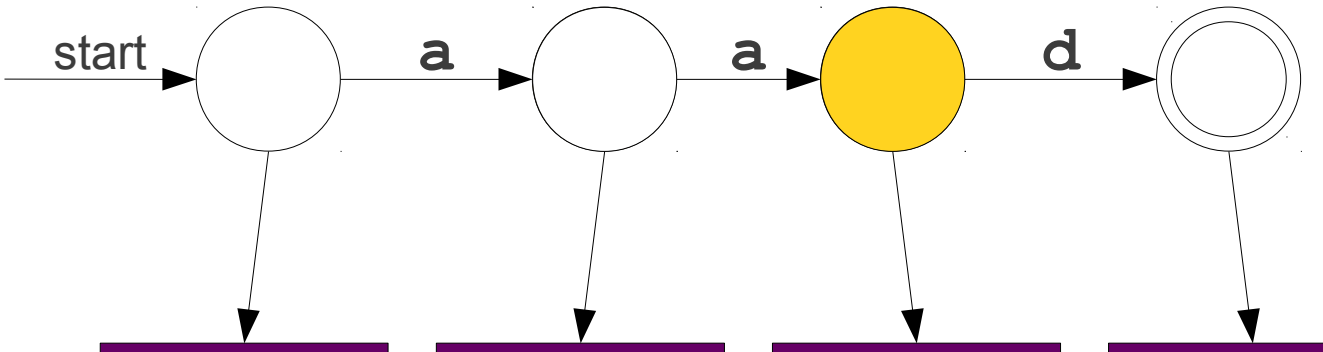
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

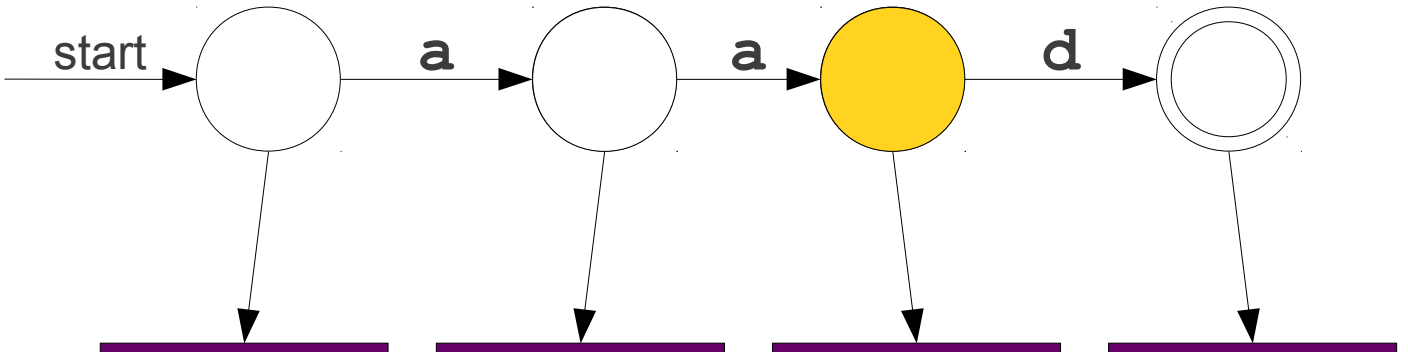
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

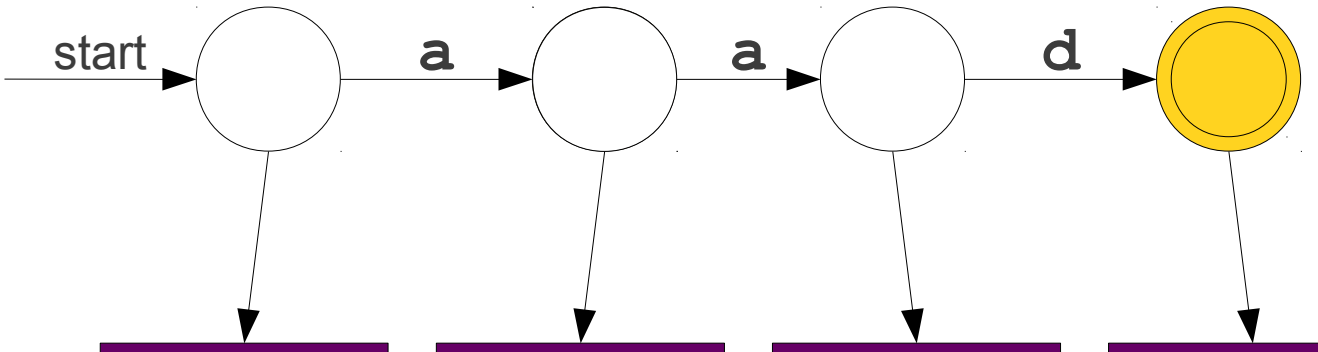
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

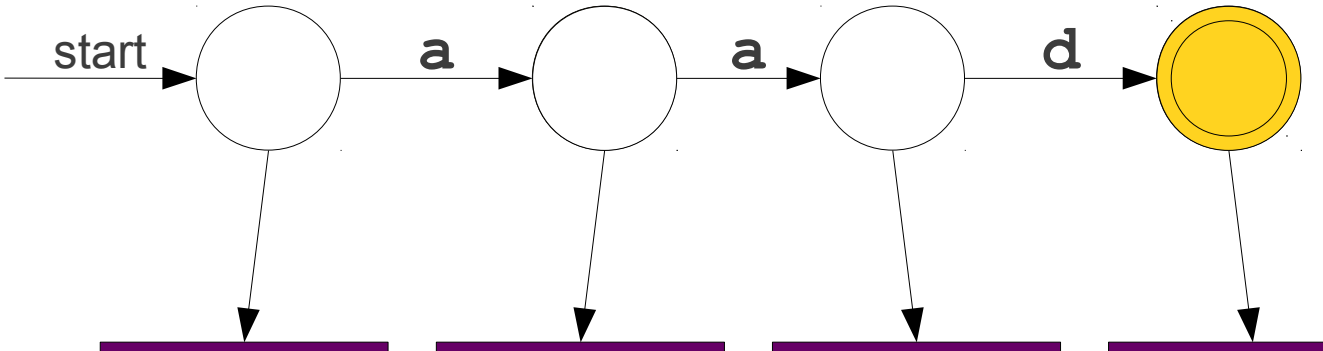
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

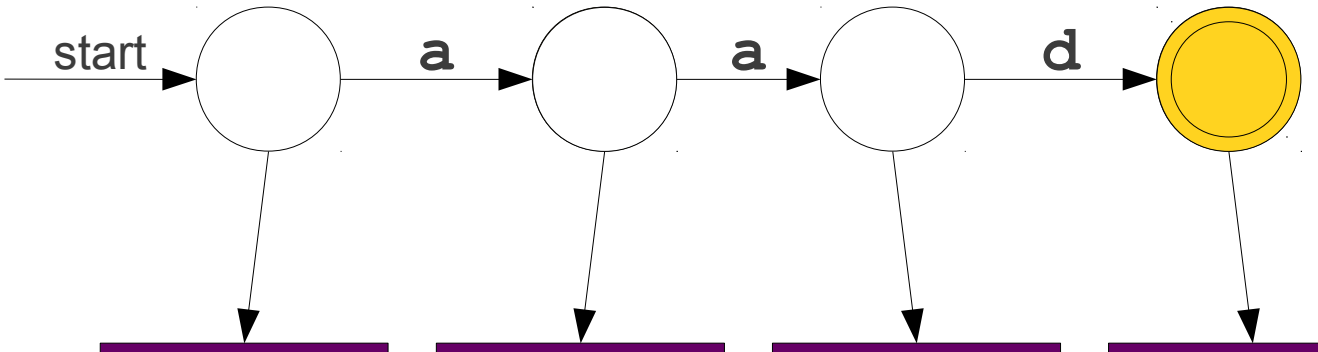
$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	
A→·Bb @1	A→B·a @1	S→A· @1	
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
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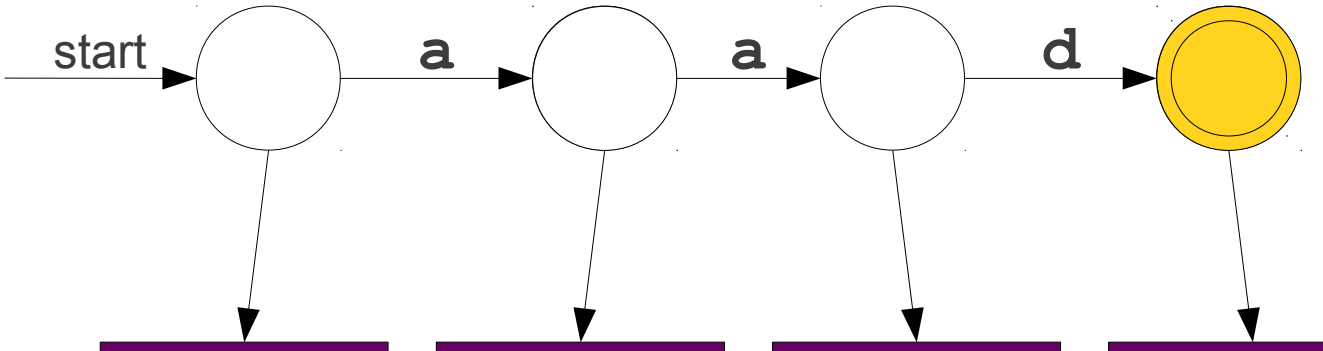
An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

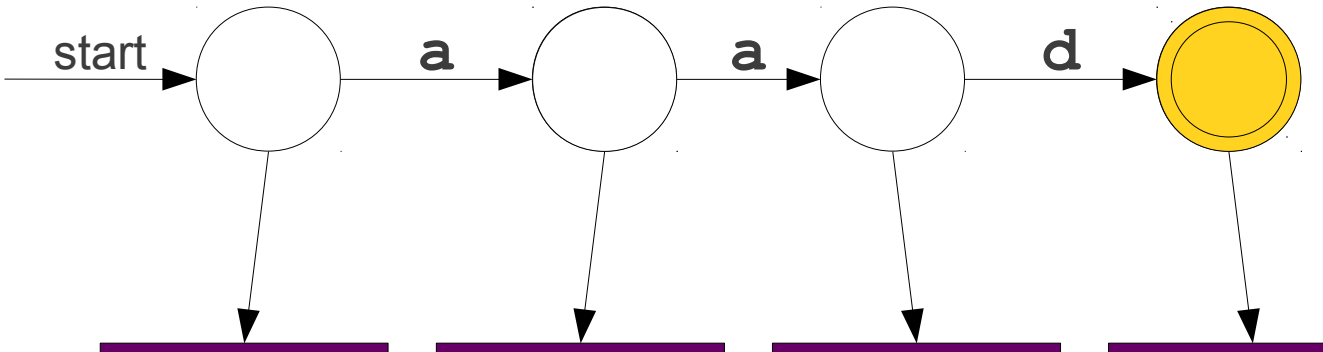
$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$

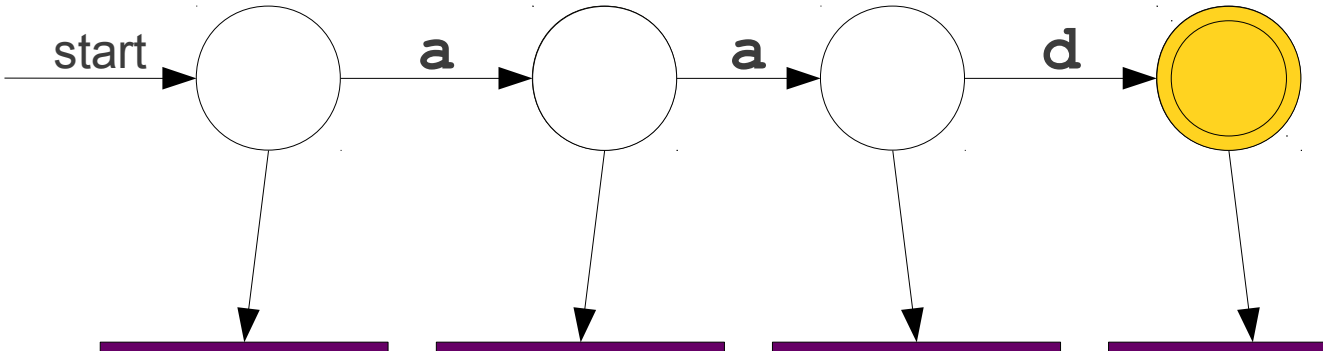


$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

S → A
A → Ba
A → Bb
A → Cab
A → Ad
B → a
C → a

An Observation

SCAN
COMPLETE
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$

$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$

$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$

$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$

$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

S → A
A → Ba
A → Bb
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A → Ad
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C → a

An Observation

SCAN
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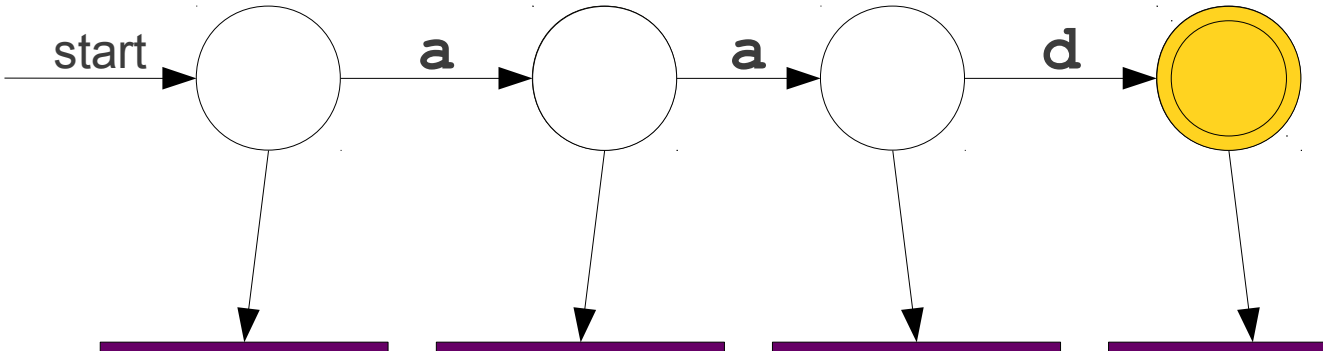
PREDICT



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

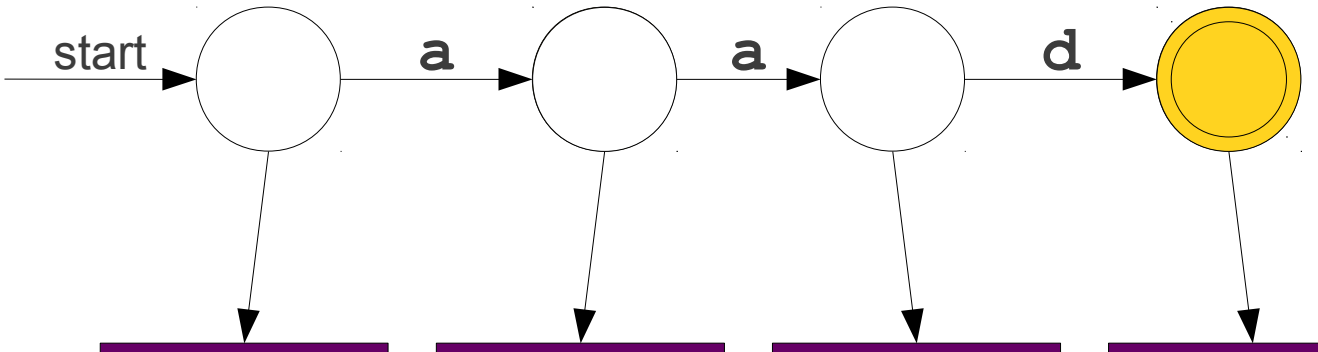
$B_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$C_{1\text{-}2} \rightarrow a_{1\text{-}2}$
$A_{1\text{-}3} \rightarrow B_{1\text{-}2}\ a_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow A_{1\text{-}3}$
$A_{1\text{-}4} \rightarrow A_{1\text{-}3}\ d_{3\text{-}4}$
$S_{1\text{-}4} \rightarrow A_{1\text{-}4}$

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$



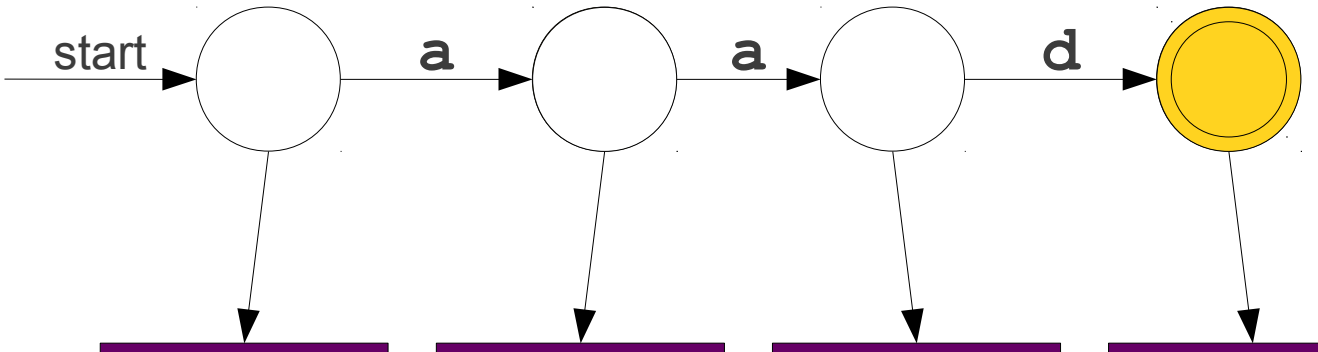
$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

$S \rightarrow A$
$A \rightarrow Ba$
$A \rightarrow Bb$
$A \rightarrow Cab$
$A \rightarrow Ad$
$B \rightarrow a$
$C \rightarrow a$

An Observation

SCAN
COMPLETE
PREDICT

$B_{1-2} \rightarrow a_{1-2}$
$C_{1-2} \rightarrow a_{1-2}$
$A_{1-3} \rightarrow B_{1-2}\ a_{2-3}$
$S_{1-3} \rightarrow A_{1-3}$
$A_{1-4} \rightarrow A_{1-3}\ d_{3-4}$
$S_{1-4} \rightarrow A_{1-4}$
$S \rightarrow S_{1-4}$



$Items_1$	$Items_2$	$Items_3$	$Items_4$
S→·A @1	B→a· @1	A→Ba· @1	A→Ad· @1
A→·Ba @1	C→a· @1	A→Ca·b@1	S→A· @1
A→·Bb @1	A→B·a @1	S→A· @1	A→A·d @1
A→·Cab@1	A→B·b @1	A→A·d @1	
A→·Ad @1	A→C·ab@1		
B→·a @1			
C→·a @1			

What Just Happened?

- The input to the Earley parser is a context-free grammar and a string.
- We can think of that string as a **finite automaton** rather than a string.
- The output of the Earley parser is context-free grammar describing all valid ways of parsing the string.
- We can think of this as a context-free grammar for the subset of the input grammar that is accepted by the automaton.
- What happens when we supply an arbitrary automaton?

Parsing as Intersection

- We can think of parsing as computing the **intersection of a context-free language and a regular language**.
- Result from formal language theory: This intersection is always a context-free language.
- Earley parsing is a *constructive algorithm* for finding this intersection!

Interpreting the Intersection

- As a **parse forest grammar**.
  - The output grammar describes all possible parse trees that would be accepted by the automaton.
- As a **filtered grammar**.
  - The output grammar is the input grammar “filtered” to just those strings matched by the automaton.

Why is this Interesting?

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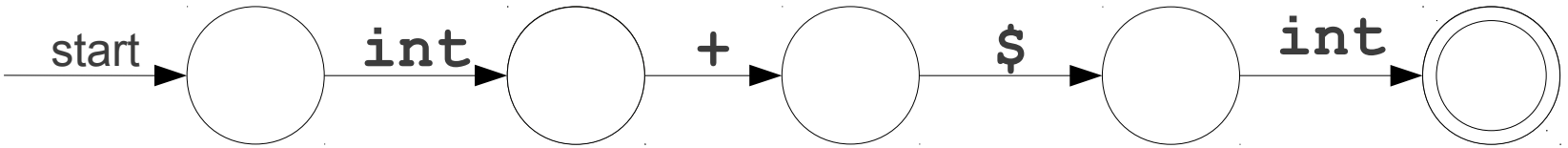

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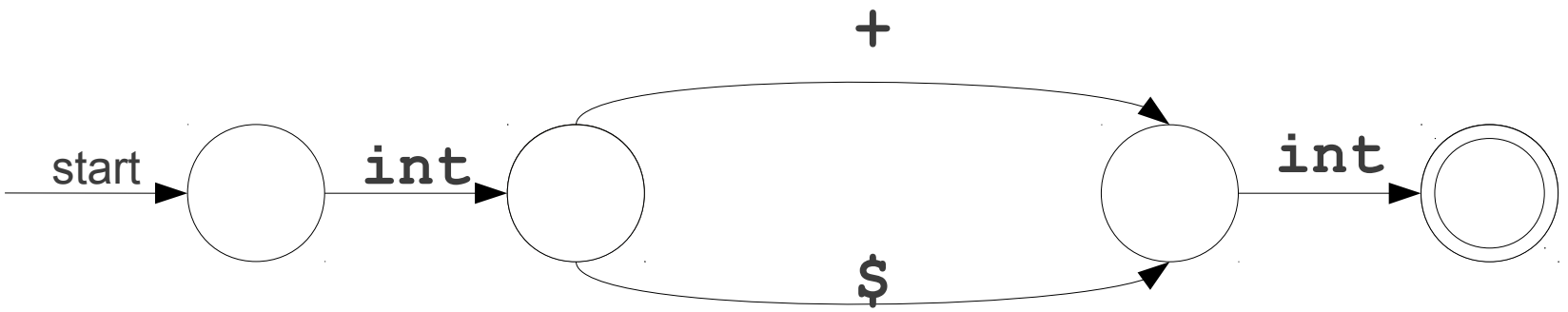

Why is this Interesting?

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```
public class identifier {    .*    }
```

Why is this Interesting?

Why is this Interesting?

Parsing as intersection immediately provides polynomial-time algorithms for tasks that would seem problematic otherwise [...] without imposing restrictions on the CF grammar used in the parsing. Although its basic component is old, it is a relatively new and little-studied subject[.]

- Grune and Jacobs, pg 441

What Needs to Change?

- **Completer** and **predictor** steps are both fine.
- **Scanner** step needs to work across transitions, not from character-to-character.
- More formally:
  - For each item **A** → ***α*·t*ω*** **@n** in the *k*th item set, if there is a transition on t from state *k* to state *j*, add **A** → ***α*t·*ω*** **@n** to the *j*th item set.
- Additionally, accept if *any accepting state* has the item **S** → **E**· **@1**.
- Cannot scan from left-to-right; must consider all sets on each iteration.

A Simple Question

- Given the (ambiguous) grammar

  S → **E**

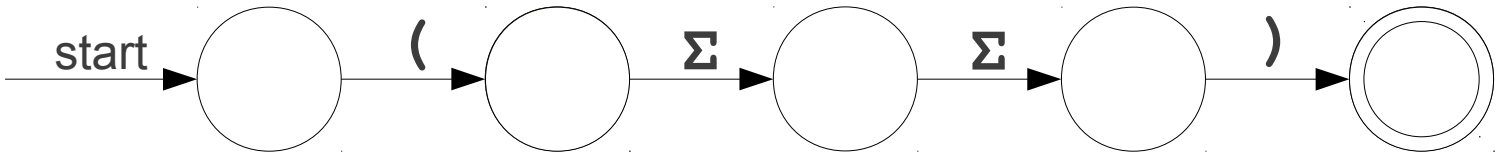
  E → **E** `+` **E**

  E → `int`

  E → `(` **E** `)`

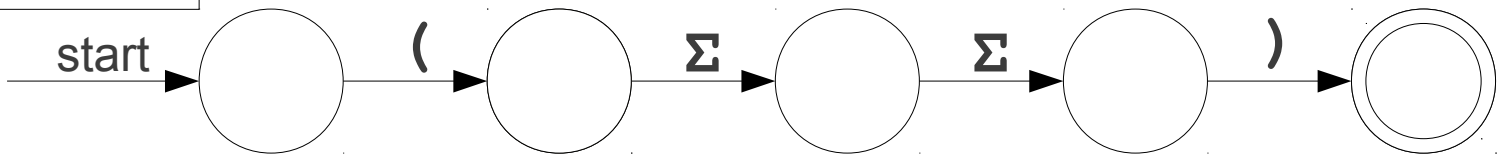
- Determine whether there is some string generated by the grammar that has the form `(` **??** `)`, where **?** represents any symbol in the grammar.

Our Automaton



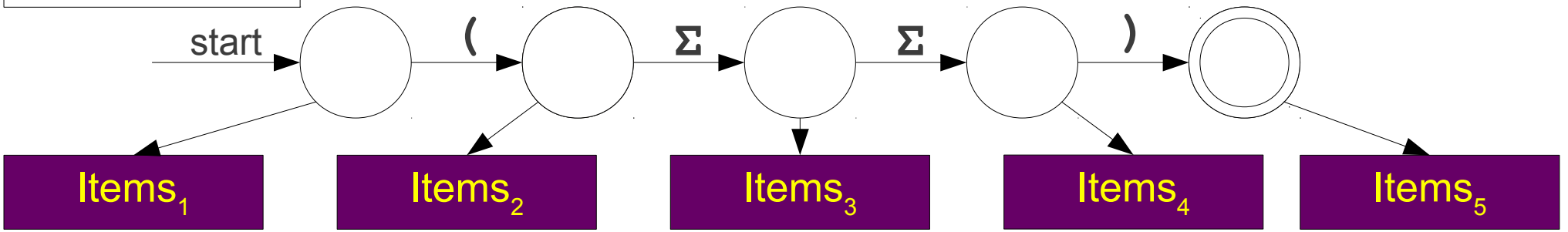
S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs



S → E
E → E + E
E → **int**
E → (E)

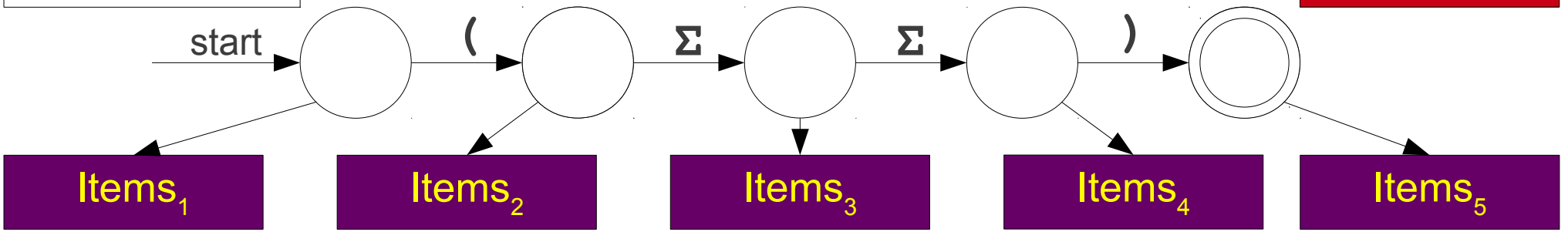
Earley on DFAs



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

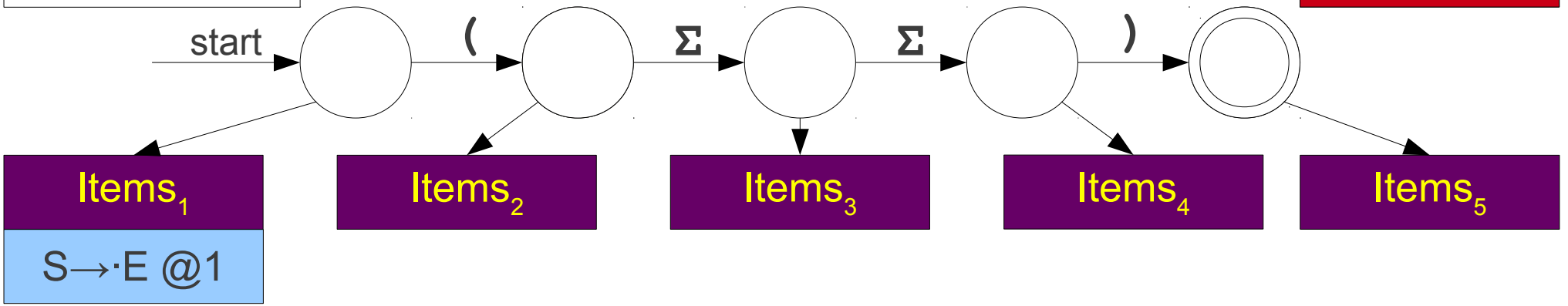
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

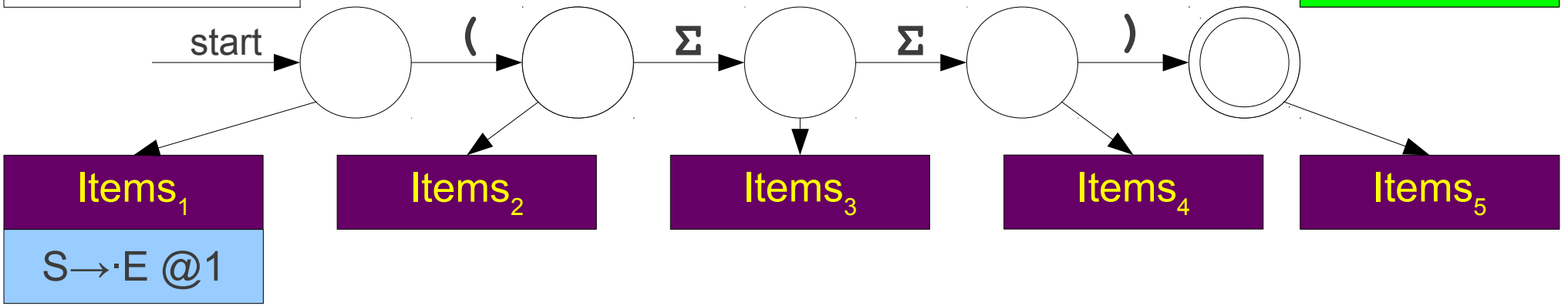
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

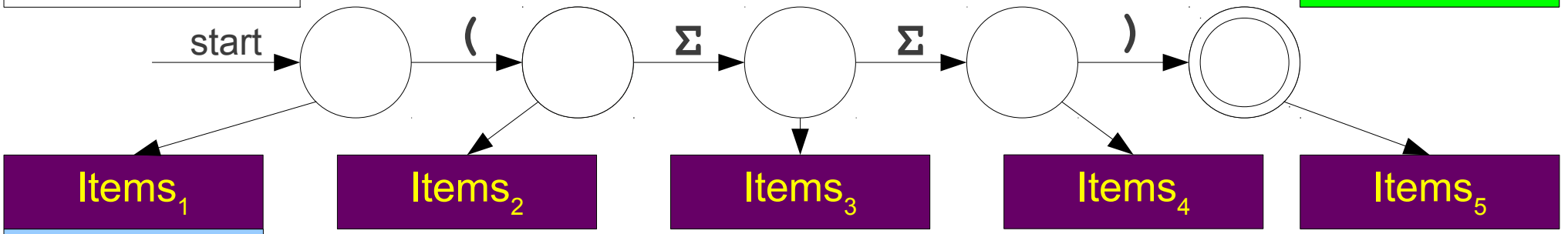
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT

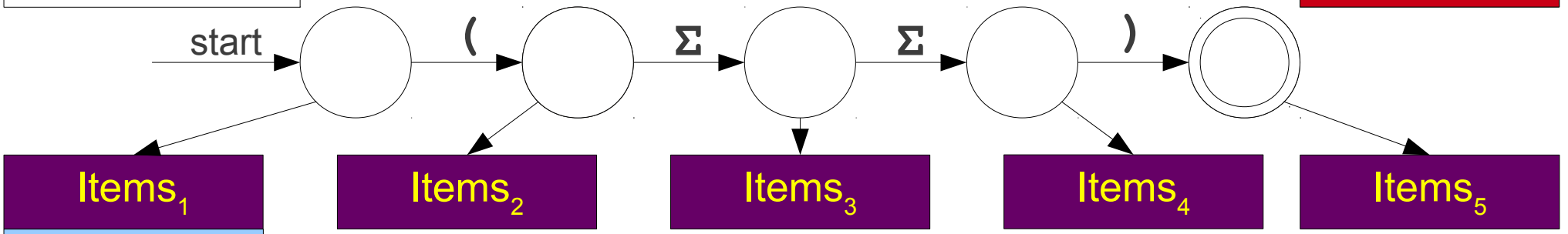


Items$_1$	Items$_2$	Items$_3$	Items$_4$	Items$_5$
S→·E @1				
E→·E+E @1				
E→·(E) @1				
E→·**int** @1				

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT

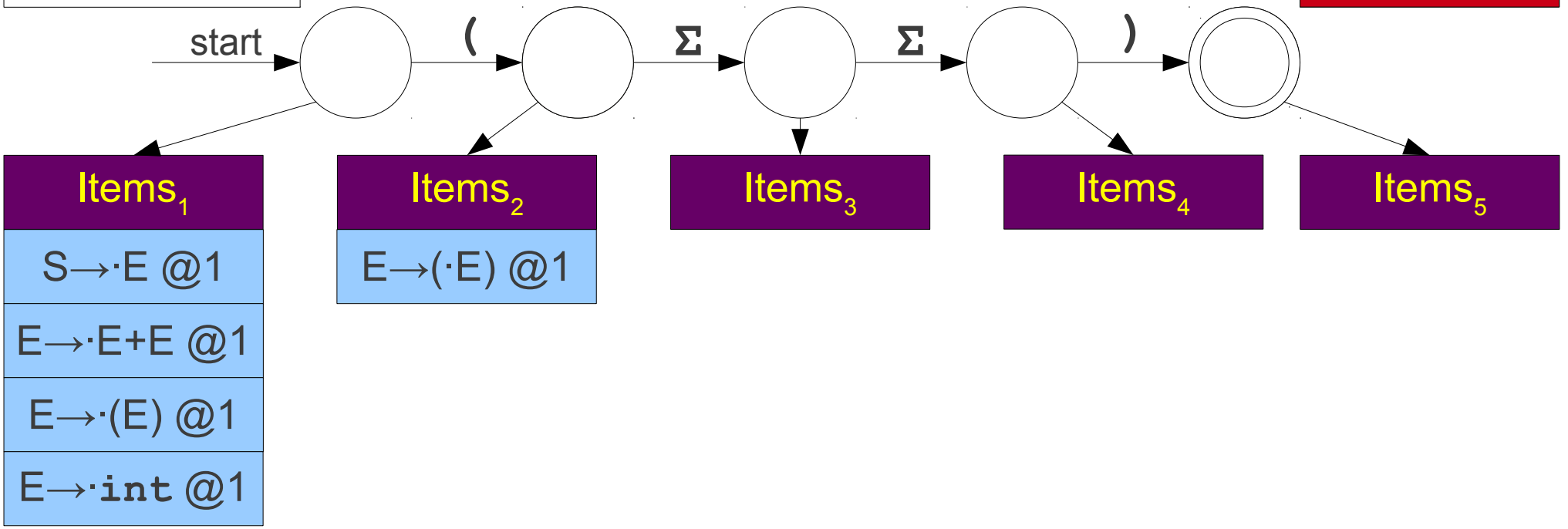


Items$_1$	Items$_2$	Items$_3$	Items$_4$	Items$_5$
S→·E @1				
E→·E+E @1				
E→·(E) @1				
E→·**int** @1				

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

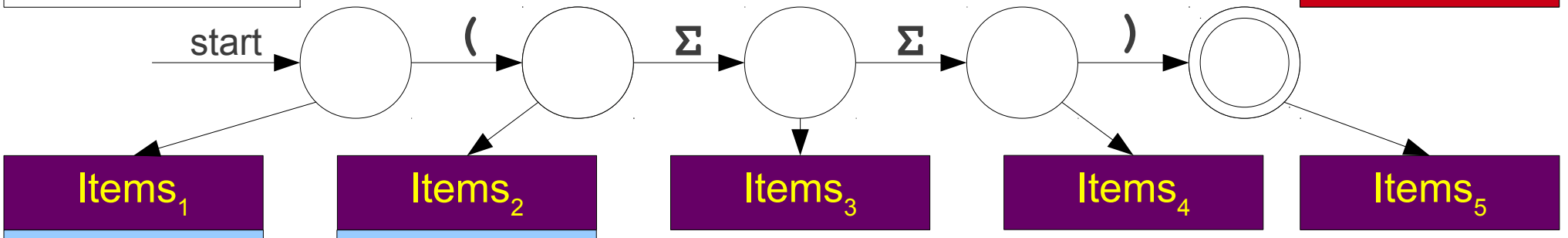
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT

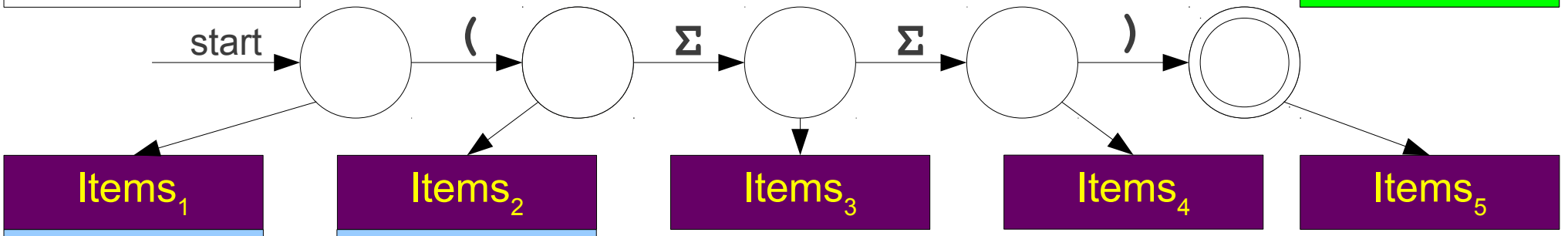


$Items_1$	$Items_2$	$Items_3$	$Items_4$	$Items_5$
S→·E @1	E→(·E) @1			
E→·E+E @1				
E→·(E) @1				
E→·**int** @1				

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT

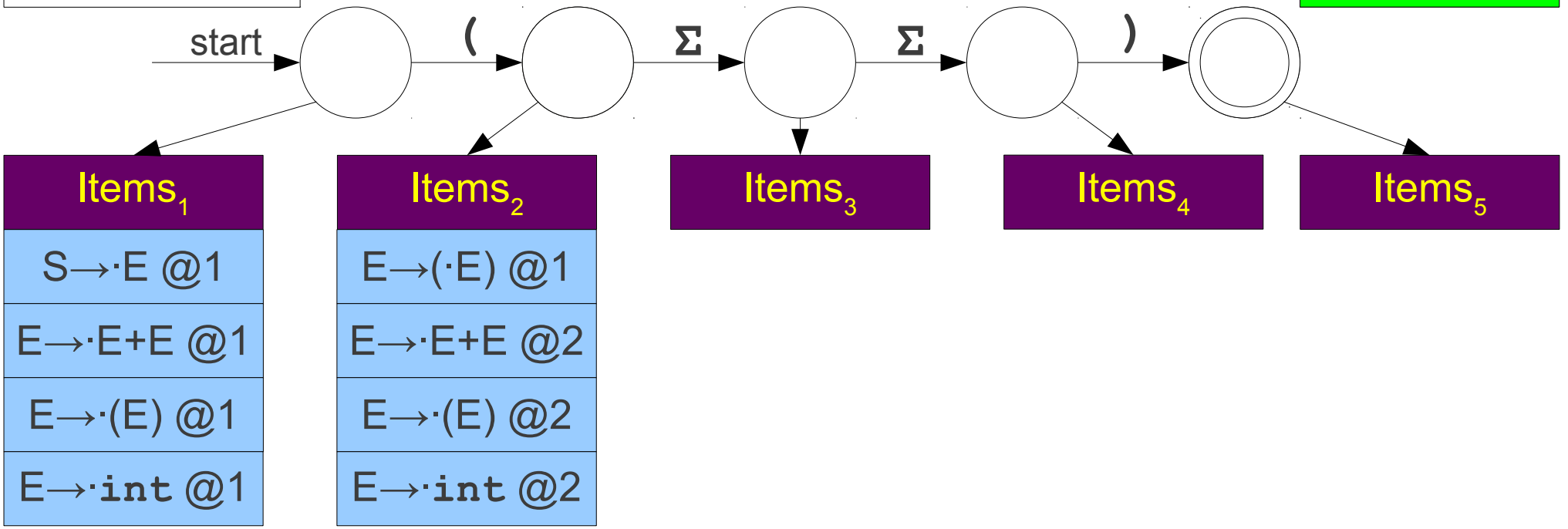


Items$_1$	Items$_2$	Items$_3$	Items$_4$	Items$_5$
S→·E @1	E→(·E) @1			
E→·E+E @1				
E→·(E) @1				
E→·**int** @1				

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

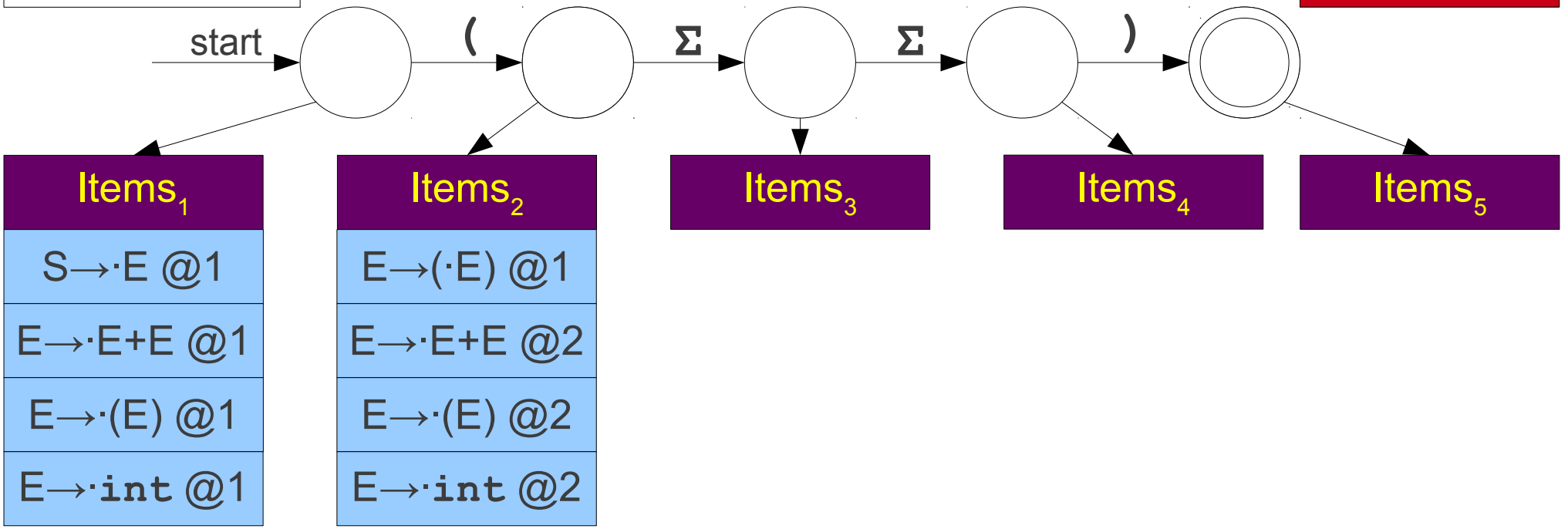
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

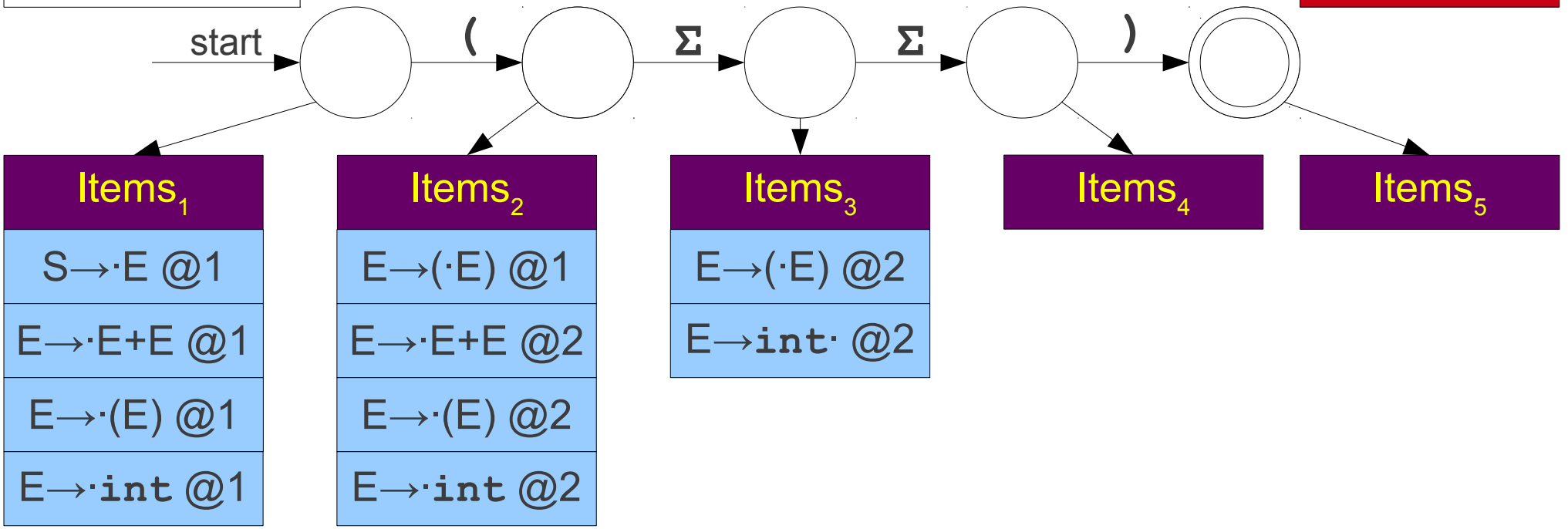
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

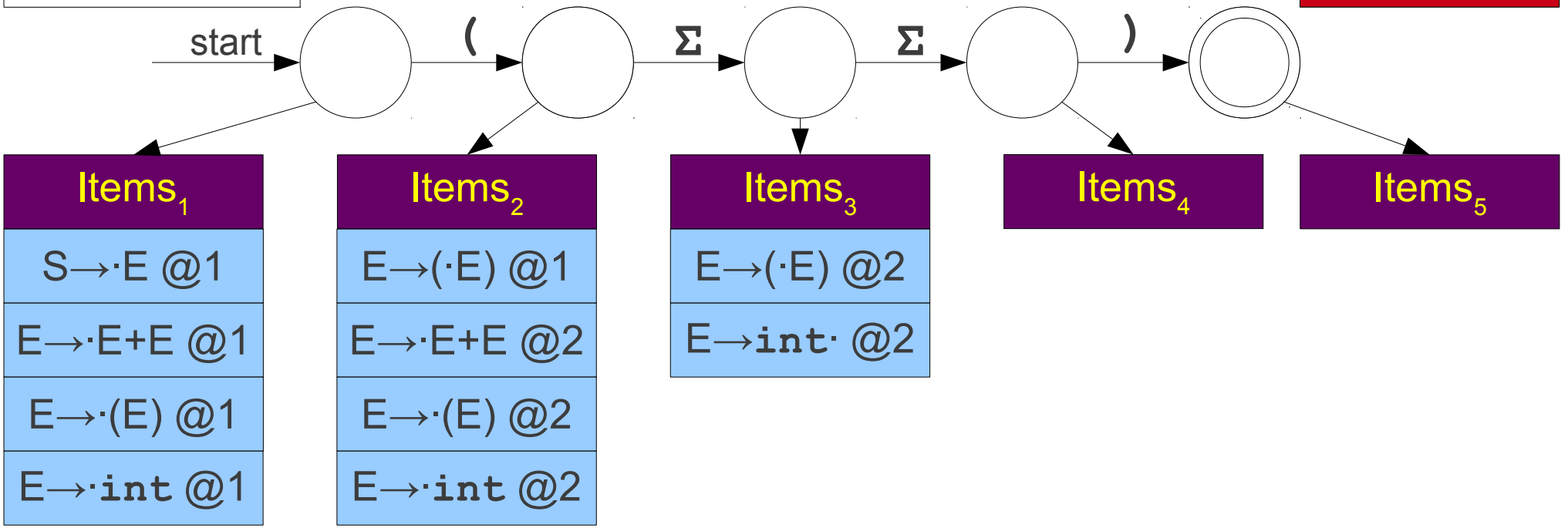
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

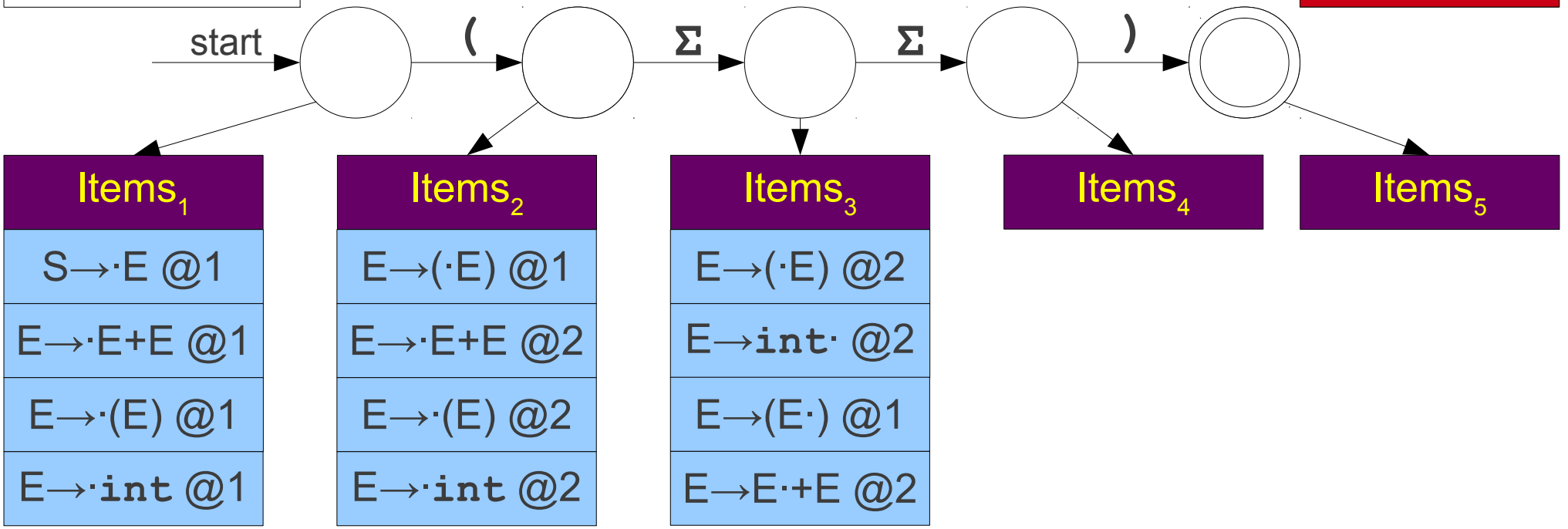
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

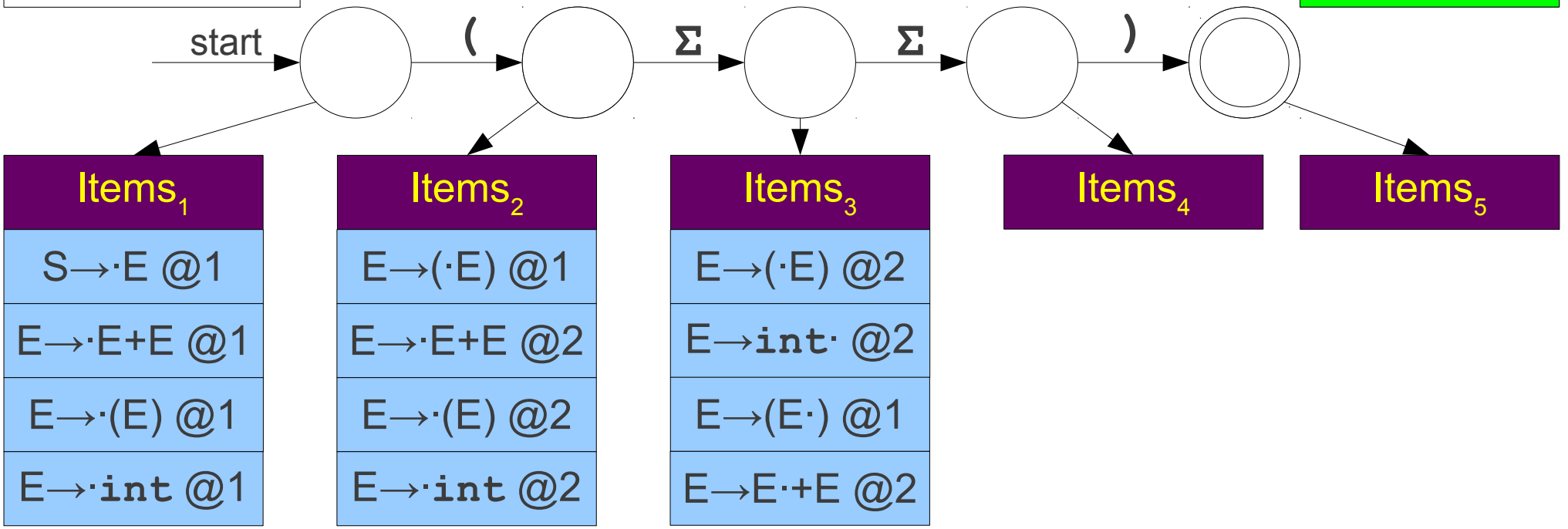
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

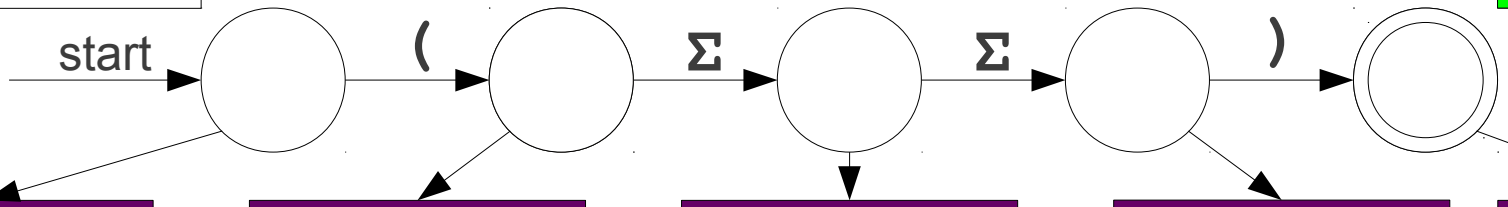
SCAN
COMPLETE
PREDICT



S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

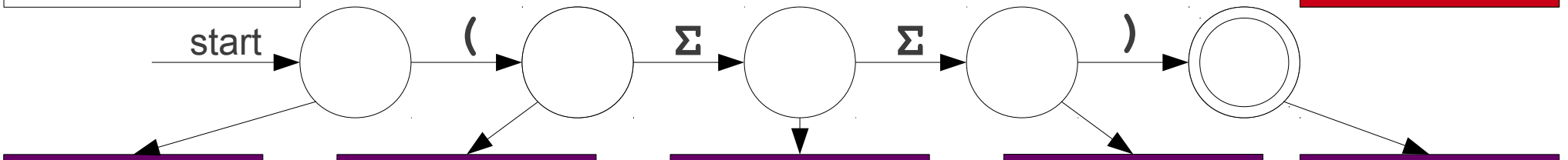
Items$_4$

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

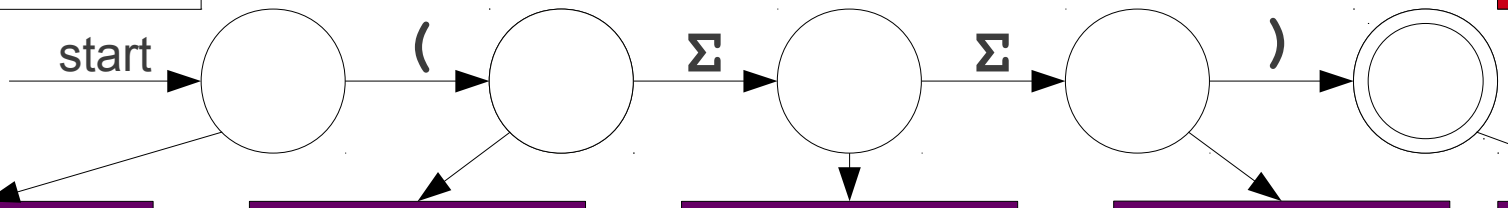
Items$_4$

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

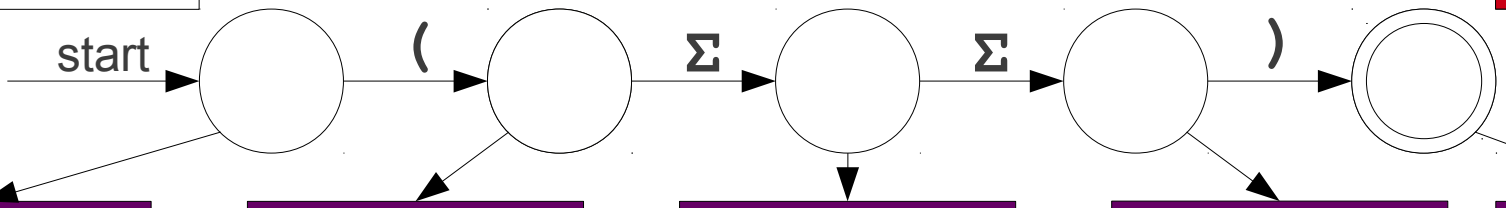
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

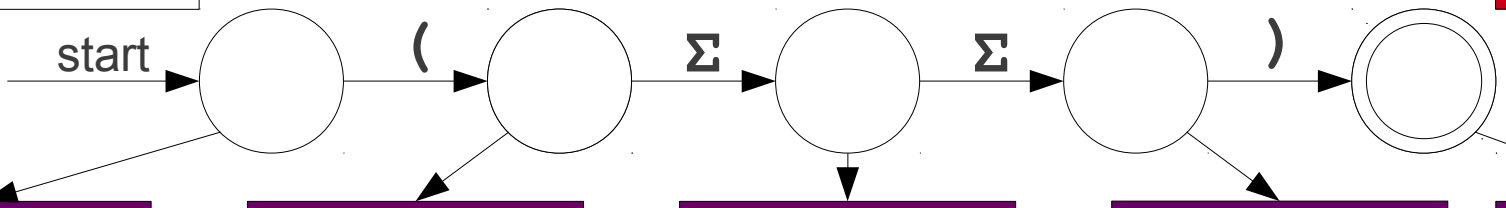
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

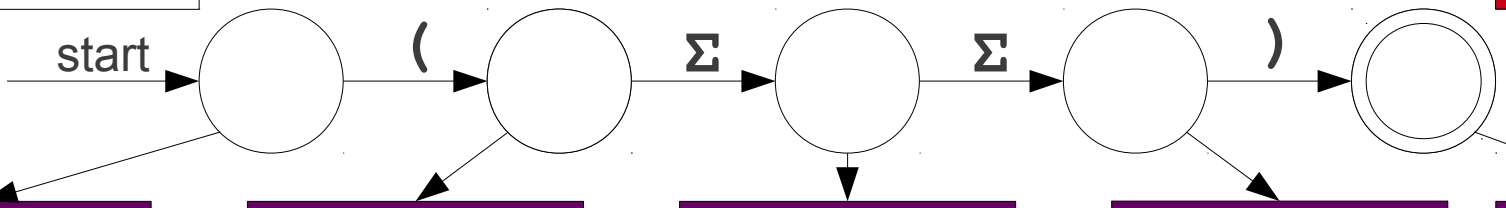
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

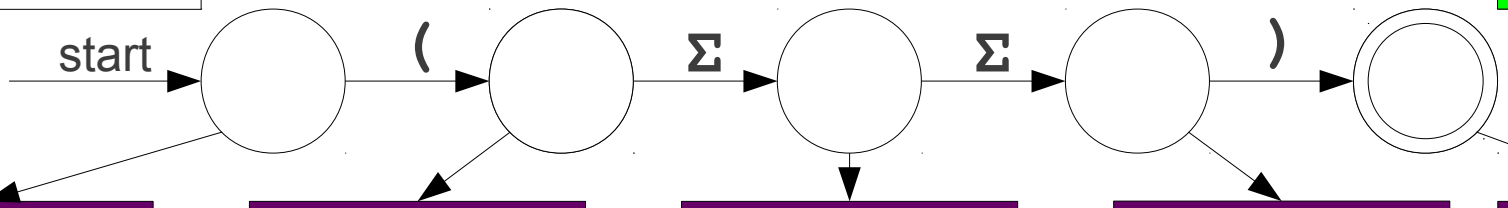
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

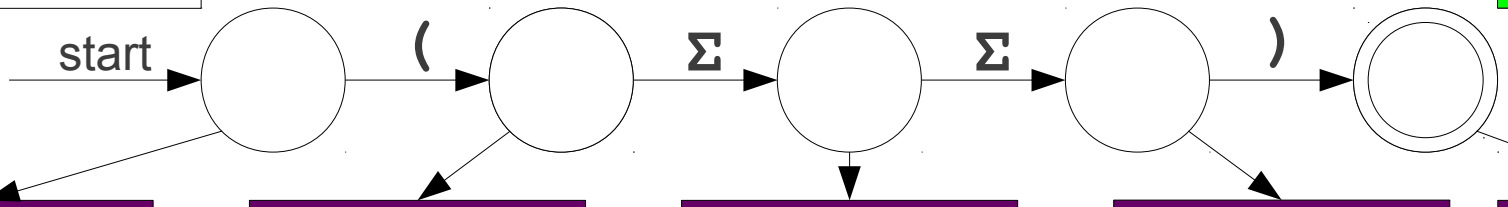
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3

Items$_5$

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

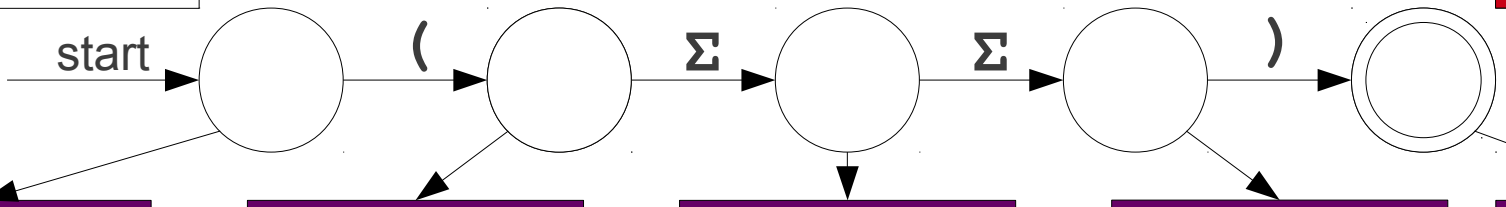
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3
E→·E+E @4
E→·(E) @4
E→·**int** @4

Items$_5$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items_1
$S \rightarrow \cdot E$ @1
$E \rightarrow \cdot E+E$ @1
$E \rightarrow \cdot (E)$ @1
$E \rightarrow \cdot \texttt{int}$ @1

Items_2
$E \rightarrow (\cdot E)$ @1
$E \rightarrow \cdot E+E$ @2
$E \rightarrow \cdot (E)$ @2
$E \rightarrow \cdot \texttt{int}$ @2

Items_3
$E \rightarrow (\cdot E)$ @2
$E \rightarrow \texttt{int}\cdot$ @2
$E \rightarrow (E\cdot)$ @1
$E \rightarrow E\cdot+E$ @2
$E \rightarrow \cdot E+E$ @3
$E \rightarrow \cdot (E)$ @3
$E \rightarrow \cdot \texttt{int}$ @3

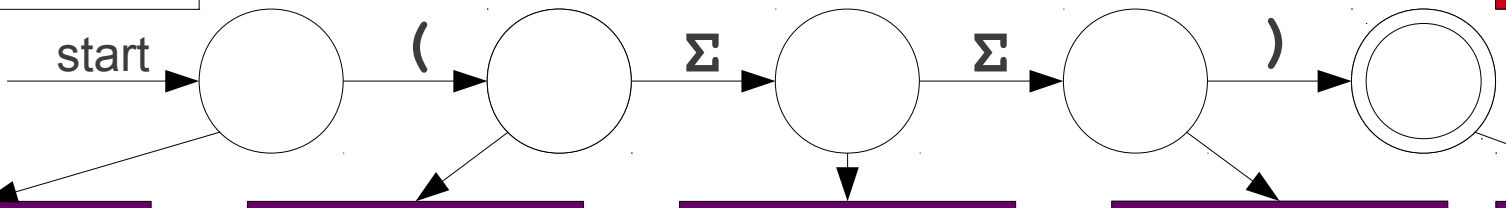
Items_4
$E \rightarrow (E)\cdot$ @1
$E \rightarrow (\cdot E)$ @3
$E \rightarrow \texttt{int}\cdot$ @3
$E \rightarrow E+\cdot E$ @2
$S \rightarrow E\cdot$ @1
$E \rightarrow E\cdot+E$ @1
$E \rightarrow (E\cdot)$ @2
$E \rightarrow E\cdot+E$ @3
$E \rightarrow \cdot E+E$ @4
$E \rightarrow \cdot (E)$ @4
$E \rightarrow \cdot \texttt{int}$ @4

Items_5

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

Earley on DFAs

SCAN
COMPLETE
PREDICT



$Items_1$
$S \rightarrow \cdot E$ @1
$E \rightarrow \cdot E+E$ @1
$E \rightarrow \cdot (E)$ @1
$E \rightarrow \cdot \texttt{int}$ @1

$Items_2$
$E \rightarrow (\cdot E)$ @1
$E \rightarrow \cdot E+E$ @2
$E \rightarrow \cdot (E)$ @2
$E \rightarrow \cdot \texttt{int}$ @2

$Items_3$
$E \rightarrow (\cdot E)$ @2
$E \rightarrow \texttt{int}\cdot$ @2
$E \rightarrow (E\cdot)$ @1
$E \rightarrow E\cdot+E$ @2
$E \rightarrow \cdot E+E$ @3
$E \rightarrow \cdot (E)$ @3
$E \rightarrow \cdot \texttt{int}$ @3

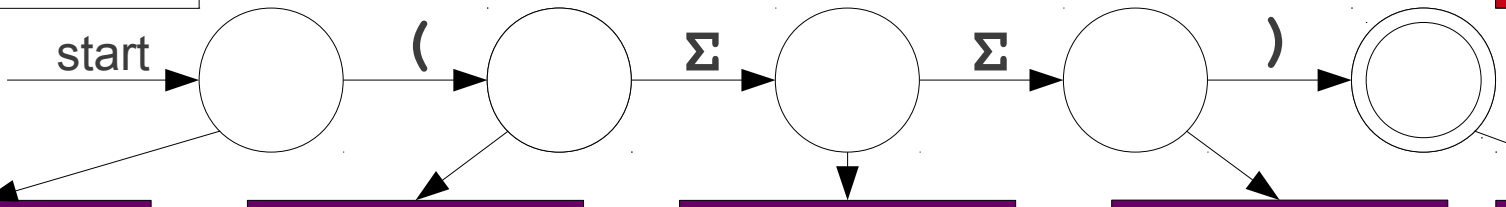
$Items_4$
$E \rightarrow (E)\cdot$ @1
$E \rightarrow (\cdot E)$ @3
$E \rightarrow \texttt{int}\cdot$ @3
$E \rightarrow E+\cdot E$ @2
$S \rightarrow E\cdot$ @1
$E \rightarrow E\cdot+E$ @1
$E \rightarrow (E\cdot)$ @2
$E \rightarrow E\cdot+E$ @3
$E \rightarrow \cdot E+E$ @4
$E \rightarrow \cdot (E)$ @4
$E \rightarrow \cdot \texttt{int}$ @4

$Items_5$
$E \rightarrow (E)\cdot$ @2

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

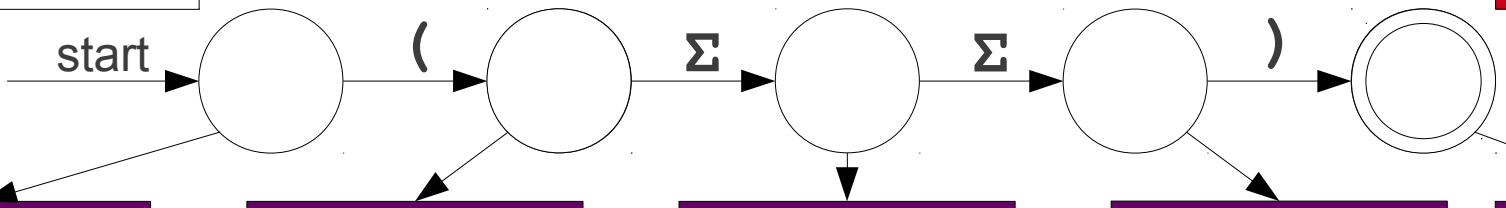
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3
E→·E+E @4
E→·(E) @4
E→·**int** @4

Items$_5$
E→(E)· @2

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

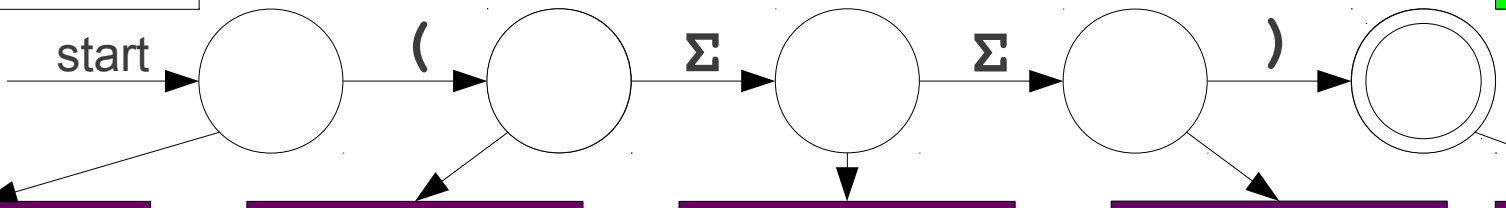
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3
E→·E+E @4
E→·(E) @4
E→·**int** @4

Items$_5$
E→(E)· @2
E→(E·) @1
E→E·+E @2

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

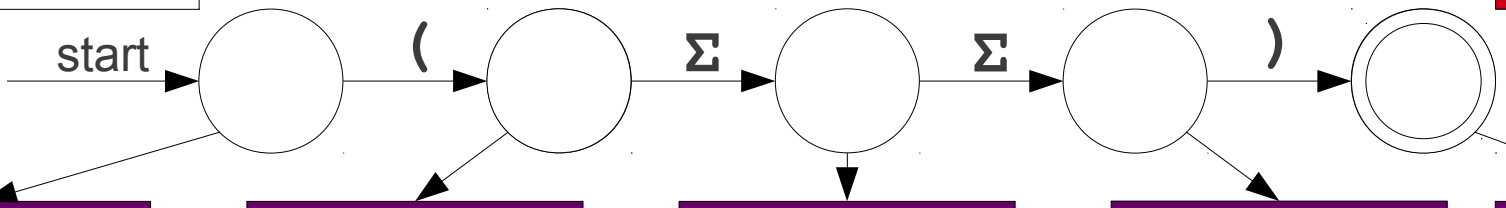
Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3
E→·E+E @4
E→·(E) @4
E→·**int** @4

Items$_5$
E→(E)· @2
E→(E·) @1
E→E·+E @2

S → E
E → E + E
E → **int**
E → (E)

Earley on DFAs

SCAN
COMPLETE
PREDICT



Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

Items$_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

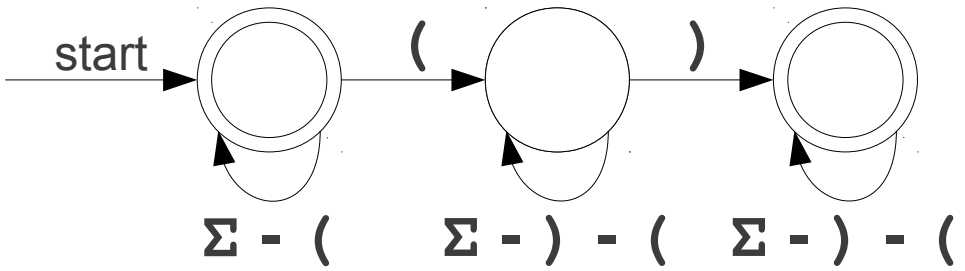
Items$_3$
E→(·E) @2
E→**int**· @2
E→(E·) @1
E→E·+E @2
E→·E+E @3
E→·(E) @3
E→·**int** @3

Items$_4$
E→(E)· @1
E→(·E) @3
E→**int**· @3
E→E+·E @2
S→E· @1
E→E·+E @1
E→(E·) @2
E→E·+E @3
E→·E+E @4
E→·(E) @4
E→·**int** @4

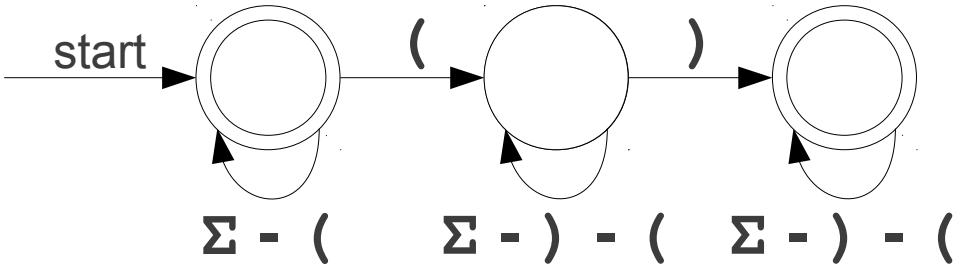
Items$_5$
E→(E)· @2
E→(E·) @1
E→E·+E @2

Parsing DFAs

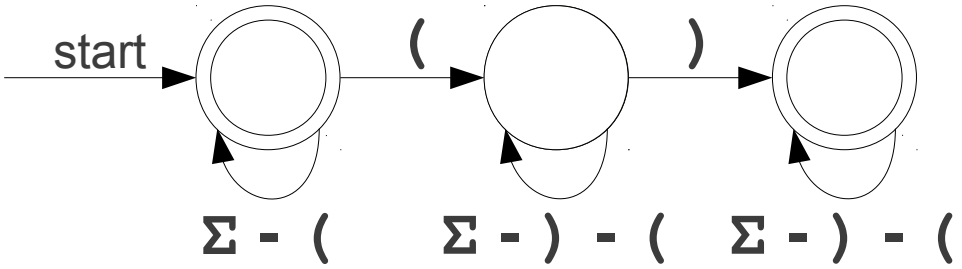
- Similar logic to parsing strings: build a parse forest grammar.
- Intuitively, filters grammar through DFA to produce a new grammar.
- Example: Filter grammar for expressions to only allow one pair of parentheses:



S → E
E → E + E
E → **int**
E → (E)

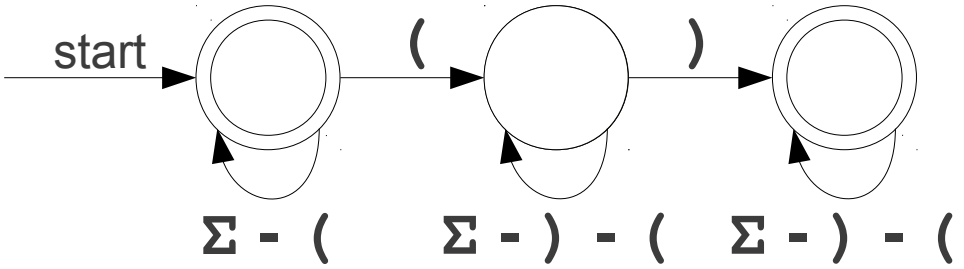


S → E
E → E + E
E → **int**
E → (E)



Items$_1$ Items$_2$ Items$_3$

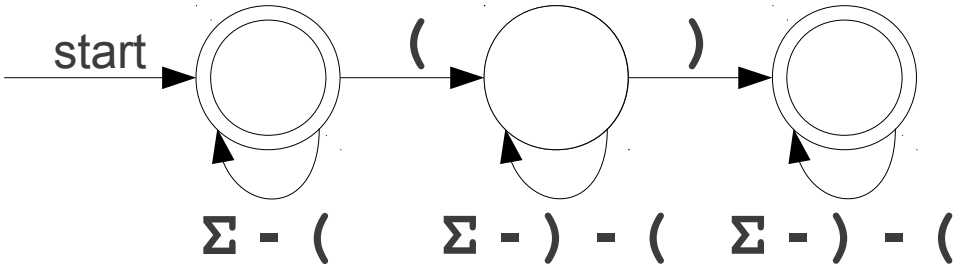
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

Items$_1$ Items$_2$ Items$_3$

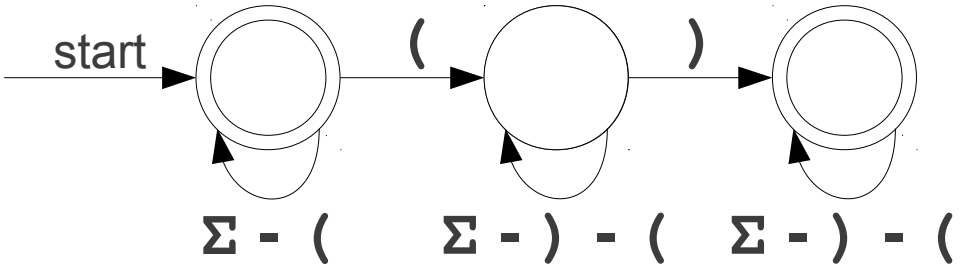
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

Items$_1$	Items$_2$	Items$_3$
S→·E @1		

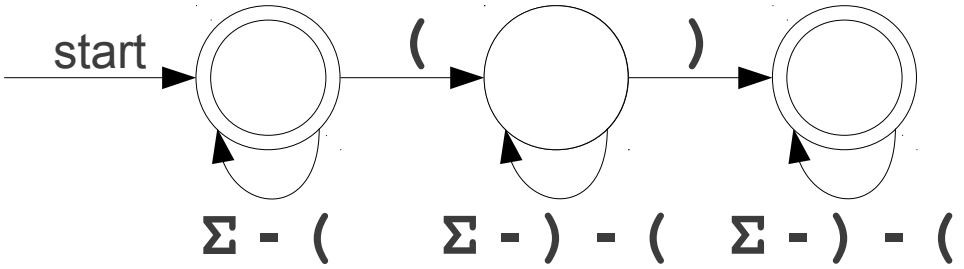
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

Items$_1$	Items$_2$	Items$_3$
S→·E @1		

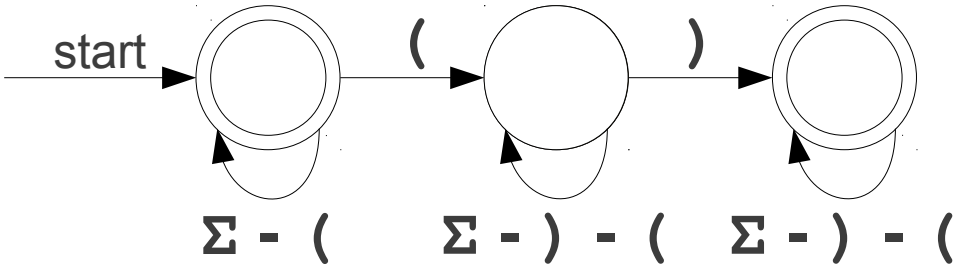
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

Items$_1$	Items$_2$	Items$_3$
S→·E @1		
E→·E+E @1		
E→·(E) @1		
E→·**int** @1		

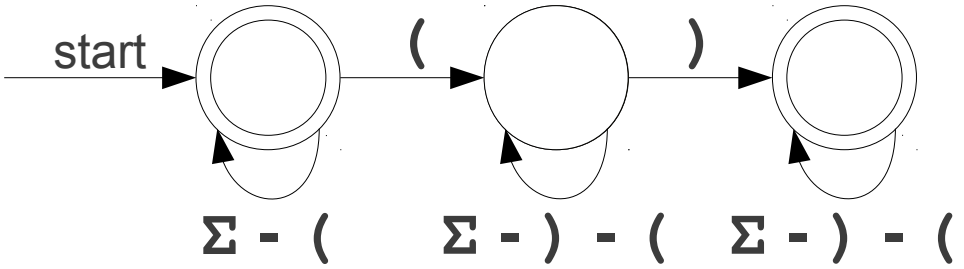
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

$Items_1$	$Items_2$	$Items_3$
S→·E @1		
E→·E+E @1		
E→·(E) @1		
E→·**int** @1		

S → E
E → E + E
E → **int**
E → (E)



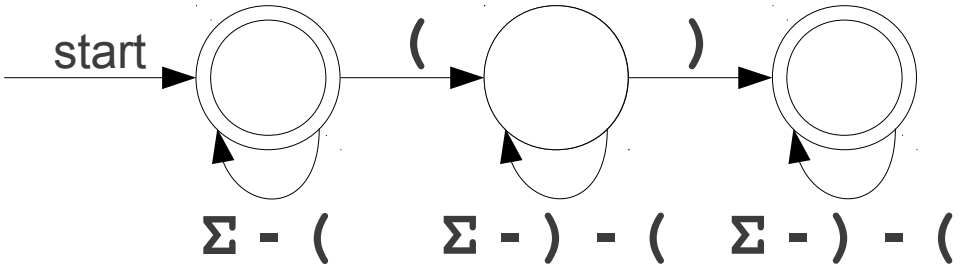
SCAN
COMPLETE
PREDICT

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1

$Items_2$
E→(·E) @1

$Items_3$

S → E
E → E + E
E → **int**
E → (E)



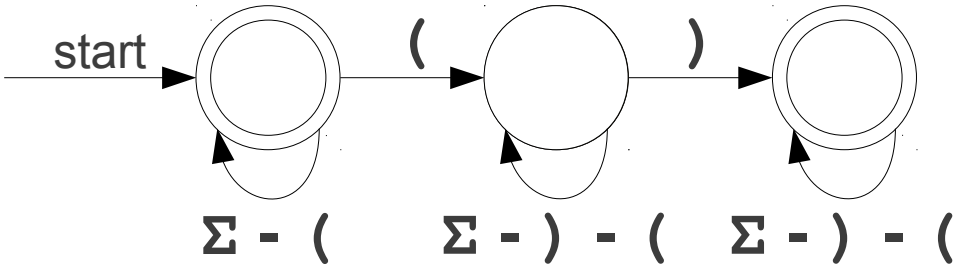
SCAN
COMPLETE
PREDICT

Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1

Items$_2$
E→(·E) @1

Items$_3$

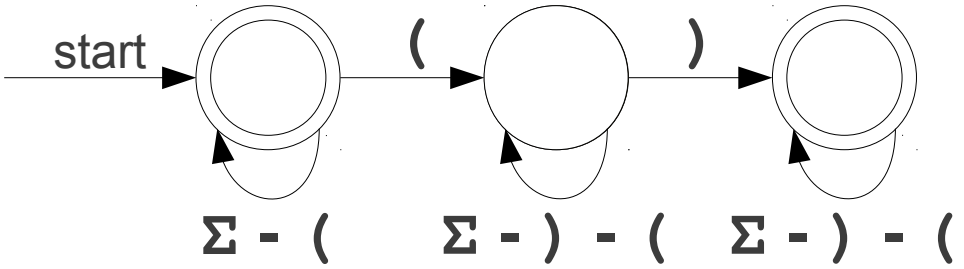
S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

Items$_1$	Items$_2$	Items$_3$
S→·E @1	E→(·E) @1	
E→·E+E @1		
E→·(E) @1		
E→·**int** @1		
E→**int** · @1		

S → E
E → E + E
E → **int**
E → (E)



SCAN
COMPLETE
PREDICT

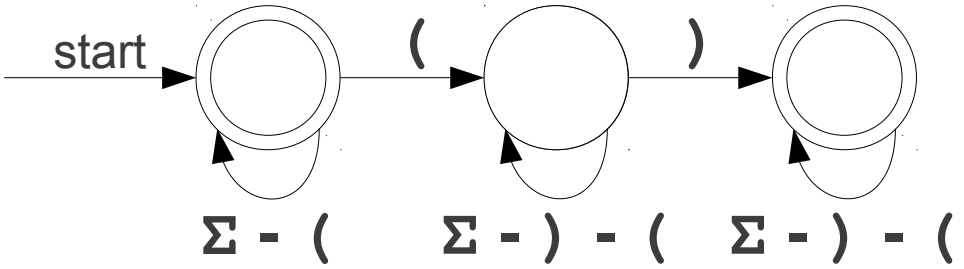
$E_{1\text{-}1} \rightarrow$ **int**$_{1\text{-}1}$

Items$_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1

Items$_2$
E→(·E) @1

Items$_3$

S → E
E → E + E
E → **int**
E → (E)

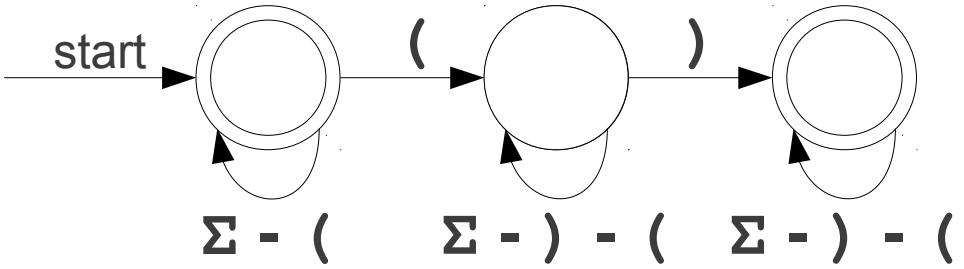


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow$ **int**$_{1\text{-}1}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1		
E→·(E) @1		
E→·**int** @1		
E→**int** · @1		
S→E· @1		
E→E·+E @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

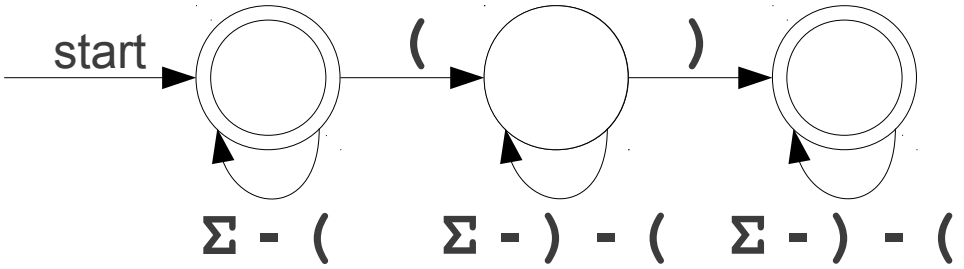
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1

$Items_2$
E→(·E) @1

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

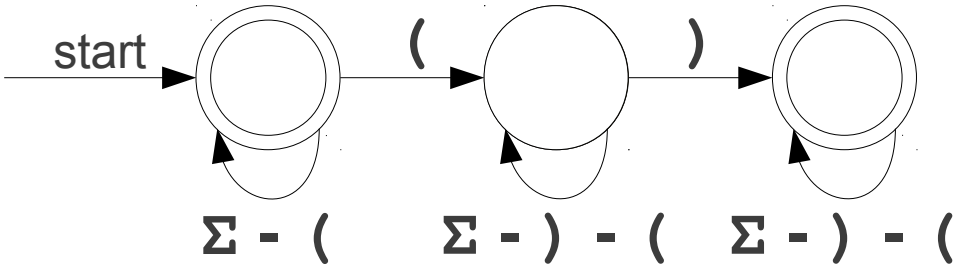
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1

$Items_2$
E→(·E) @1

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

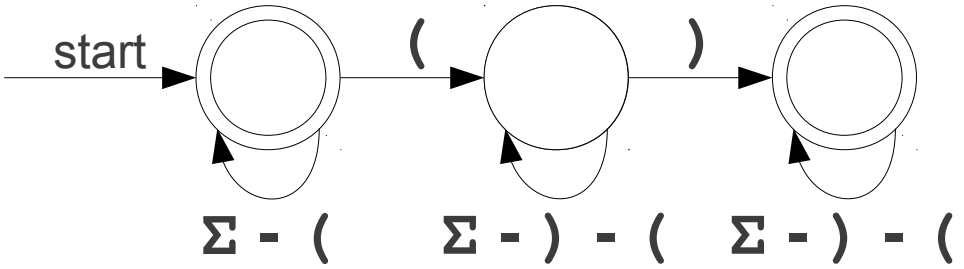
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

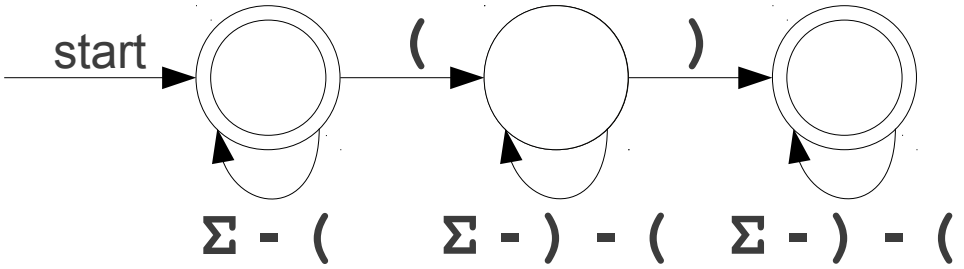
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

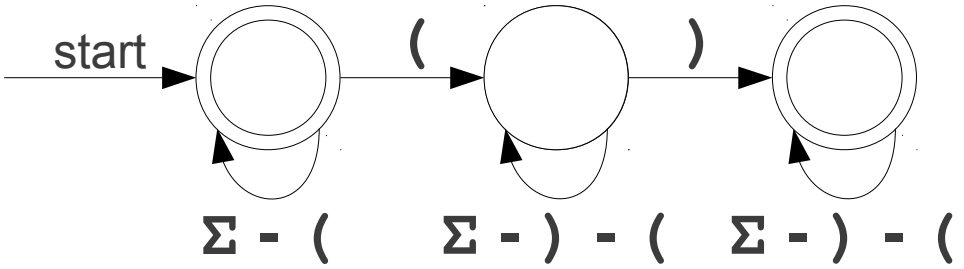
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

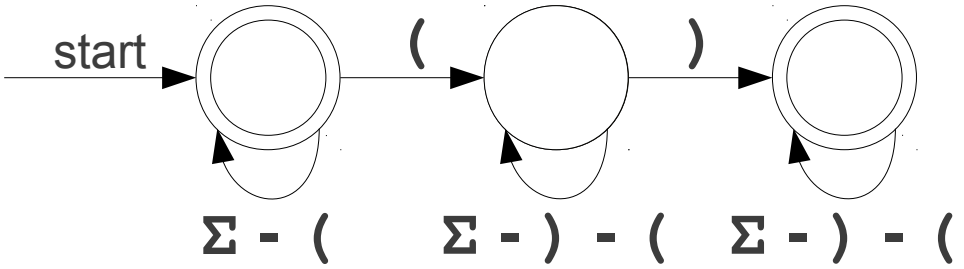
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

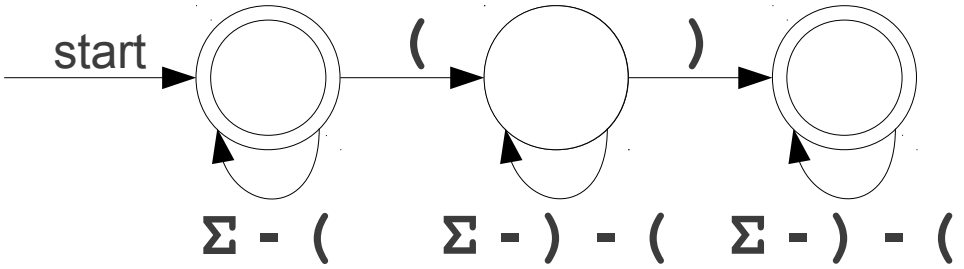
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

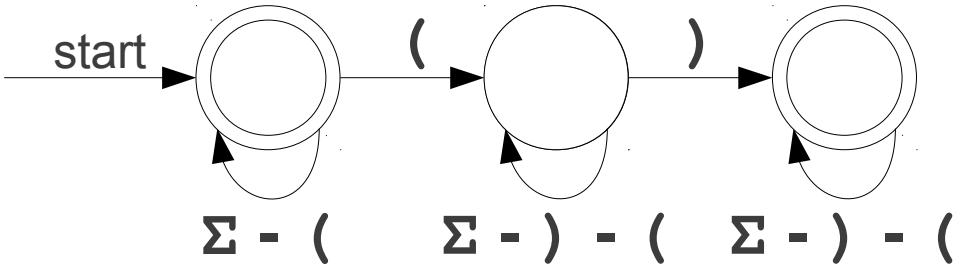
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2

$Items_3$

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

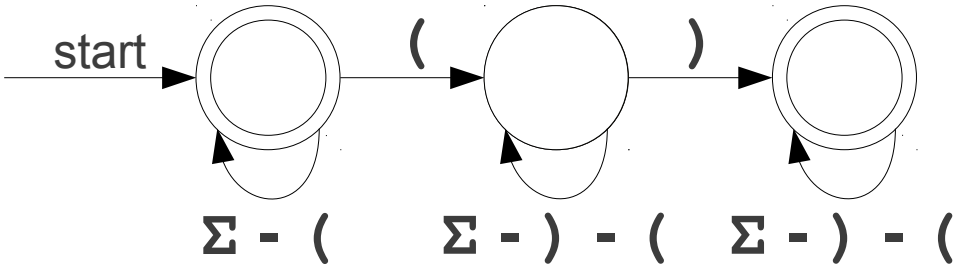


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1	E→·E+E @2	
E→·(E) @1	E→·(E) @2	
E→·**int** @1	E→·**int** @2	
E→**int** · @1	E→**int** · @2	
S→E· @1	E→(E·) @1	
E→E·+E @1	E→E·+E @2	
E→E+·E @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

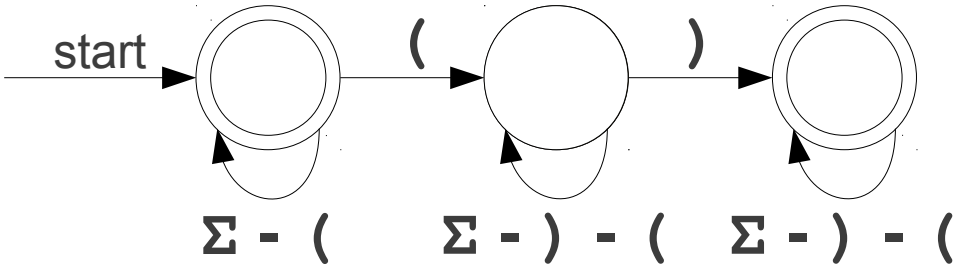


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1	E→·E+E @2	
E→·(E) @1	E→·(E) @2	
E→·**int** @1	E→·**int** @2	
E→**int** · @1	E→**int** · @2	
S→E· @1	E→(E·) @1	
E→E·+E @1	E→E·+E @2	
E→E+·E @1		
E→E+E· @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

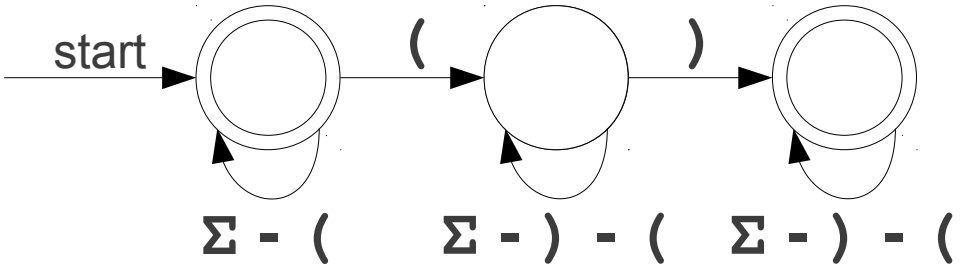


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1	E→·E+E @2	
E→·(E) @1	E→·(E) @2	
E→·**int** @1	E→·**int** @2	
E→**int** · @1	E→**int** · @2	
S→E· @1	E→(E·) @1	
E→E·+E @1	E→E·+E @2	
E→E+·E @1		
E→E+E· @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

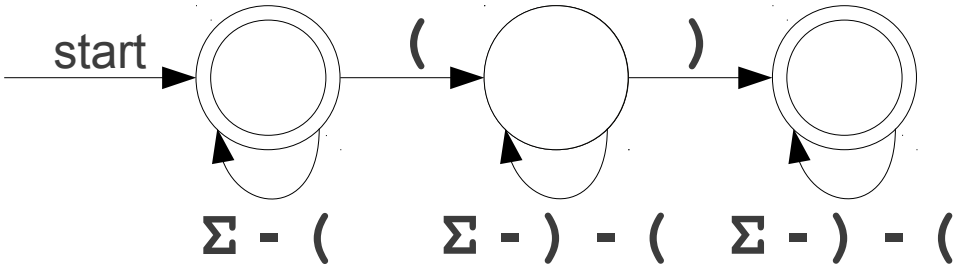


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1	E→·E+E @2	
E→·(E) @1	E→·(E) @2	
E→·**int** @1	E→·**int** @2	
E→**int** · @1	E→**int** · @2	
S→E· @1	E→(E·) @1	
E→E·+E @1	E→E·+E @2	
E→E+·E @1		
E→E+E· @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

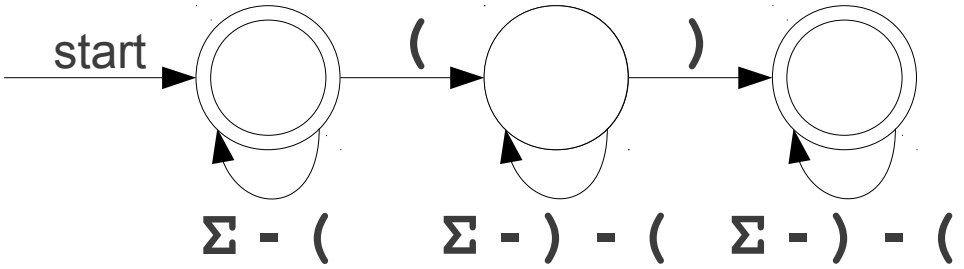


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	
E→·E+E @1	E→·E+E @2	
E→·(E) @1	E→·(E) @2	
E→·**int** @1	E→·**int** @2	
E→**int** · @1	E→**int** · @2	
S→E· @1	E→(E·) @1	
E→E·+E @1	E→E·+E @2	
E→E+·E @1		
E→E+E· @1		

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

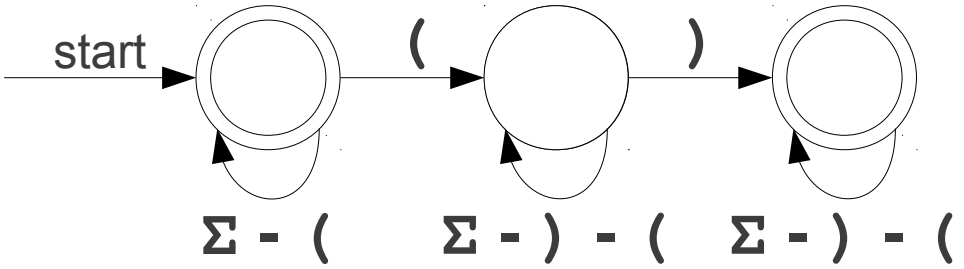
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2

$Items_3$
E→(E)· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

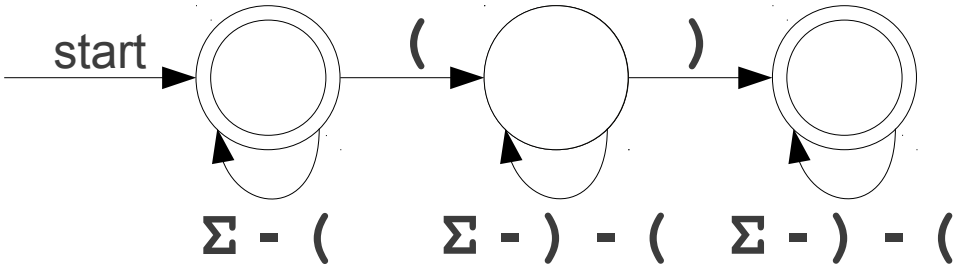
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

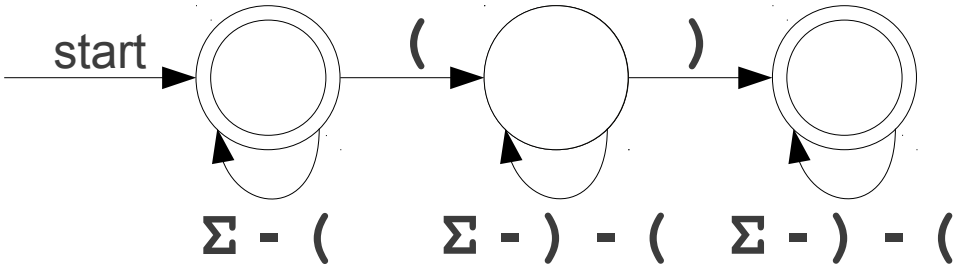
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

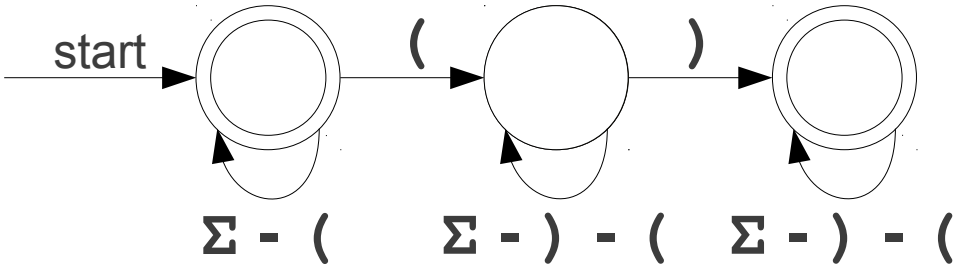
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

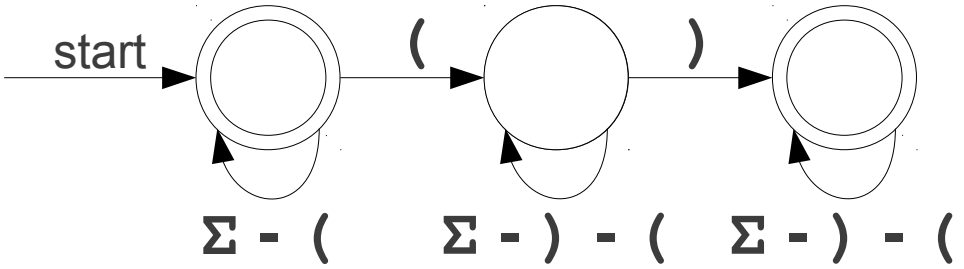
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

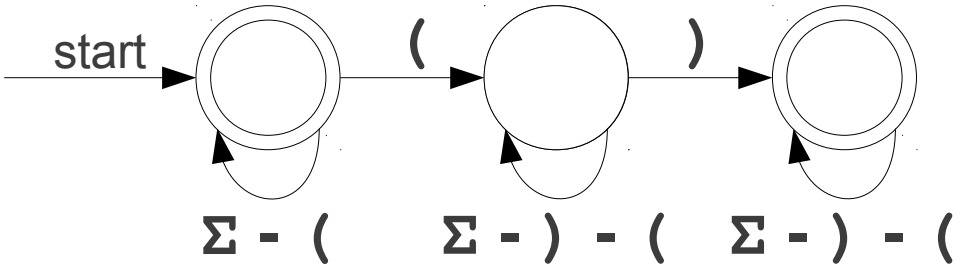
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

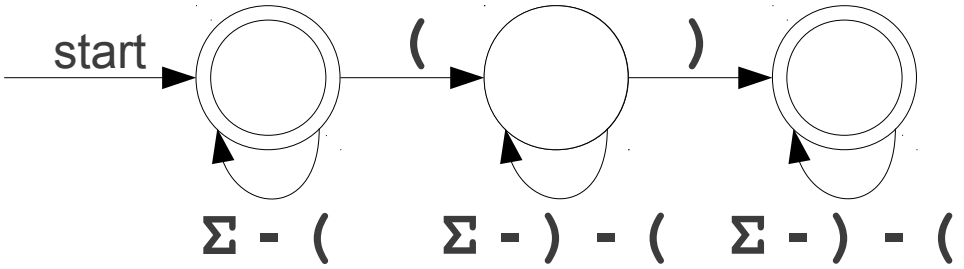
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

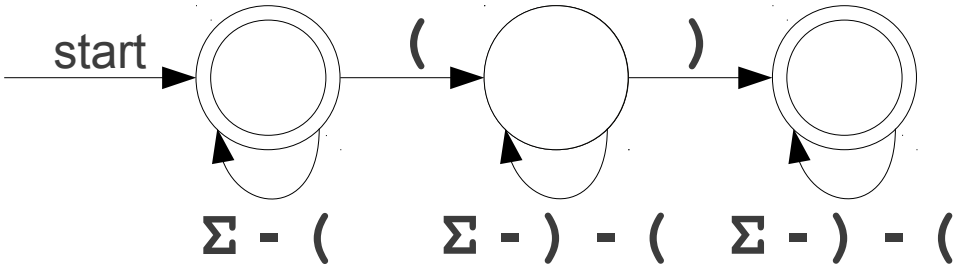
$E_{1-1} \rightarrow \texttt{int}_{1-1}$
$S_{1-1} \rightarrow E_{1-1}$
$E_{2-2} \rightarrow \texttt{int}_{2-2}$
$E_{1-1} \rightarrow E_{1-1} +_{1-1} E_{1-1}$
$E_{1-3} \rightarrow (_{1-2} E_{2-2})_{2-3}$
$S_{1-3} \rightarrow E_{1-3}$
$E_{1-3} \rightarrow E_{1-1} +_{1-1} E_{1-3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

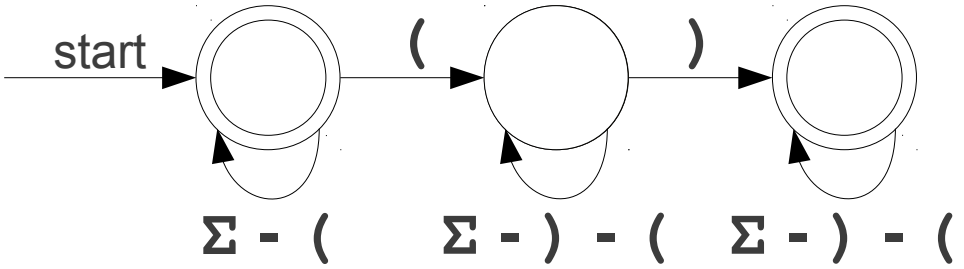
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

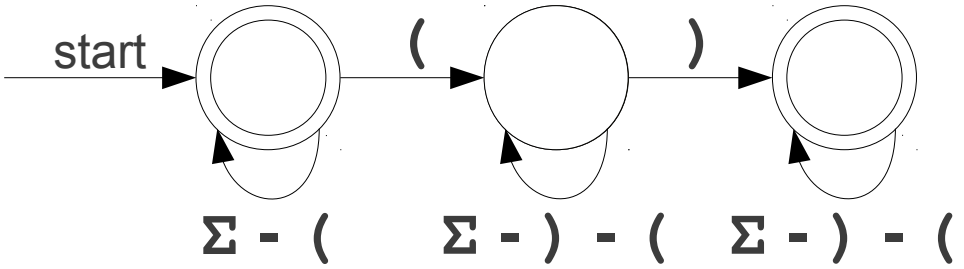
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

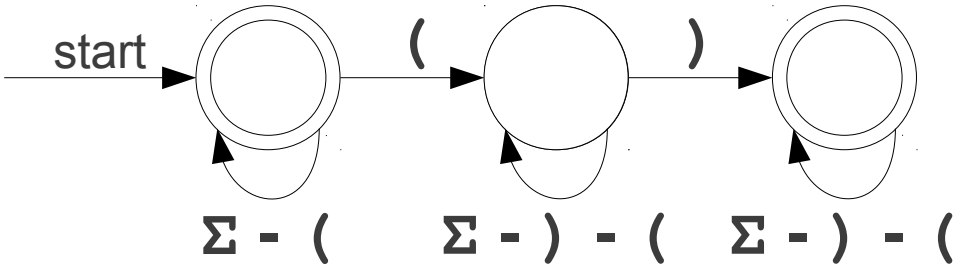
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

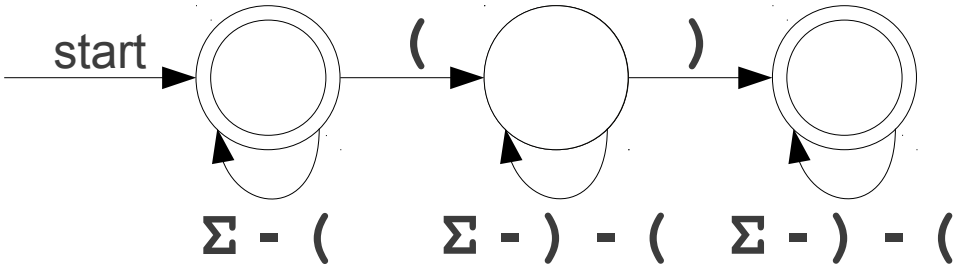
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

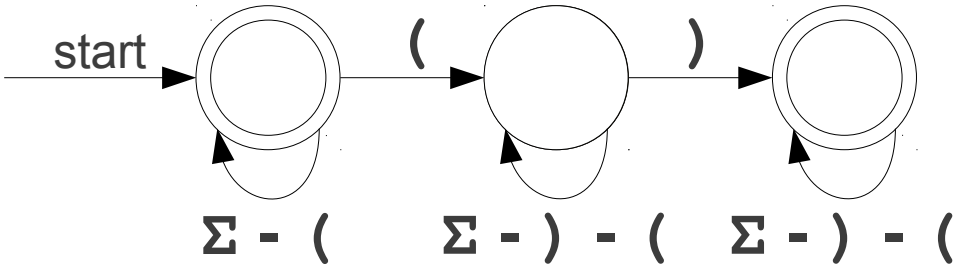
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

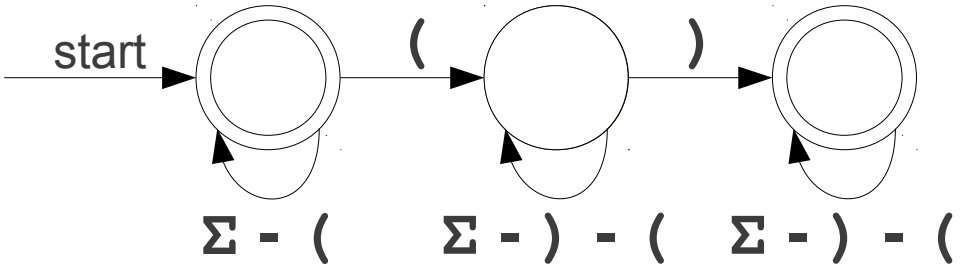
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

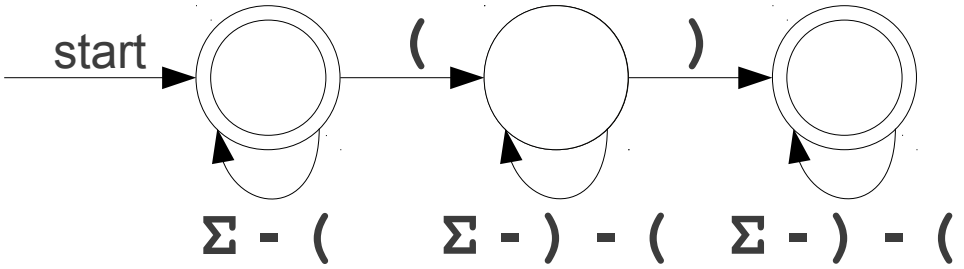


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	E→(E)· @1
E→·E+E @1	E→·E+E @2	S→E· @1
E→·(E) @1	E→·(E) @2	E→E·+E @1
E→·**int** @1	E→·**int** @2	E→E+E· @1
E→**int** · @1	E→**int** · @2	E→E+·E @1
S→E· @1	E→(E·) @1	E→·E+E @3
E→E·+E @1	E→E·+E @2	E→·(E) @3
E→E+·E @1	E→E+·E @2	E→·**int** @3
E→E+E· @1	E→E+E· @2	

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

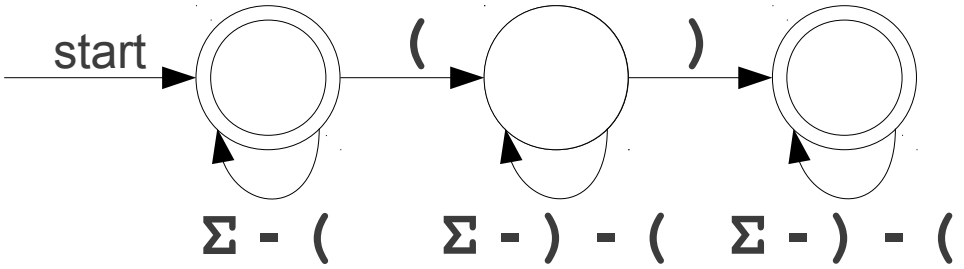
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

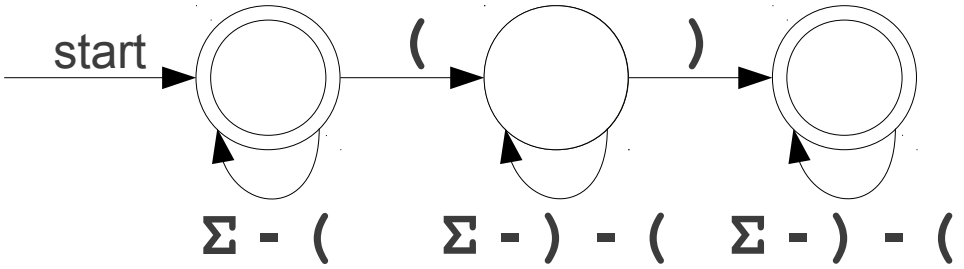


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	E→(E)· @1
E→·E+E @1	E→·E+E @2	S→E· @1
E→·(E) @1	E→·(E) @2	E→E·+E @1
E→·**int** @1	E→·**int** @2	E→E+E· @1
E→**int** · @1	E→**int** · @2	E→E+·E @1
S→E· @1	E→(E·) @1	E→·E+E @3
E→E·+E @1	E→E·+E @2	E→·(E) @3
E→E+·E @1	E→E+·E @2	E→·**int** @3
E→E+E· @1	E→E+E· @2	E→**int**· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

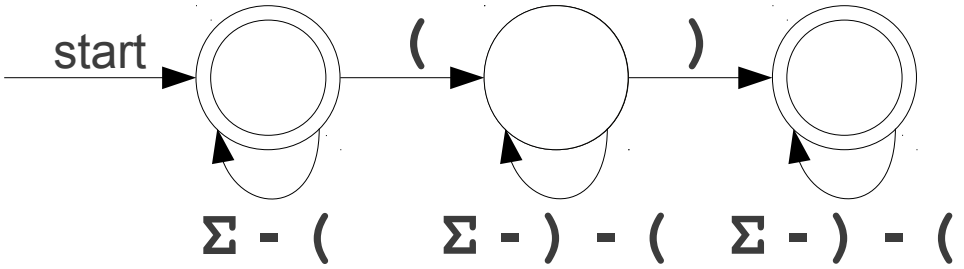


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	E→(E)· @1
E→·E+E @1	E→·E+E @2	S→E· @1
E→·(E) @1	E→·(E) @2	E→E·+E @1
E→·**int** @1	E→·**int** @2	E→E+E· @1
E→**int** · @1	E→**int** · @2	E→E+·E @1
S→E· @1	E→(E·) @1	E→·E+E @3
E→E·+E @1	E→E·+E @2	E→·(E) @3
E→E+·E @1	E→E+·E @2	E→·**int** @3
E→E+E· @1	E→E+E· @2	E→**int**· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$

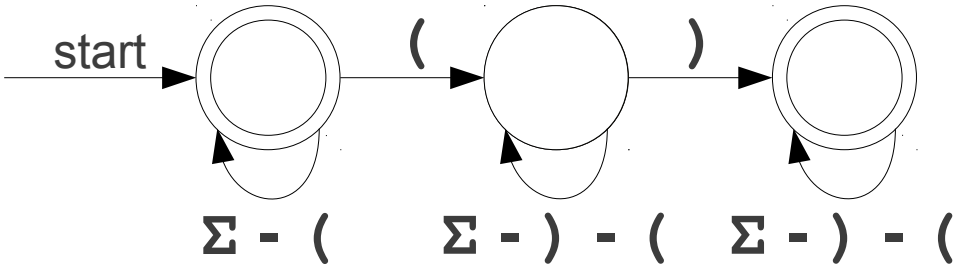


SCAN
COMPLETE
PREDICT

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$

$Items_1$	$Items_2$	$Items_3$
S→·E @1	E→(·E) @1	E→(E)· @1
E→·E+E @1	E→·E+E @2	S→E· @1
E→·(E) @1	E→·(E) @2	E→E·+E @1
E→·**int** @1	E→·**int** @2	E→E+E· @1
E→**int** · @1	E→**int** · @2	E→E+·E @1
S→E· @1	E→(E·) @1	E→·E+E @3
E→E·+E @1	E→E·+E @2	E→·(E) @3
E→E+·E @1	E→E+·E @2	E→·**int** @3
E→E+E· @1	E→E+E· @2	E→**int**· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

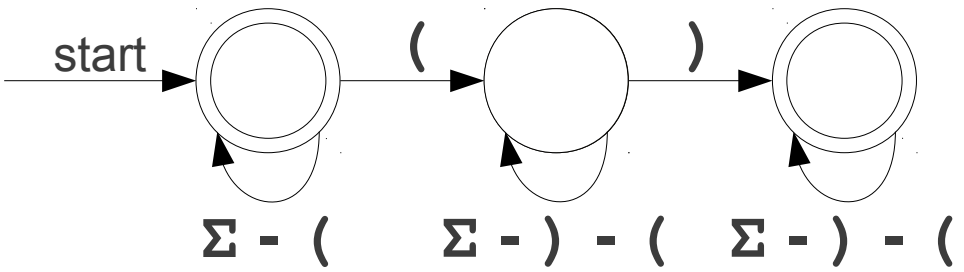
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

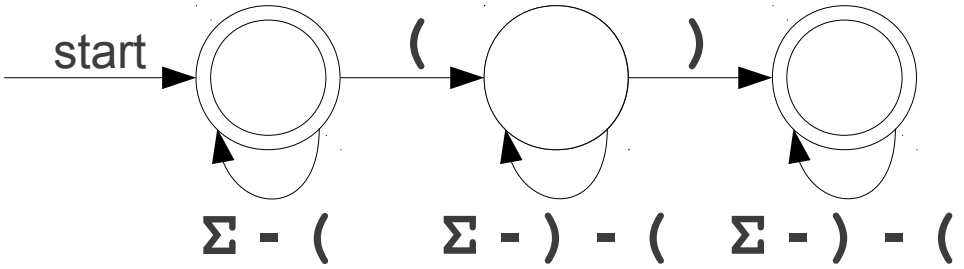
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

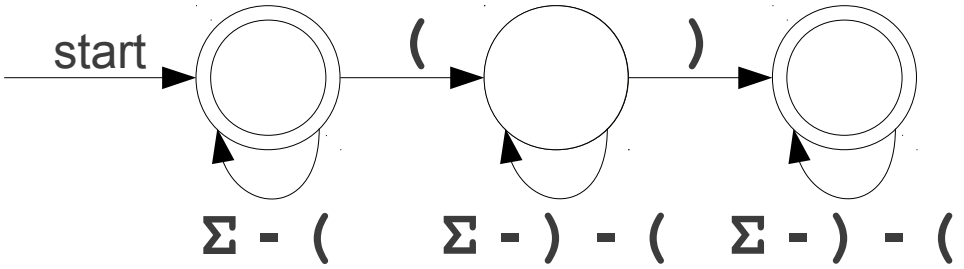
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

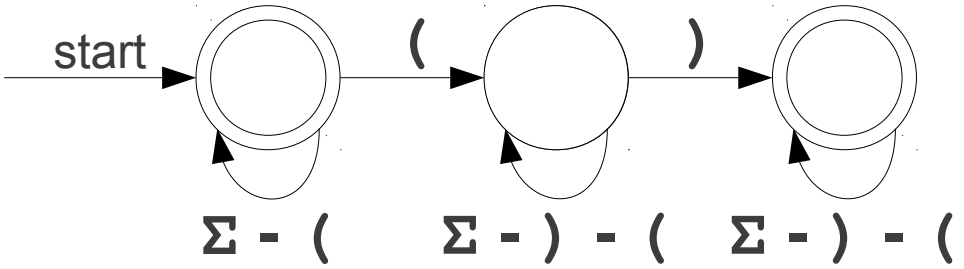
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

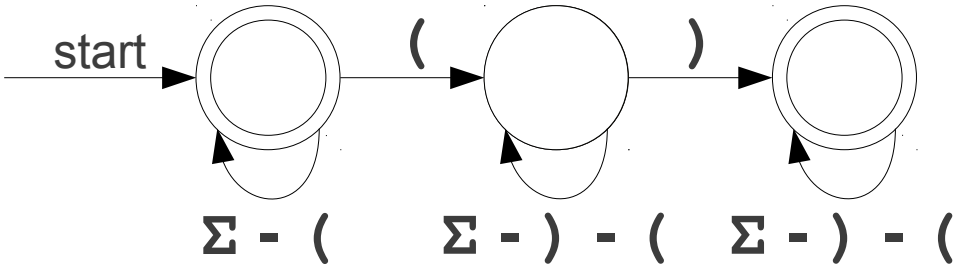
$E_{1-1} \rightarrow \texttt{int}_{1-1}$
$S_{1-1} \rightarrow E_{1-1}$
$E_{2-2} \rightarrow \texttt{int}_{2-2}$
$E_{1-1} \rightarrow E_{1-1} +_{1-1} E_{1-1}$
$E_{1-3} \rightarrow (_{1-2} E_{2-2})_{2-3}$
$S_{1-3} \rightarrow E_{1-3}$
$E_{1-3} \rightarrow E_{1-1} +_{1-1} E_{1-3}$
$E_{2-2} \rightarrow E_{2-2} +_{2-2} E_{2-2}$
$E_{3-3} \rightarrow \texttt{int}_{3-3}$
$E_{1-3} \rightarrow E_{1-3} +_{1-3} E_{3-3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

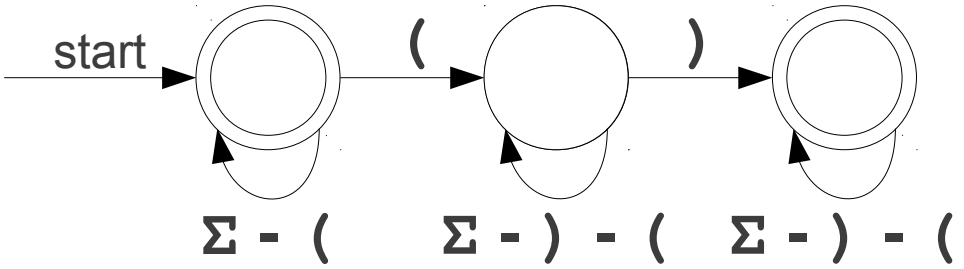
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

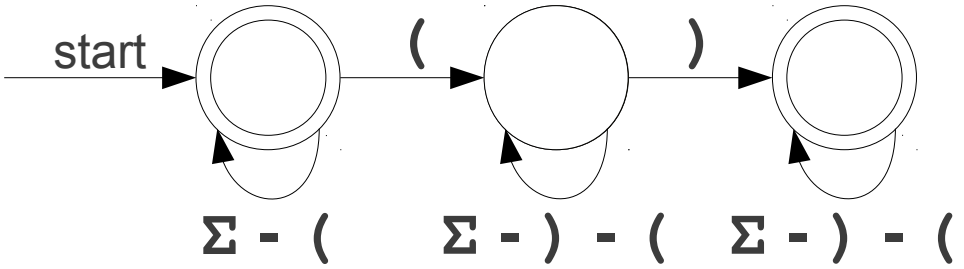
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3
E→E+E· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

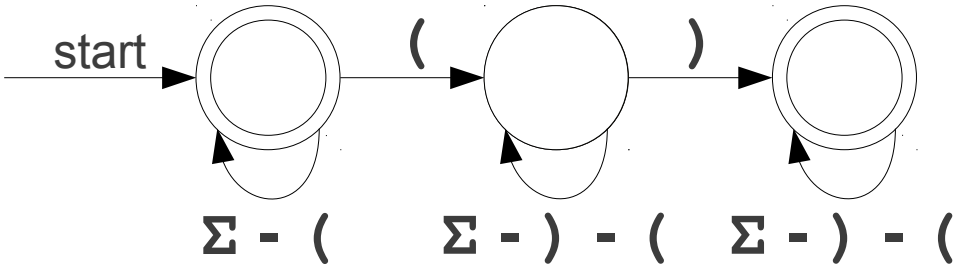
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$
$E_{3\text{-}3} \rightarrow E_{3\text{-}3} +_{3\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3
E→E+E· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

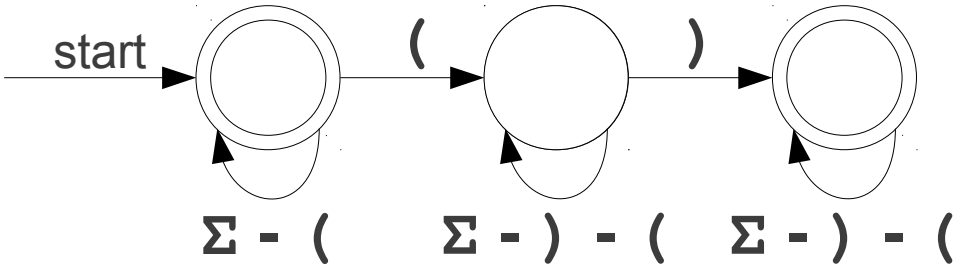
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$
$E_{3\text{-}3} \rightarrow E_{3\text{-}3} +_{3\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3
E→E+E· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \texttt{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

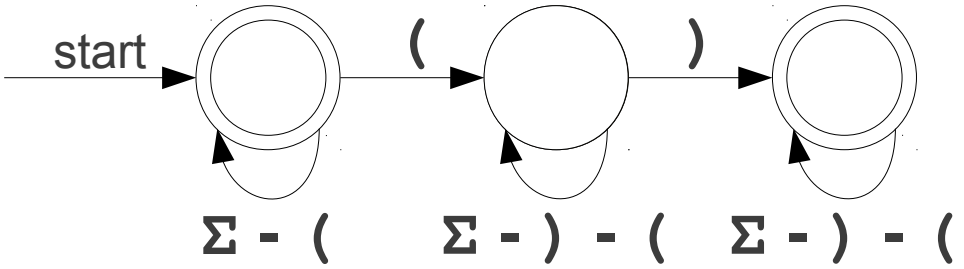
$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$
$E_{3\text{-}3} \rightarrow E_{3\text{-}3} +_{3\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3
E→E+E· @3

$S \rightarrow E$
$E \rightarrow E + E$
$E \rightarrow \mathbf{int}$
$E \rightarrow (E)$



SCAN
COMPLETE
PREDICT

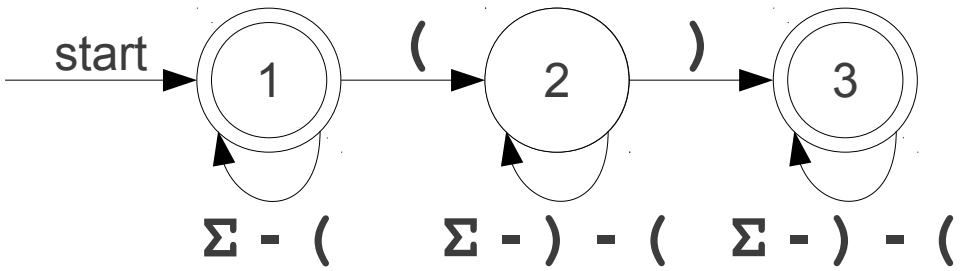
$S \rightarrow S_{1\text{-}1} \mid S_{1\text{-}3}$
$E_{1\text{-}1} \rightarrow \mathbf{int}_{1\text{-}1}$
$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$
$E_{2\text{-}2} \rightarrow \mathbf{int}_{2\text{-}2}$
$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$
$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$
$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$
$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$
$E_{3\text{-}3} \rightarrow \mathbf{int}_{3\text{-}3}$
$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$
$E_{3\text{-}3} \rightarrow E_{3\text{-}3} +_{3\text{-}3} E_{3\text{-}3}$

$Items_1$
S→·E @1
E→·E+E @1
E→·(E) @1
E→·**int** @1
E→**int** · @1
S→E· @1
E→E·+E @1
E→E+·E @1
E→E+E· @1

$Items_2$
E→(·E) @1
E→·E+E @2
E→·(E) @2
E→·**int** @2
E→**int** · @2
E→(E·) @1
E→E·+E @2
E→E+·E @2
E→E+E· @2

$Items_3$
E→(E)· @1
S→E· @1
E→E·+E @1
E→E+E· @1
E→E+·E @1
E→·E+E @3
E→·(E) @3
E→·**int** @3
E→**int**· @3
E→E+E· @1
E→E·+E @3
E→E+·E @3
E→E+E· @3

Analyzing the Result



$S \rightarrow S_{1\text{-}1}$

$S_{1\text{-}1} \rightarrow E_{1\text{-}1}$

$E_{1\text{-}1} \rightarrow \texttt{int}_{1\text{-}1}$

$E_{1\text{-}1} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}1}$

$S \rightarrow S_{1\text{-}3}$

$S_{1\text{-}3} \rightarrow E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow E_{1\text{-}1} +_{1\text{-}1} E_{1\text{-}3}$

$E_{1\text{-}3} \rightarrow E_{1\text{-}3} +_{1\text{-}3} E_{3\text{-}3}$

$E_{1\text{-}3} \rightarrow (_{1\text{-}2} E_{2\text{-}2})_{2\text{-}3}$

$E_{2\text{-}2} \rightarrow \texttt{int}_{2\text{-}2}$

$E_{2\text{-}2} \rightarrow E_{2\text{-}2} +_{2\text{-}2} E_{2\text{-}2}$

$E_{3\text{-}3} \rightarrow \texttt{int}_{3\text{-}3}$

$E_{3\text{-}3} \rightarrow E_{3\text{-}3} +_{3\text{-}3} E_{3\text{-}3}$

Summary

- The **Earley algorithm** can be used to efficiently parse arbitrary CFGs.
- A **parse forest grammar** is a CFG encoding a (possibly infinite) family of parse trees.
- **Intersection parsing** treats parsing as the intersection of a CFG and a regular language.
- The **Earley-on-DFA algorithm** can be used to filter CFGs through a DFA to produce a new CFG.

Where to Go from Here

- GLR Parsing
  - **Generalized LR**
  - Conceptually similar to Earley, except based on an LR(0) automaton.
  - (Optionally) used by **`bison`**.
- Fast Earley Parsers
  - Many research papers discuss how to speed up Earley parsers; many are quite good.

Next Time

- **Semantic Analysis**
  - Overview of semantic analysis.
  - Scoping and symbol tables.
  - Introduction to types and type-checking.