# Bottom-Up Parsing, Part II

# Announcements

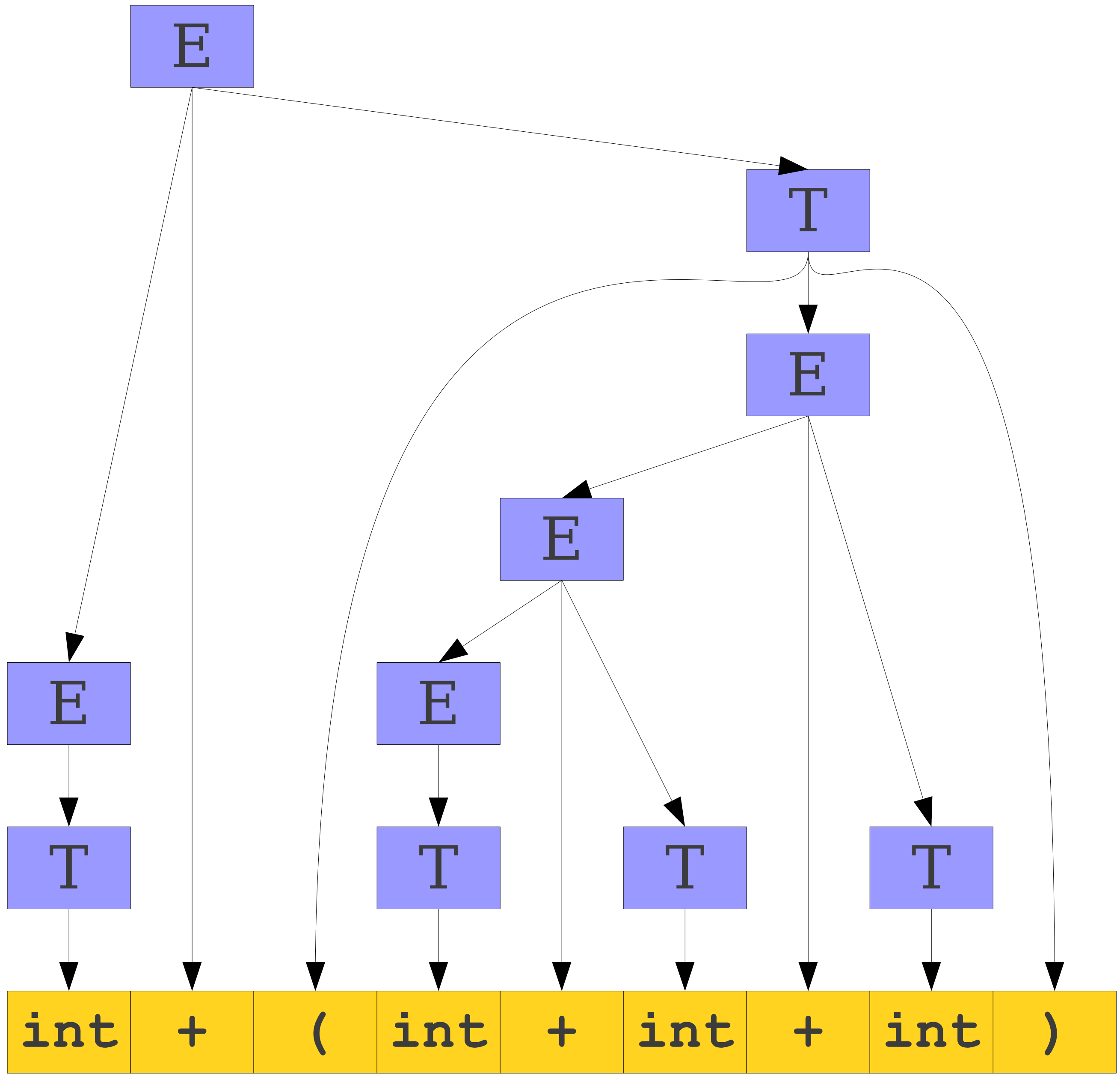
- Programming Assignment 1 due tonight at 11:59PM.
- Programming Assignment 2 (parsing) out, due Friday, July 20th at 11:59PM.
  - Play around with the **`bison`** parser generator!
  - See how real parsers are written!

# Announcements

- C++ review session tonight in Gates B12 from 7:00PM – 8:30PM.
    - Covers classes and inheritance.
    - Extremely valuable for the second programming assignment, especially if you have not seen C++ inheritance before.

# One View of a Bottom-Up Parse

**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



# A Second View of a Bottom-Up Parse

**E** → **T**
**E** → **E** + **T**
**T** → `int`
**T** → `(`**E**`)`

$$
\begin{aligned}
& \texttt{int + (int + int + int)} \\
\Rightarrow\ & \mathbf{T}\ \texttt{+ (int + int + int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (int + int + int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{T}\ \texttt{+ int + int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{E}\ \texttt{+ int + int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{E}\ \texttt{+}\ \mathbf{T}\ \texttt{+ int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{E}\ \texttt{+ int)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{E}\ \texttt{+}\ \mathbf{T}\texttt{)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+ (}\mathbf{E}\texttt{)} \\
\Rightarrow\ & \mathbf{E}\ \texttt{+}\ \mathbf{T} \\
\Rightarrow\ & \mathbf{E}
\end{aligned}
$$

# A Second View of a Bottom-Up Parse

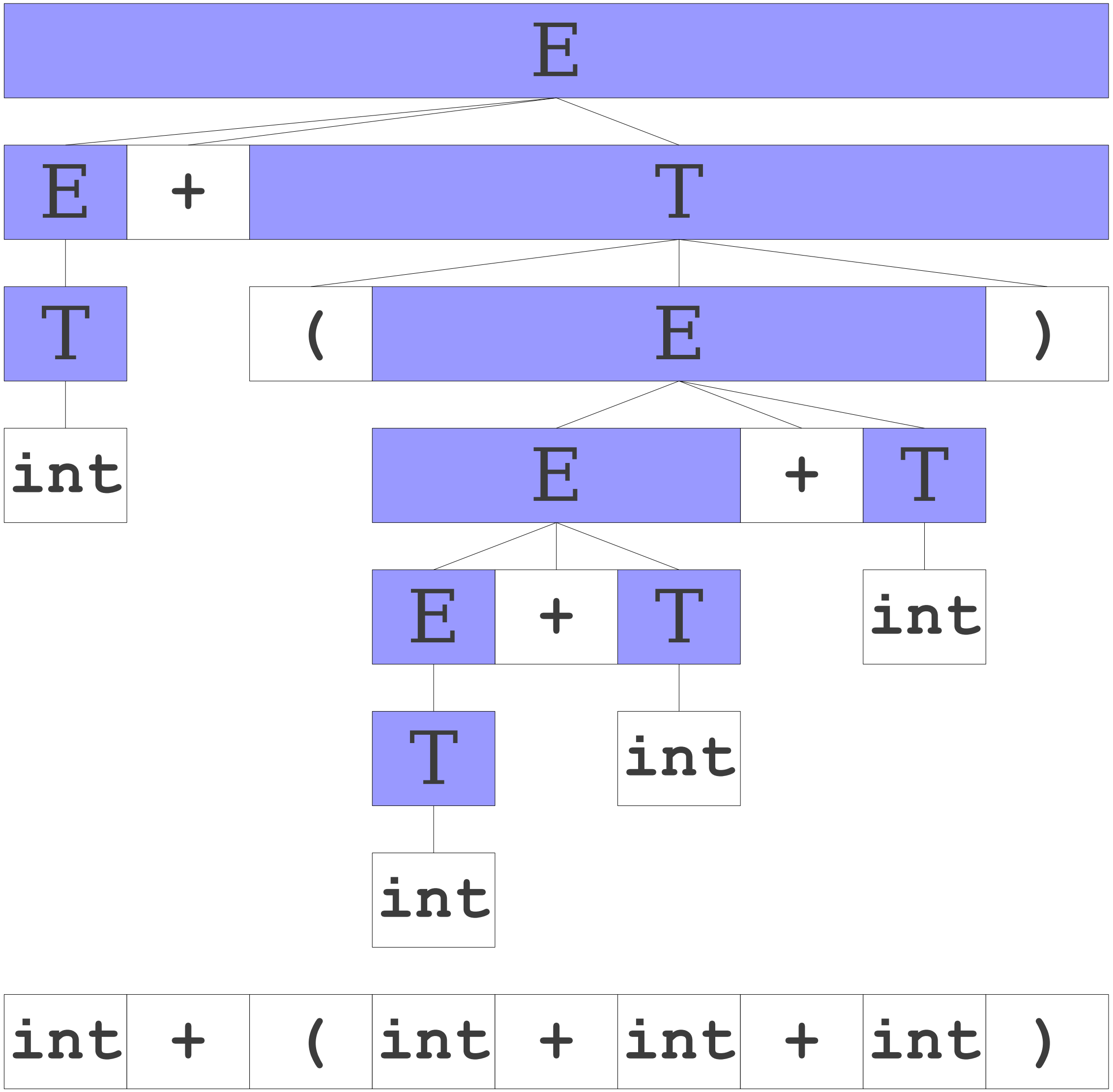
**E** → **T**
**E** → **E** `+` **T**
**T** → `int`
**T** → `(`**E**`)`

$\quad$ int + (int + int + int)
⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ E + (**T** + int + int)
⇒ E + (**E** + int + int)
⇒ E + (E + **T** + int)
⇒ E + (**E** + int)
⇒ E + (E + **T**)
⇒ E + (**E**)
⇒ E + **T**
⇒ **E**

A left-to-right, bottom-up parse is a rightmost derivation traced in reverse.

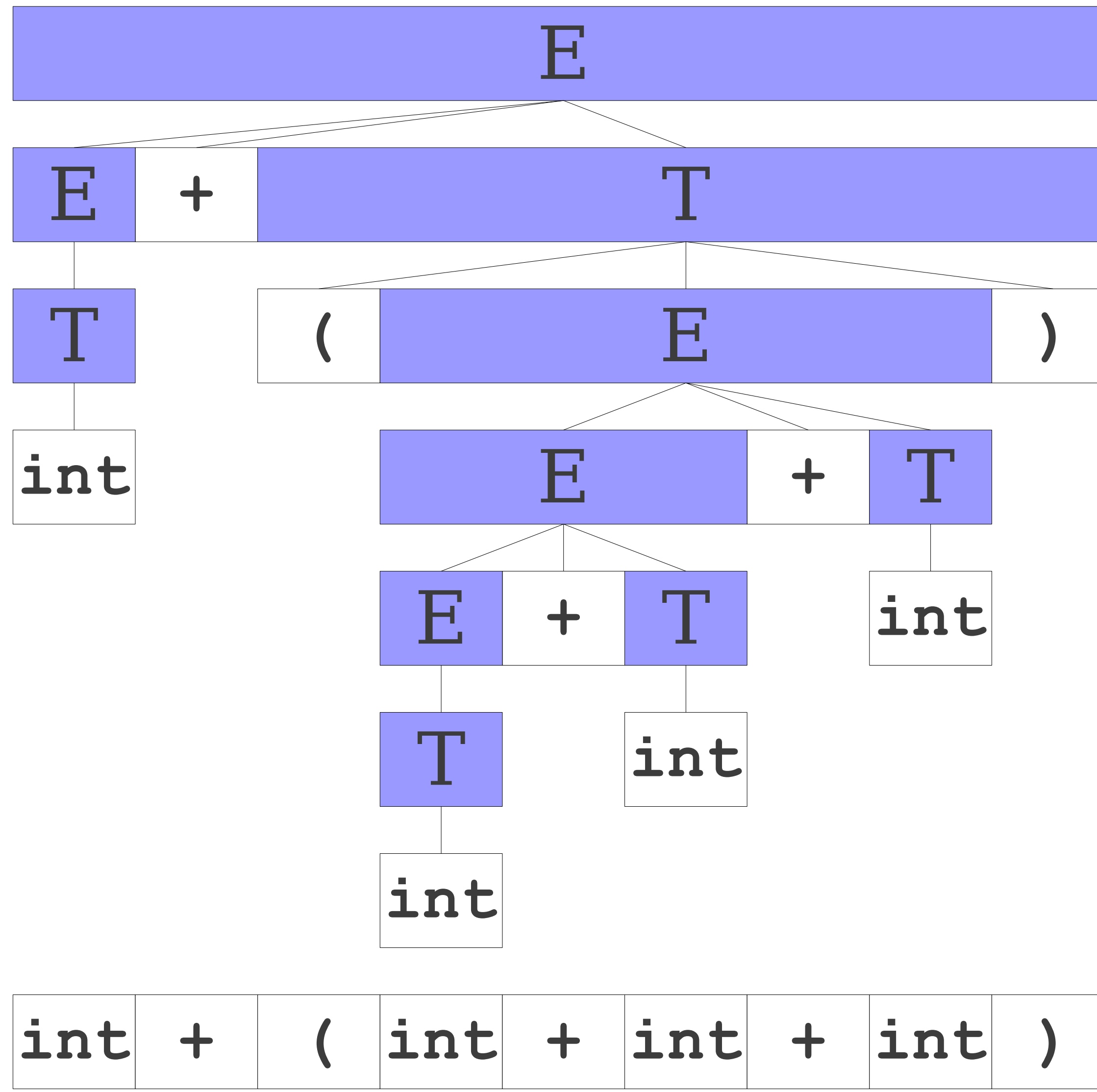
# A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



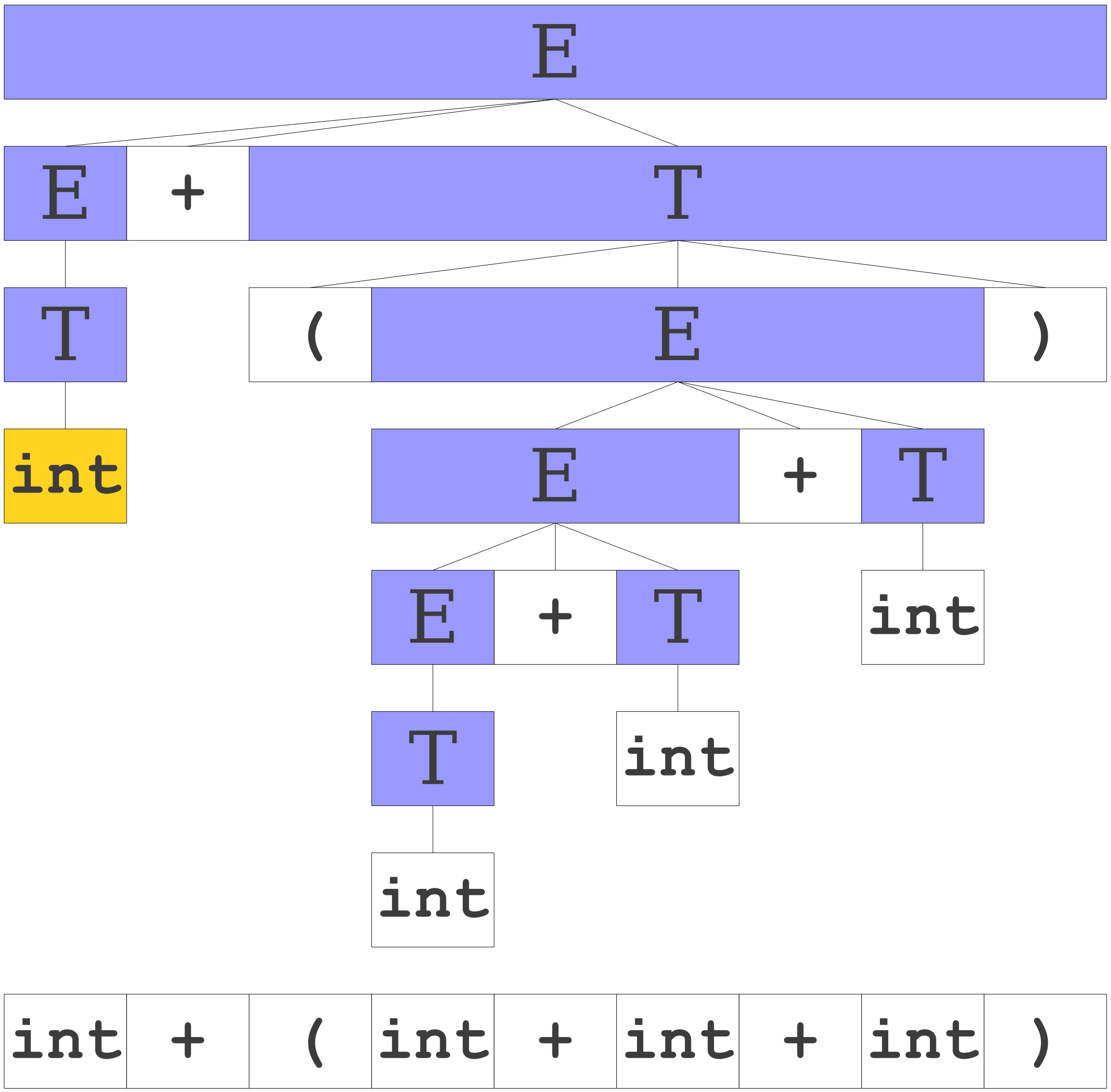
# A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



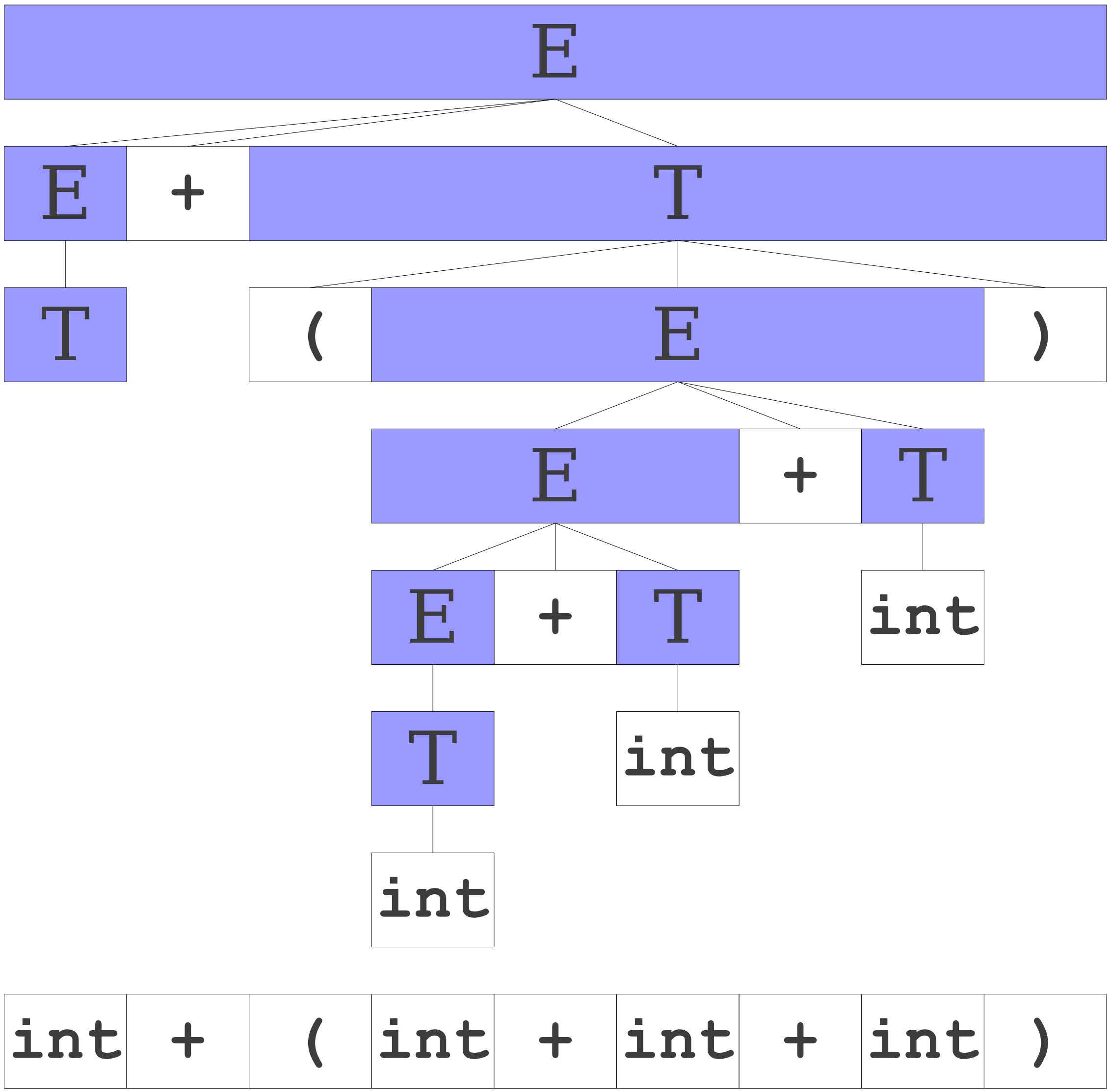
# A Third View of a Bottom-Up Parse

int + (int + int + int)
⇒ T + (int + int + int)
⇒ E + (int + int + int)
⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



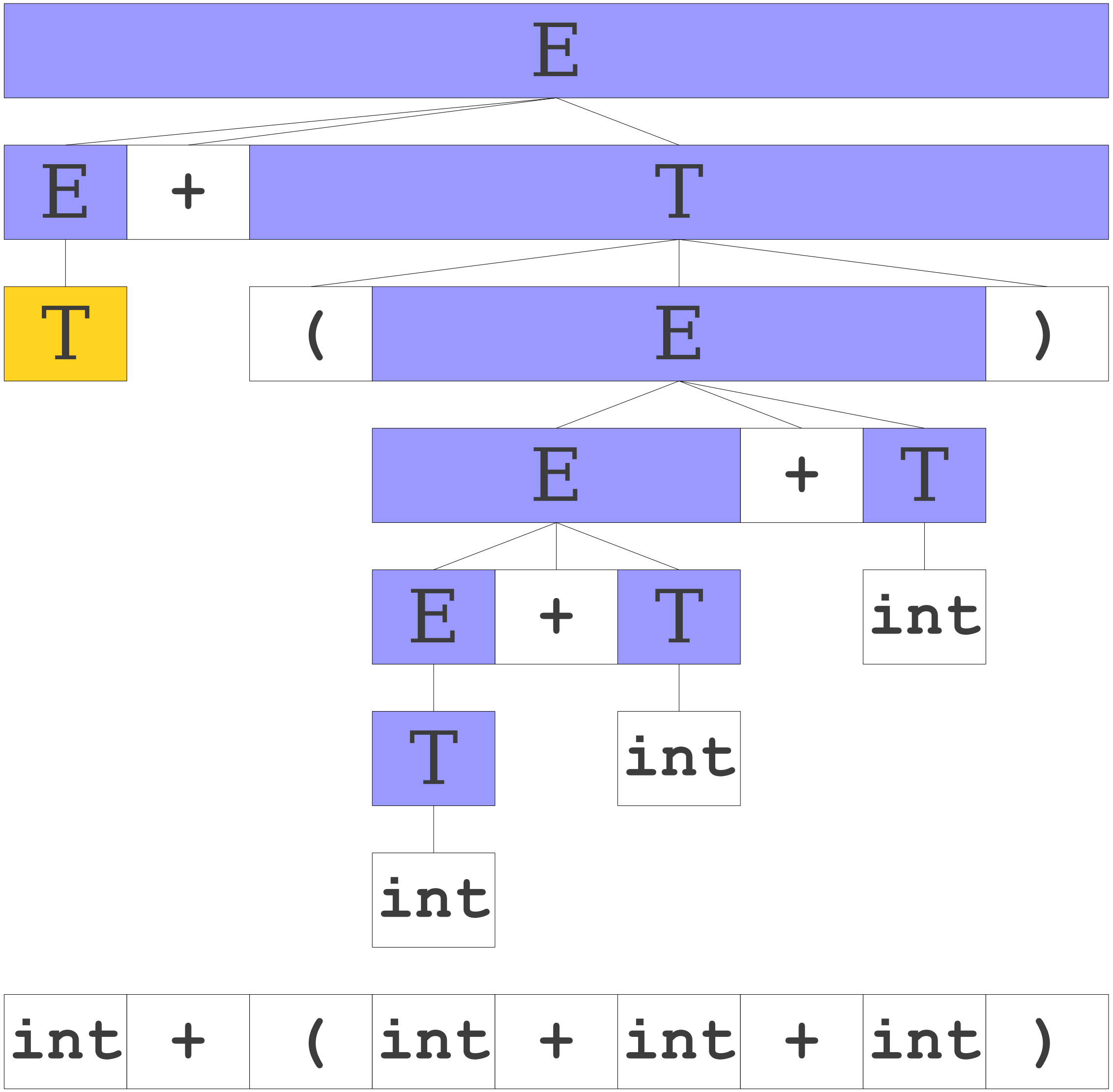
# A Third View of a Bottom-Up Parse

⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



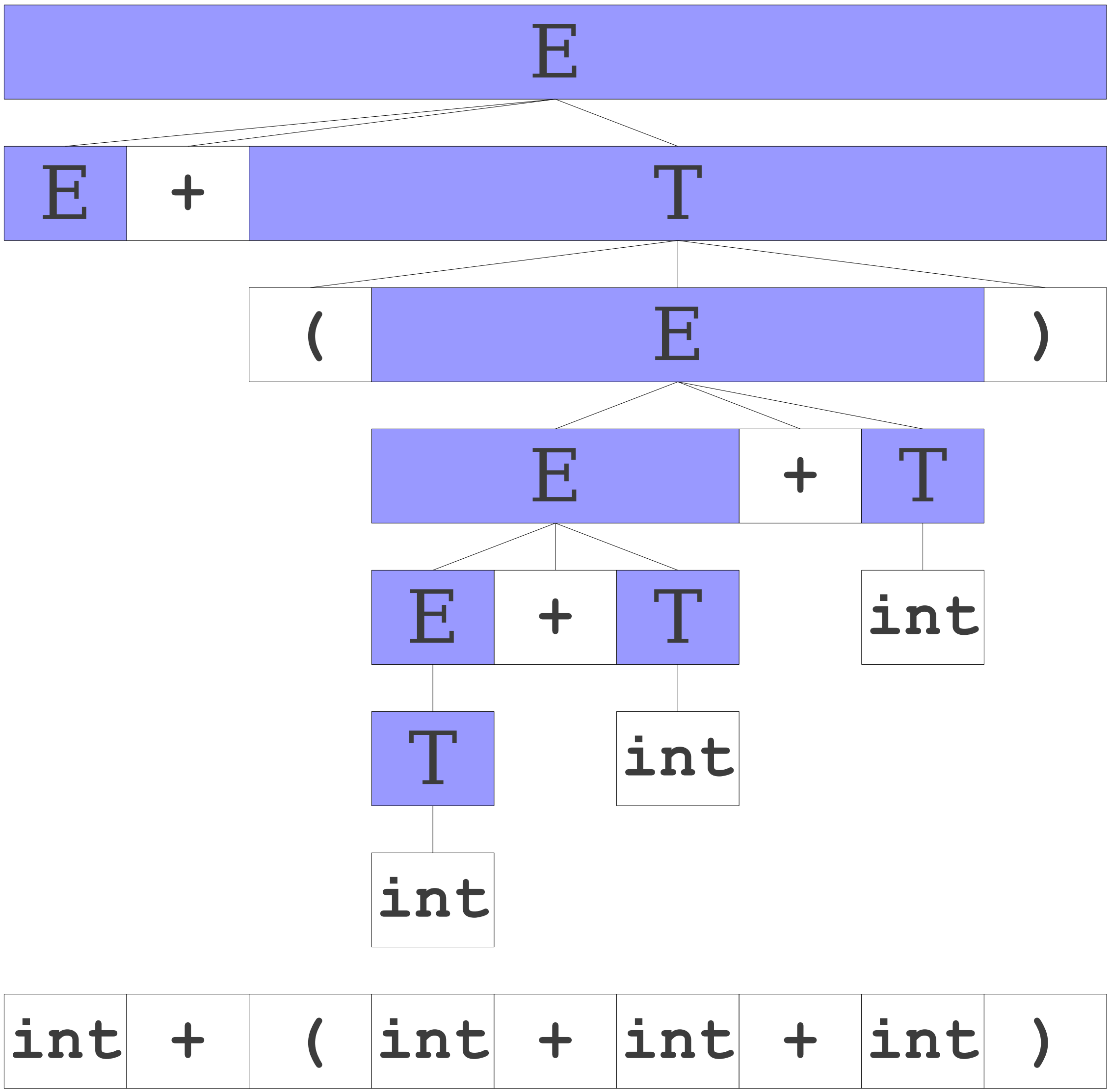
# A Third View of a Bottom-Up Parse

⇒ **T** + (int + int + int)
⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



# A Third View of a Bottom-Up Parse

⇒ **E** + (int + int + int)
⇒ **E** + (**T** + int + int)
⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



# A Third View of a Bottom-Up Parse

⇒ E + (int + int + int)

⇒ E + (T + int + int)

⇒ E + (E + int + int)

⇒ E + (E + T + int)

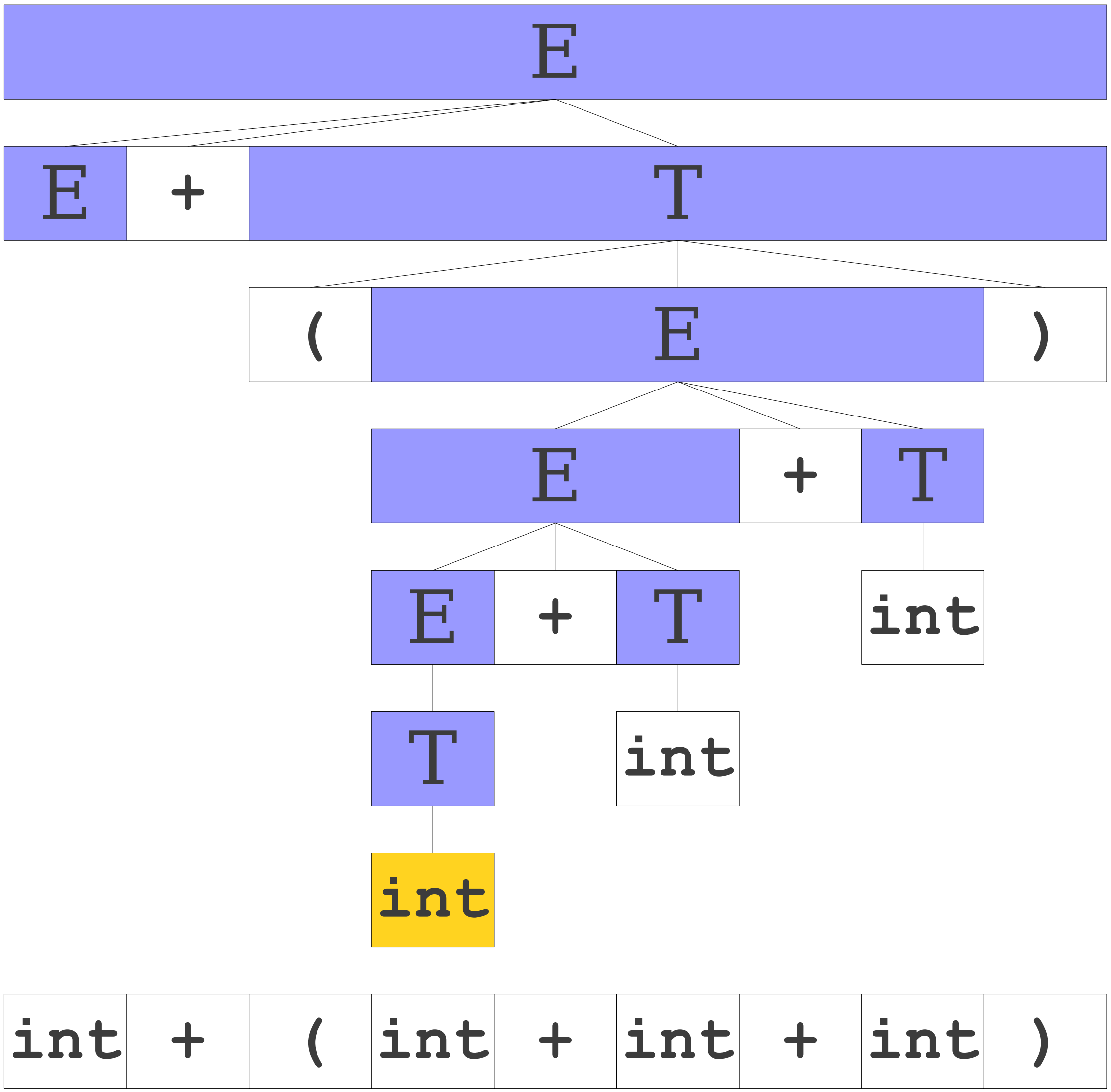
⇒ E + (E + int)

⇒ E + (E + T)

⇒ E + (E)

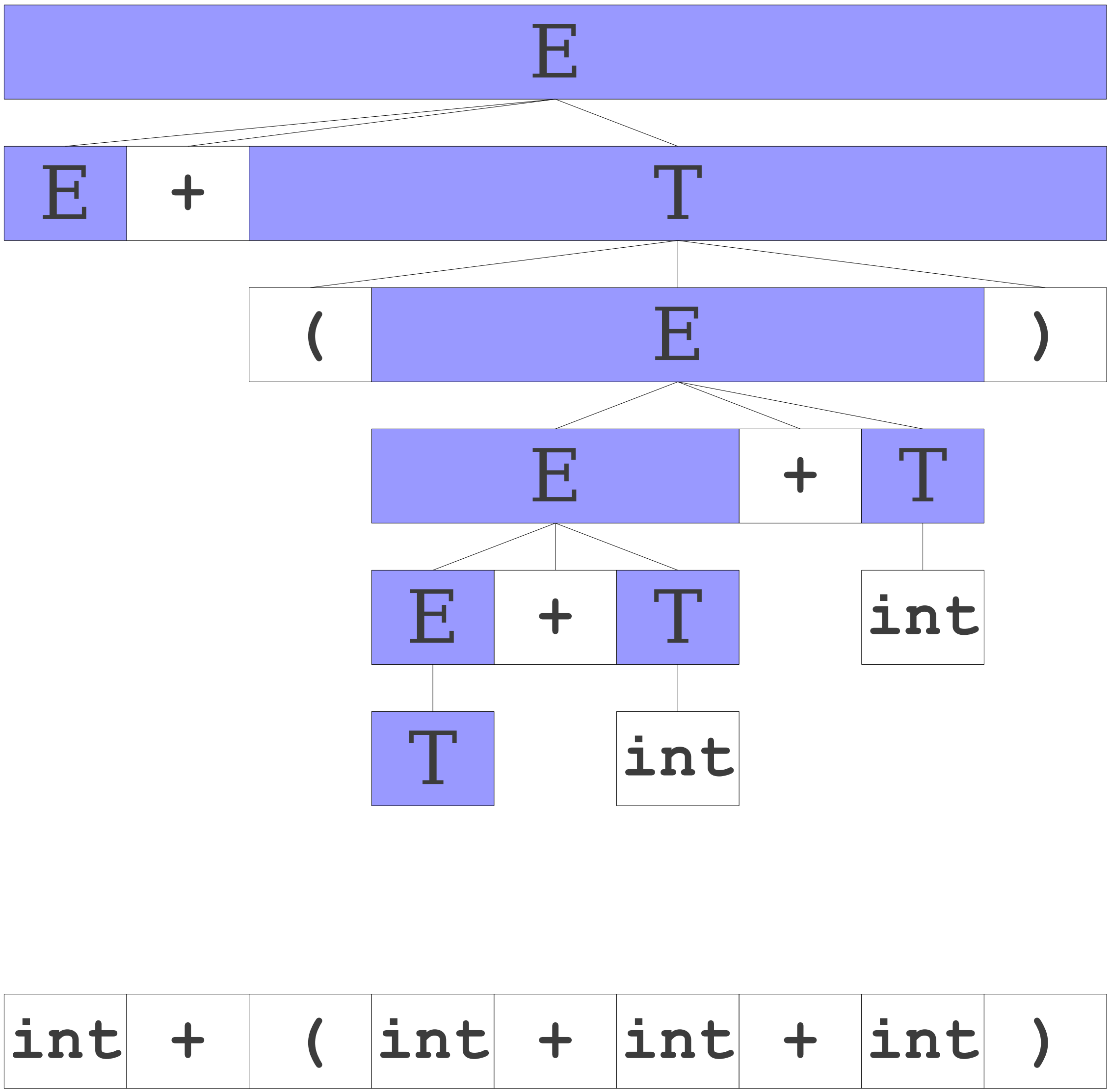
⇒ E + T

⇒ E



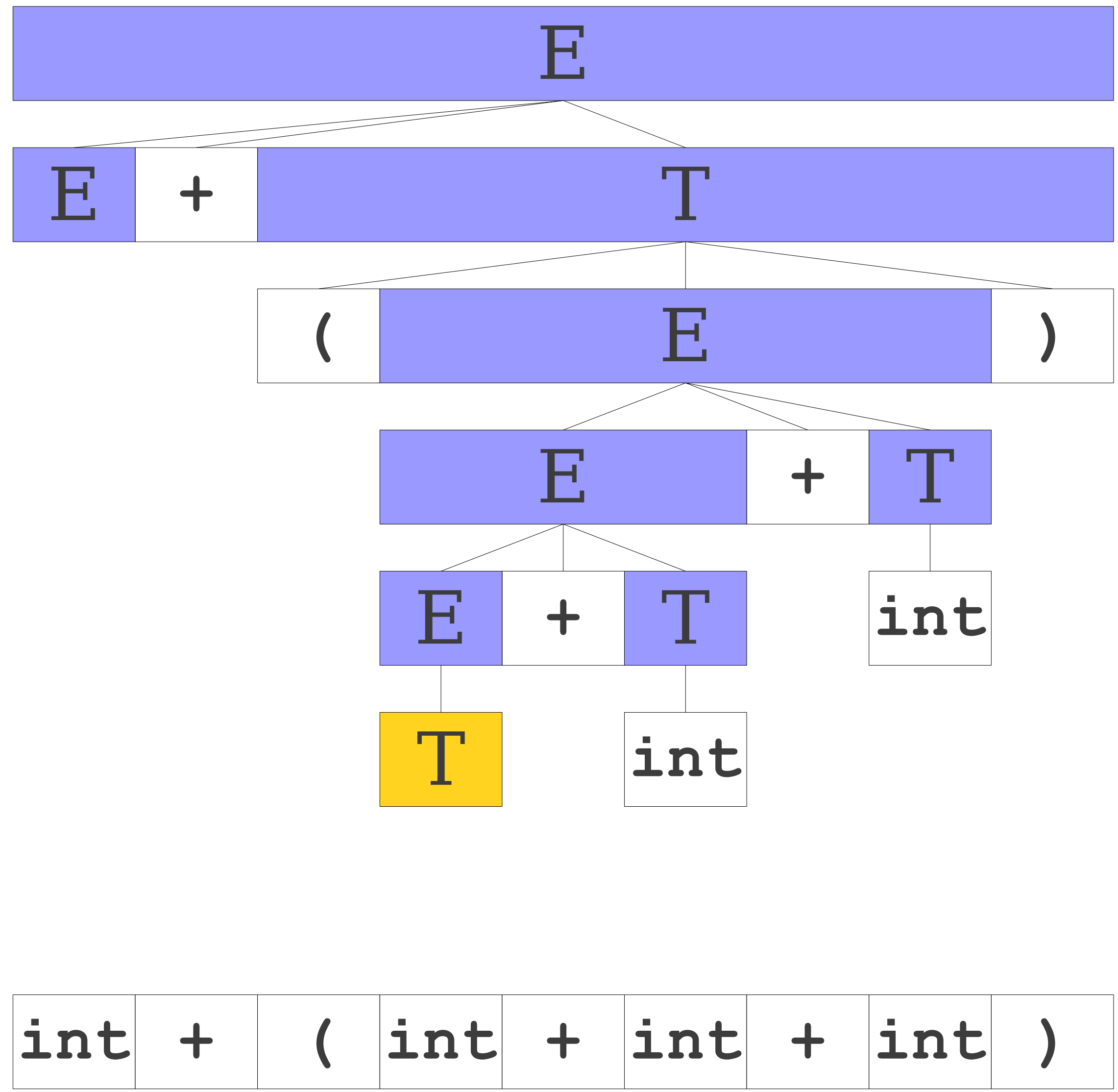
# A Third View of a Bottom-Up Parse

⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



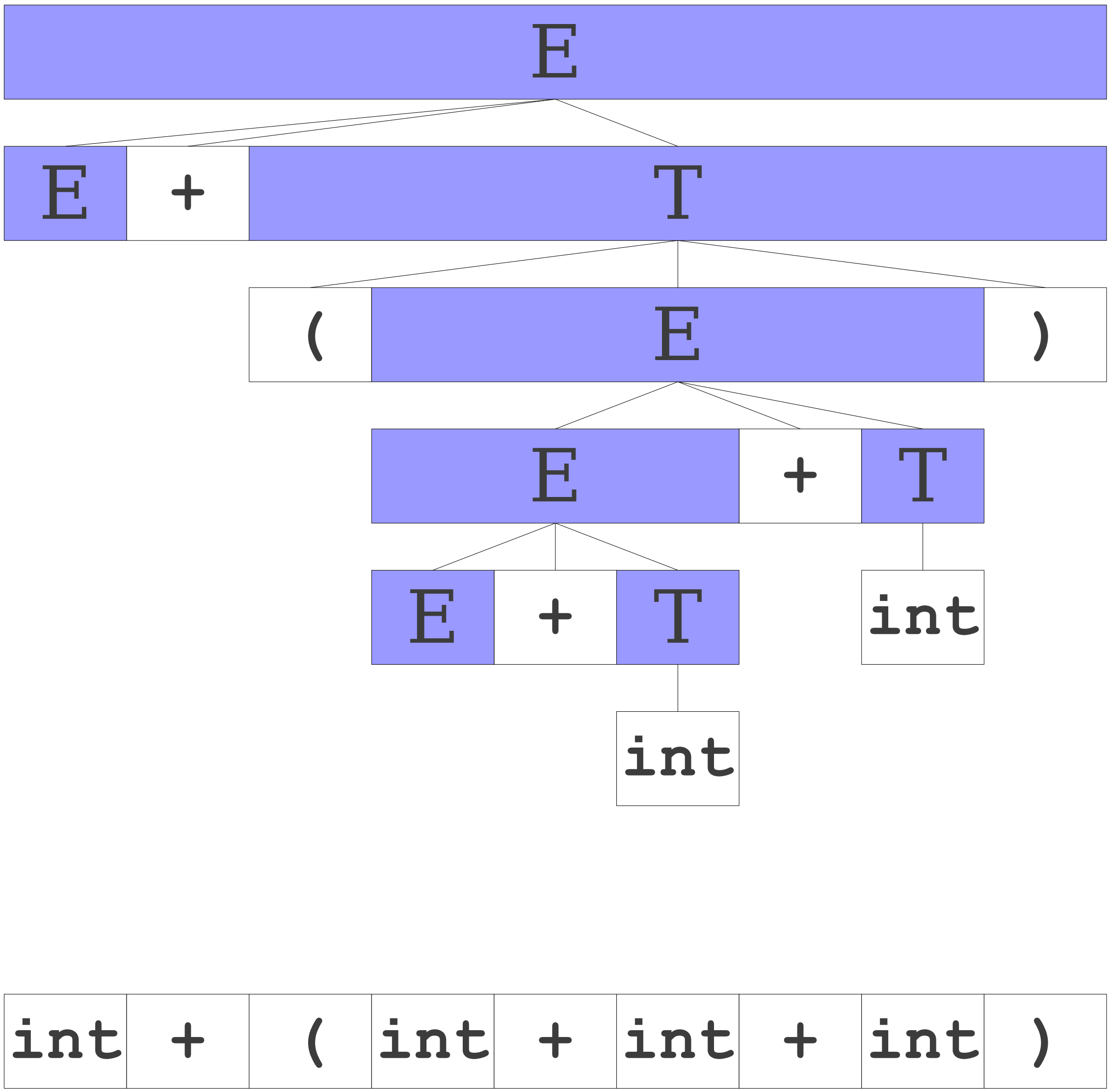
# A Third View of a Bottom-Up Parse

⇒ E + (T + int + int)
⇒ E + (E + int + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



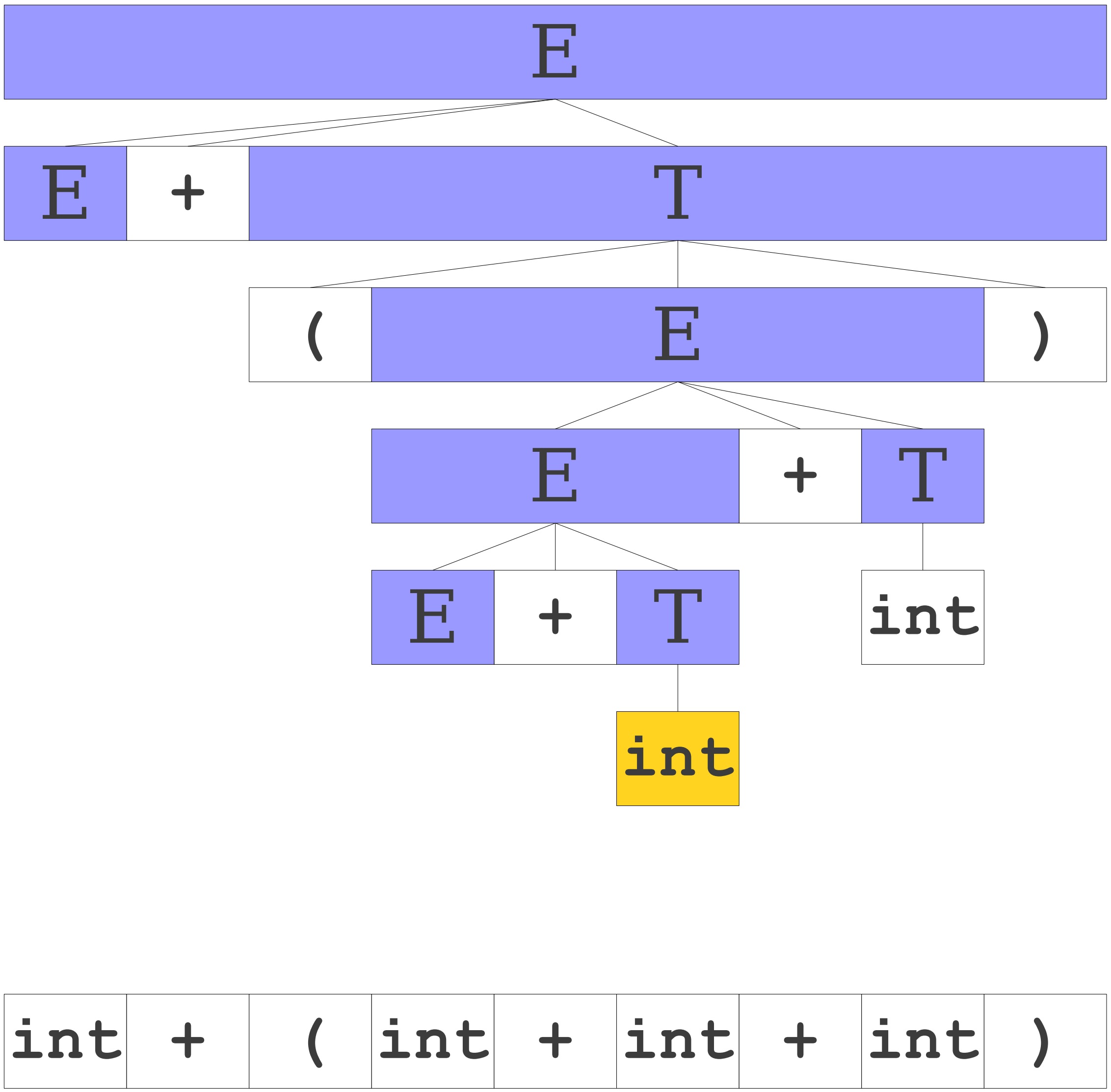
# A Third View of a Bottom-Up Parse

⇒ **E** + (**E** + int + int)
⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**

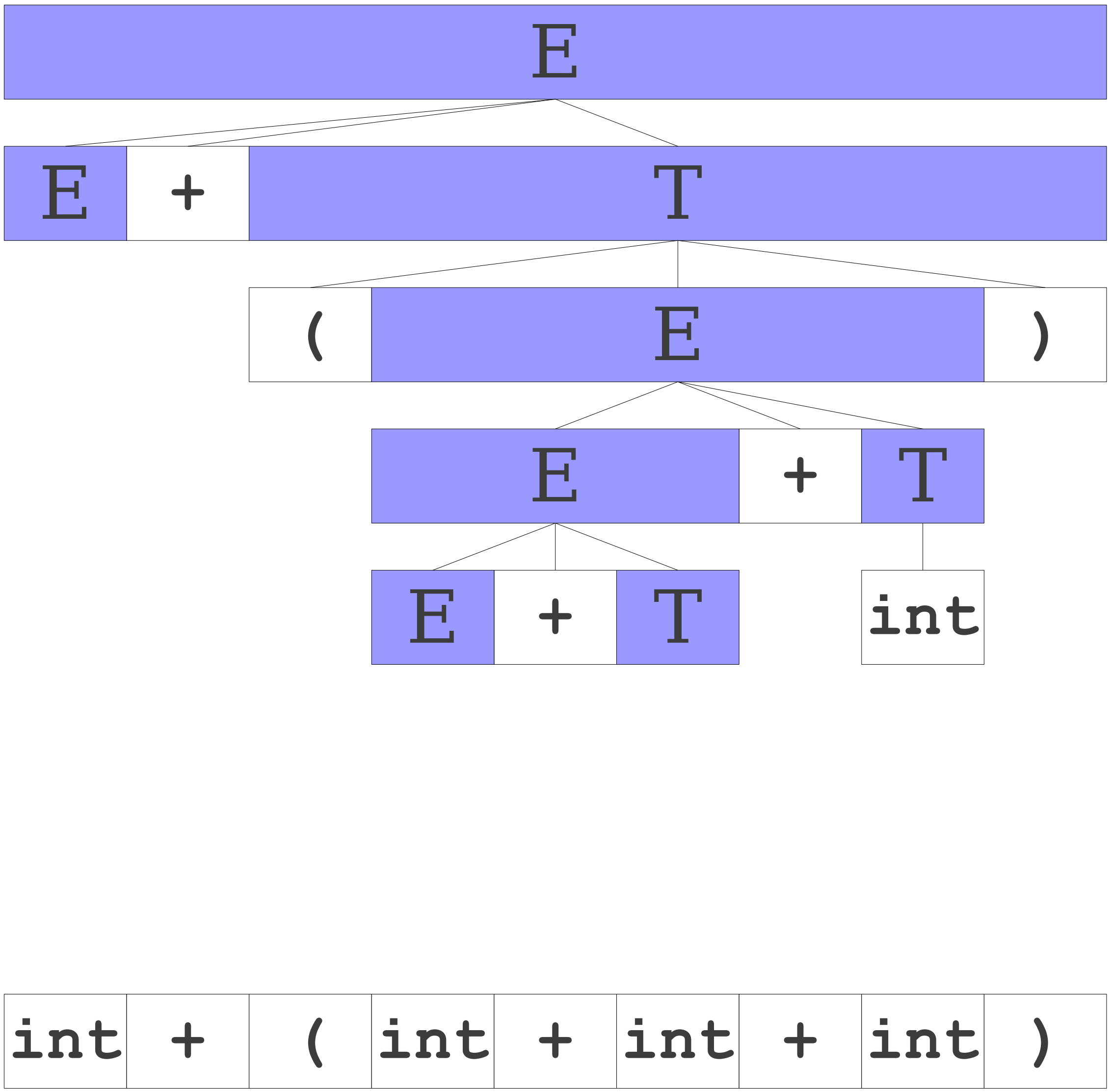


# A Third View of a Bottom-Up Parse

⇒ E + (E + **int** + int)
⇒ E + (E + T + int)
⇒ E + (E + int)
⇒ E + (E + T)
⇒ E + (E)
⇒ E + T
⇒ E



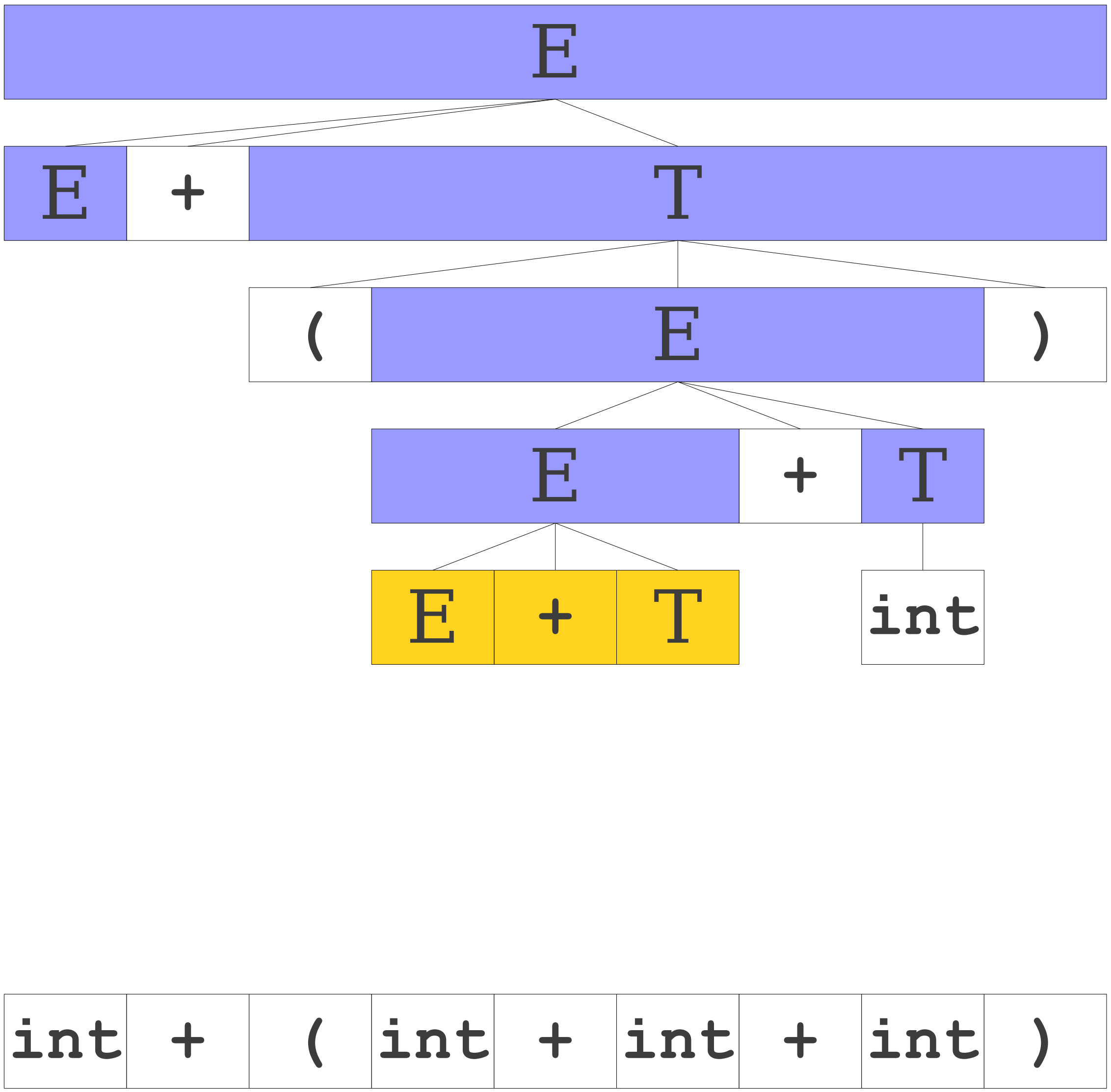
# A Third View of a Bottom-Up Parse



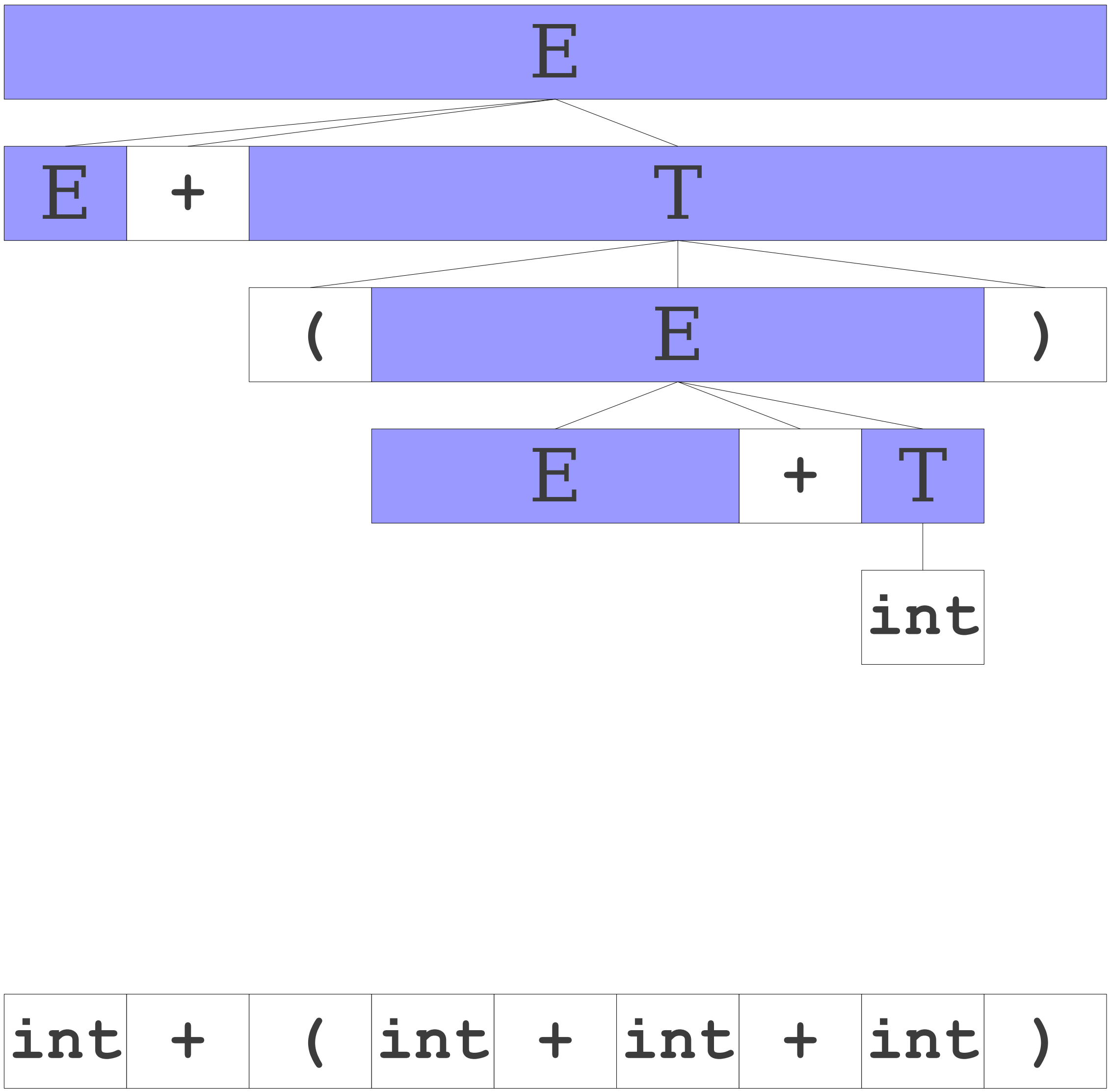

⇒ **E + (E + T + int)**
⇒ **E + (E + int)**
⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

# A Third View of a Bottom-Up Parse

⇒ **E** + (**E** + **T** + int)
⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



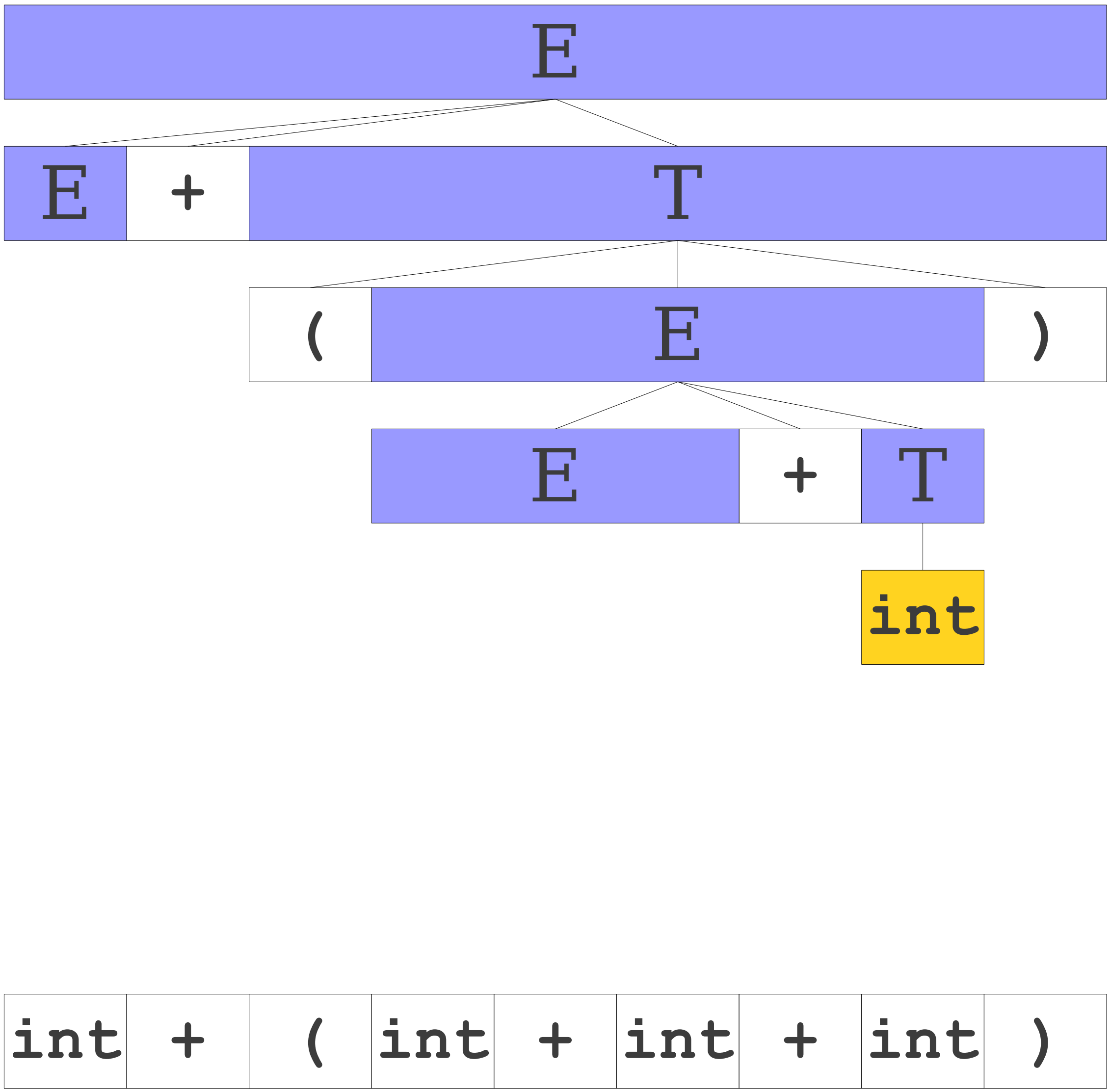
# A Third View of a Bottom-Up Parse



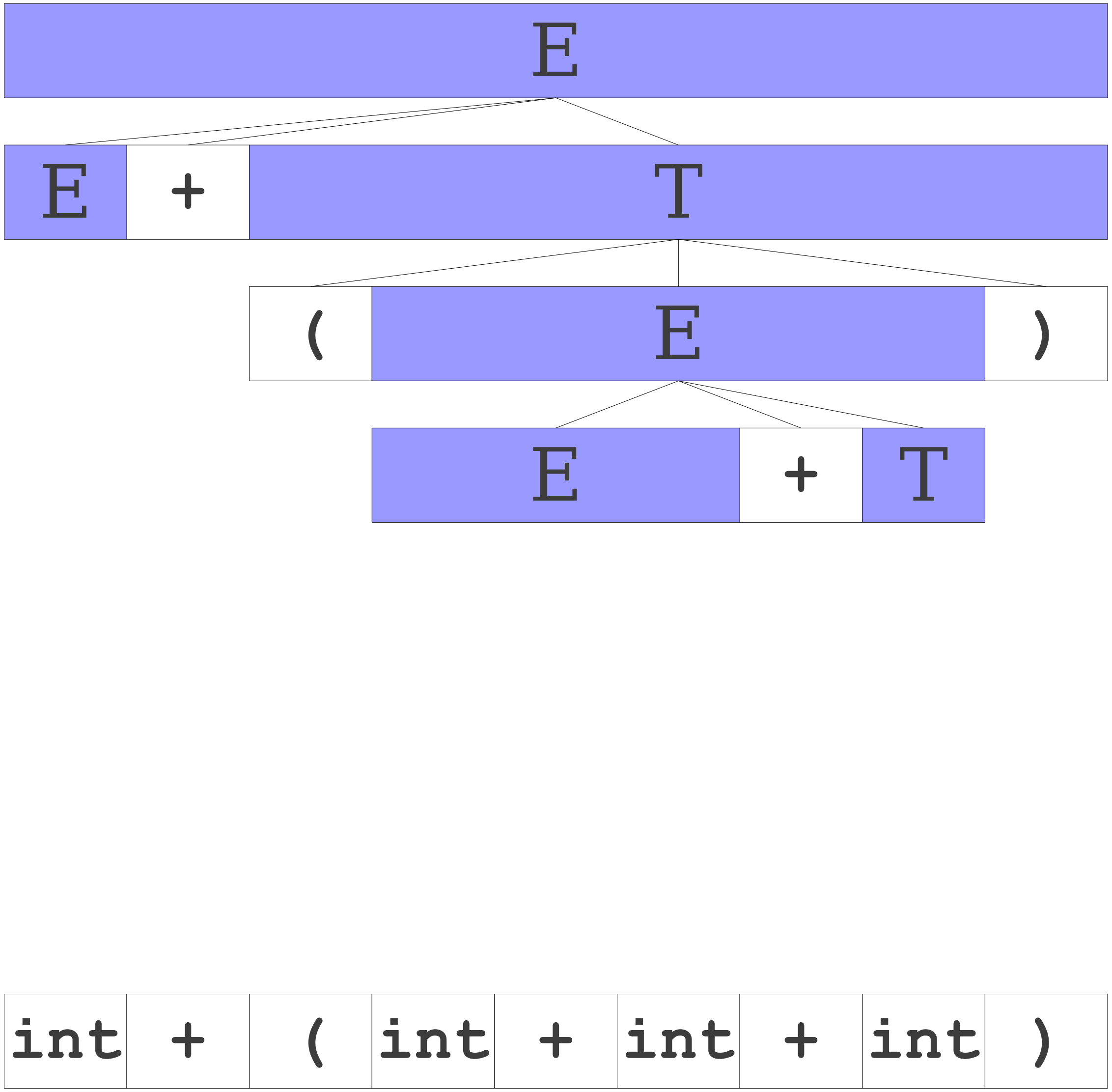

⇒ **E + (E + int)**
⇒ **E + (E + T)**
⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

# A Third View of a Bottom-Up Parse

⇒ **E** + (**E** + int)
⇒ **E** + (**E** + **T**)
⇒ **E** + (**E**)
⇒ **E** + **T**
⇒ **E**



# A Third View of a Bottom-Up Parse



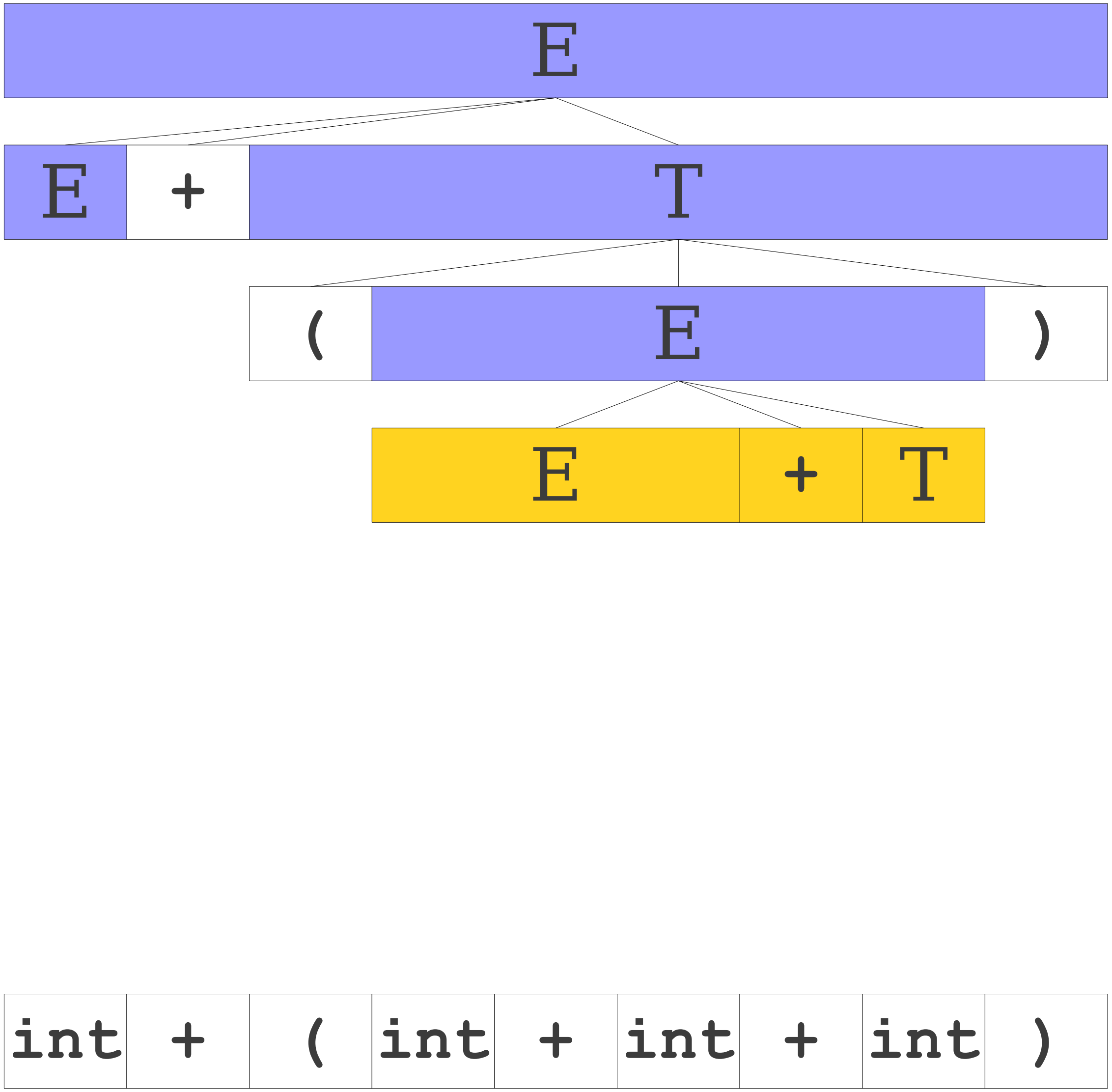

⇒ **E + (E + T)**

⇒ **E + (E)**

⇒ **E + T**

⇒ **E**

# A Third View of a Bottom-Up Parse



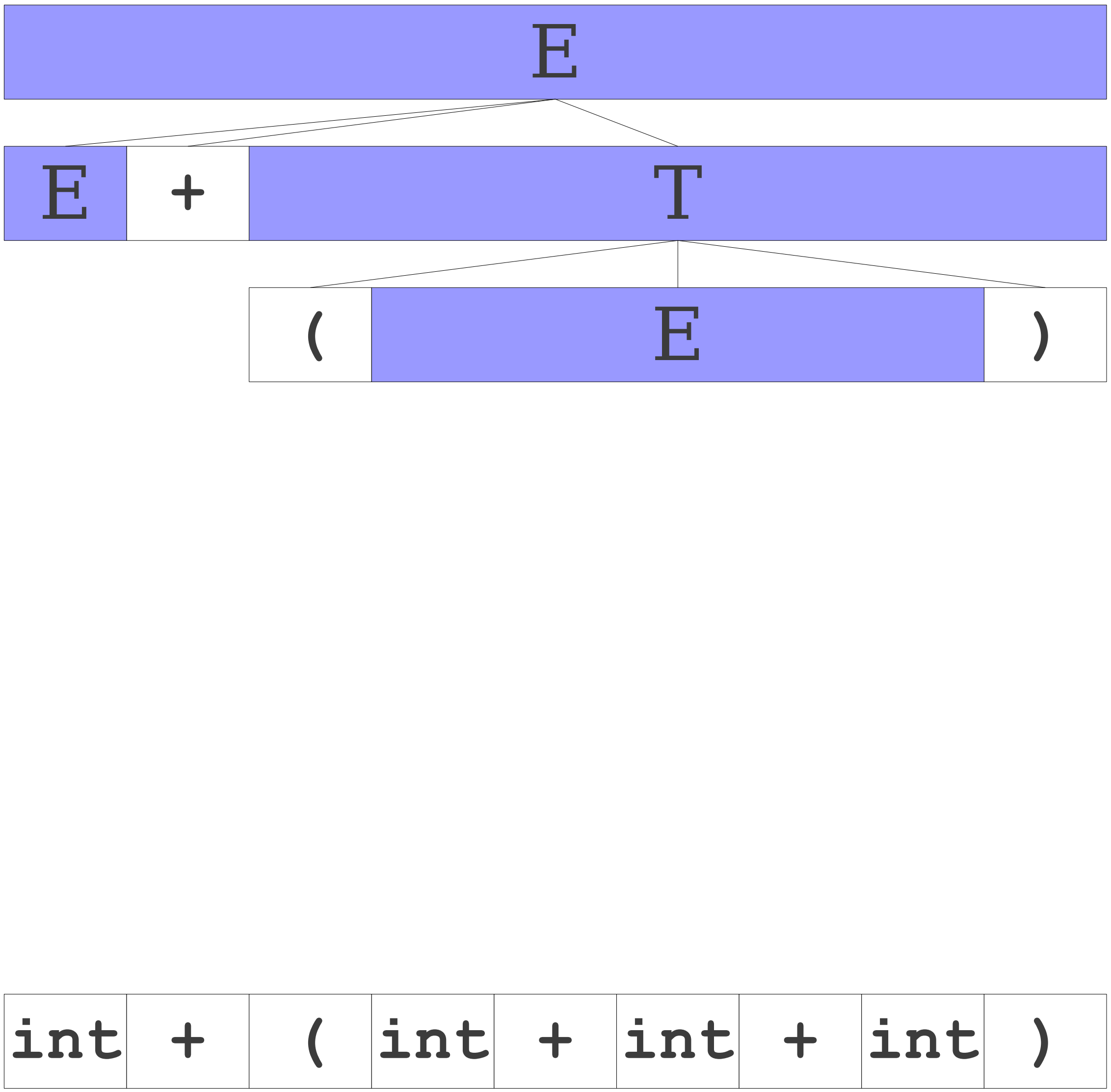

⇒ **E + (E + T)**

⇒ **E + (E)**

⇒ **E + T**

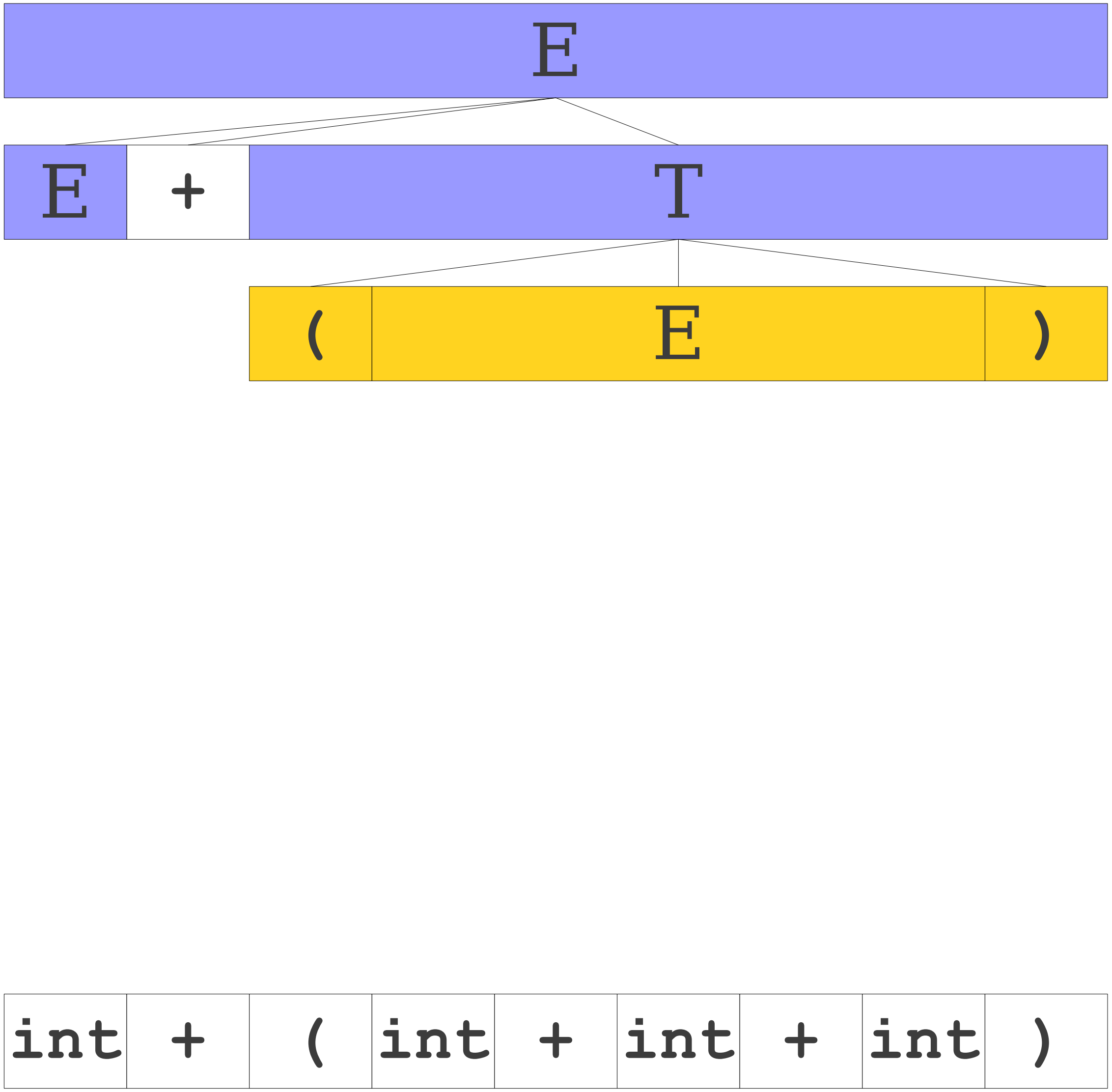
⇒ **E**

# A Third View of a Bottom-Up Parse




⇒ **E + (E)**
⇒ **E + T**
⇒ **E**

# A Third View of a Bottom-Up Parse

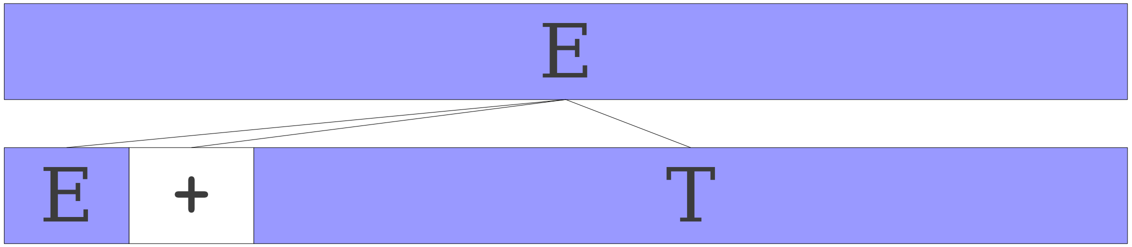



⇒ **E + (E)**

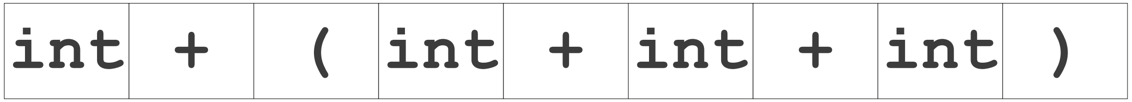
⇒ **E + T**

⇒ **E**

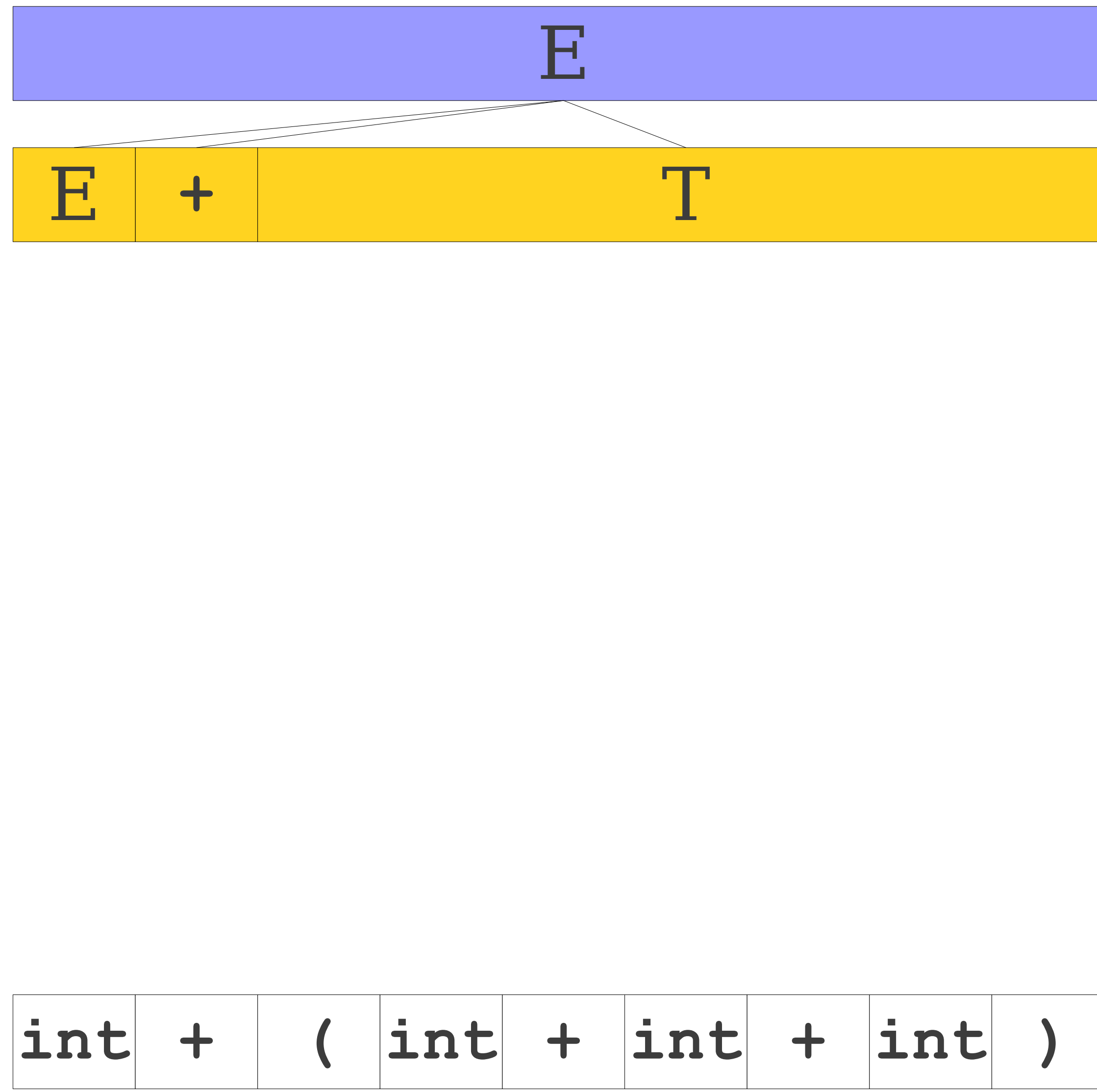
# A Third View of a Bottom-Up Parse




⇒ **E + T**
⇒ **E**



# A Third View of a Bottom-Up Parse




⇒ **E + T**

⇒ **E**

# A Third View of a Bottom-Up Parse

| E |
|---|

⇒ **E**

| int | + | ( | int | + | int | + | int | ) |
|---|---|---|---|---|---|---|---|---|

# Handles

- The **handle** of a parse tree *T* is the leftmost complete cluster of leaf nodes.
- A left-to-right, bottom-up parse works by iteratively searching for a handle, then reducing the handle.

# Question One:

Where are handles?

# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

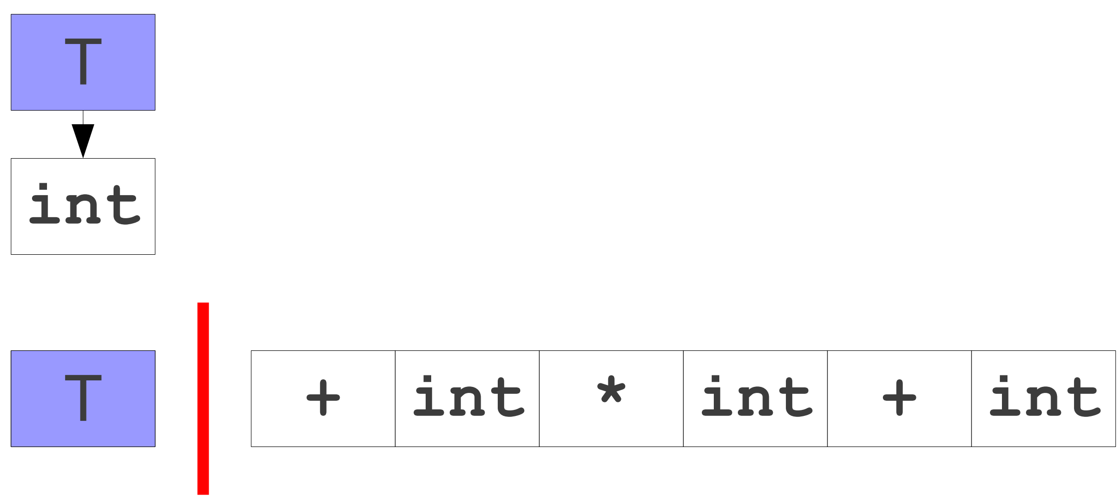
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| `int` | \| | `+` | `int` | `*` | `int` | `+` | `int` |

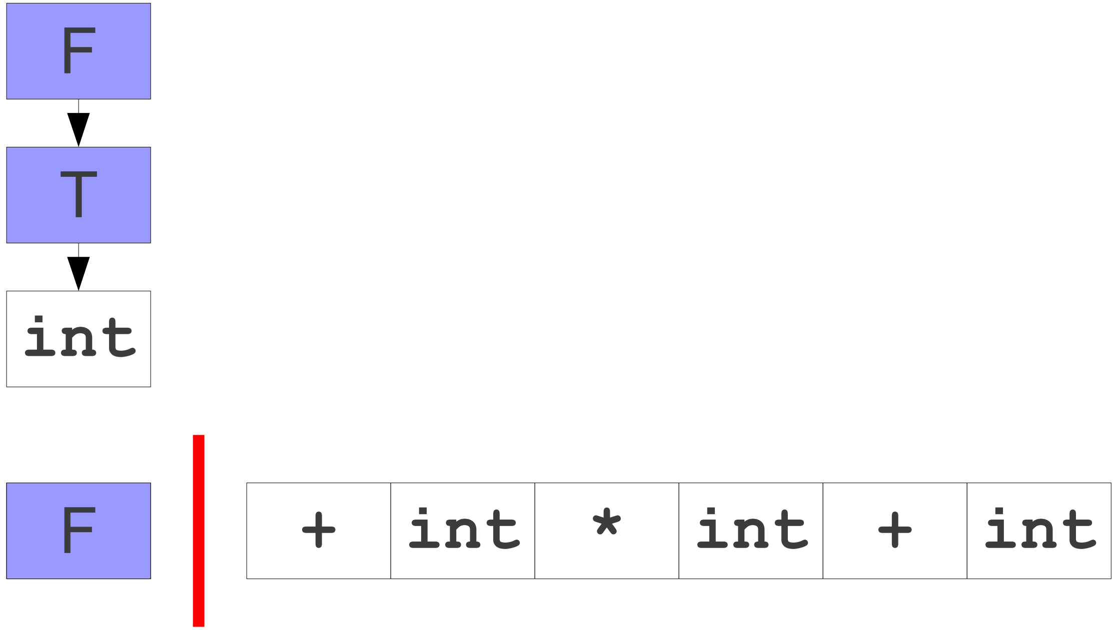
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)



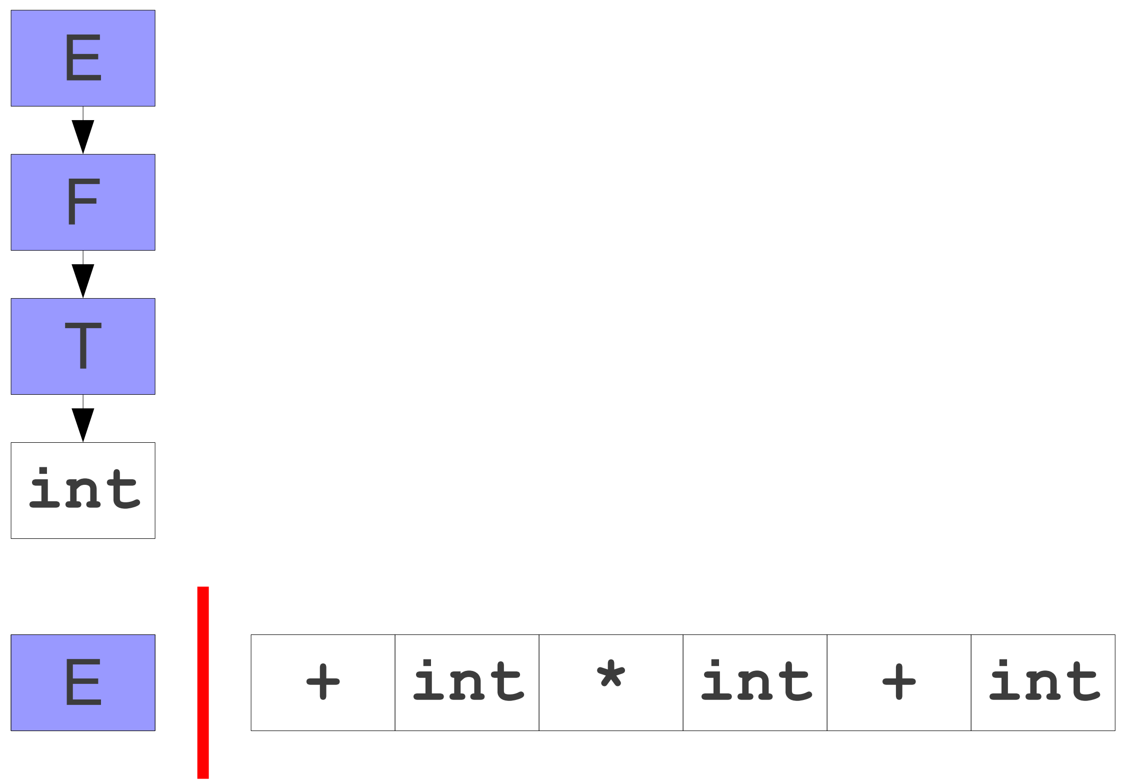
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



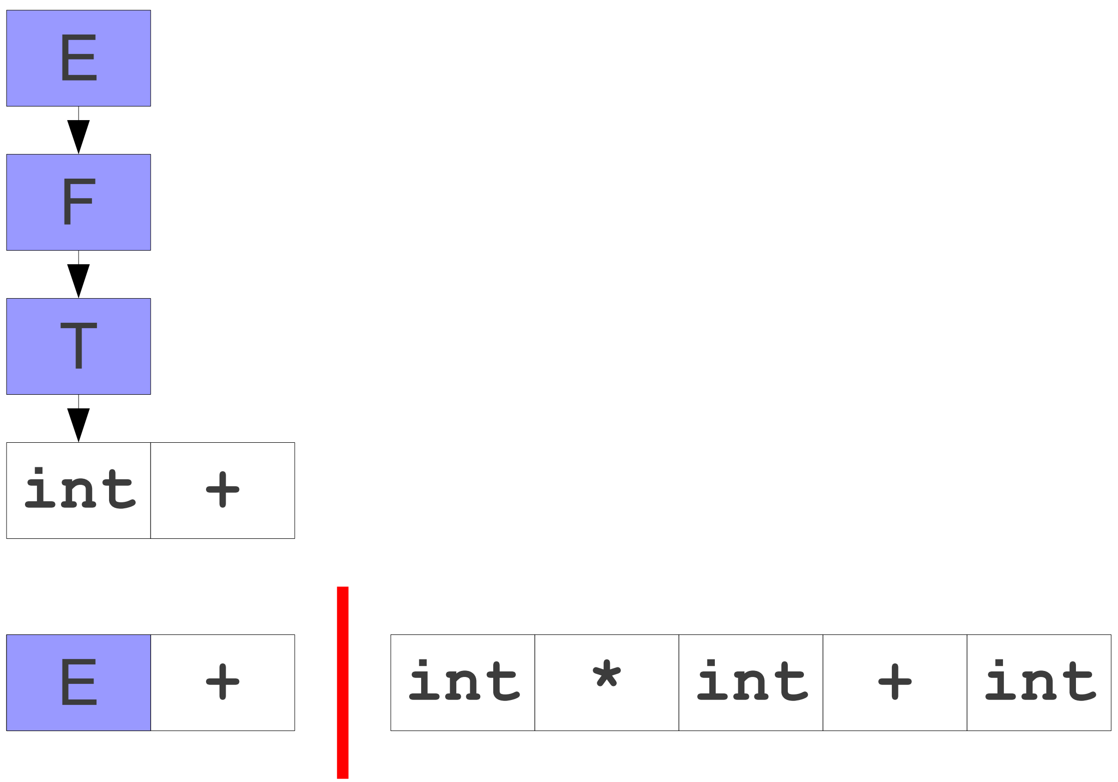
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)



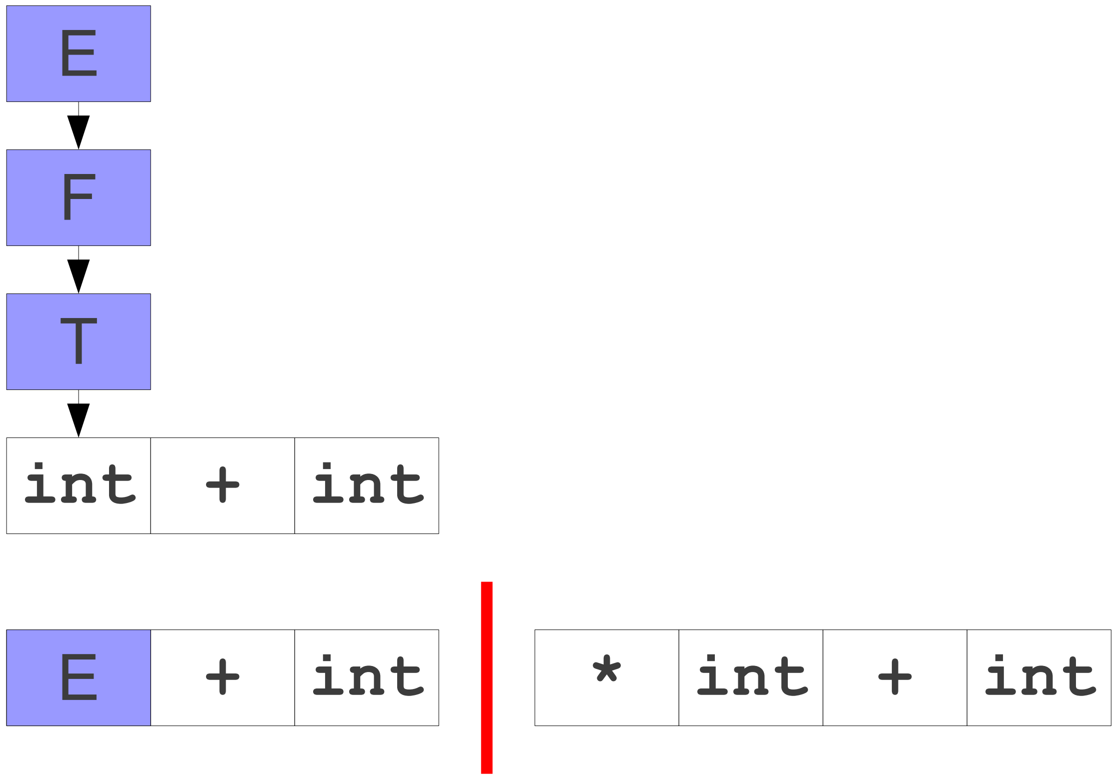
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



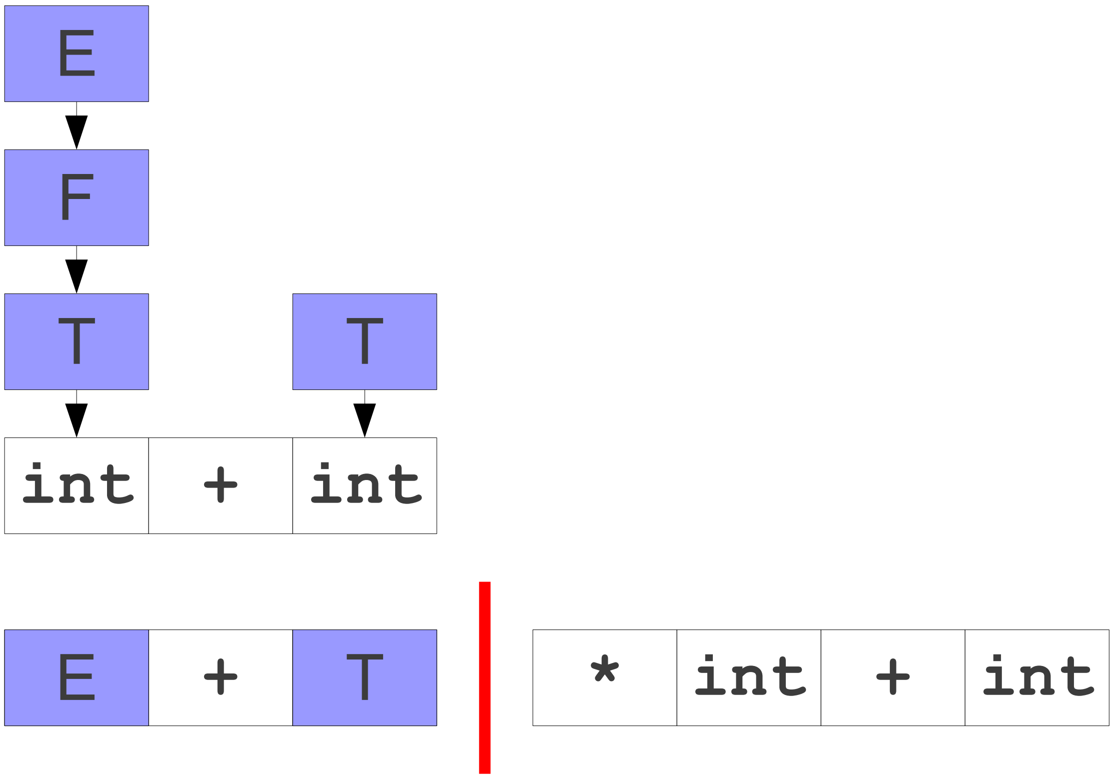
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



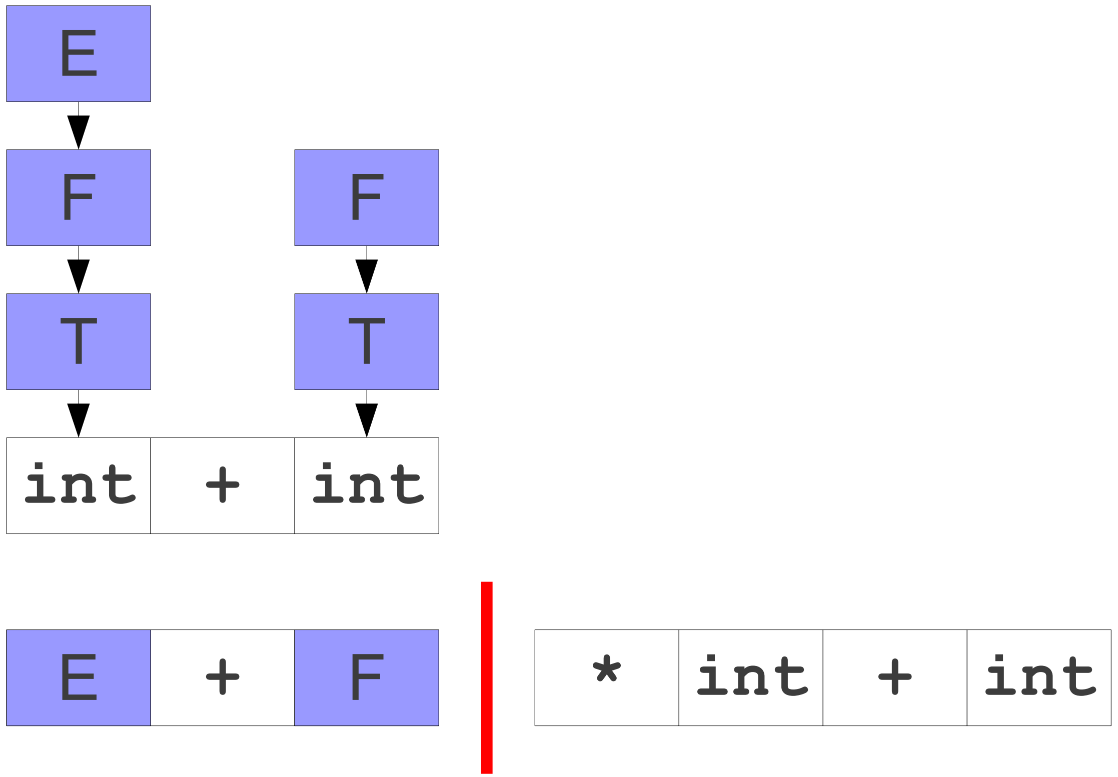
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



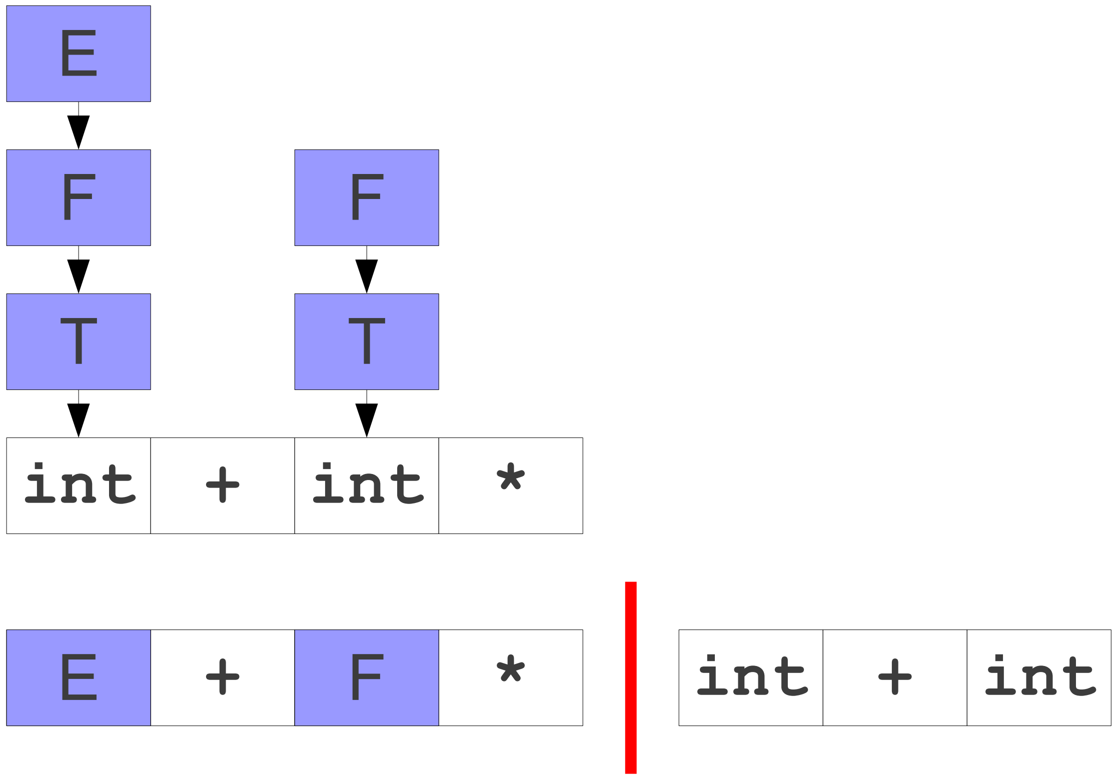
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → `int`
**T** → (**E**)



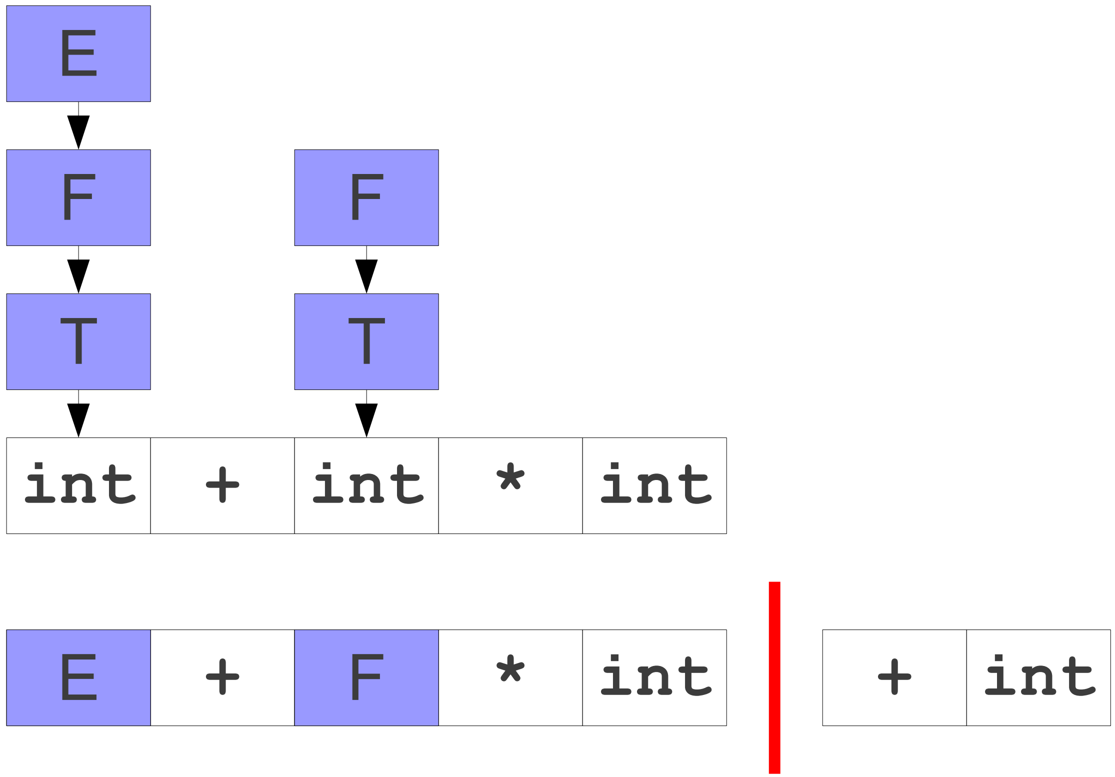
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)



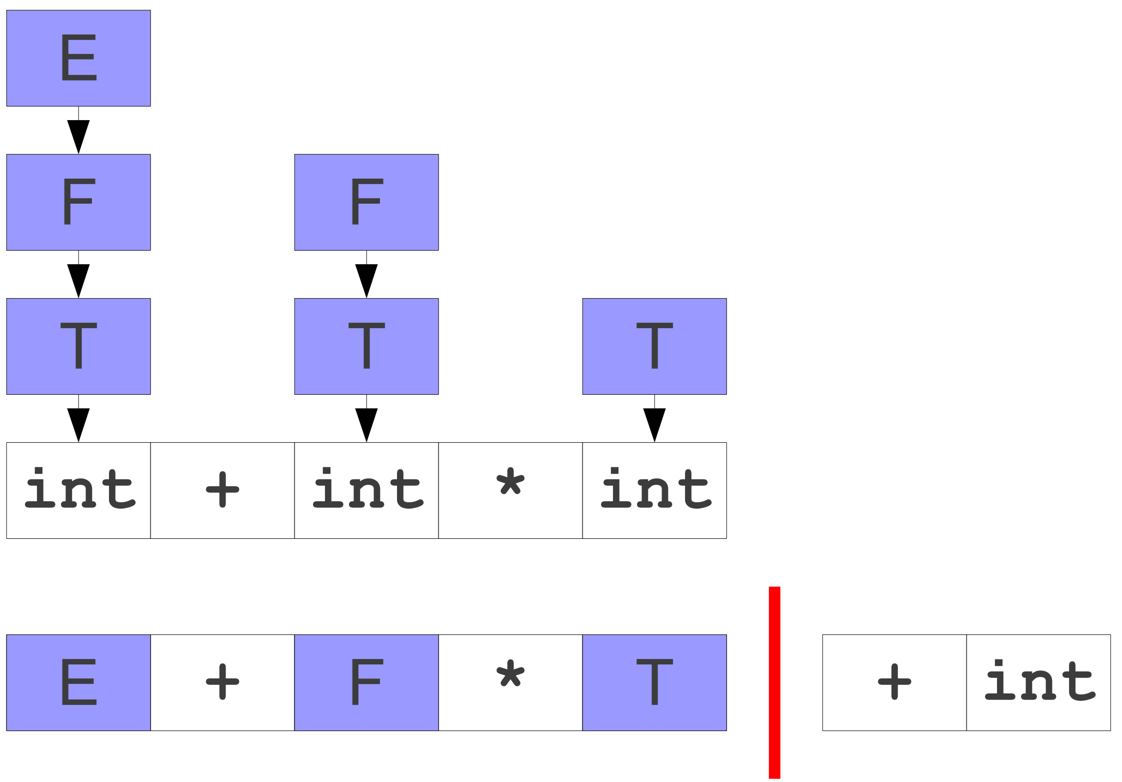
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



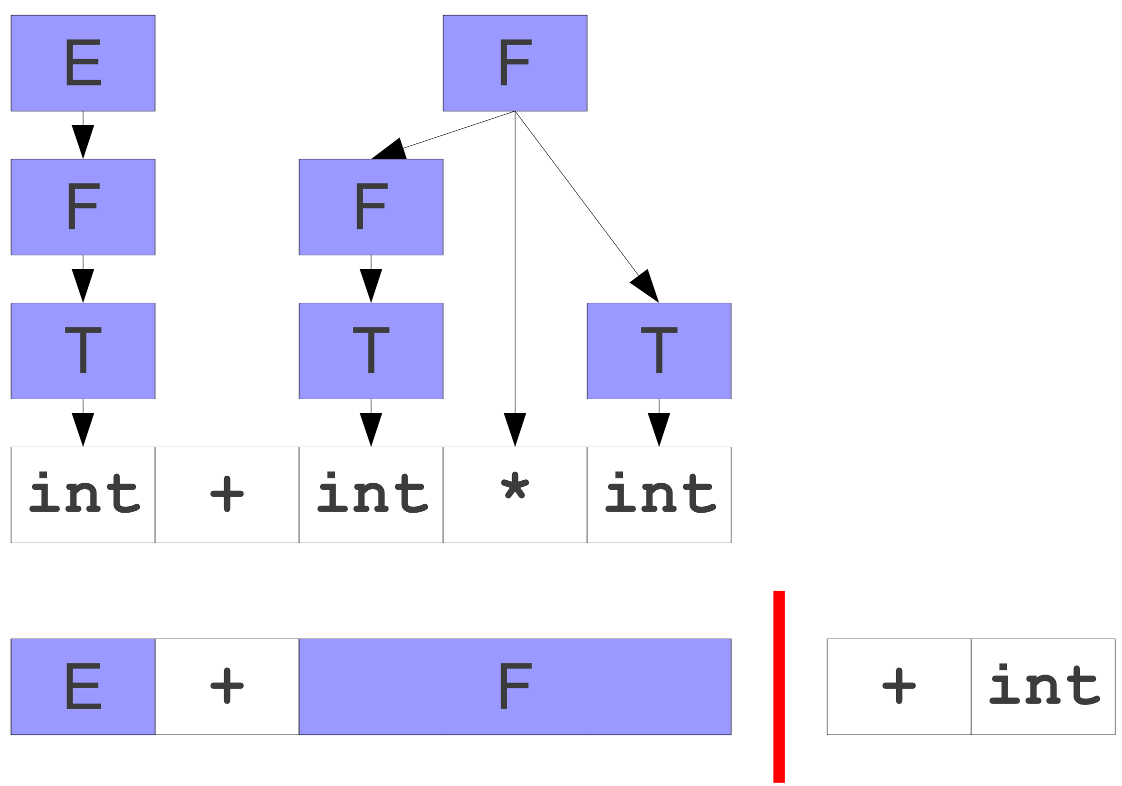
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



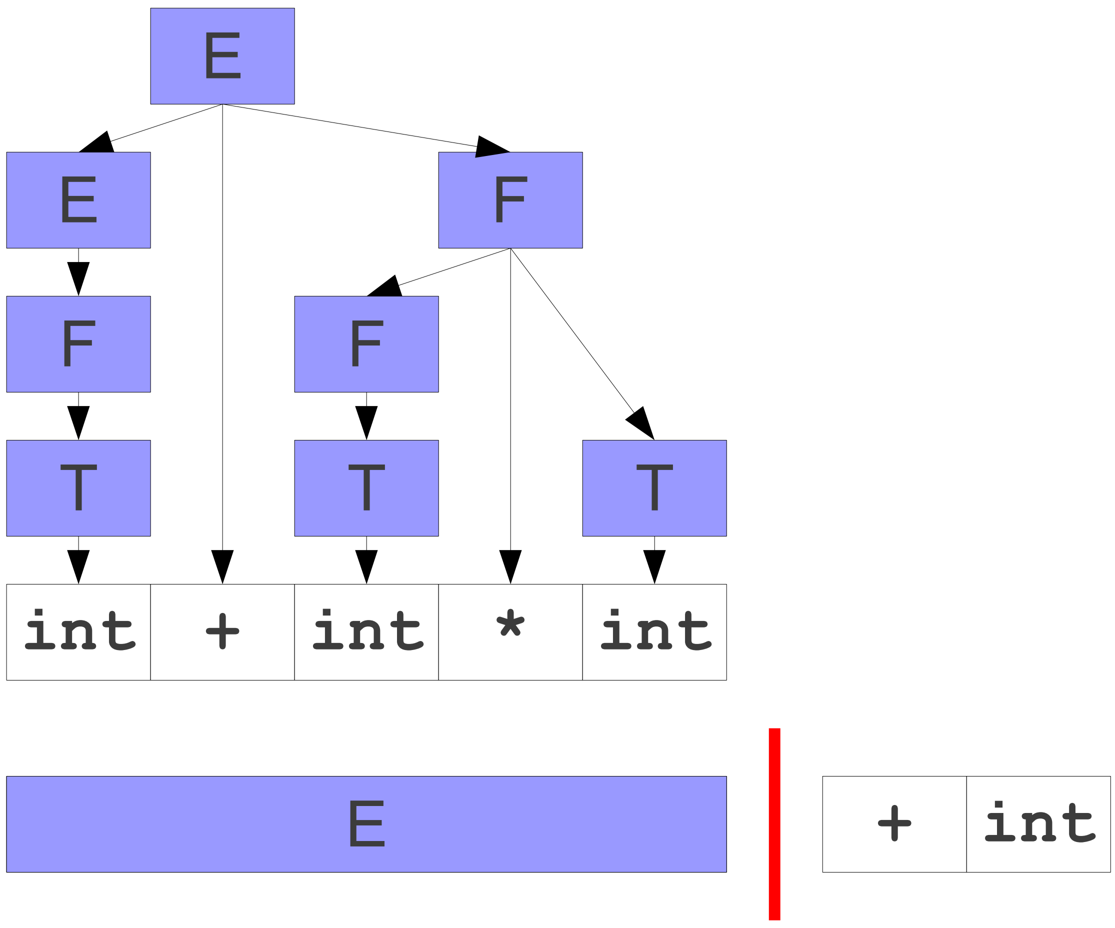
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



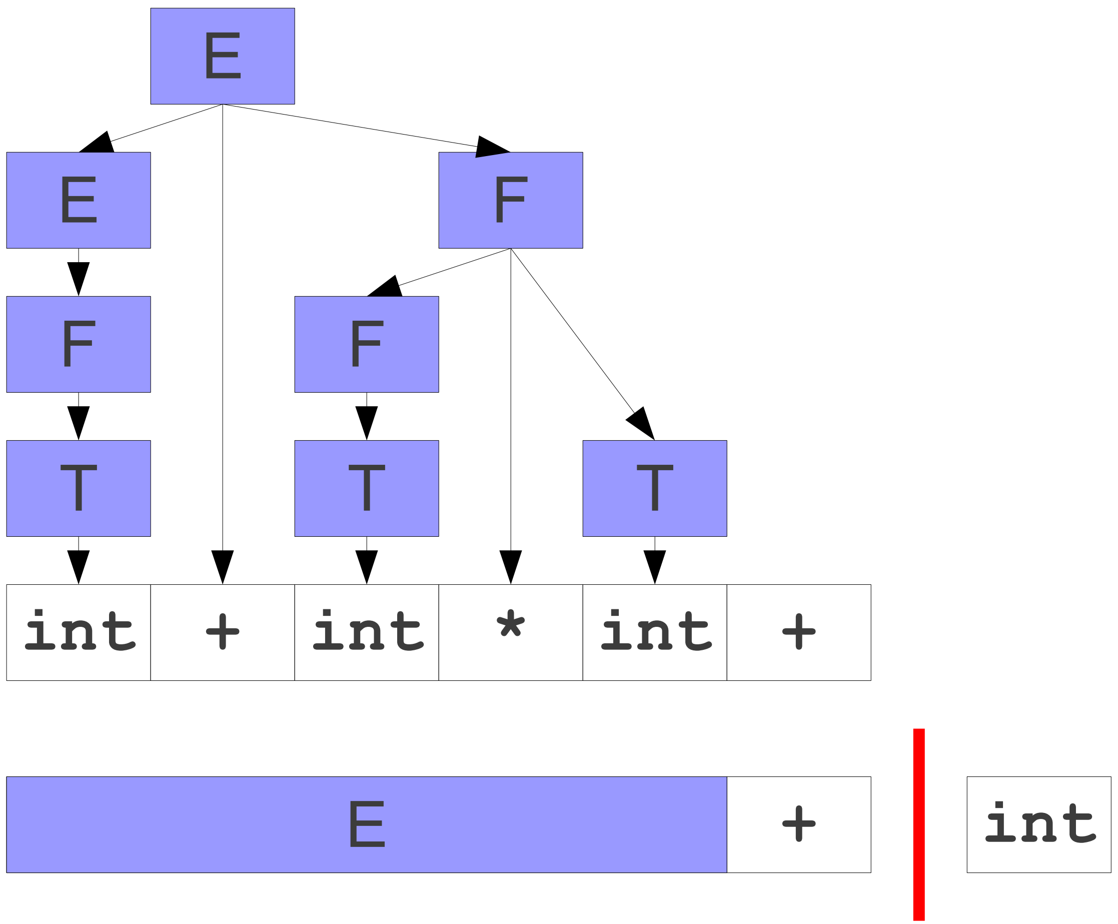
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



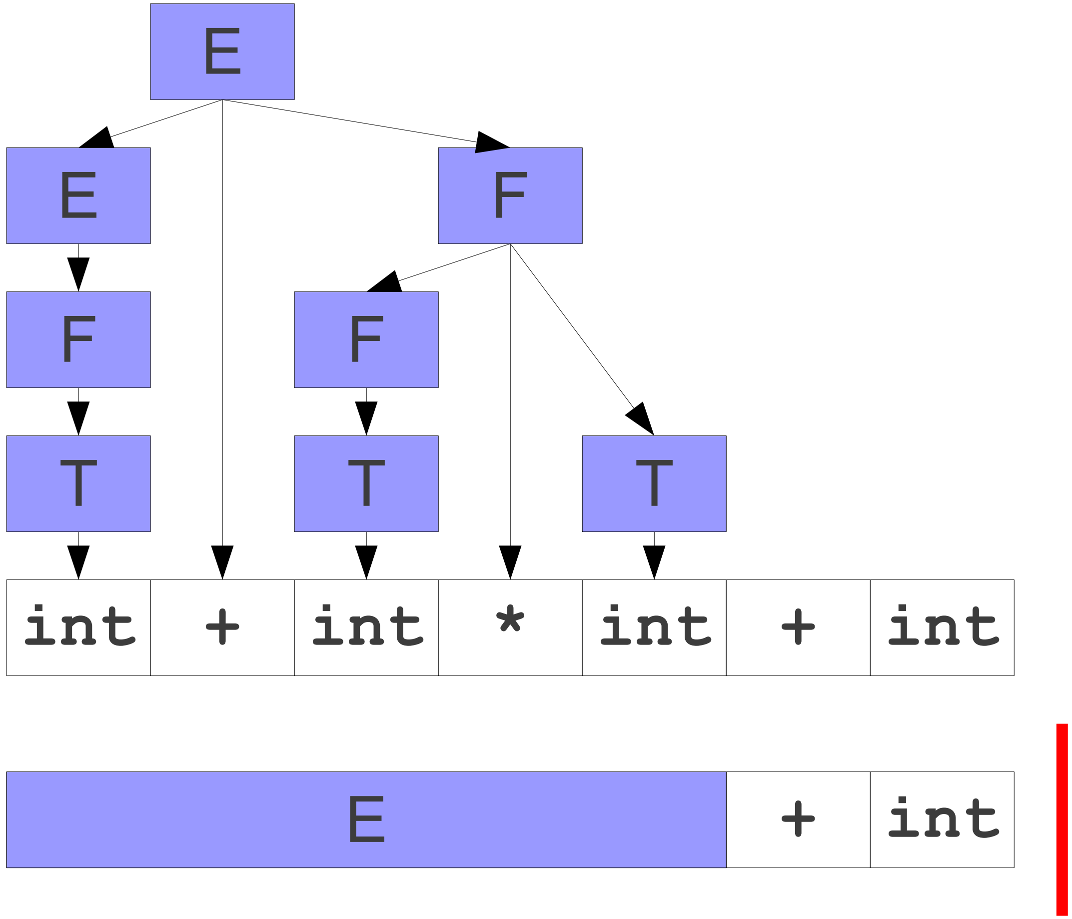
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



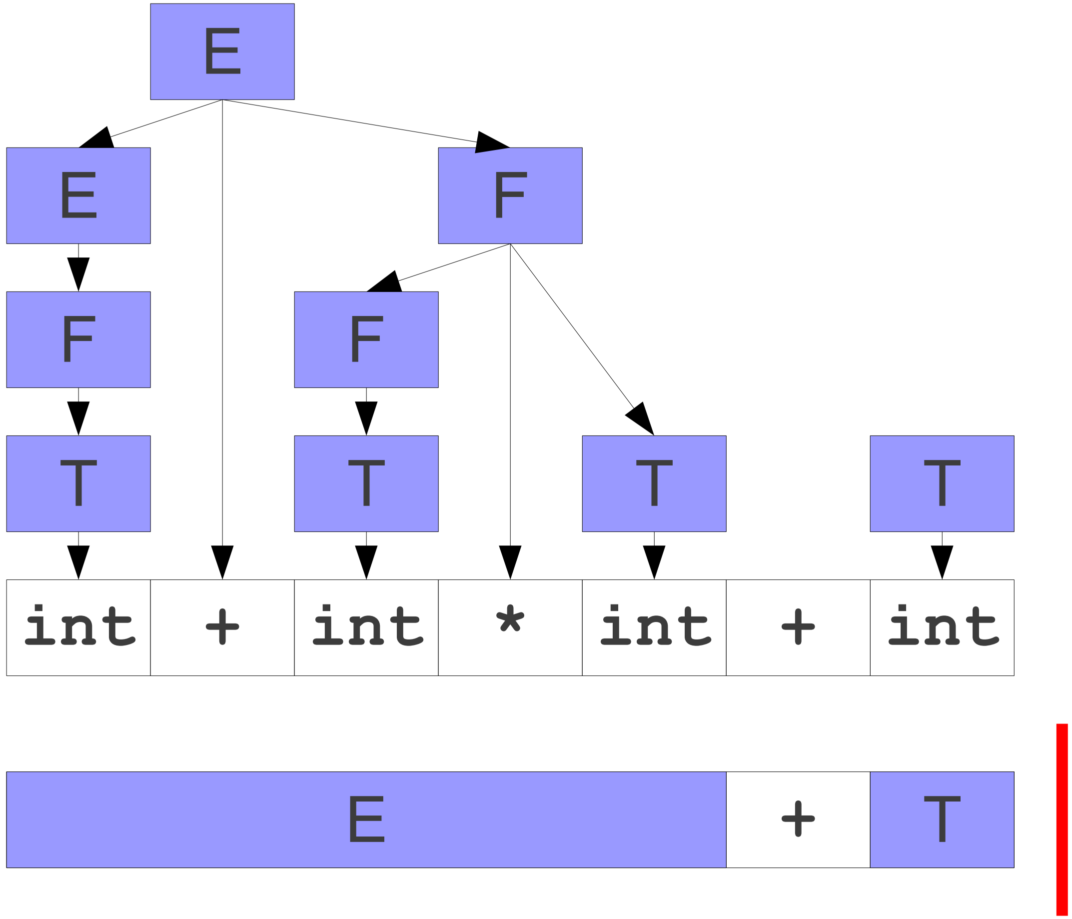
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)



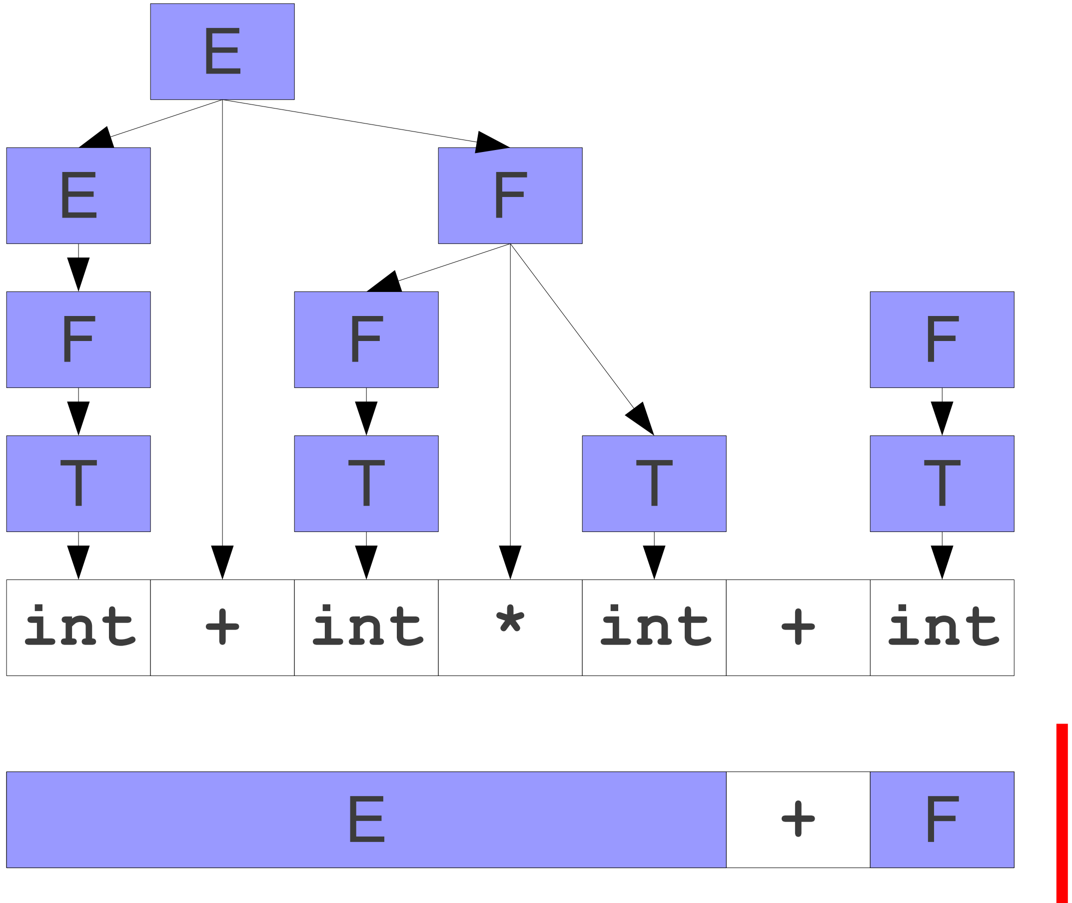
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



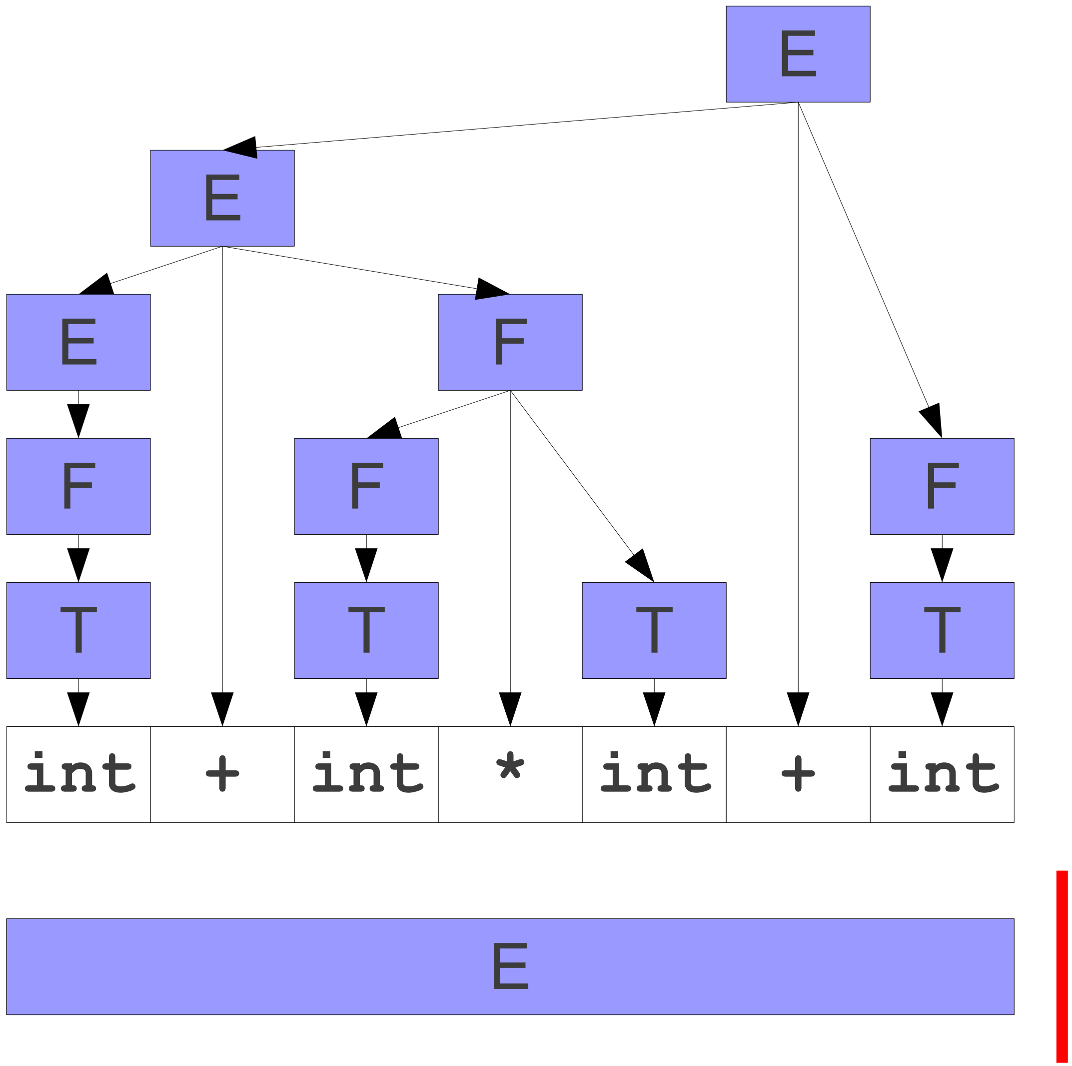
# A Sample Shift/Reduce Parse

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



# A Sample Shift/Reduce Parse

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



# An Important Corollary

- Since reductions are always at the right side of the left area, we never need to shift from the left to the right.
- No need to "uncover" something to do a reduction.
- Consequently, shift/reduce parsing means
  - **Shift**: Move a terminal from the right to the left area.
  - **Reduce**: Replace some number of symbols at the right side of the left area.

# Finding Handles

- Where do we look for handles?
  - **At the top of the stack.**
- How do we search for handles?
  - What algorithm do we use to try to discover a handle?
- How do we recognize handles?
  - Once we've found a possible handle, how do we confirm that it's correct?

# Question Two:

How do we search for handles?

# Searching for Handles

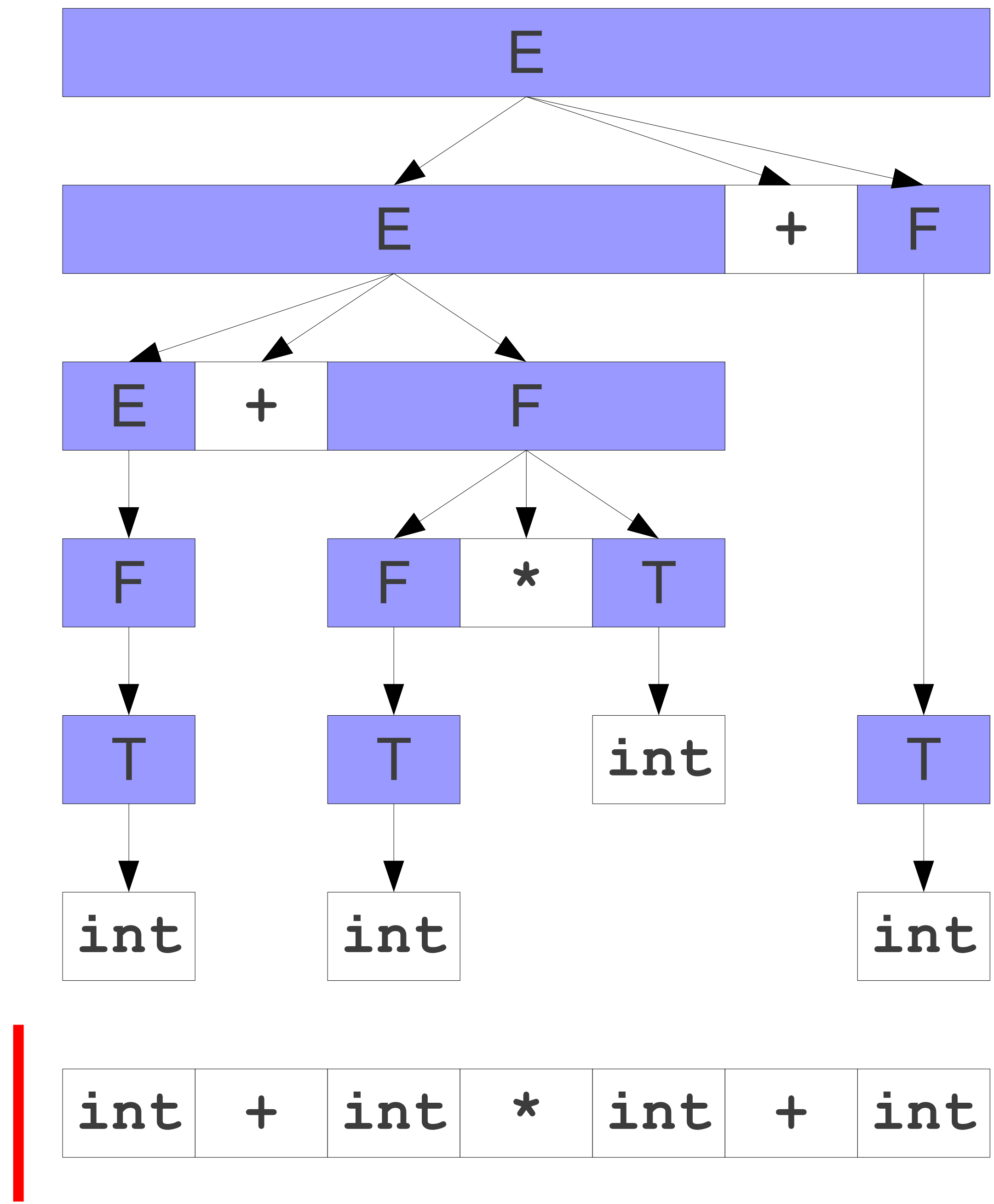
- When using a shift/reduce parser, we must decide whether to shift or reduce at each point.
- We only want to reduce when we know we have a handle.
- **Question:** How can we tell that we might be looking at a handle?

# Exploring the Left Side

- The handle will always appear at the end of string in the left side of the parser.
- Can *any* string appear on the left side of the parser, or are there restrictions on what sorts of strings can appear there?
- If we can find a pattern to the strings that can appear on the left side, we might be able to exploit it to detect handles.

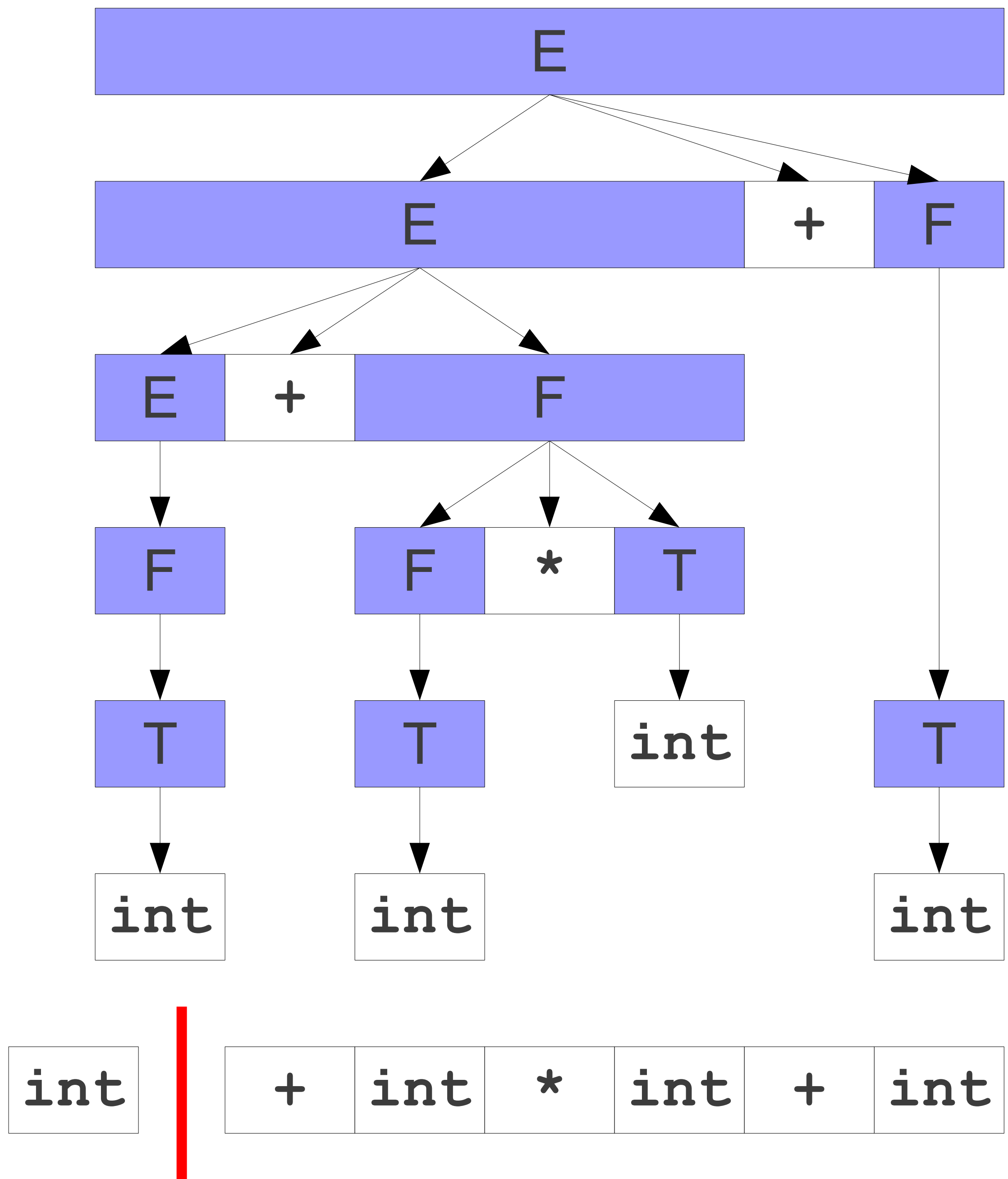
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



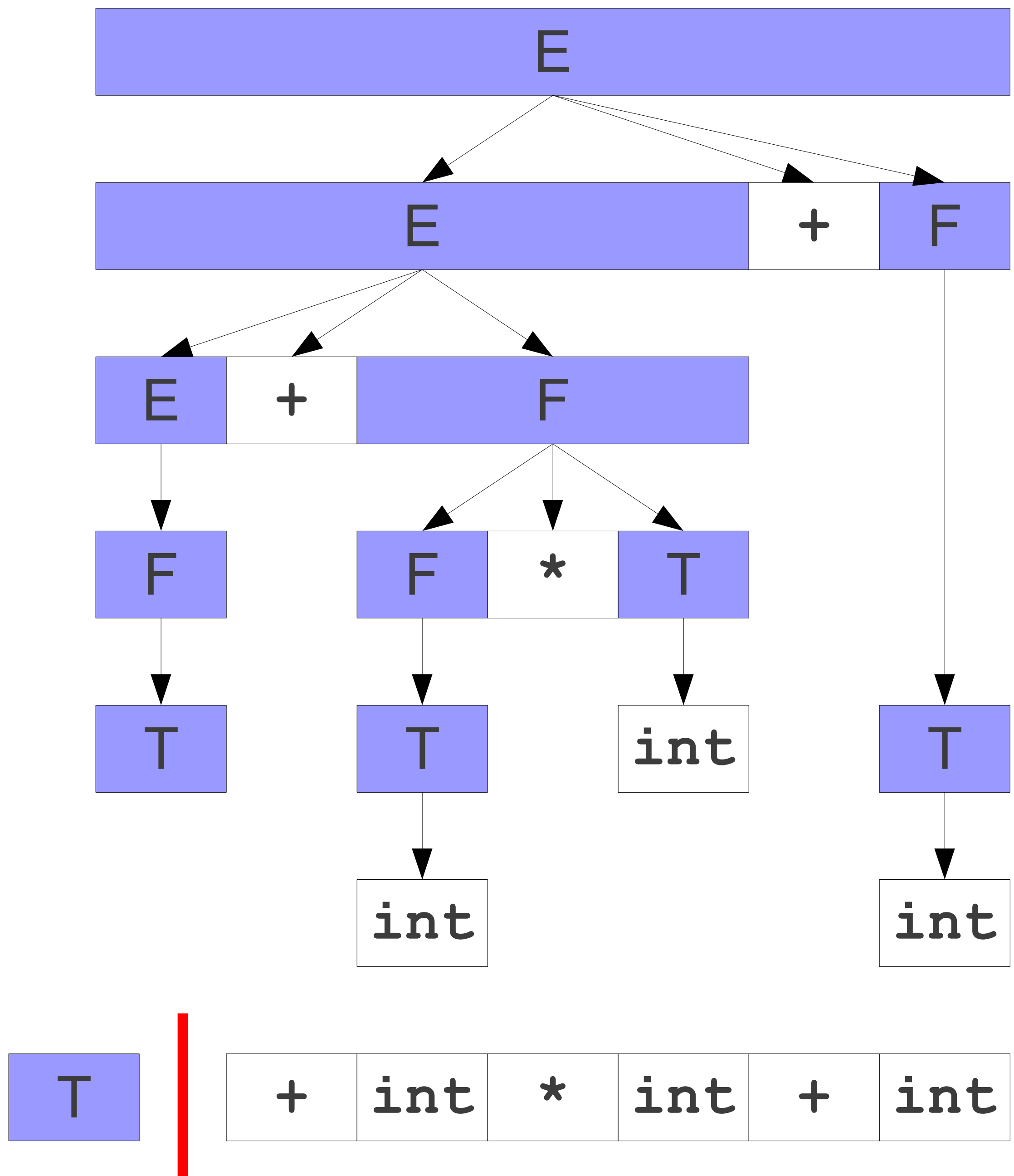
# Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



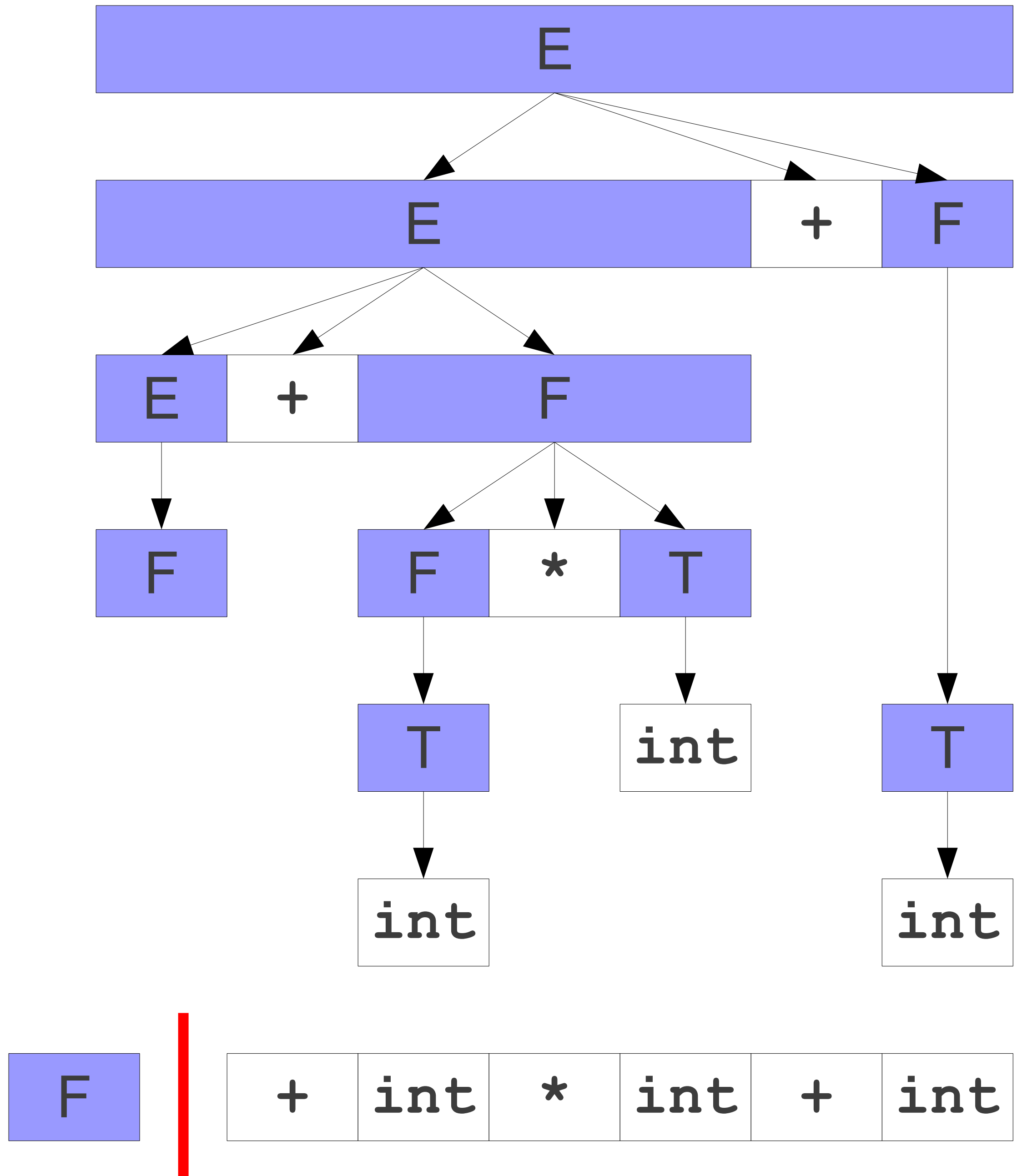
# Another Look at Handles

$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow (E)$



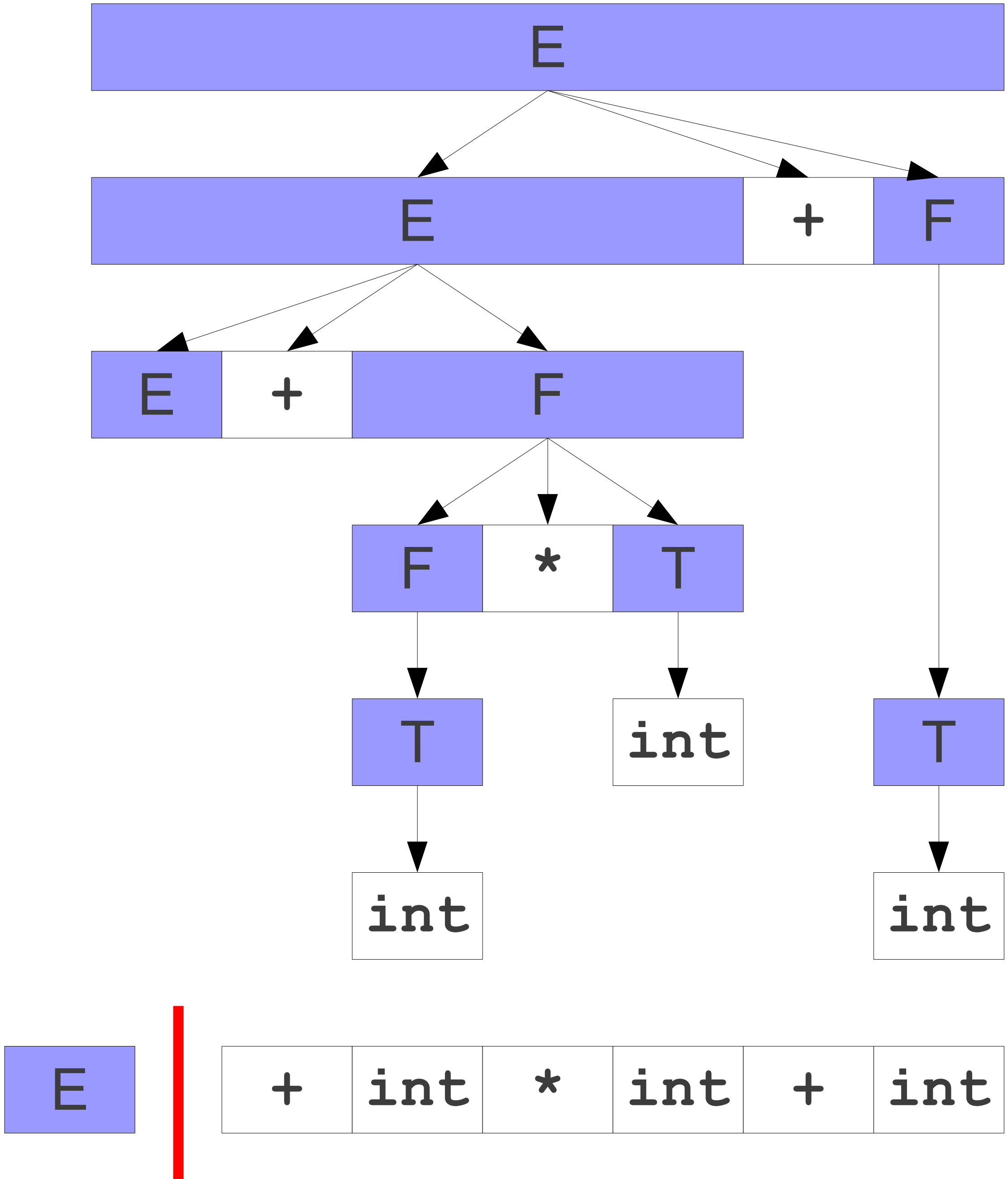
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



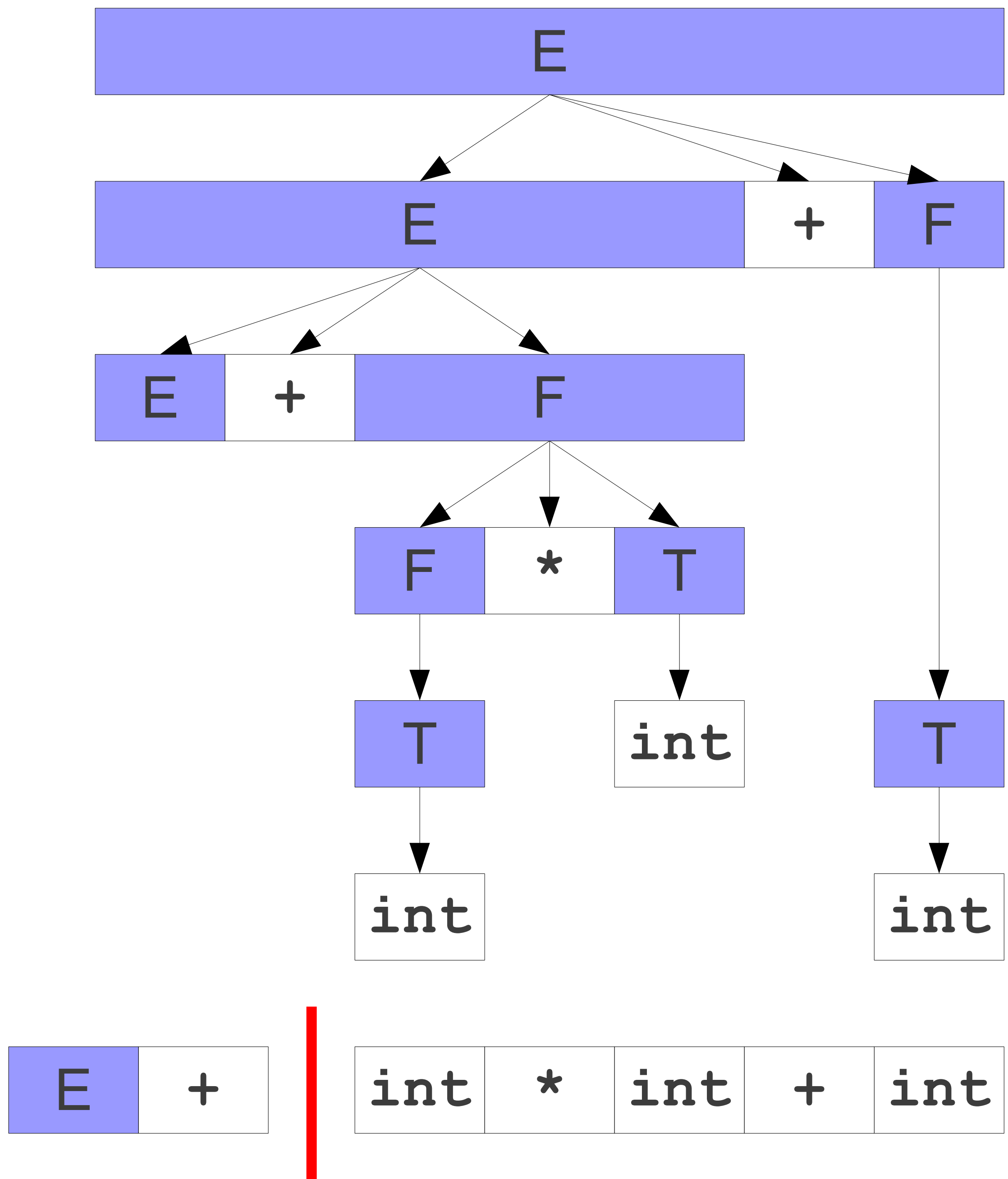
# Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



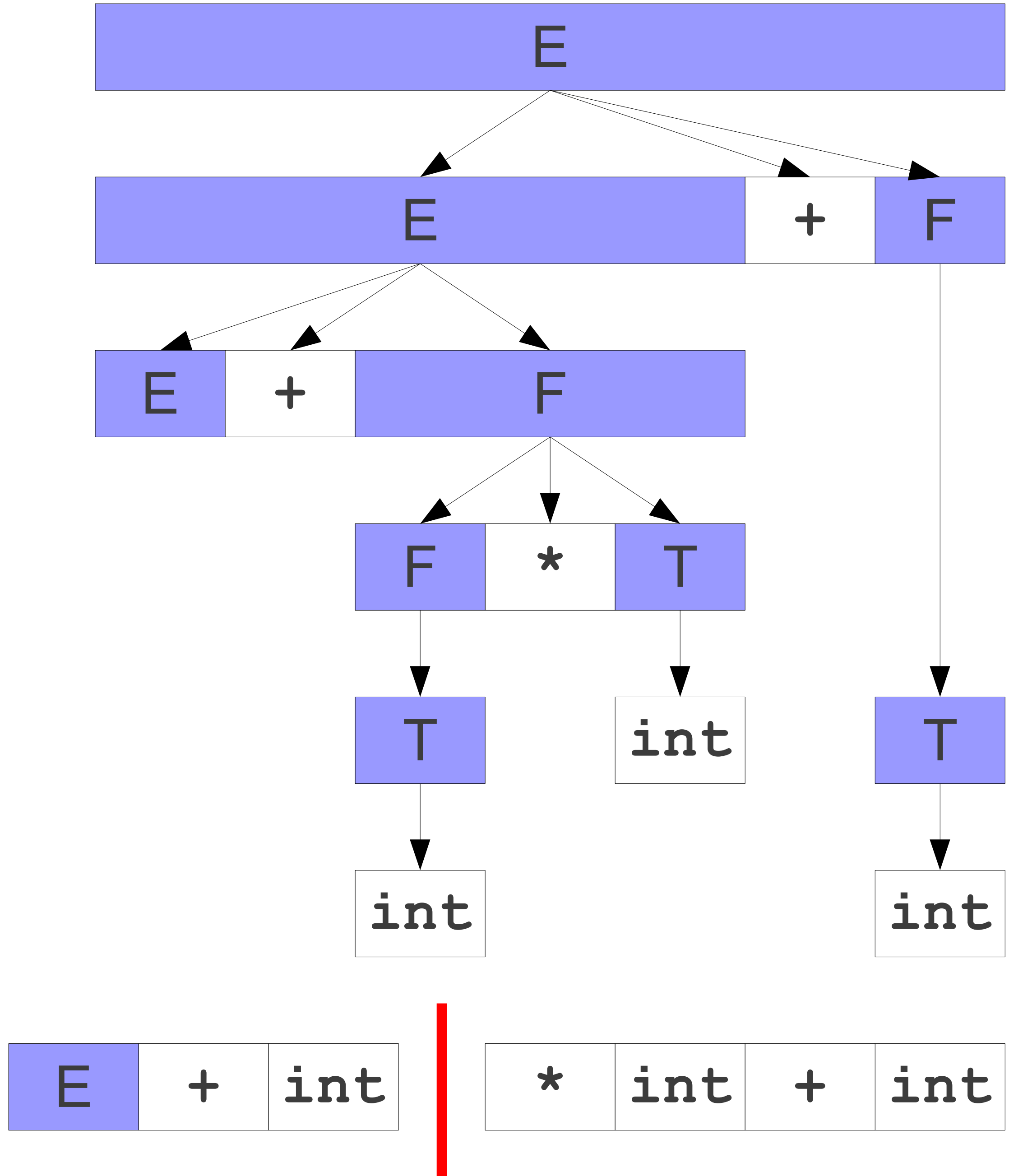
# Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)



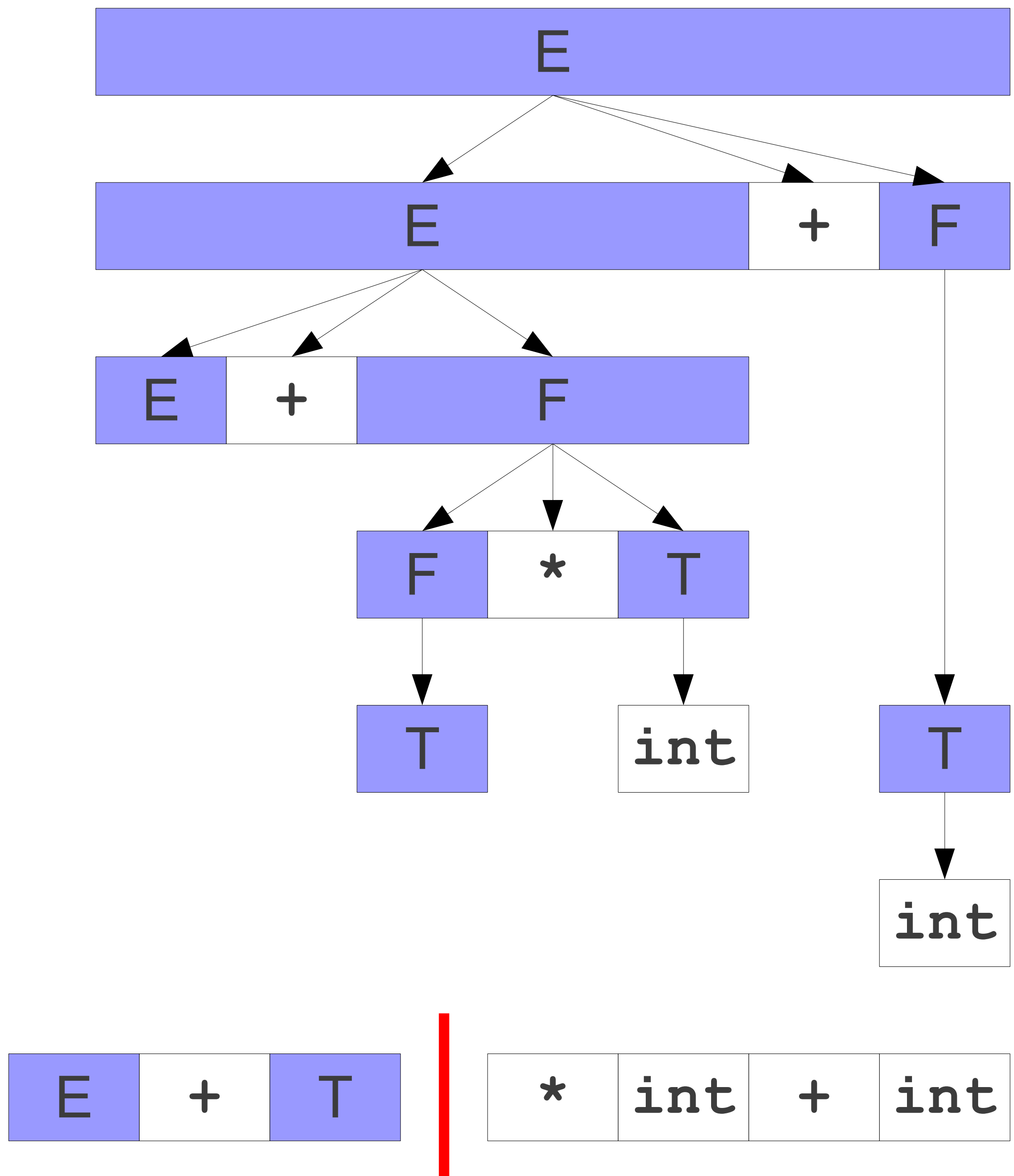
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



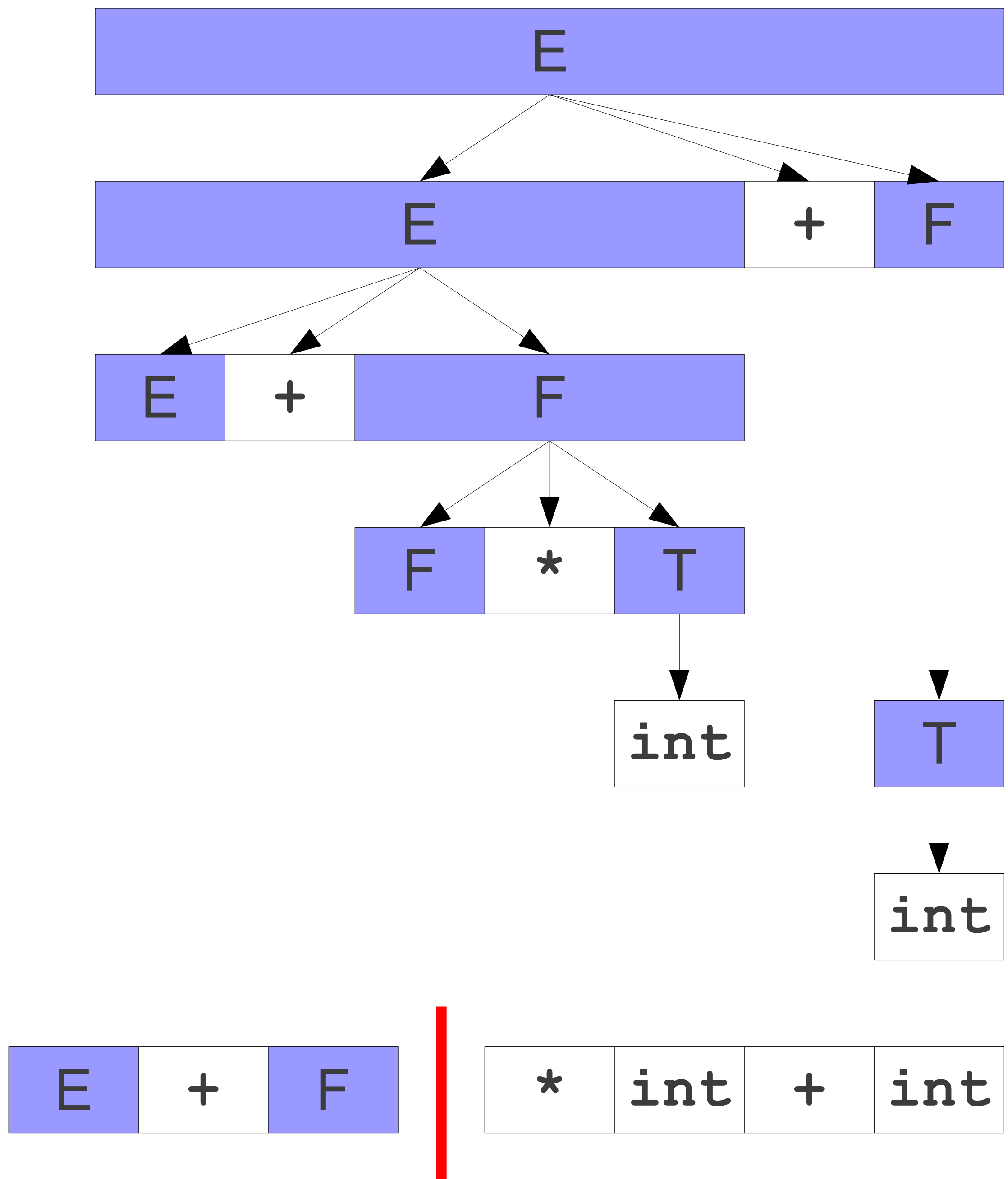
# Another Look at Handles

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



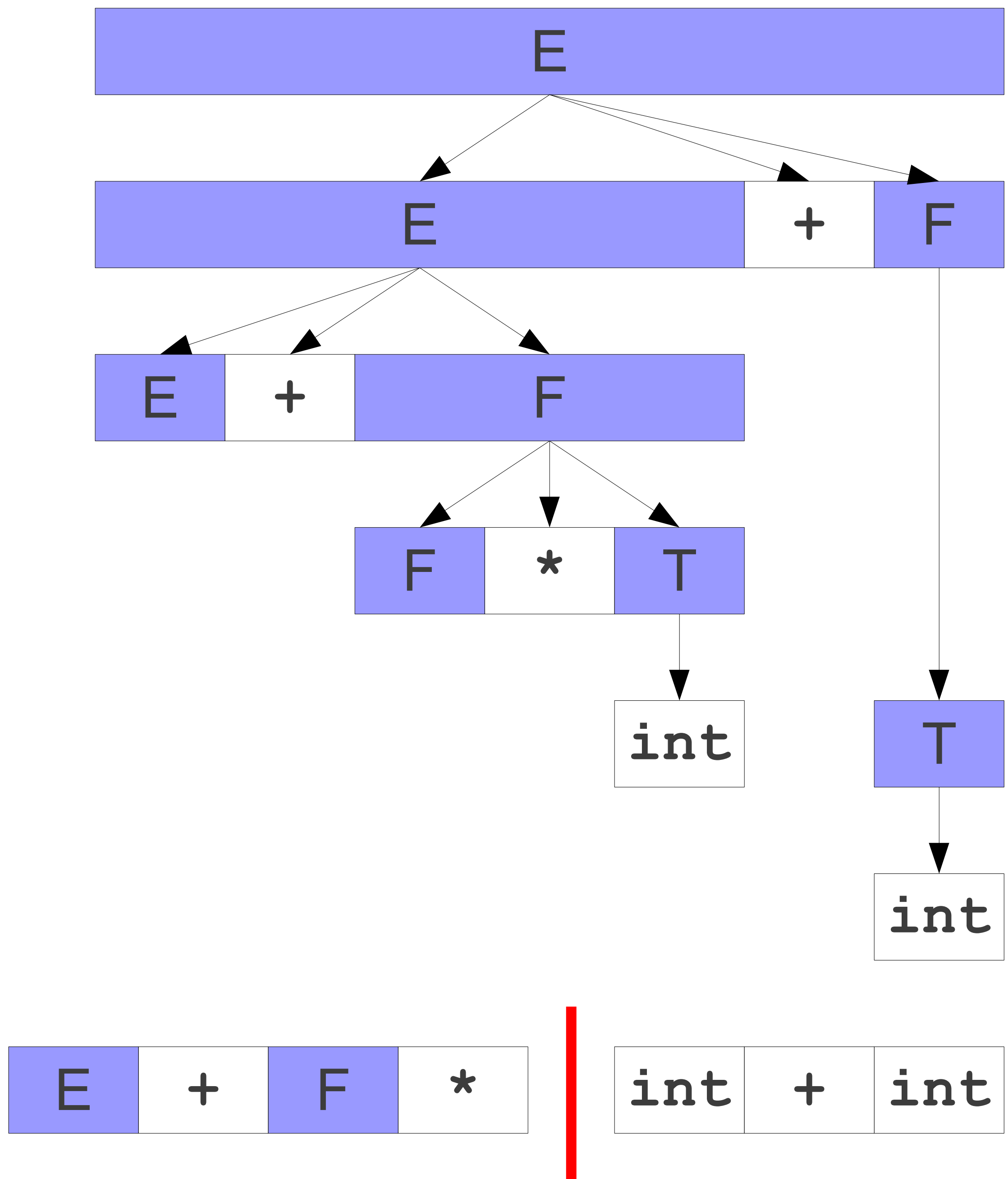
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



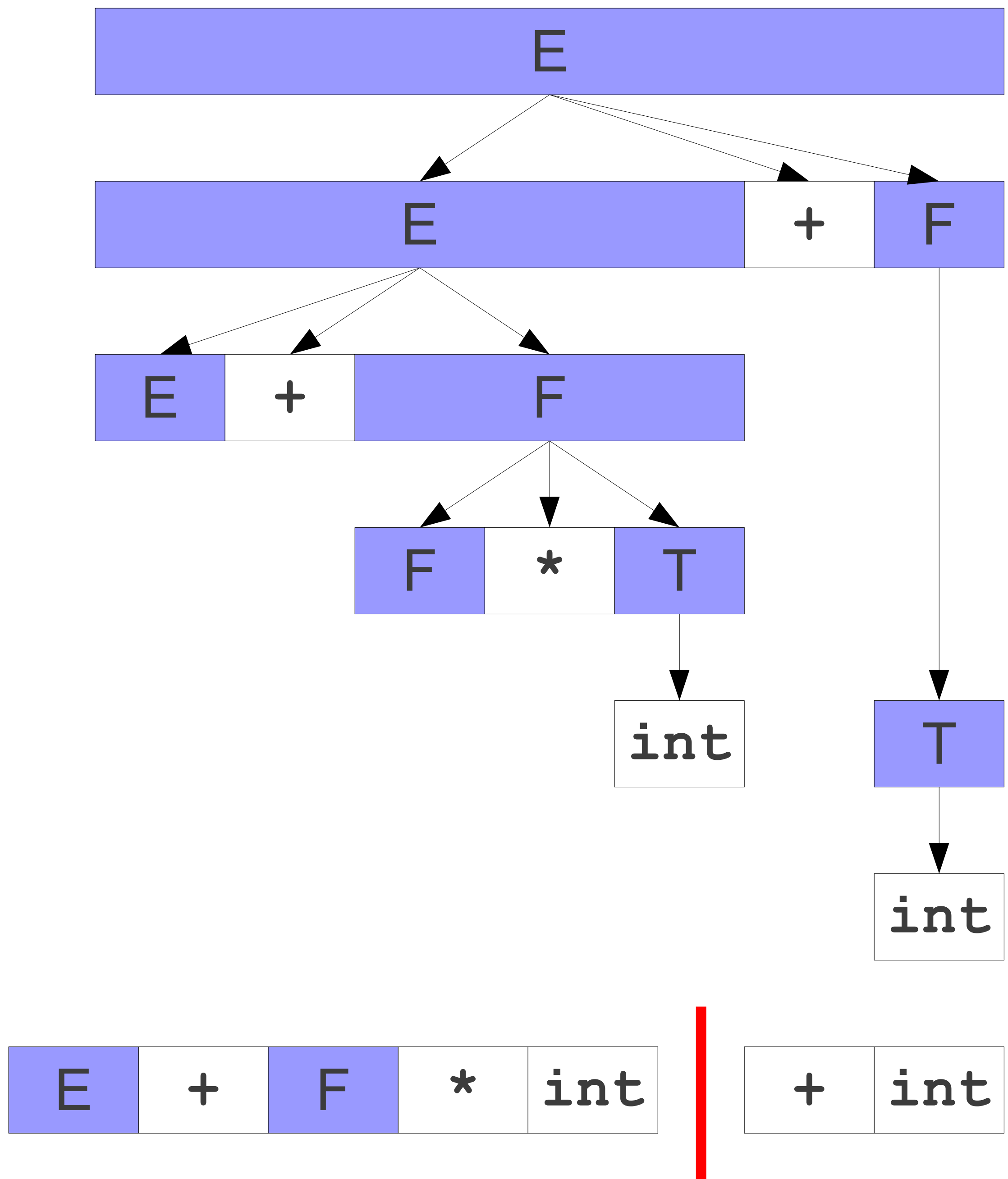
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



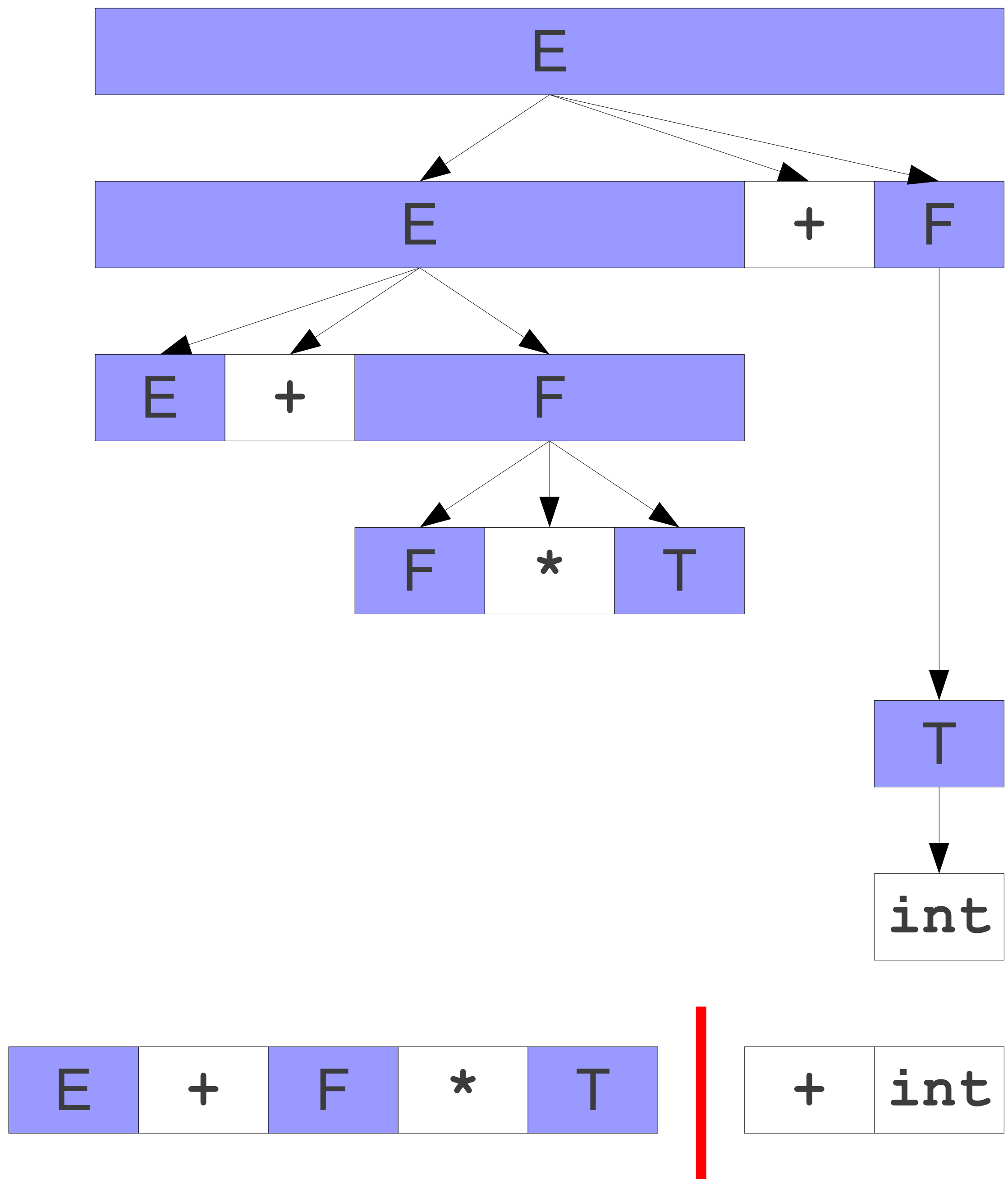
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → `int`
**T** → **(E)**



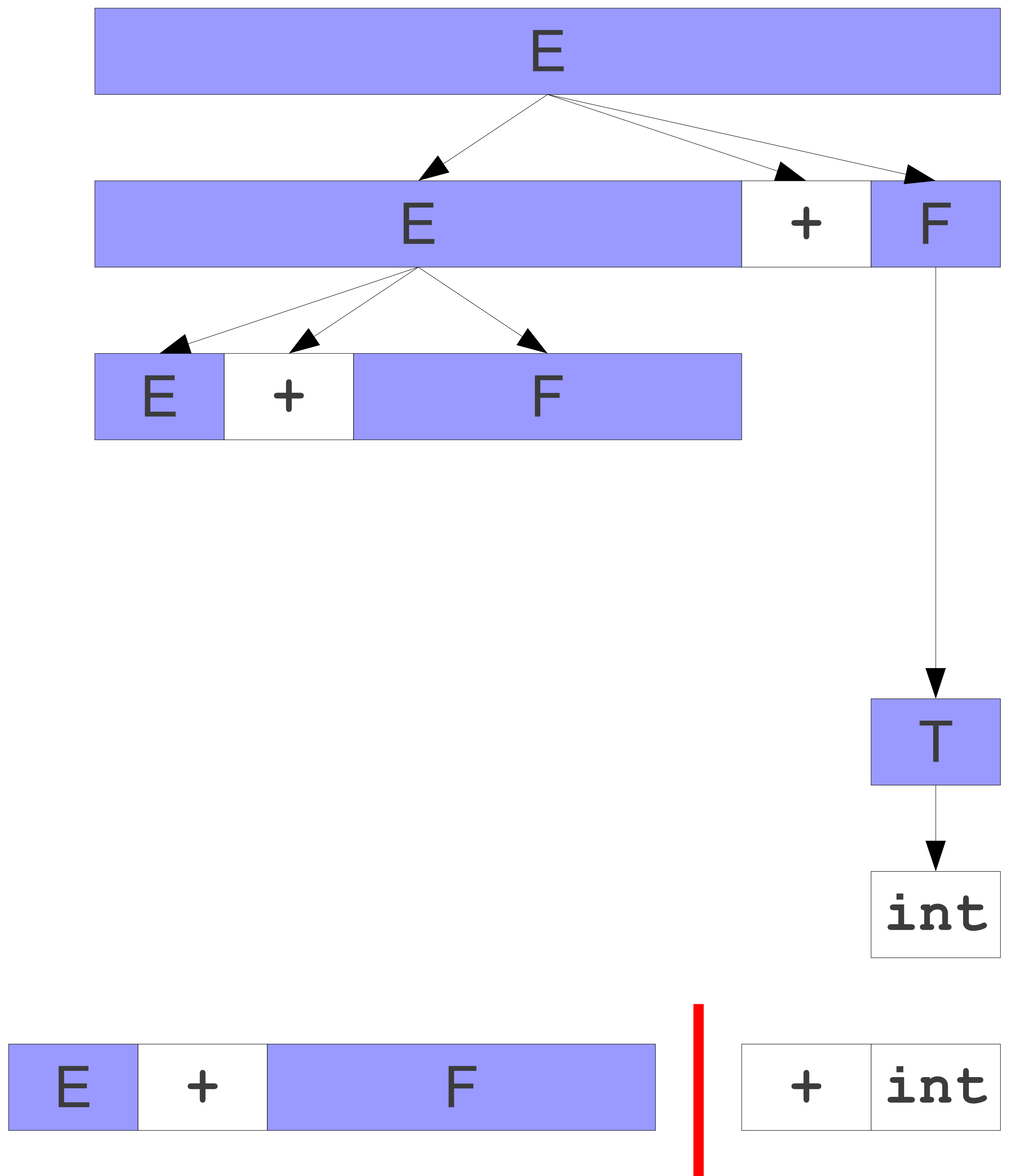
# Another Look at Handles

E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

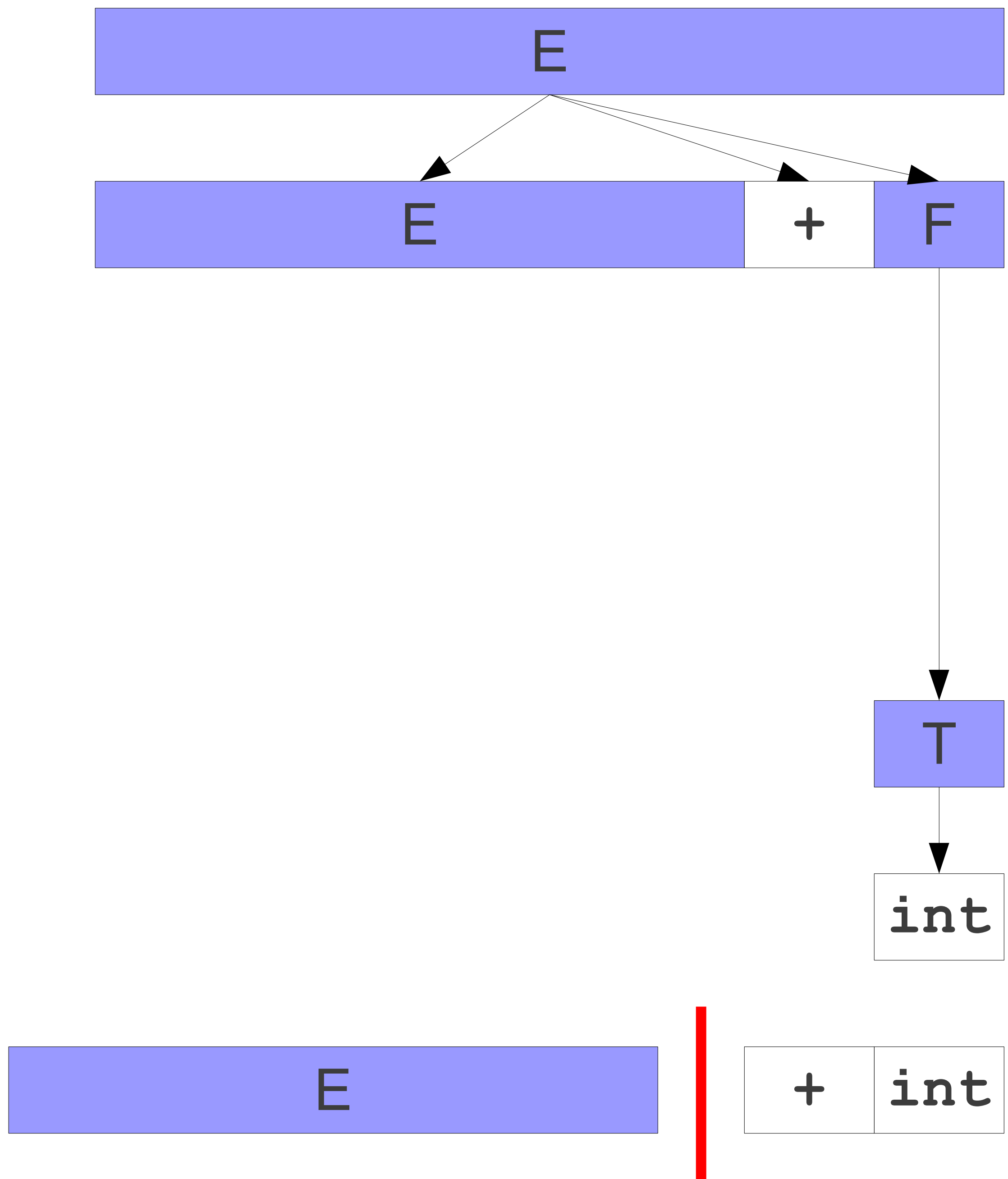


# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**



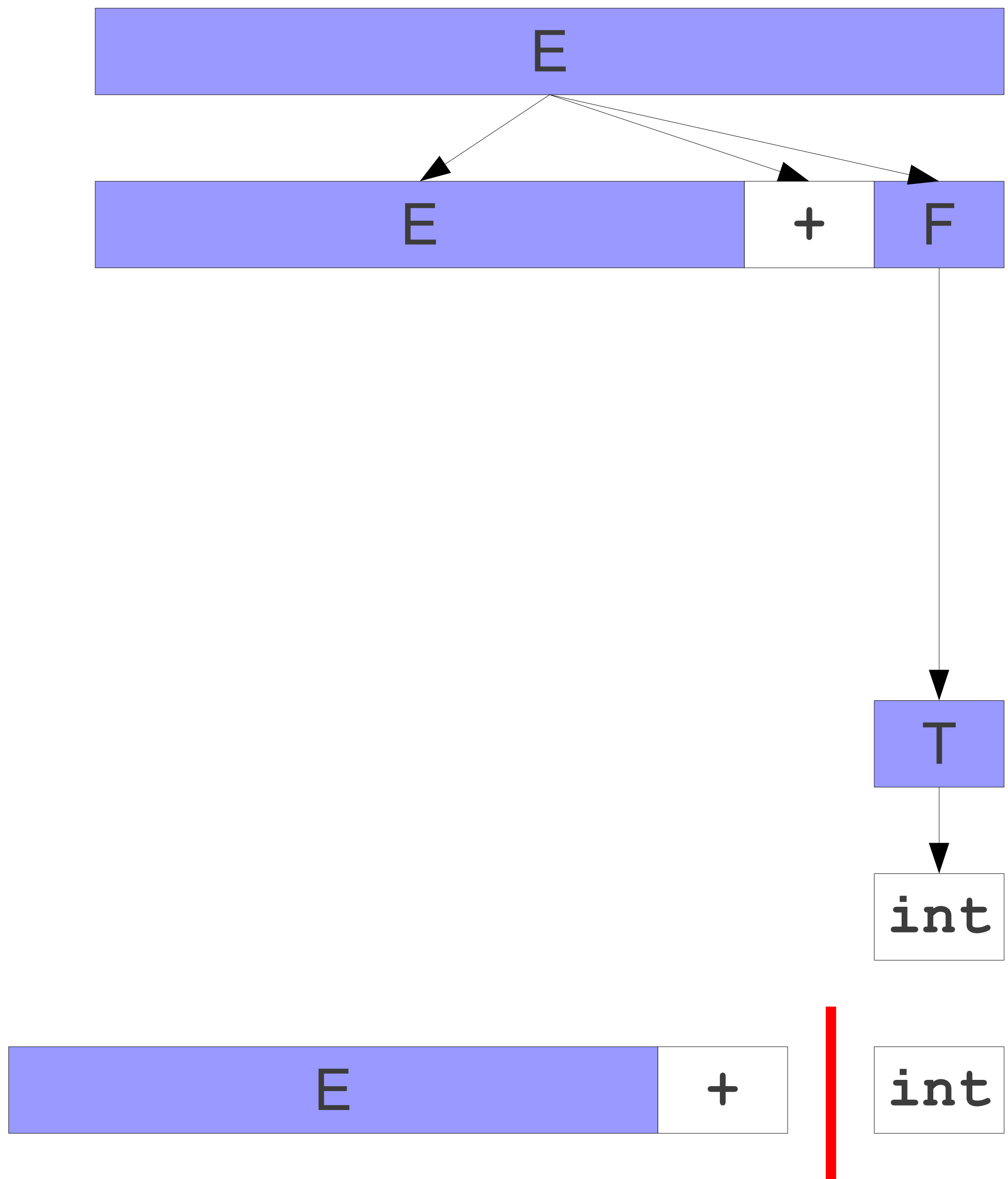
# Another Look at Handles




**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

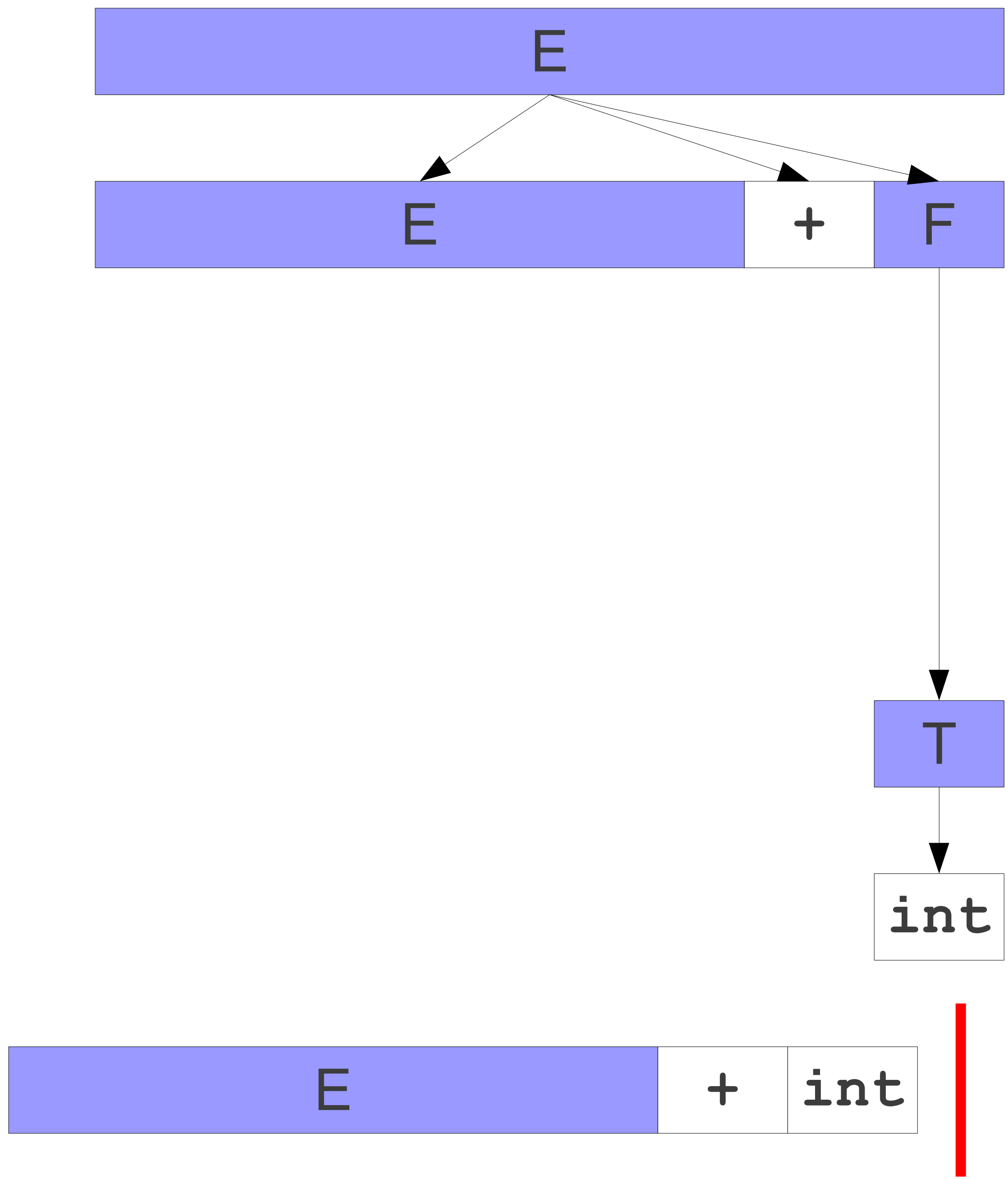
# Another Look at Handles

**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**

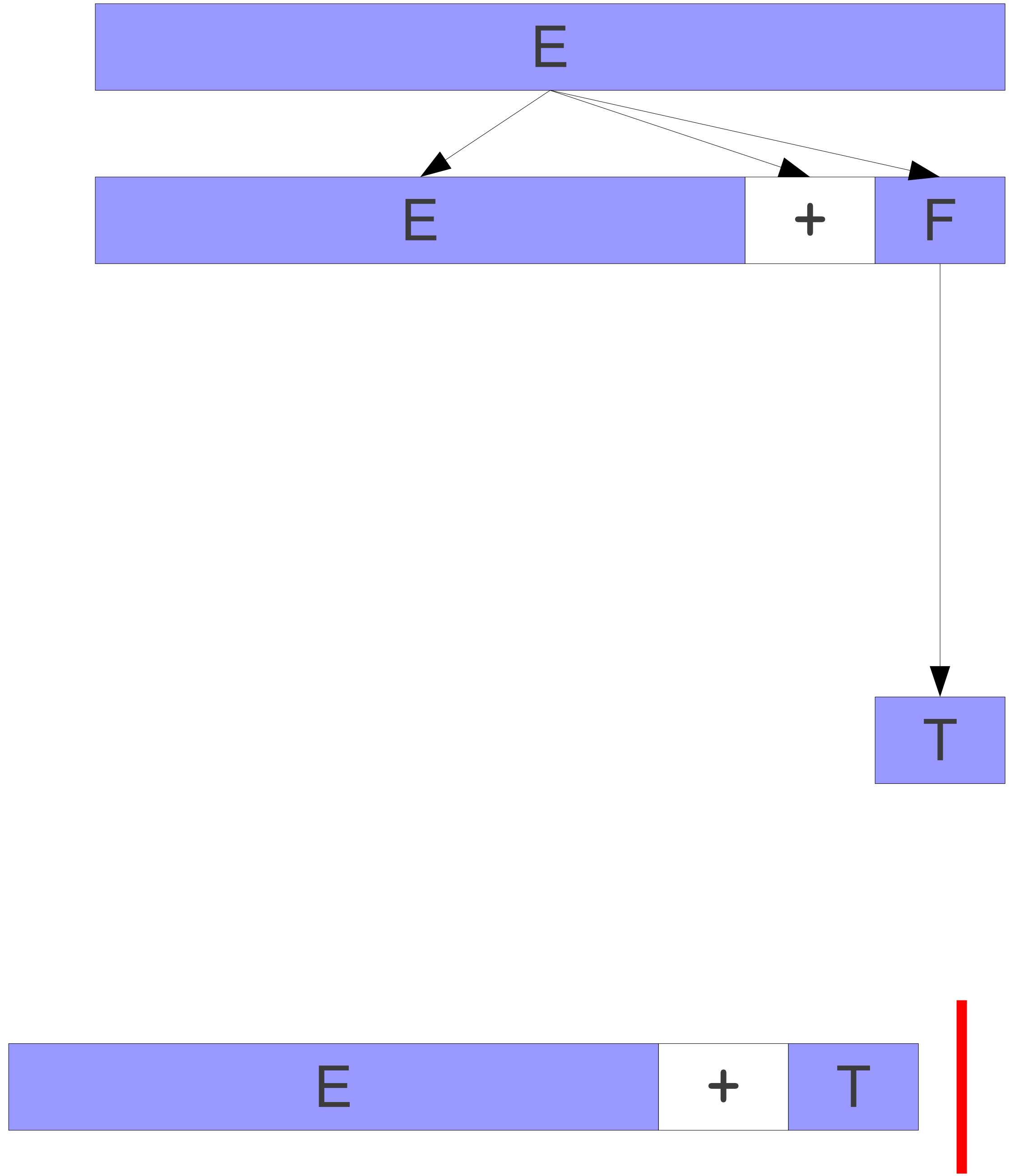


# Another Look at Handles

**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

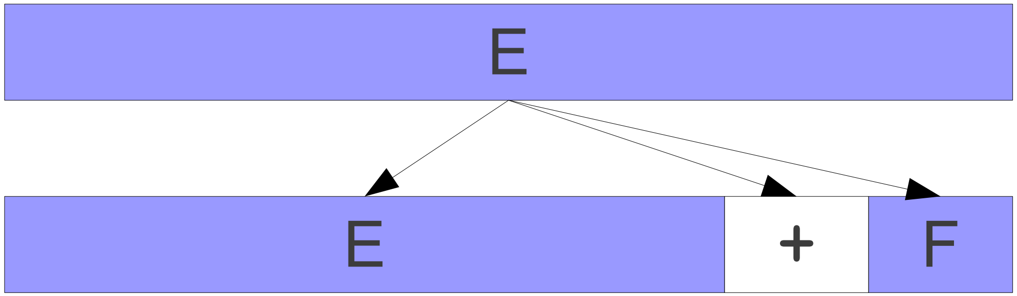


# Another Look at Handles




**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

# Another Look at Handles




**E** → **F**

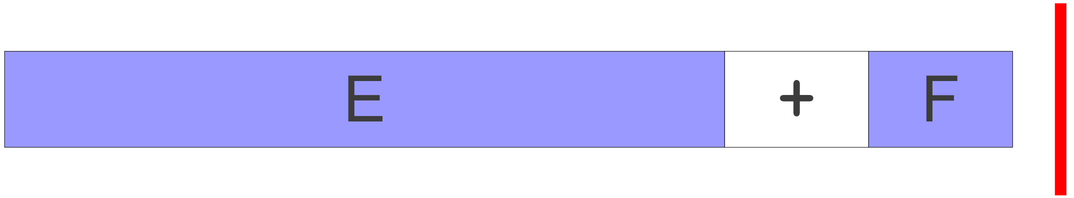
**E** → **E** `+` **F**

**F** → **F** `*` **T**

**F** → **T**

**T** → `int`

**T** → `(`**E**`)`



# Another Look at Handles


**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

# Tracking Our Position

**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

S → · E

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| |
|---|
| S → · E |
| E → · E + F |

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| $S \rightarrow \cdot E$ |
| $E \rightarrow \cdot E + F$ |
| $E \rightarrow \cdot E + F$ |
| $E \rightarrow \cdot F$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |
| F → · T |

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |
| F → · T |
| T → · int |

| | | | | | | |
|---|---|---|---|---|---|---|
| int | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |
| F → · T |
| T → int · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| int | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |
| F → · T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| T | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |
| F → T · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| T | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → · F |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| F | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |
| E → F · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| F | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **`int`**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → · E + F |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | \| | + | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E · + F |

| E | \| | + | int | * | int | + | int |
|---|---|---|---|---|---|---|---|

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

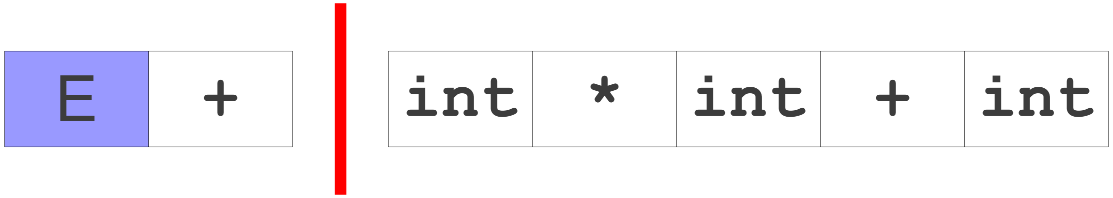
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | \| | int | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |
| F → · T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | \| | int | * | int | + | int |

# Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |
| F → · T |
| T → · int |

E + | int * int + int

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |
| F → · T |
| T → int · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | int | \| | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |
| F → · T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | T | \| | * | int | + | int |

# Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |
| F → T · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | T | \| | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **`int`**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | \| | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F · * T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | \| | * | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * · T |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | * | \| | int | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → `int`
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * · T |
| T → · `int` |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | * | \| | int | + | int |

# Tracking Our Position

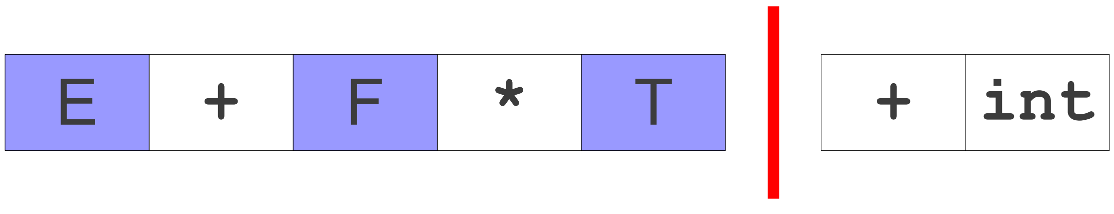
**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * · T |
| T → int · |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → `int`
**T** → (**E**)

| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * · T |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

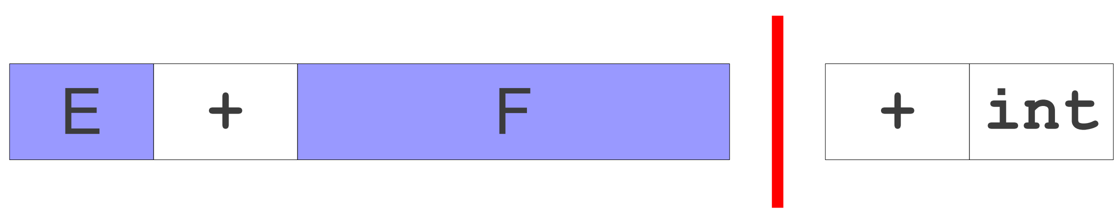
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * T · |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | * | T | \| | + | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

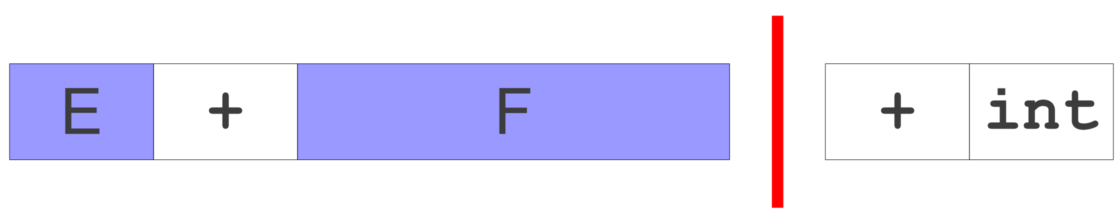
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

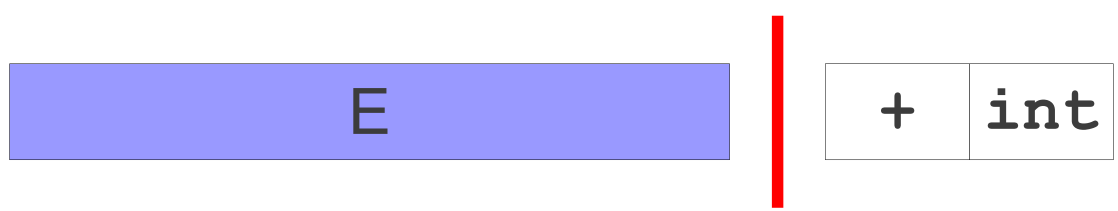
| |
|---|
| S → · E |
| E → · E + F |
| E → E + F · |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

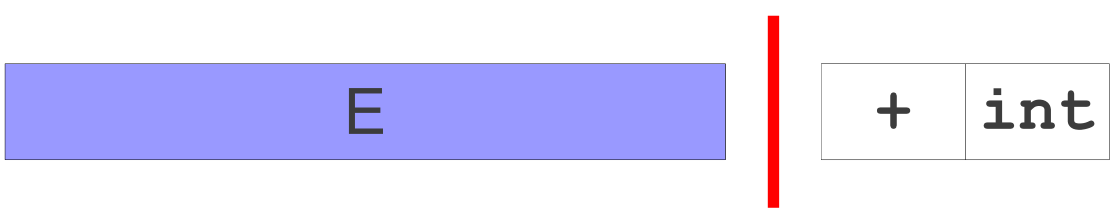
| |
|---|
| S → · E |
| E → · E + F |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

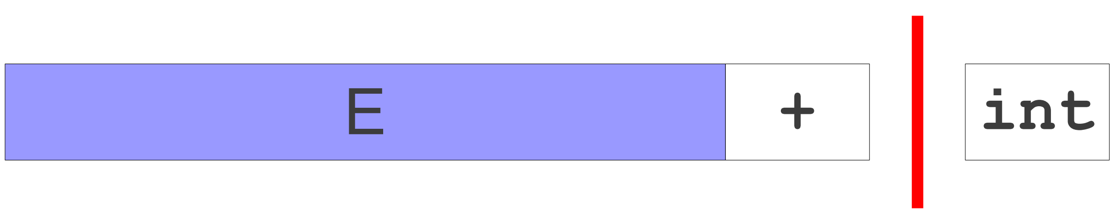
| |
|---|
| S → · E |
| E → E · + F |



# Tracking Our Position

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

S → · E
E → E + · F



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → E + · F |
| F → · T |

| | | | |
|---|---|---|---|
| E | + | \| | int |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → E + · F |
| F → · T |
| T → · **int** |

| | | | |
|---|---|---|---|
| E | + | \| | **int** |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **`int`**
**T** → **(E)**

| |
|---|
| S → · E |
| E → E + · F |
| F → · T |
| T → `int` · |

| | | |
|---|---|---|
| E | + | `int` |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → E + · F |
| F → · T |

# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → `int`
**T** → (**E**)

| |
|---|
| S → · E |
| E → E + · F |
| F → T · |

| | | |
|---|---|---|
| E | + | T |

# Tracking Our Position

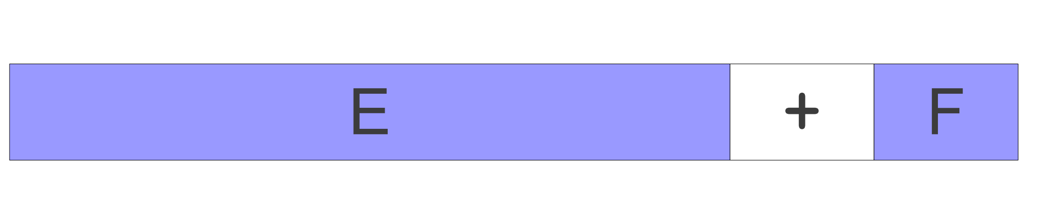
$S \rightarrow E$
$E \rightarrow F$
$E \rightarrow E + F$
$F \rightarrow F * T$
$F \rightarrow T$
$T \rightarrow$ int
$T \rightarrow (E)$

| |
|---|
| $S \rightarrow \cdot E$ |
| $E \rightarrow E + \cdot F$ |

# Tracking Our Position

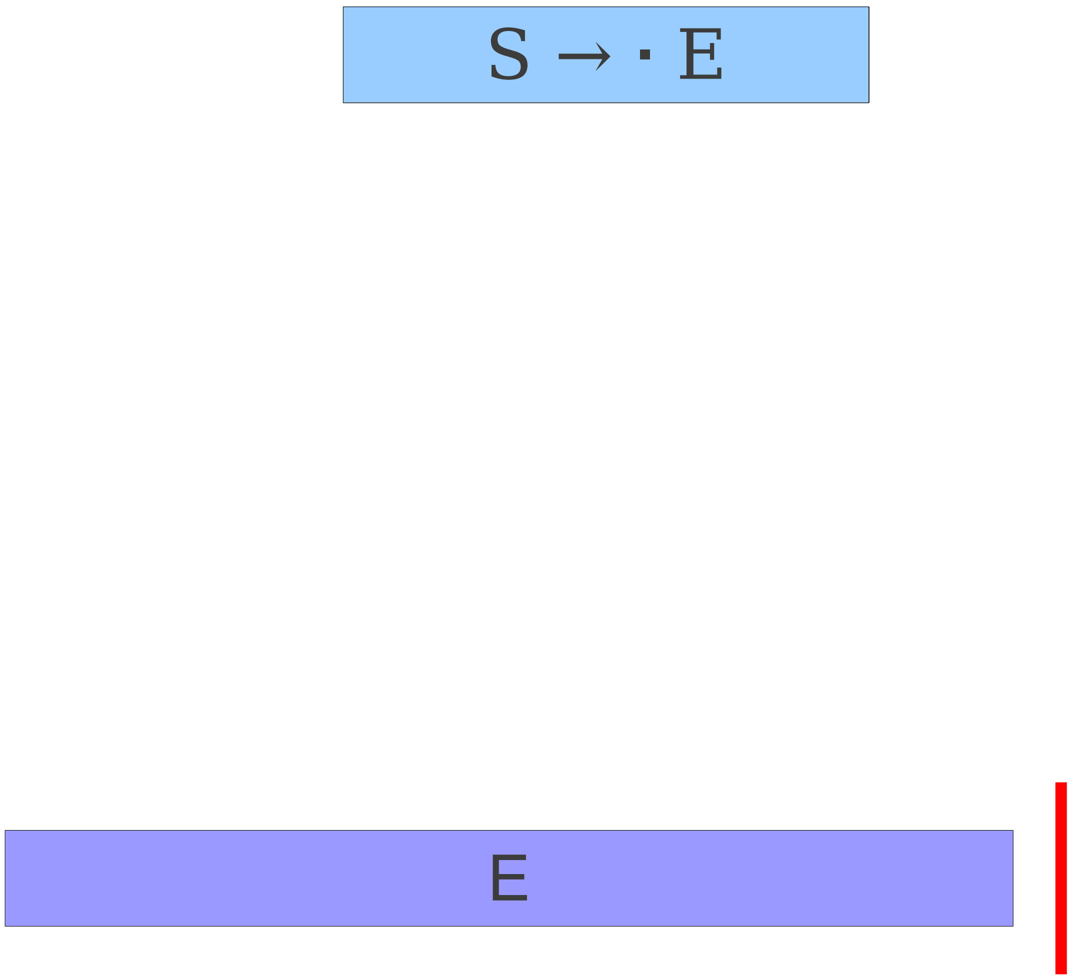
**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

| |
|---|
| S → · E |
| E → E + F · |



# Tracking Our Position

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`



# Tracking Our Position

S → E ·

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

E

# Generating Left-Hand Sides

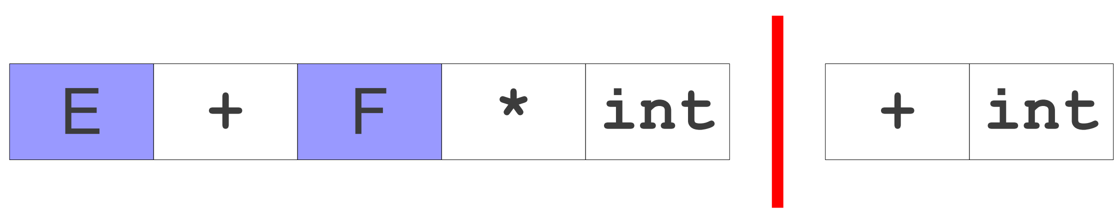
- At any instant in time, the contents of the left side of the parser can be described using the following process:
  - Trace out, from the start symbol, the series of productions that have not yet been completed and where we are in each production.
  - For each production, in order, output all of the symbols up to the point where we change from one production to the next.

# Recognizing Left-Hand Sides

- Given that we have a procedure for *generating* left-hand sides, can we build a procedure for *recognizing* those left-hand sides?
- Idea: At each point, track
  - Which production we are in, and
  - Where we are in that production.
- At each point, we can do one of two things:
  - Match the next symbol of the candidate left-hand side with the next symbol in the current production, or
  - If the next symbol of the candidate left-hand side is a nonterminal, nondeterministically guess which production to try next.

# Recognizing Left-Hand Sides

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**



# Recognizing Left-Hand Sides

$S \rightarrow \cdot E$

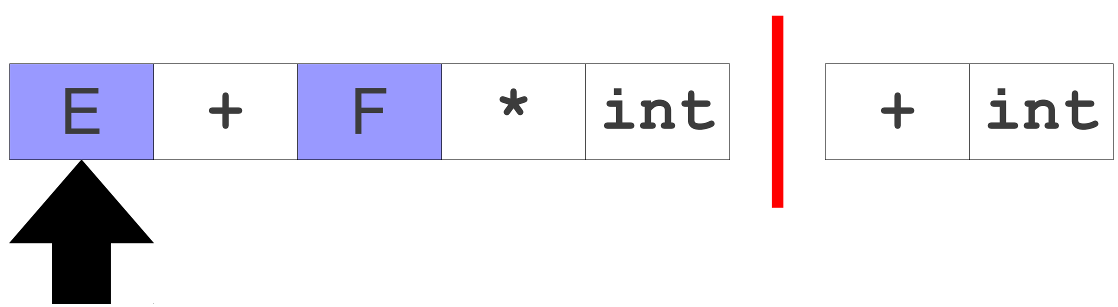
**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| E | + | F | * | int | \| | + | int |

# Recognizing Left-Hand Sides

S → · E

**S** → **E**
**E** → **F**
**E** → **E** `+` **F**
**F** → **F** `*` **T**
**F** → **T**
**T** → `int`
**T** → `(`**E**`)`



# Recognizing Left-Hand Sides

S → · E

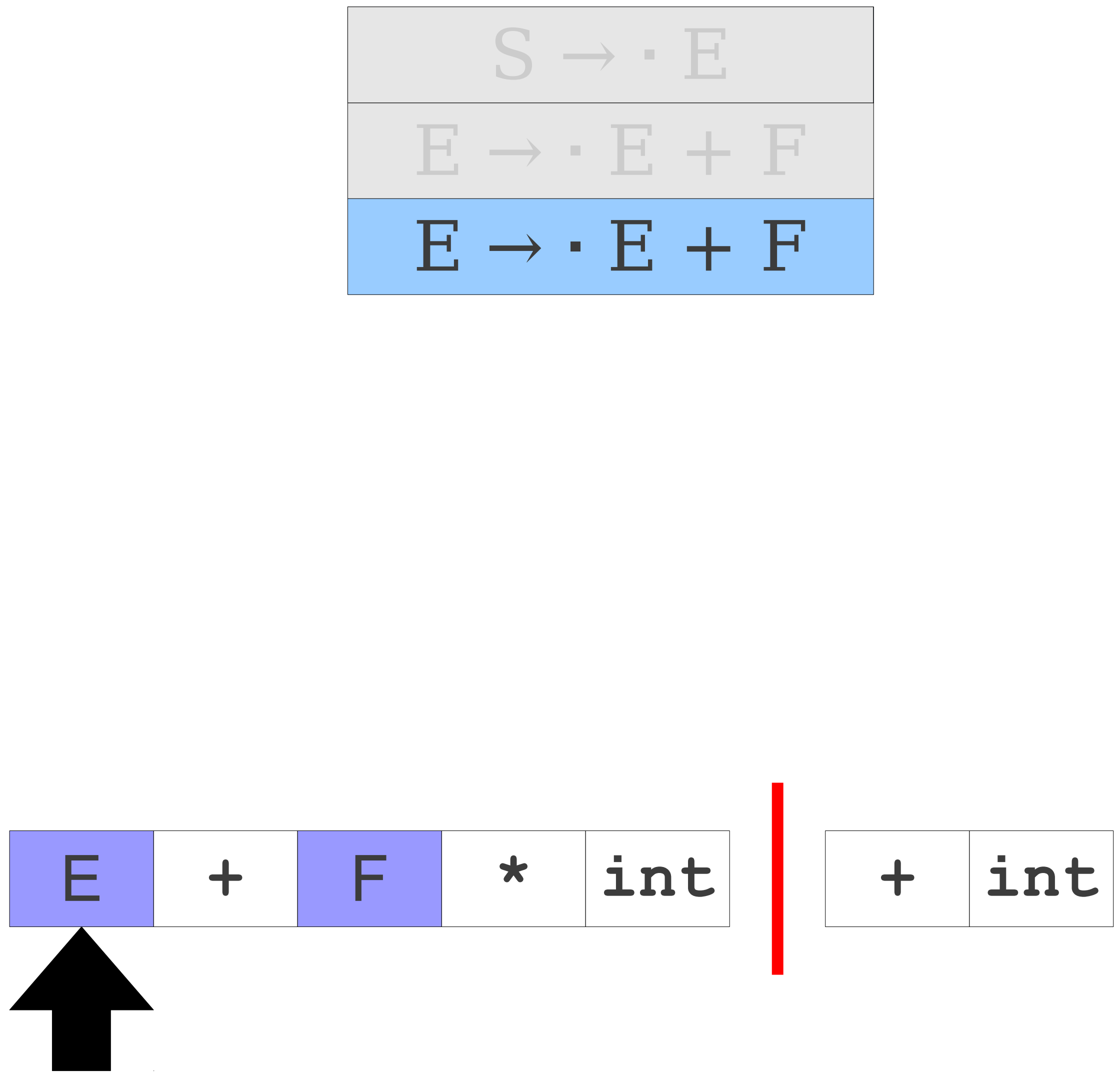
E → · E + F

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

| E | + | F | * | int | | | + | int |
|---|---|---|---|---|---|---|---|

# Recognizing Left-Hand Sides

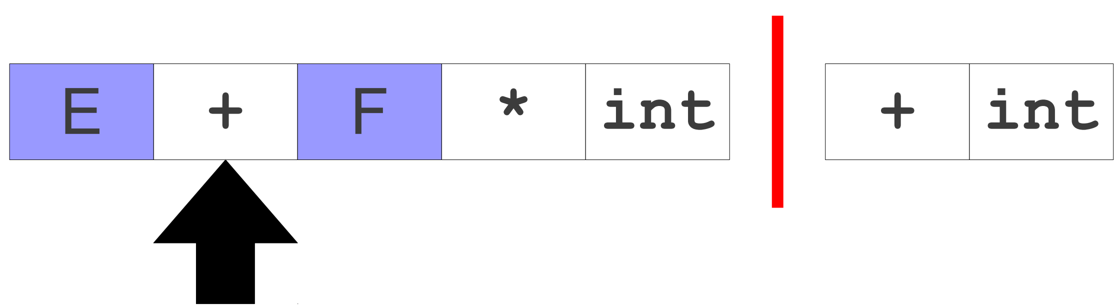
**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)



# Recognizing Left-Hand Sides

S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

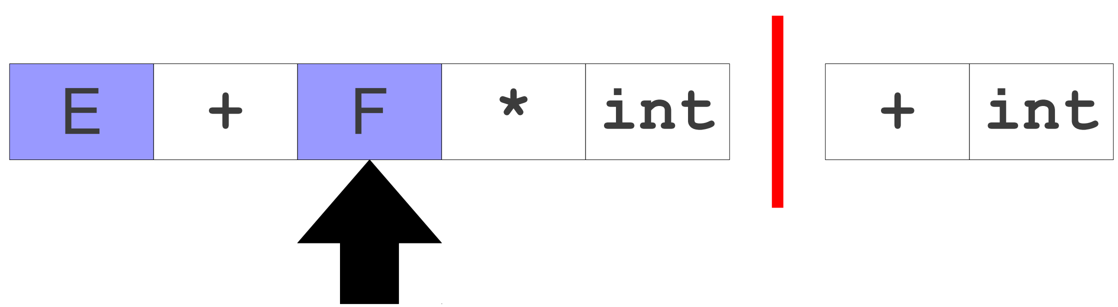
S → · E
E → · E + F
E → E · + F



# Recognizing Left-Hand Sides

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

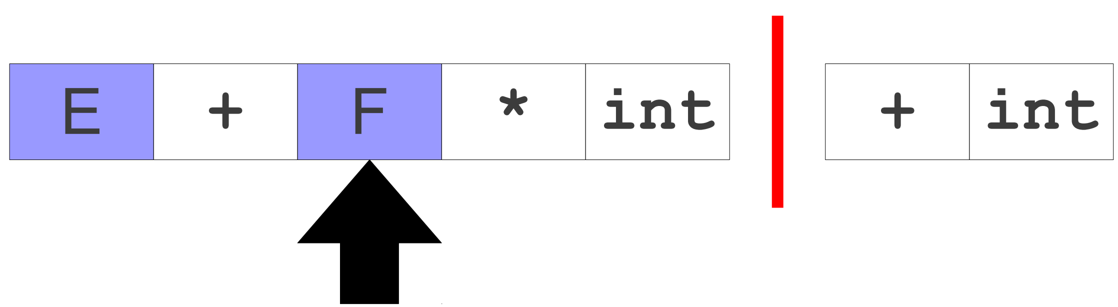
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |



# Recognizing Left-Hand Sides

**S** → **E**
**E** → **F**
**E** → **E** **+** **F**
**F** → **F** ***** **T**
**F** → **T**
**T** → **int**
**T** → **(E)**

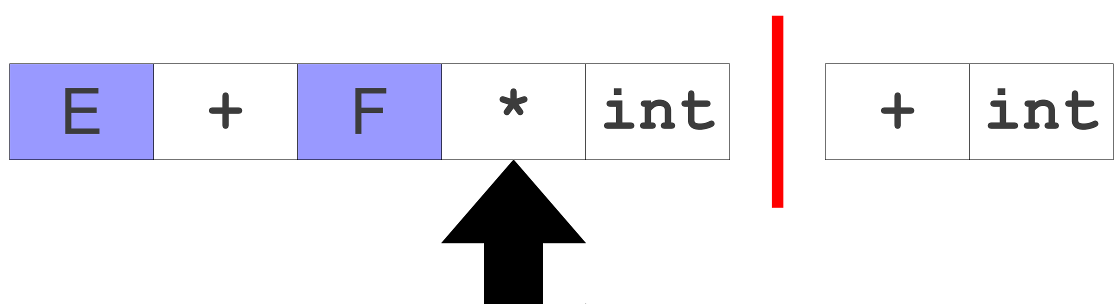
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → · F * T |



# Recognizing Left-Hand Sides

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

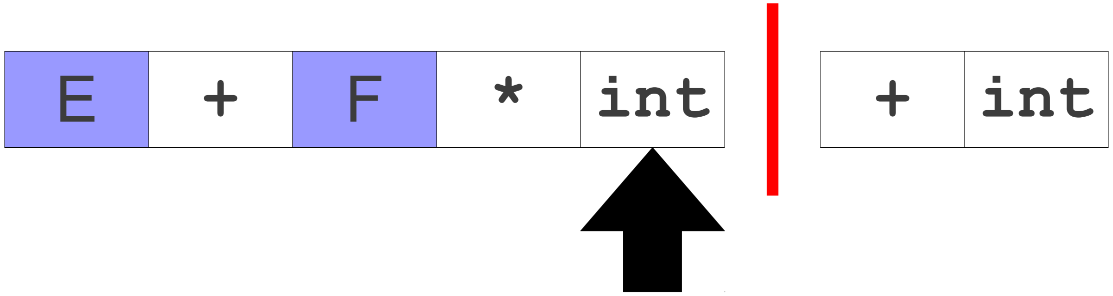
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F · * T |



# Recognizing Left-Hand Sides

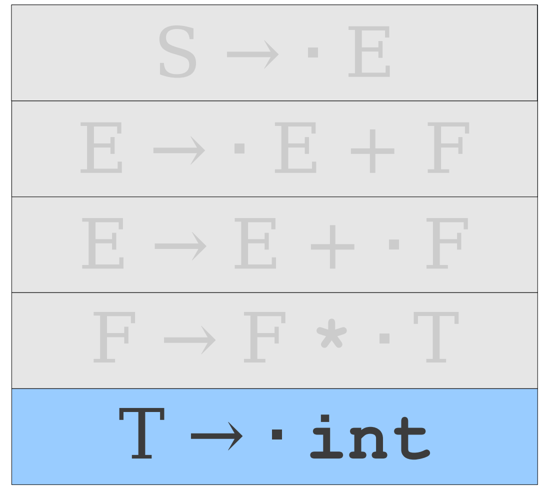
**S** → **E**
**E** → **F**
**E** → **E + F**
**F** → **F * T**
**F** → **T**
**T** → **int**
**T** → **(E)**

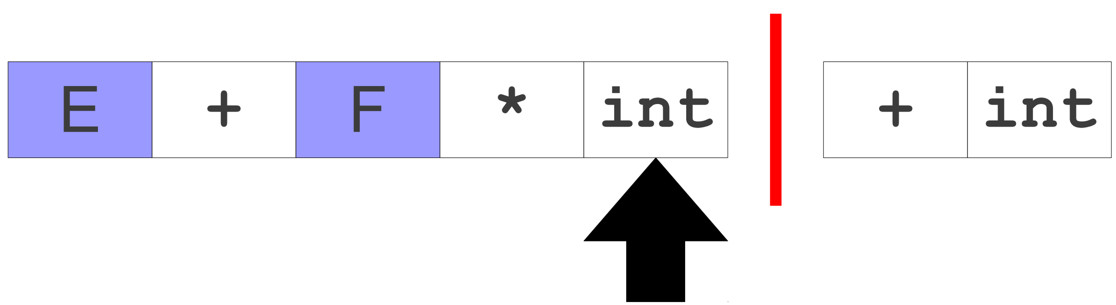
| |
|---|
| S → · E |
| E → · E + F |
| E → E + · F |
| F → F * · T |



# Recognizing Left-Hand Sides

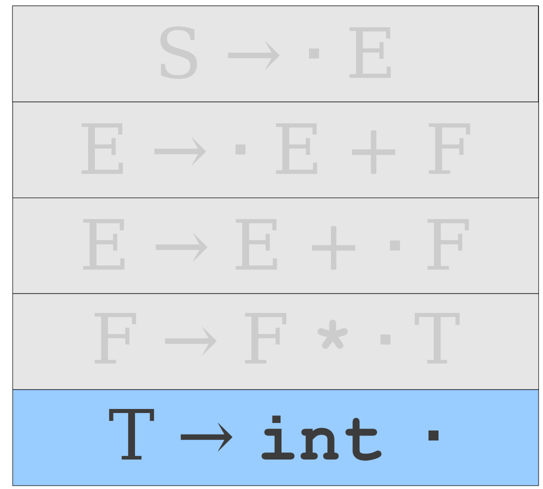
S → E
E → F
E → E + F
F → F * T
F → T
T → int
T → (E)

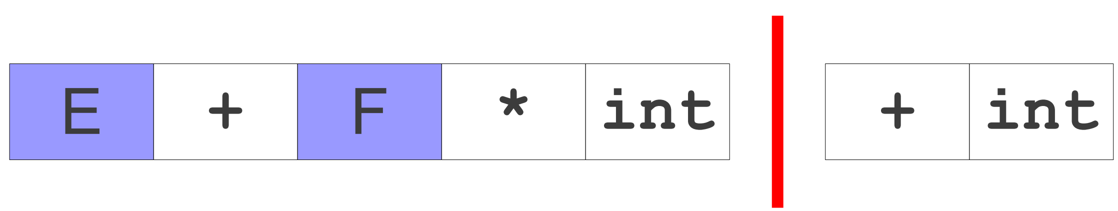


# Recognizing Left-Hand Sides

**S** → **E**
**E** → **F**
**E** → **E** + **F**
**F** → **F** * **T**
**F** → **T**
**T** → int
**T** → (**E**)

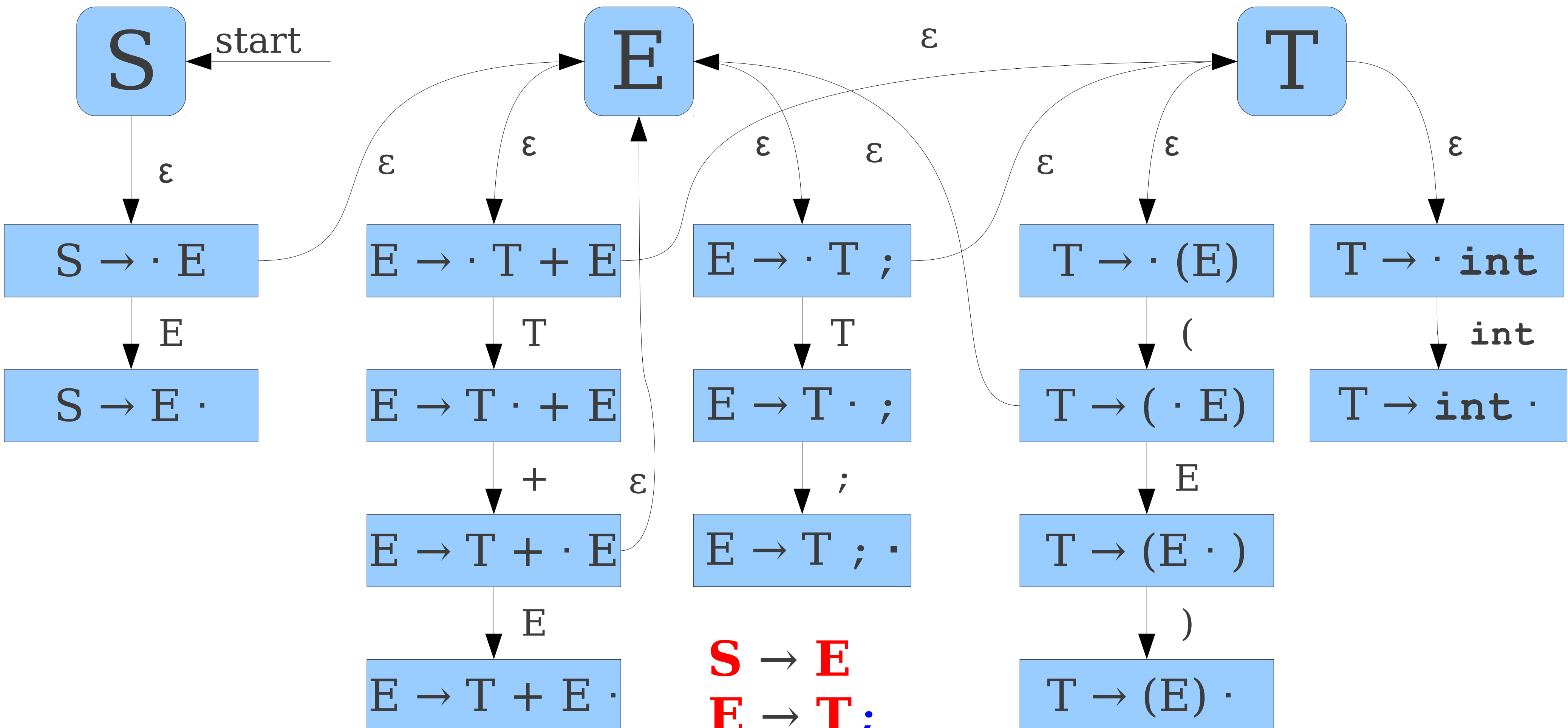


# An Important Result

- There are only finitely many productions, and within those productions only finitely many positions.
- At any point in time, we only need to track where we are in one production.
- There are only finitely many options we can take at any one point.
- **We can use a finite automaton as our recognizer.**

# An Automaton for Left Areas




S → E
E → T;
E → T + E
T → int
T → (E)

# Constructing the Automaton

- Create a state for each nonterminal.
- For each production $\mathbf{A} \to \boldsymbol{\gamma}$:
  - Construct states $\mathbf{A} \to \boldsymbol{\alpha} \cdot \boldsymbol{\omega}$ for each possible way of splitting $\boldsymbol{\gamma}$ into two substrings $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$.
  - Add transitions on $\boldsymbol{x}$ between $\mathbf{A} \to \boldsymbol{\alpha} \cdot \boldsymbol{x\omega}$ and $\mathbf{A} \to \boldsymbol{\alpha x} \cdot \boldsymbol{\omega}$.
- For each state $\mathbf{A} \to \boldsymbol{\alpha} \cdot \mathbf{B}\boldsymbol{\omega}$ for nonterminal $\mathbf{B}$, add an ε-transition from $\mathbf{A} \to \boldsymbol{\alpha} \cdot \mathbf{B}\boldsymbol{\omega}$ to $\mathbf{B}$.

# Why This Matters

- Our initial goal was to find handles.
- When running this automaton, if we ever end up in a state with a rule of the form

$$\mathbf{A} \to \boldsymbol{\omega} \cdot$$

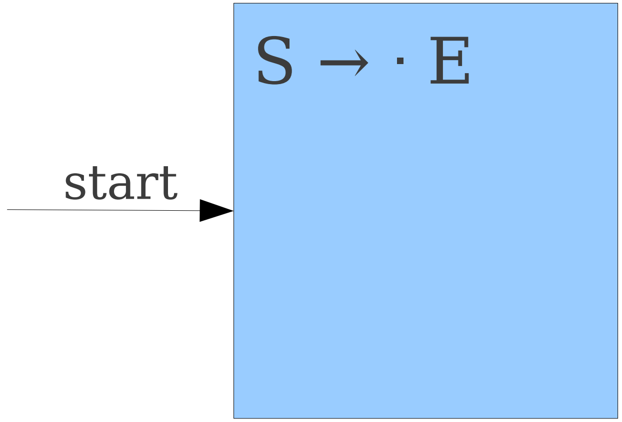
- Then we might be looking at a handle.
- This automaton can be used to discover possible handle locations!

# Adding Determinism

- Typically, this handle-finding automaton is implemented deterministically.
- We could construct a deterministic parsing automaton by constructing the nondeterministic automaton and applying the subset construction, but there is a more direct approach.

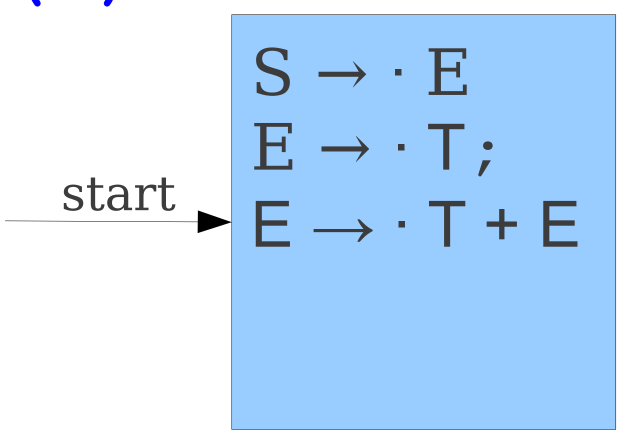
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



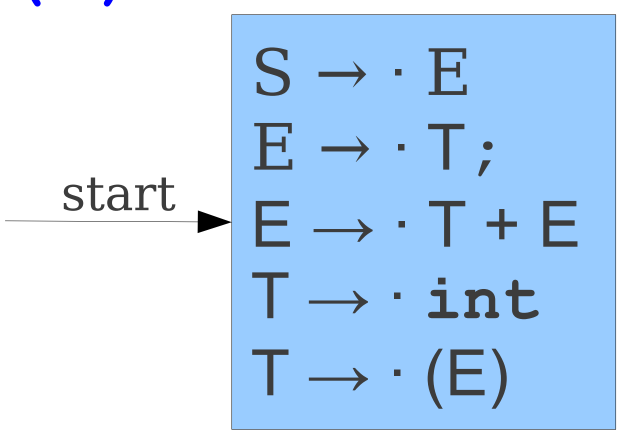
# A Deterministic Automaton

- $S \rightarrow E$
- $E \rightarrow T$ `;`
- $E \rightarrow T$ `+` $E$
- $T \rightarrow$ `int`
- $T \rightarrow$ `(`$E$`)`



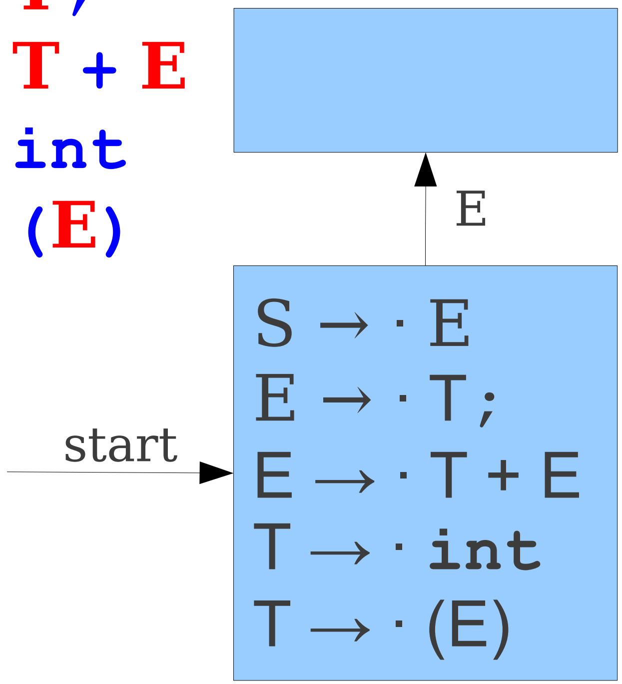
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



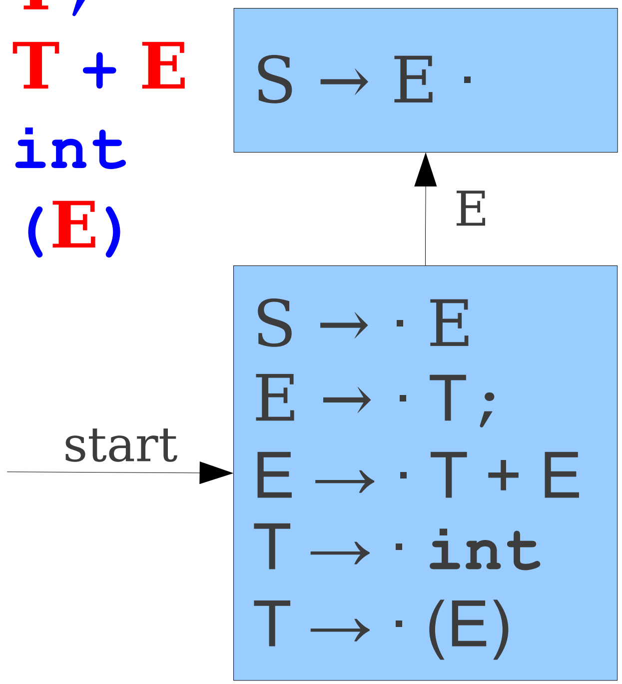
# A Deterministic Automaton

**S** → **E**
**E** → **T** **;**
**E** → **T** **+** **E**
**T** → **int**
**T** → **(E)**



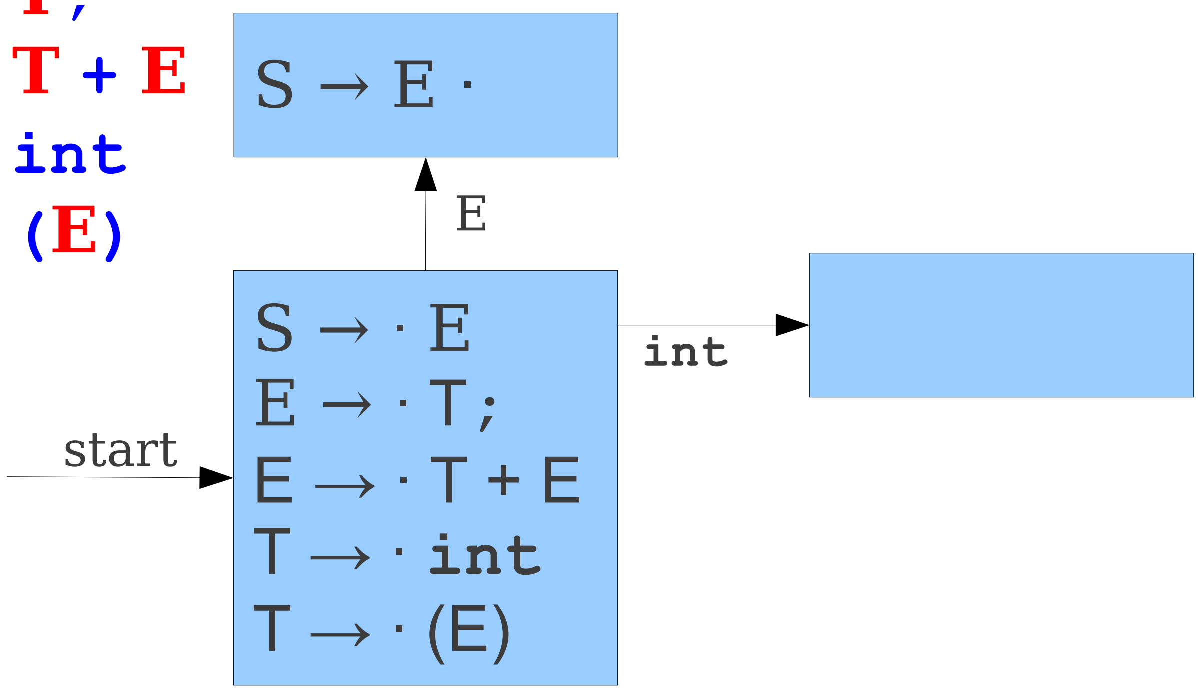
# A Deterministic Automaton

$S \to E$
$E \to T;$
$E \to T + E$
$T \to \texttt{int}$
$T \to (E)$



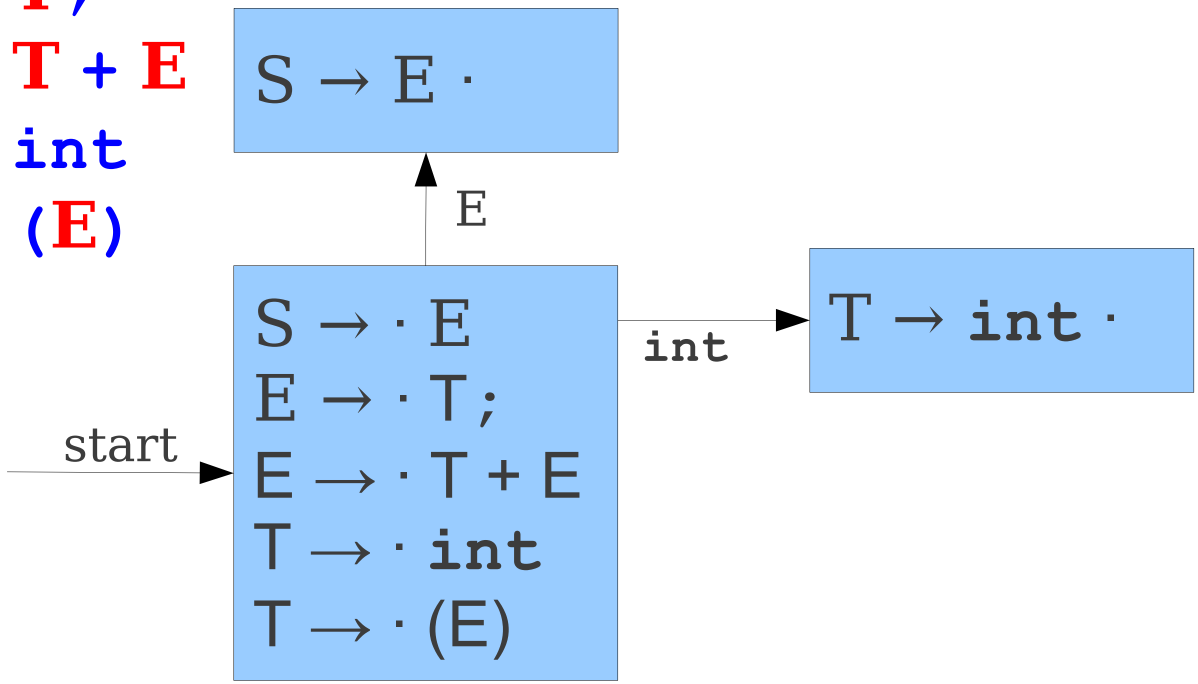
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



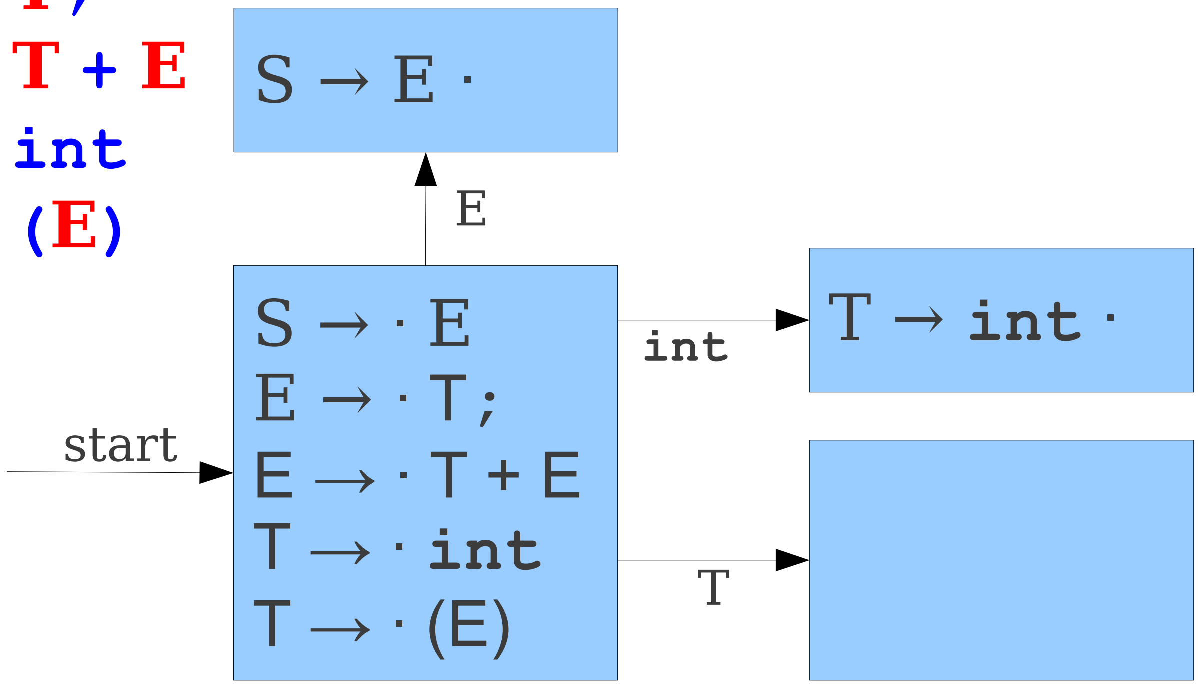
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → (**E**)



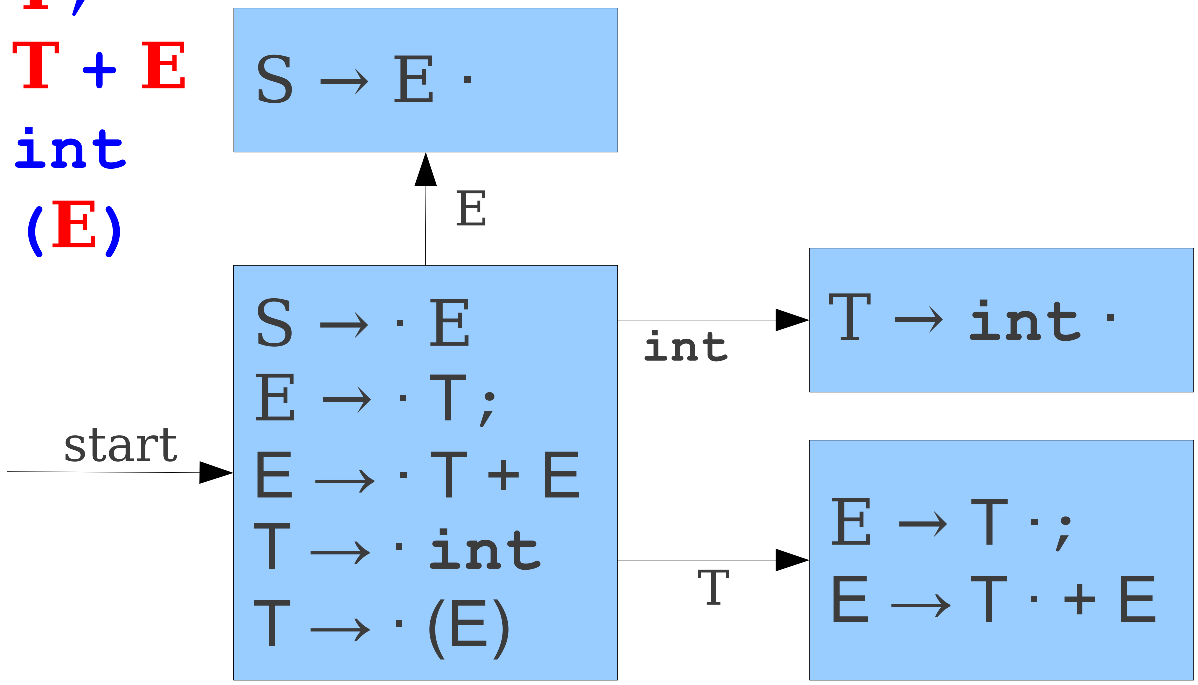
# A Deterministic Automaton

**S** → **E**
**E** → **T** **;**
**E** → **T** **+** **E**
**T** → **int**
**T** → **(E)**



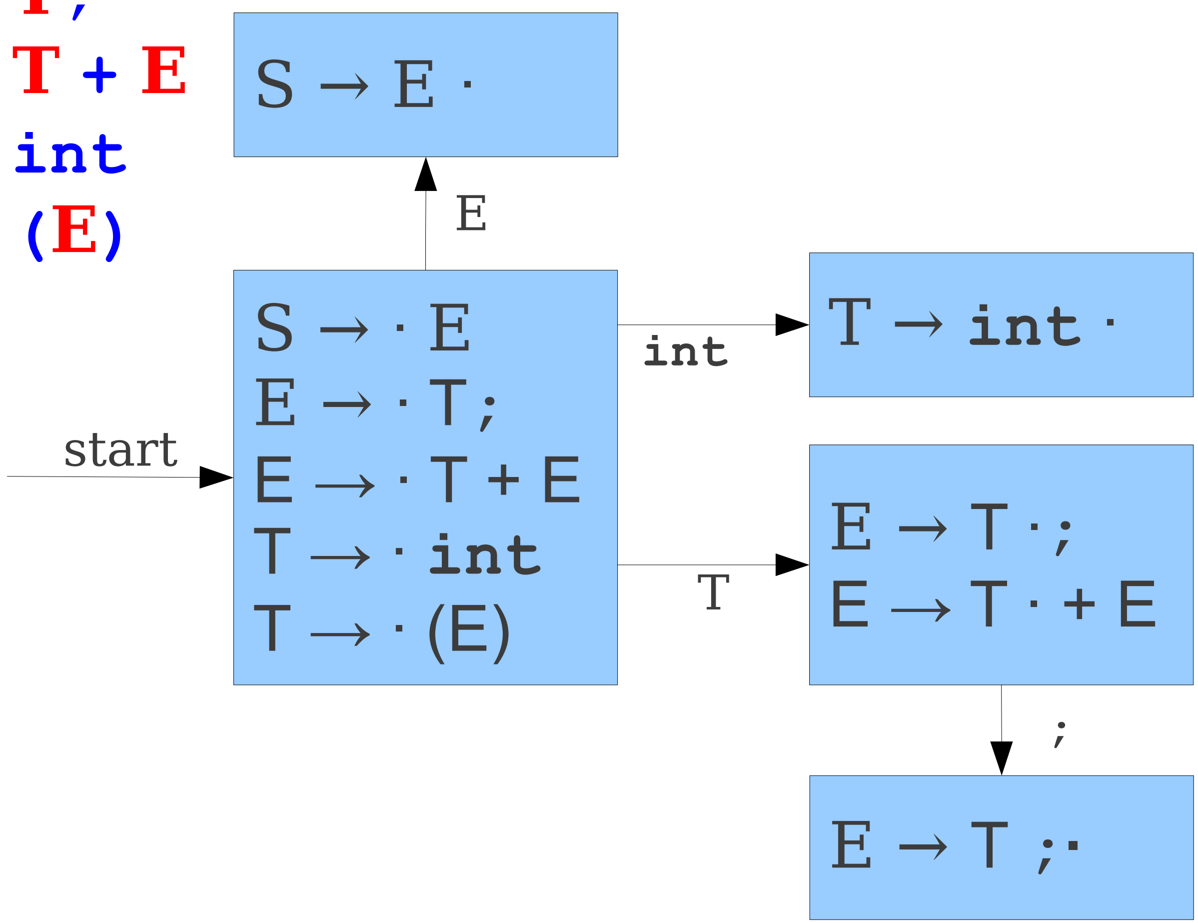
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → ( **E** )



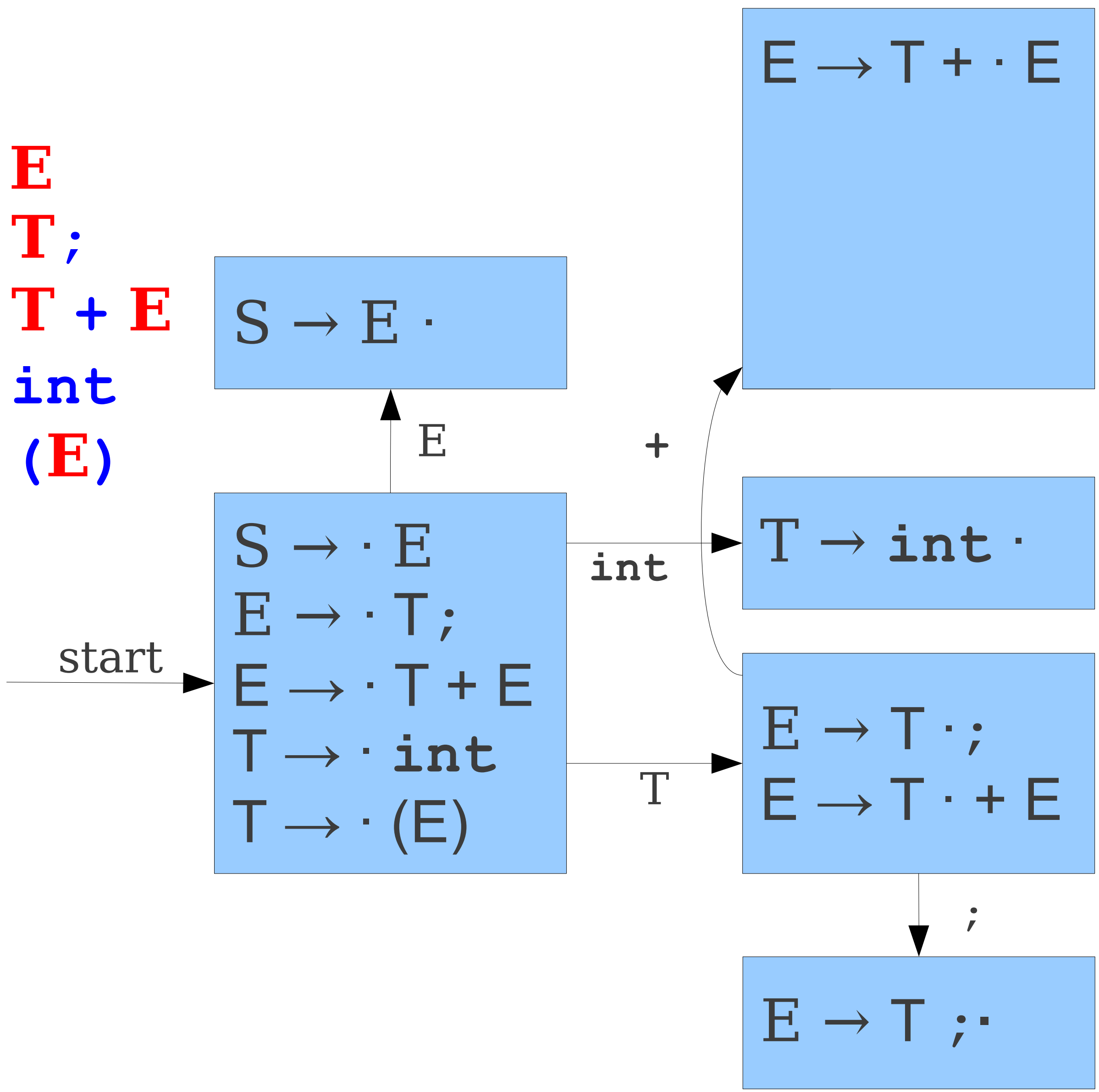
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



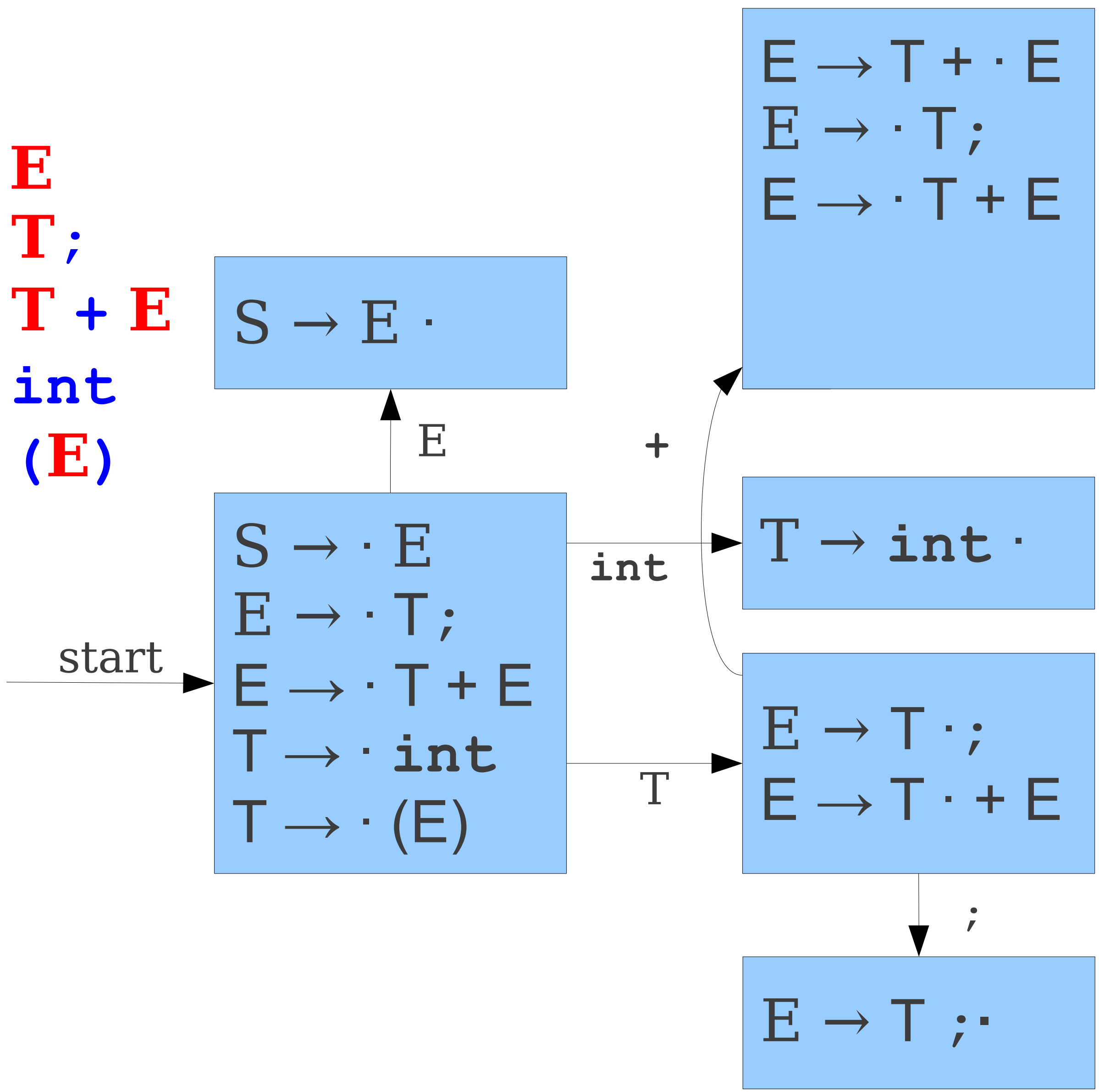
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



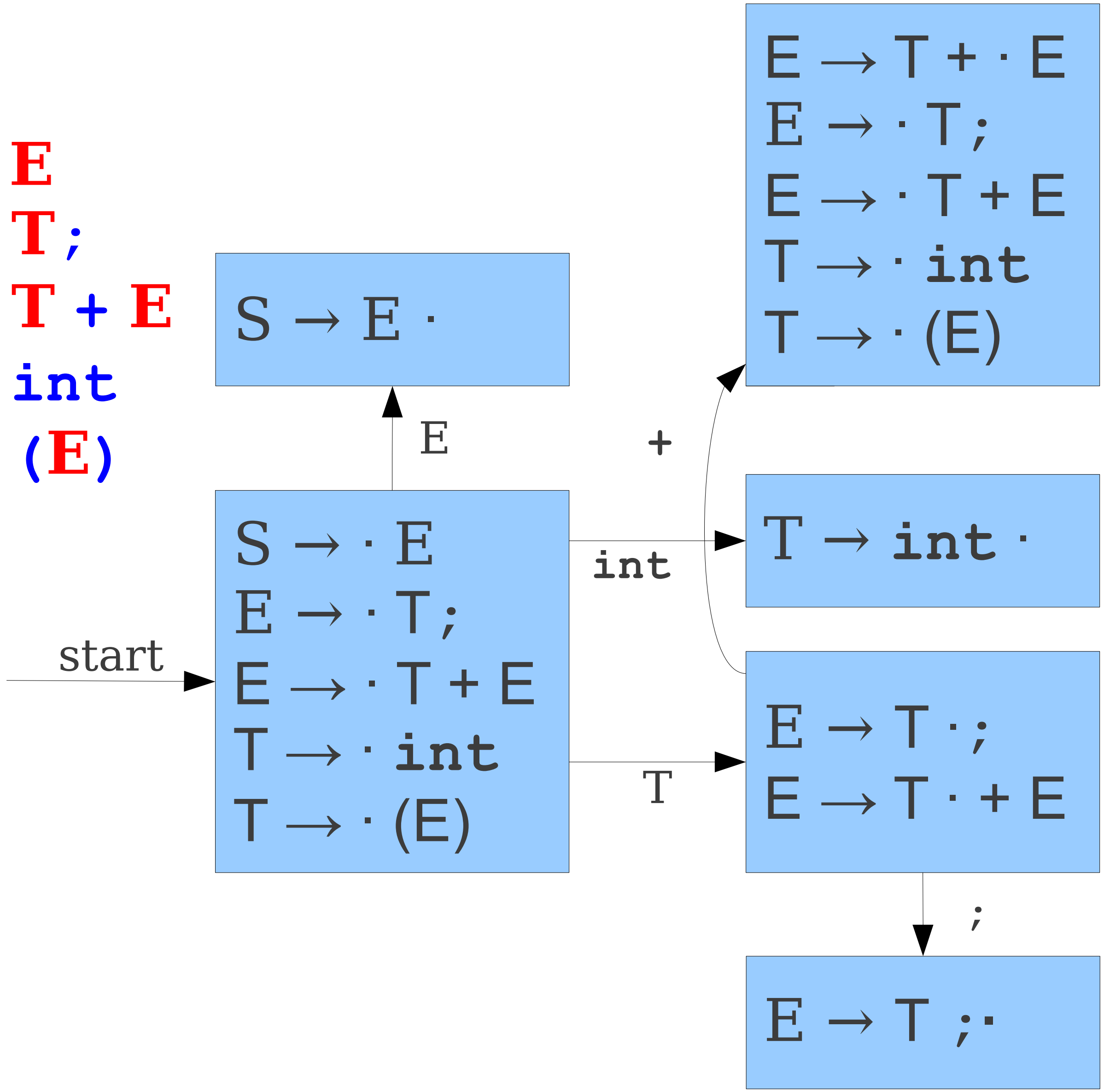
# A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



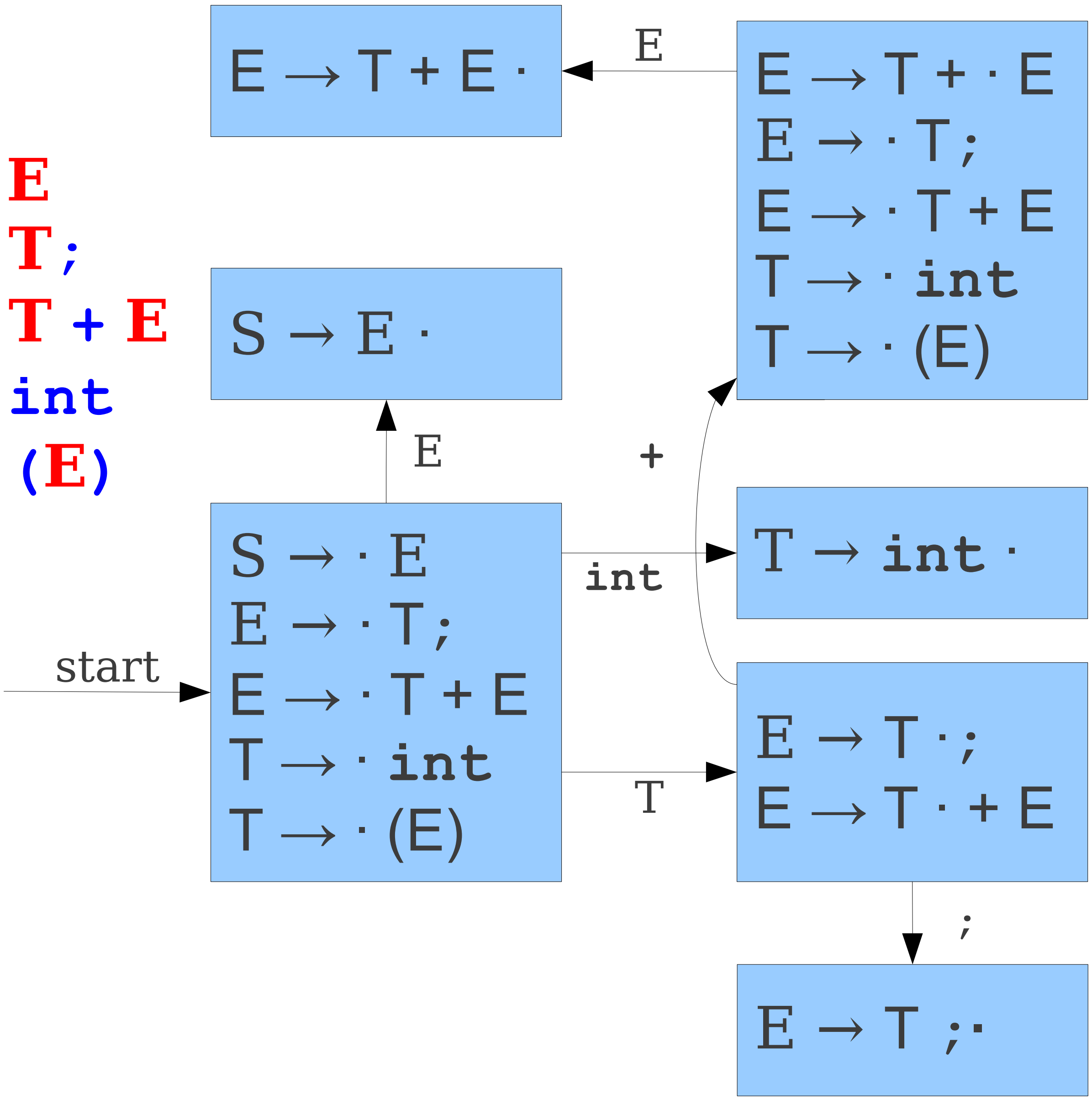
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



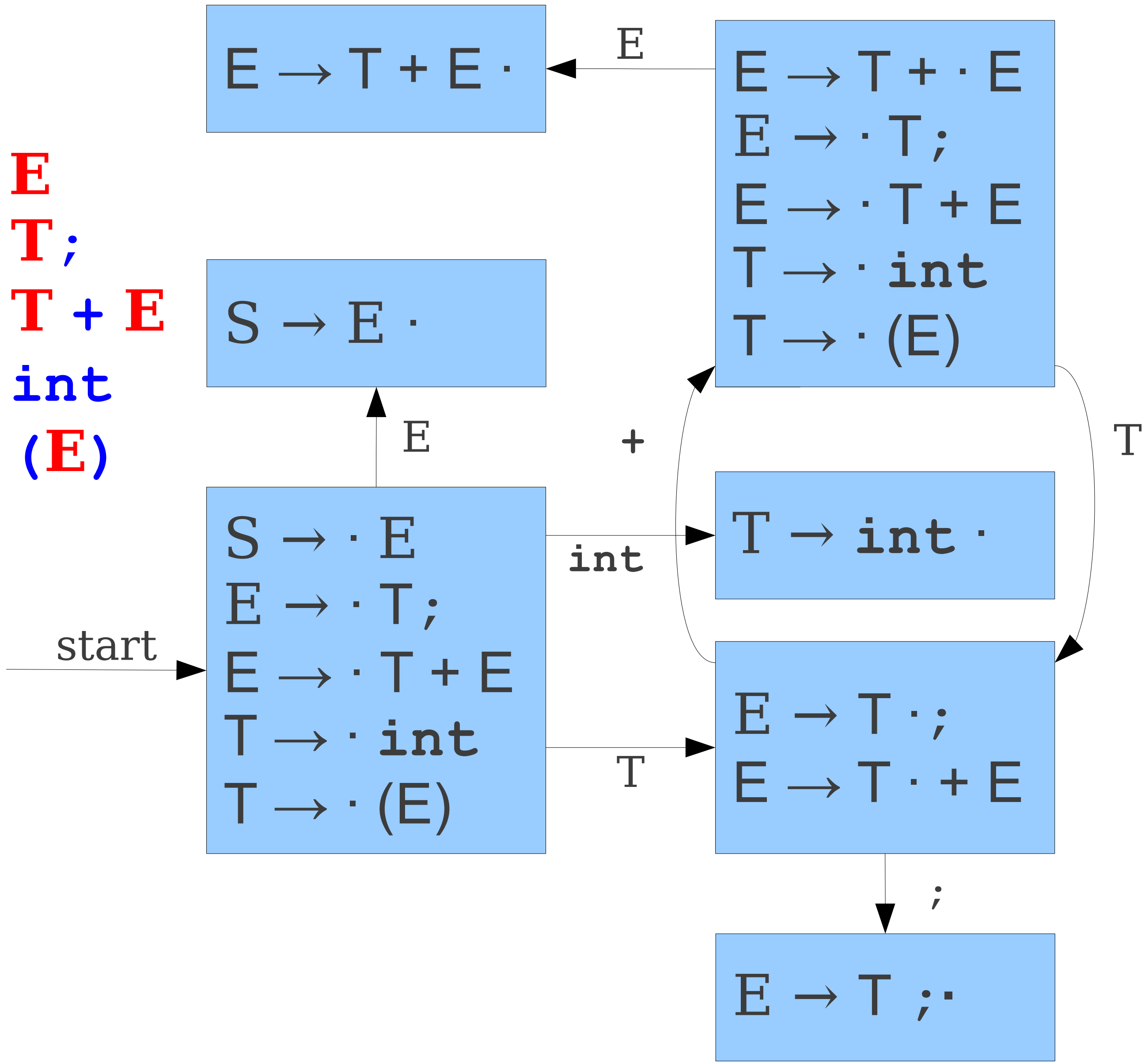
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** **+** **E**
**T** → `int`
**T** → `(`**E**`)`



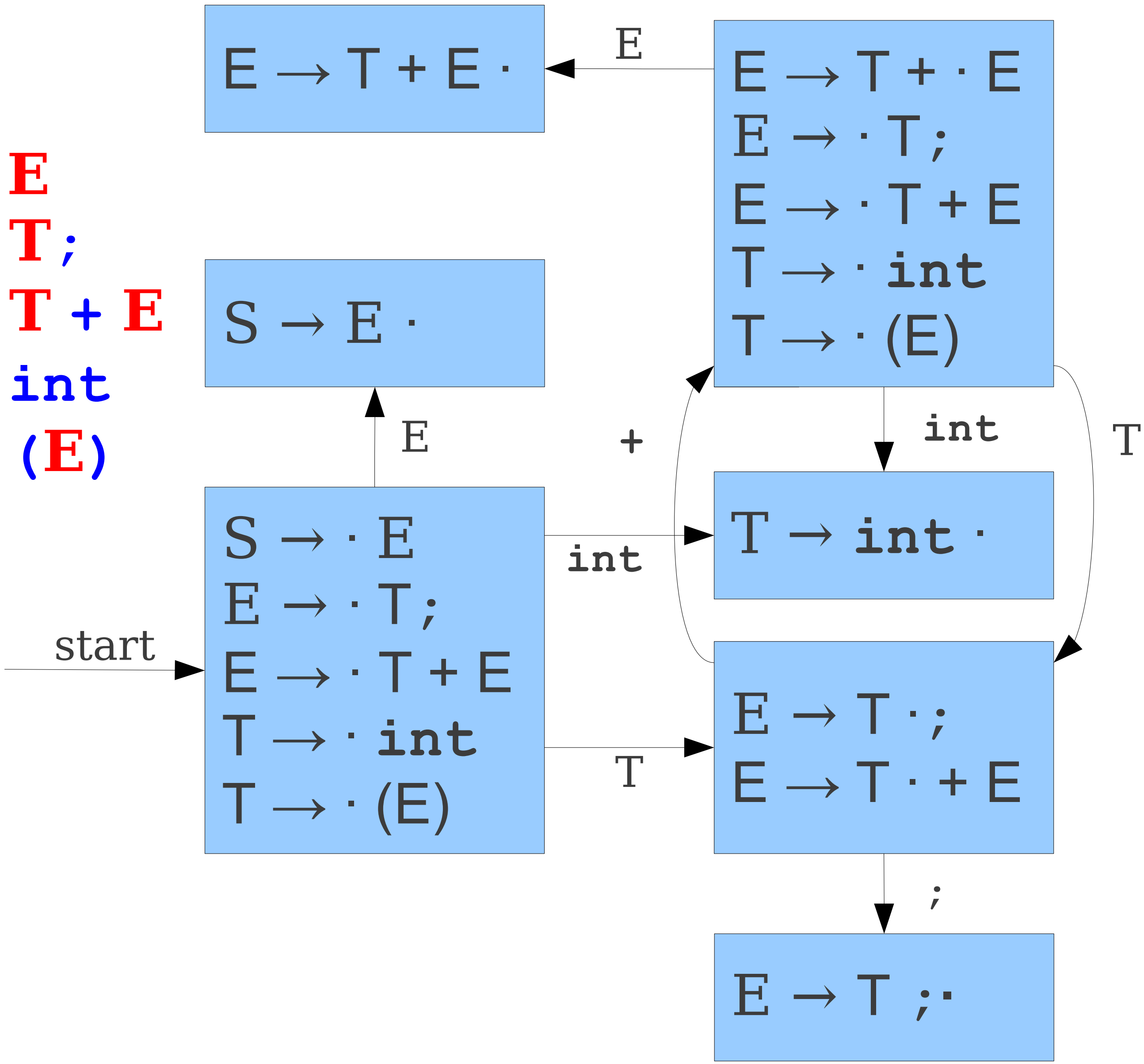
# A Deterministic Automaton

**S** → **E**
**E** → **T**;
**E** → **T** + **E**
**T** → `int`
**T** → (**E**)



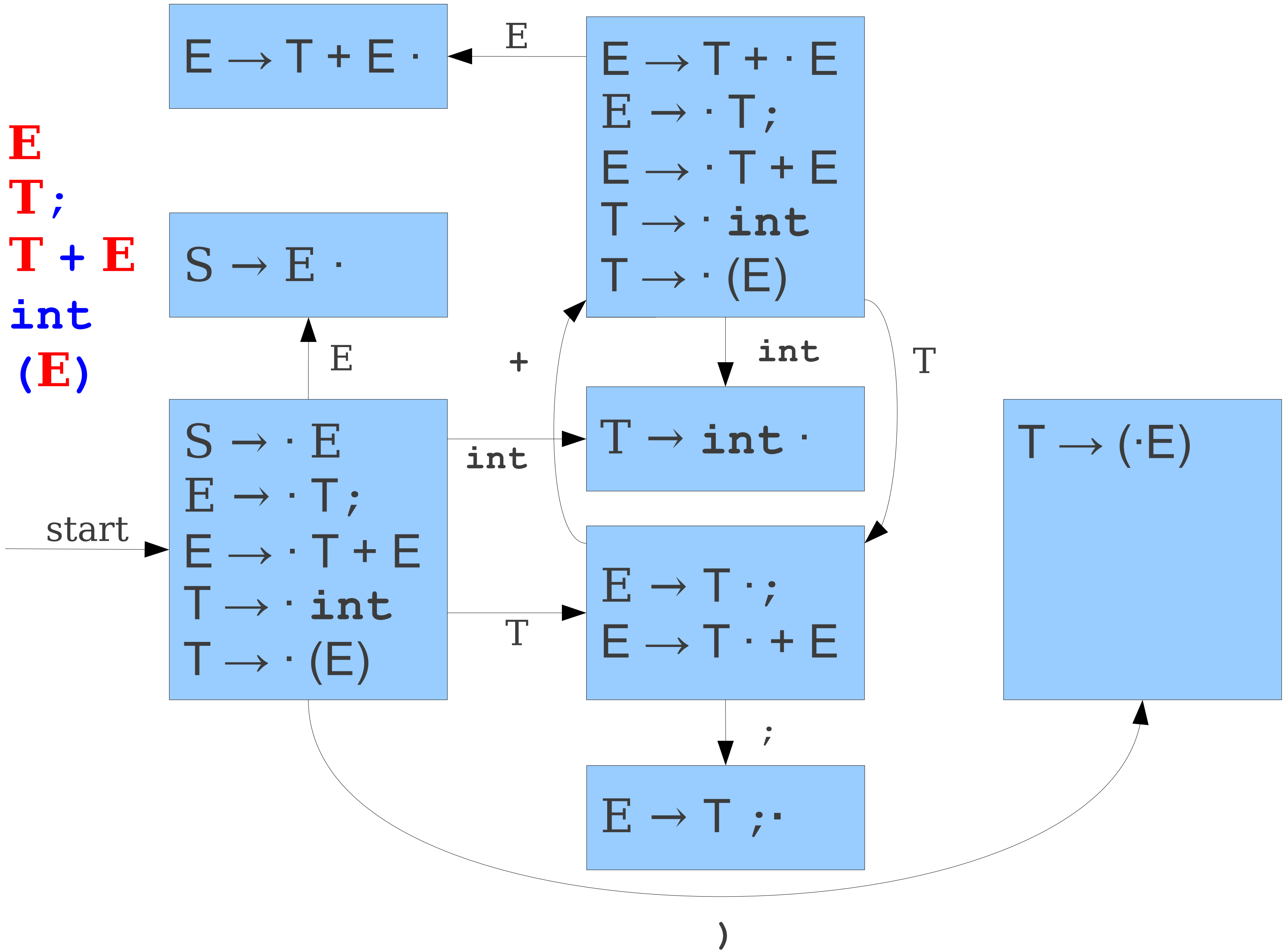
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



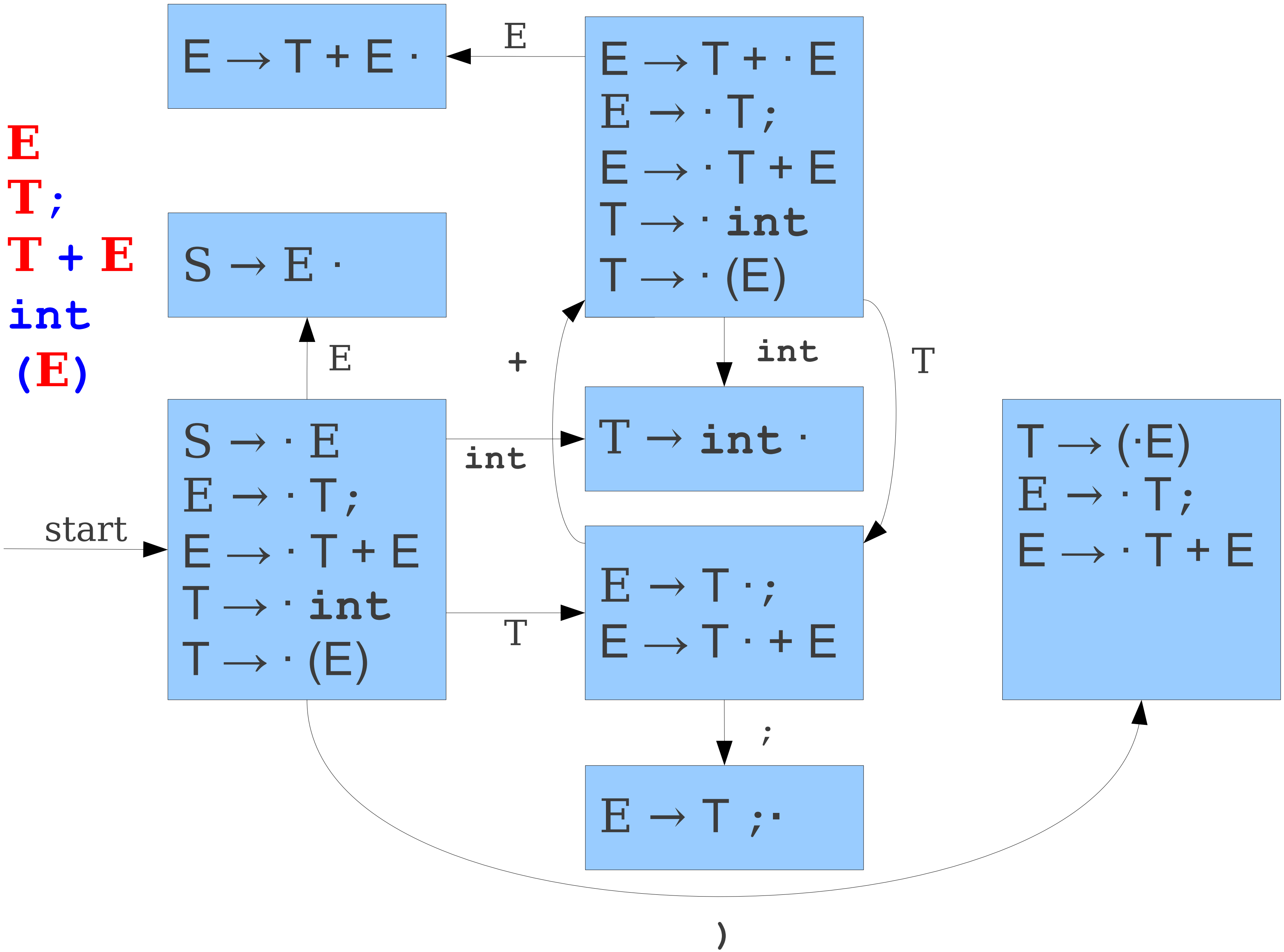
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



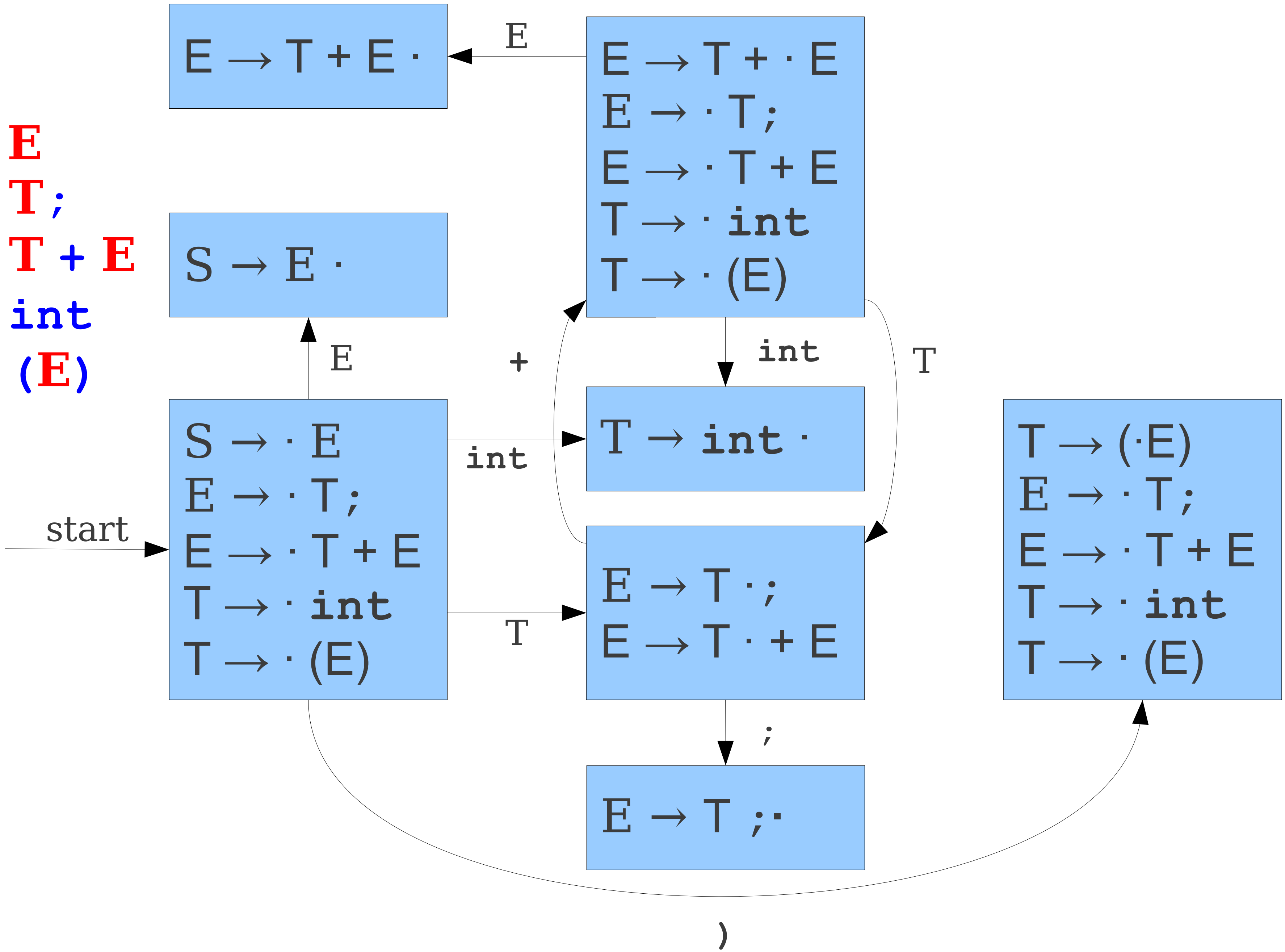
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



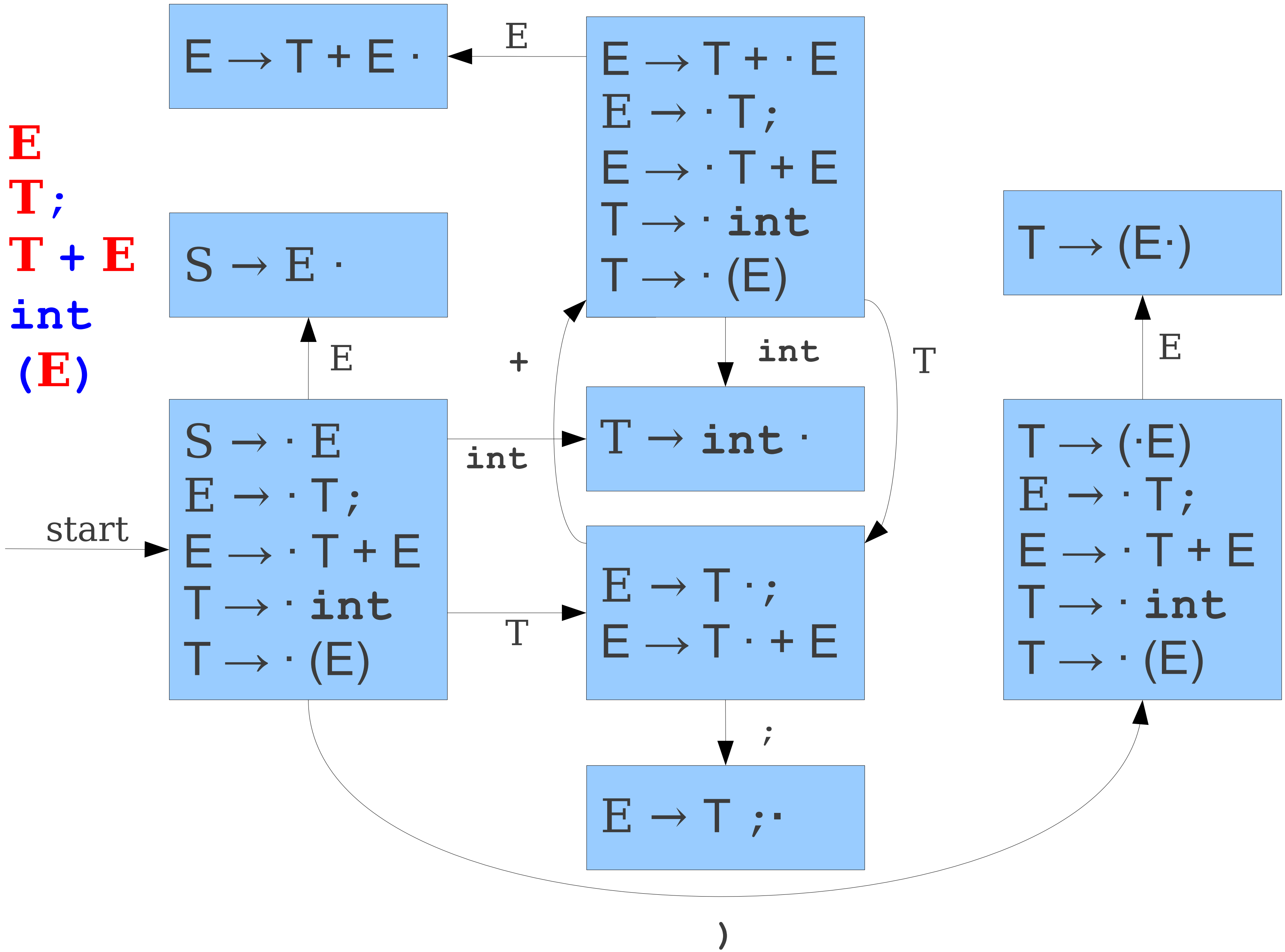
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



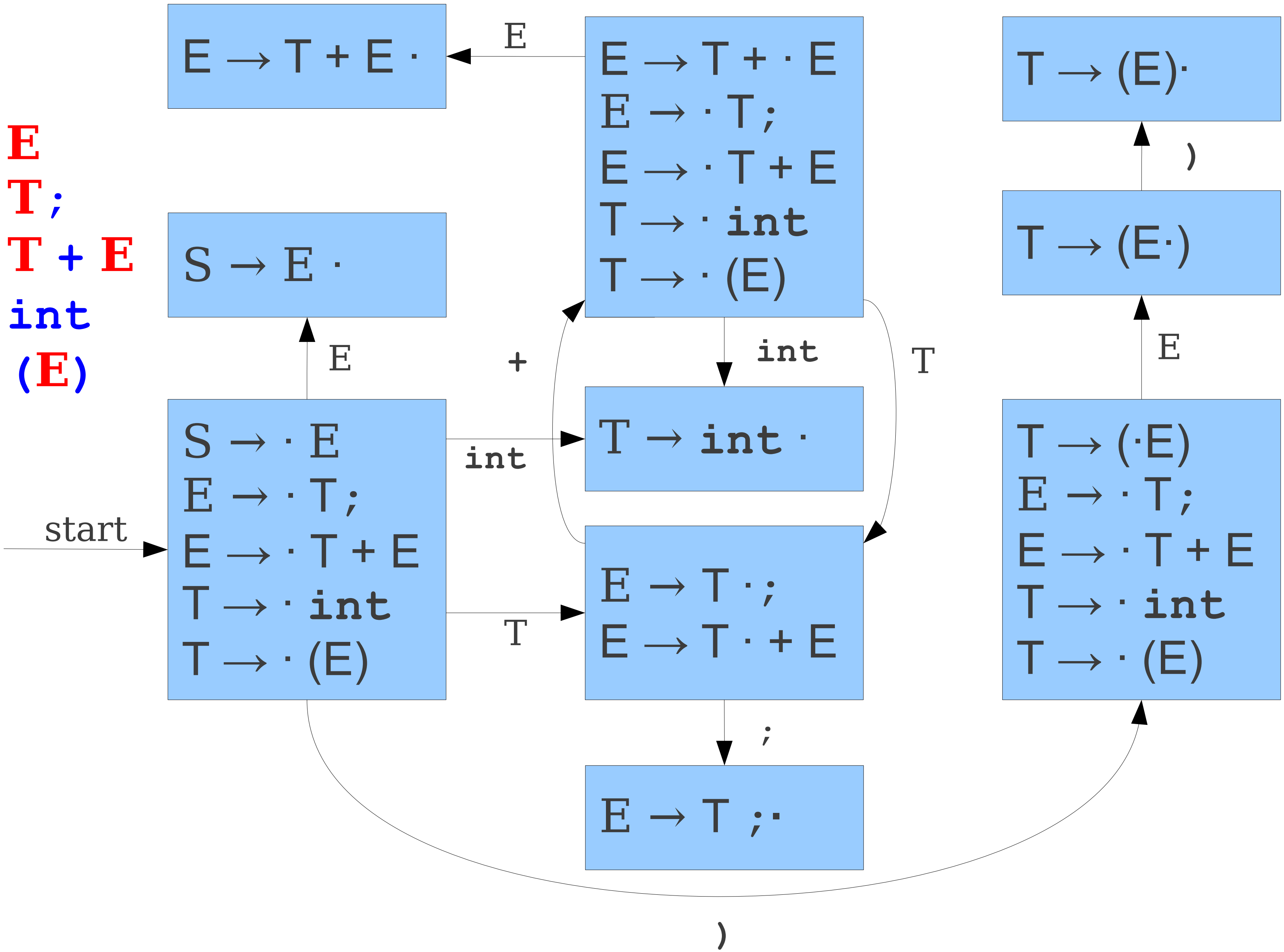
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



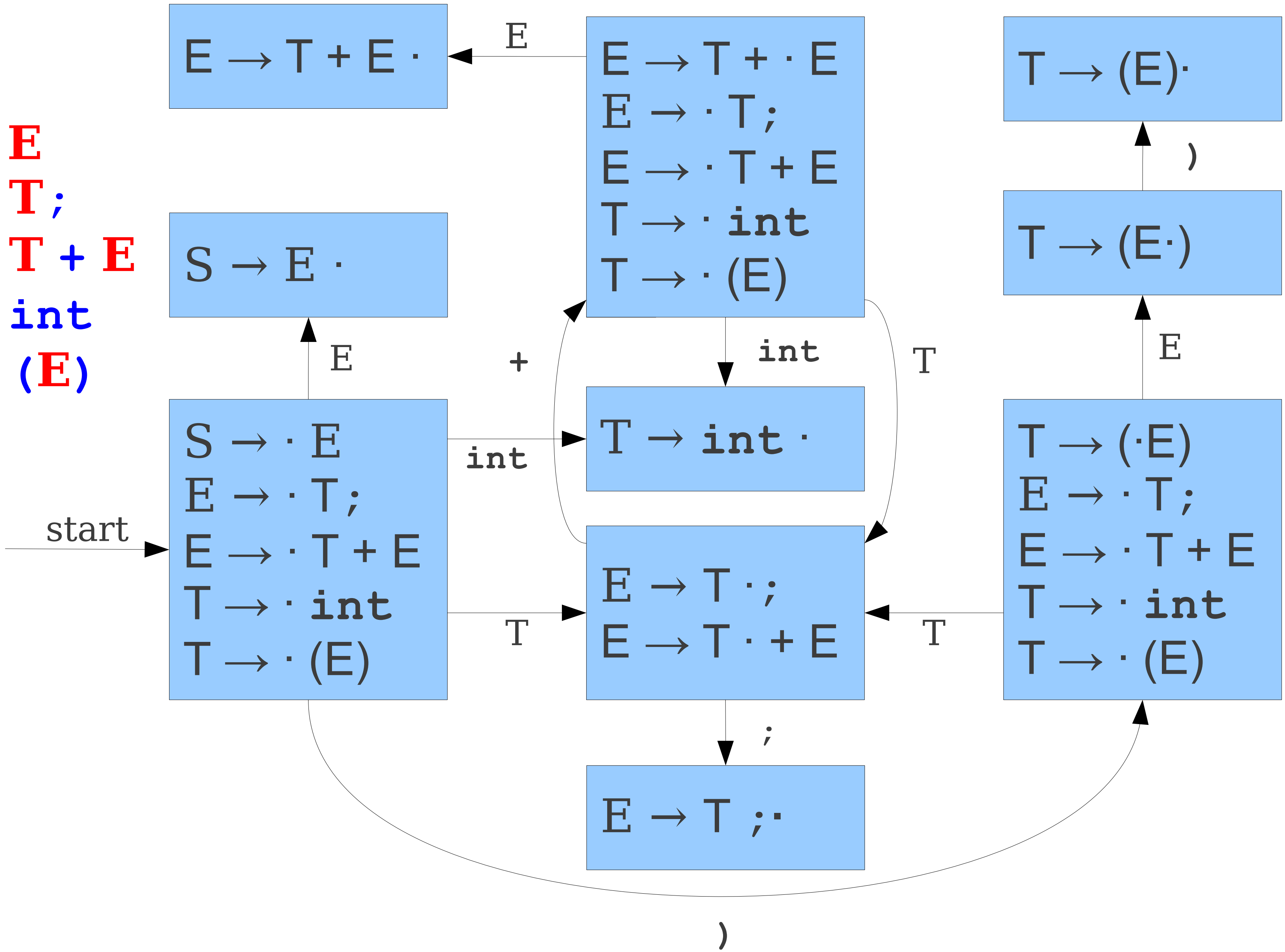
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



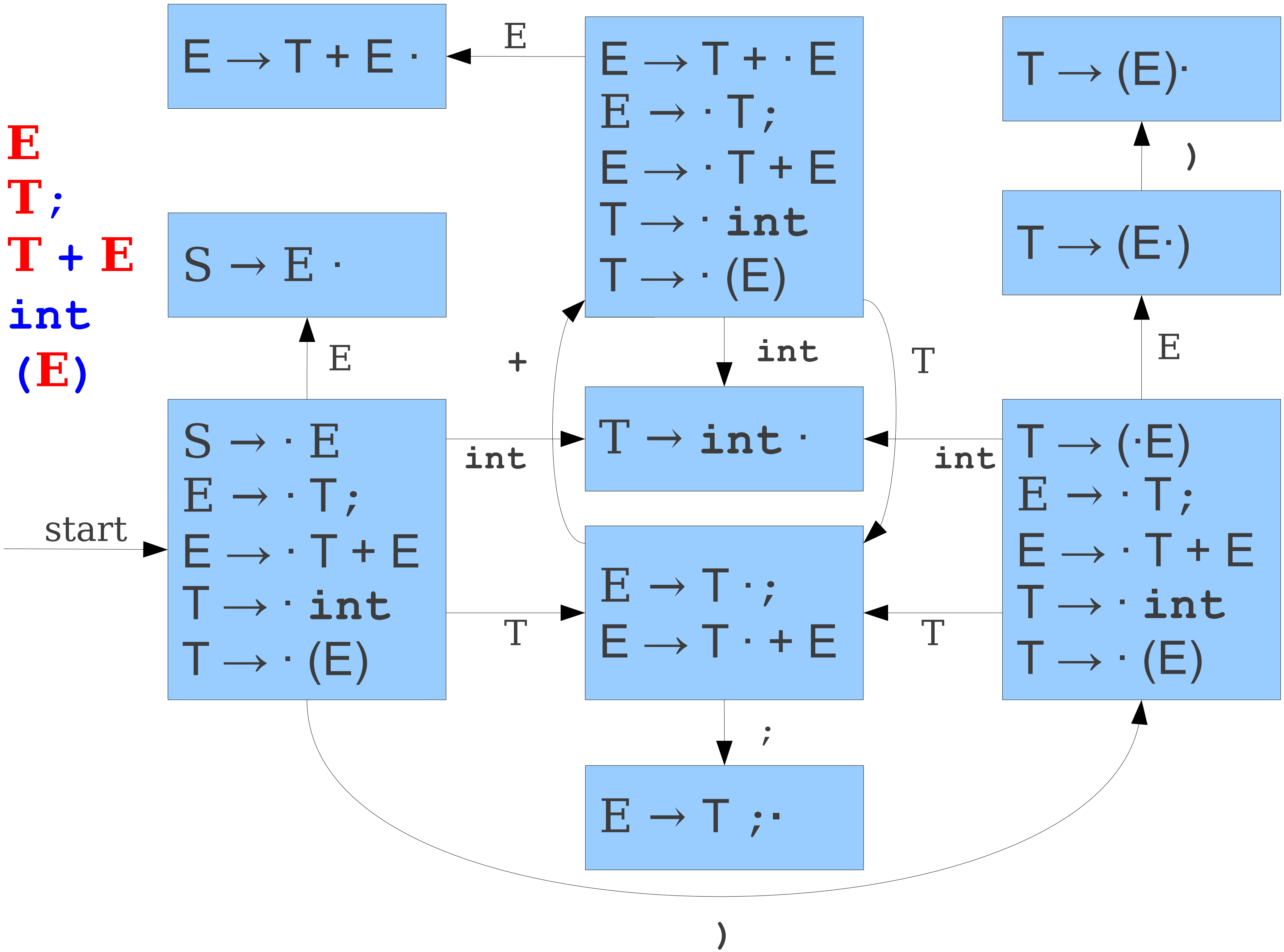
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



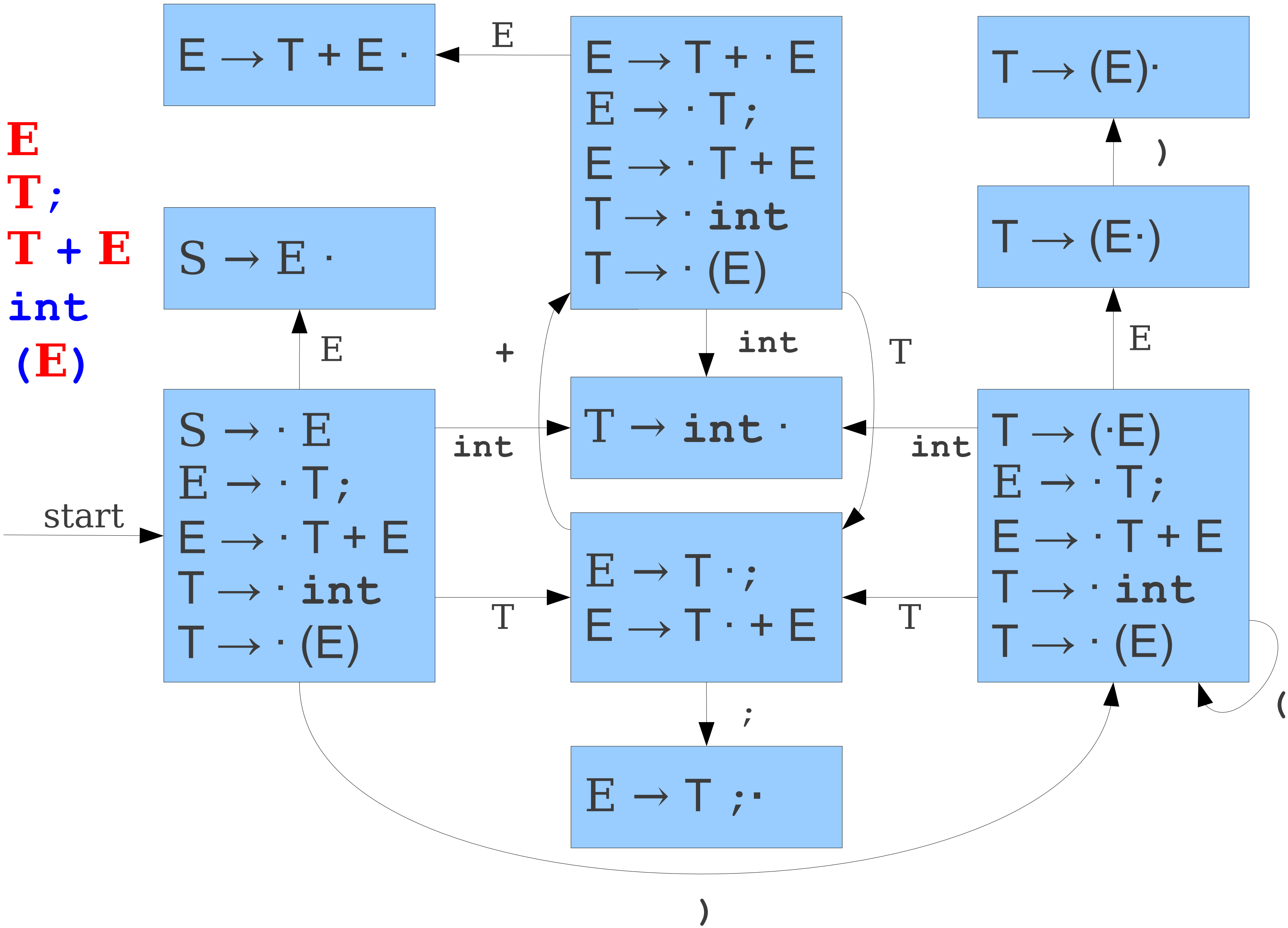
# A Deterministic Automaton

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



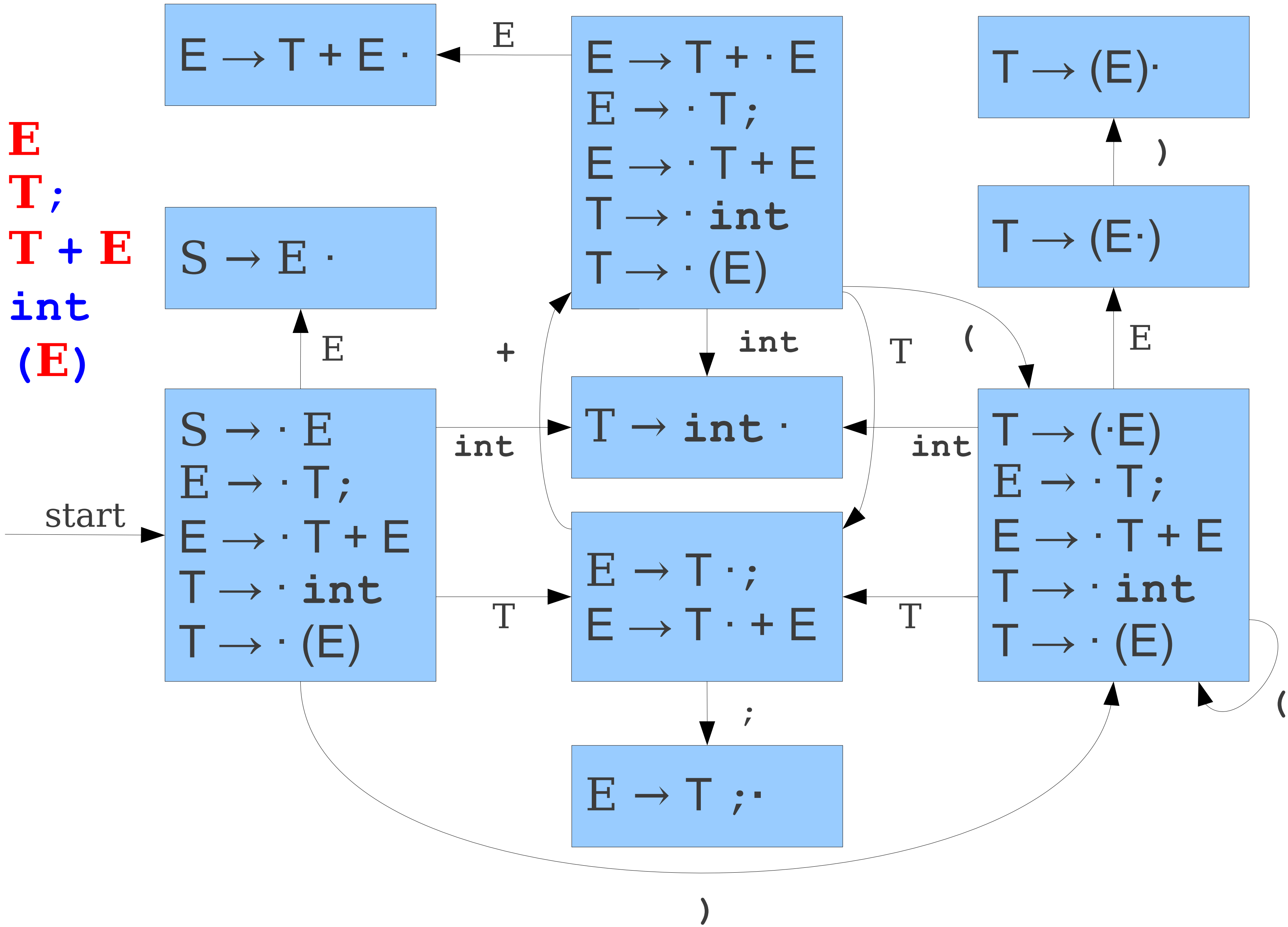
# A Deterministic Automaton

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# A Deterministic Automaton

S → E
E → T;
E → T + E
T → int
T → (E)



# Constructing the Automaton II

- Begin in a state containing **S** → · **A**, where **S** is the augmented start symbol.
- Compute the **closure** of the state:
  - If **A** → ***α*** · **B*ω*** is in the state, add **B** → · ***γ*** to the state for each production **B** → ***γ***.
  - Yet another fixed-point iteration!
- Repeat until no new states are added:
  - If a state contains a production **A** → ***α*** · ***xω*** for symbol ***x***, add a transition on ***x*** from that state to the state containing the closure of **A** → ***αx*** · ***ω***
- This is equivalent to a subset construction on the NFA.

# Handle-Finding Automata

- Handling-finding automata can be very large.
- NFA has states proportional to the size of the grammar, so DFA can have size exponential in the size of the grammar.
  - There are grammars that can exhibit this worst-case.
- Automata are almost always generated by tools like **`bison`**.

# Finding Handles

- Where do we look for handles?
  - **At the top of the stack.**
- How do we search for handles?
  - **Build a handle-finding automaton**.
- How do we recognize handles?
  - Once we've found a possible handle, how do we confirm that it's correct?

# Question Three:

How do we recognize handles?

# Handle Recognition

- Our automaton will tell us all places where a handle might be.
- However, if the automaton says that there might be a handle at a given point, we need a way to confirm this.
- We'll thus use **predictive bottom-up parsing**:
  - Have a deterministic procedure for guessing where handles are.
- There are many predictive algorithms, each of which recognize different grammars.

# Our First Algorithm: **LR(0)**

- Bottom-up predictive parsing with:
  - **L**: **L**eft-to-right scan of the input.
  - **R**: **R**ightmost derivation.
  - (**0**): Zero tokens of lookahead.
- Use the handle-finding automaton, without any lookahead, to predict where handles are.

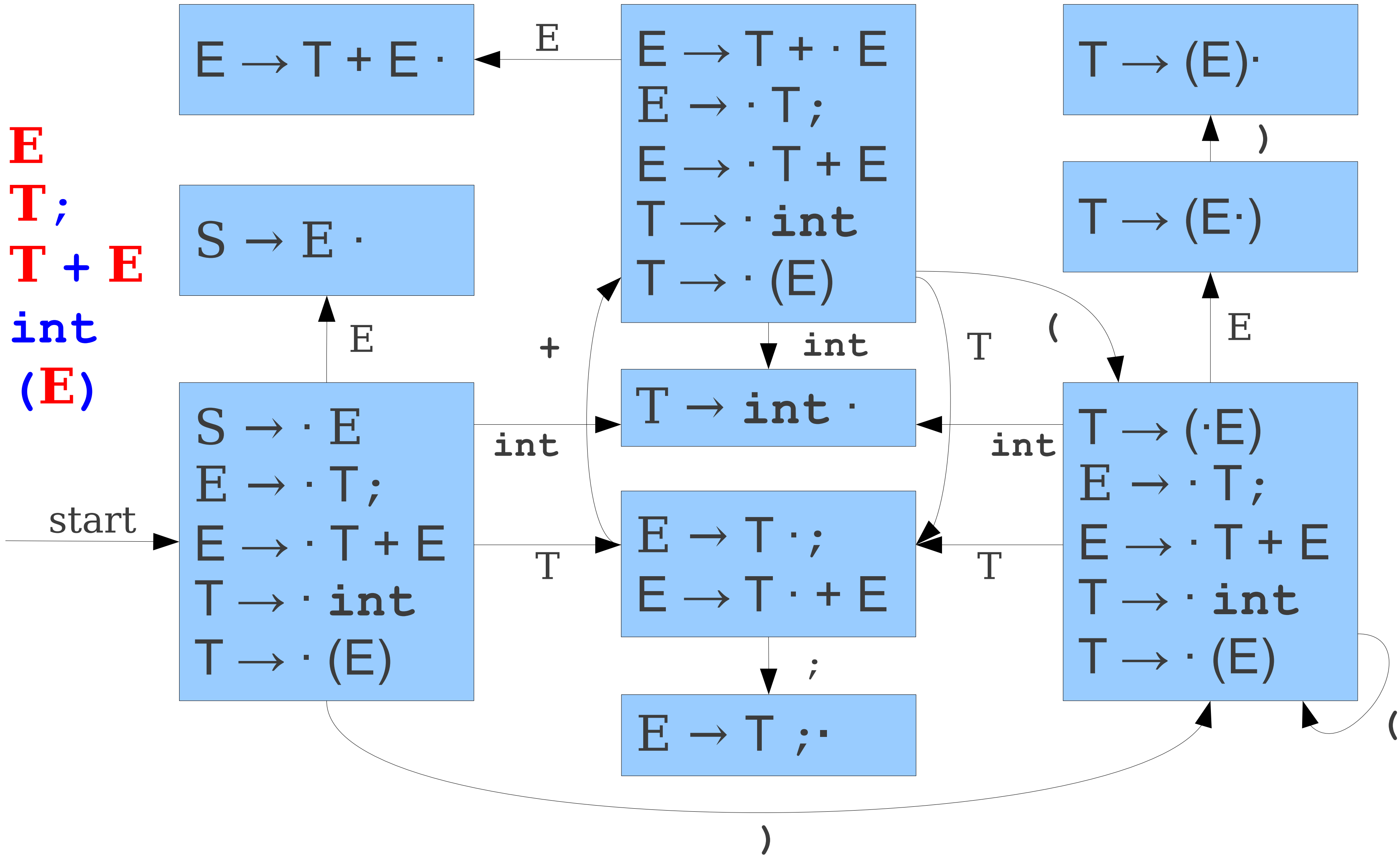
# LR(0) Parsing

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| `int` | `+` | `(` | `int` | `+` | `int` | `;` | `)` | `;` |

# LR(0) Parsing

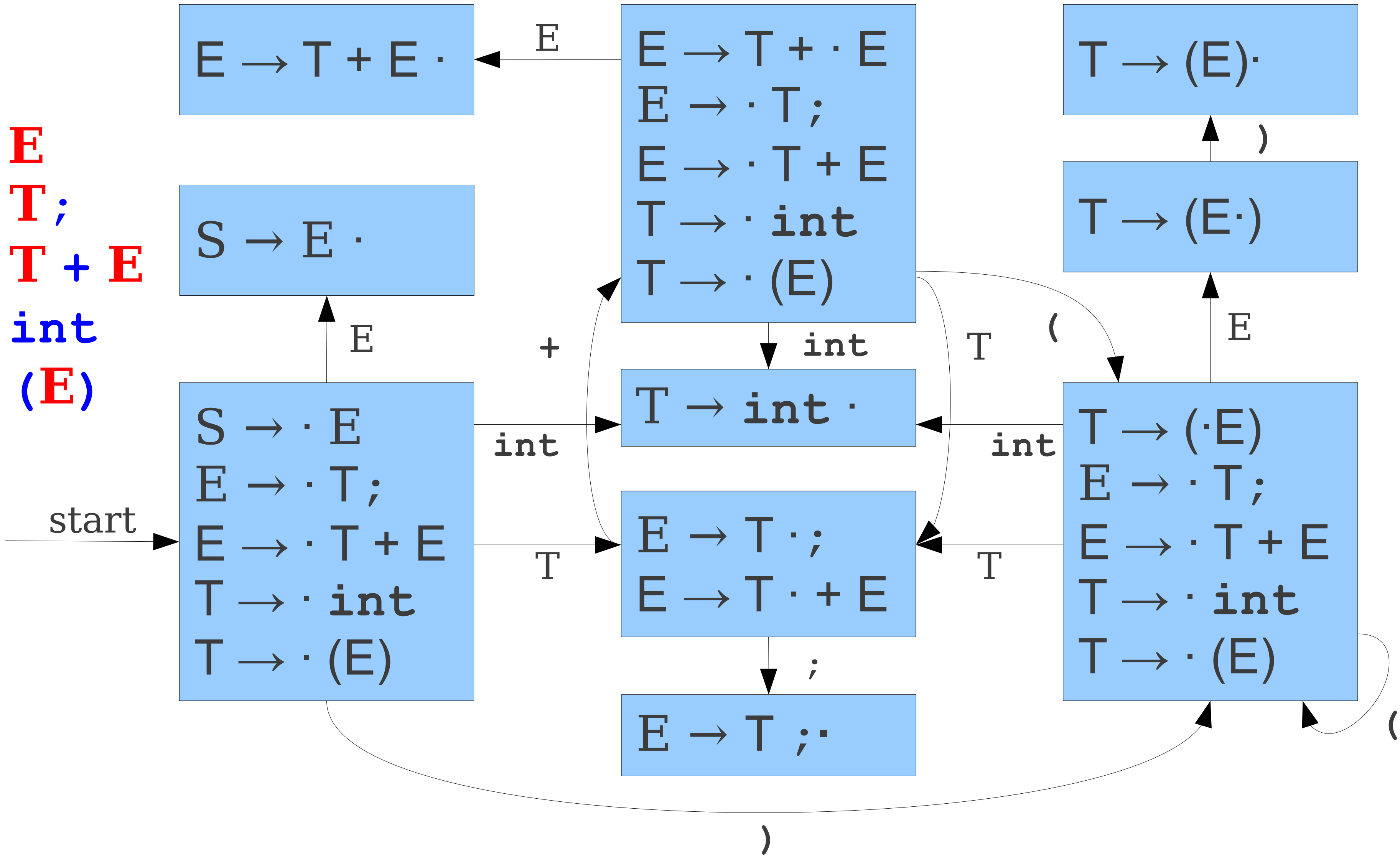
**S → E**
**E → T ;**
**E → T + E**
**T → int**
**T → (E)**




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | ; | ) | ; |

# LR(0) Parsing

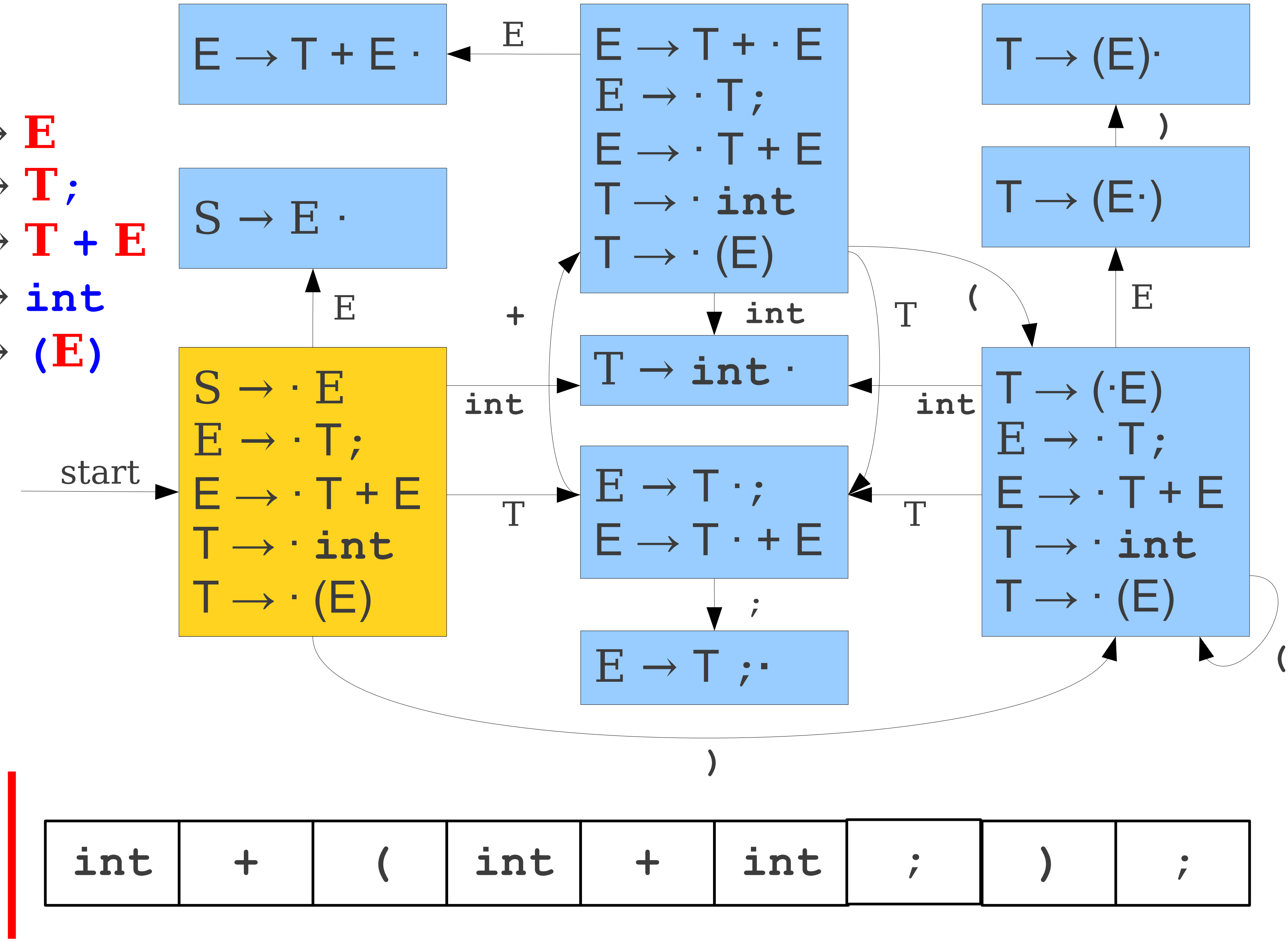
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | ; | ) | ; |

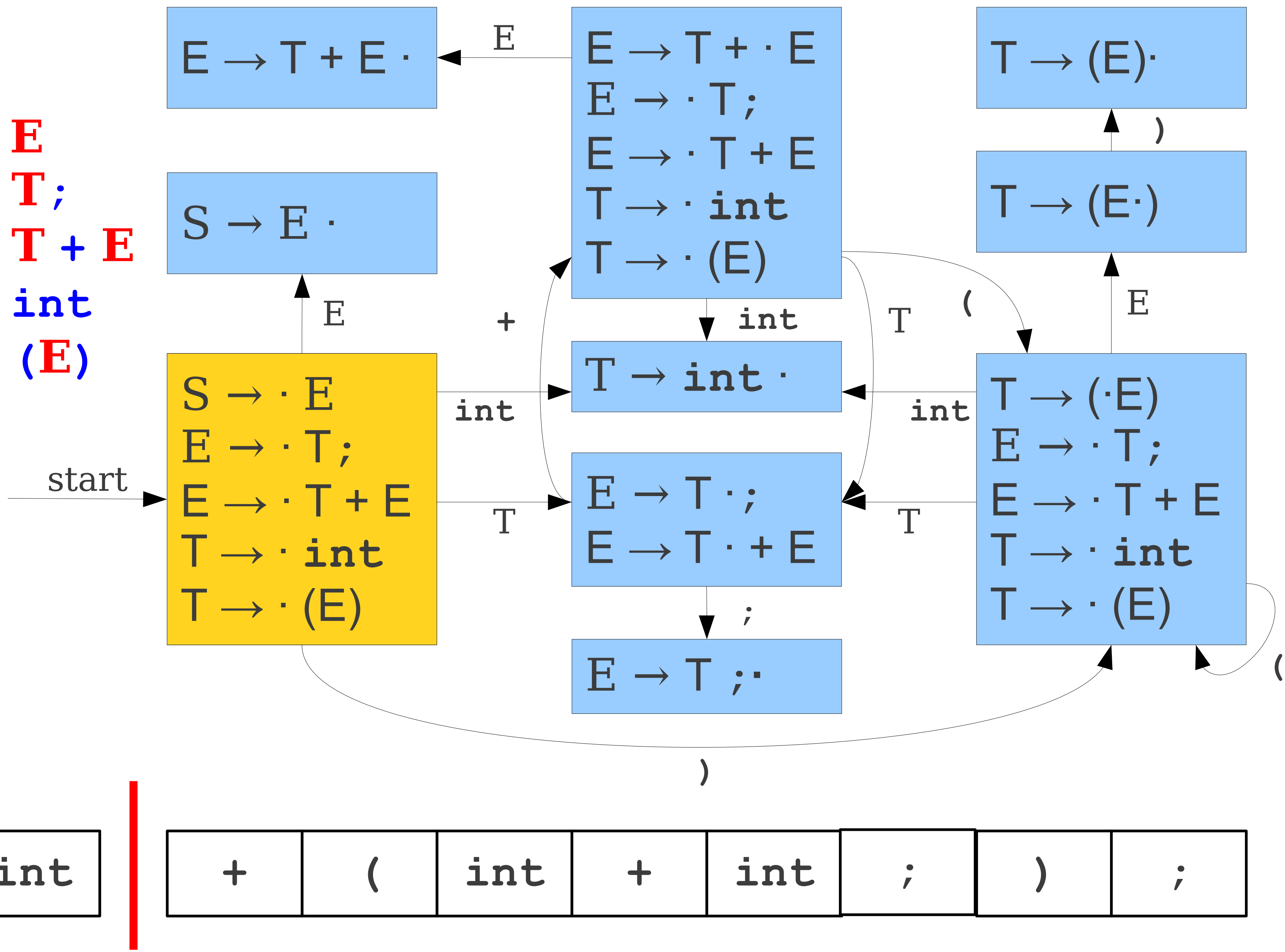
# LR(0) Parsing

**S** → **E**
**E** → **T ;**
**E** → **T + E**
**T** → **int**
**T** → **(E)**



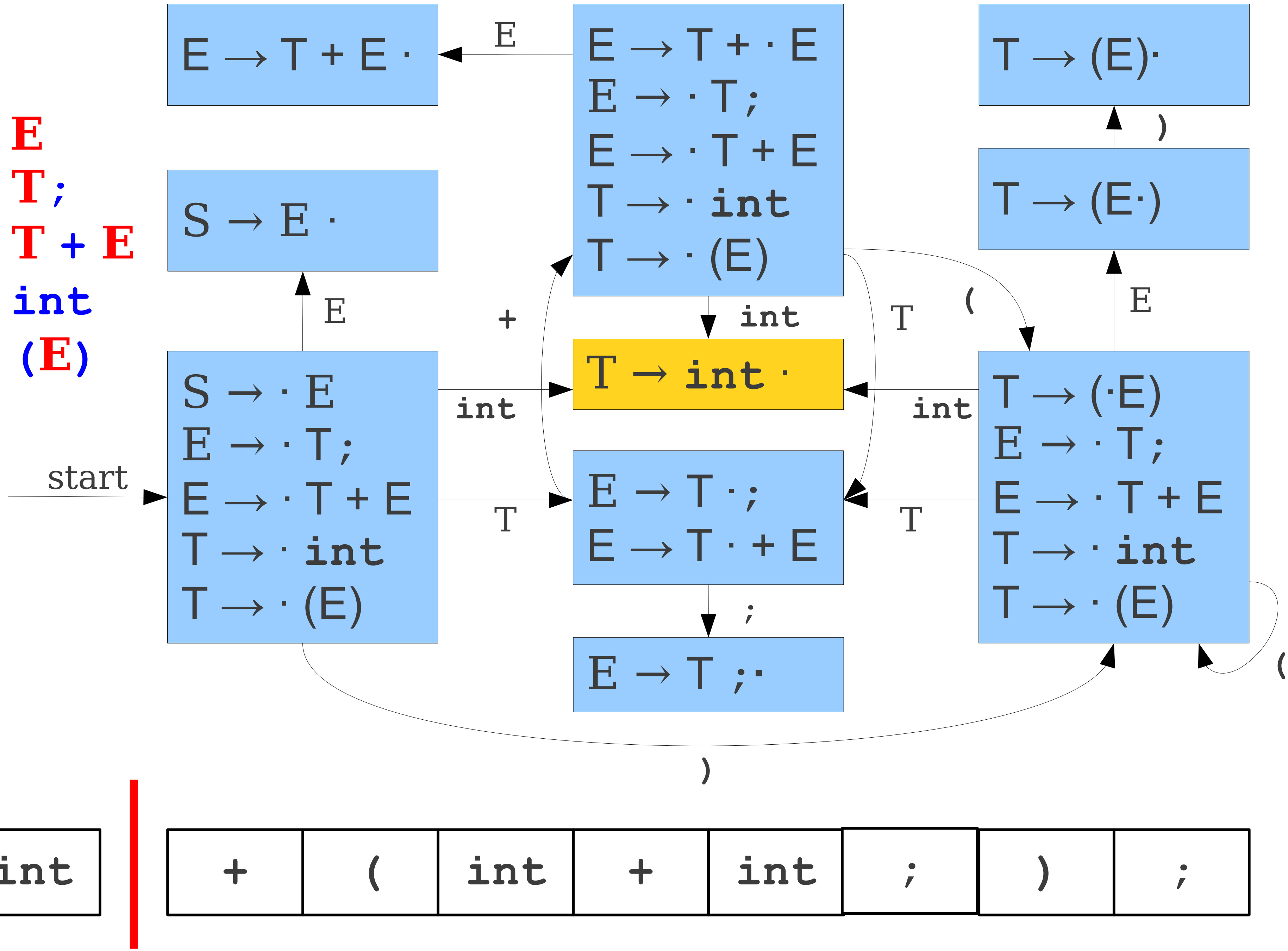
# LR(0) Parsing

$S \to E$
$E \to T;$
$E \to T + E$
$T \to \texttt{int}$
$T \to (E)$



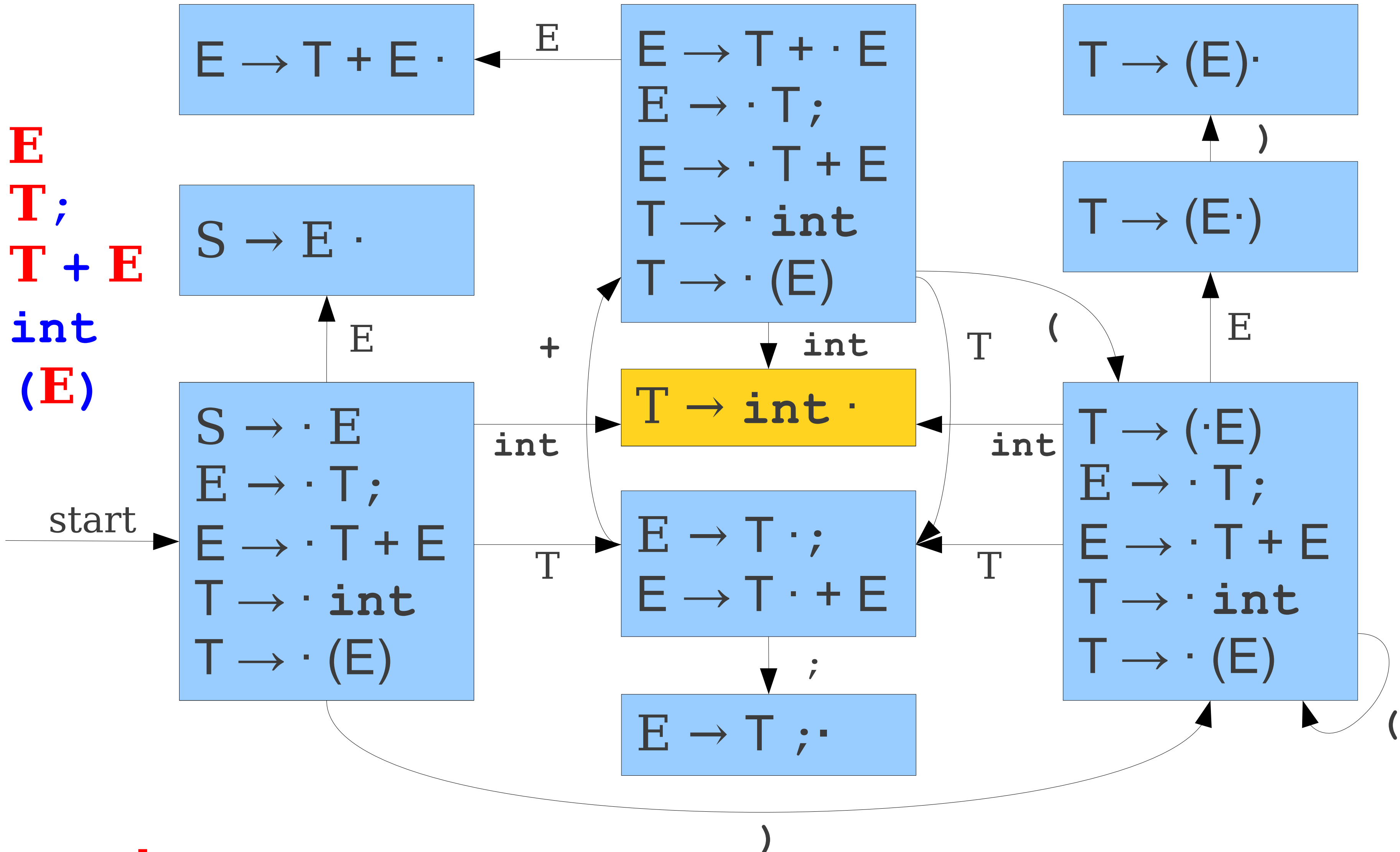
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



# LR(0) Parsing

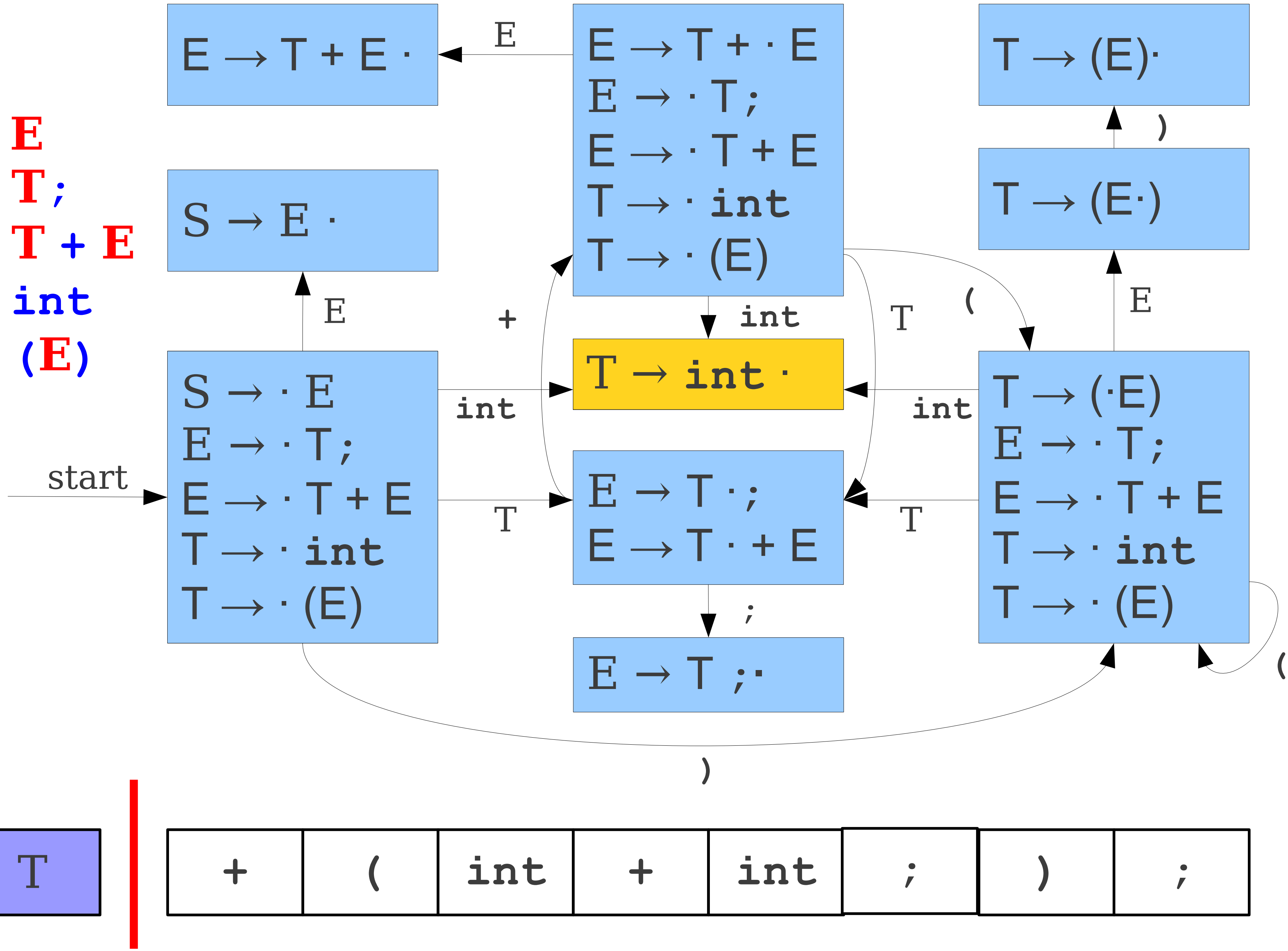
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)




| | | | | | | | |
|---|---|---|---|---|---|---|---|
| + | ( | int | + | int | ; | ) | ; |

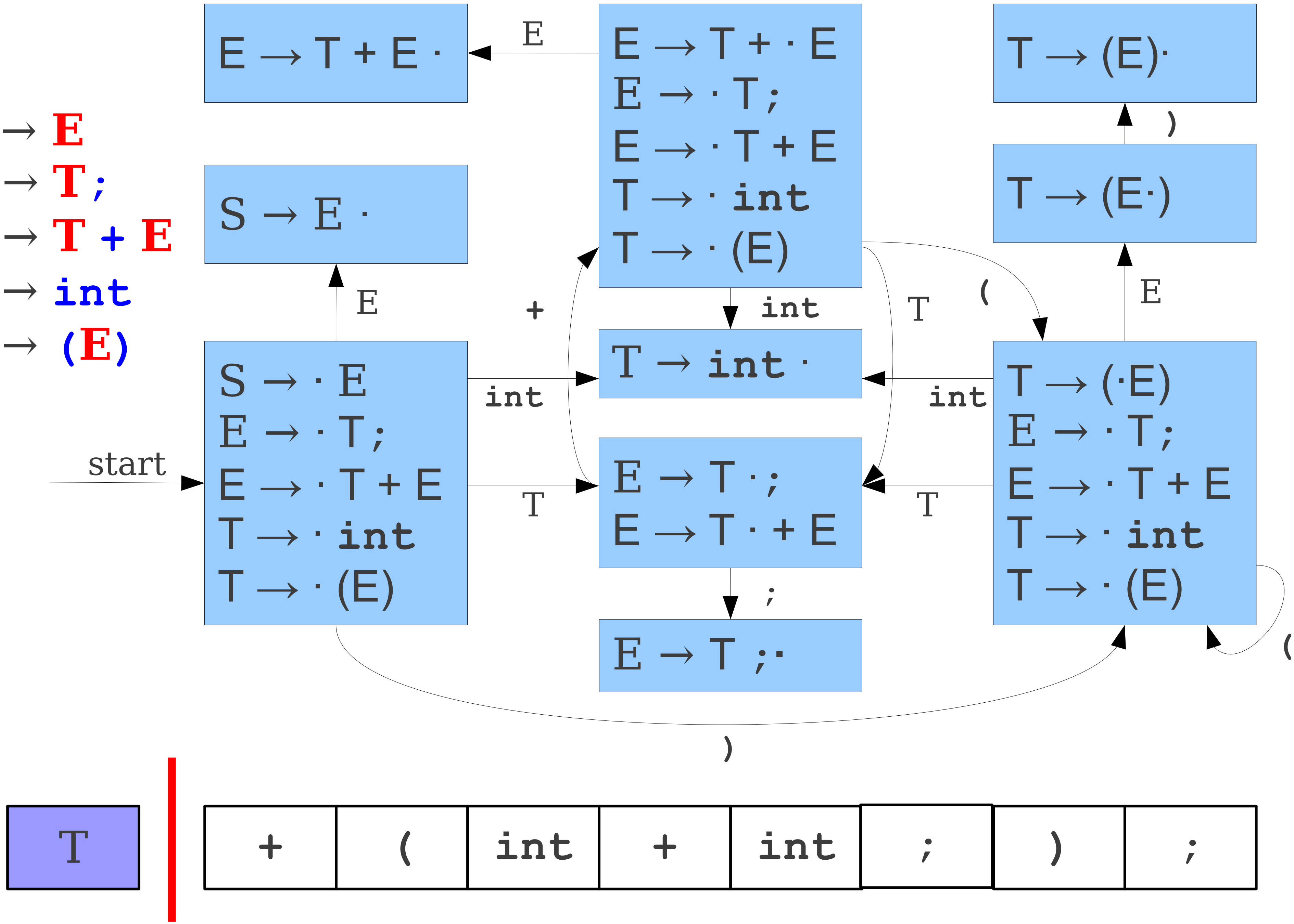
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



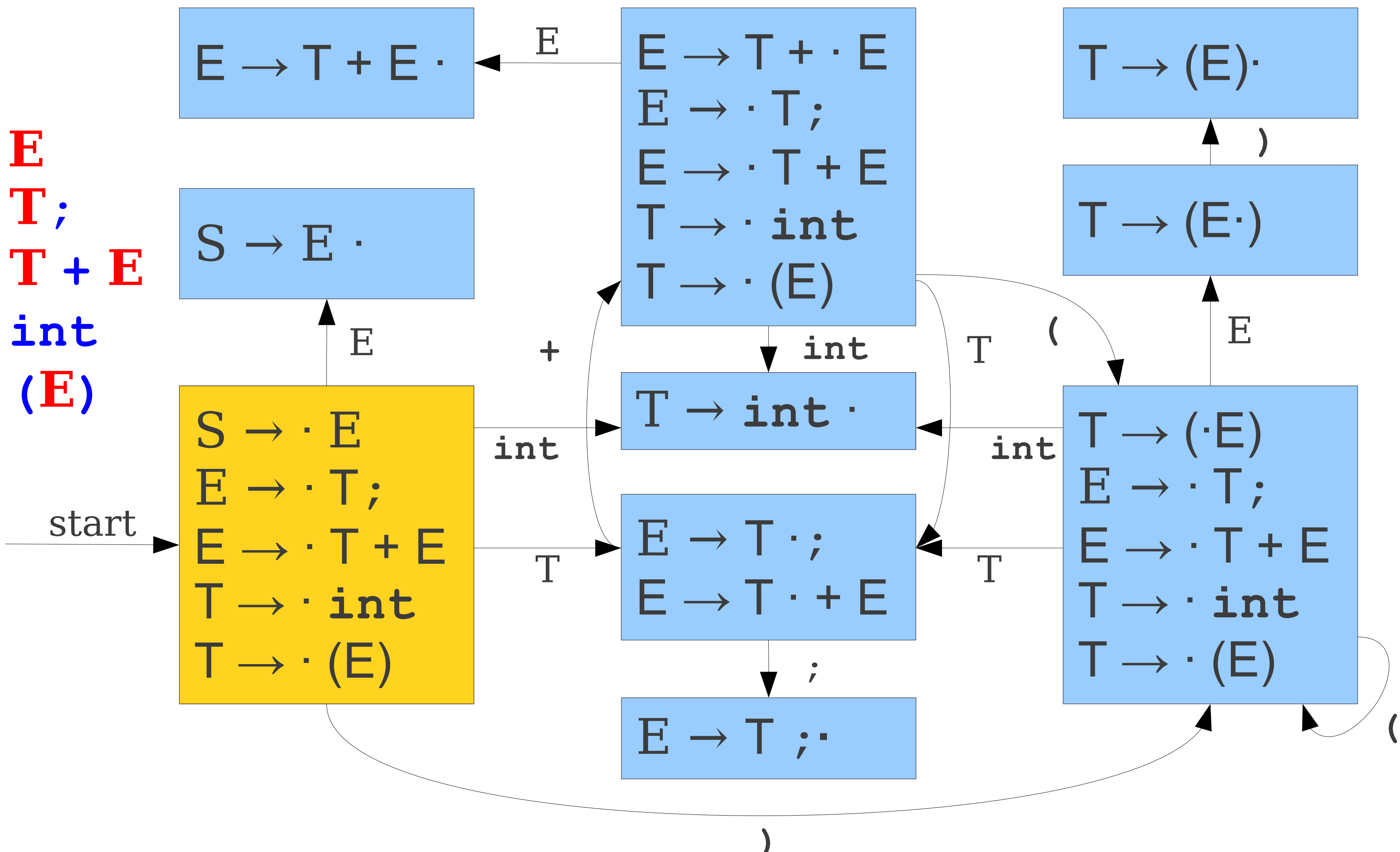
# LR(0) Parsing

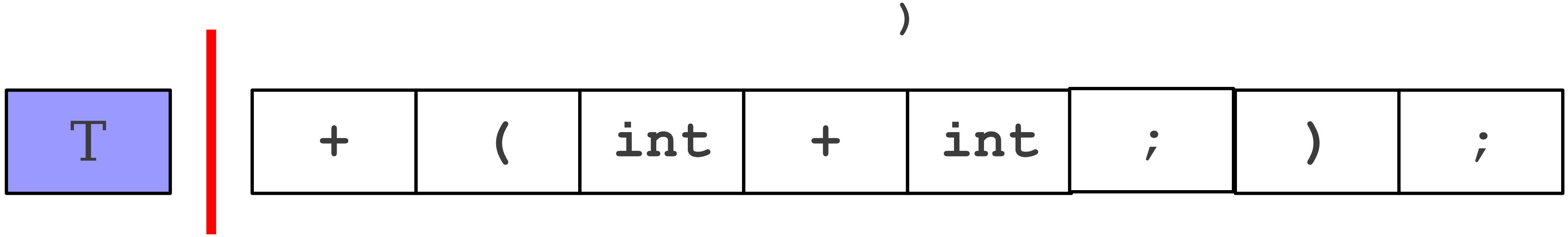
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



# LR(0) Parsing

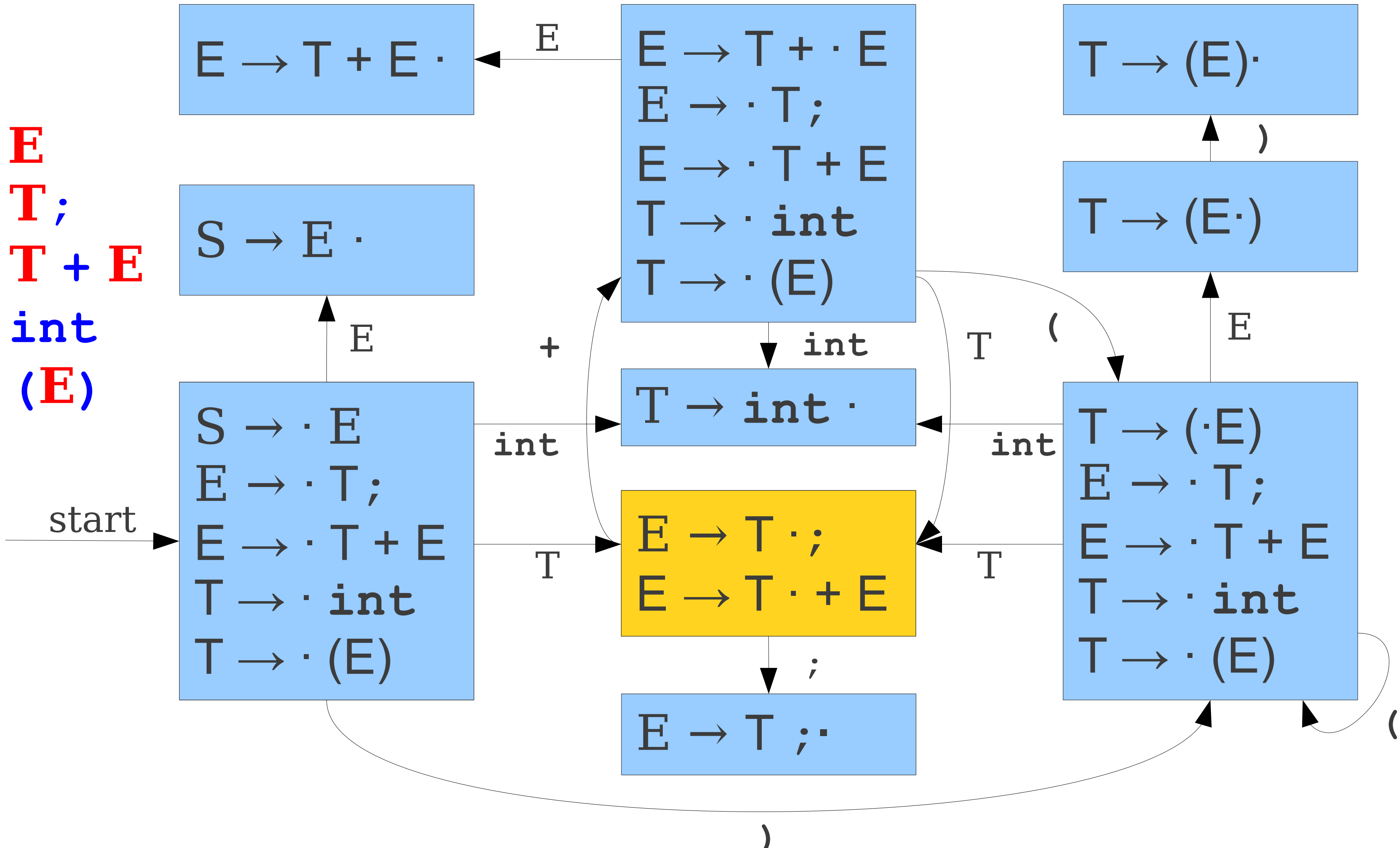
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



# LR(0) Parsing

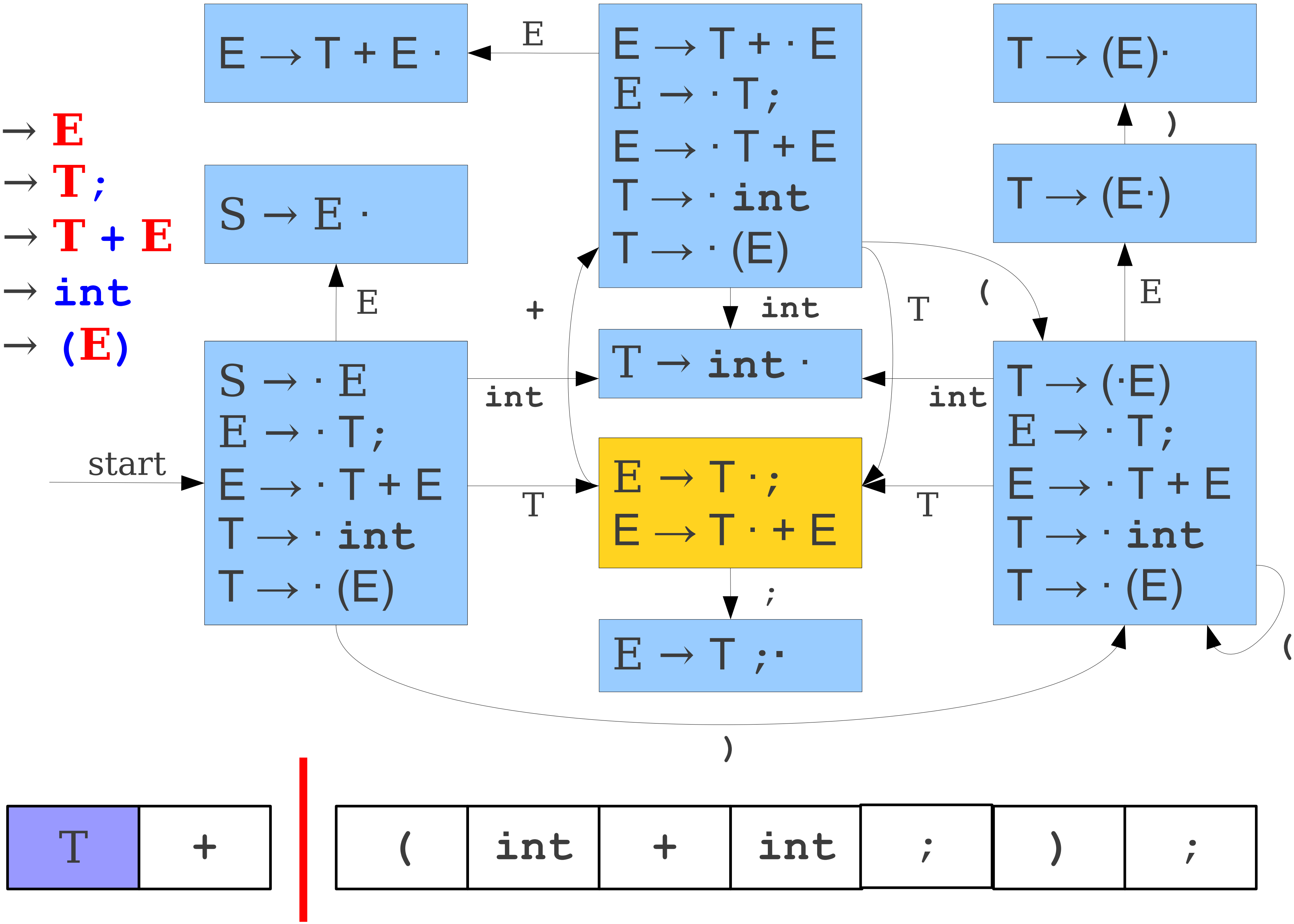
S → E
E → T;
E → T + E
T → int
T → (E)




| T | + | ( | int | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|

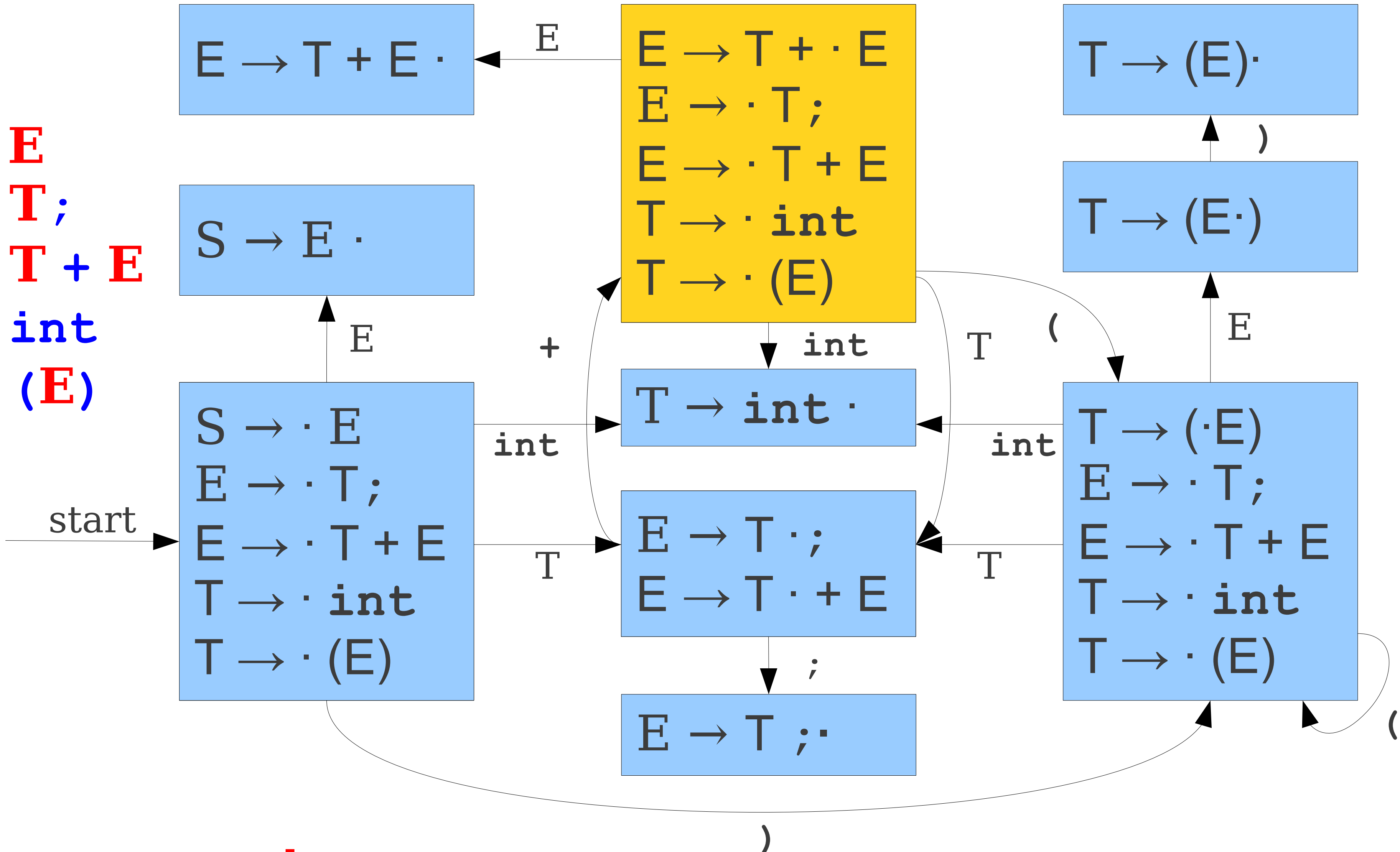
# LR(0) Parsing

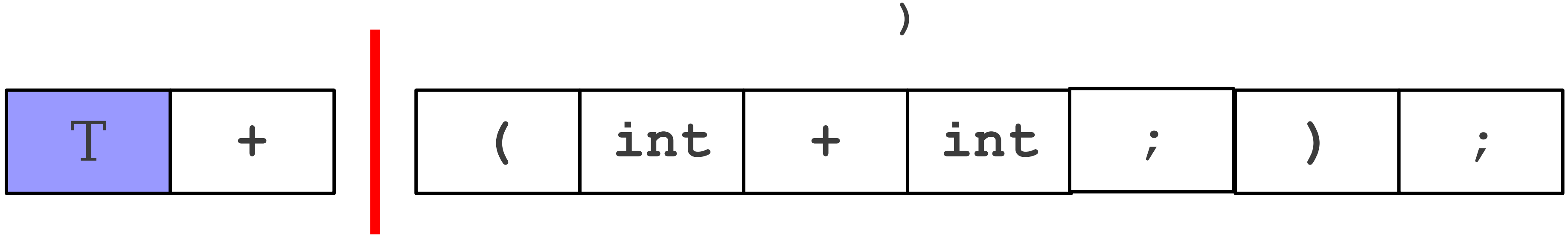
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



# LR(0) Parsing

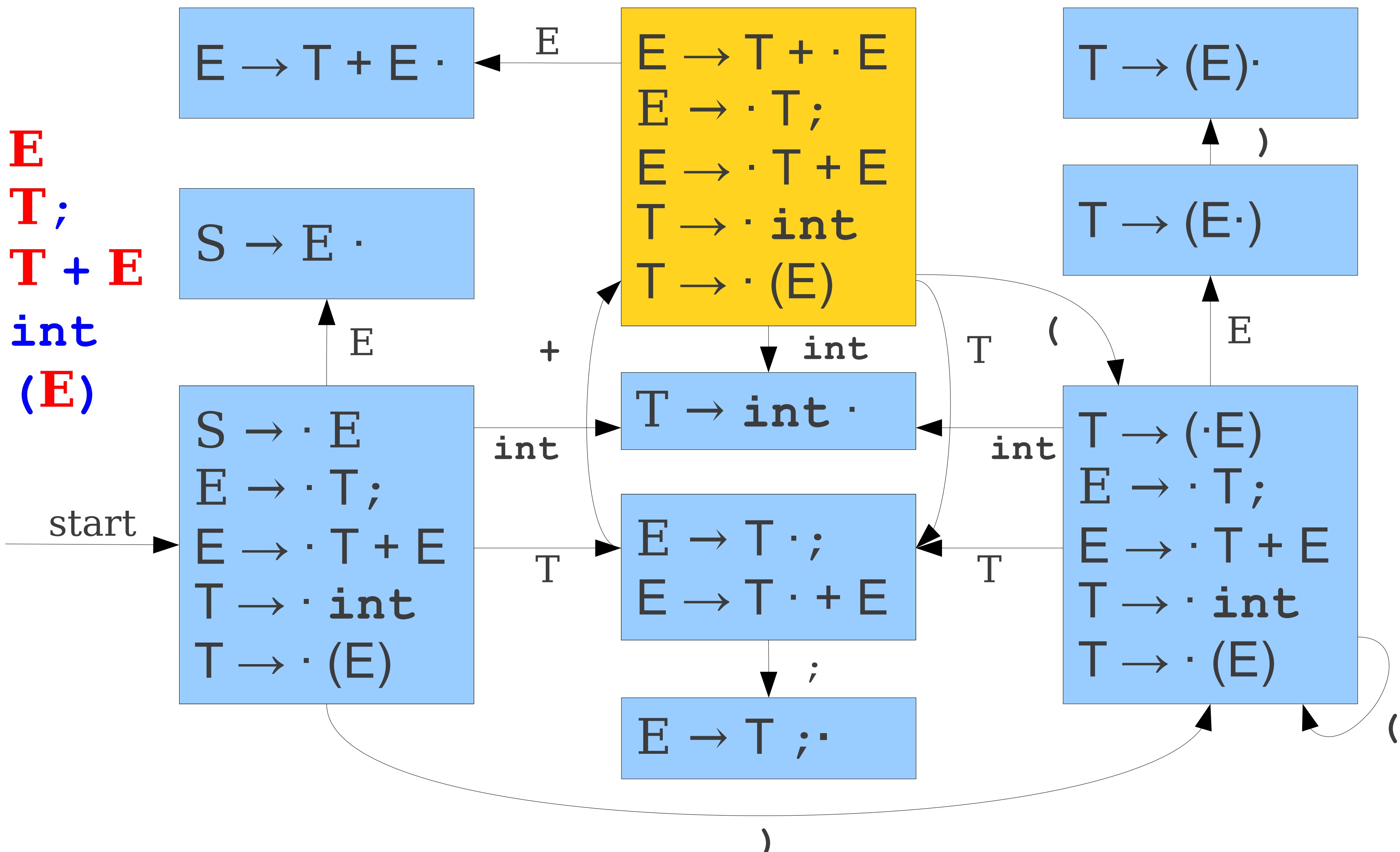
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)

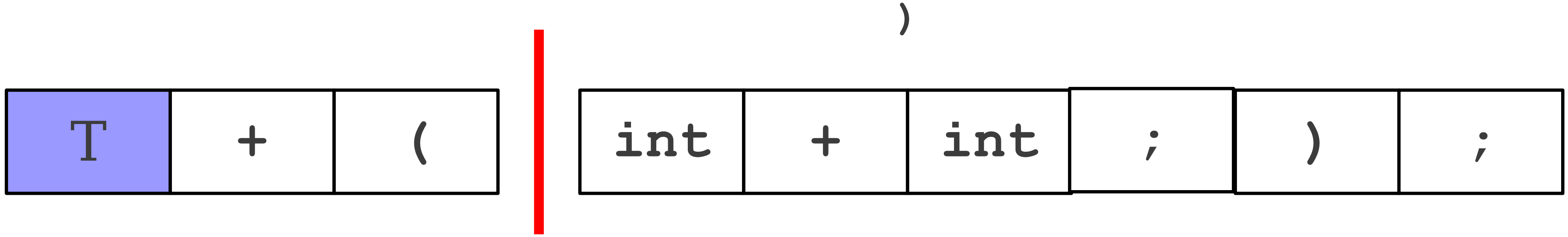


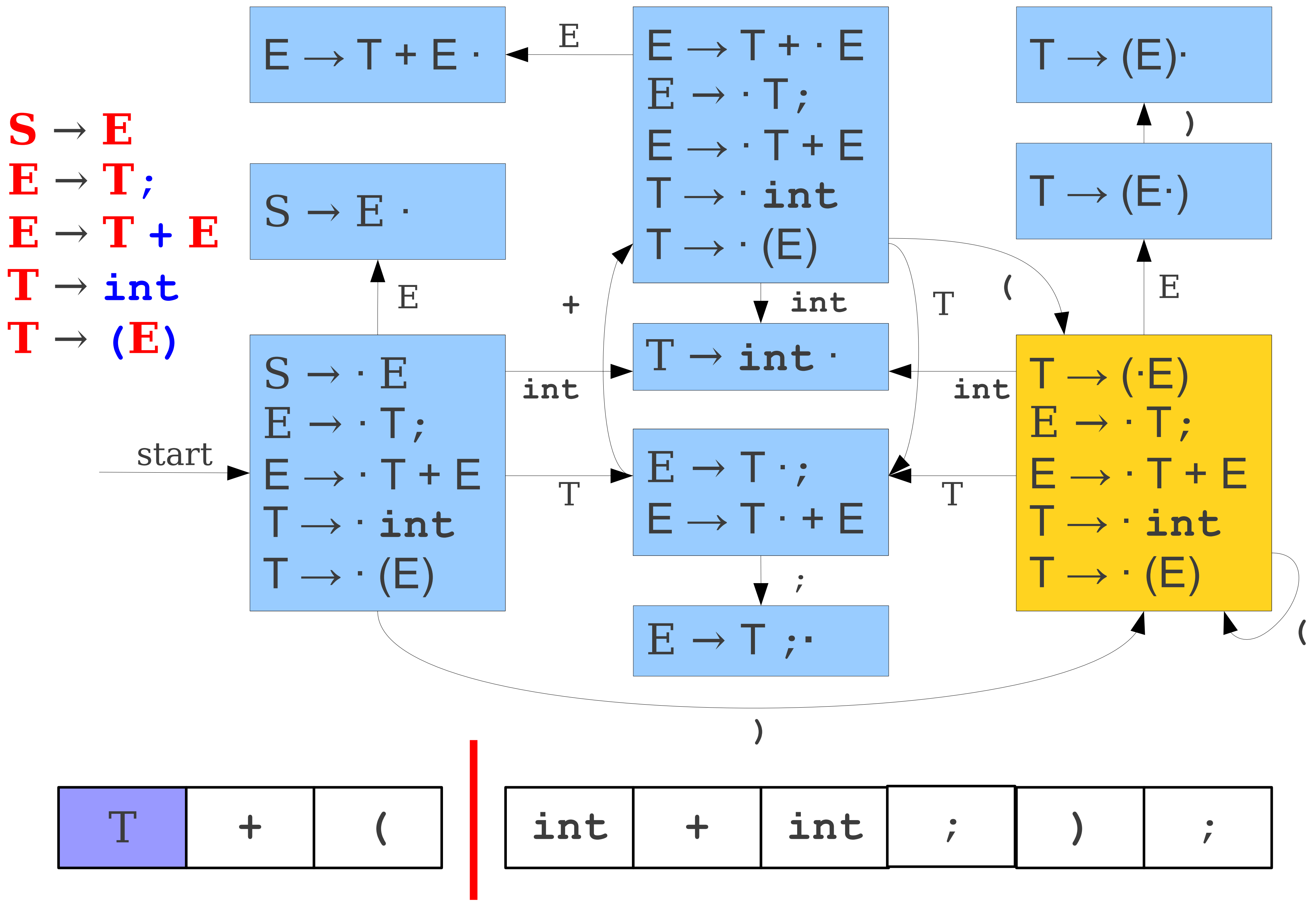
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



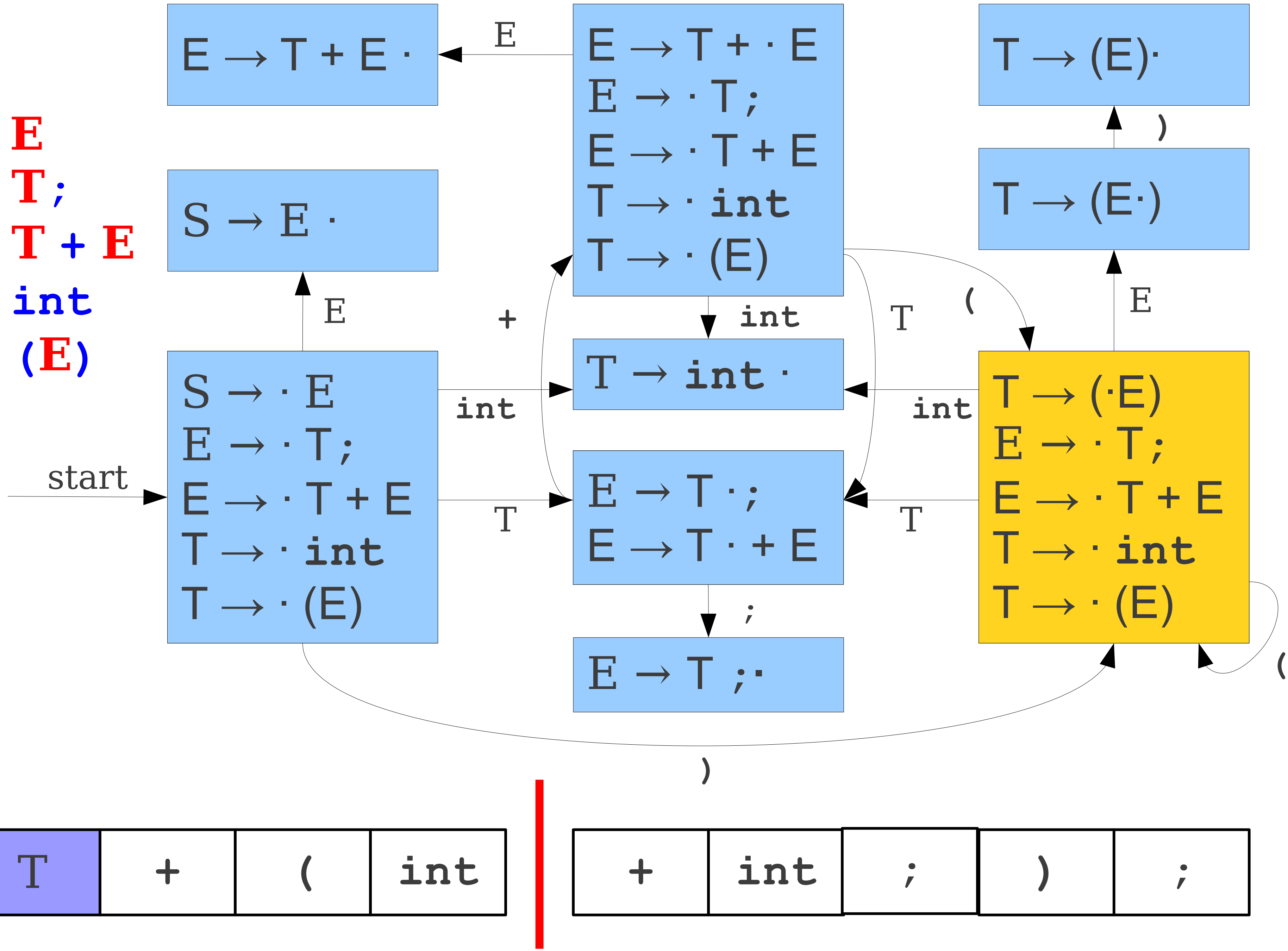



# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
start
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
S → E ·
E → T + E ·
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
T → int ·
E → T ·;
E → T · + E
E → T ;·
T → (E)·
T → (E·)
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
E
int
T
+
;
(
)
T
+
(
int
+
int
;
)
;

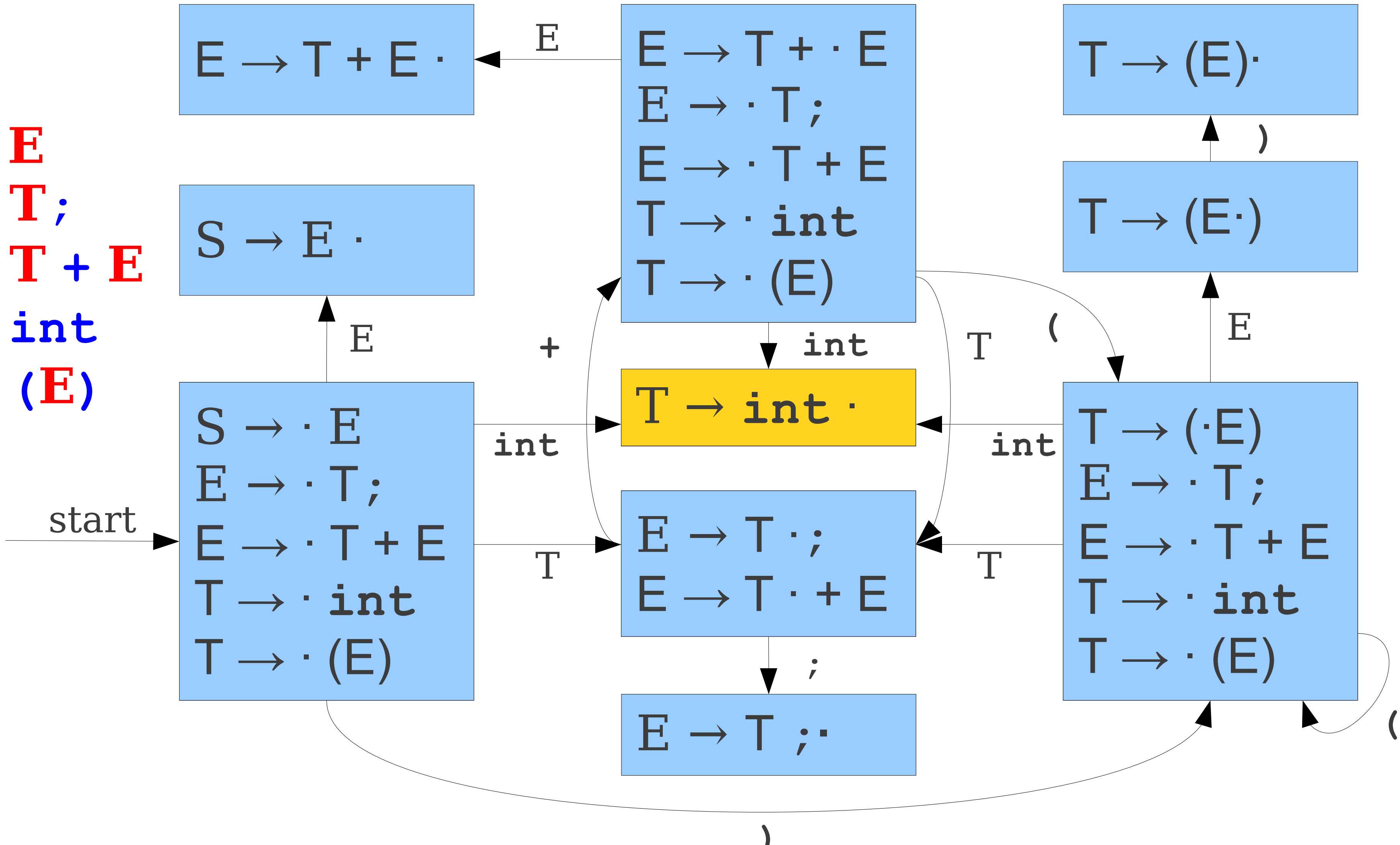
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



# LR(0) Parsing

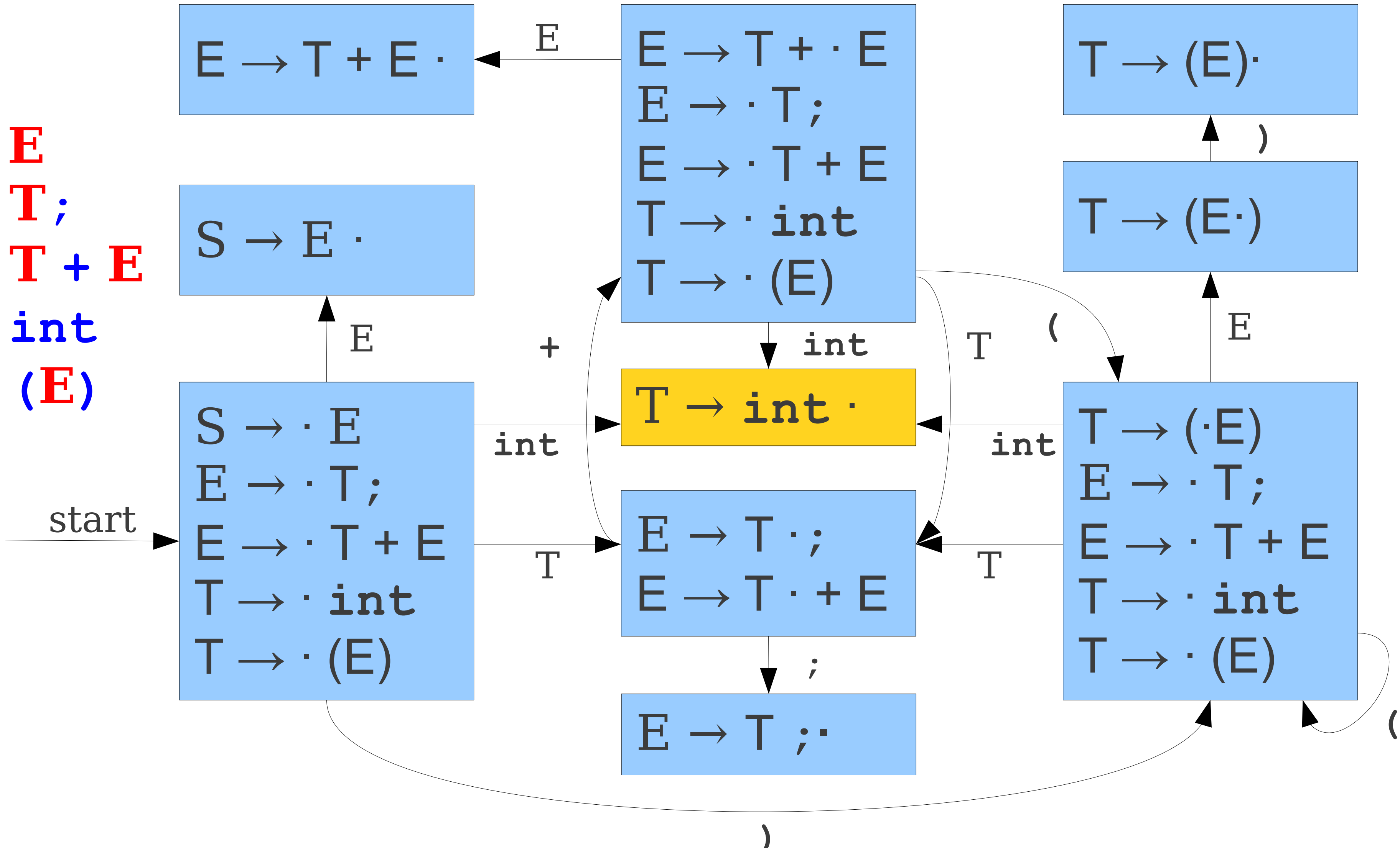
S → E
E → T;
E → T + E
T → int
T → (E)

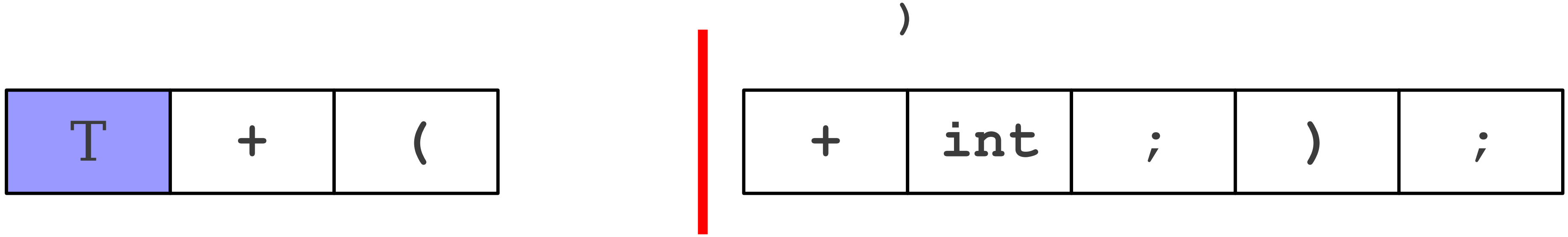



| T | + | ( | int | | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|---|

# LR(0) Parsing

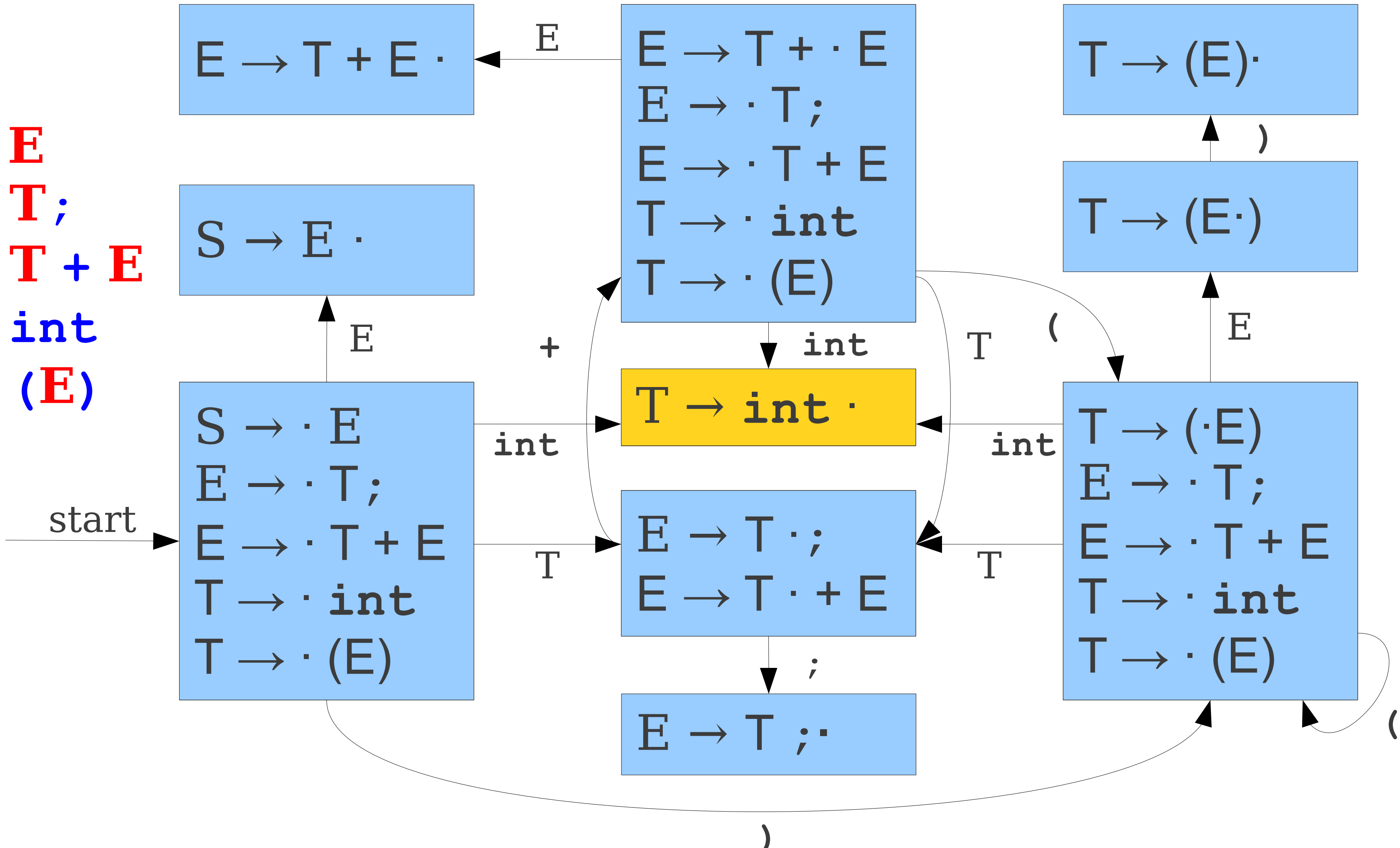
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → **( E )**



# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

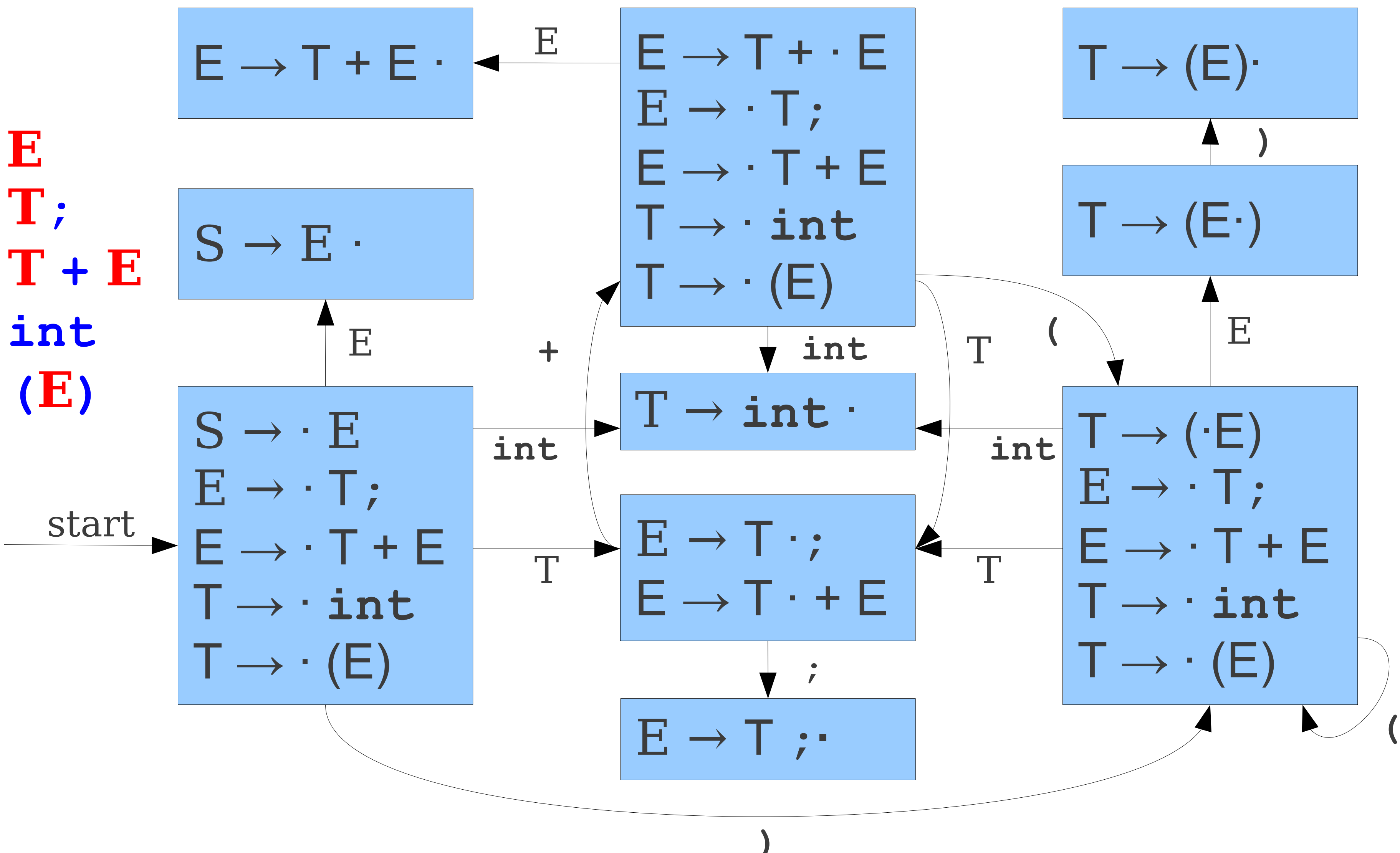



| | | | |
|---|---|---|---|
| T | + | ( | T |

| | | | | |
|---|---|---|---|---|
| + | int | ; | ) | ; |

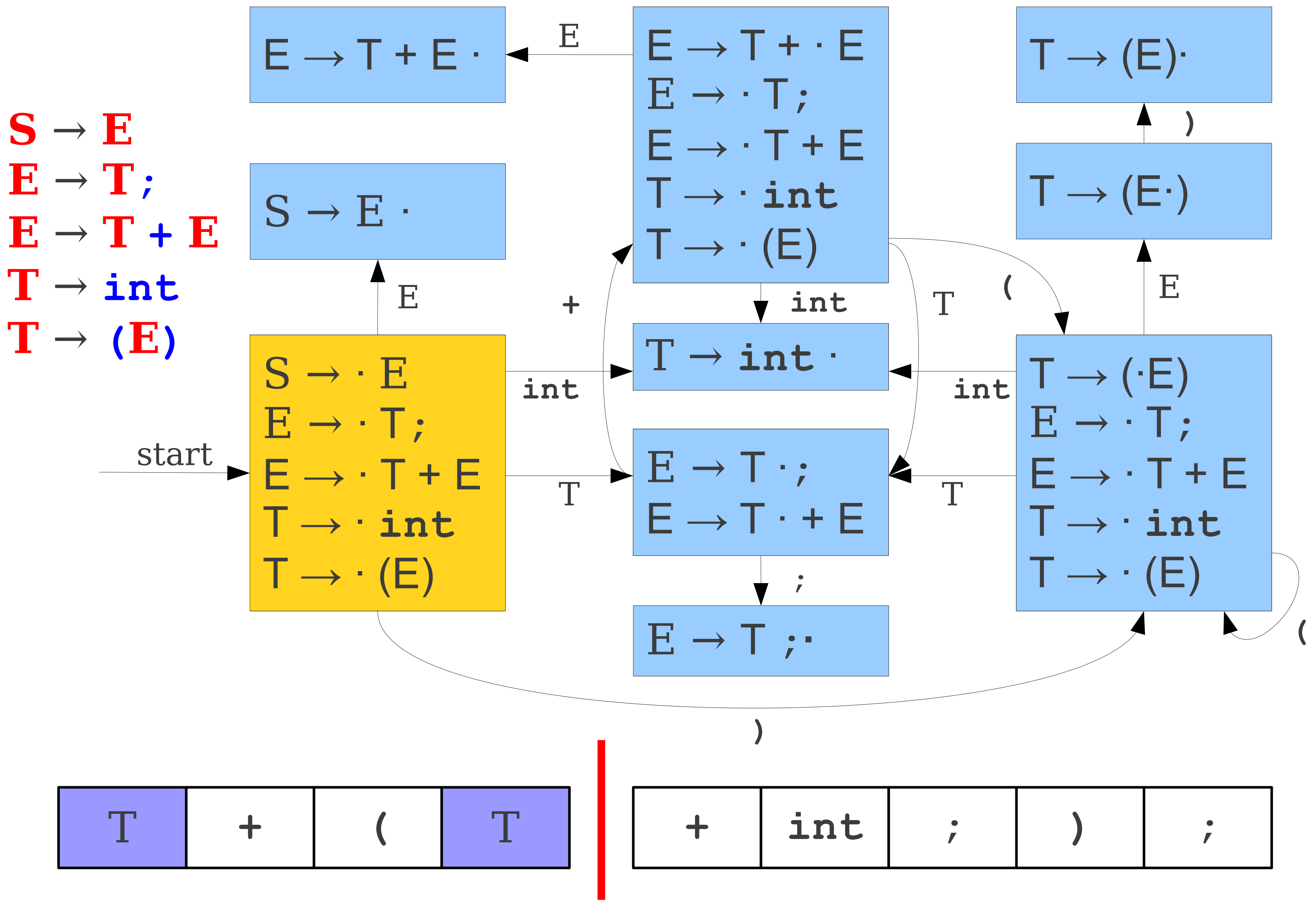
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



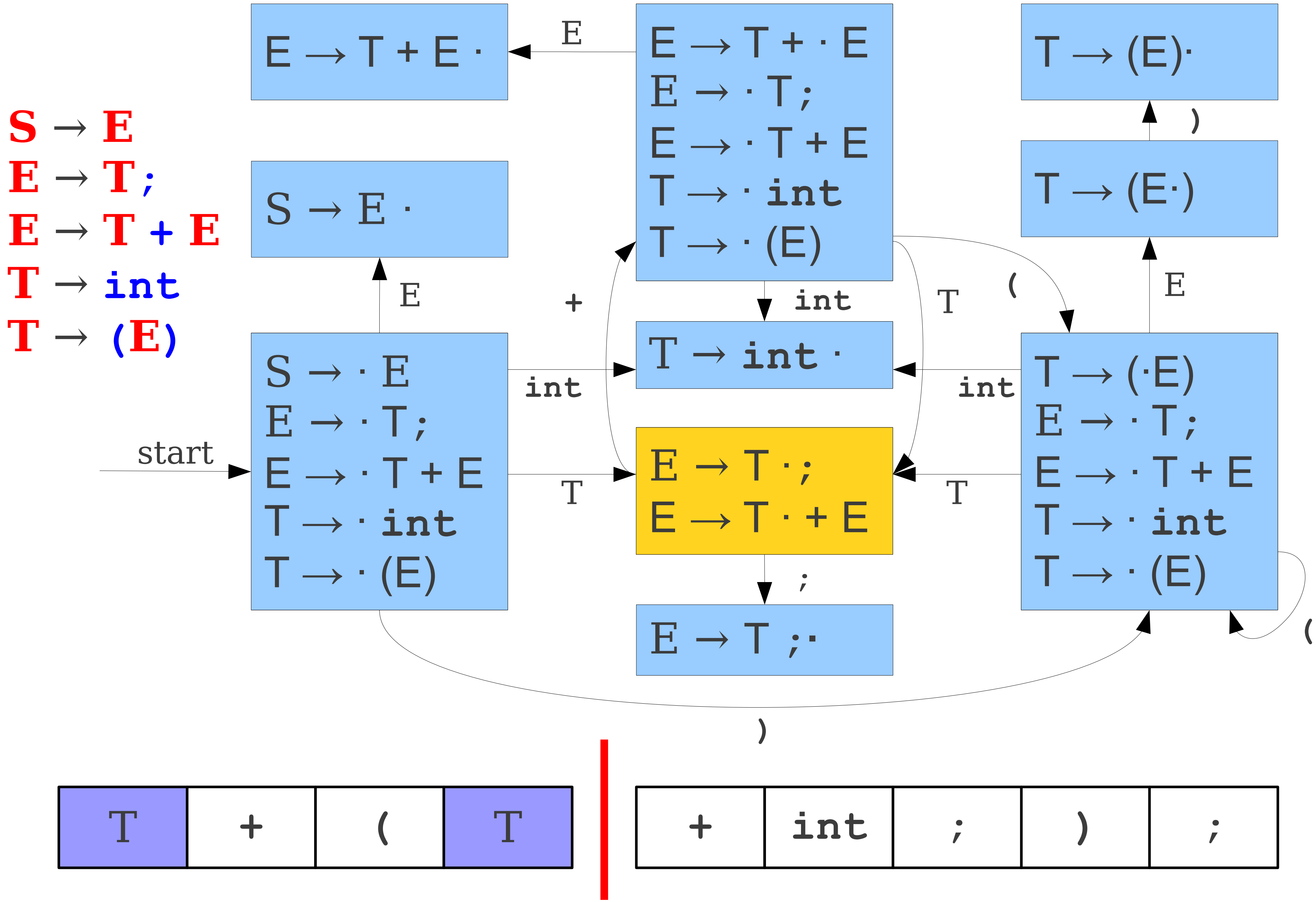

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| T | + | ( | T | \| | + | int | ; | ) | ; |

# LR(0) Parsing



S → E
E → T;
E → T + E
T → int
T → (E)
E → T + E ·
E
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
T → (E)·
)
S → E ·
T → (E·)
E
+
int
T
(
E
T → int ·
int
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
start
int
T
E → T ·;
E → T · + E
T
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
;
E → T ;·
(
)
T
+
(
T
+
int
;
)
;

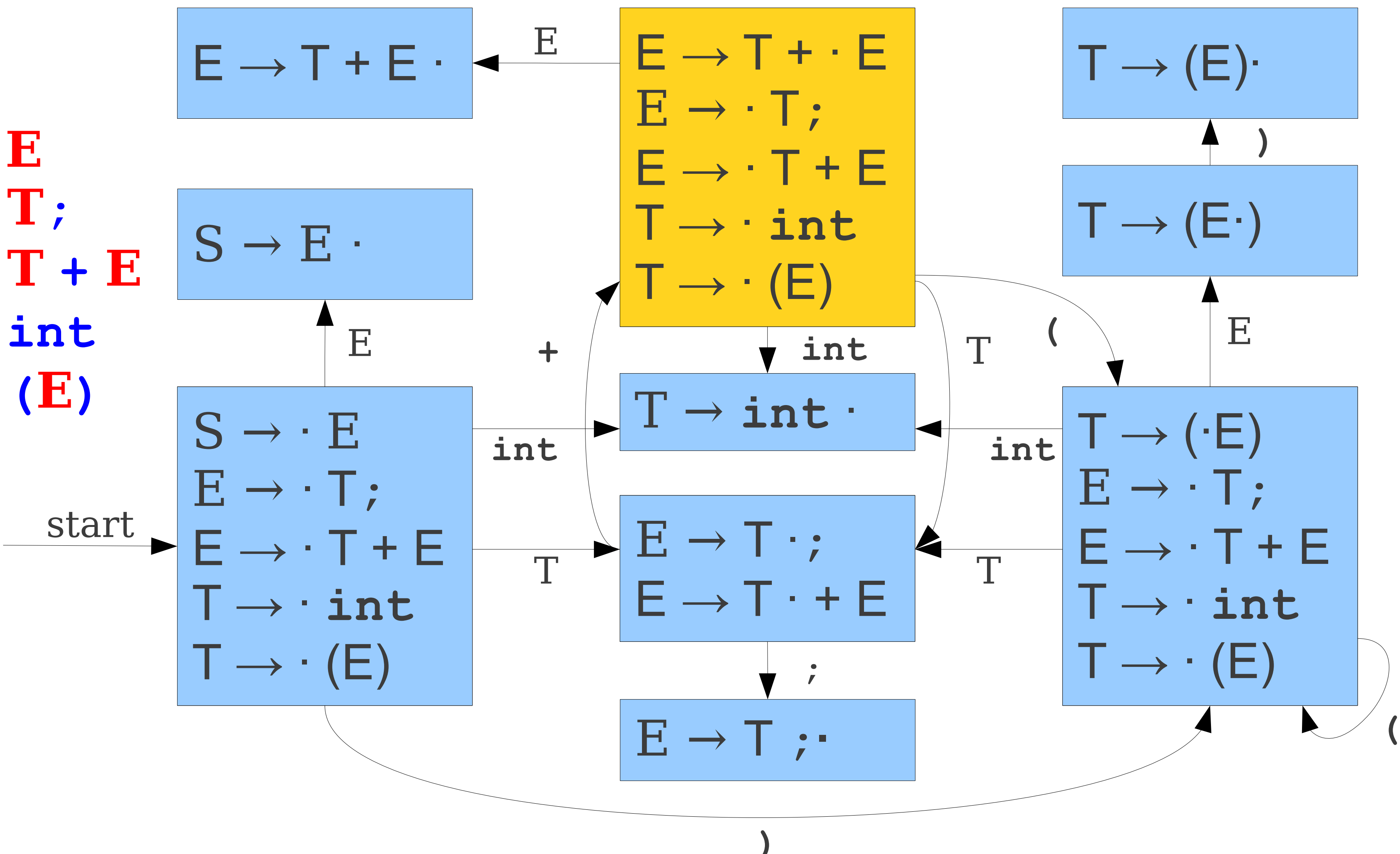
# LR(0) Parsing

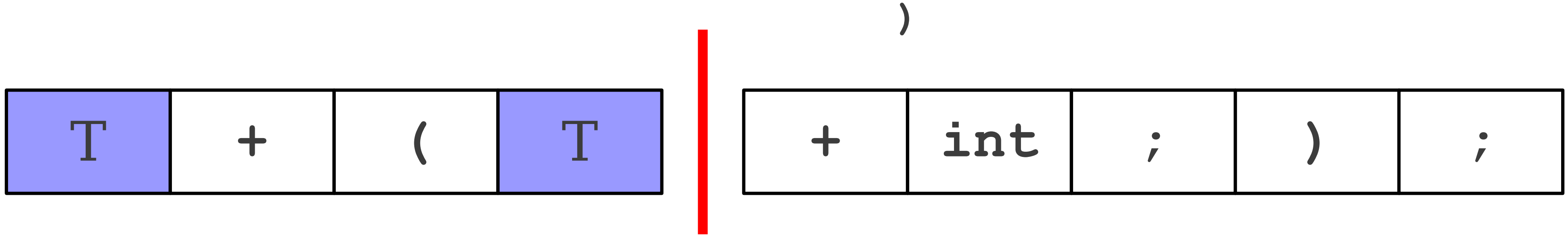


S → E
E → T;
E → T + E
T → int
T → (E)
E → T + E ·
E
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
T → (E)·
)
T → (E·)
S → E ·
E
+
int
T
(
E
T → int ·
int
int
start
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
T
E → T ·;
E → T · + E
T
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
;
E → T ;·
(
)
T
+
(
T
+
int
;
)
;

# LR(0) Parsing

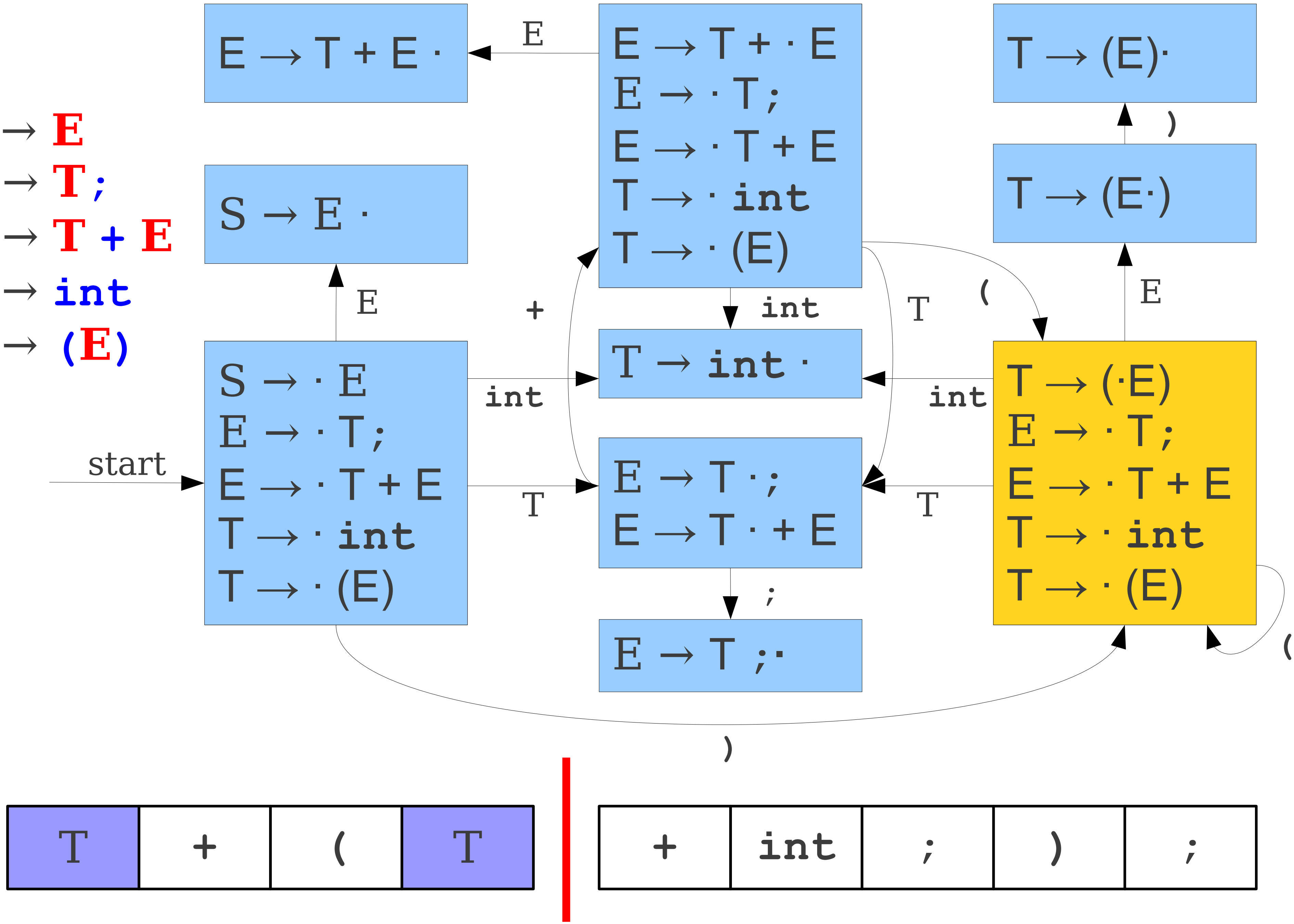
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



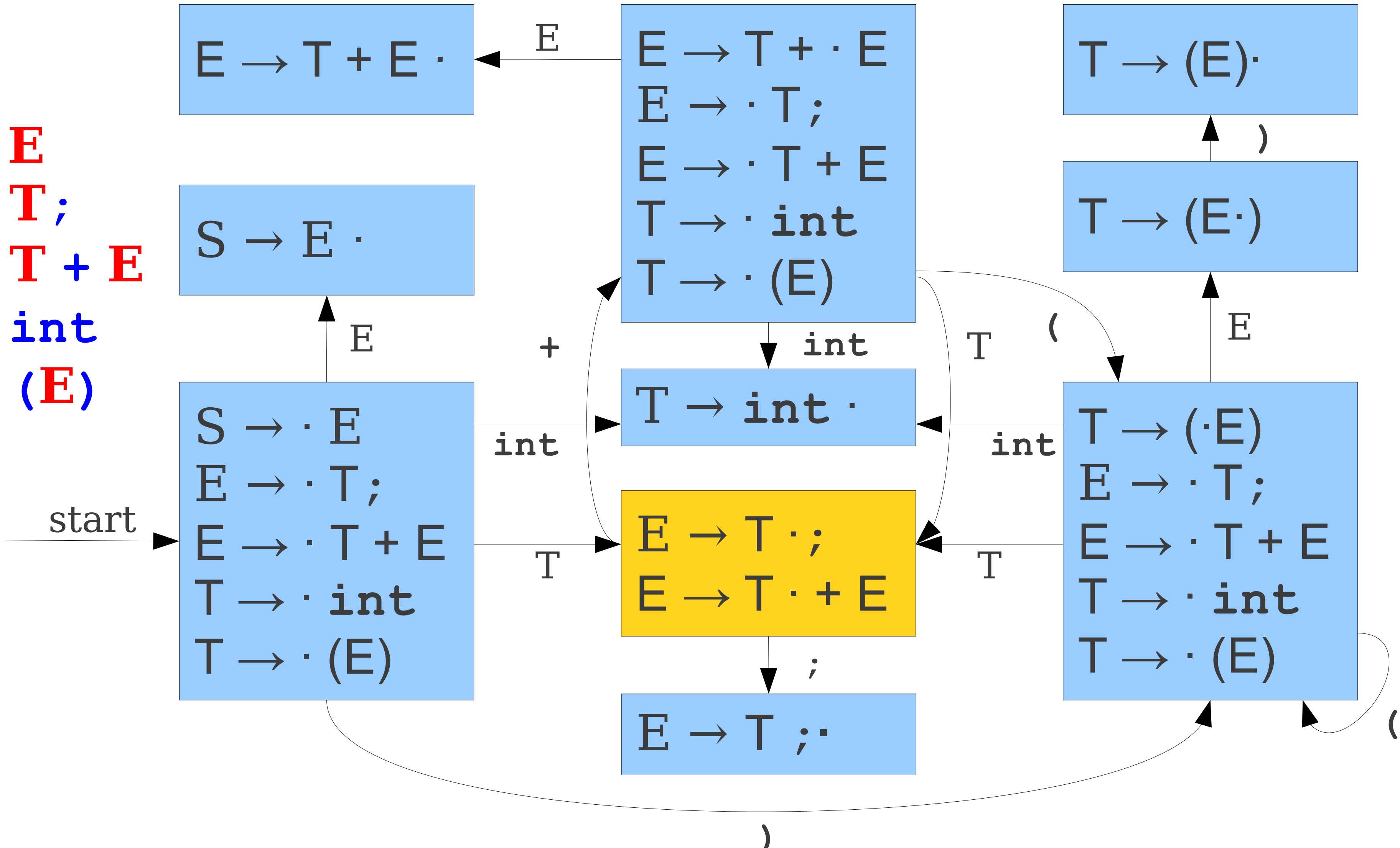
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



# LR(0) Parsing

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`

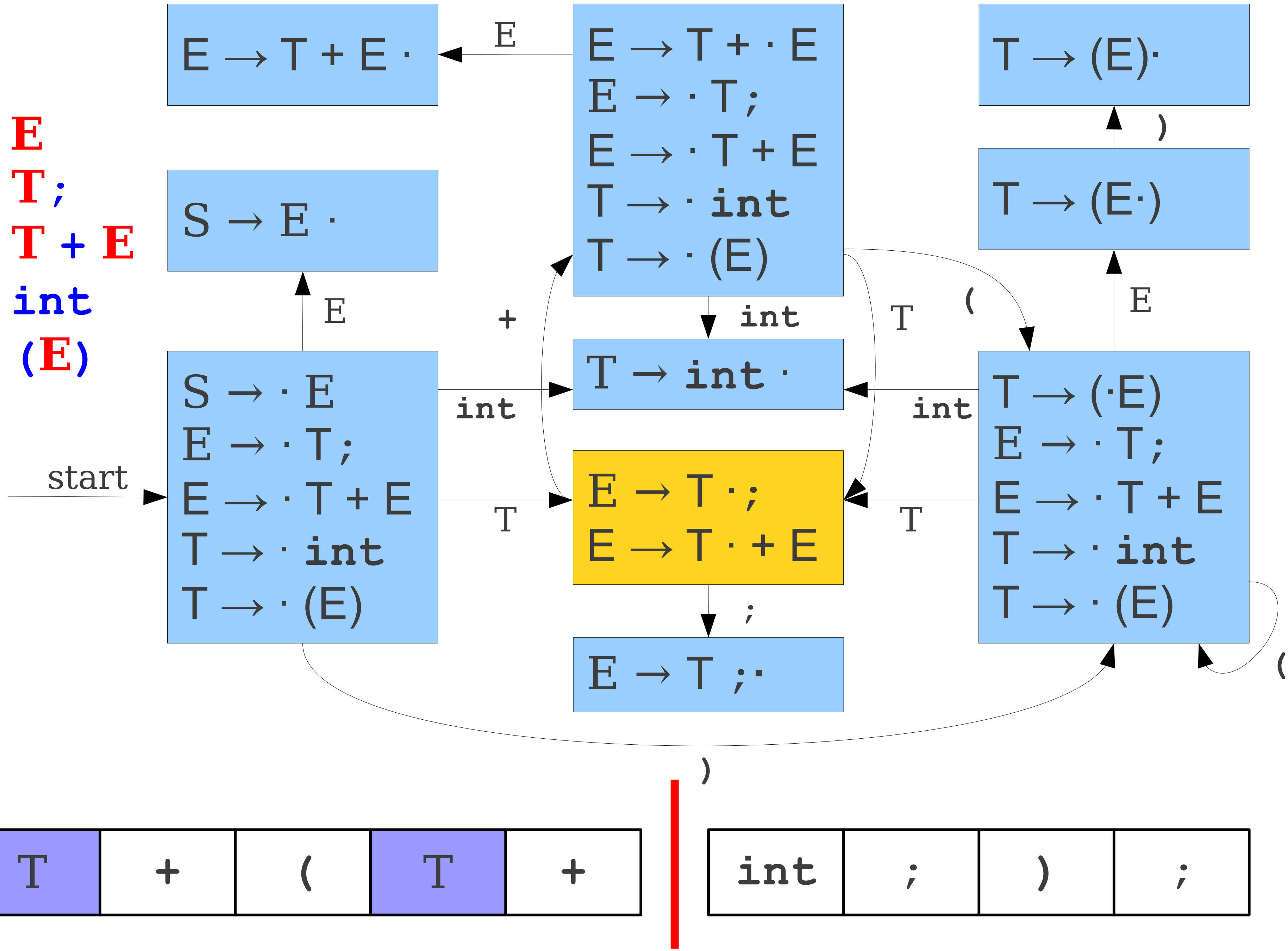



| T | + | ( | T |
|---|---|---|---|

| + | int | ; | ) | ; |
|---|---|---|---|---|

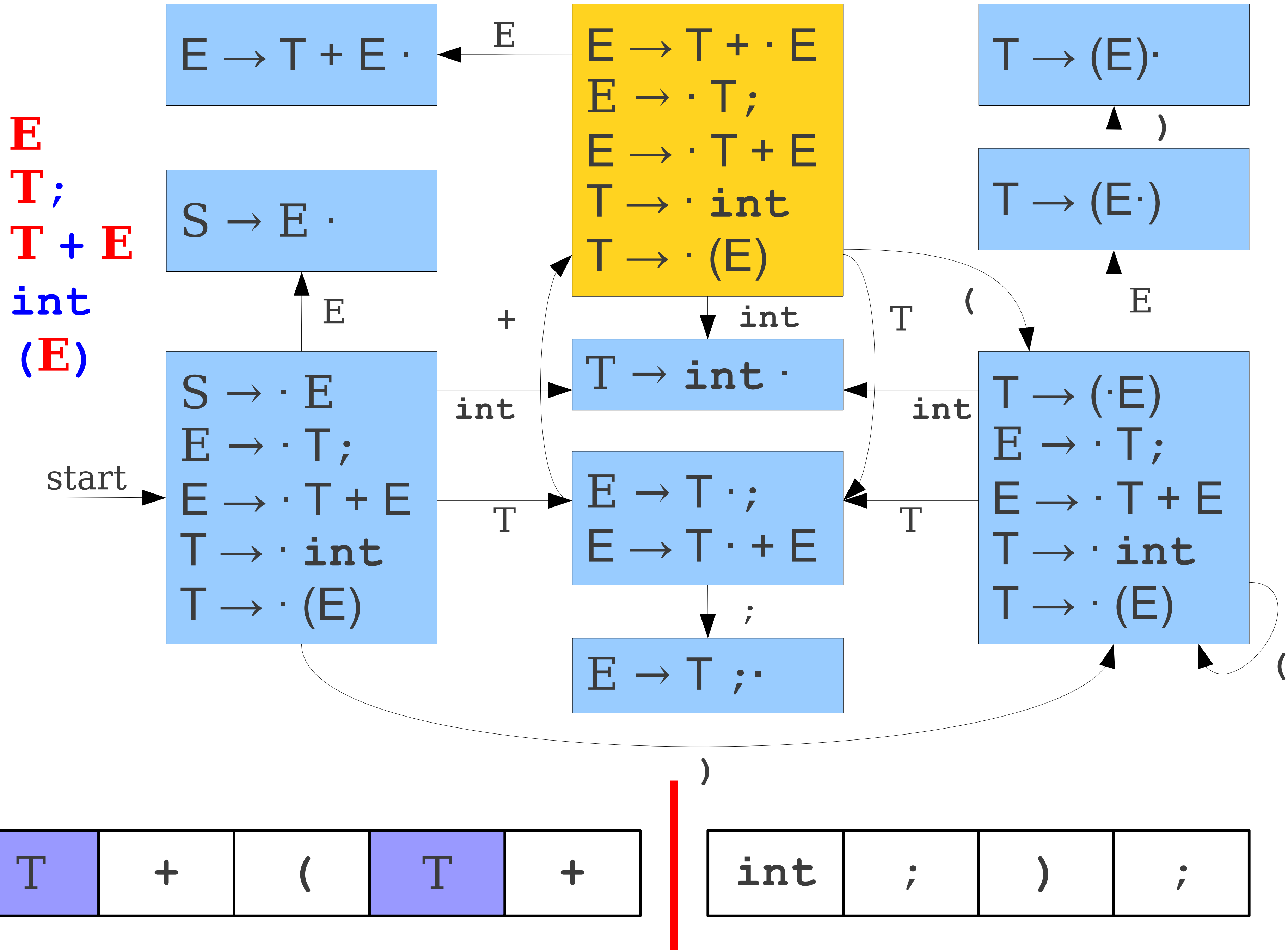
# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



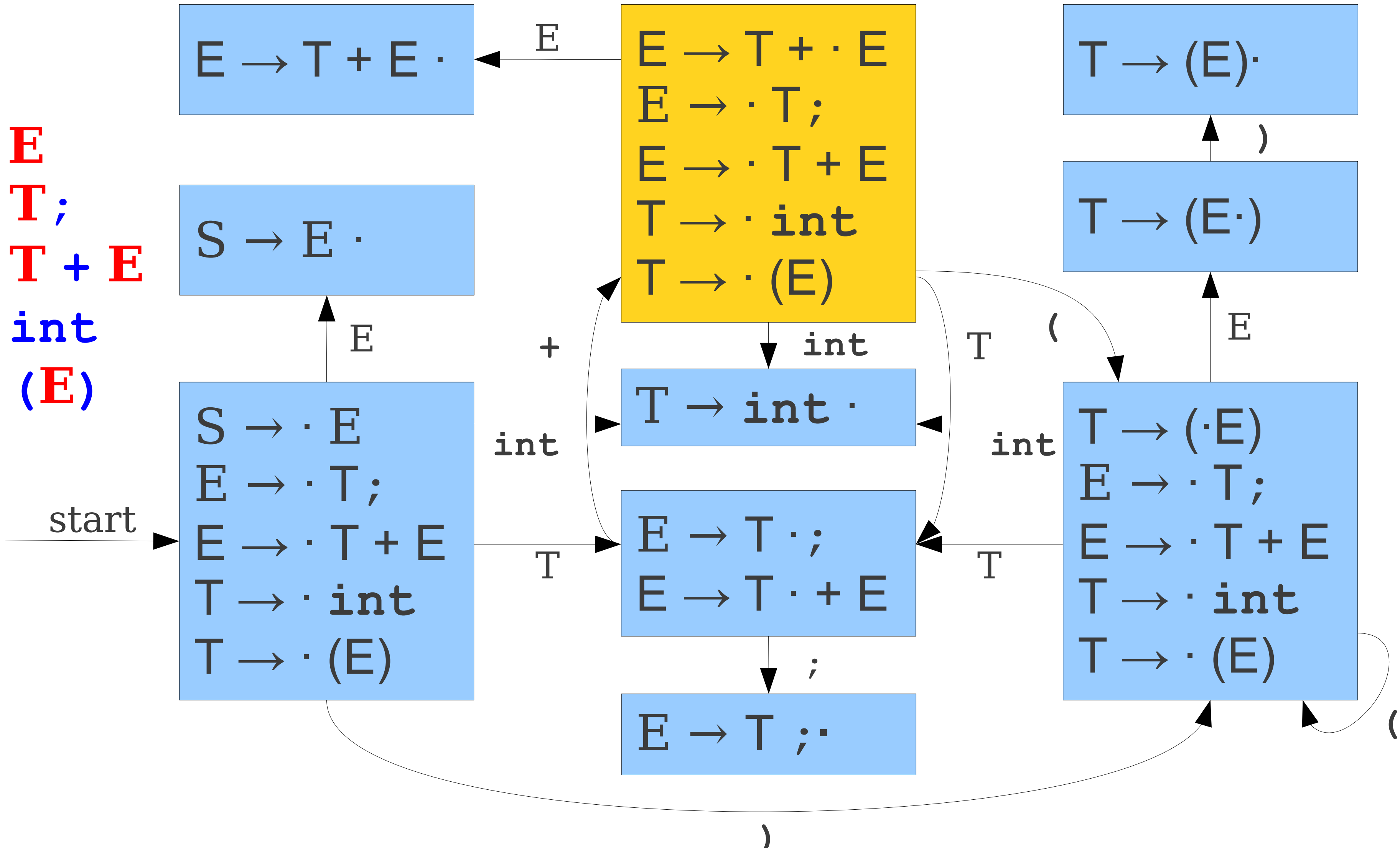
# LR(0) Parsing

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



# LR(0) Parsing

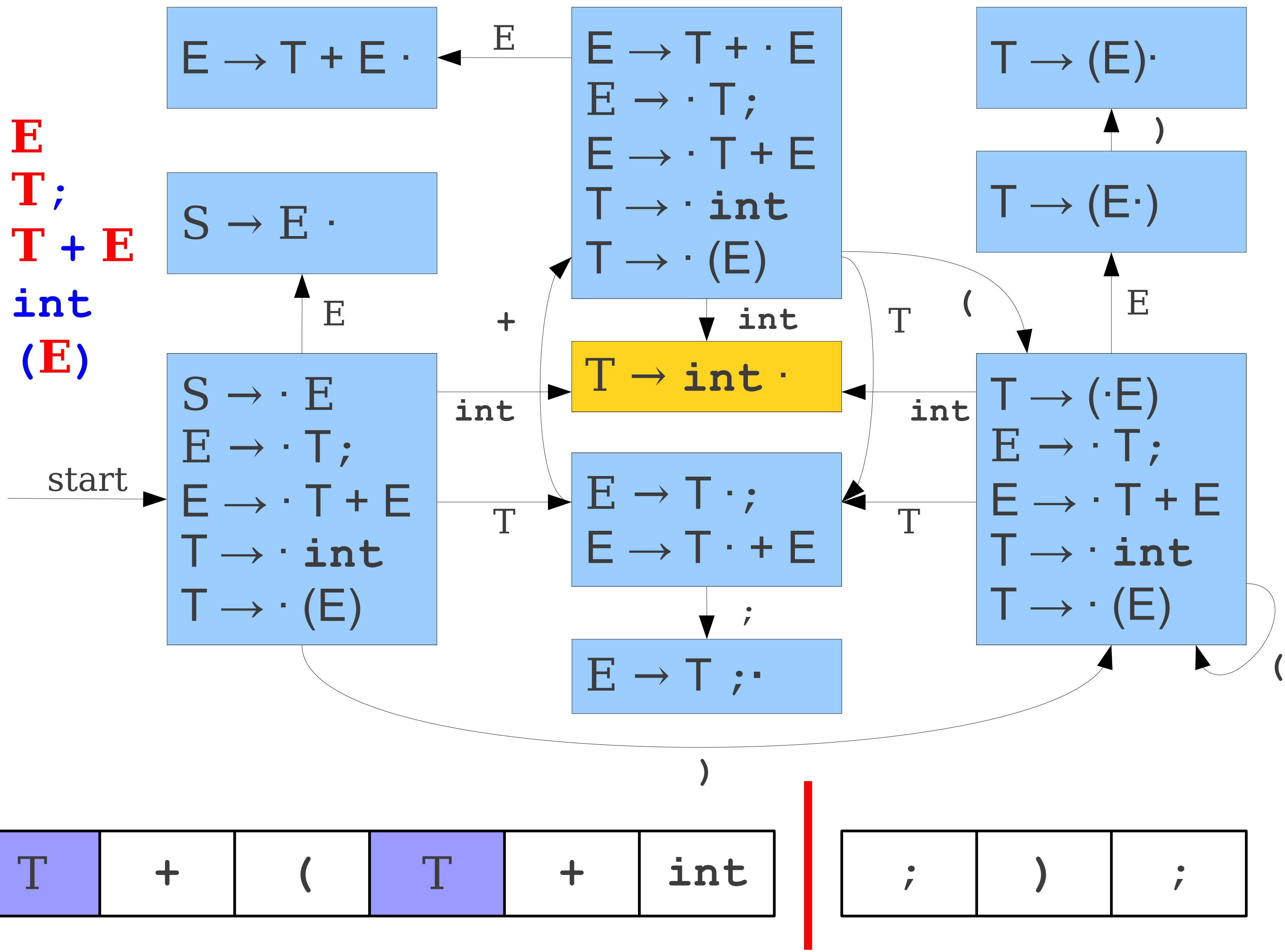
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)




| T | + | ( | T | + | int | | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|---|

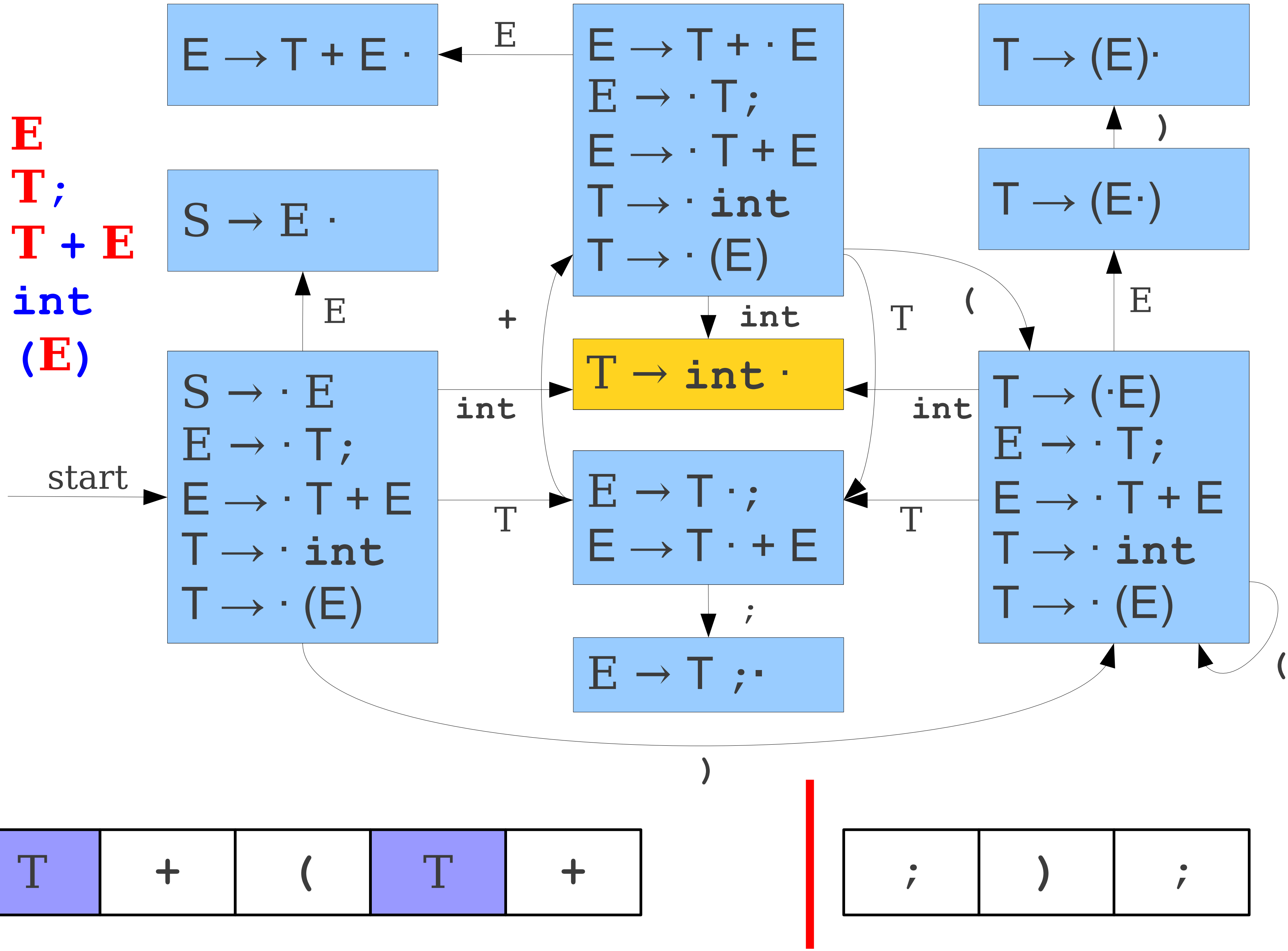
# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



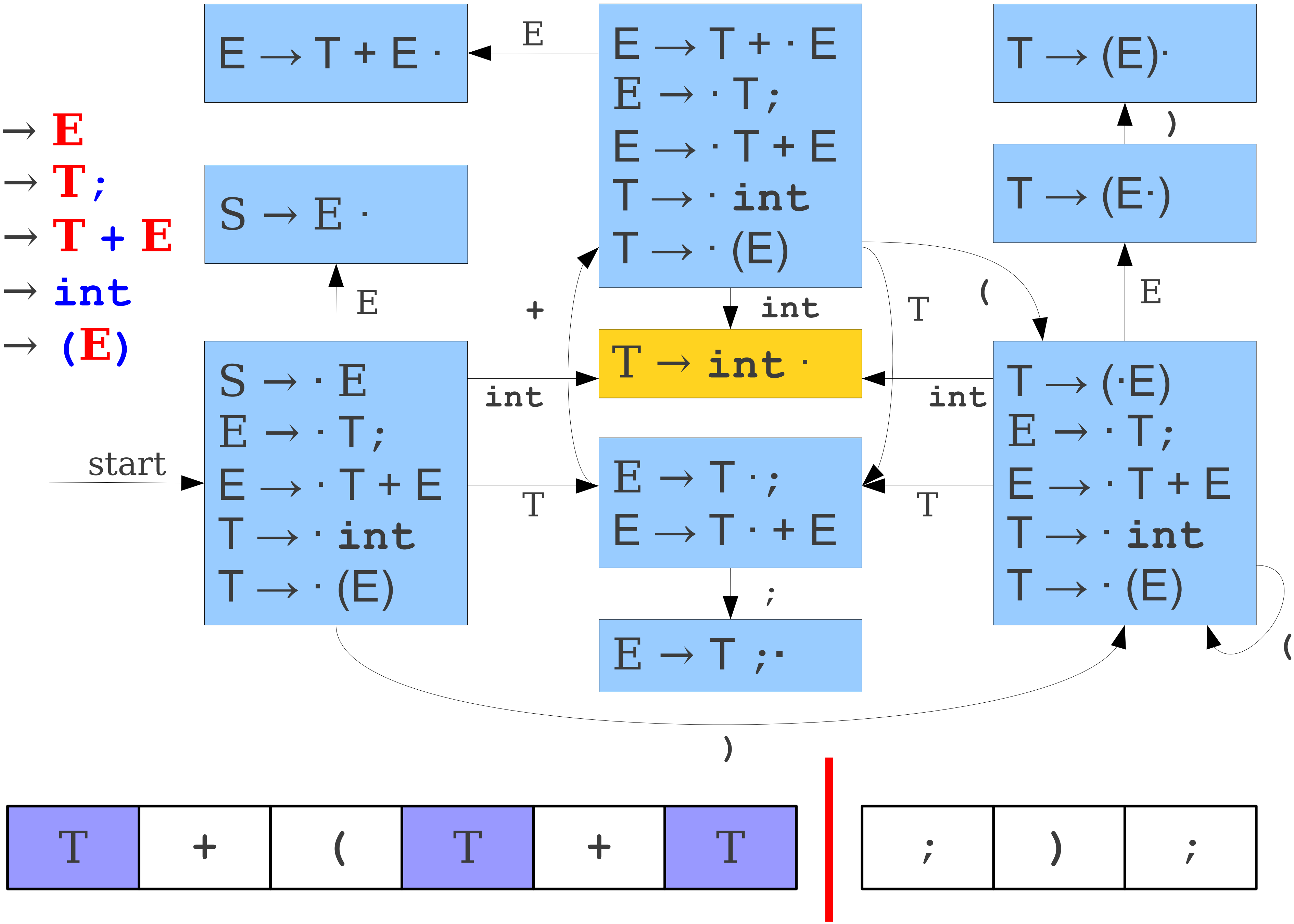
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



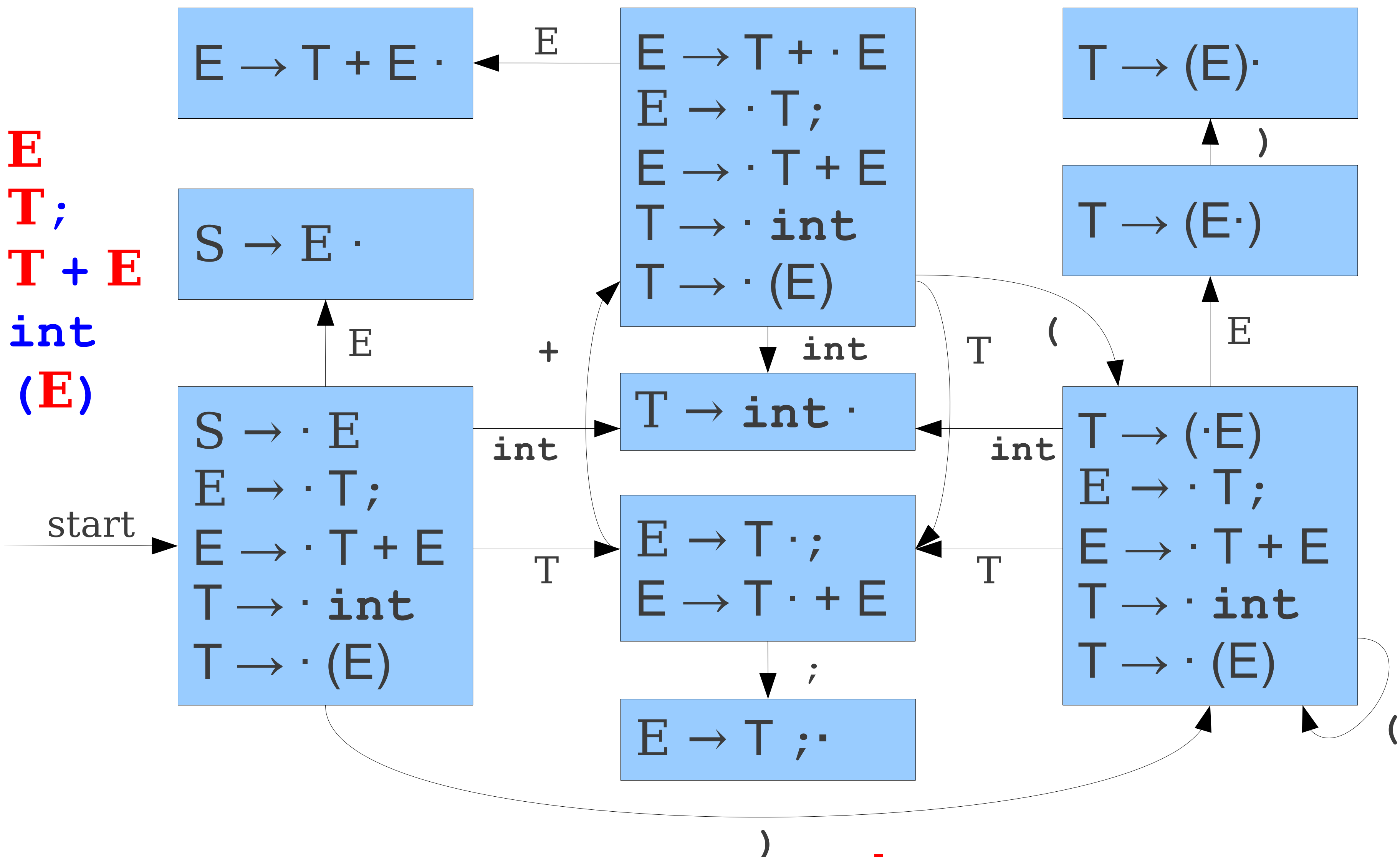
# LR(0) Parsing

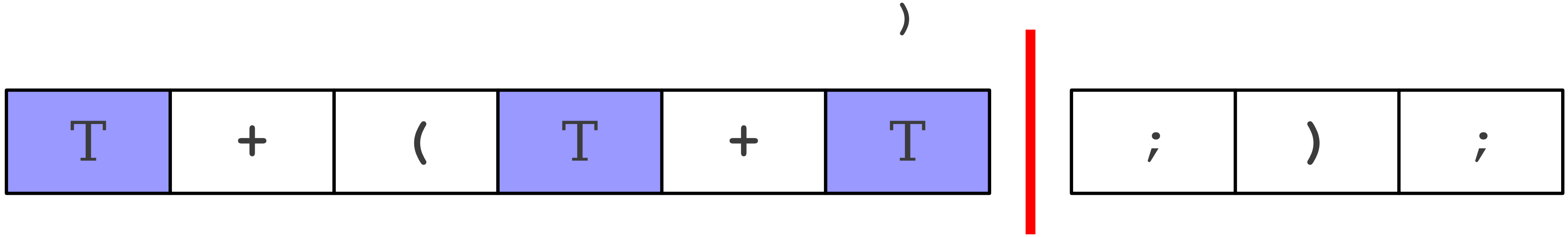
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

**S → E**
**E → T;**
**E → T + E**
**T → int**
**T → (E)**



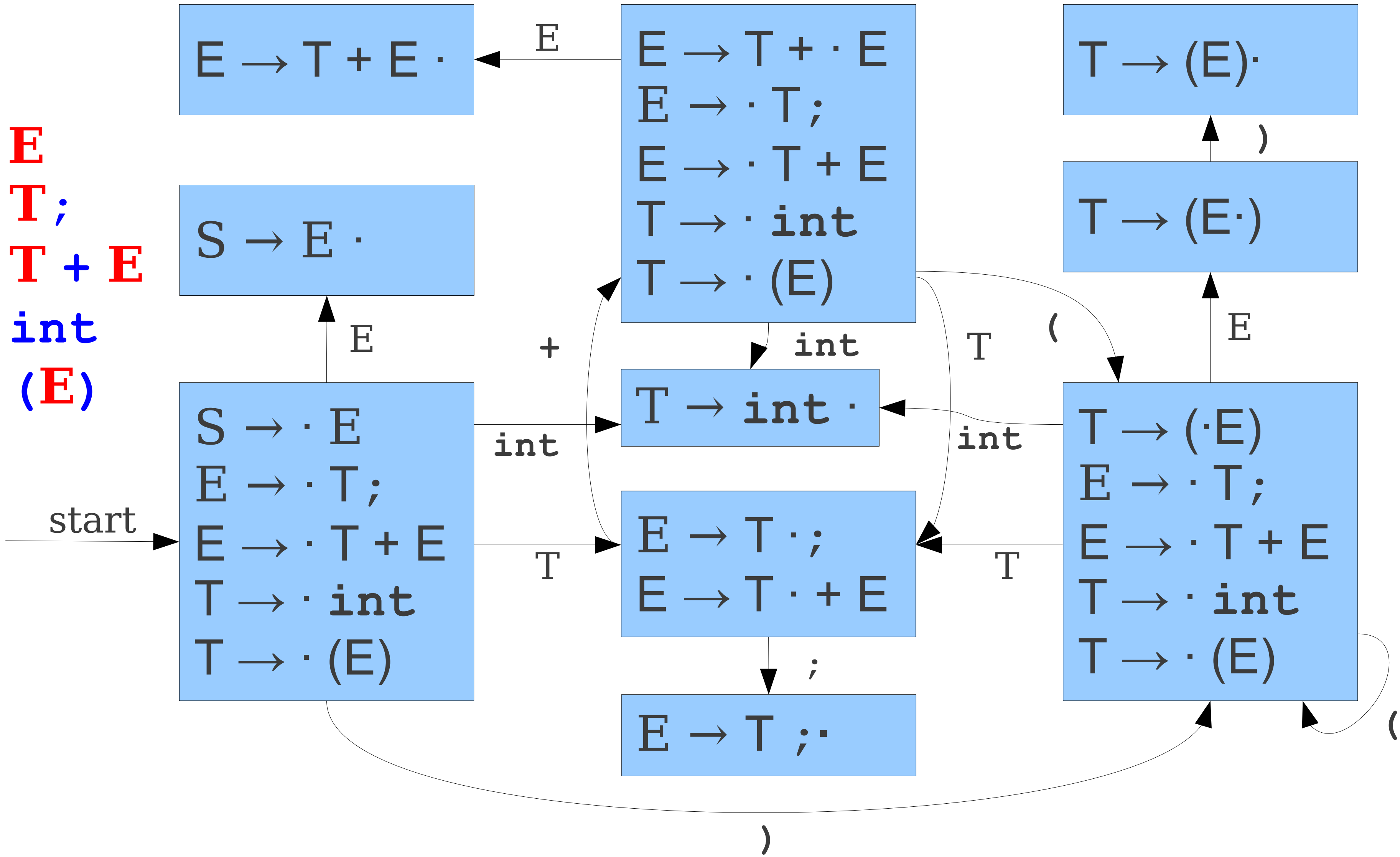



# An Optimization

- Rather than restart the automaton on each reduction, remember what state we were in for each symbol.
- When applying a reduction, restart the automaton from the last known good state.

# LR(0) Parsing

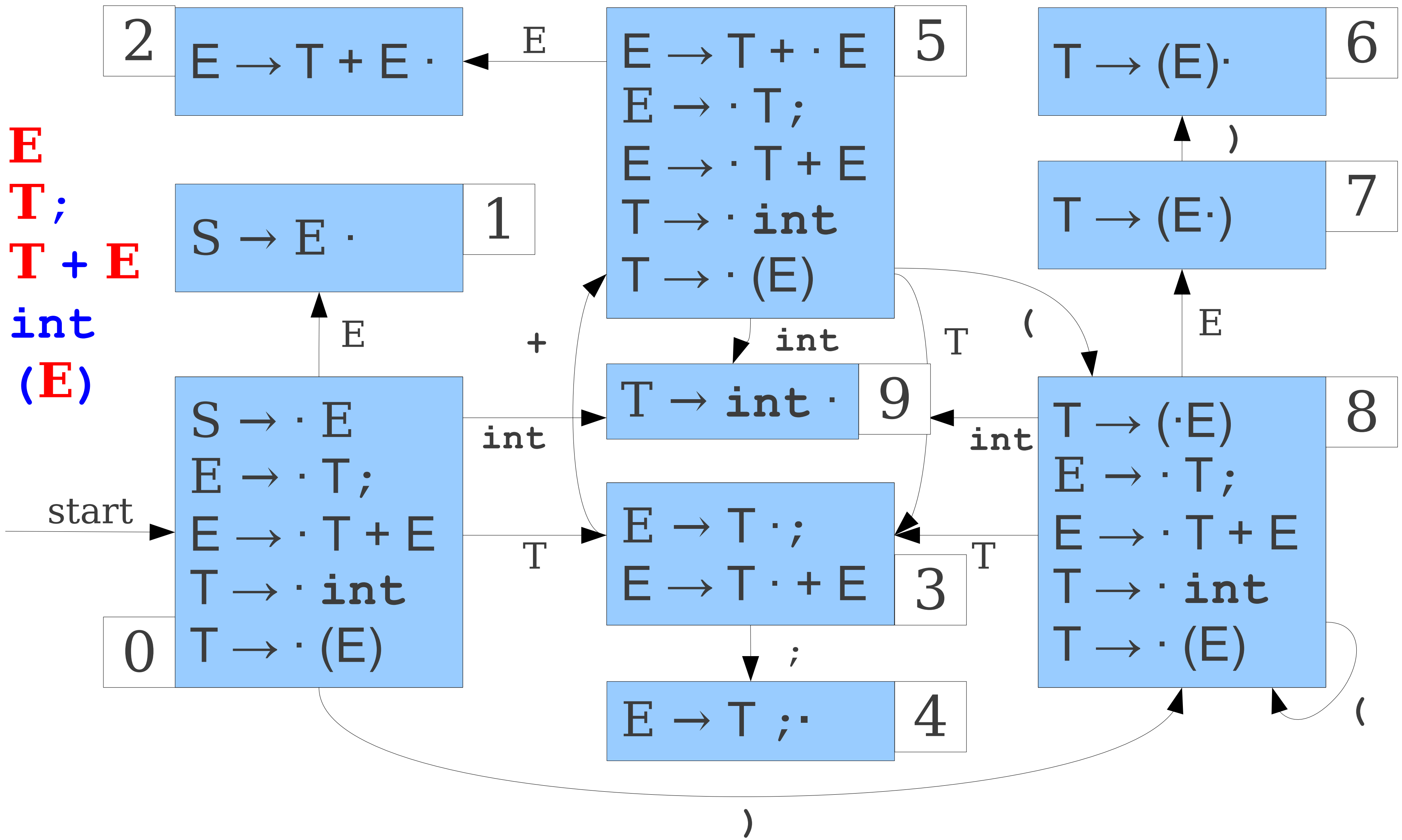
**S** → **E**
**E** → **T** `;`
**E** → **T** + **E**
**T** → `int`
**T** → `(`**E**`)`




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | ; | ) | ; |

# LR(0) Parsing

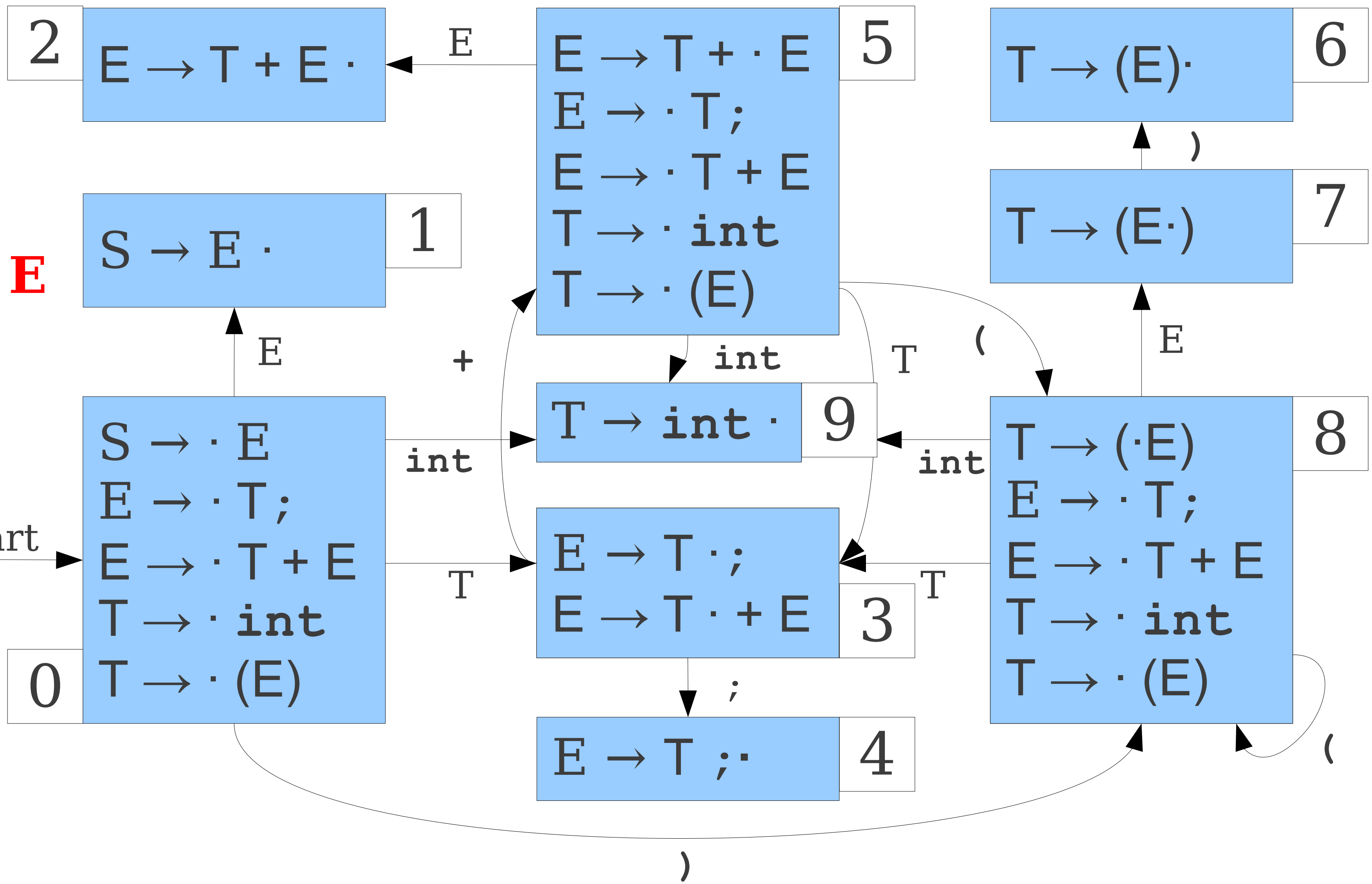
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )




| int | + | ( | int | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|

# LR(0) Parsing

**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(` **E** `)`

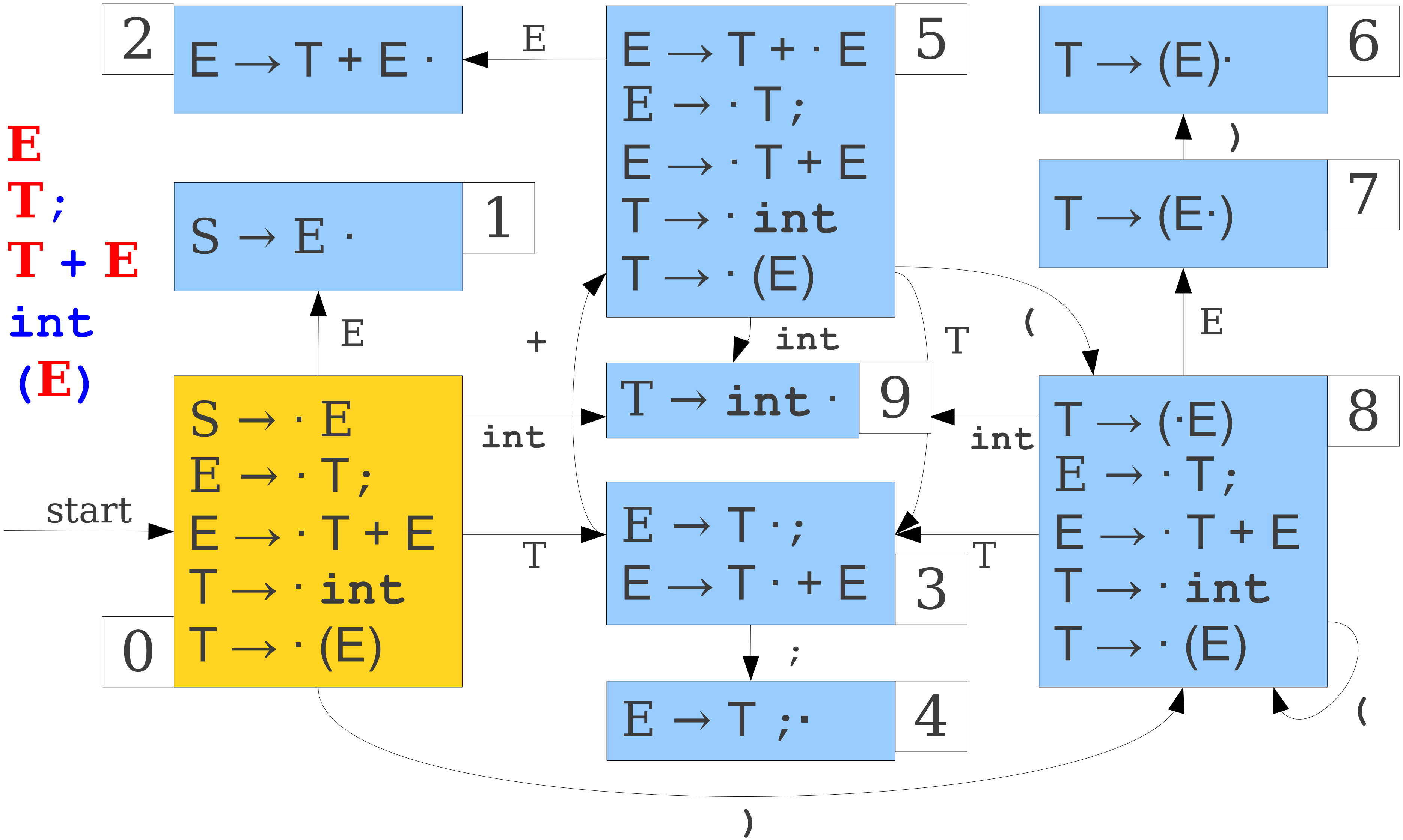



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | ; | ) | ; |

**0**

# LR(0) Parsing

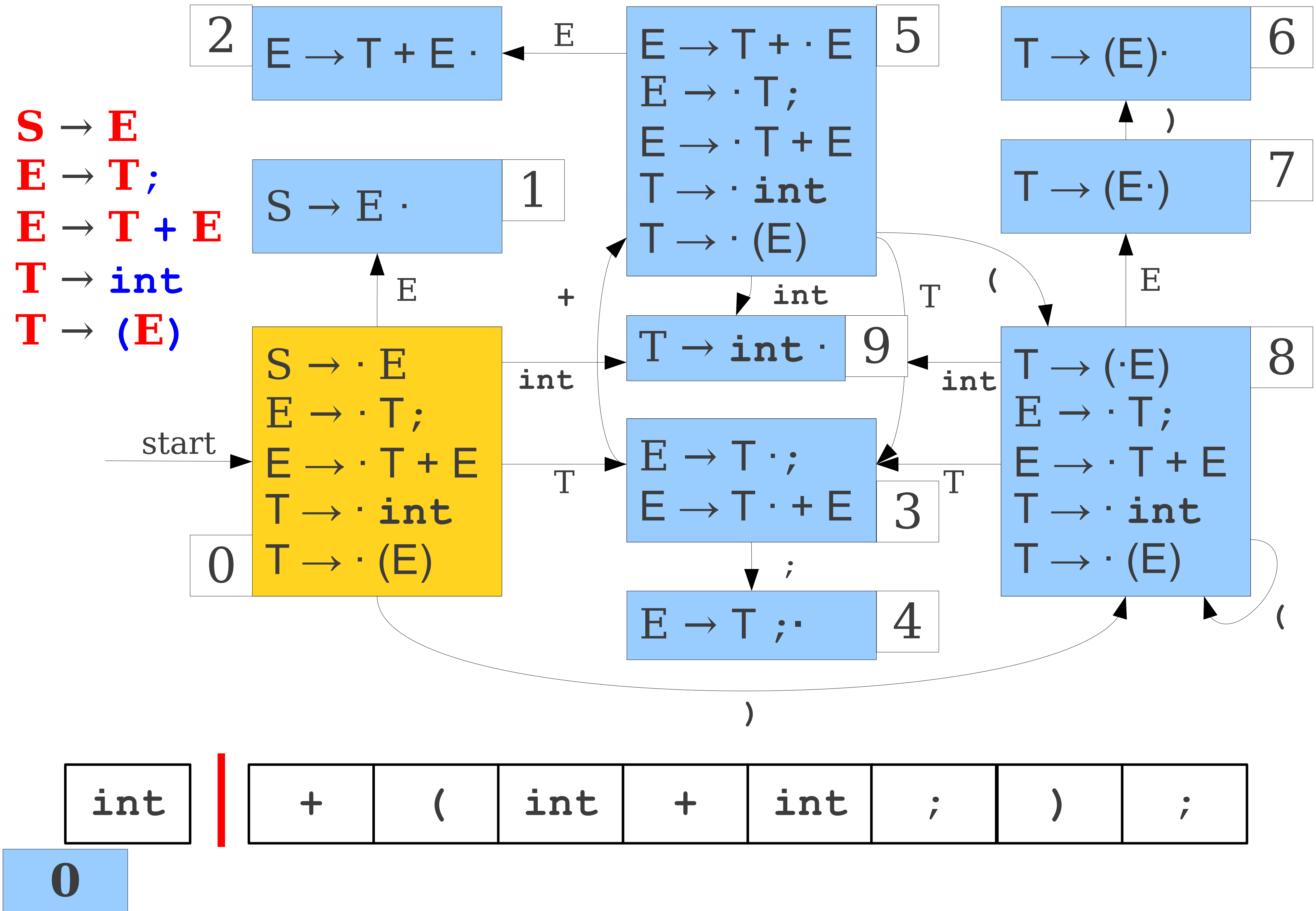
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | ; | ) | ; |

**0**

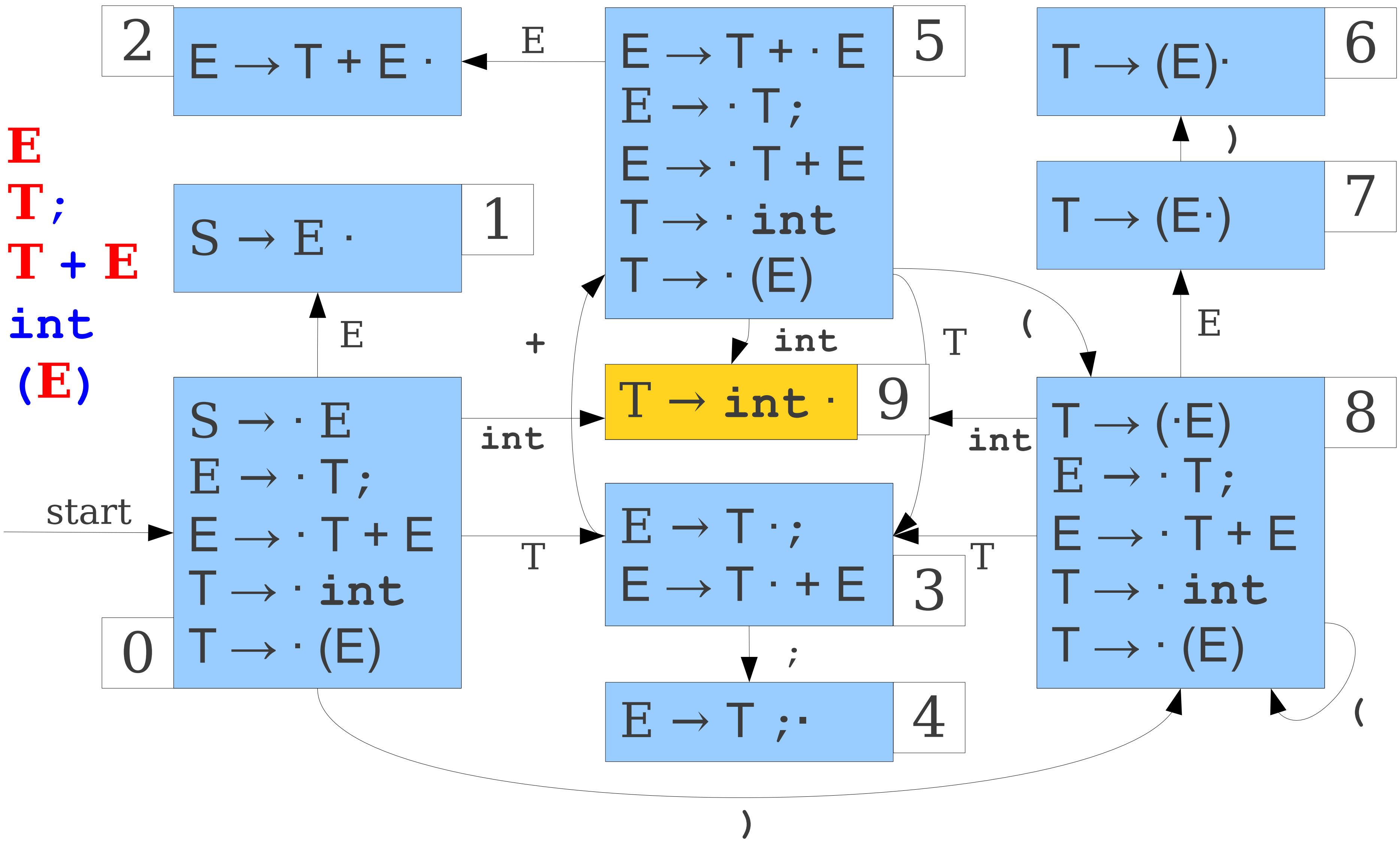
# LR(0) Parsing



S → E
E → T;
E → T + E
T → int
T → (E)
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
1
S → E ·
2
E → T + E ·
3
E → T · ;
E → T · + E
4
E → T ;·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
7
T → (E·)
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
9
T → int ·
E
E
int
T
+
int
T
(
E
)
int
T
;
(
(
int
+
(
int
+
int
;
)
;
0

# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)

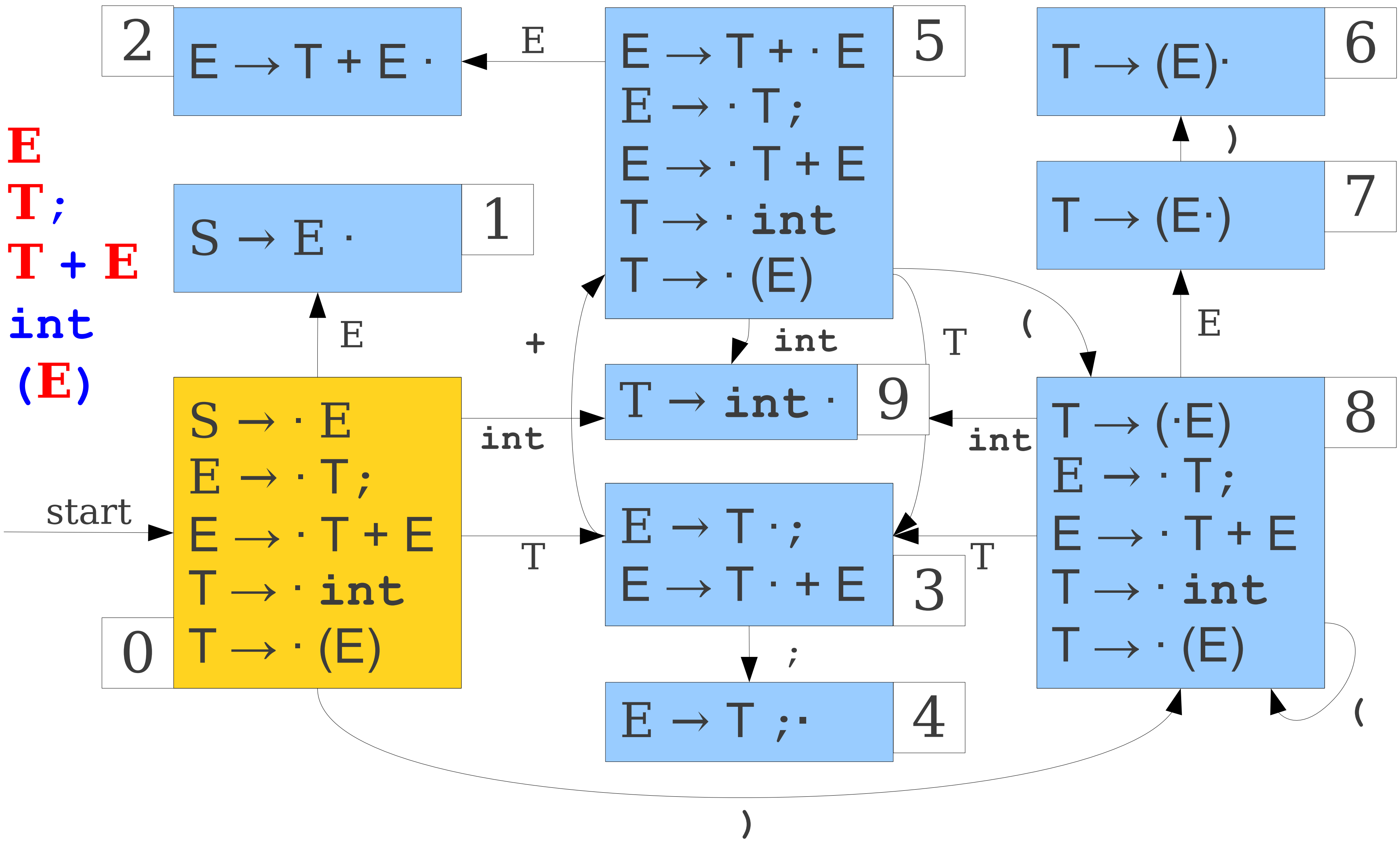



| int | + | ( | int | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|

**0**

# LR(0) Parsing

**S → E**
**E → T;**
**E → T + E**
**T → int**
**T → (E)**

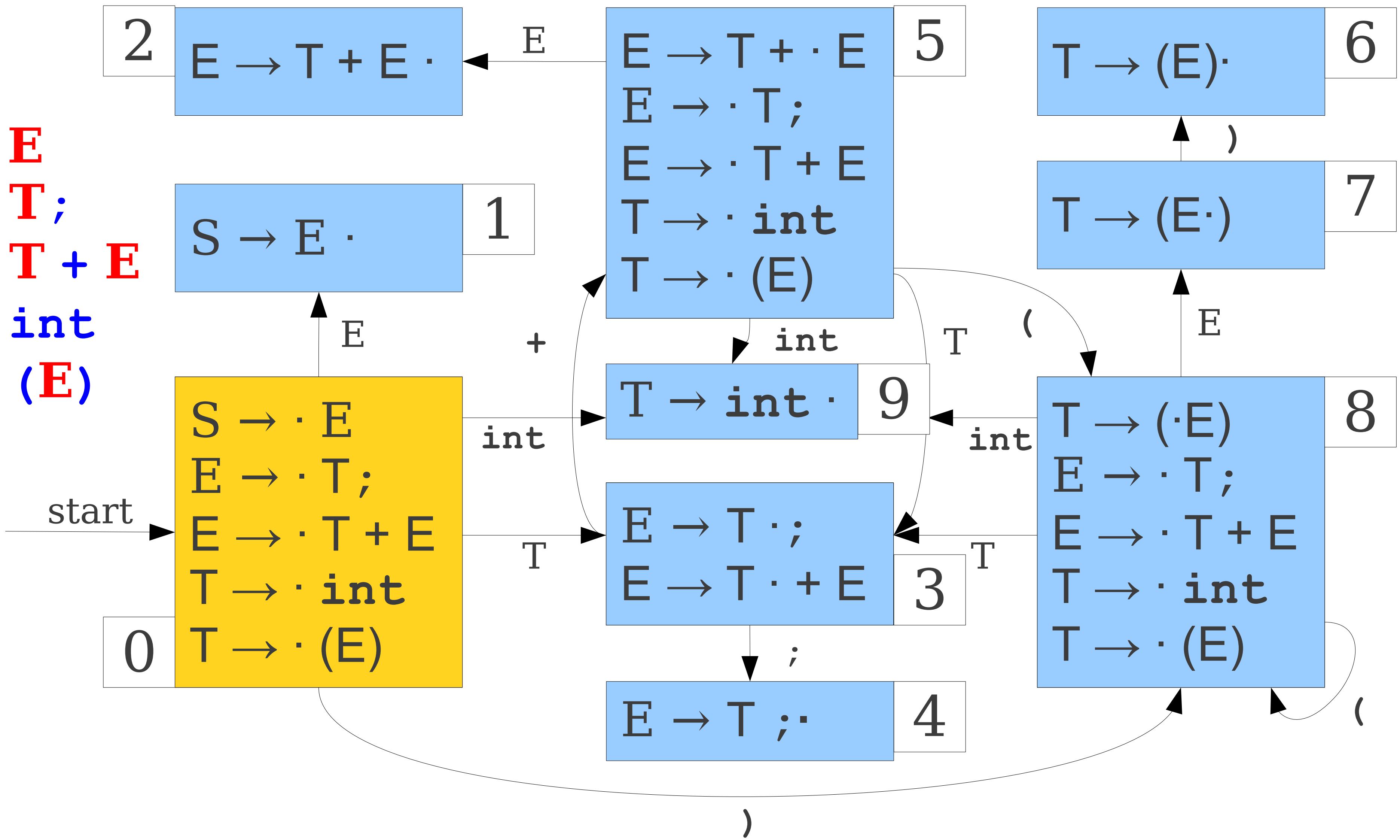



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| + | ( | int | + | int | ; | ) | ; |

**0**

# LR(0) Parsing

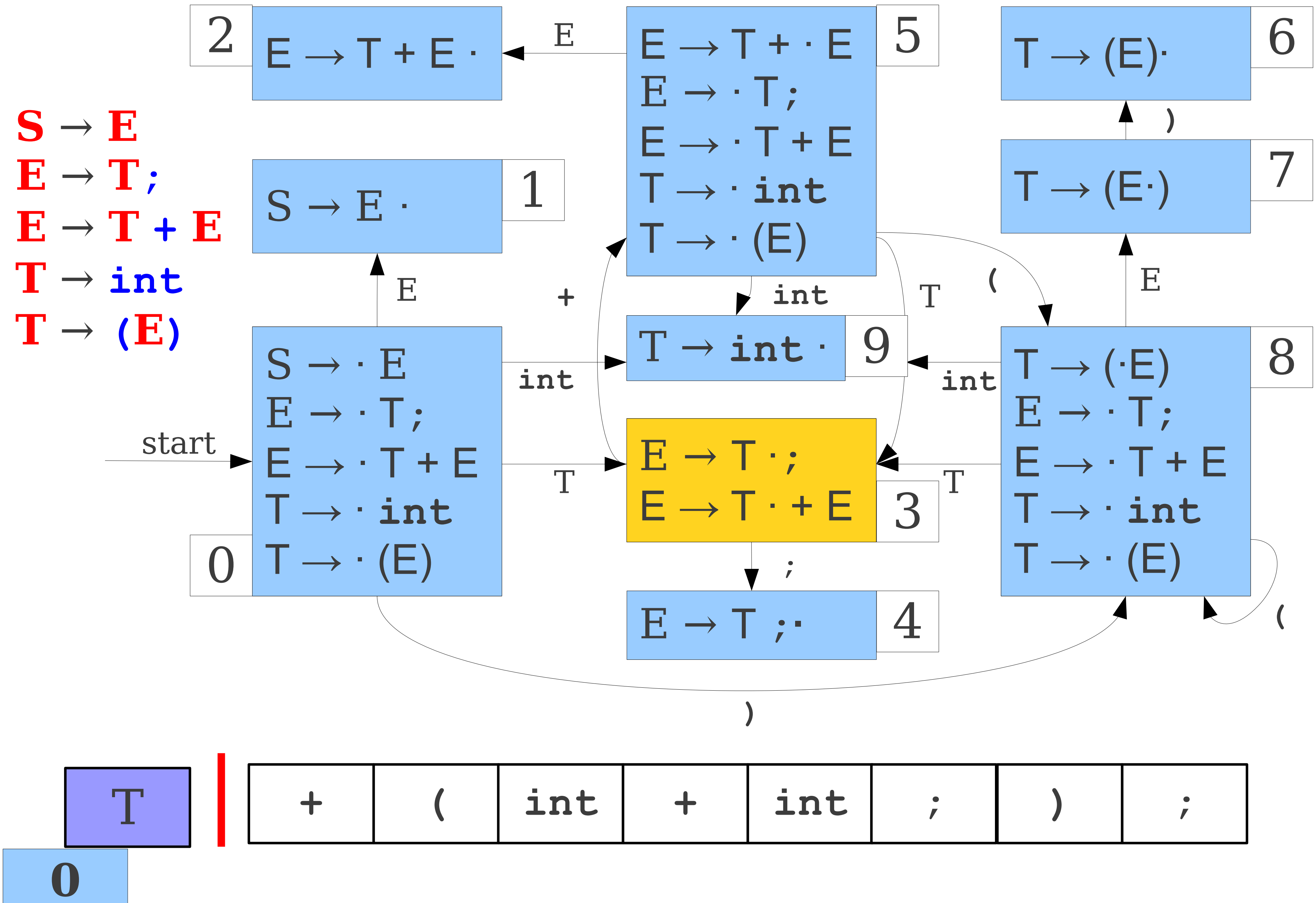
S → E
E → T;
E → T + E
T → int
T → (E)




| T | + | ( | int | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|

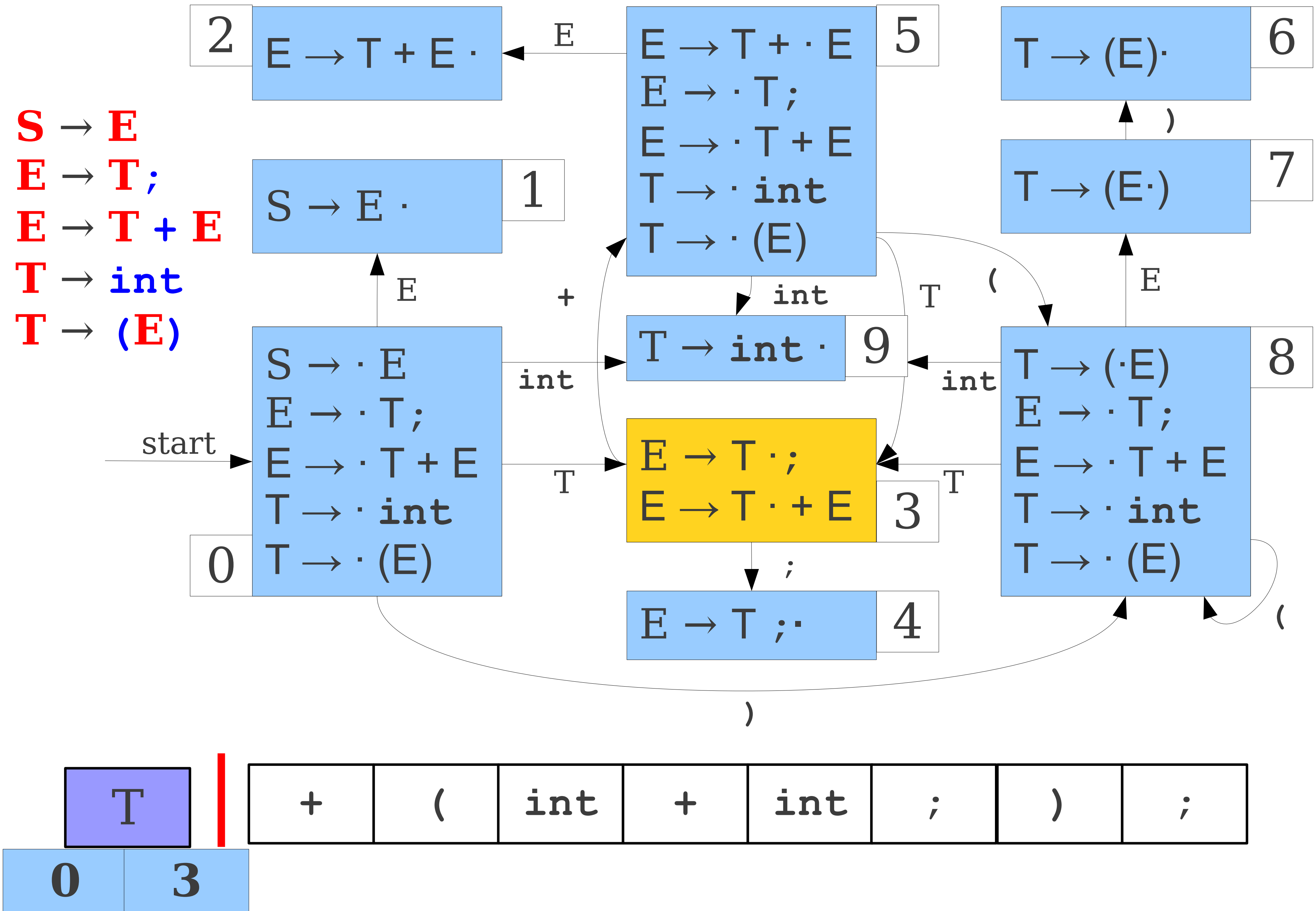
0

# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
7
T → (E·)
1
S → E ·
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
start
9
T → int ·
3
E → T ·;
E → T · + E
4
E → T ;·
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
E
+
int
T
(
)
;
T
0
+
(
int
+
int
;
)
;

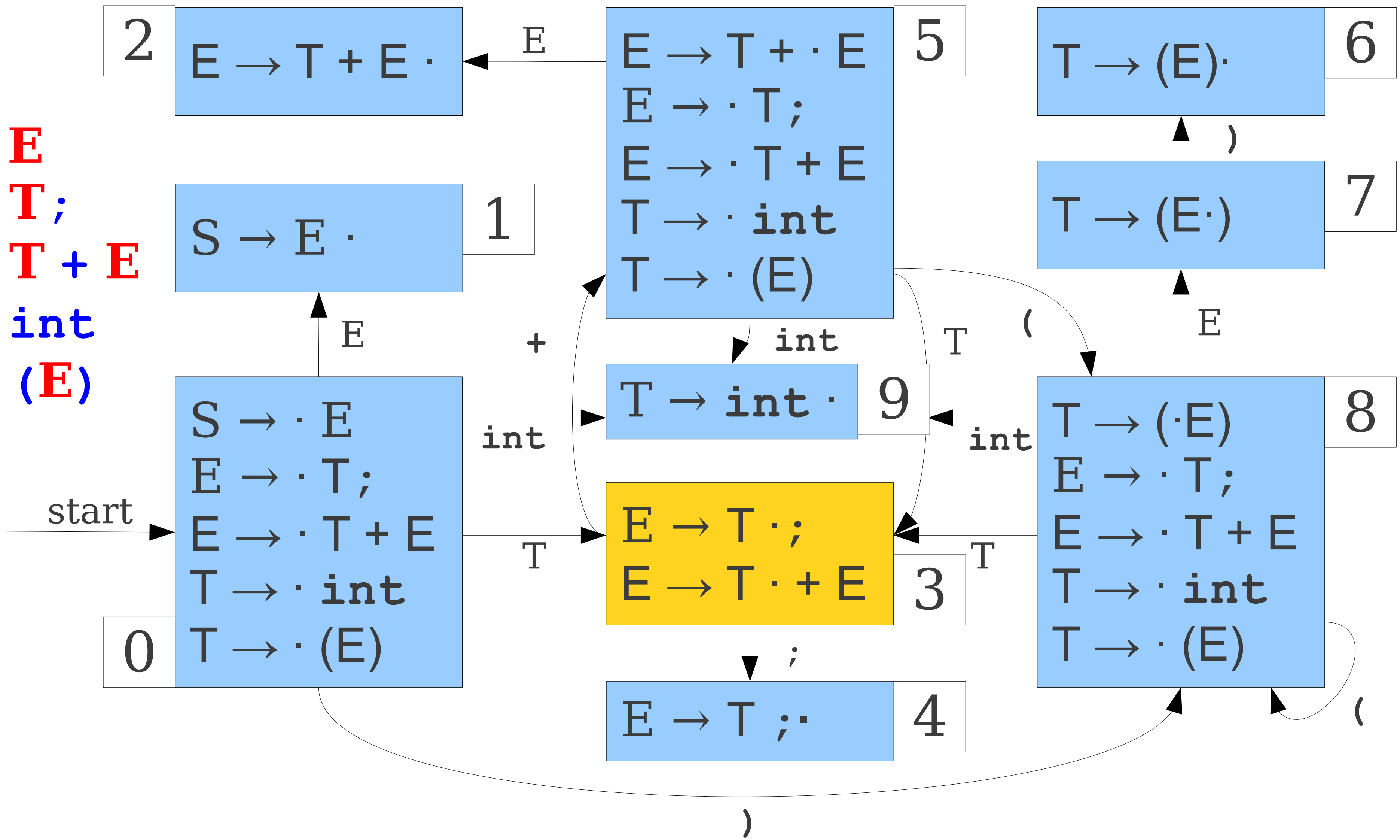
# LR(0) Parsing

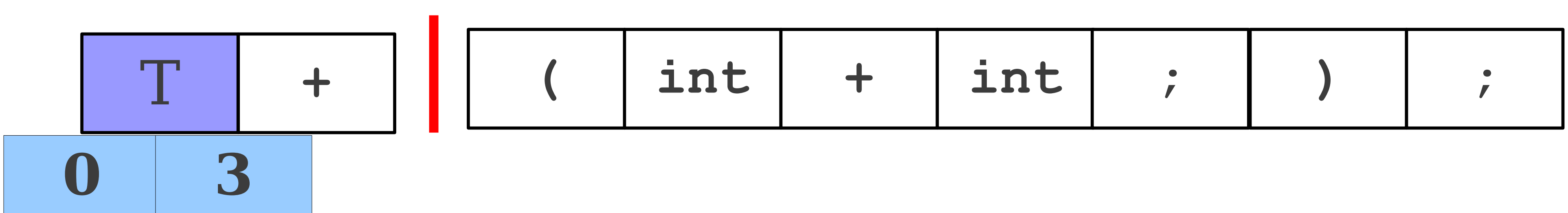
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

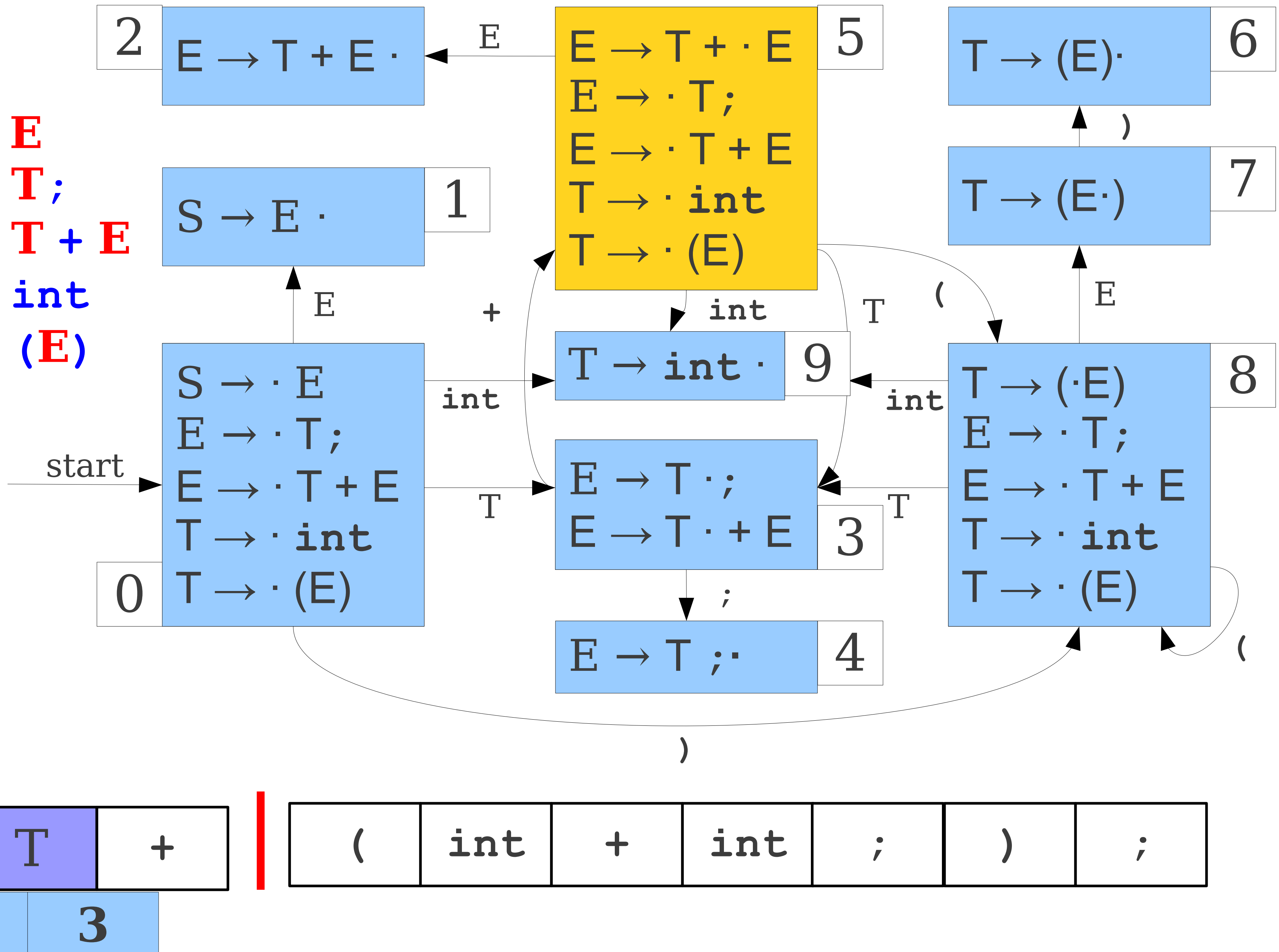
**S** → **E**
**E** → **T** `;`
**E** → **T** `+` **E**
**T** → `int`
**T** → `(`**E**`)`



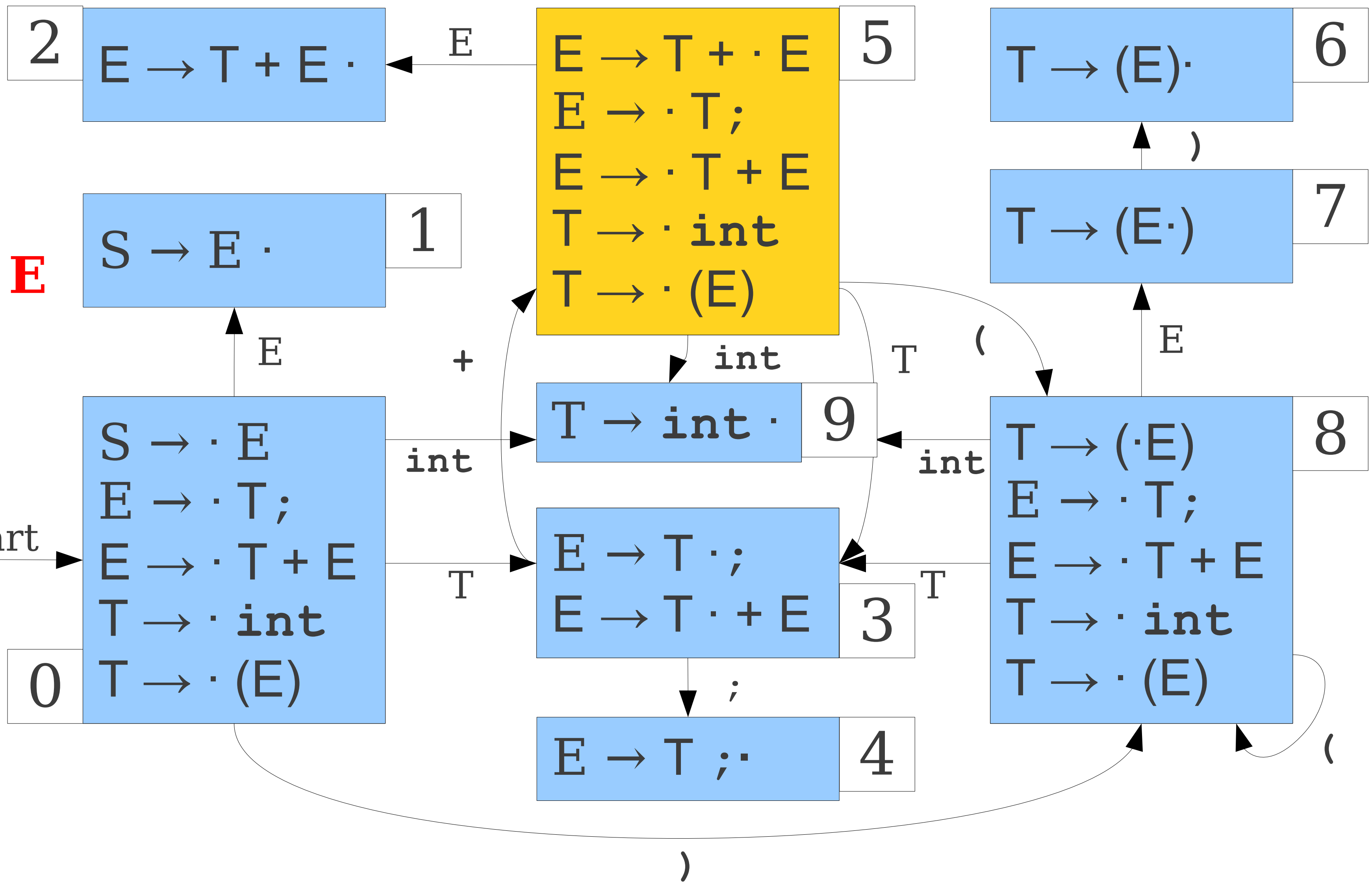
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

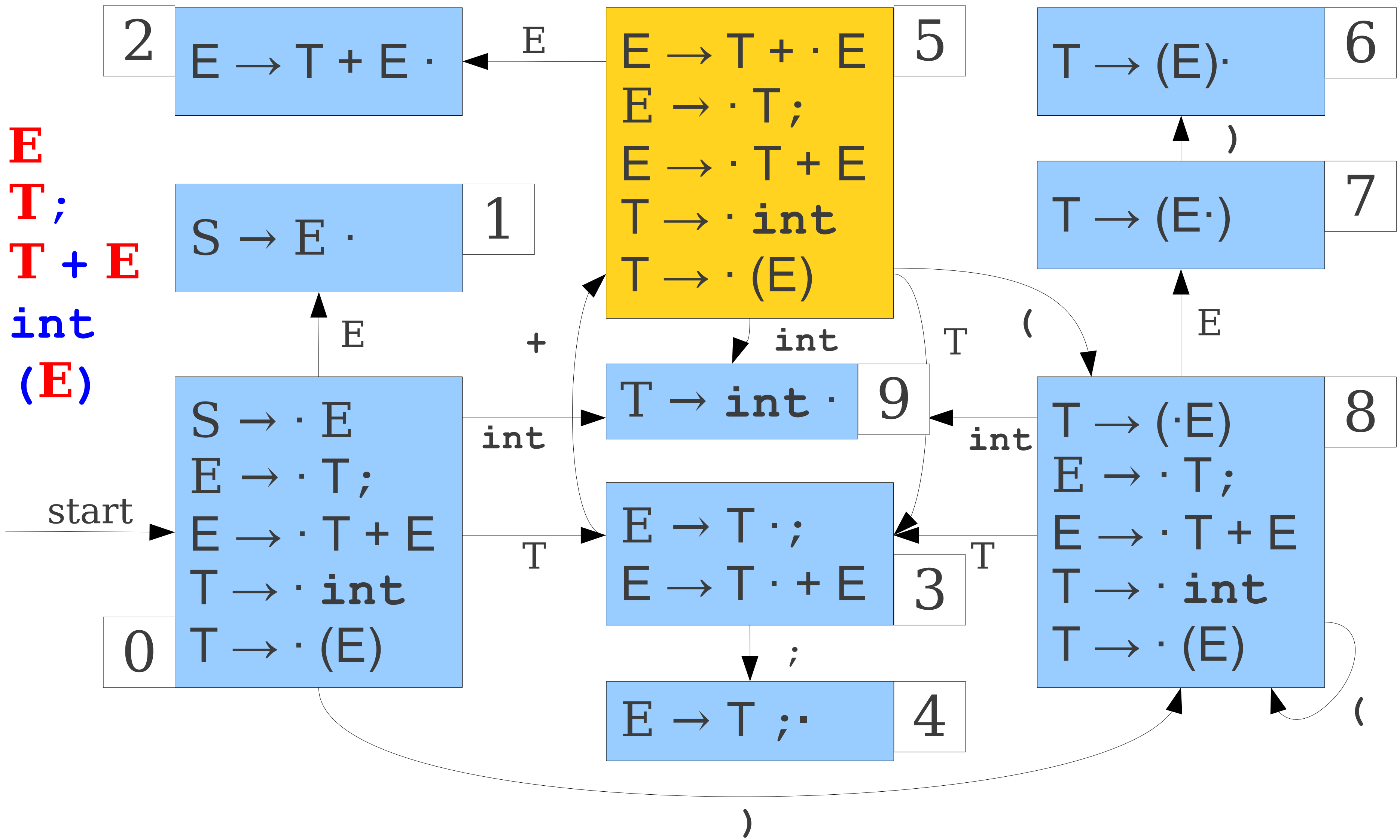
S → E
E → T;
E → T + E
T → int
T → (E)

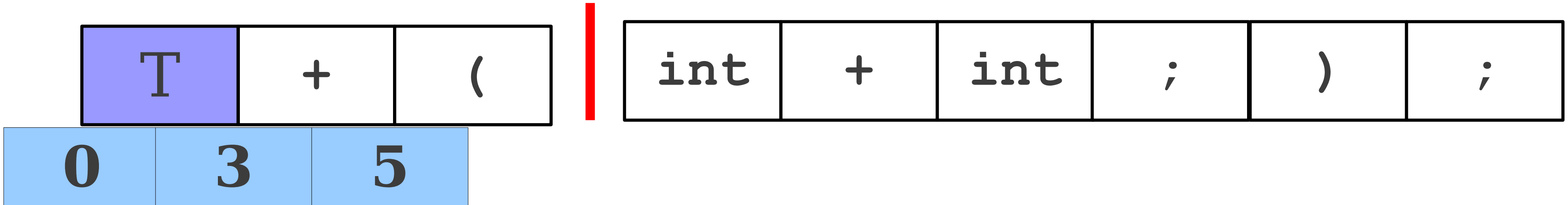



| T | + | | ( | int | + | int | ; | ) | ; |
|---|---|---|---|---|---|---|---|---|---|
| **0** | **3** | **5** | | | | | | | |

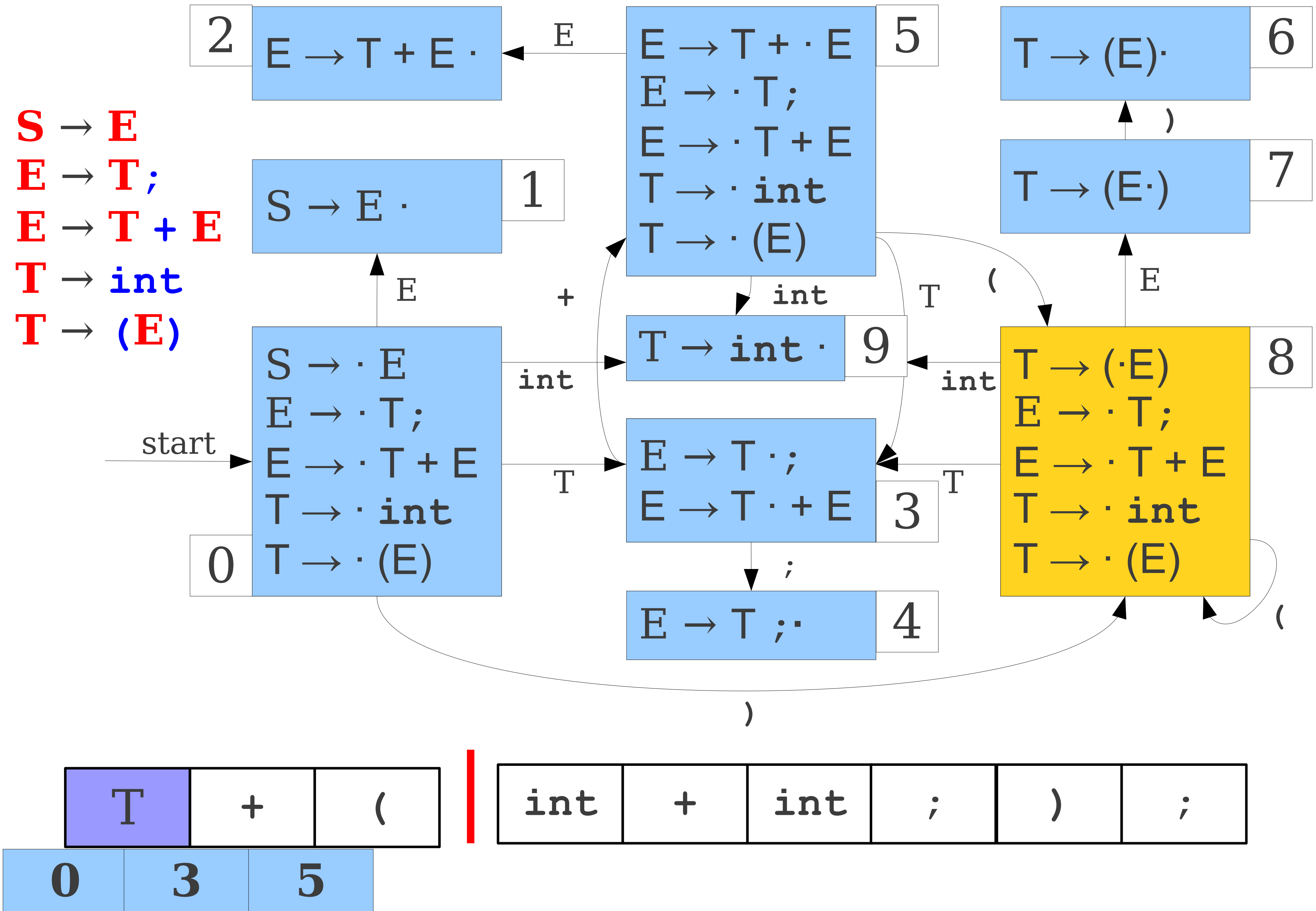
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



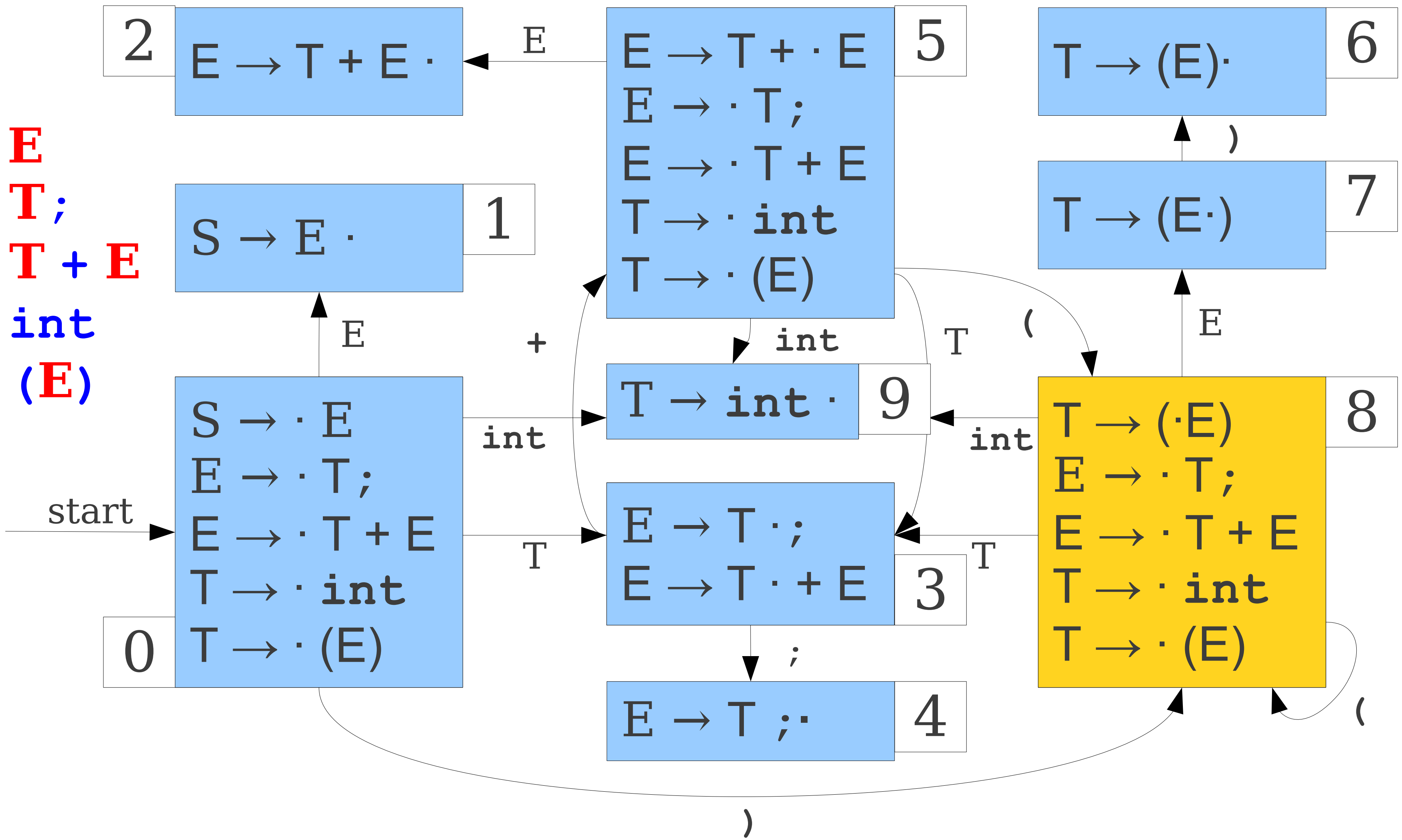



# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
7
T → (E·)
1
S → E ·
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
start
9
T → int ·
3
E → T · ;
E → T · + E
4
E → T ;·
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
E
E
+
int
int
int
T
T
T
(
(
)
)
E
;
T
+
(
int
+
int
;
)
;
0
3
5

# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )

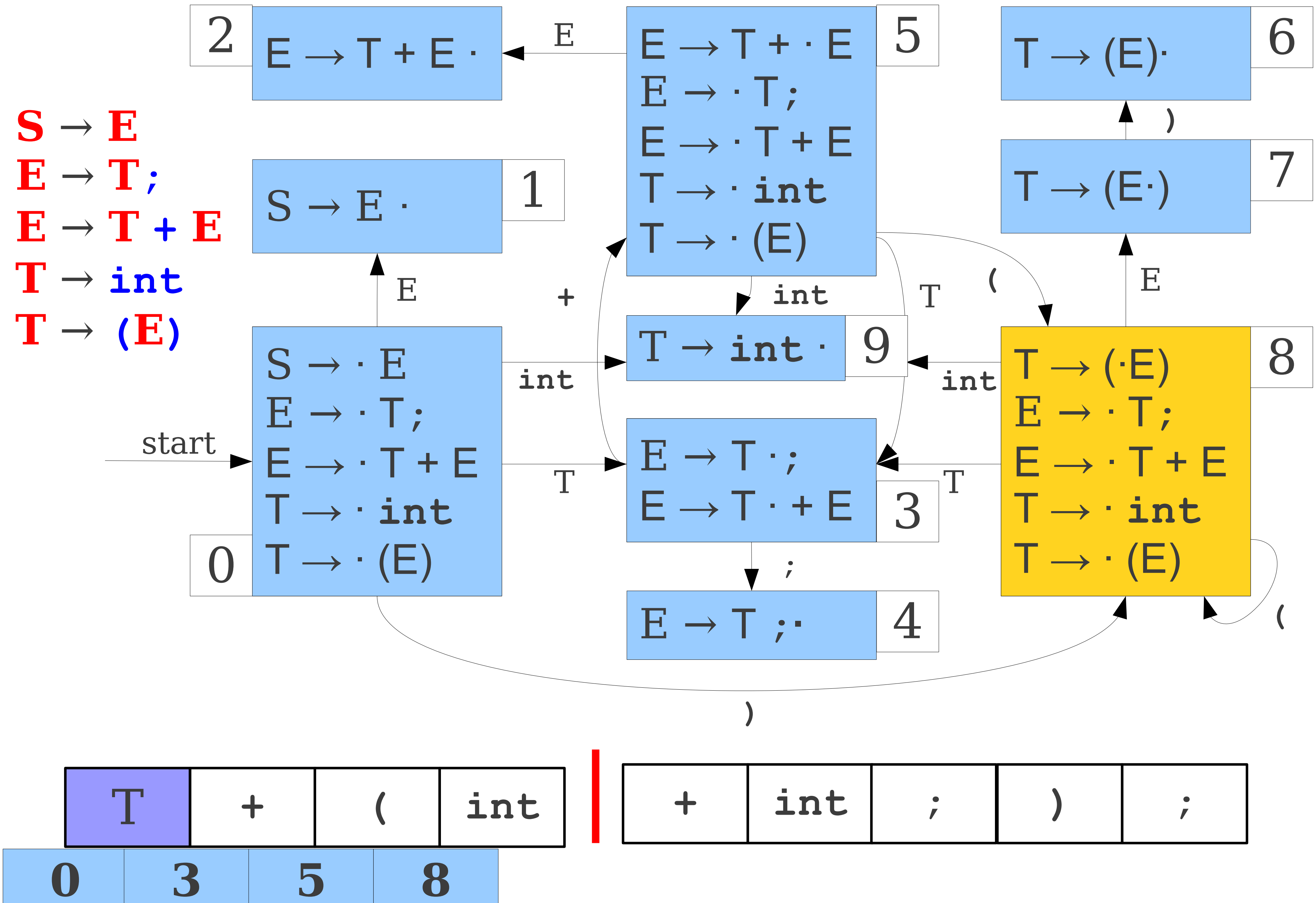



| T | + | ( |
|---|---|---|

| int | + | int | ; | ) | ; |
|---|---|---|---|---|---|

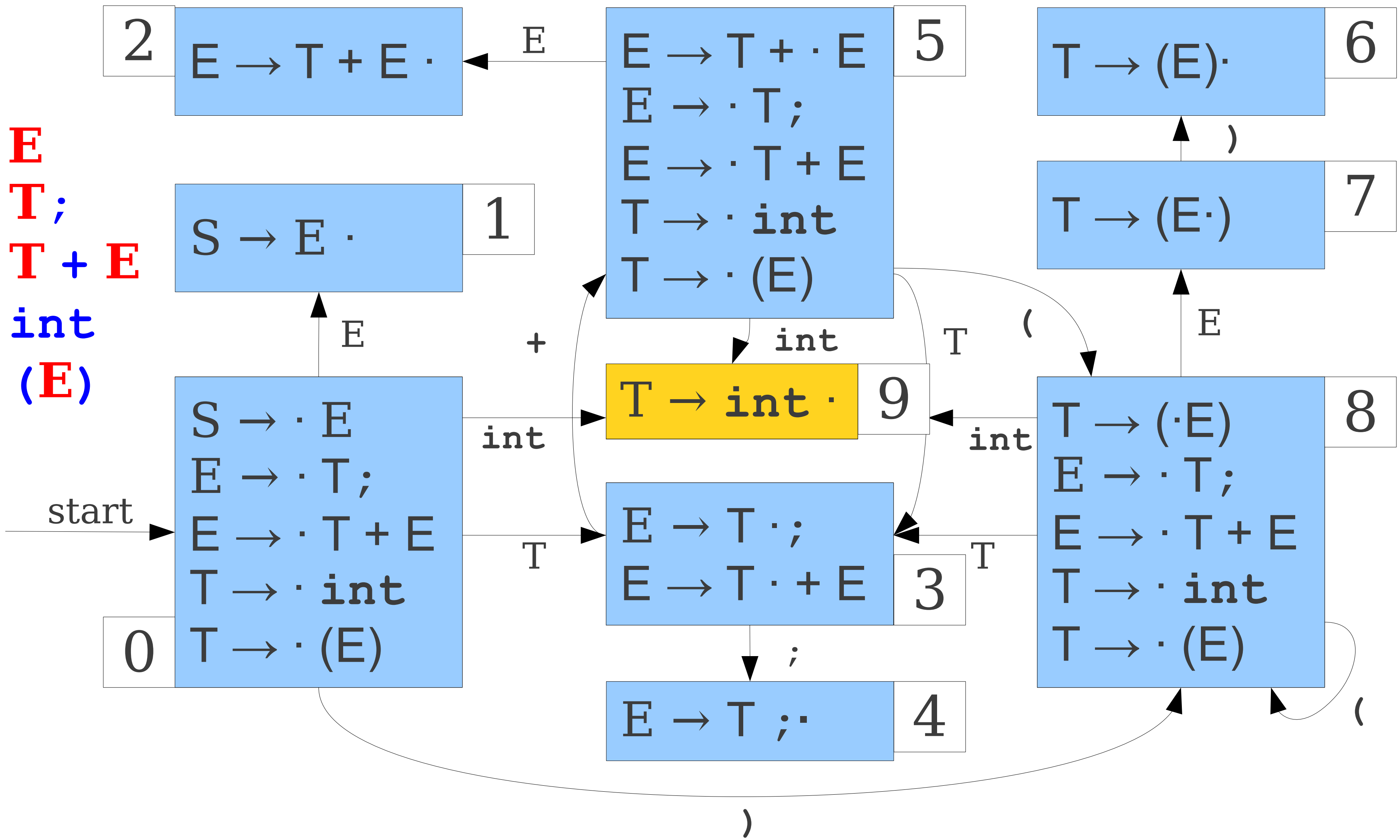
| 0 | 3 | 5 | 8 |
|---|---|---|---|

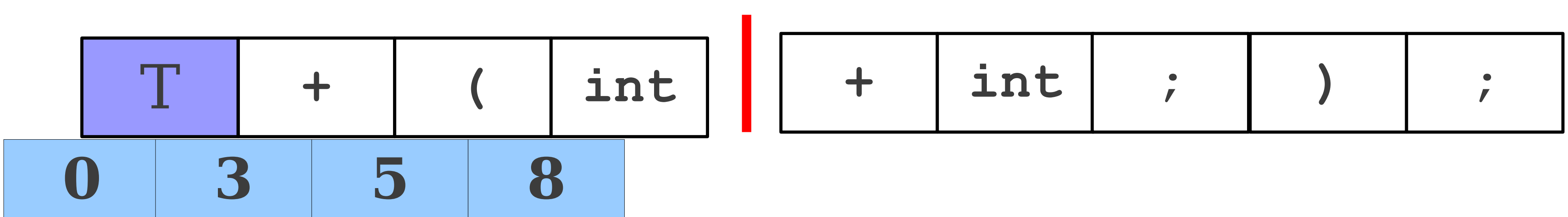
# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
1
S → E ·
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
9
T → int ·
3
E → T · ;
E → T · + E
4
E → T ;·
6
T → (E)·
7
T → (E·)
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
E
int
T
+
;
(
)
T
+
(
int
+
int
;
)
;
0
3
5
8

# LR(0) Parsing

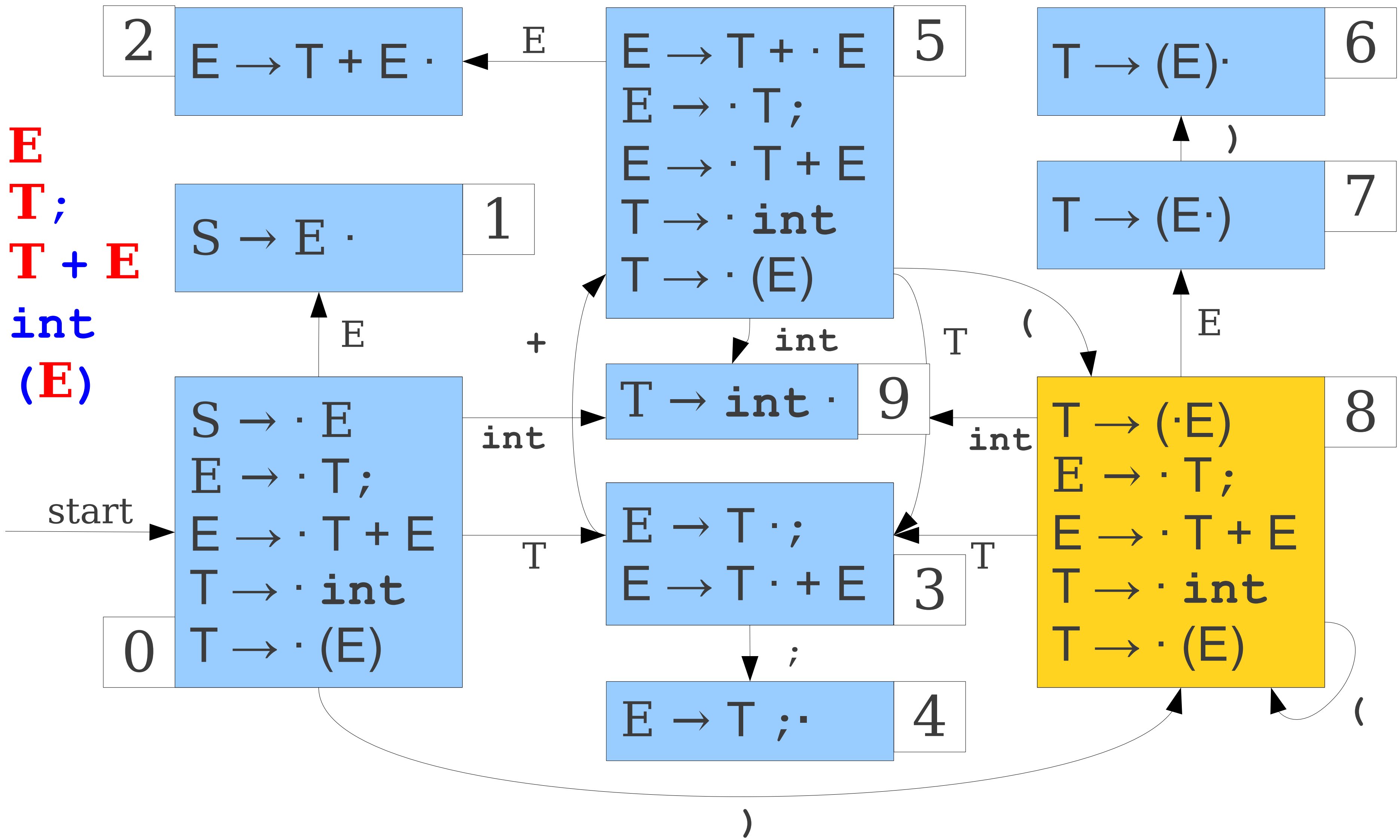
S → E
E → T;
E → T + E
T → int
T → (E)



# LR(0) Parsing

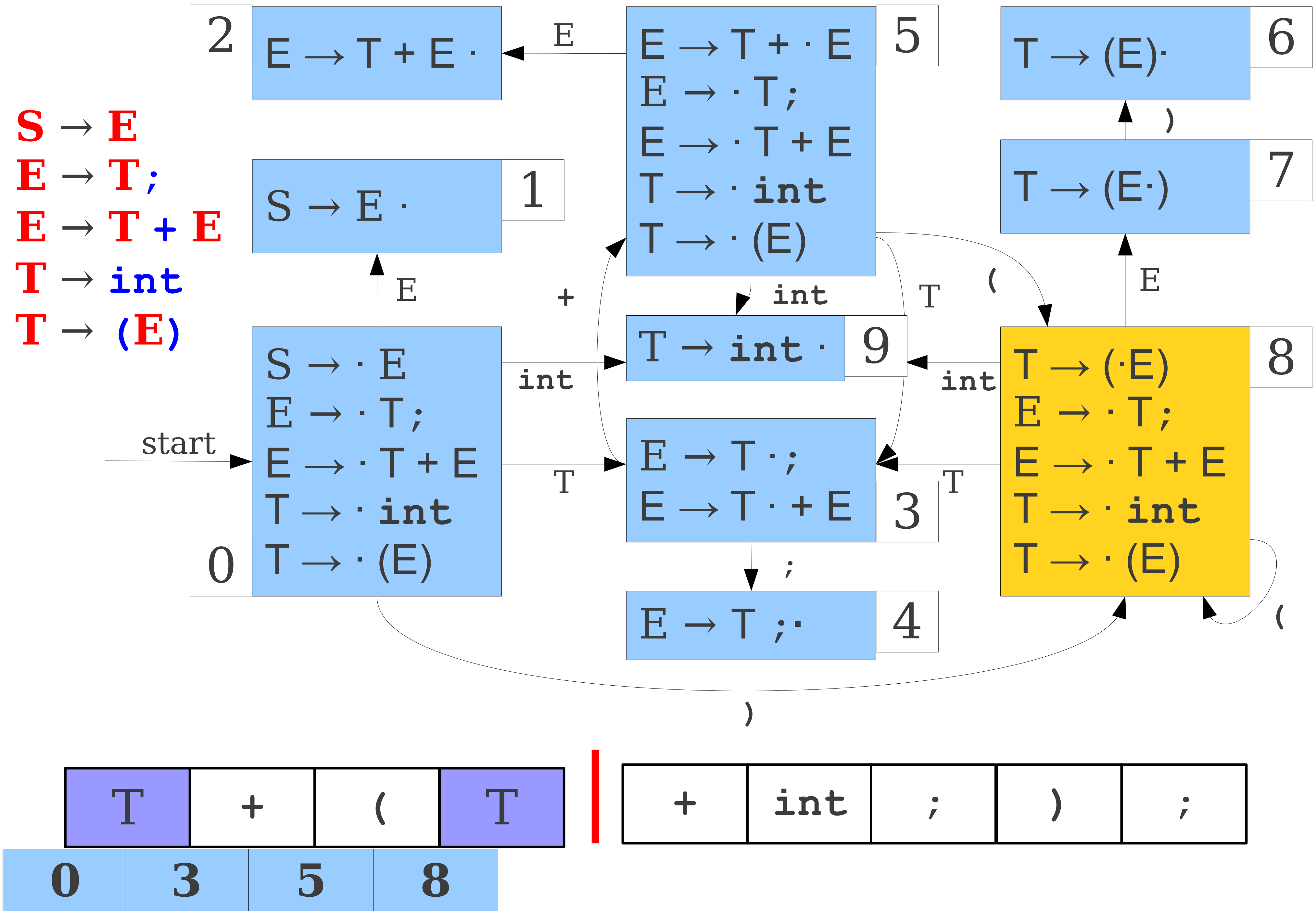
**S** → **E**
**E** → **T;**
**E** → **T + E**
**T** → **int**
**T** → **(E)**




| | | | |
|---|---|---|---|
| | T | + | ( |
| 0 | 3 | 5 | 8 |

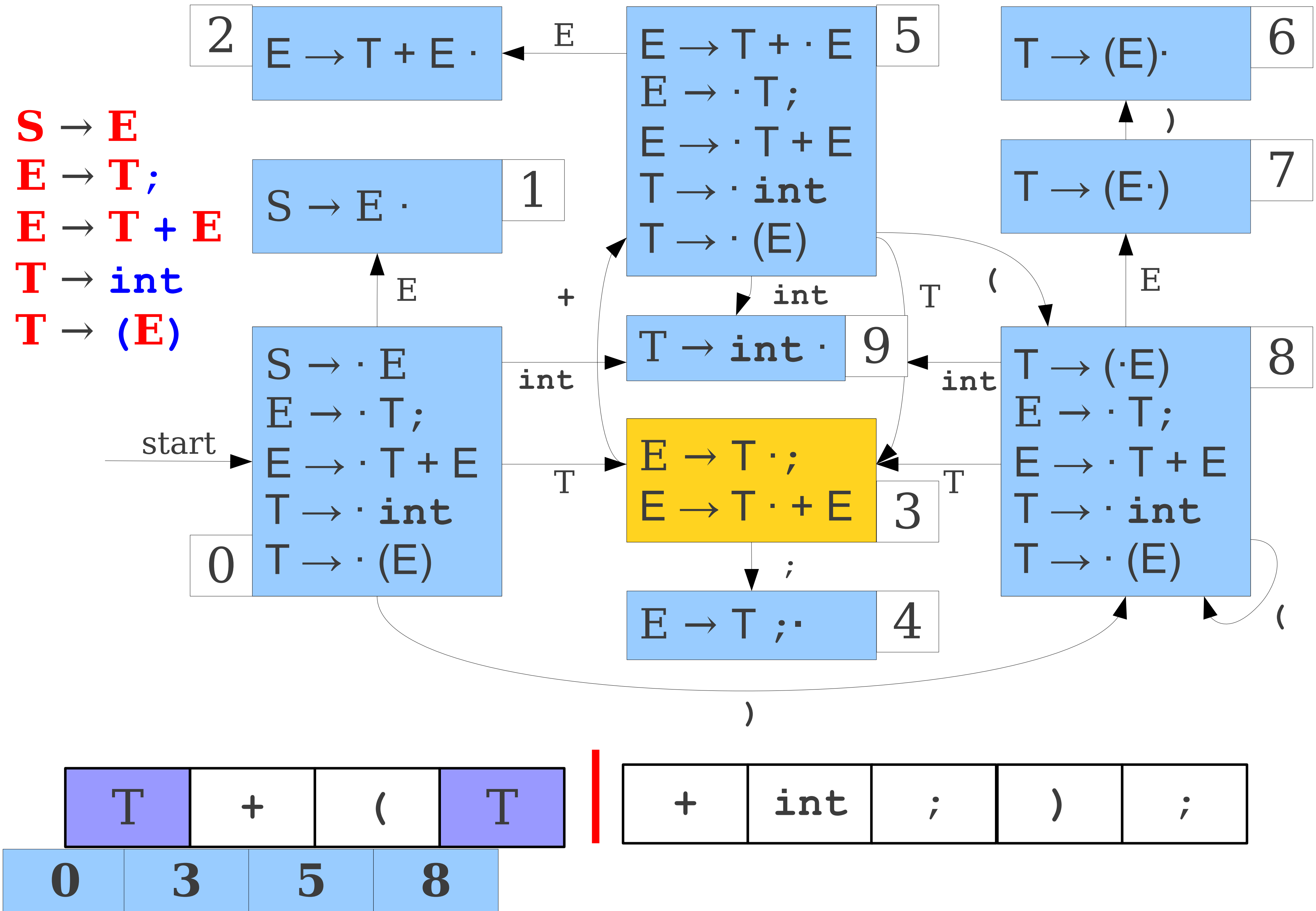
| | | | | |
|---|---|---|---|---|
| + | int | ; | ) | ; |

# LR(0) Parsing



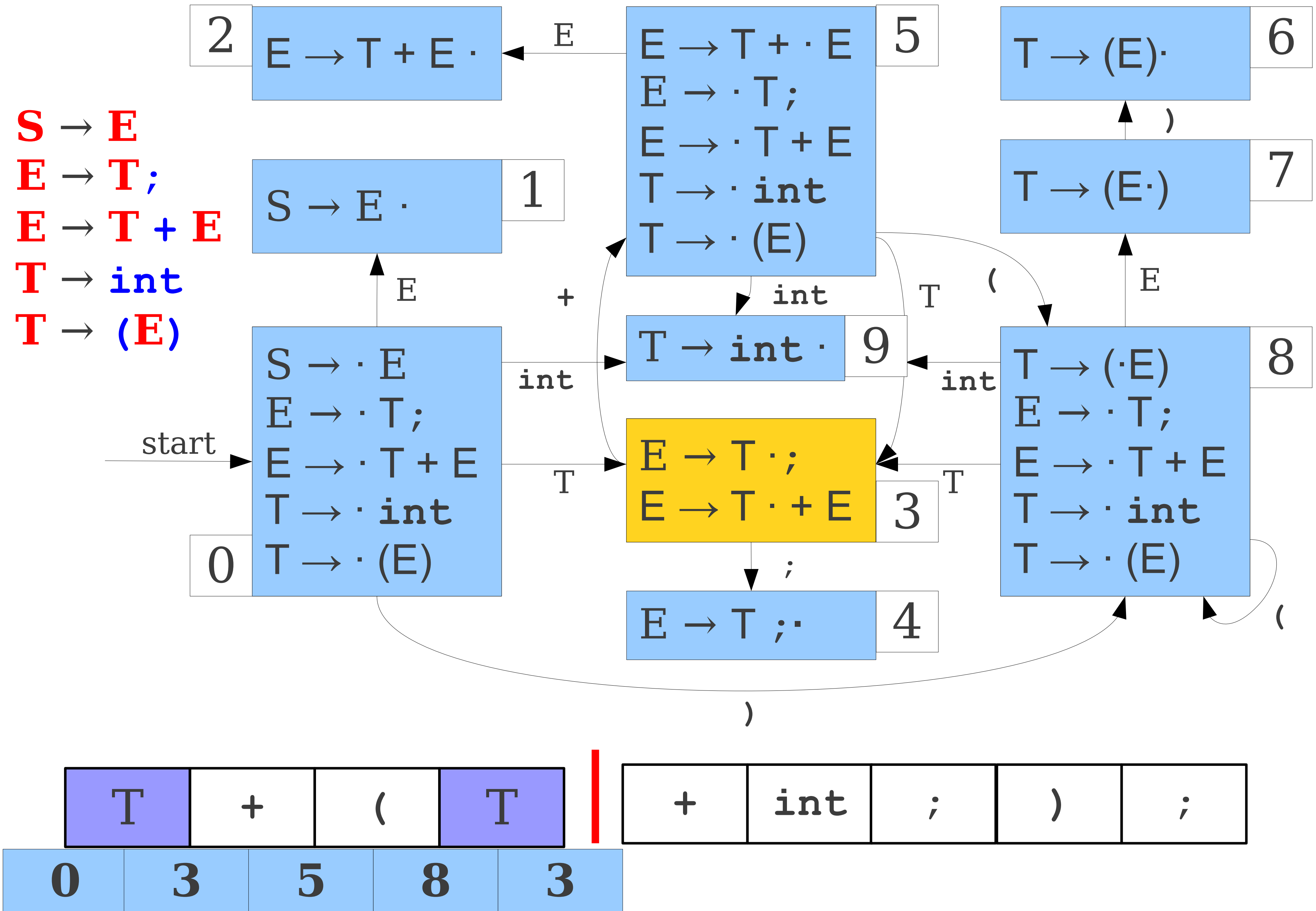
S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
E
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
)
7
T → (E·)
1
S → E ·
E
+
int
T
(
E
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
0
start
int
T → int ·
9
int
T
E → T ·;
E → T · + E
3
T
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
8
;
E → T ;·
4
(
)
T
+
(
T
+
int
;
)
;
0
3
5
8

# LR(0) Parsing



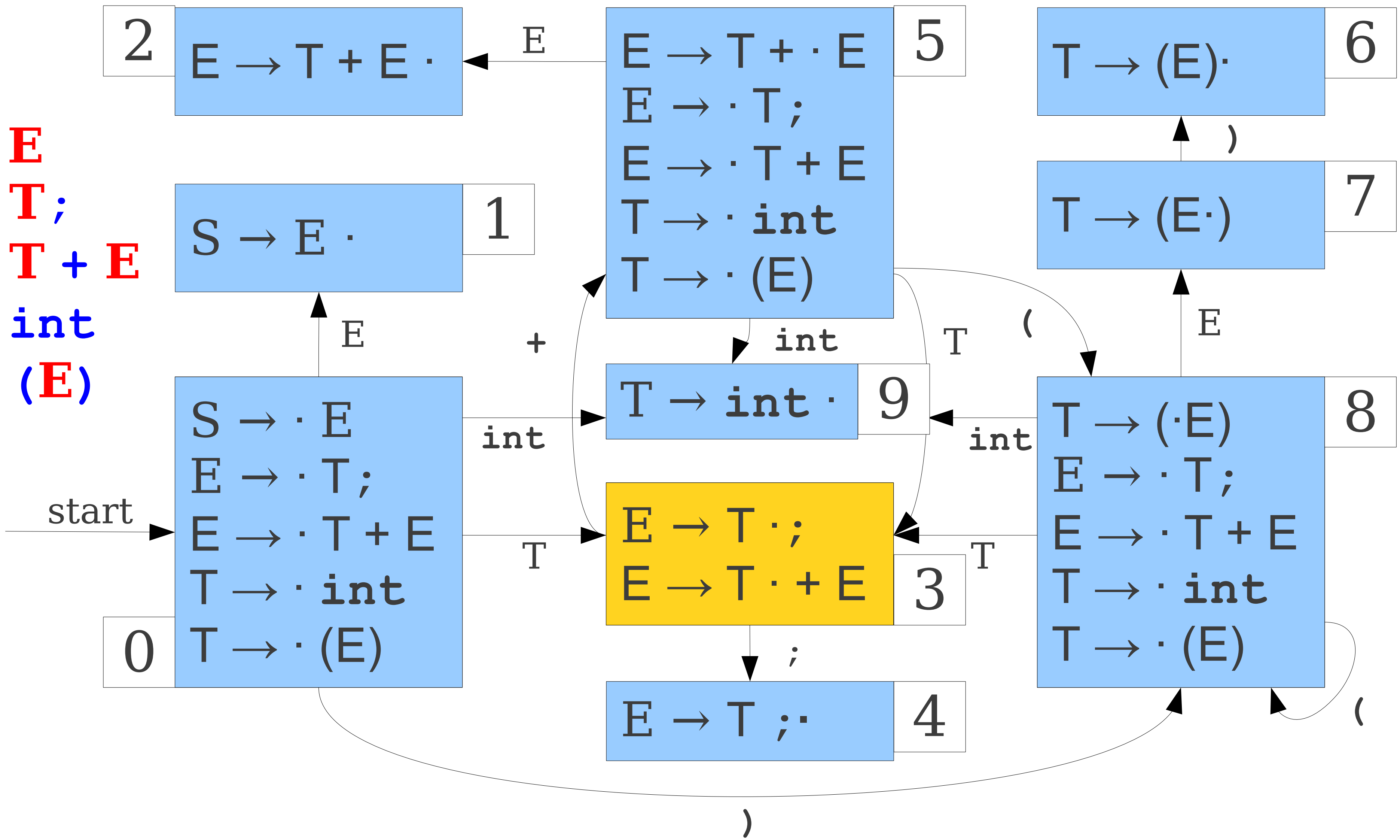
S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
E
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
5
T → (E)·
6
)
T → (E·)
7
S → E ·
1
E
+
int
T
(
E
T → int ·
9
int
int
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
0
start
T
E → T ·;
E → T · + E
3
T
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
8
;
E → T ;·
4
(
)
T
+
(
T
0
3
5
8
+
int
;
)
;

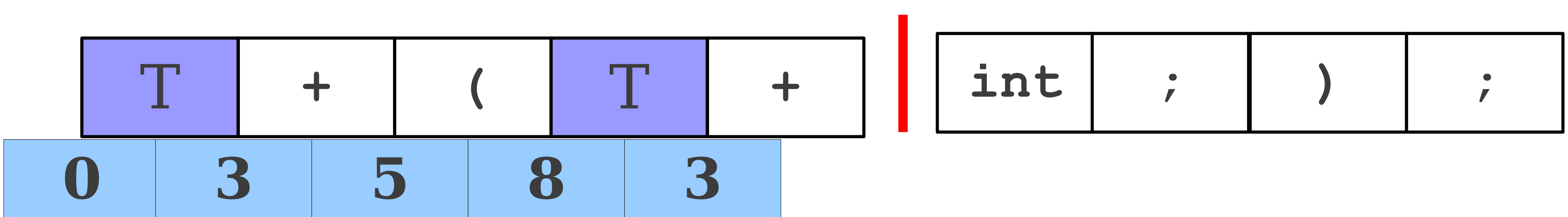
# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
1
S → E ·
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
7
T → (E·)
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
9
T → int ·
3
E → T ·;
E → T · + E
4
E → T ;·
E
E
E
T
T
T
int
int
int
+
;
(
(
)
)
T
+
(
T
+
int
;
)
;
0
3
5
8
3

# LR(0) Parsing

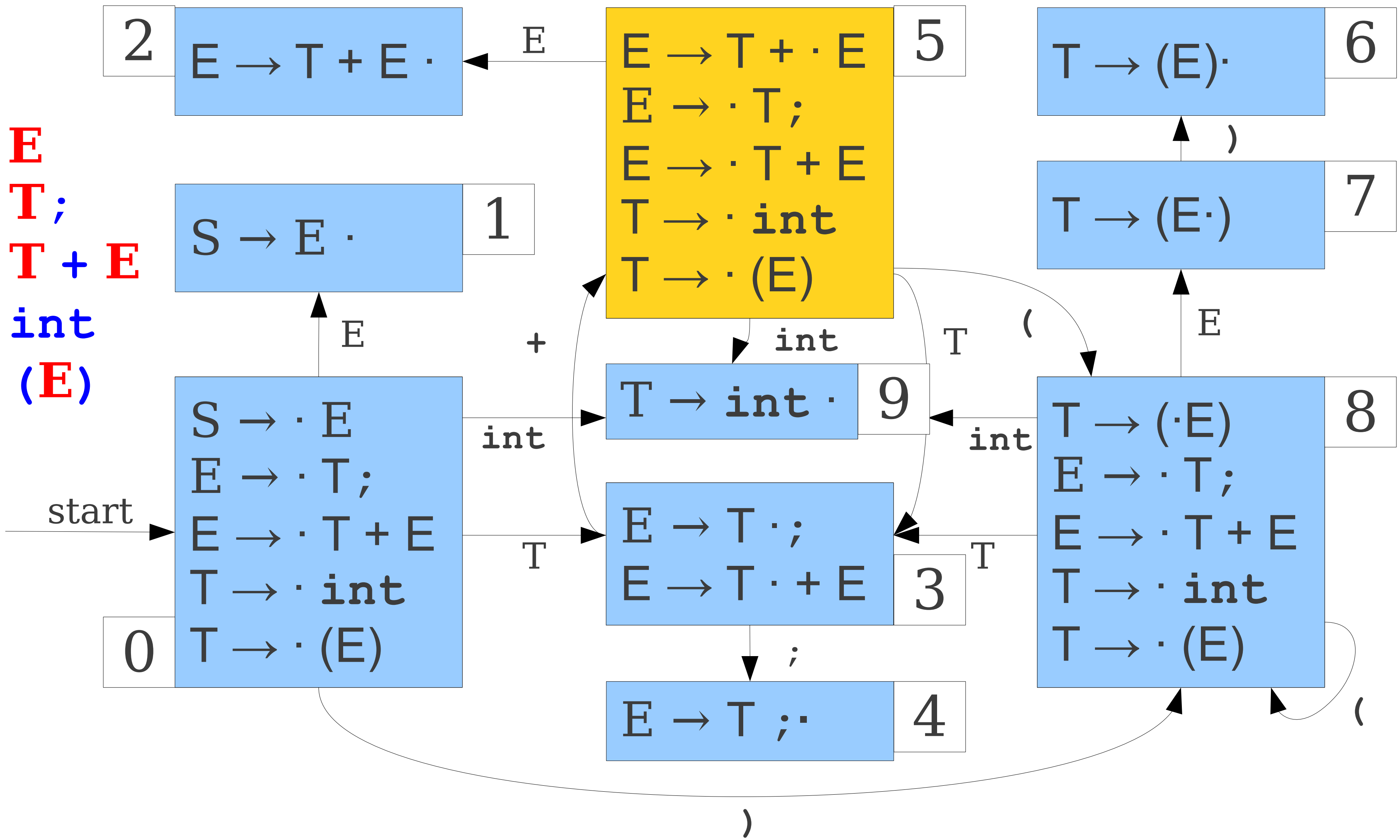
S → E
E → T;
E → T + E
T → int
T → (E)

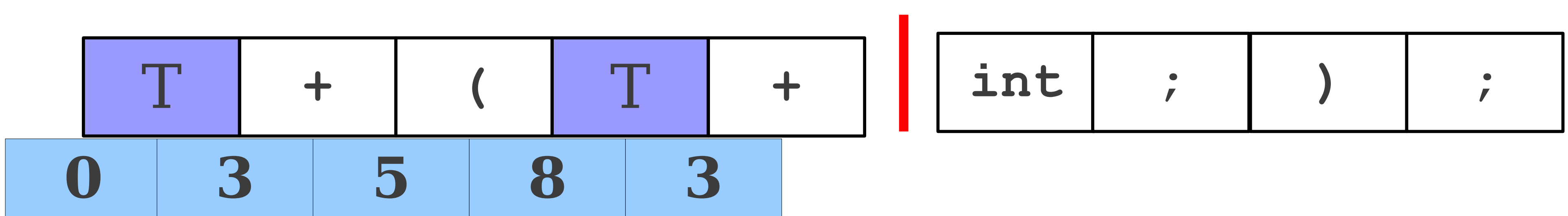


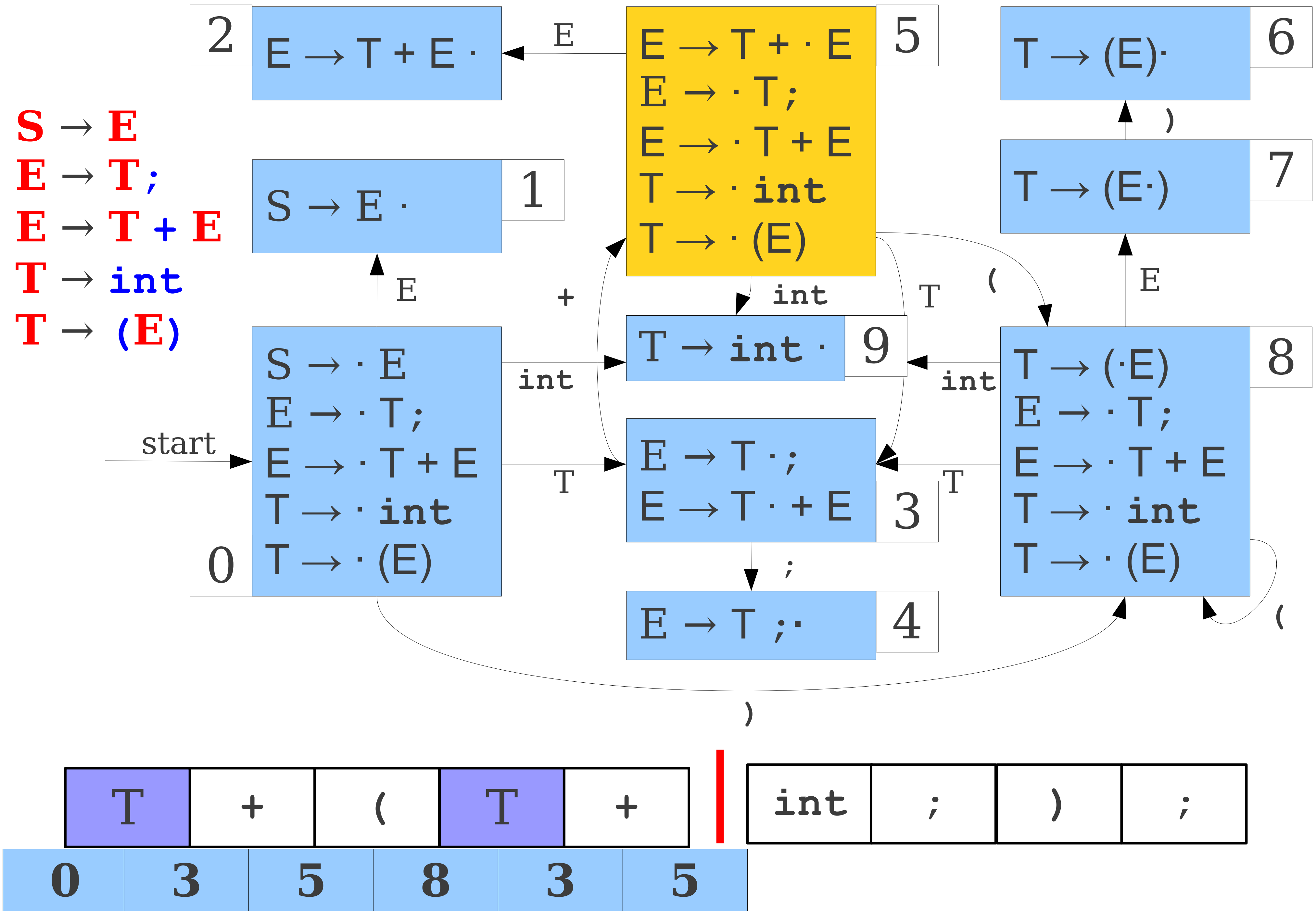
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



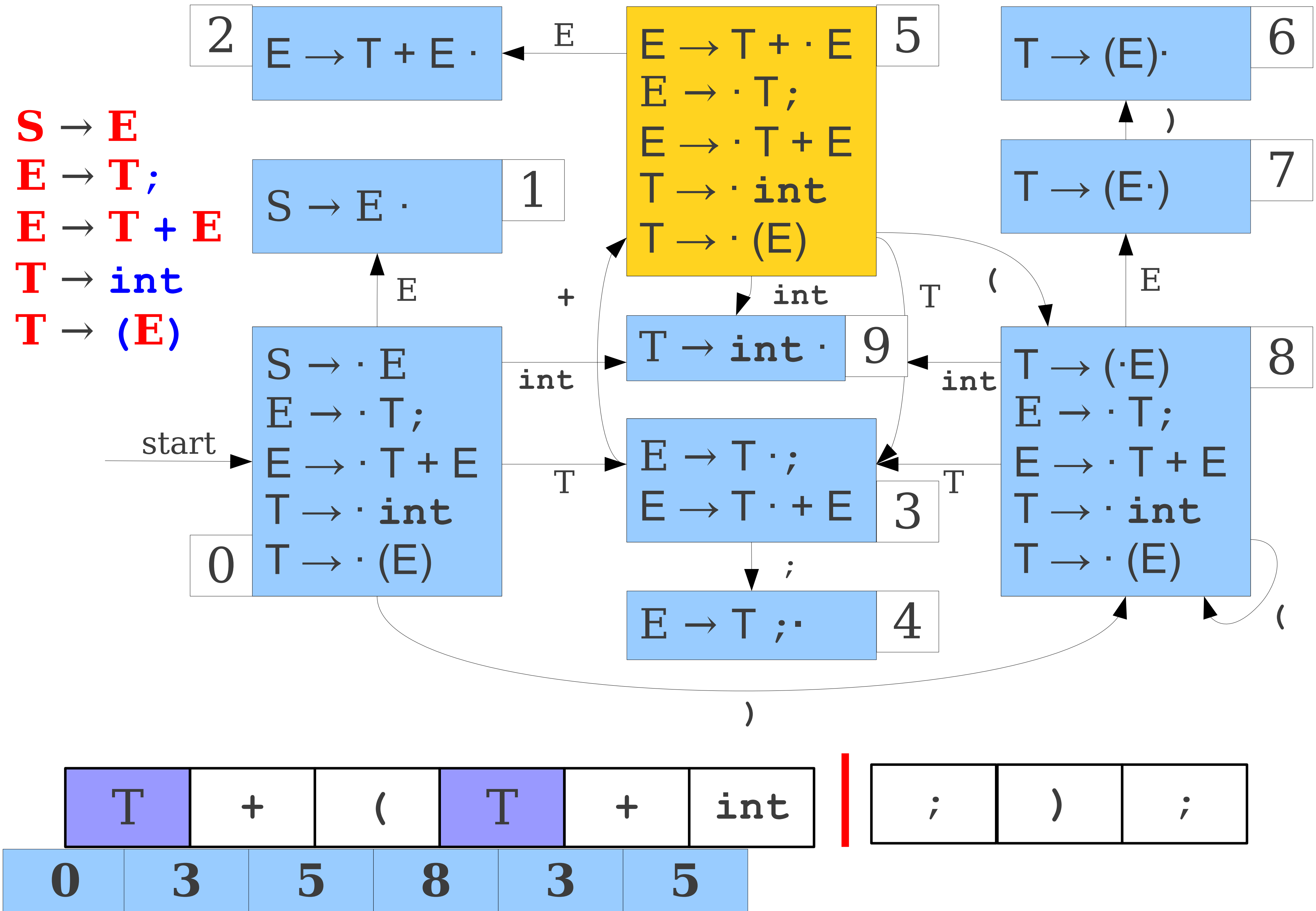
# LR(0) Parsing



S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
E
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
5
T → (E)·
6
)
T → (E·)
7
S → E ·
1
E
+
int
T
(
E
start
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
0
int
T → int ·
9
int
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
8
T
E → T ·;
E → T · + E
3
T
;
E → T ;·
4
(
)
T
+
(
T
+
int
;
)
;
0
3
5
8
3
5

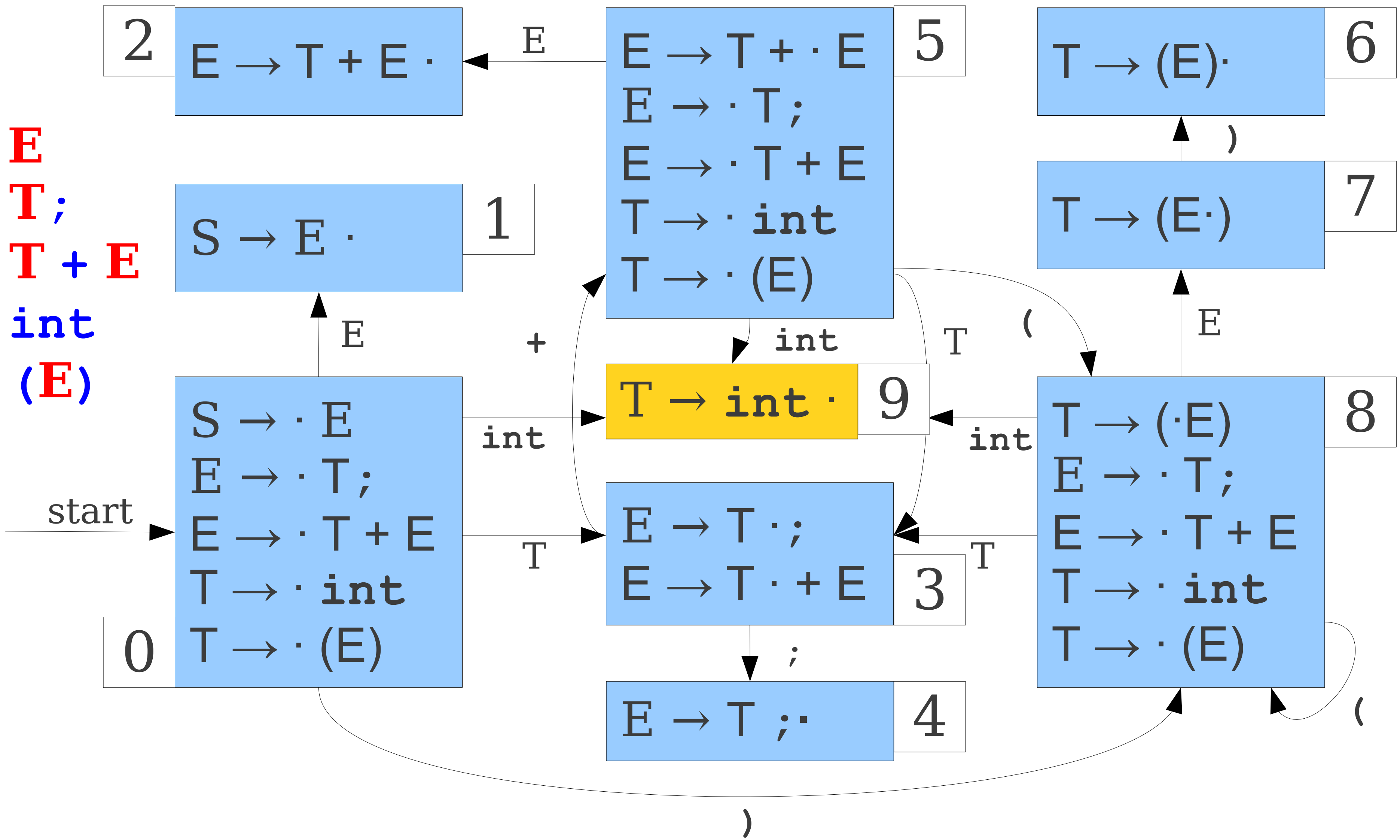
# LR(0) Parsing

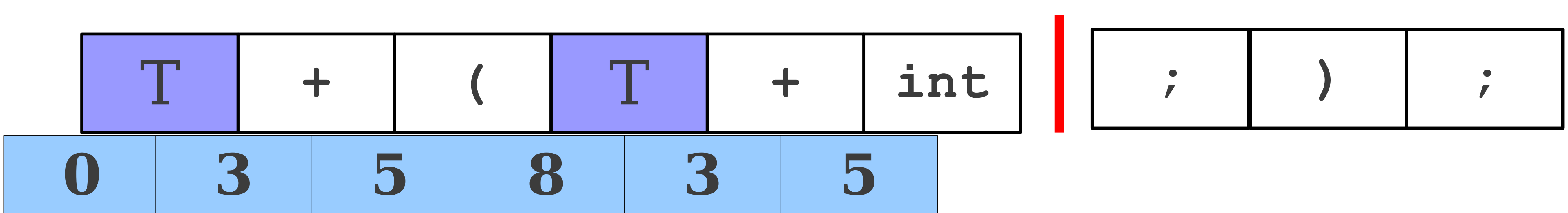
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

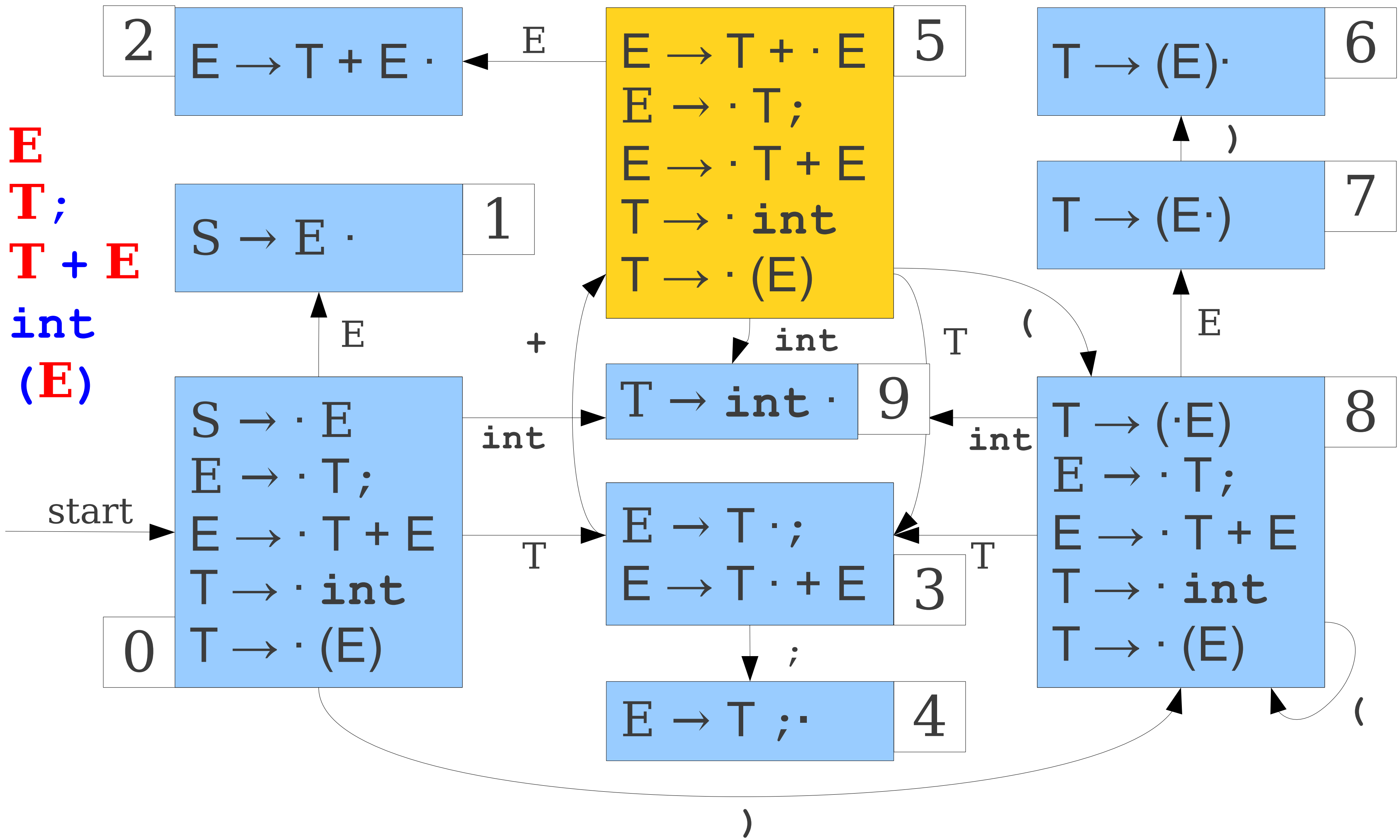
**S → E**
**E → T;**
**E → T + E**
**T → int**
**T → (E)**

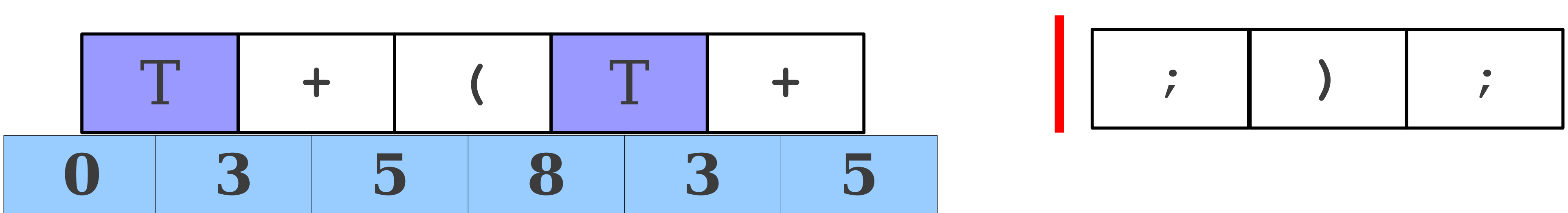


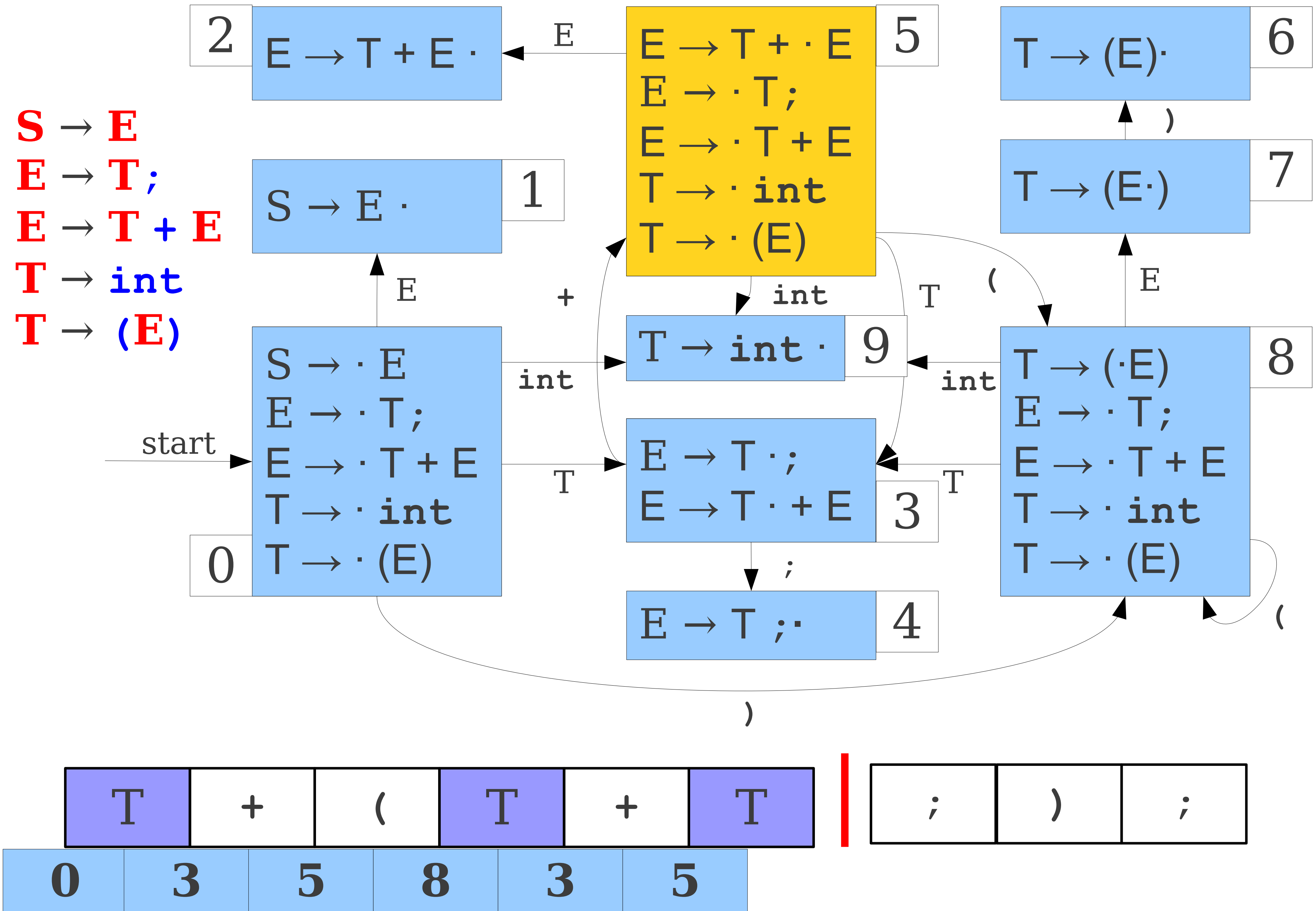
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )

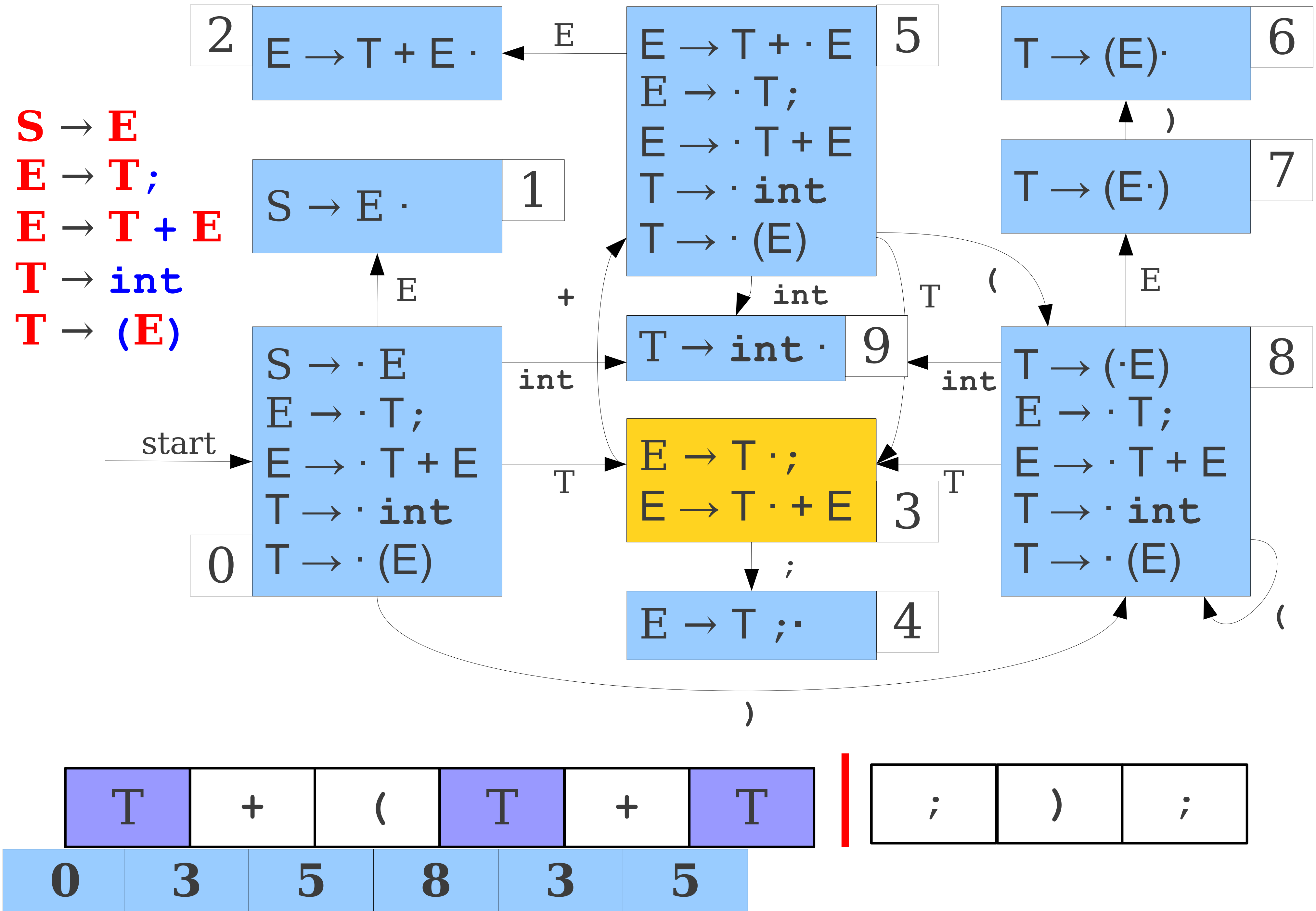


# LR(0) Parsing


S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
E
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
5
T → (E)·
6
)
T → (E·)
7
S → E ·
1
E
+
int
T
(
E
start
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
0
int
T → int ·
9
int
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
8
T
E → T ·;
E → T · + E
3
T
;
E → T ;·
4
(
)
T
+
(
T
+
T
;
)
;
0
3
5
8
3
5

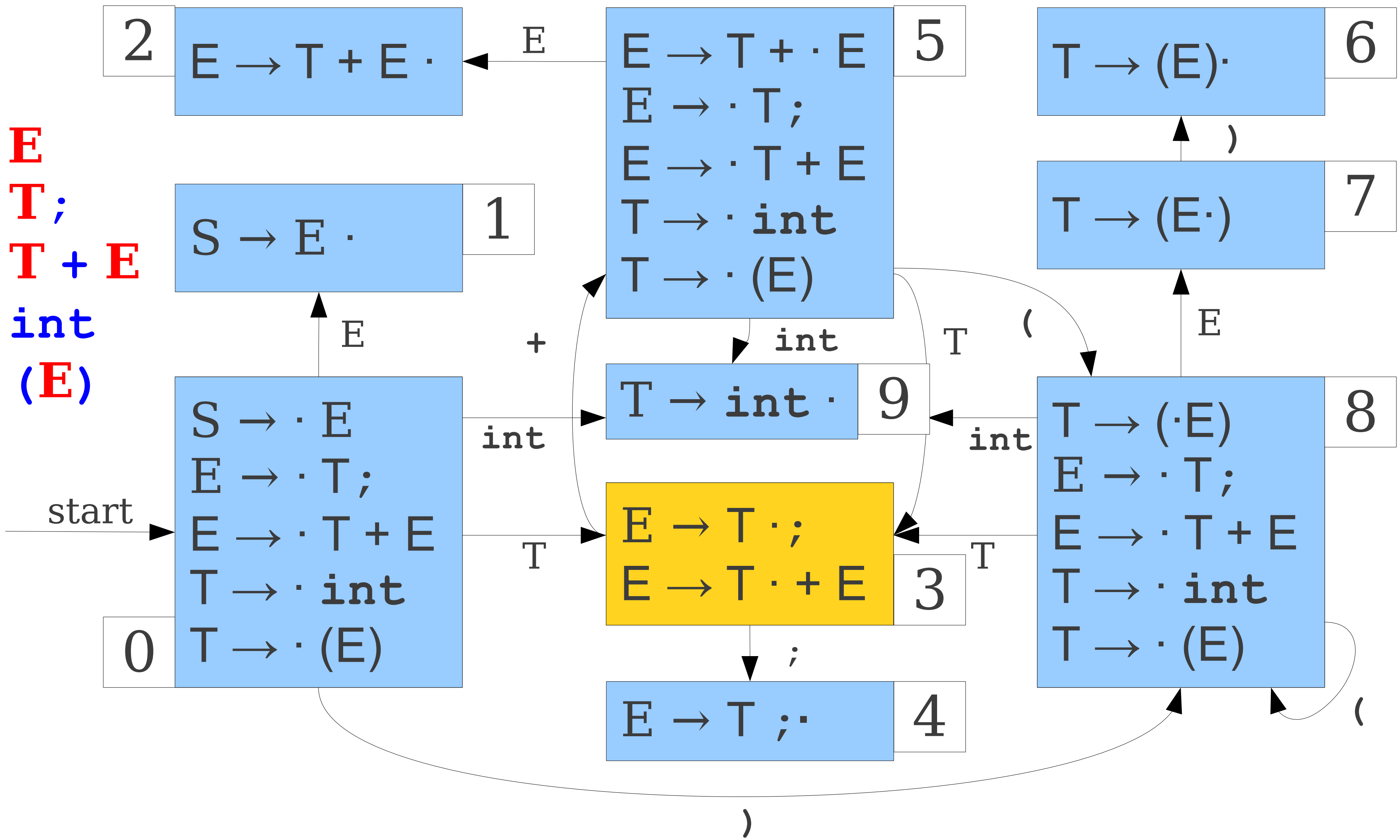
# LR(0) Parsing



S → E
E → T;
E → T + E
T → int
T → (E)
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
1
S → E ·
2
E → T + E ·
3
E → T · ;
E → T · + E
4
E → T ;·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
7
T → (E·)
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
9
T → int ·
E
int
T
+
;
(
)
T
+
(
T
+
T
;
)
;
0
3
5
8
3
5

# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → `(`**E**`)`



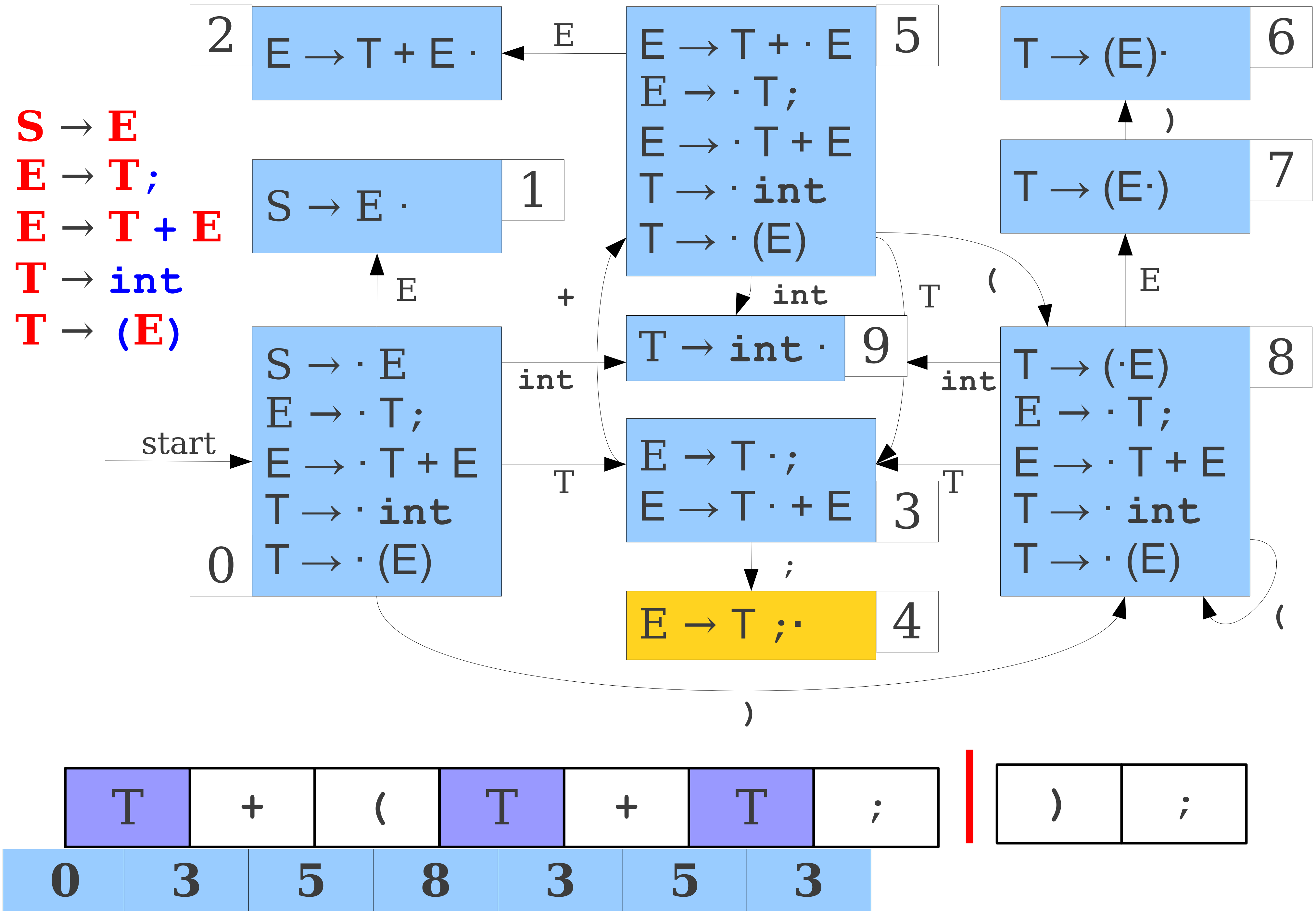

| T | + | ( | T | + | T | ; |
|---|---|---|---|---|---|---|

| ) | ; |
|---|---|

| 0 | 3 | 5 | 8 | 3 | 5 | 3 |
|---|---|---|---|---|---|---|

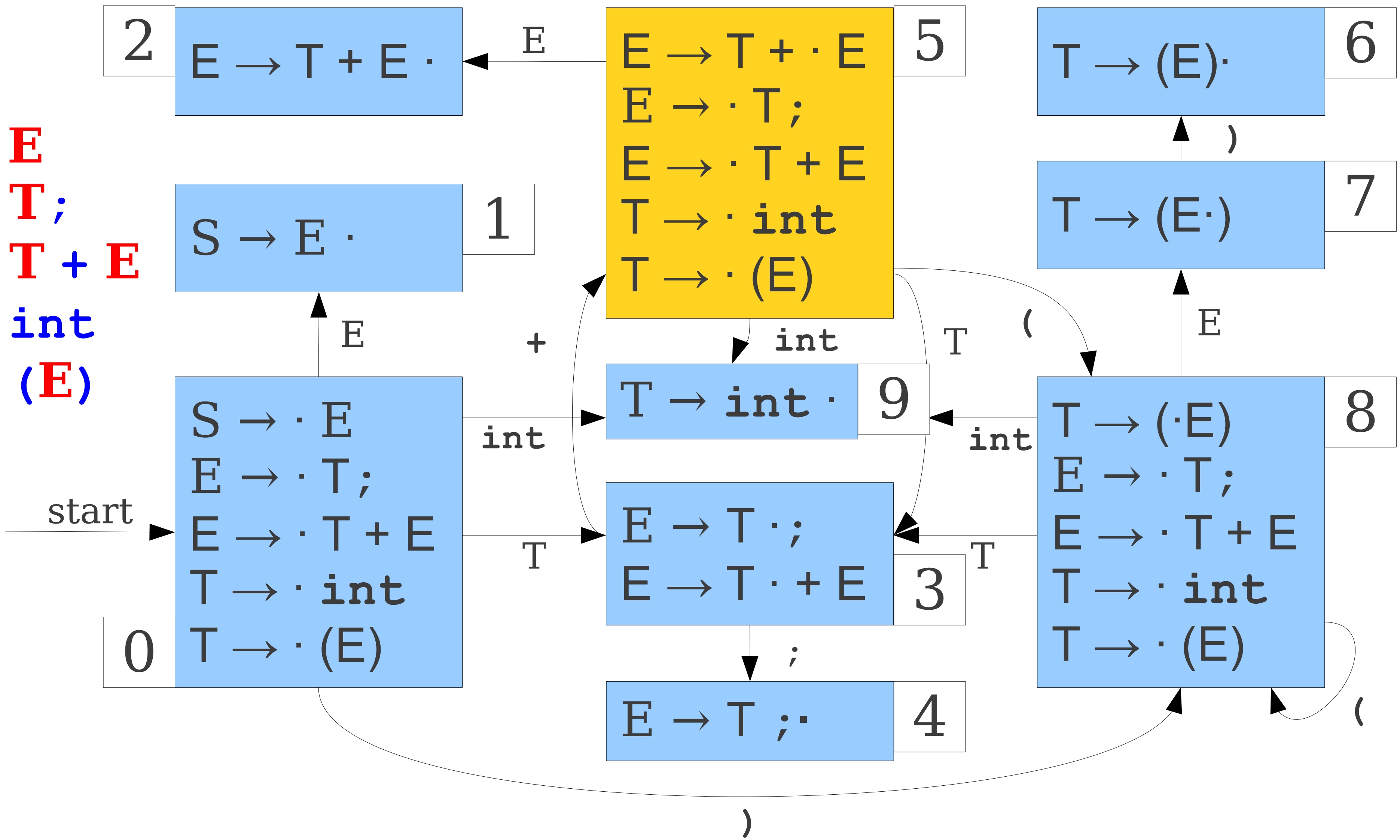
# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



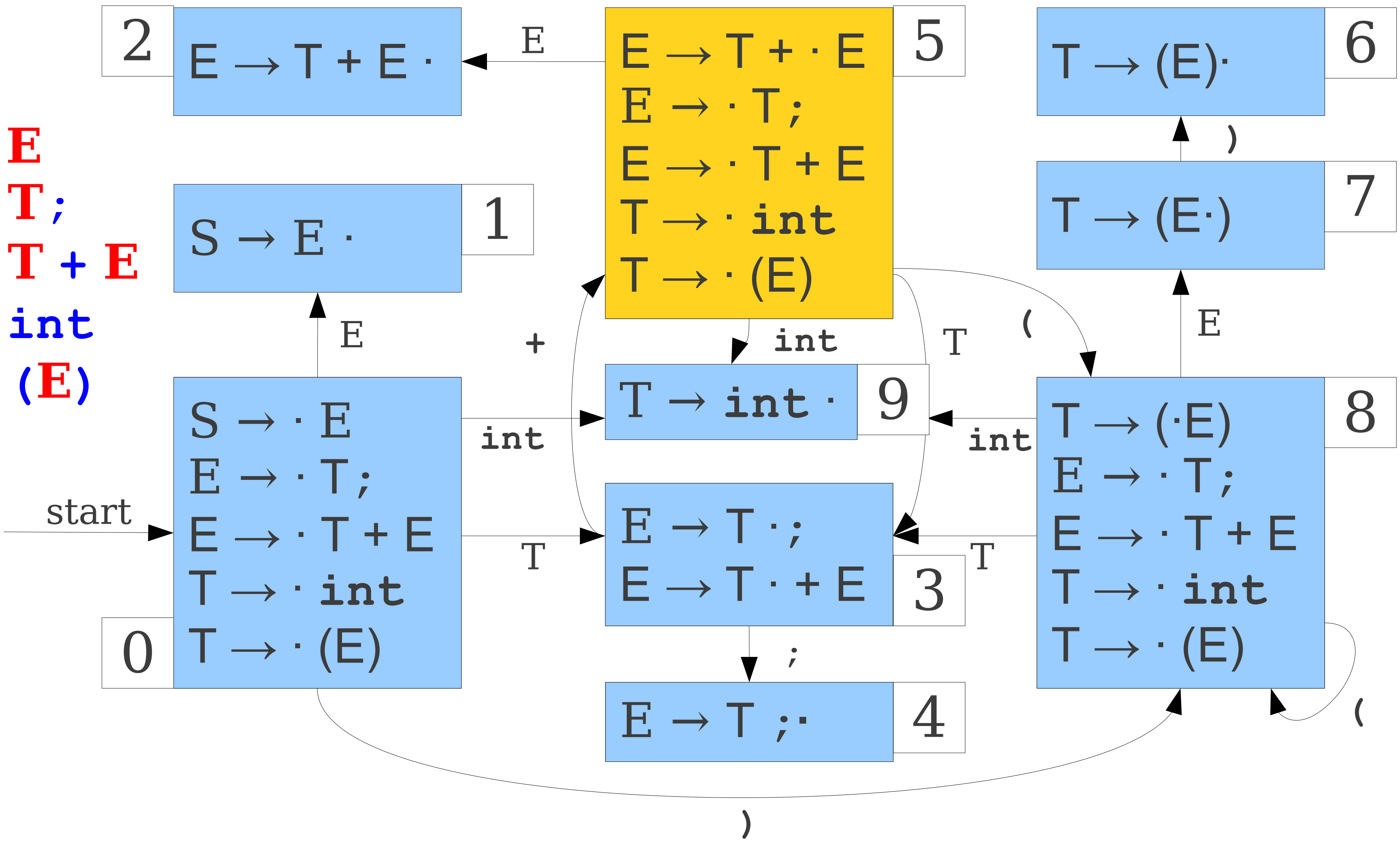

| | | | | |
|---|---|---|---|---|
| T | + | ( | T | + |

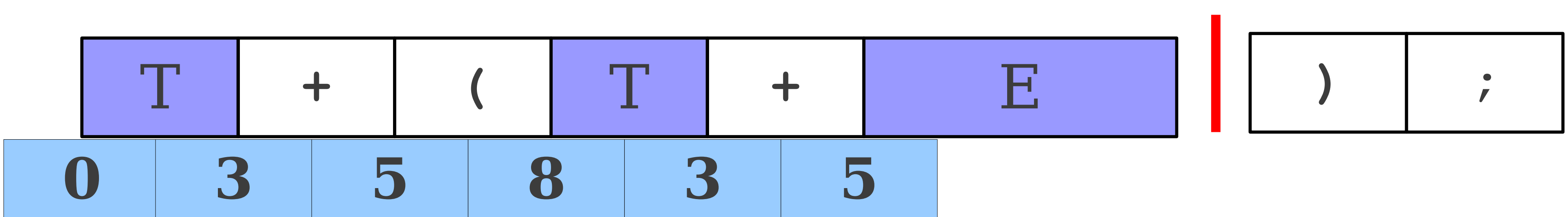
| | | | | | |
|---|---|---|---|---|---|
| 0 | 3 | 5 | 8 | 3 | 5 |

| | |
|---|---|
| ) | ; |

# LR(0) Parsing

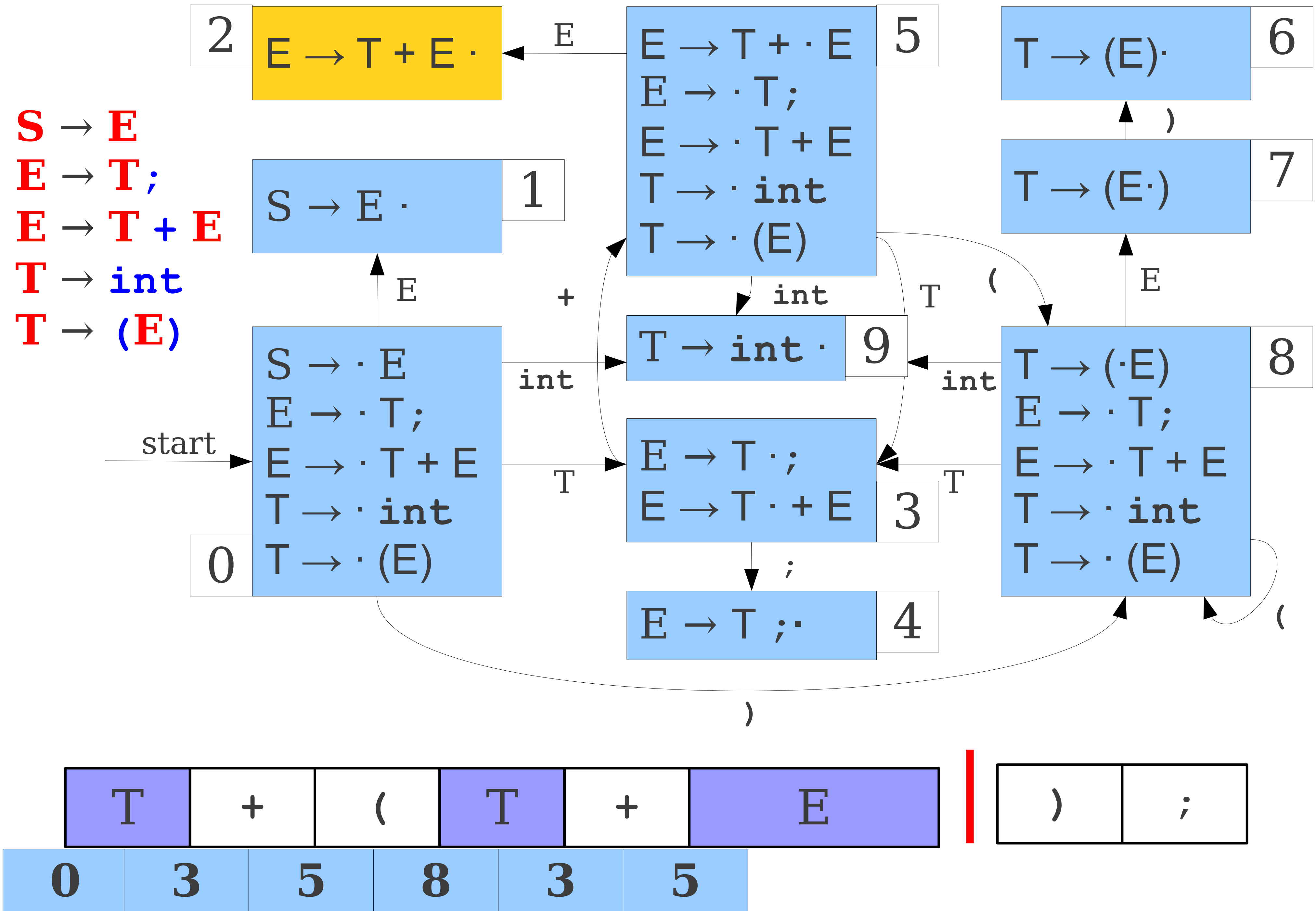
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



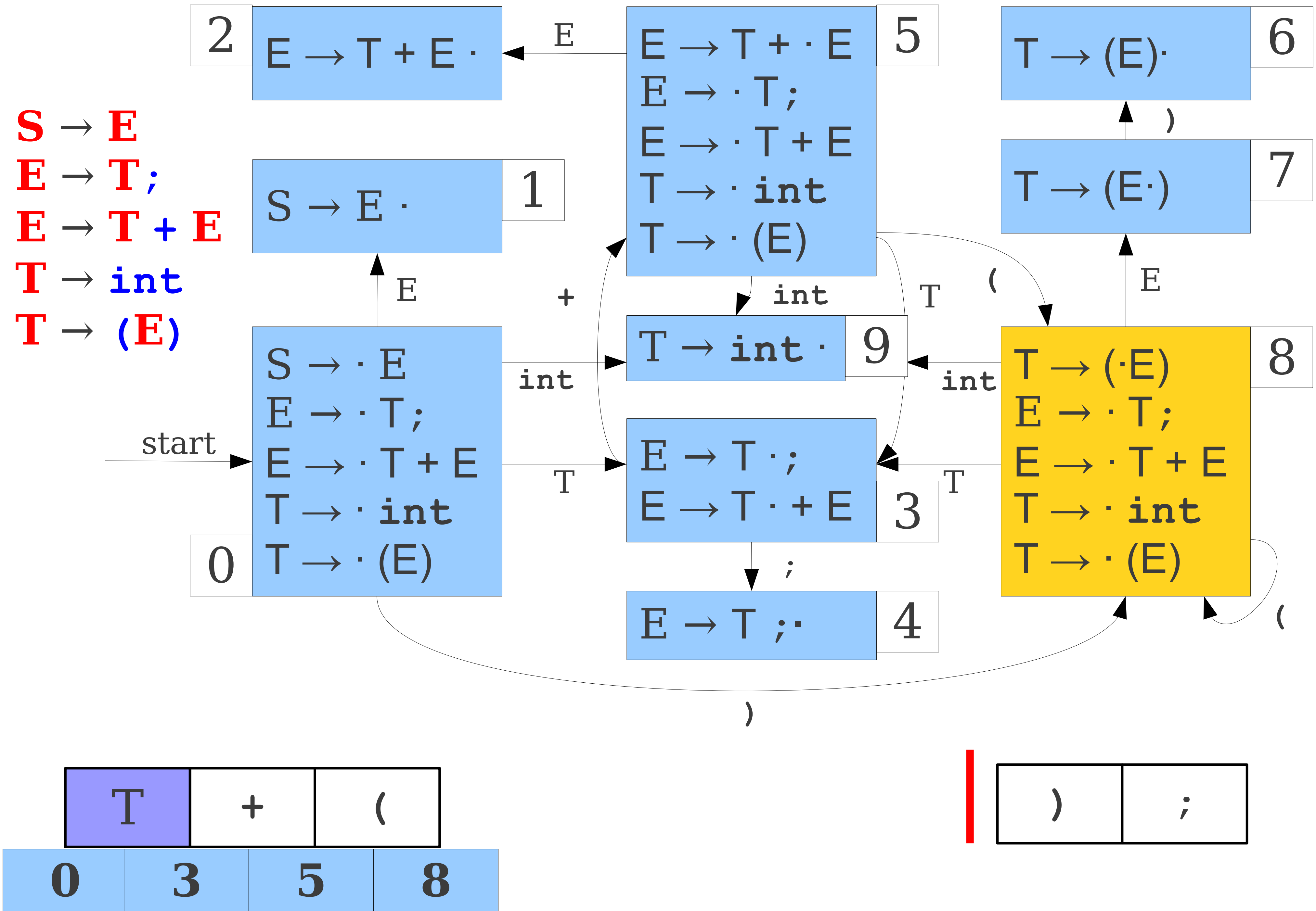
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



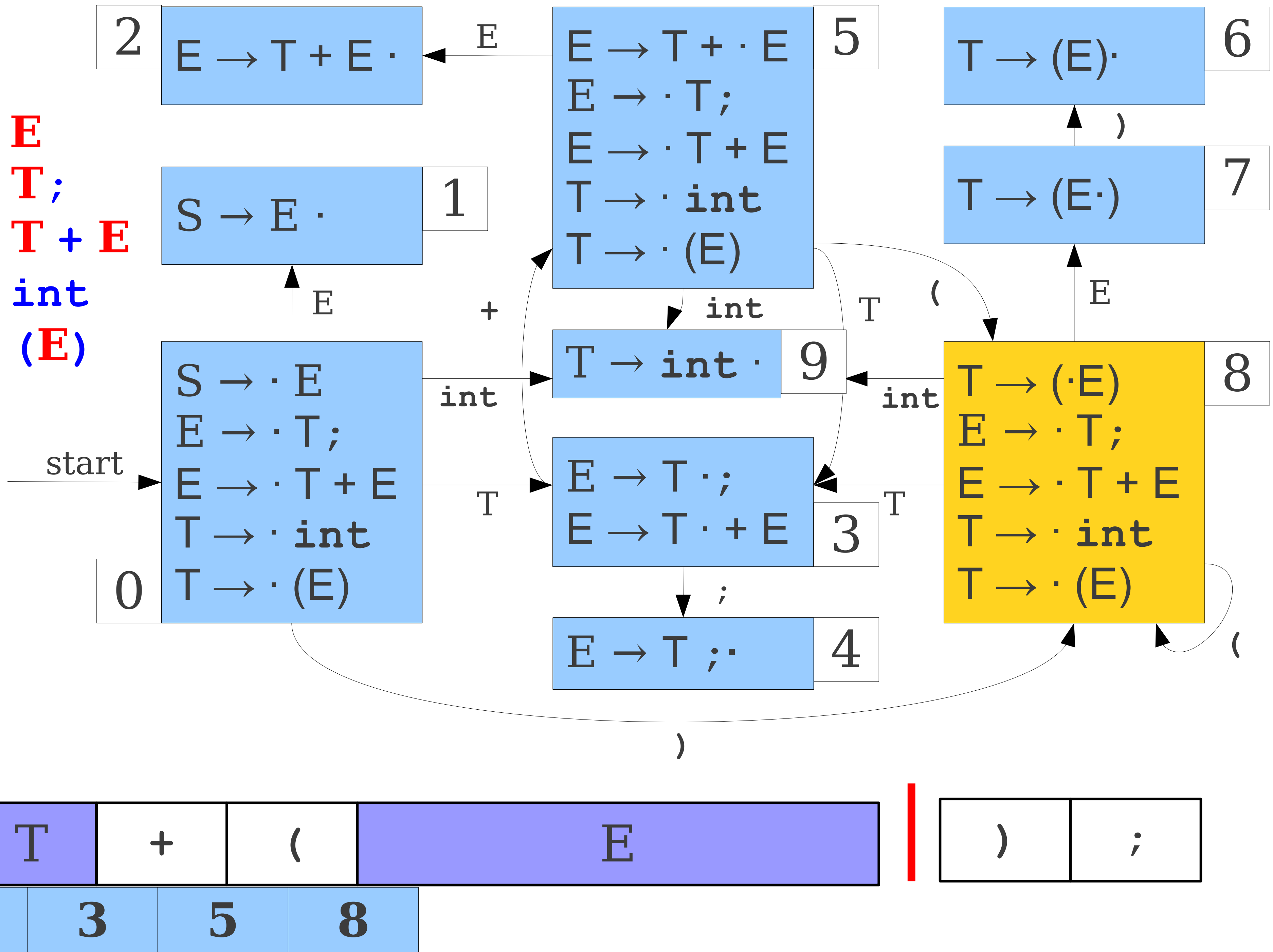
# LR(0) Parsing

S → E
E → T;
E → T + E
T → int
T → (E)



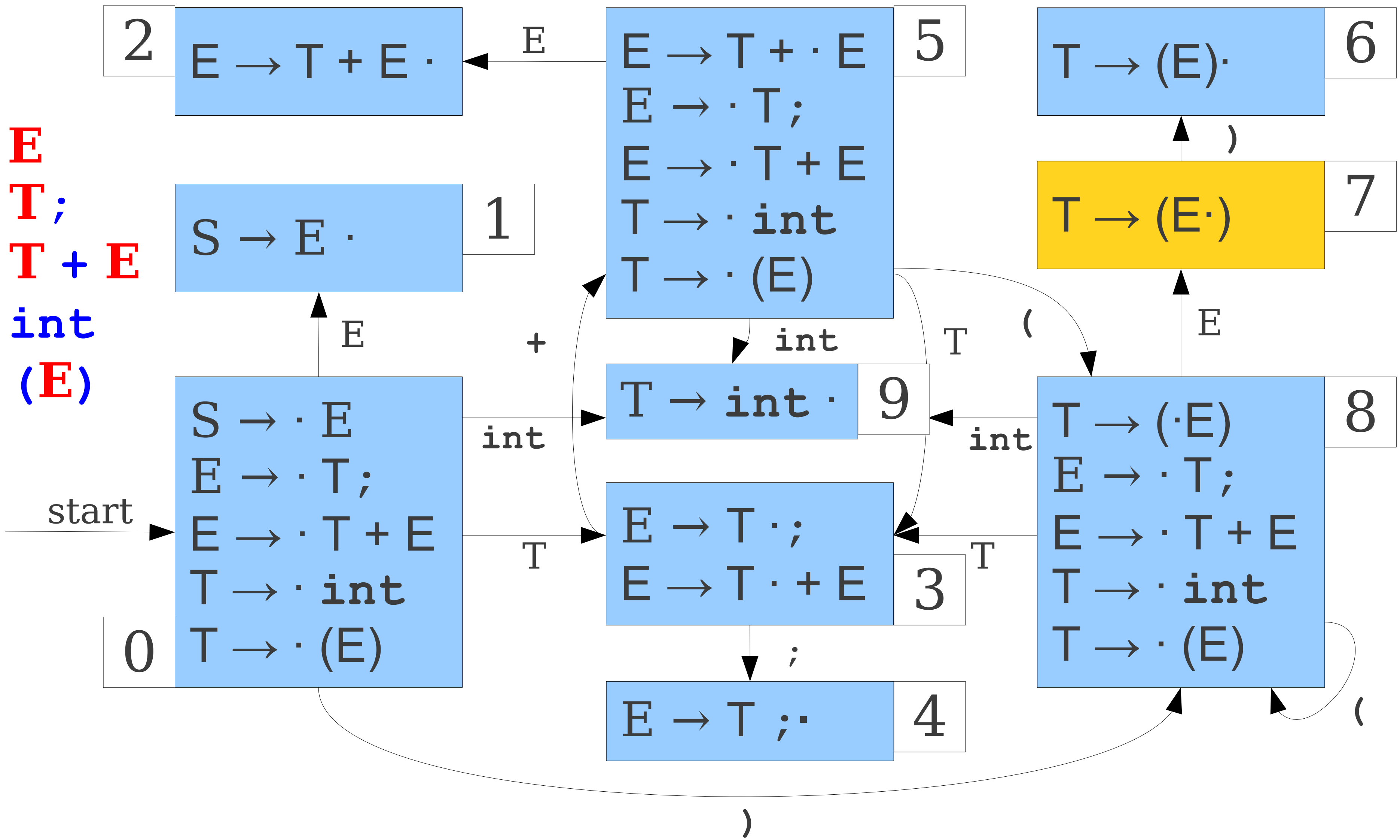
# LR(0) Parsing

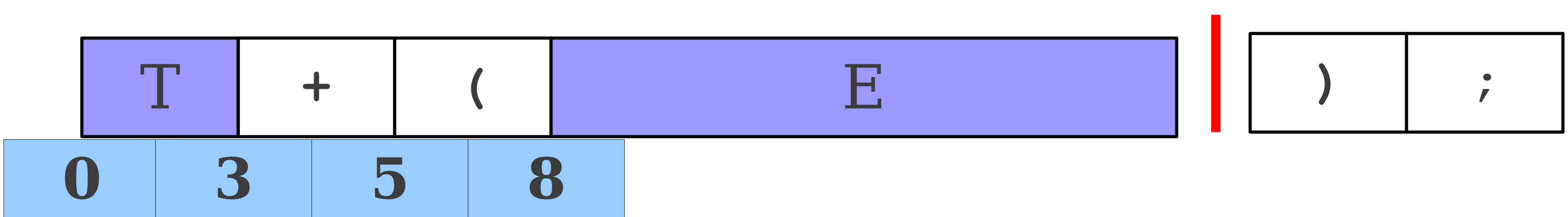
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

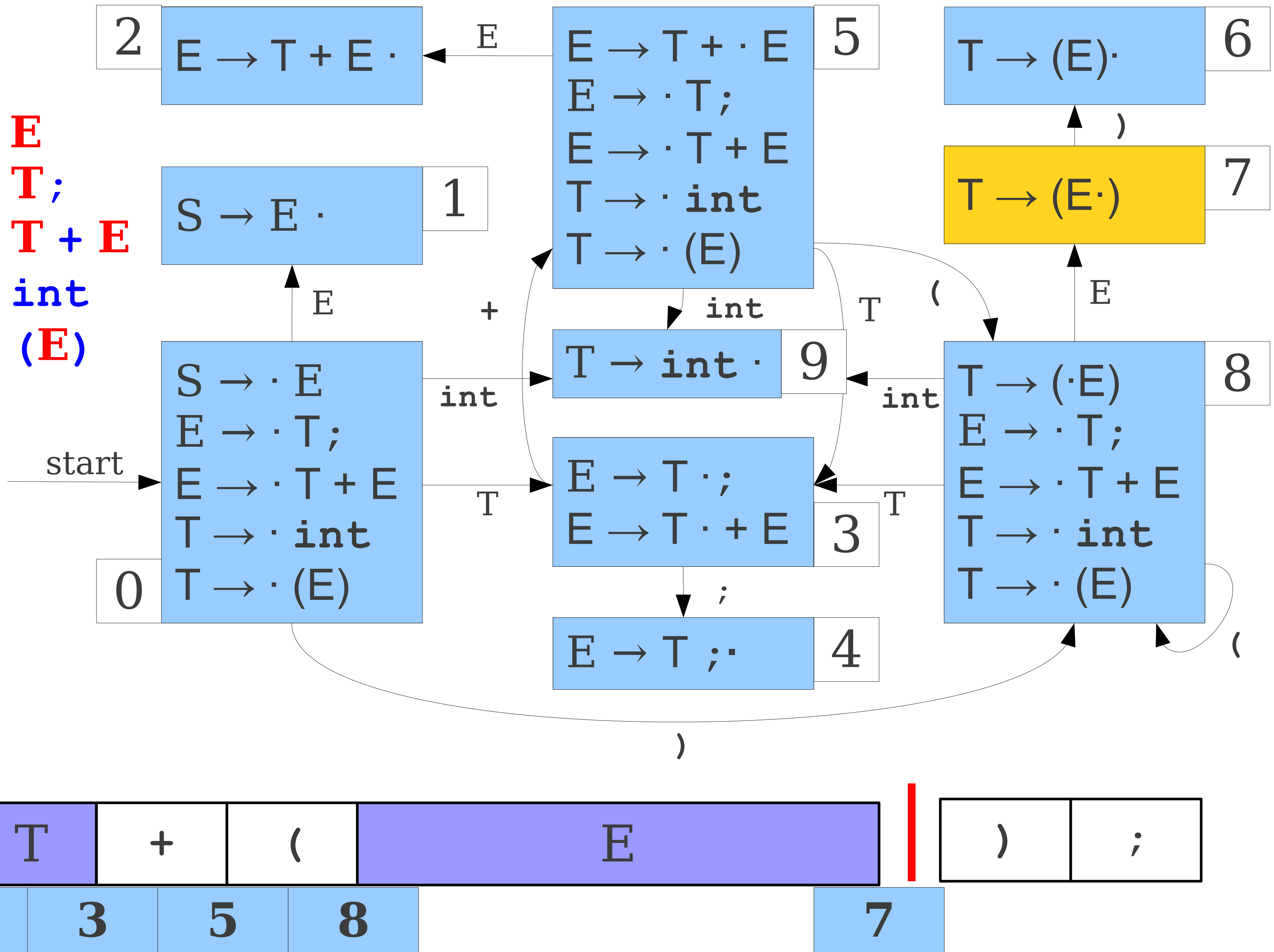
S → E
E → T;
E → T + E
T → int
T → (E)



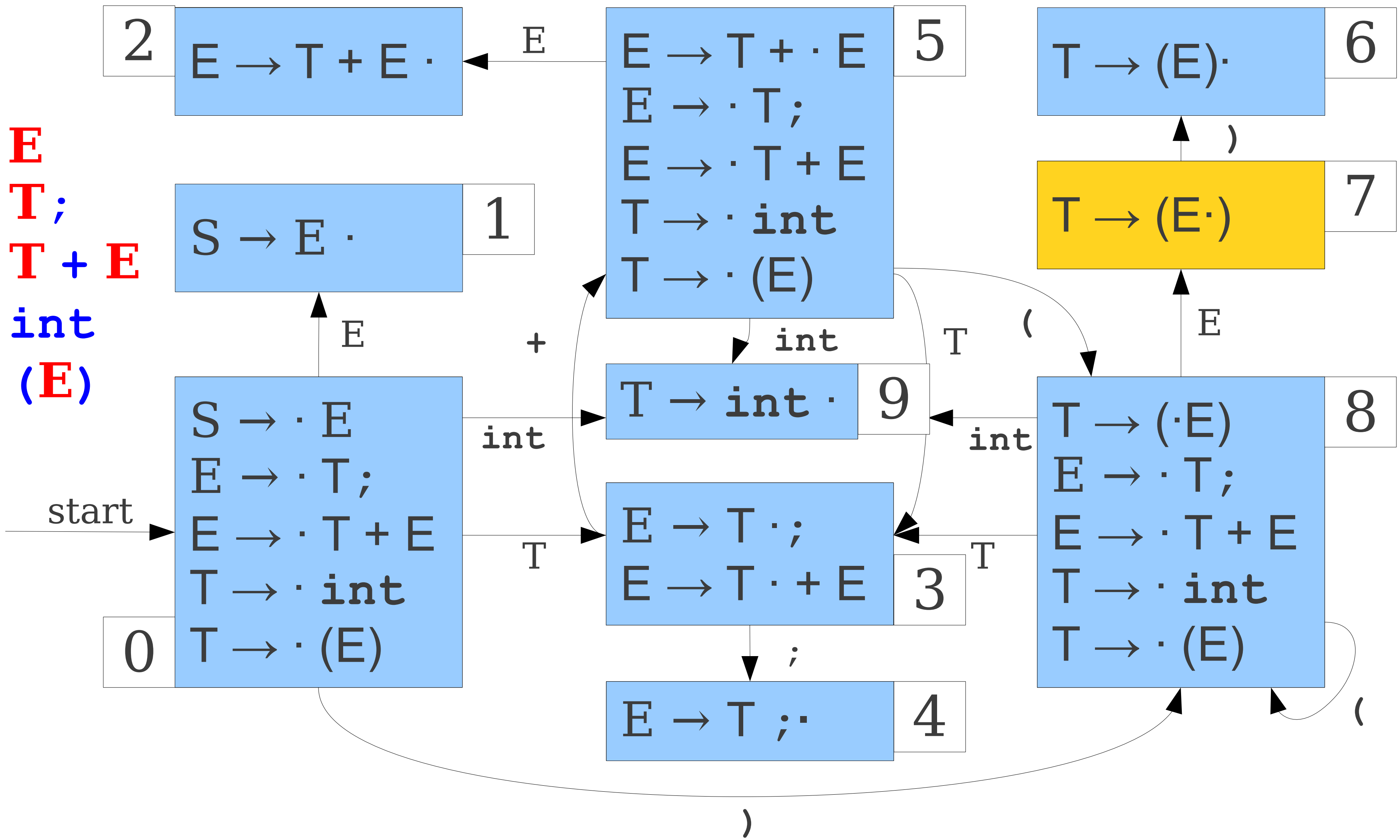
# LR(0) Parsing

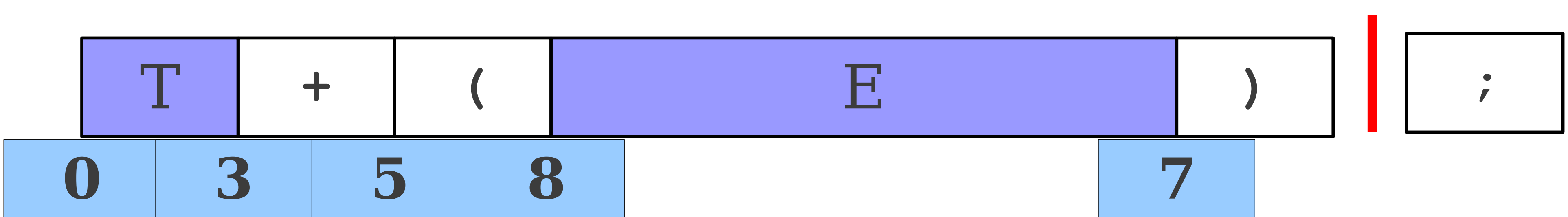
S → E
E → T;
E → T + E
T → int
T → (E)



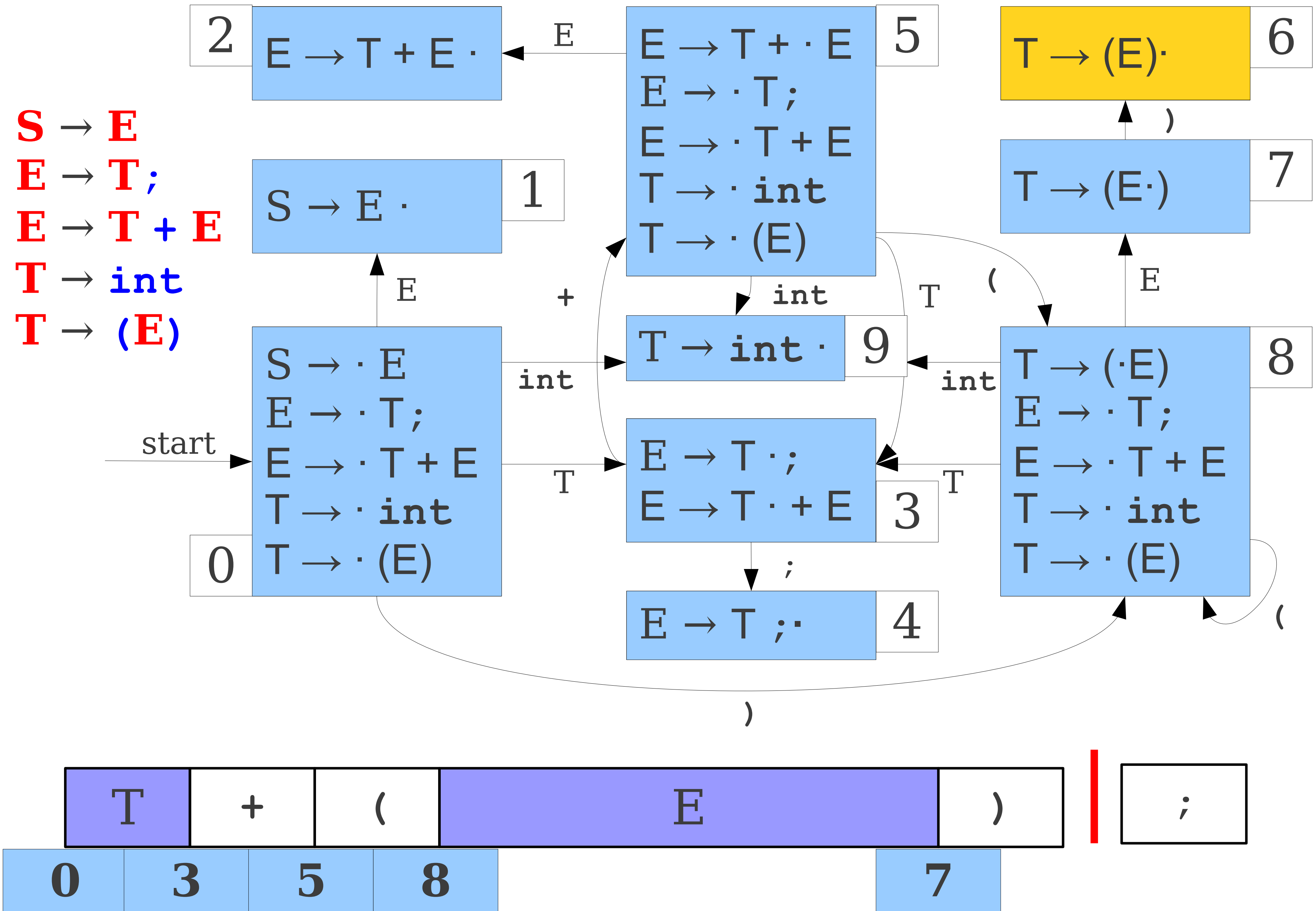
# LR(0) Parsing

**S → E**
**E → T;**
**E → T + E**
**T → `int`**
**T → (E)**



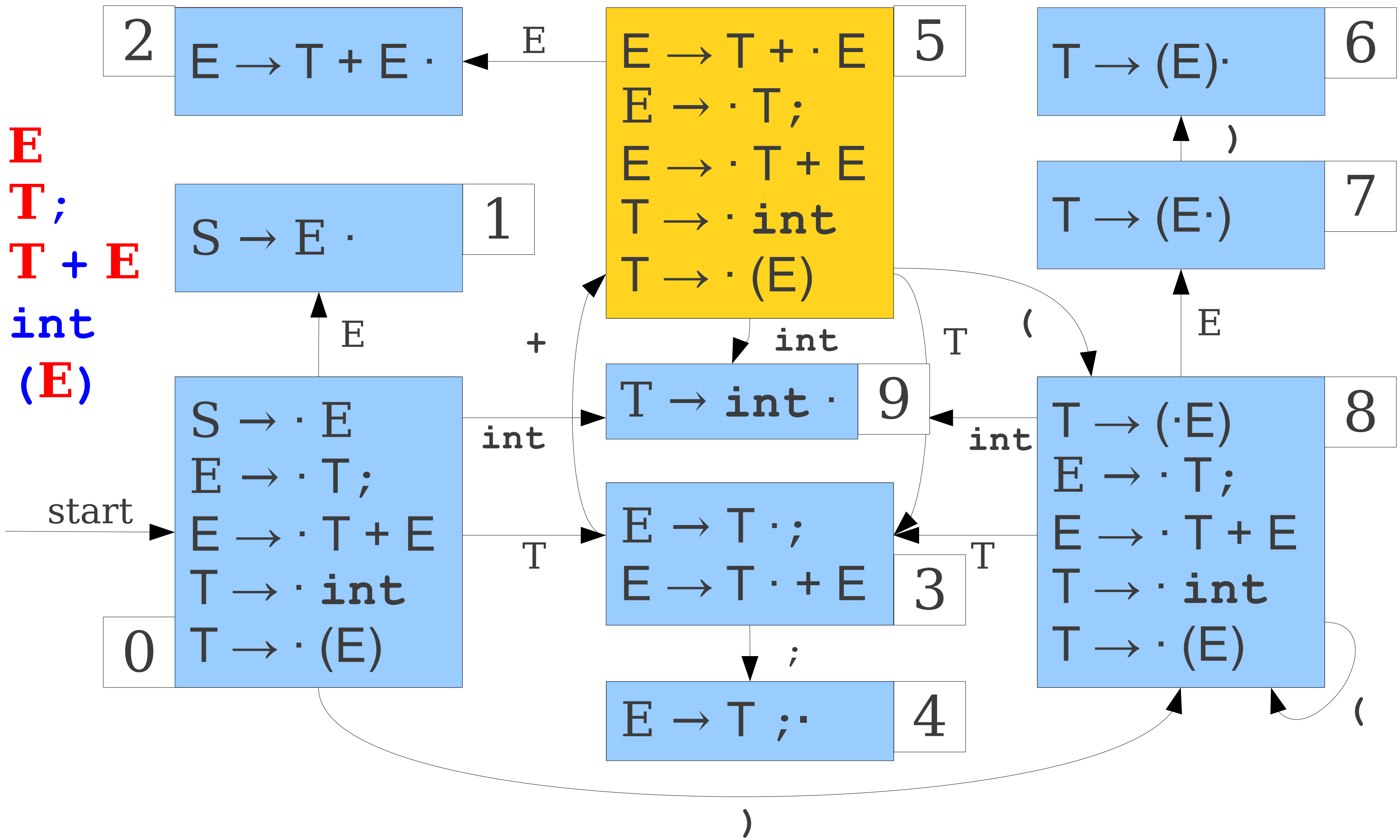



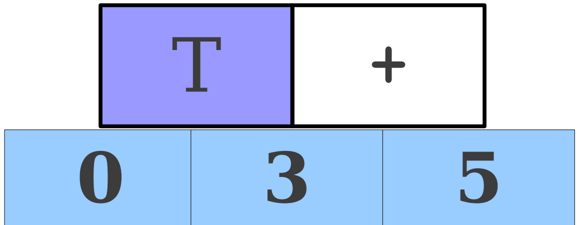
# LR(0) Parsing

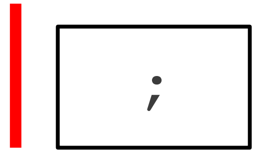

S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
1
S → E ·
7
T → (E·)
9
T → int ·
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
3
E → T ·;
E → T · + E
4
E → T ;·
start
E
+
int
T
(
)
;
T + ( E ) ;
0 3 5 8 7

# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → **(E)**

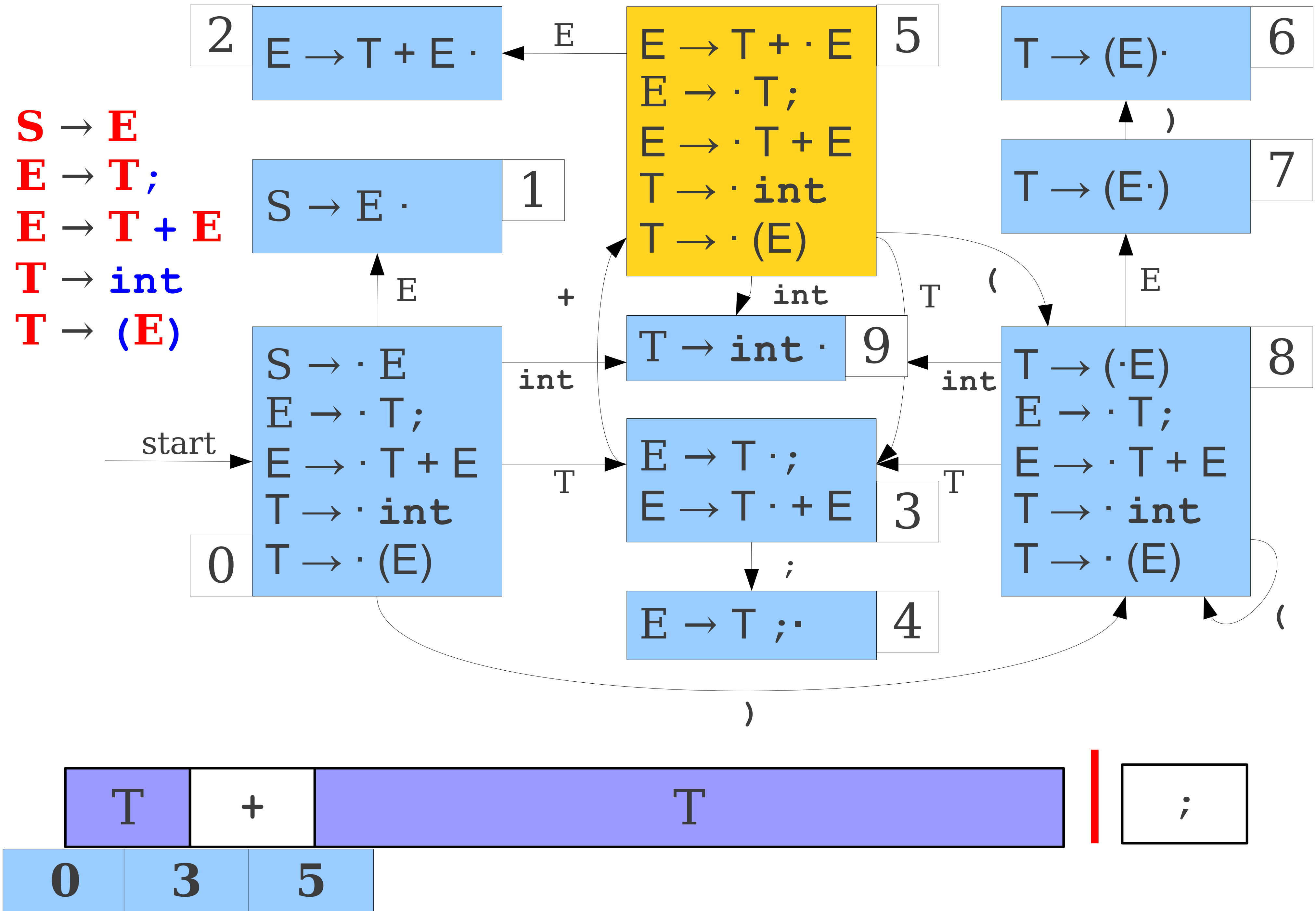


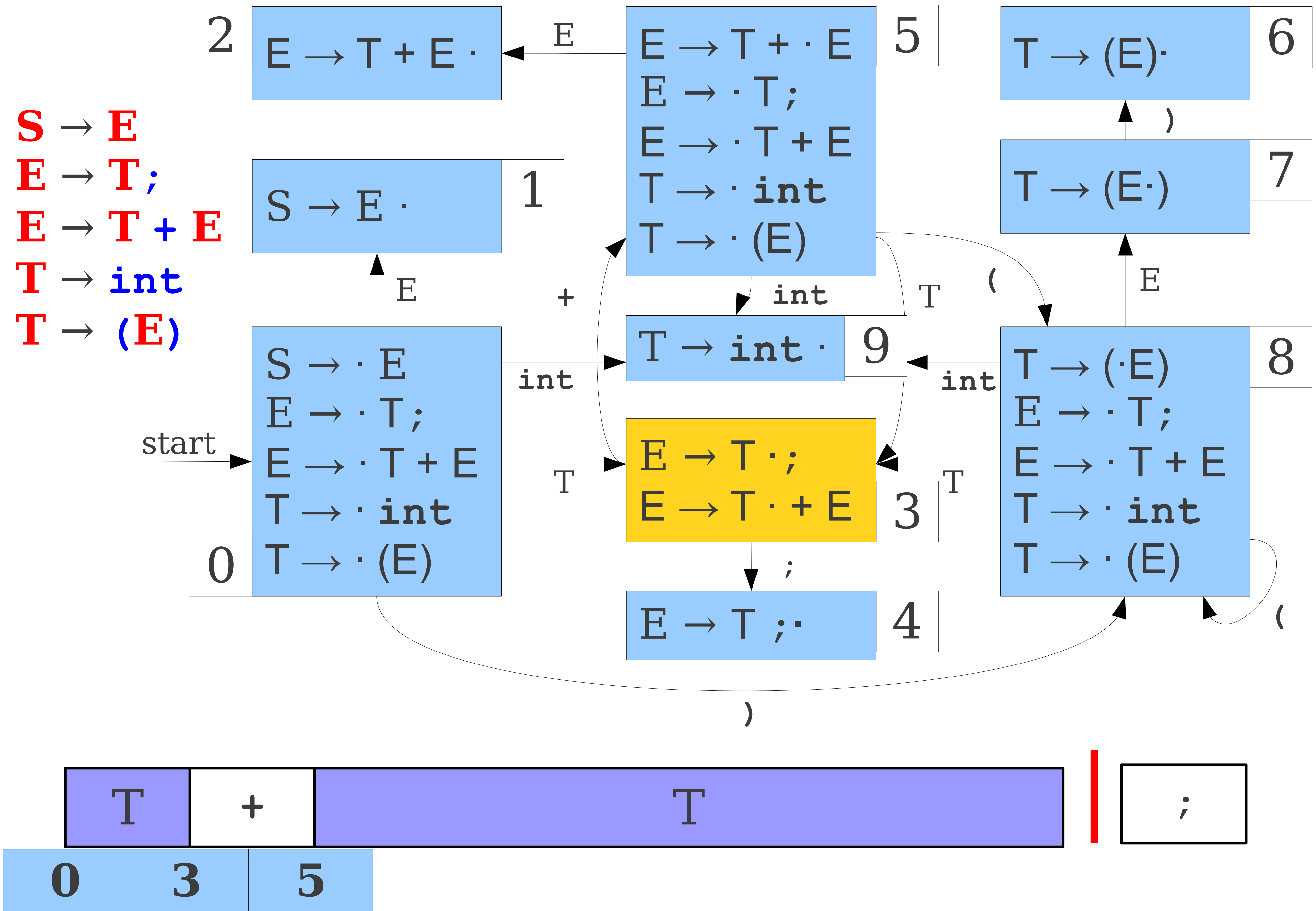



# LR(0) Parsing

**S** → **E**
**E** → **T**;
**E** → **T** + **E**
**T** → int
**T** → (**E**)



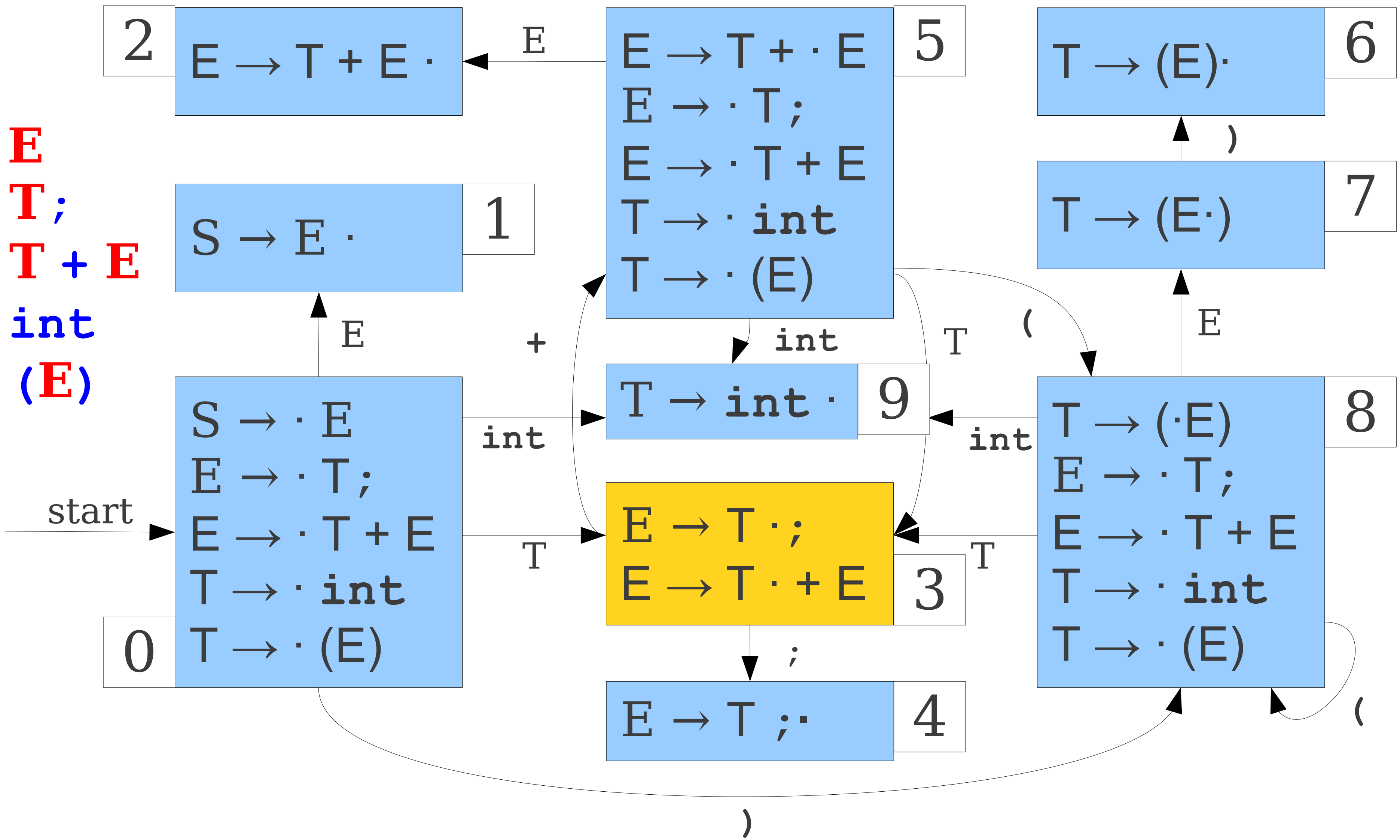
# LR(0) Parsing

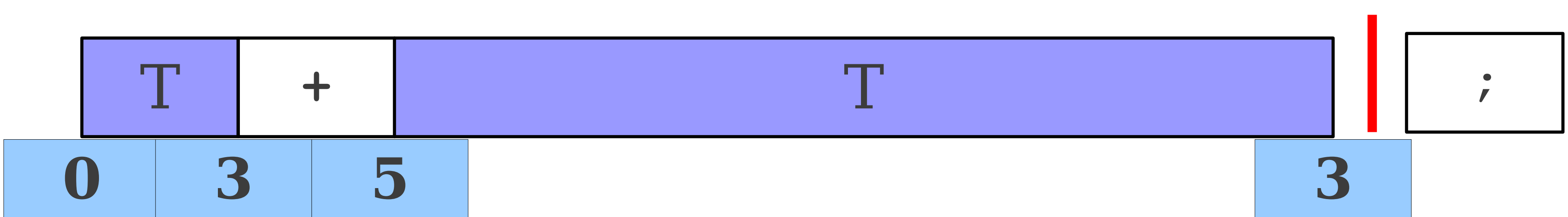


S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
E
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
)
7
T → (E·)
1
S → E ·
E
+
int
T
(
E
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
int
T
9
T → int ·
int
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
3
E → T ·;
E → T · + E
T
;
4
E → T ;·
(
)
T
+
T
;
0
3
5

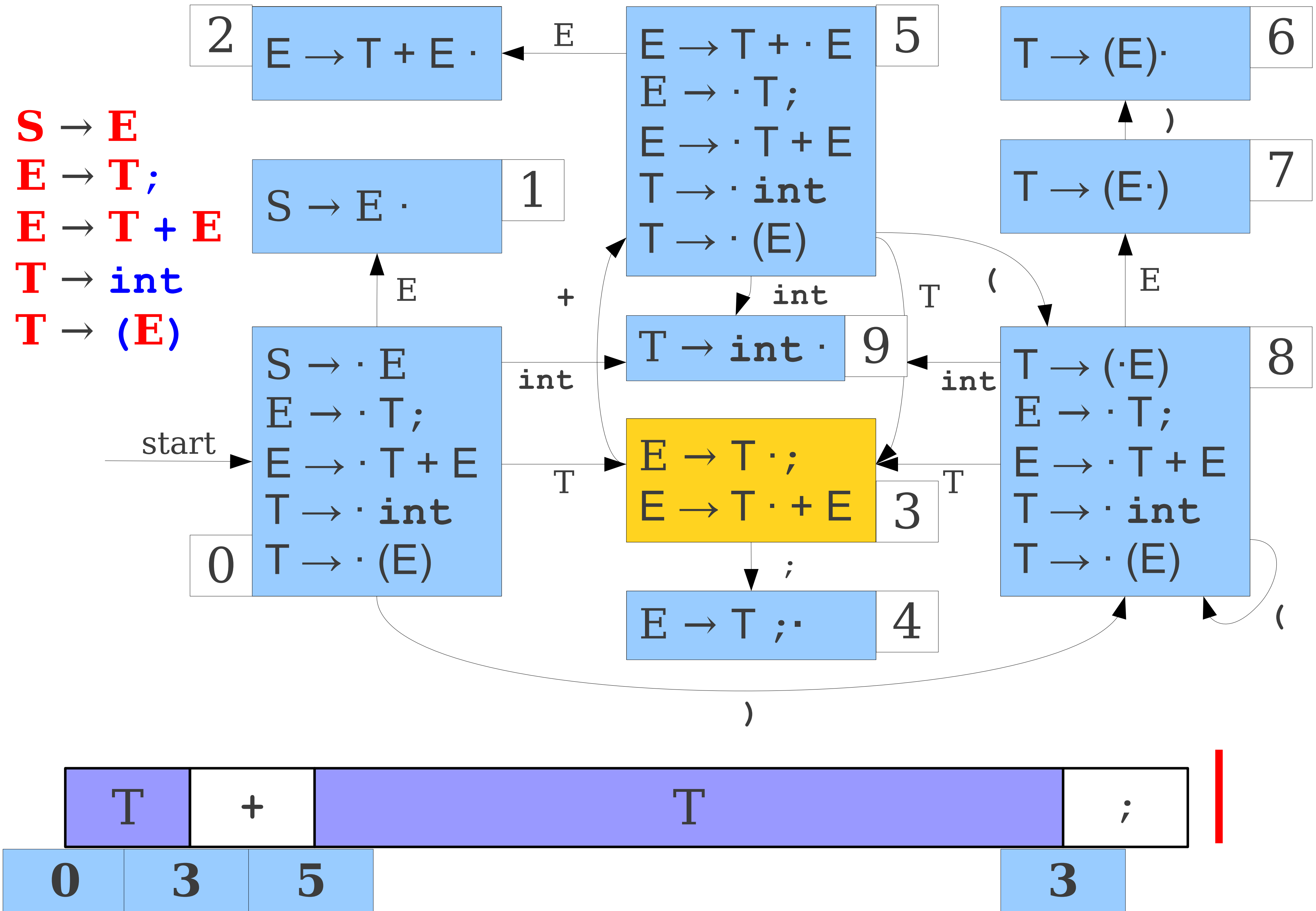
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → **(E)**



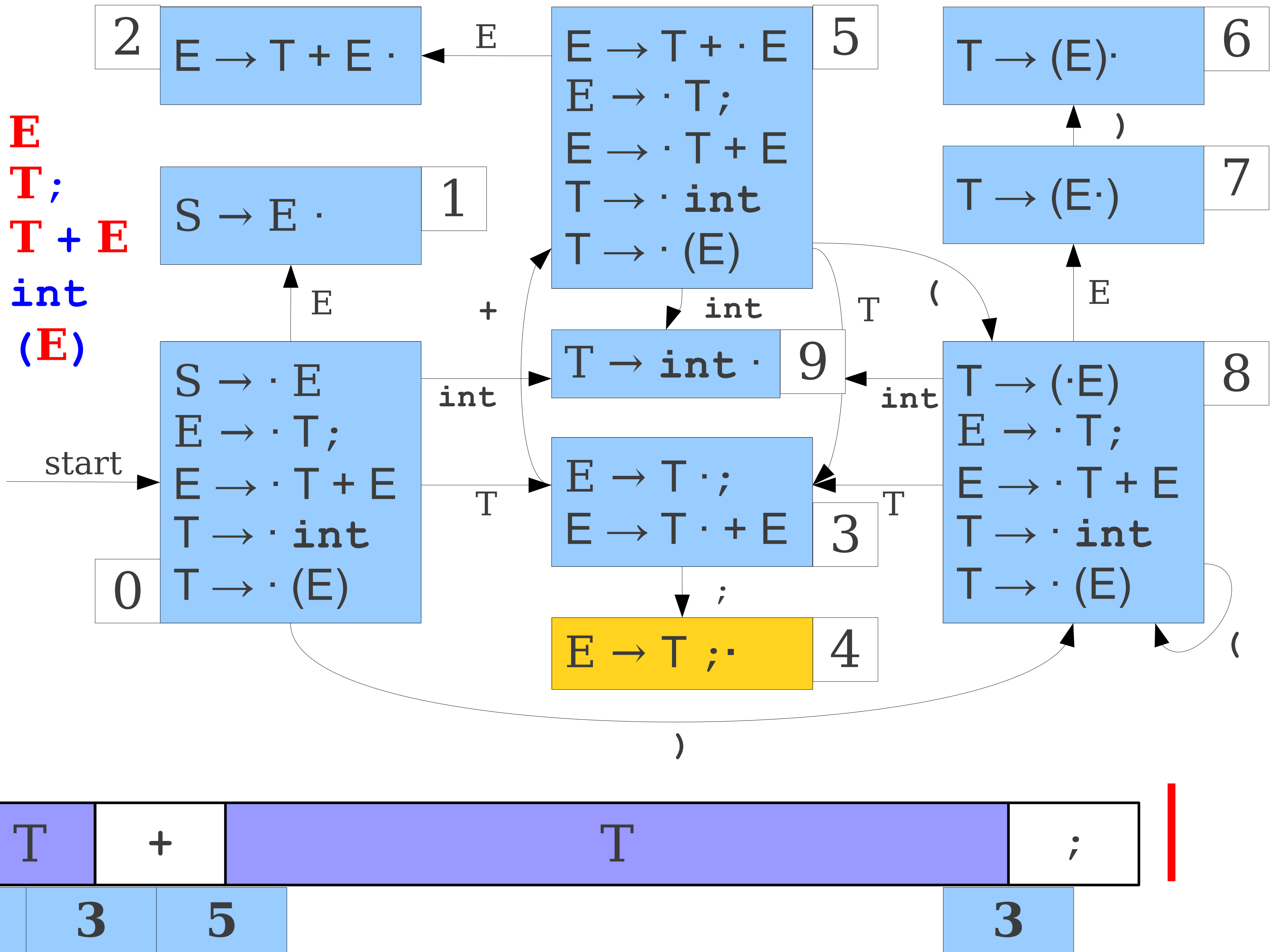
# LR(0) Parsing



S → E
E → T;
E → T + E
T → int
T → (E)
2
E → T + E ·
5
E → T + · E
E → · T;
E → · T + E
T → · int
T → · (E)
6
T → (E)·
)
7
T → (E·)
1
S → E ·
E
+
int
T
(
E
9
T → int ·
int
int
start
0
S → · E
E → · T;
E → · T + E
T → · int
T → · (E)
T
3
E → T ·;
E → T · + E
T
8
T → (·E)
E → · T;
E → · T + E
T → · int
T → · (E)
;
4
E → T ;·
(
)
T
+
T
;
0
3
5
3

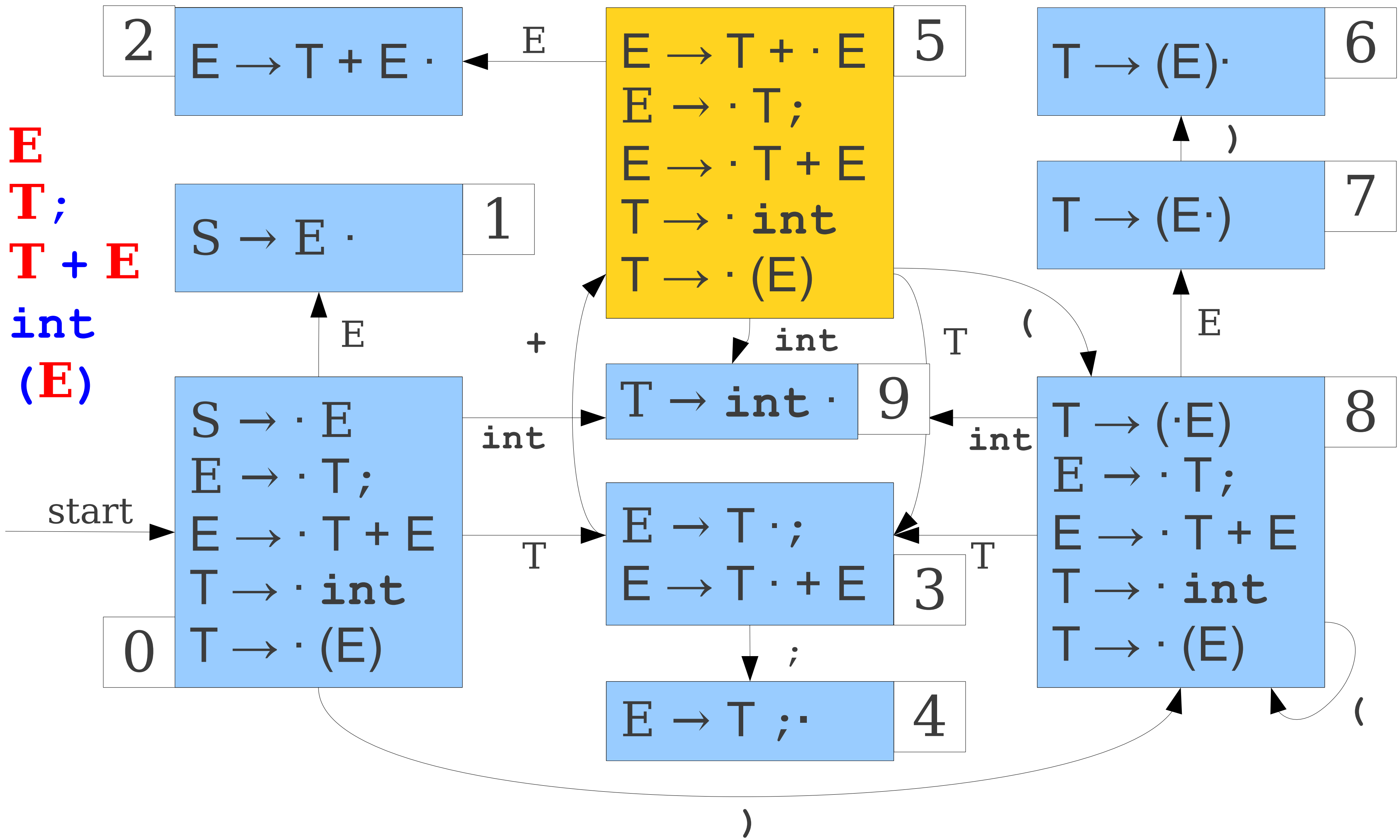
# LR(0) Parsing

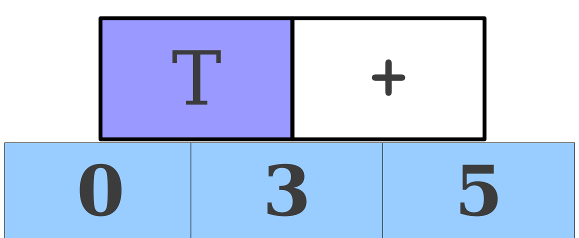
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

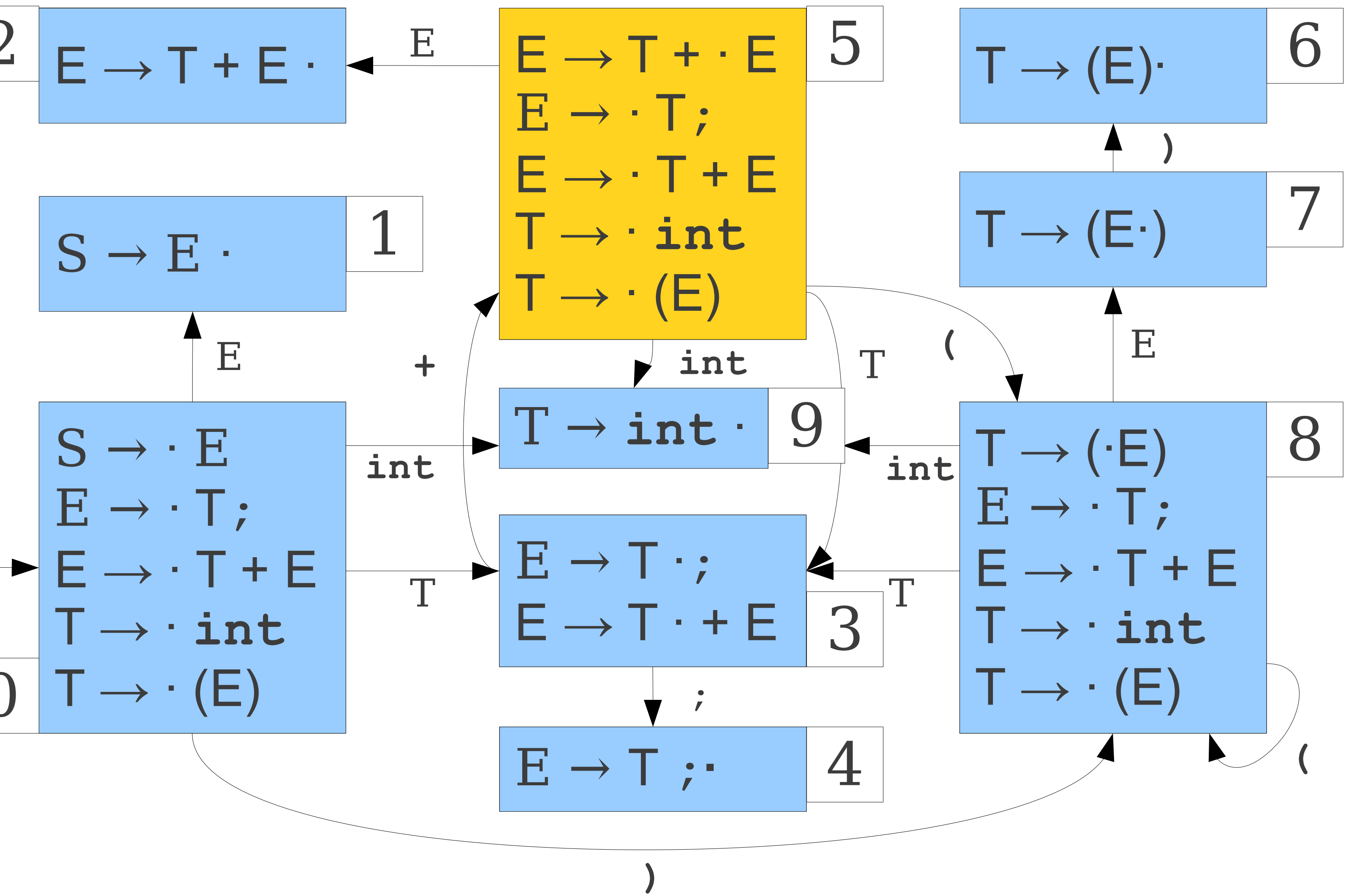
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )



# LR(0) Parsing

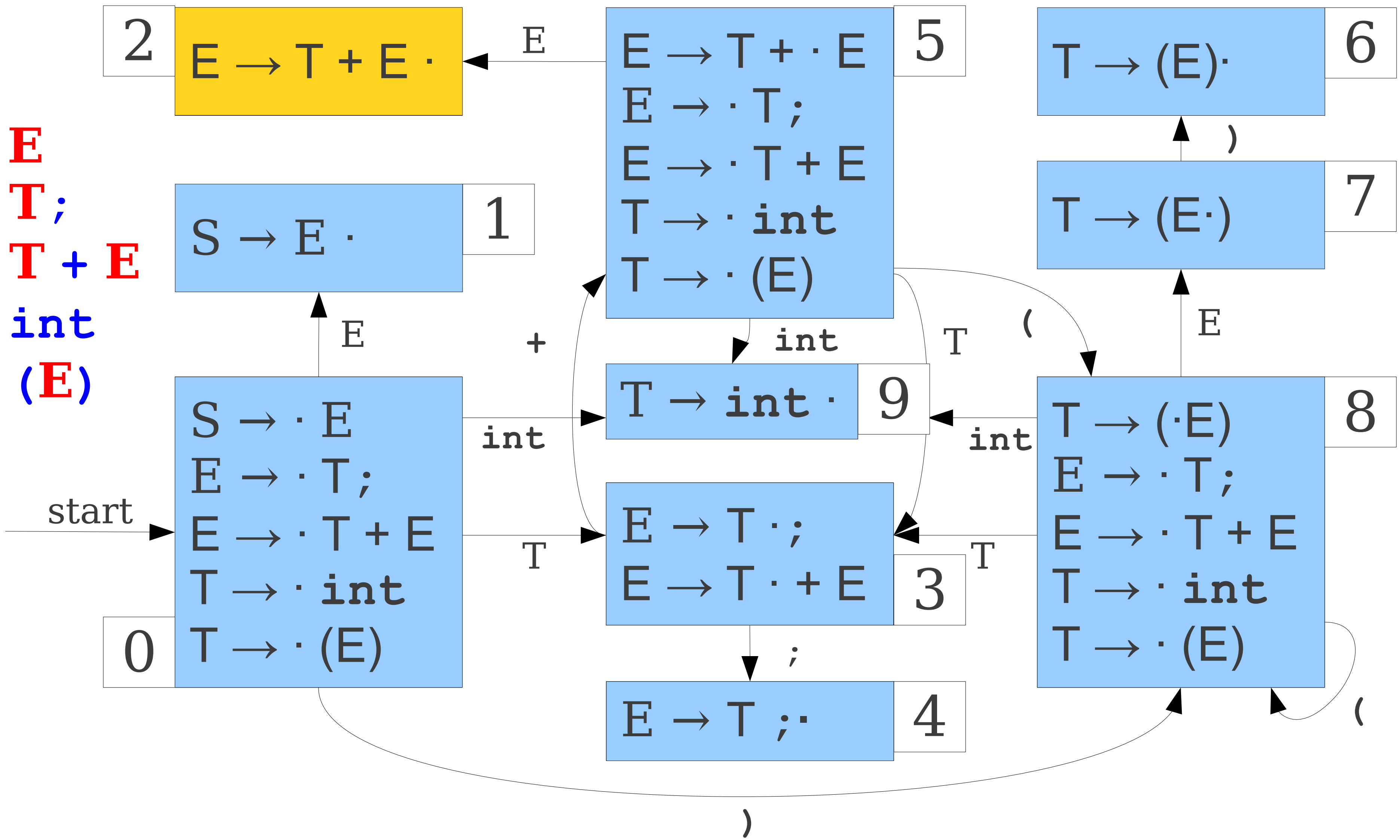
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → ( **E** )




| T | + | E |
|---|---|---|
| **0** | **3** | **5** |

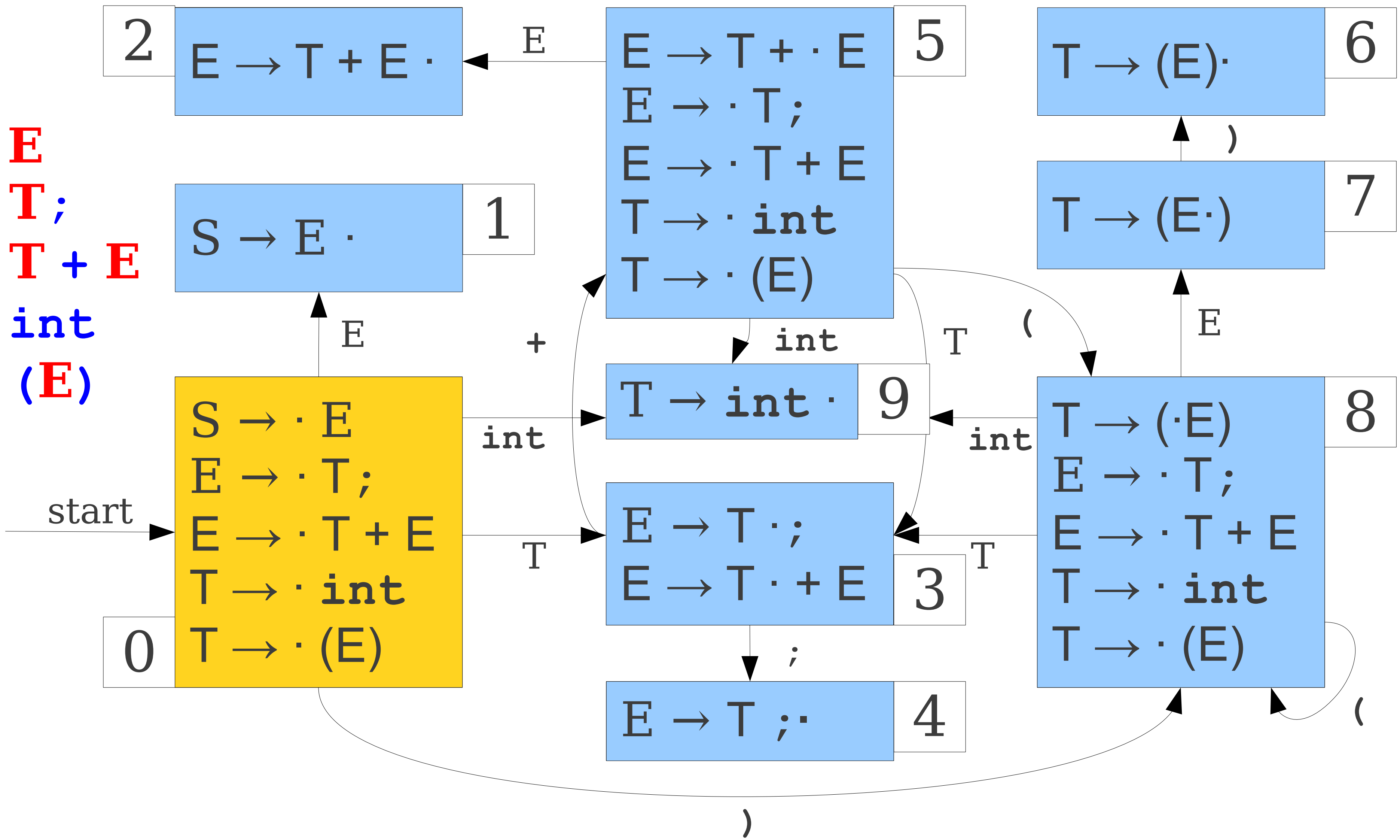
# LR(0) Parsing

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → **int**
**T** → ( **E** )




# LR(0) Parsing

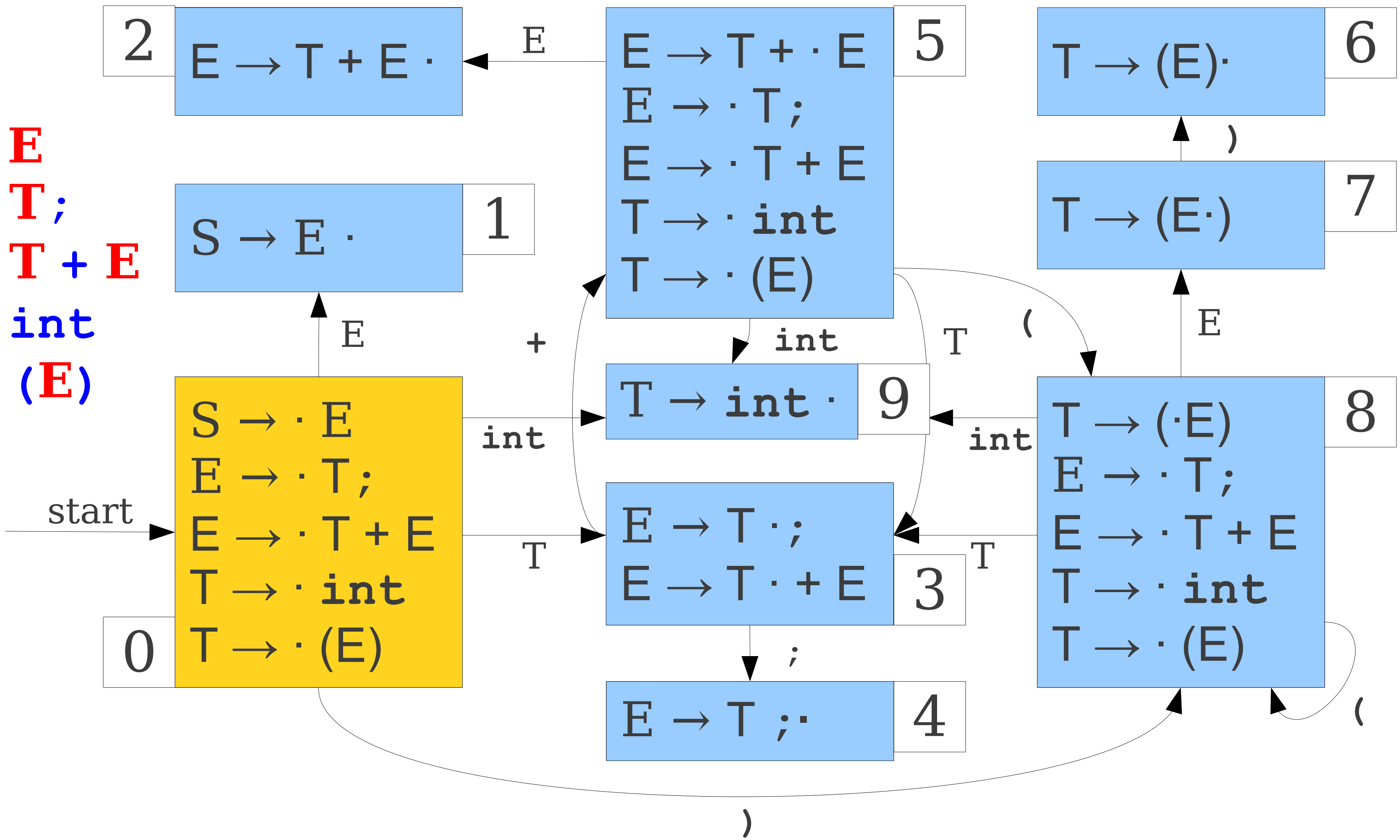
**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → `int`
**T** → `(`**E**`)`

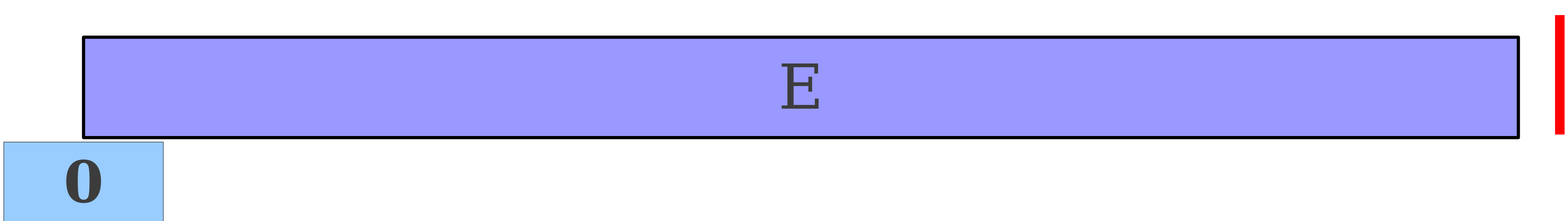



**0**

# LR(0) Parsing

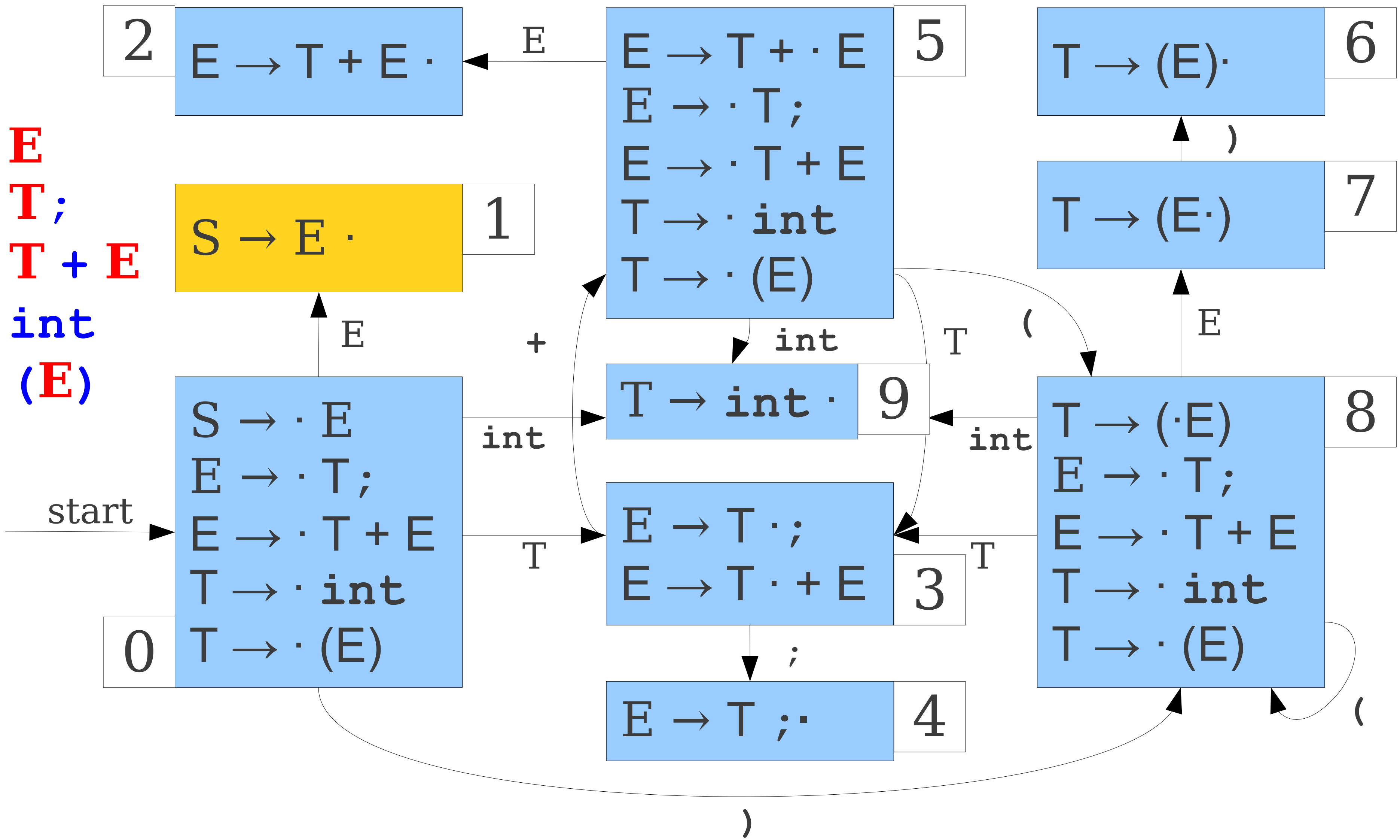
**S** → **E**
**E** → **T**;
**E** → **T** + **E**
**T** → `int`
**T** → (**E**)

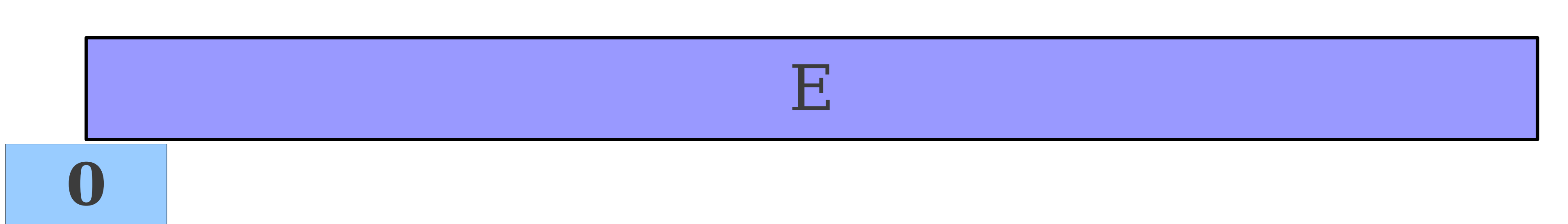


# LR(0) Parsing

**S** → **E**
**E** → **T**;
**E** → **T** + **E**
**T** → **int**
**T** → **(E)**



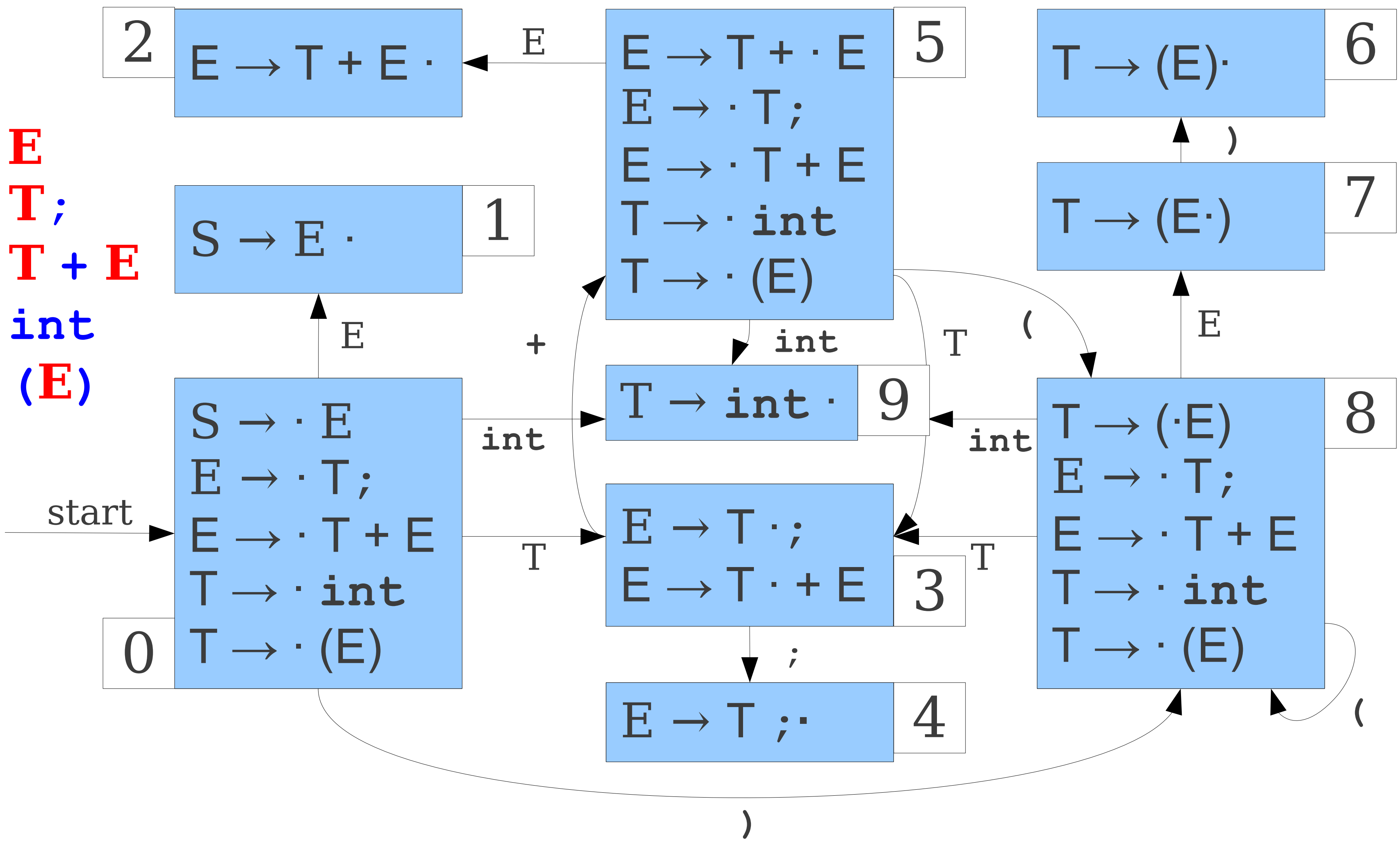



# Representing the Automaton

- LR(0) parsers are usually represented via two tables: an **action** table and a **goto** table.
- The **action** table maps each state to an action:
  - **shift**, which shifts the next terminal, and
  - **reduce A → *ω***, which performs reduction **A → *ω***.
  - Any state of the form **A → *ω*** · does that reduction; everything else shifts.
- The **goto** table maps state/symbol pairs to a next state.
  - This is just the transition table for the automaton.

# Building LR(0) Tables

S → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



# LR(0) Tables

| | int | + | ; | ( | ) | E | T | Action |
|---|---|---|---|---|---|---|---|---|
| 0 | 9 | | | 8 | | 1 | 3 | Shift |
| 1 | | | | | | | | Accept |
| 2 | | | | | | | | Reduce **E** → **T + E** |
| 3 | | 5 | 4 | | | | | Shift |
| 4 | | | | | | | | Reduce **E** → **T ;** |
| 5 | 9 | | | 8 | | 2 | 3 | Shift |
| 6 | | | | | | | | Reduce **T** → **(E)** |
| 7 | | | | | 6 | | | Shift |
| 8 | 9 | | | 8 | | 7 | 3 | Shift |
| 9 | | | | | | | | Reduce **T** → **int** |

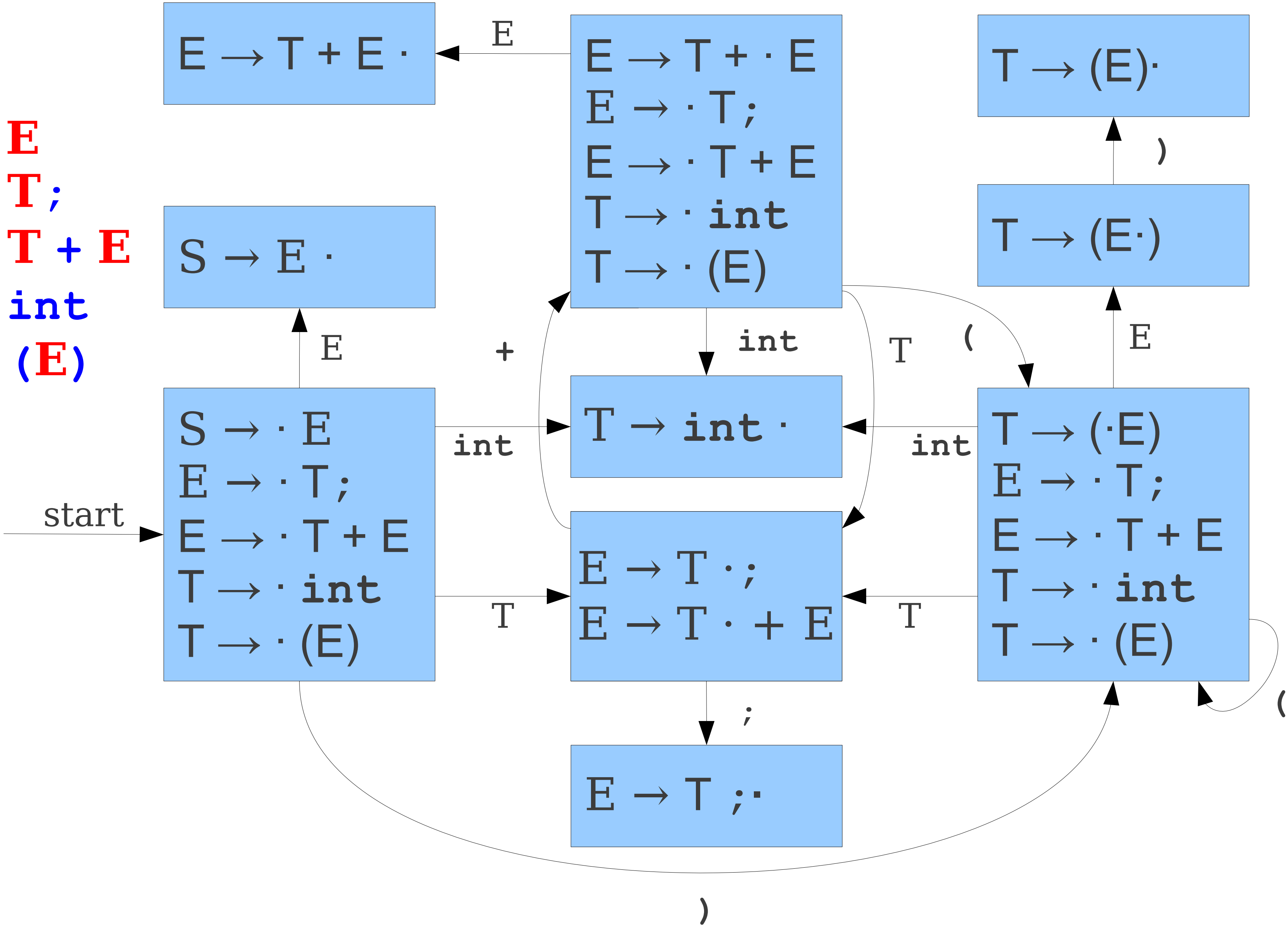
# The LR(0) Algorithm

- Maintain a stack of (symbol, state) pairs, which is initially (`?`, 1) for some dummy symbol `?`.
- While the stack is not empty:
  - Let **state** be the top state.
  - If **action[state]** is **shift**:
    - Let `t` be the next symbol in the input.
    - Push (`t`, **goto[state, `t`]**) atop the stack.
  - If **action[state]** is **reduce A** → ***ω***:
    - Remove |***ω***| symbols from the top of the stack.
    - Let **top-state** be the state on top of the stack.
    - Push (**A**, **goto[top-state, A]**) atop the stack.
  - Otherwise, report an error.

# The Limits of LR(0)

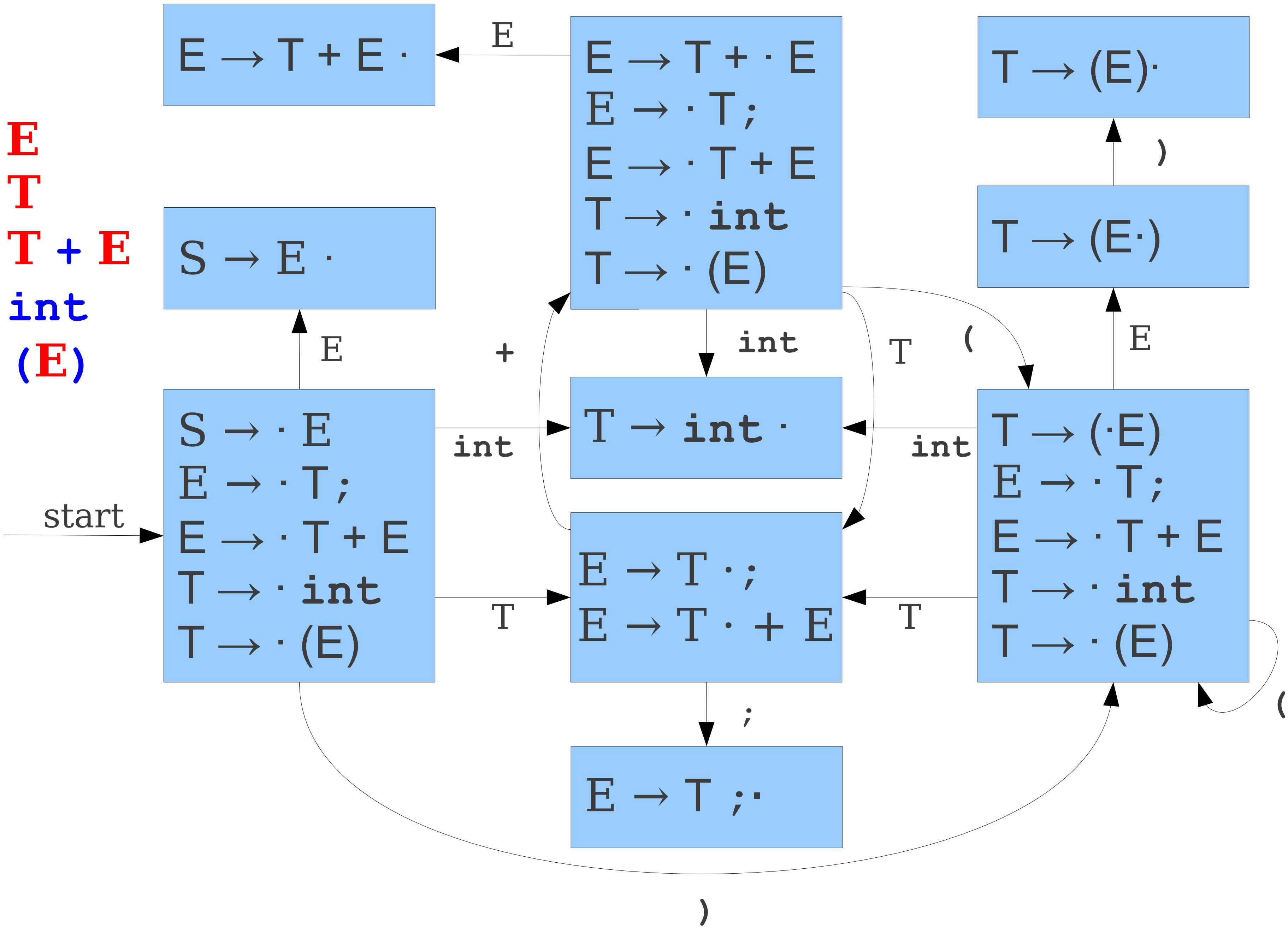
# A Non-LR(0) Grammar

**S** → **E**
**E** → **T** ;
**E** → **T** + **E**
**T** → int
**T** → ( **E** )



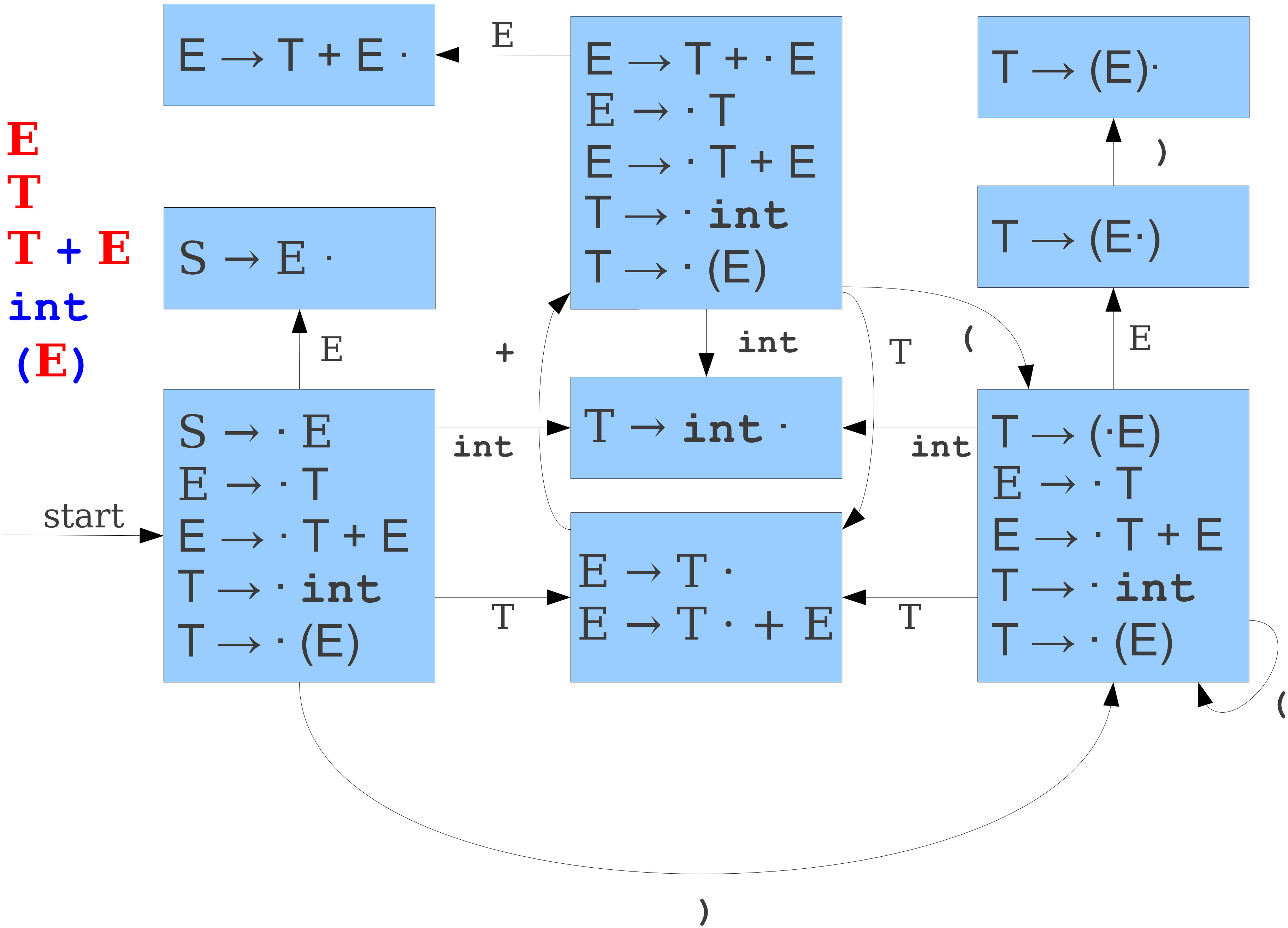
# A Non-LR(0) Grammar

**S** → **E**
**E** → **T**
**E** → **T** + **E**
**T** → `int`
**T** → (**E**)



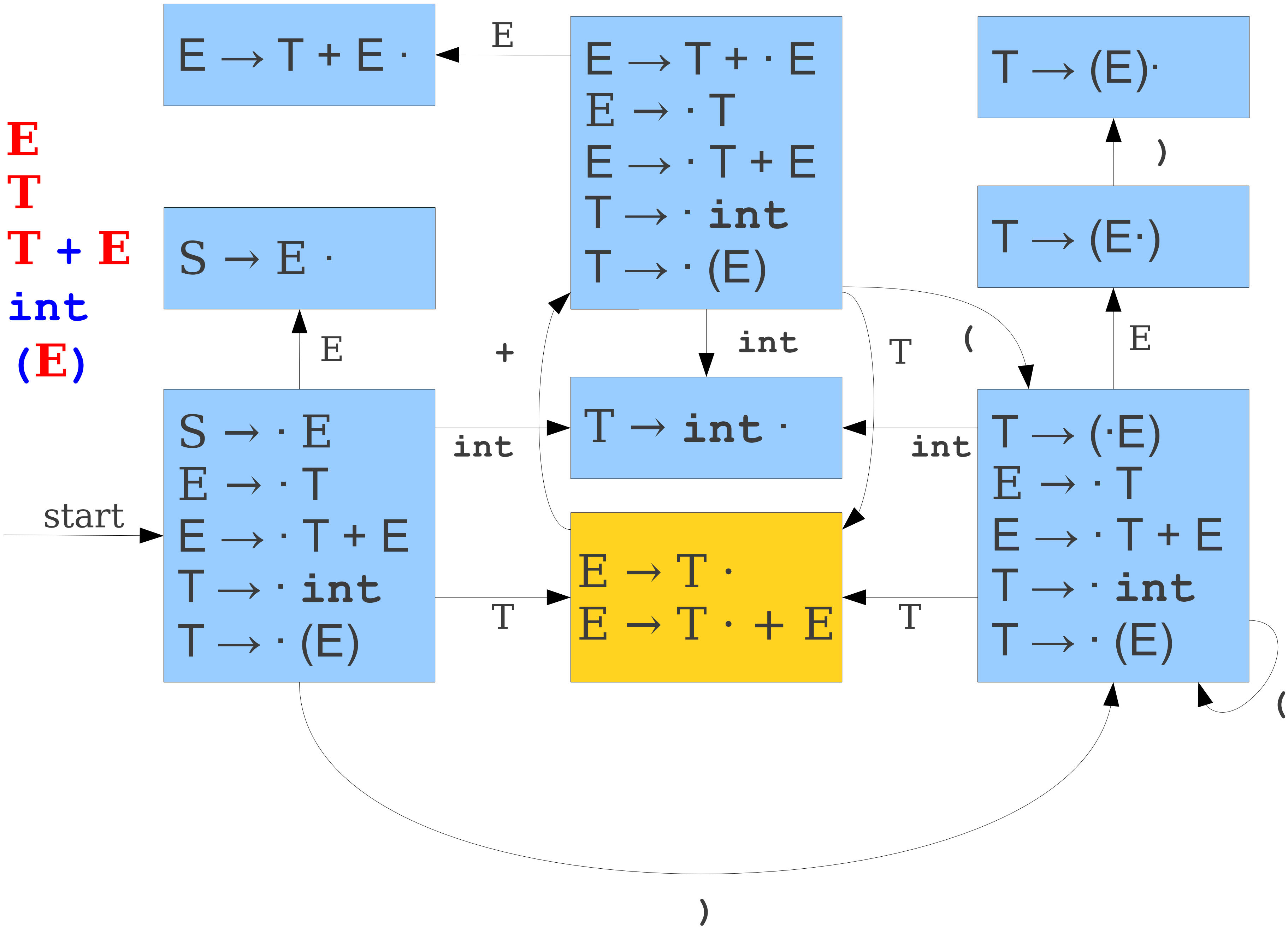
# A Non-LR(0) Grammar

**S** → **E**
**E** → **T**
**E** → **T** **+** **E**
**T** → **`int`**
**T** → **(E)**



# A Non-LR(0) Grammar

**S** → **E**
**E** → **T**
**E** → **T** + **E**
**T** → `int`
**T** → (**E**)

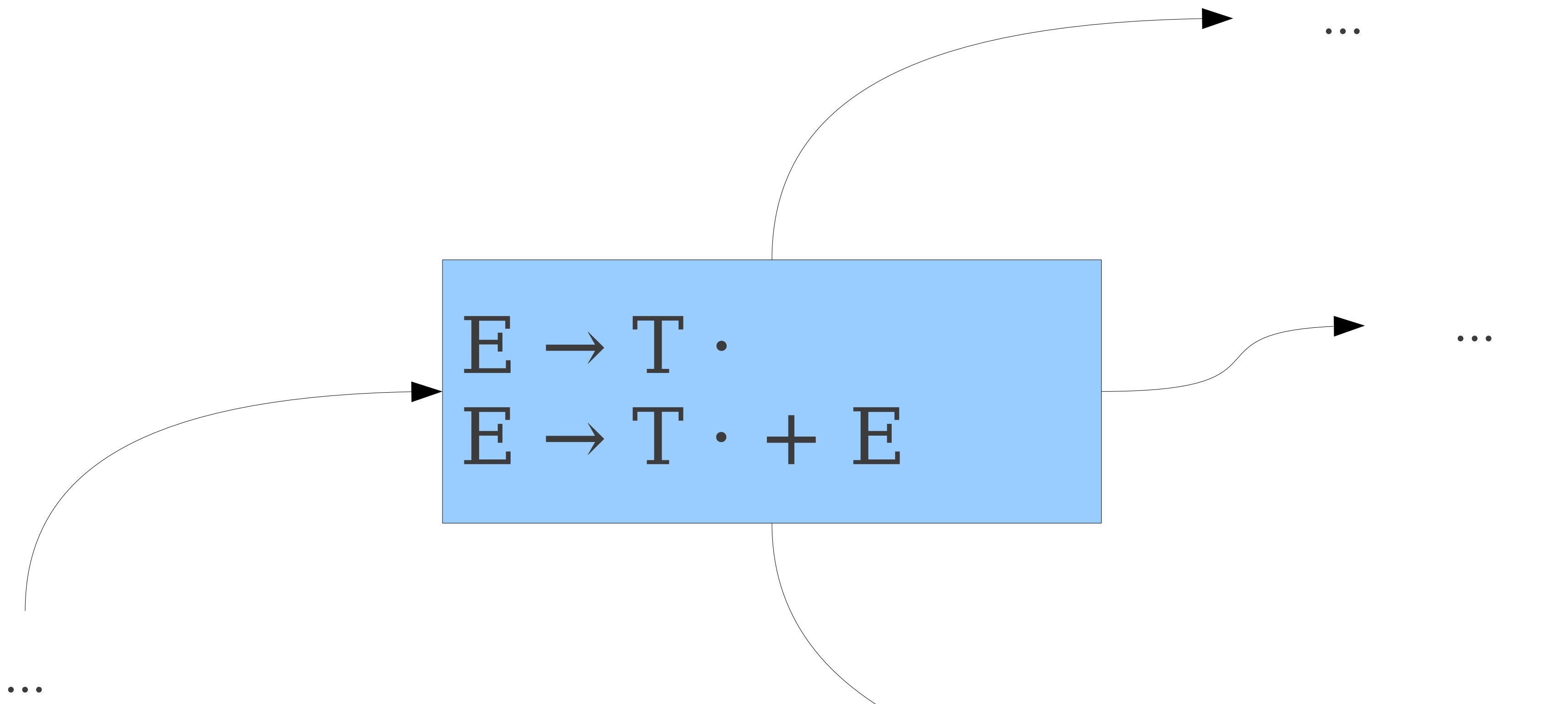


# LR Conflicts

- A **shift/reduce conflict** is an error where a shift/reduce parser cannot tell whether to shift a token or perform a reduction.
  - Often happens when two productions overlap.
- A **reduce/reduce conflict** is an error where a shift/reduce parser cannot tell which of many reductions to perform.
  - Often the result of ambiguous grammars.
- A grammar whose handle-finding automaton contains a shift/reduce conflict or a reduce/reduce conflict is not LR(0).
- Can you have a shift/shift conflict?

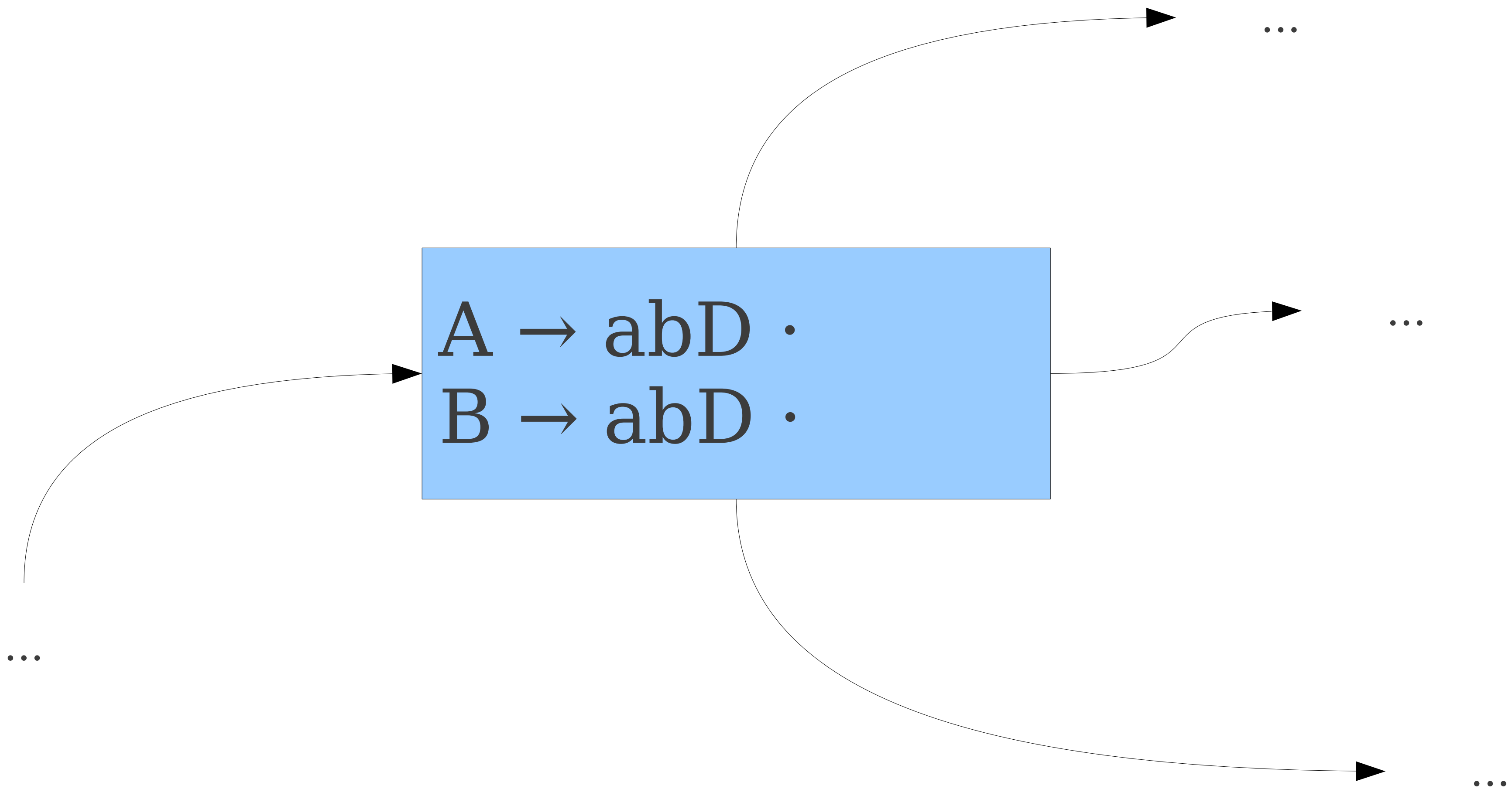
# What error is this?

# What error is this?



# What about this?

# What about this?



# And what about this?

# And what about this?

# What do these conflicts mean?

- Recall: our automaton was constructed by looking for viable prefixes.
- Each accepting state represents a point where the handle might occur.
- A **shift/reduce** conflict is a state where the handle might occur, but we might actually need to keep searching.
- A **reduce/reduce** conflict is a state where we know we have found the handle, but can't tell which reduction to apply.

# Why LR(0) is Weak

- LR(0) only accepts languages where the handle can be found with no **right context**.
- Our shift/reduce parser only looks to the left of the handle, not to the right.
- How do we exploit the tokens after a possible handle to determine what to do?

# A Powerful Parser: **LR(1)**

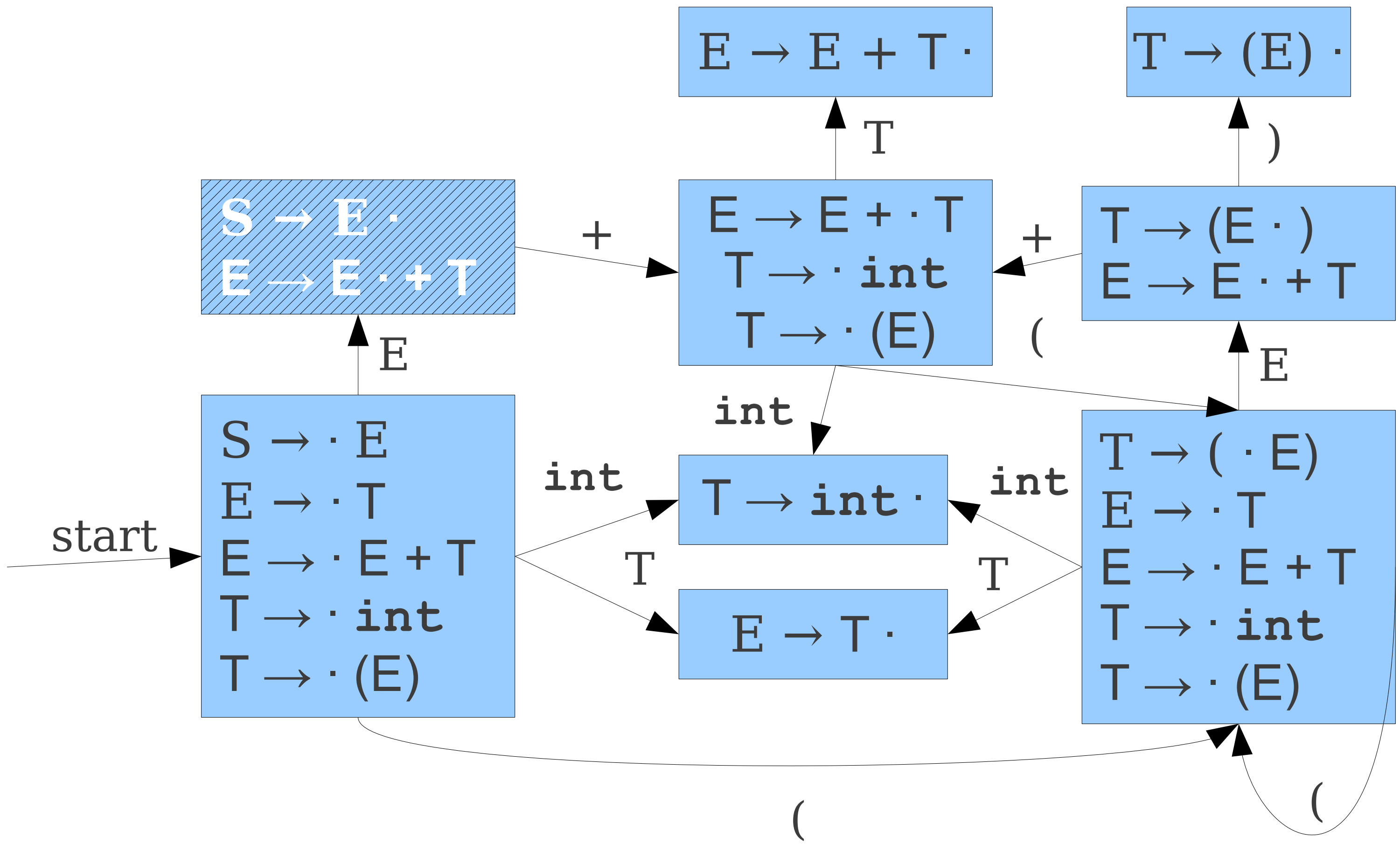
- Bottom-up predictive parsing with
  - **L**: **L**eft-to-right scan
  - **R**: **R**ightmost derivation
  - (**1**): One token lookahead
- *Substantially* more powerful than the other methods we've covered so far (more on that later).
- Tries to more intelligently find handles by using a lookahead token at each step.

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **`+`** **T**
**T** → **`int`**
**T** → **`(`E`)`**

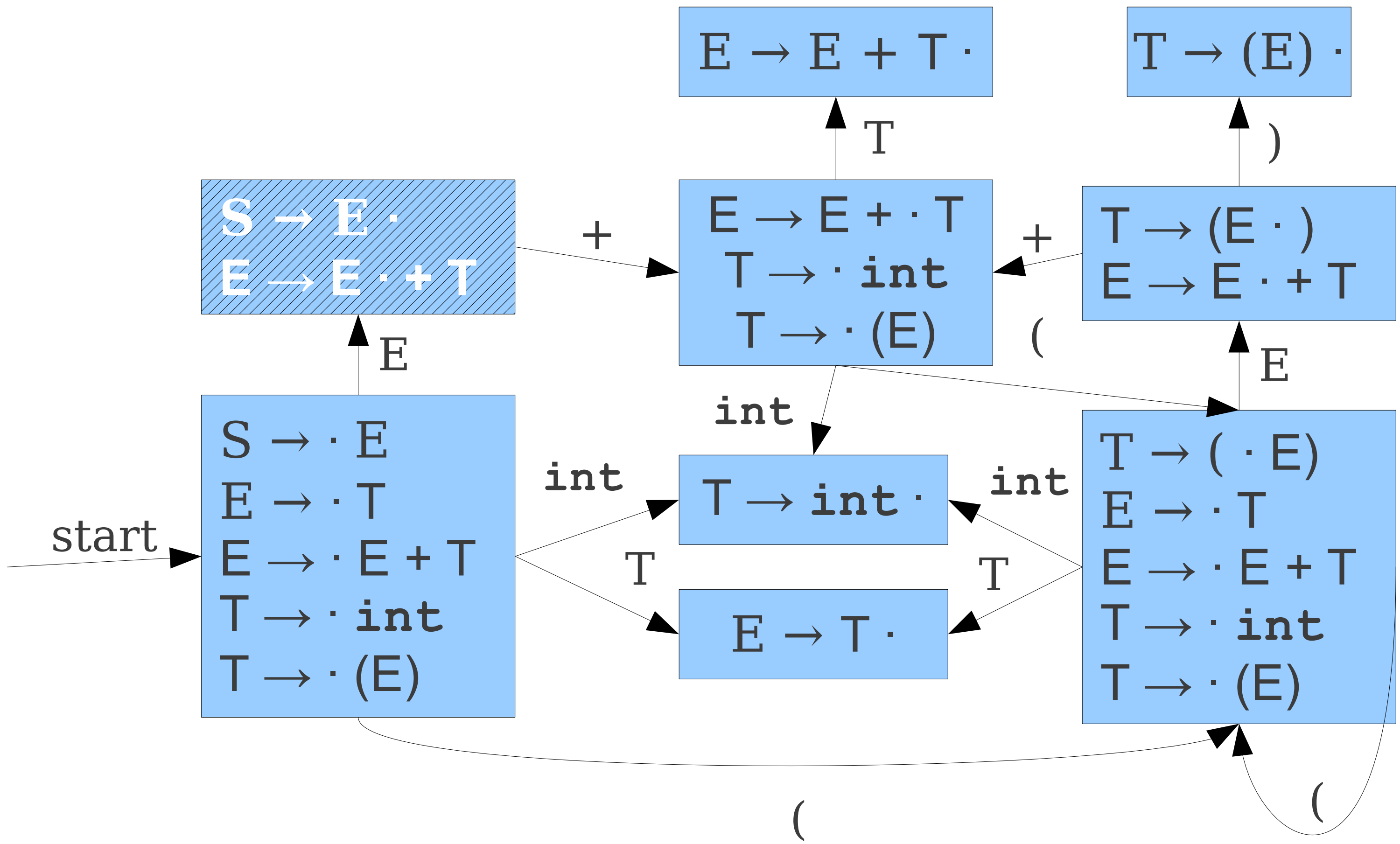
# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



# LR(1) Parsing: The Intuition

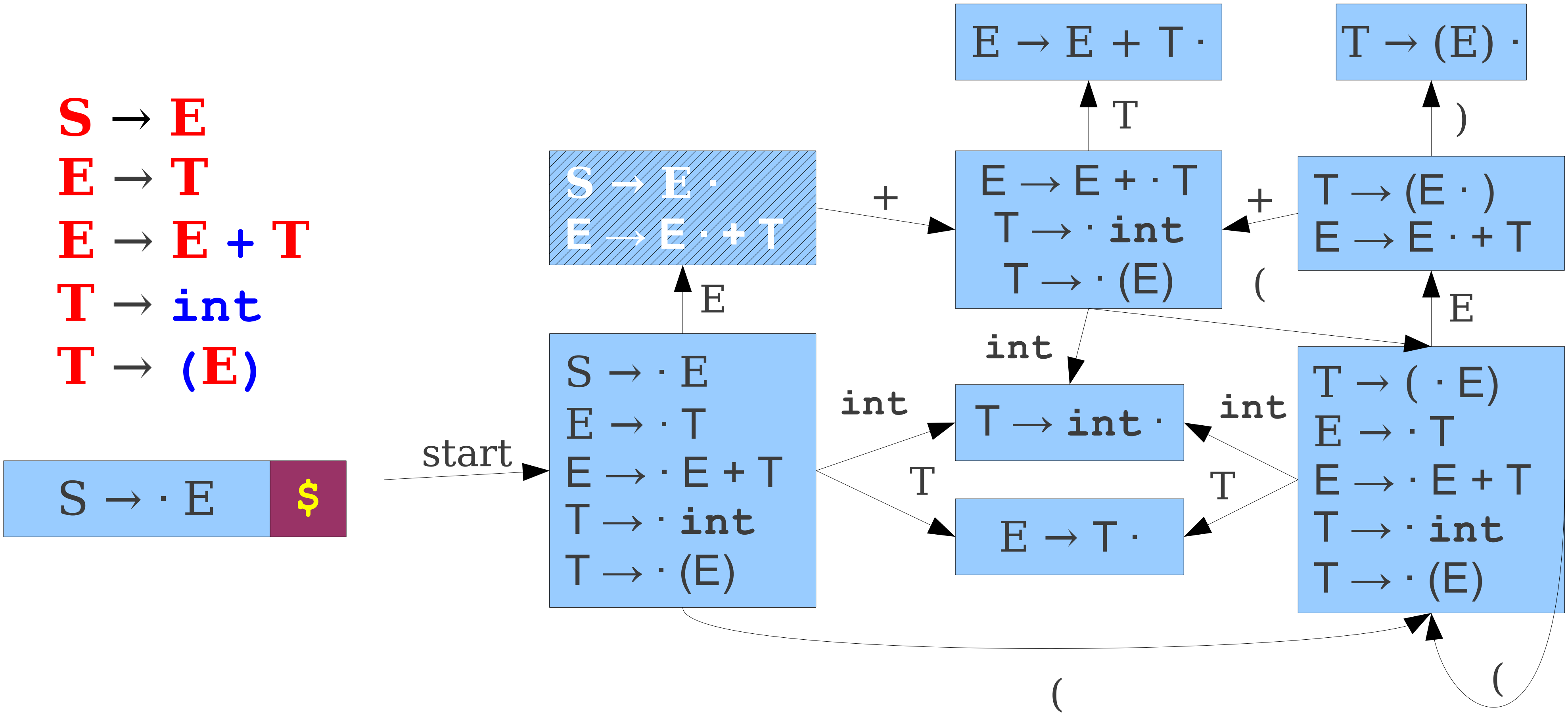
**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**

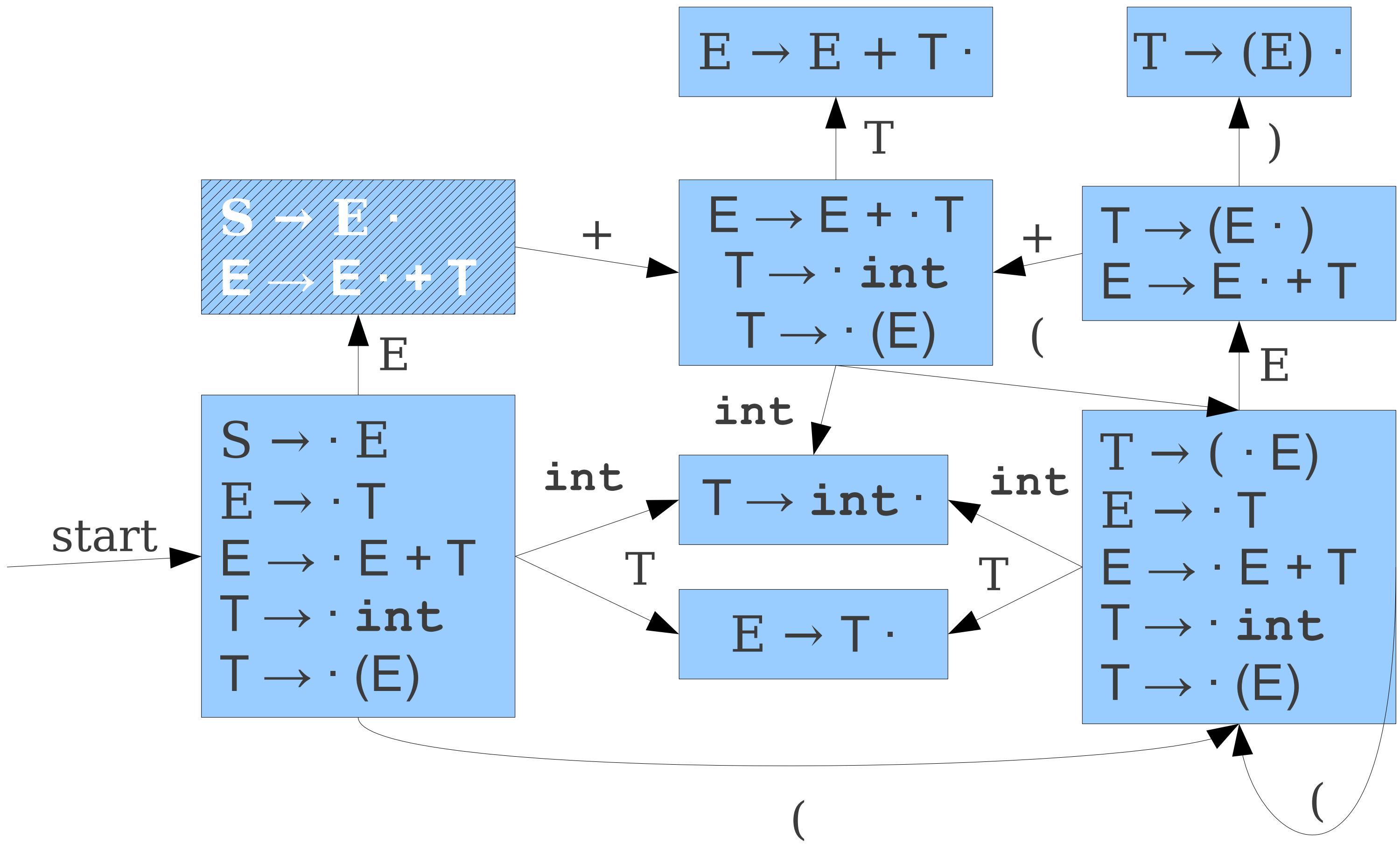



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**

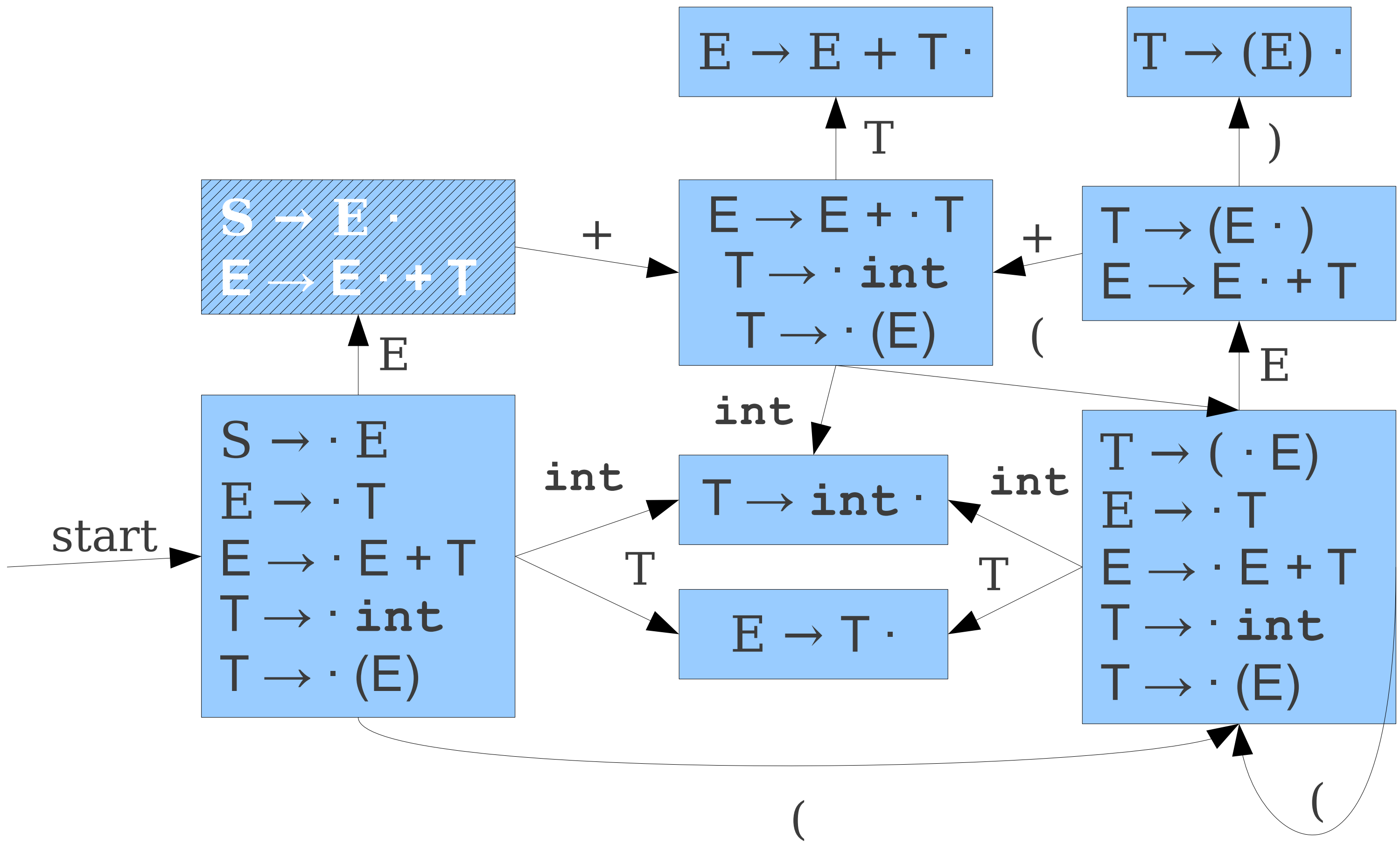
| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

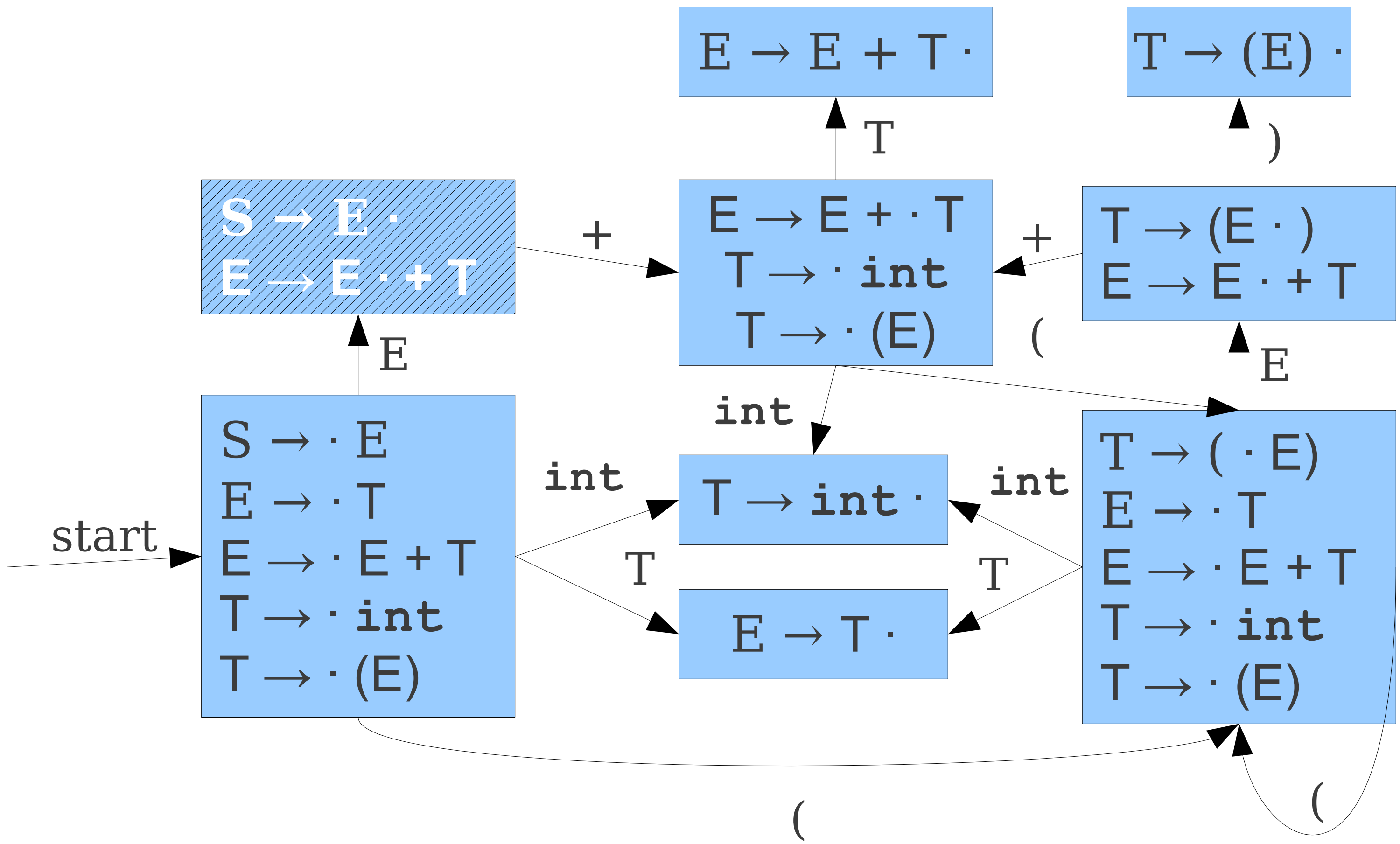



| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

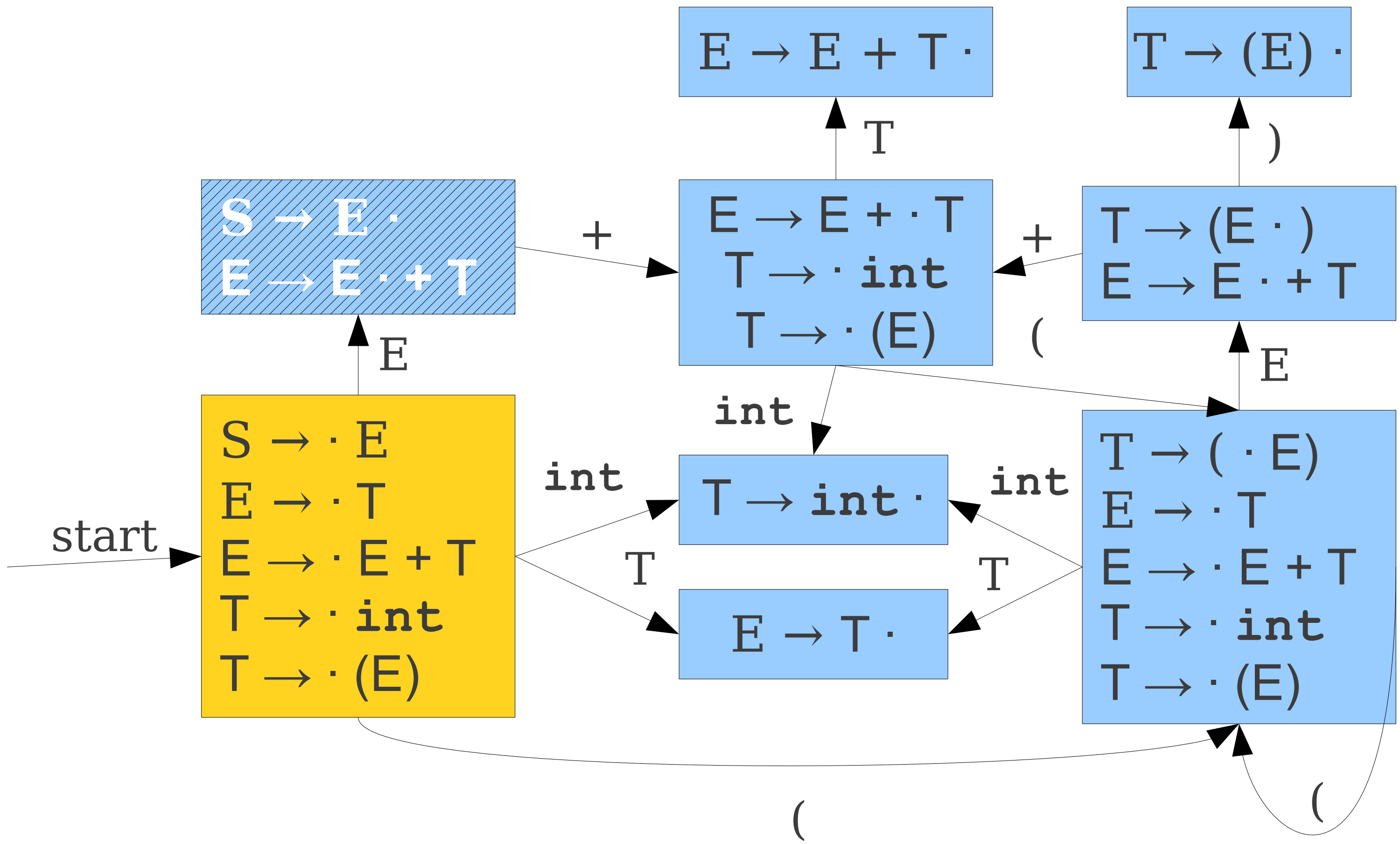



| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |
| T → · int | + |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

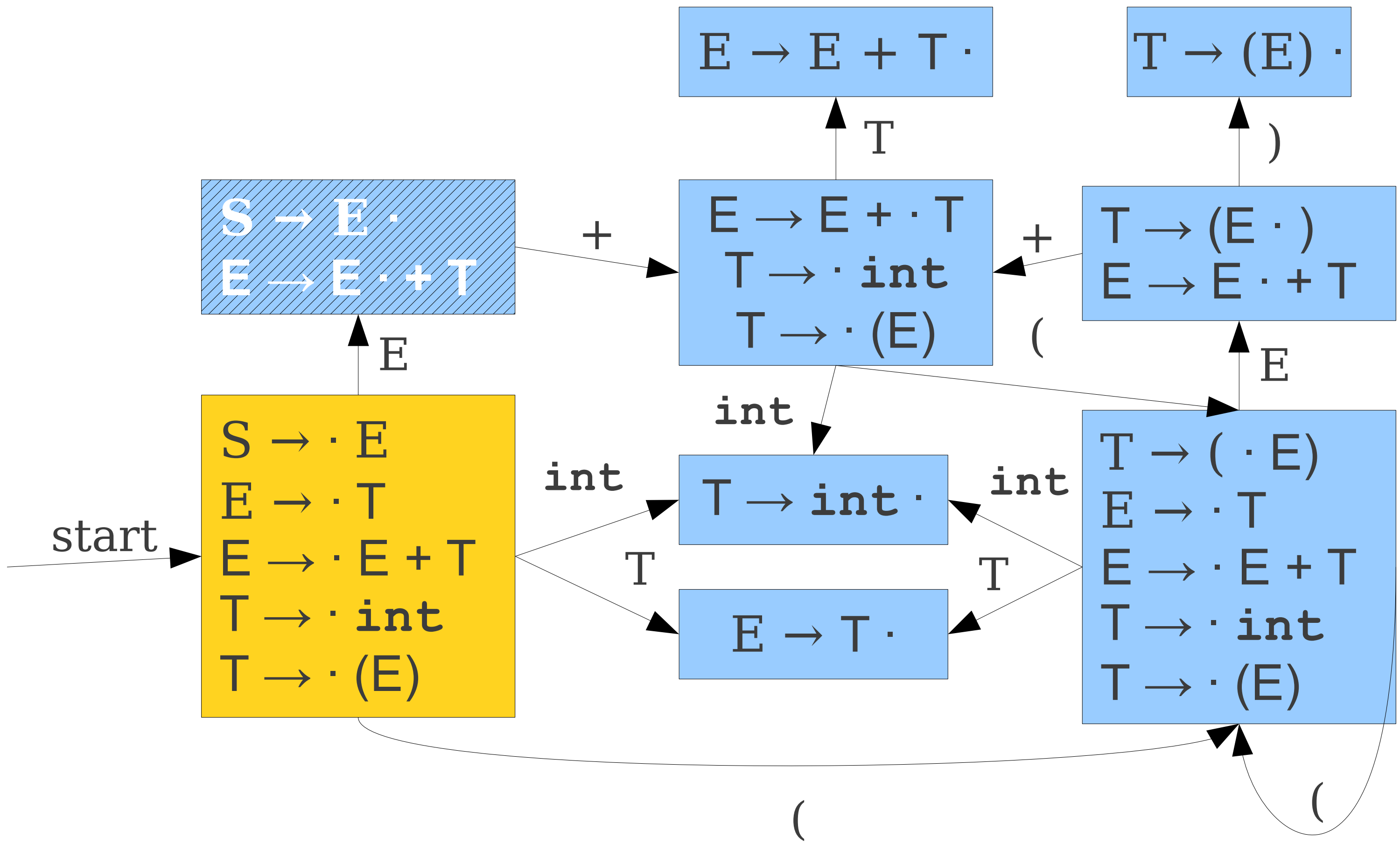



| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |
| T → · int | + |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| int | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

S → E
E → T
E → E + T
T → int
T → (E)



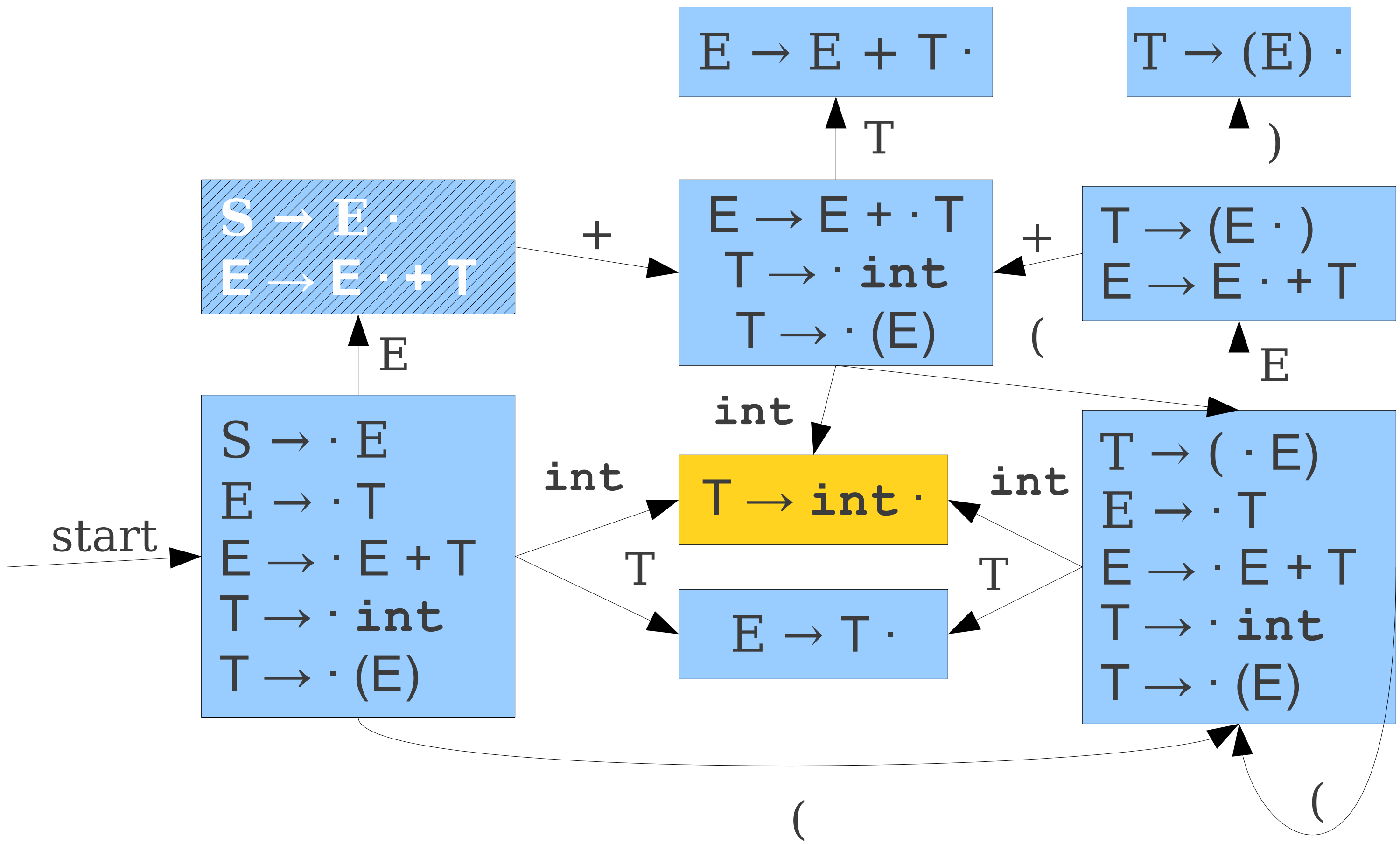

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |
| T → · int | + |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| int | \| | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

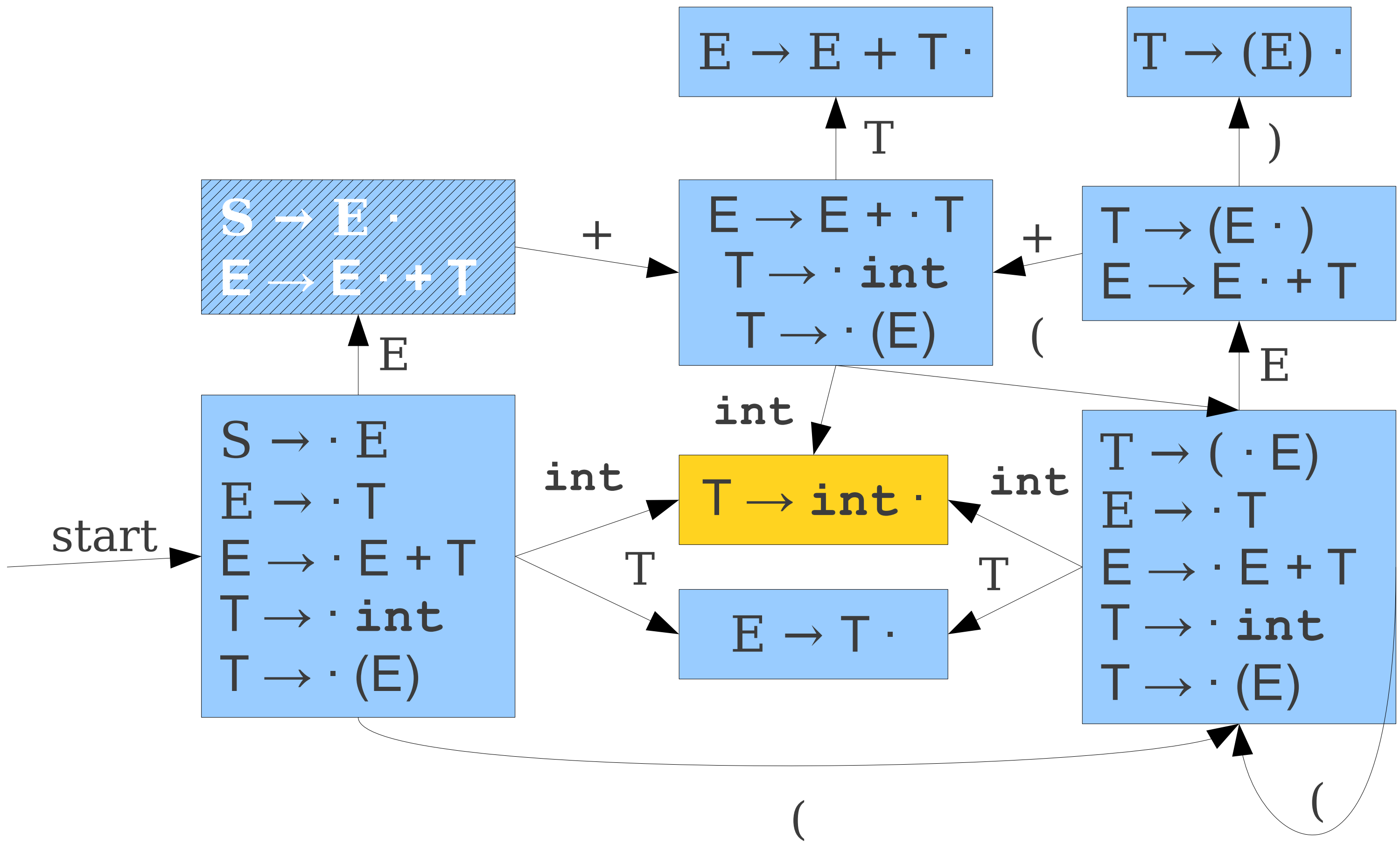
| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |
| T → · int | + |




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| int | \| | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



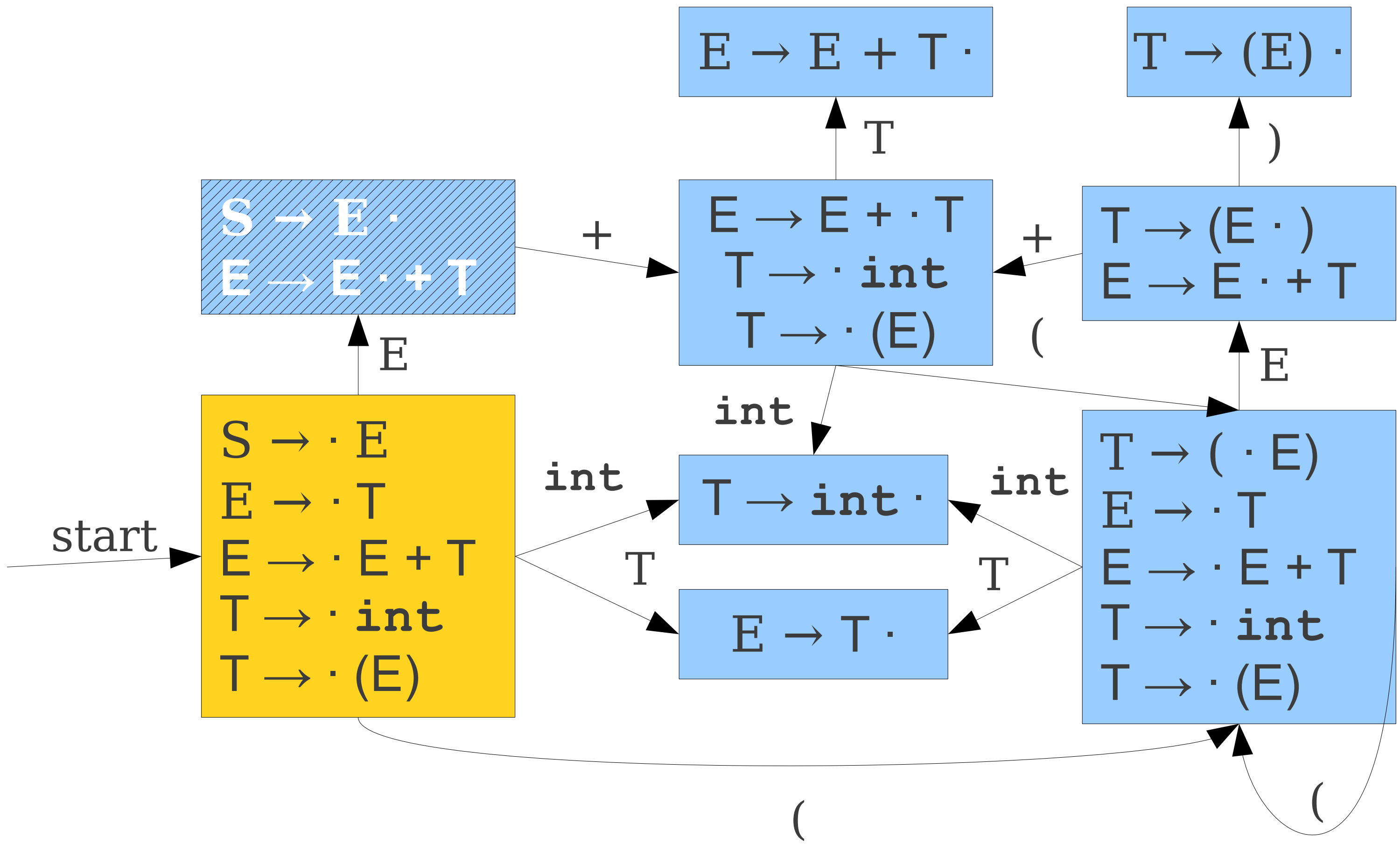

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |
| T → int · | + |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| int | \| | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |

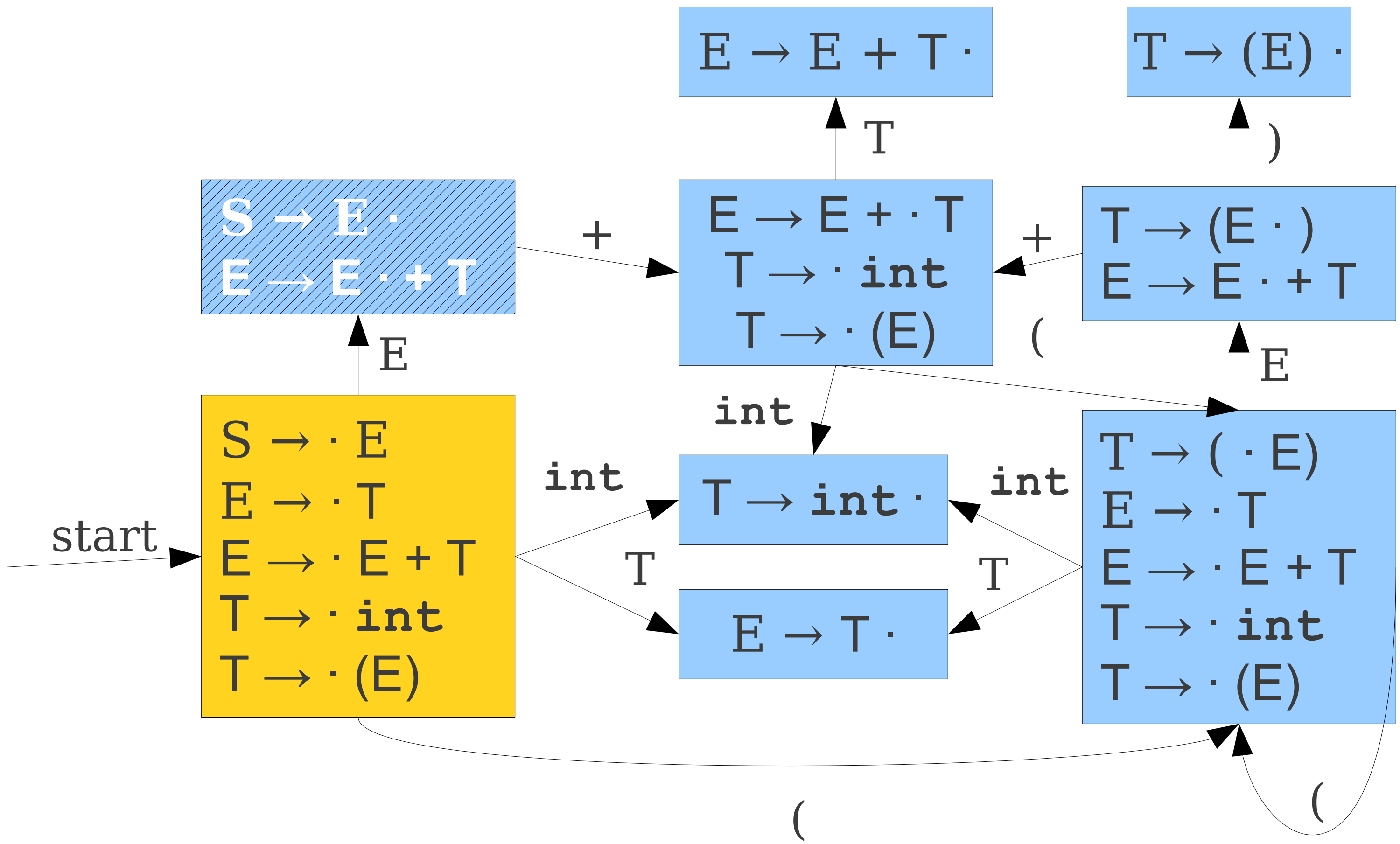



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → · T | + |

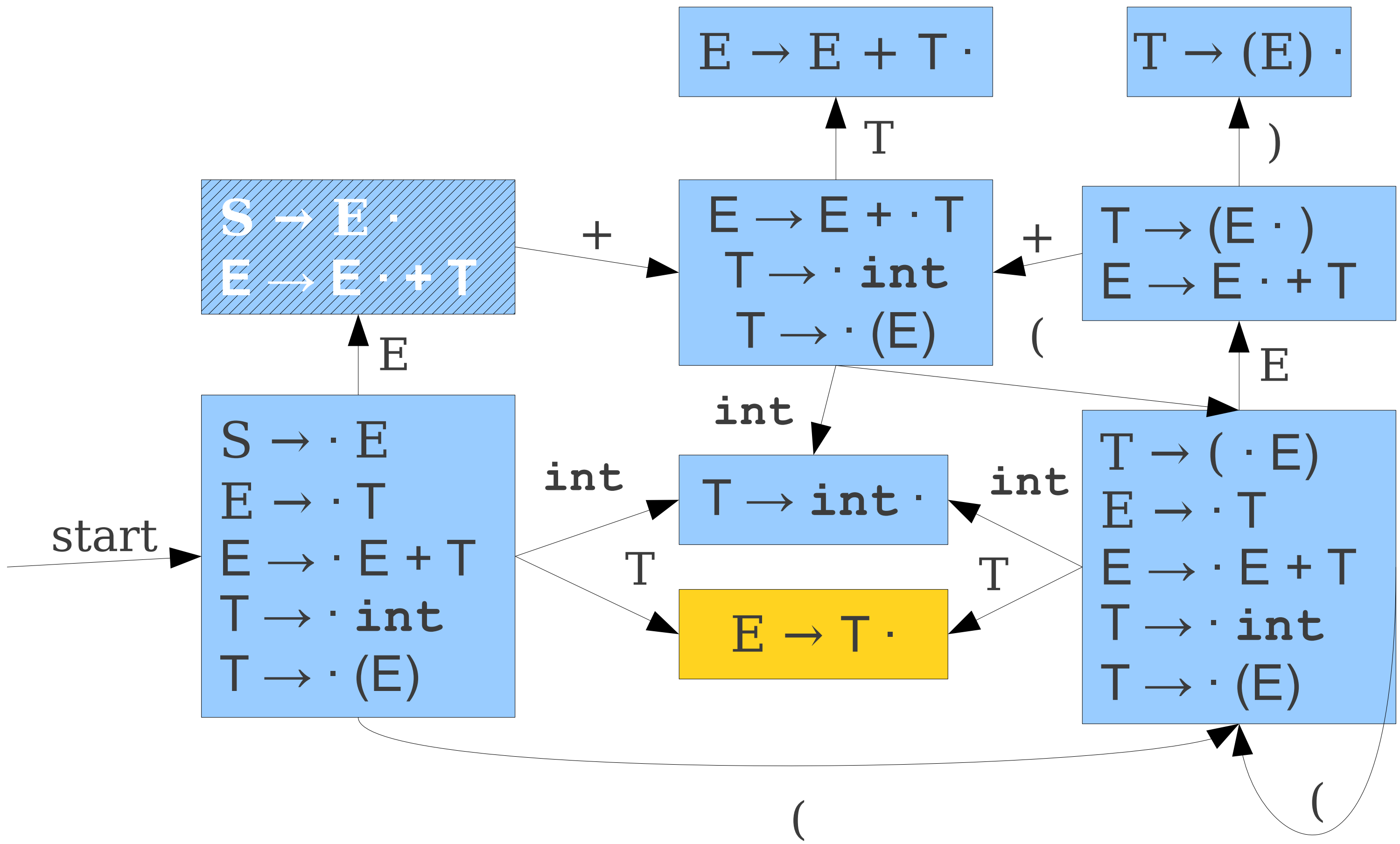



| T | + | ( | int | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

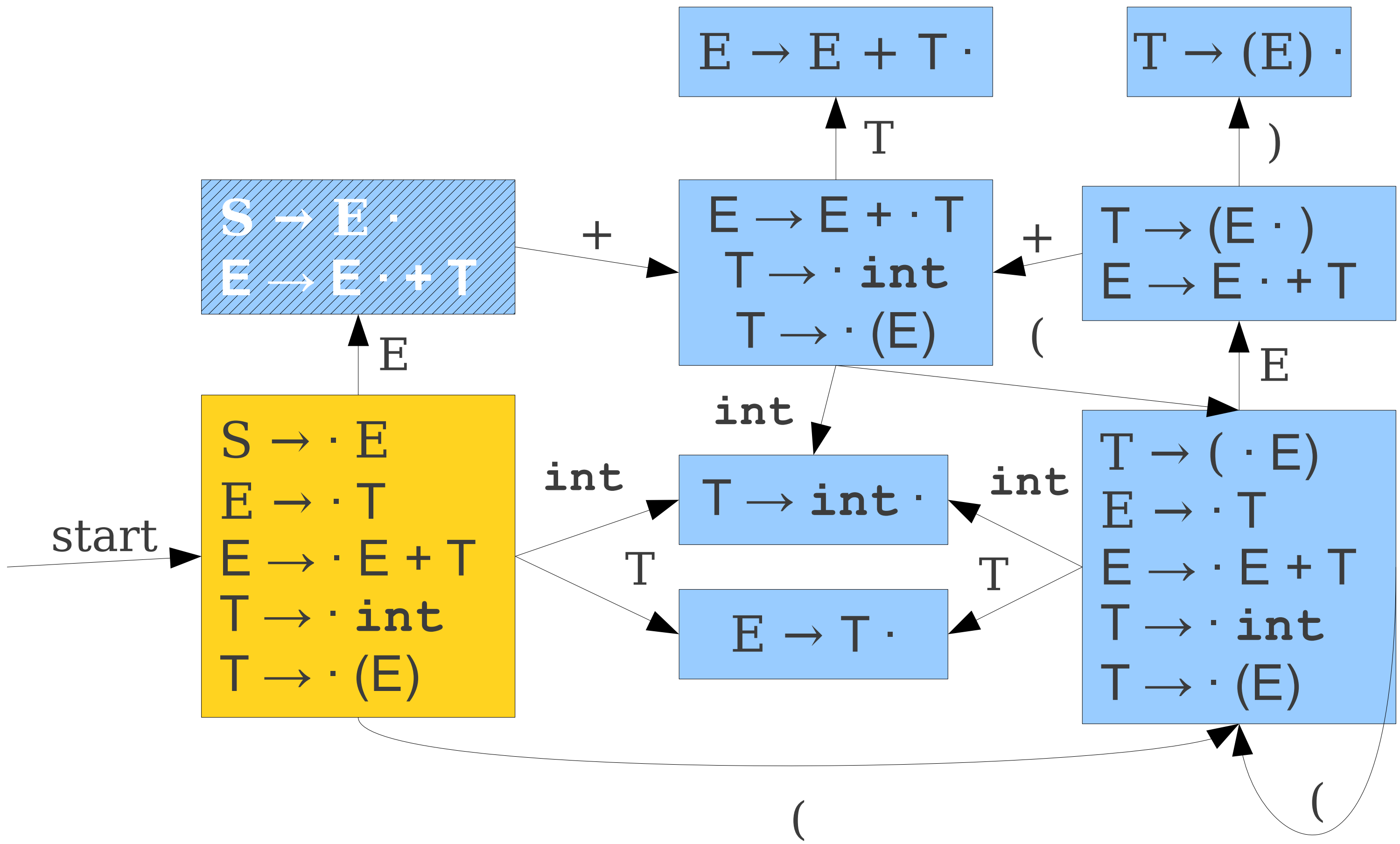
| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |
| E → T · | + |




T | + ( int + int + int ) $

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



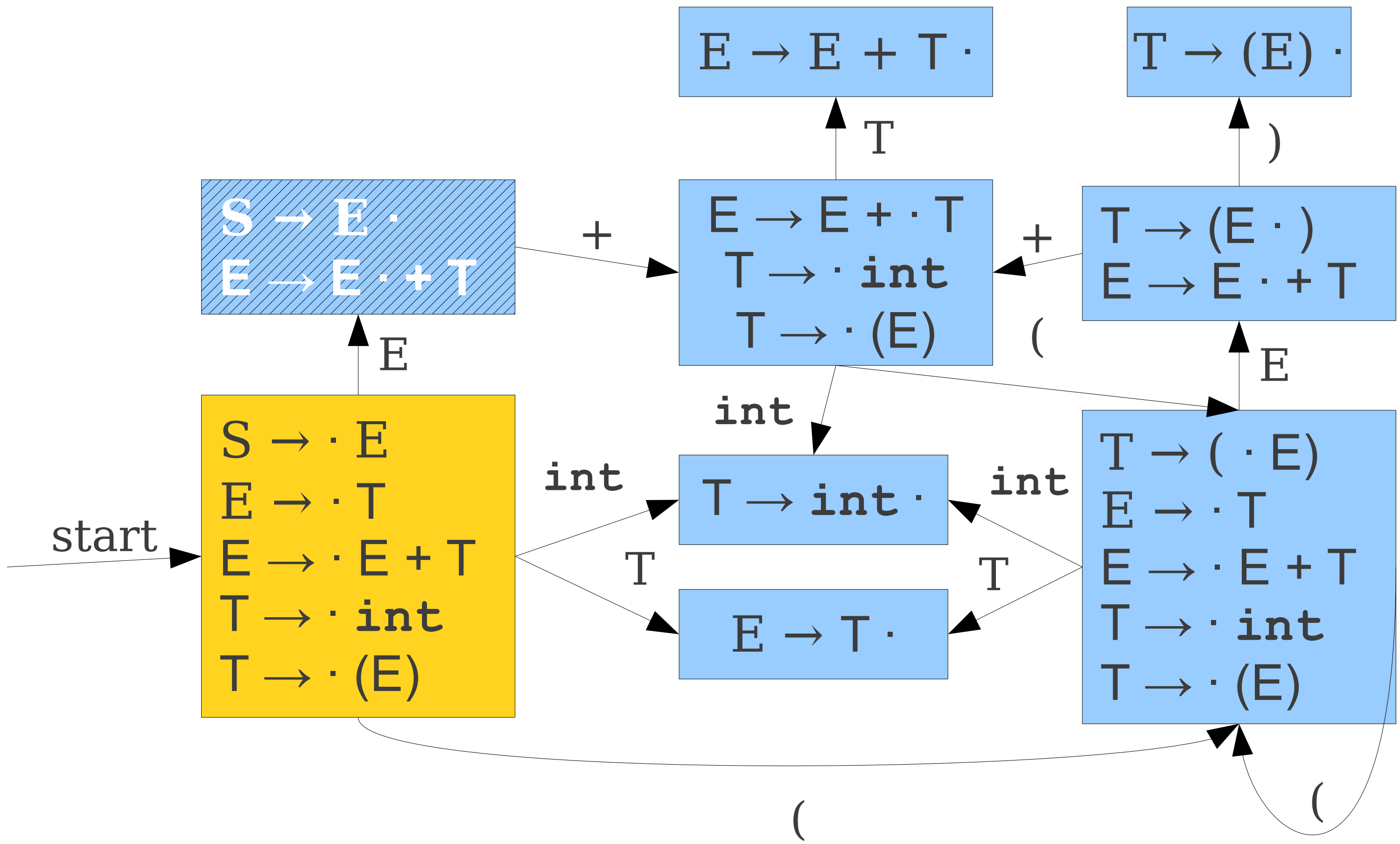

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

| | |
|---|---|
| S → · E | $ |
| E → · E + T | $ |

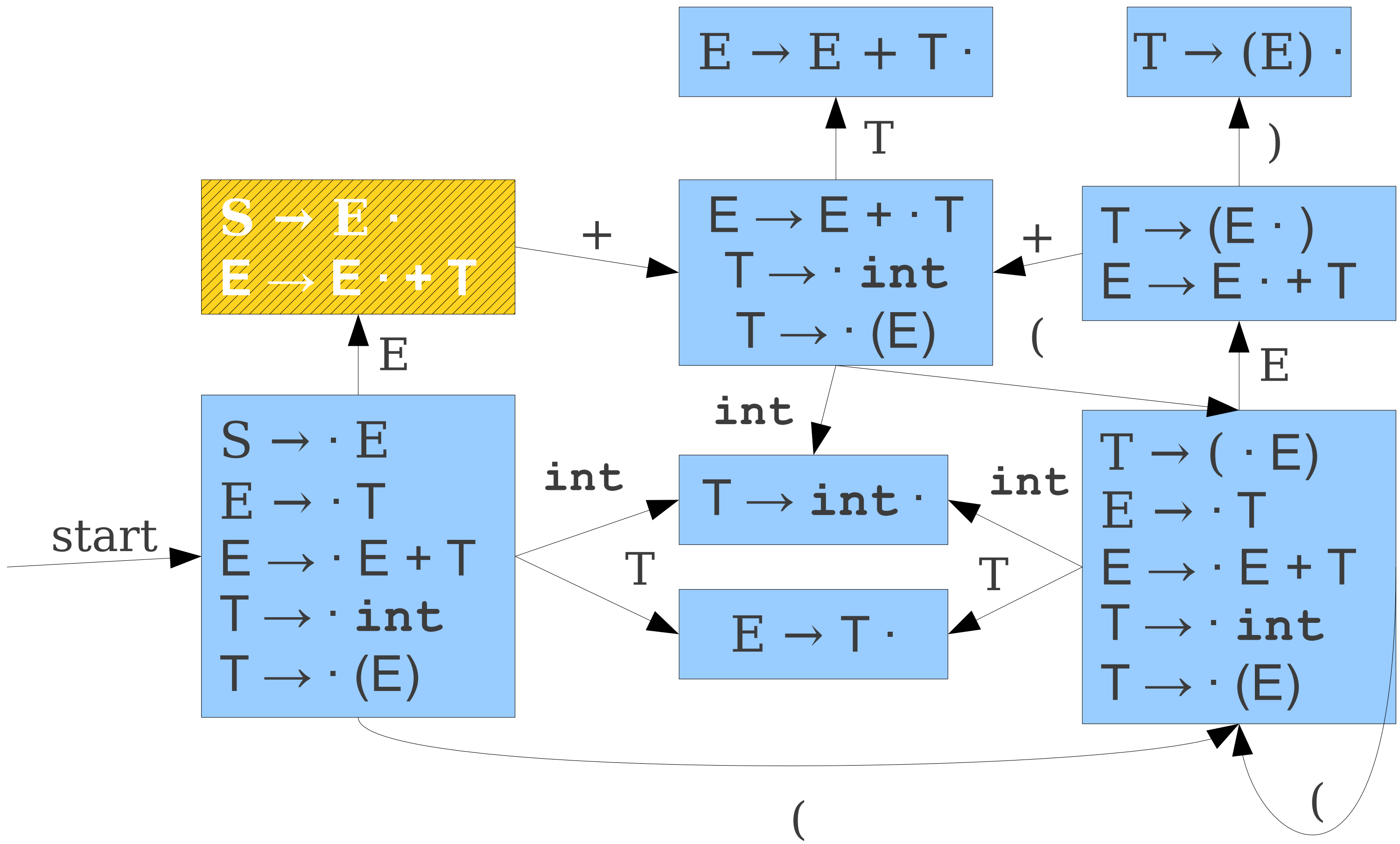



| E | | + | ( | int | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

| | |
|---|---|
| S → · E | $ |
| E → E · + T | $ |

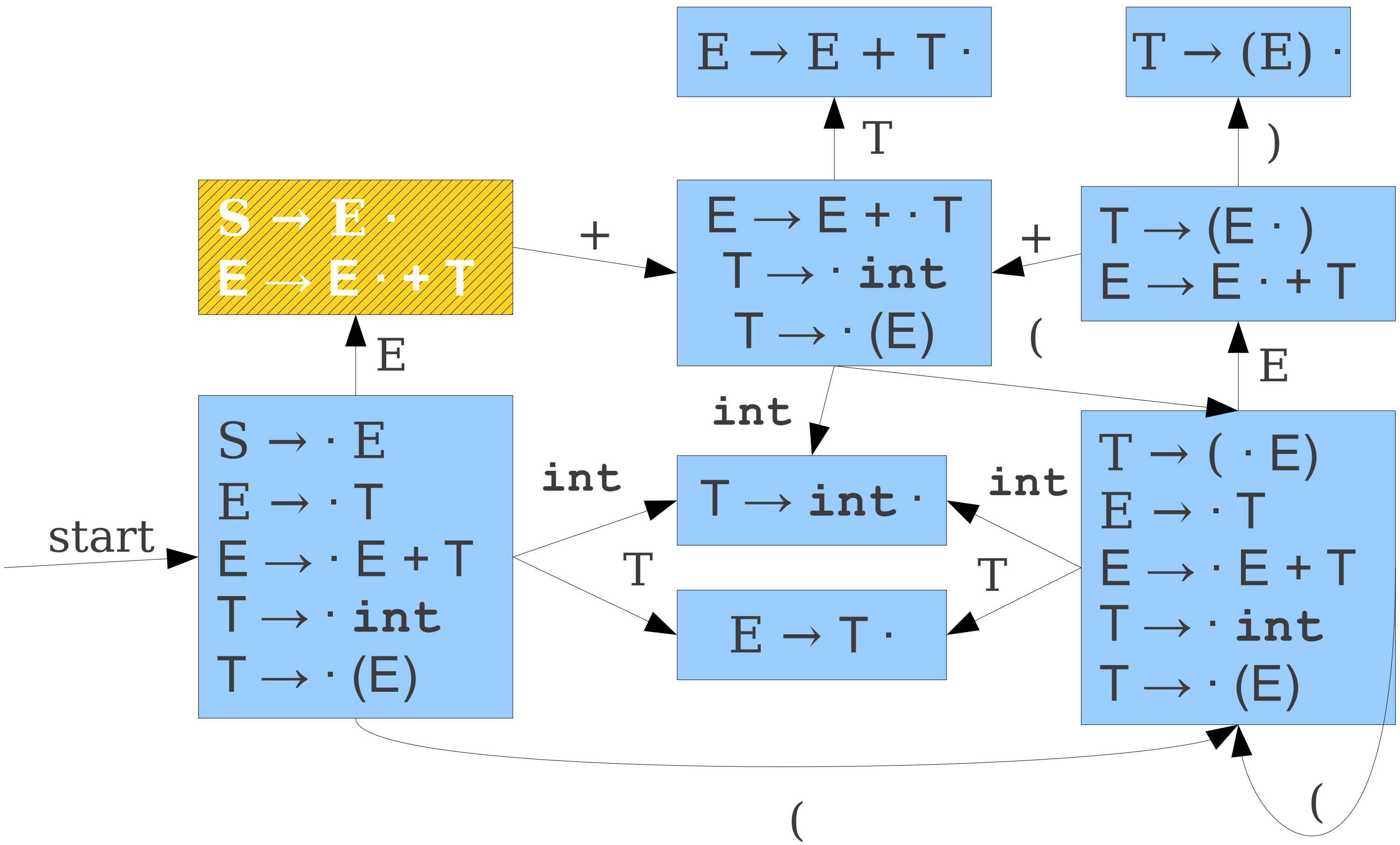



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| E | \| | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

| S → · E | $ |
|---|---|
| E → E · + T | $ |

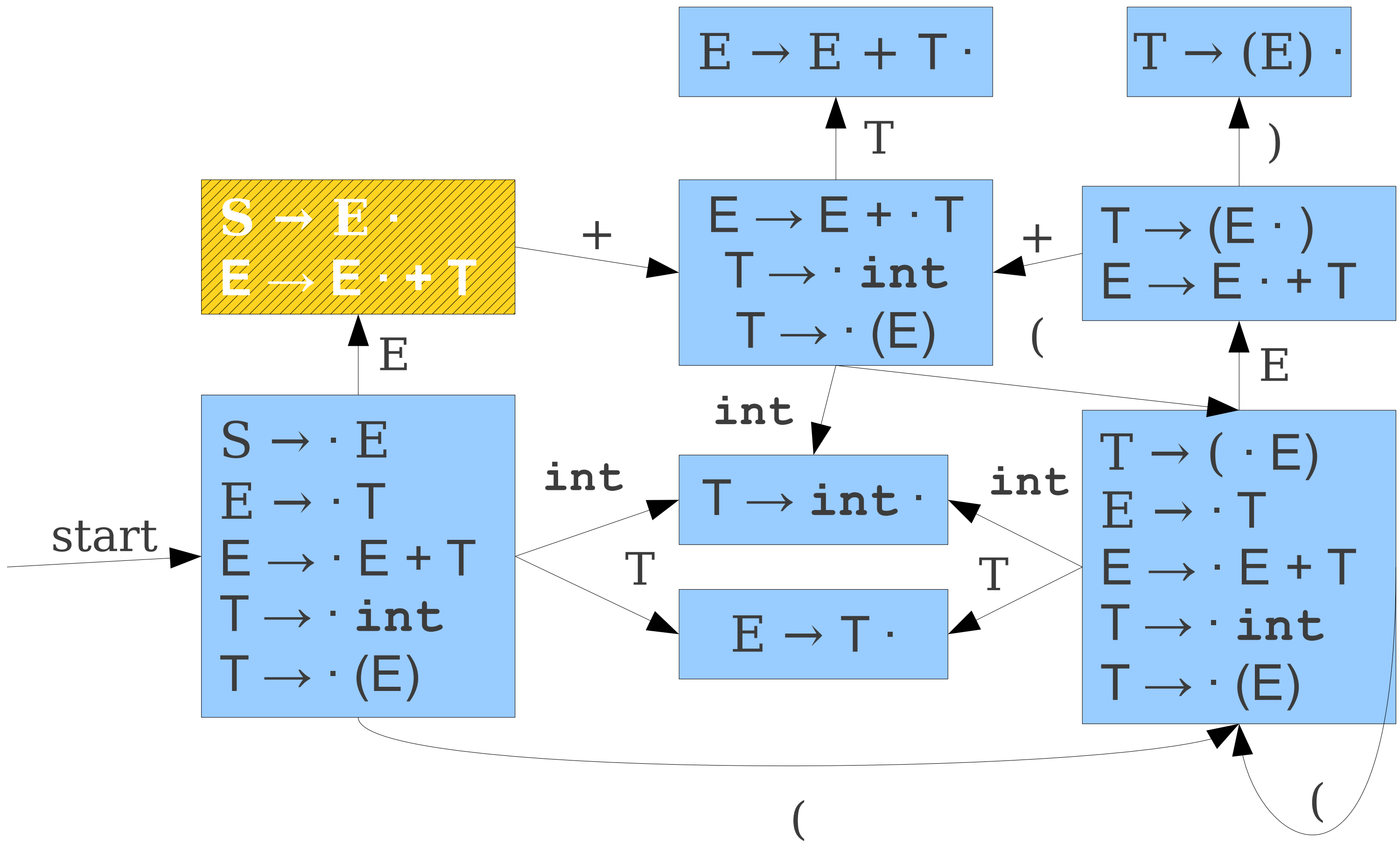



| E | + | ( | int | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**

| | |
|---|---|
| S → · E | $ |
| E → E · + T | $ |

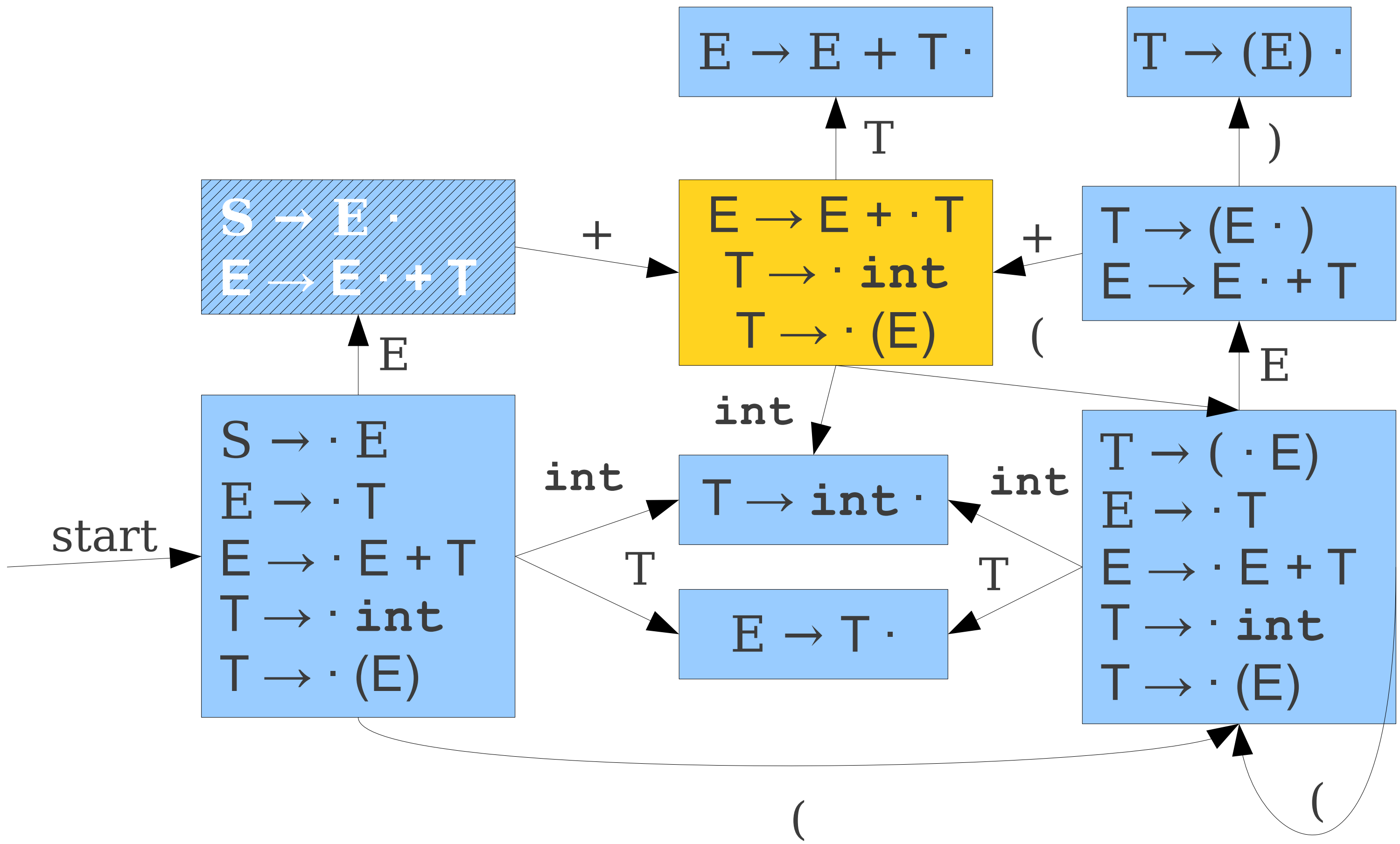



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| E | + | \| | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |

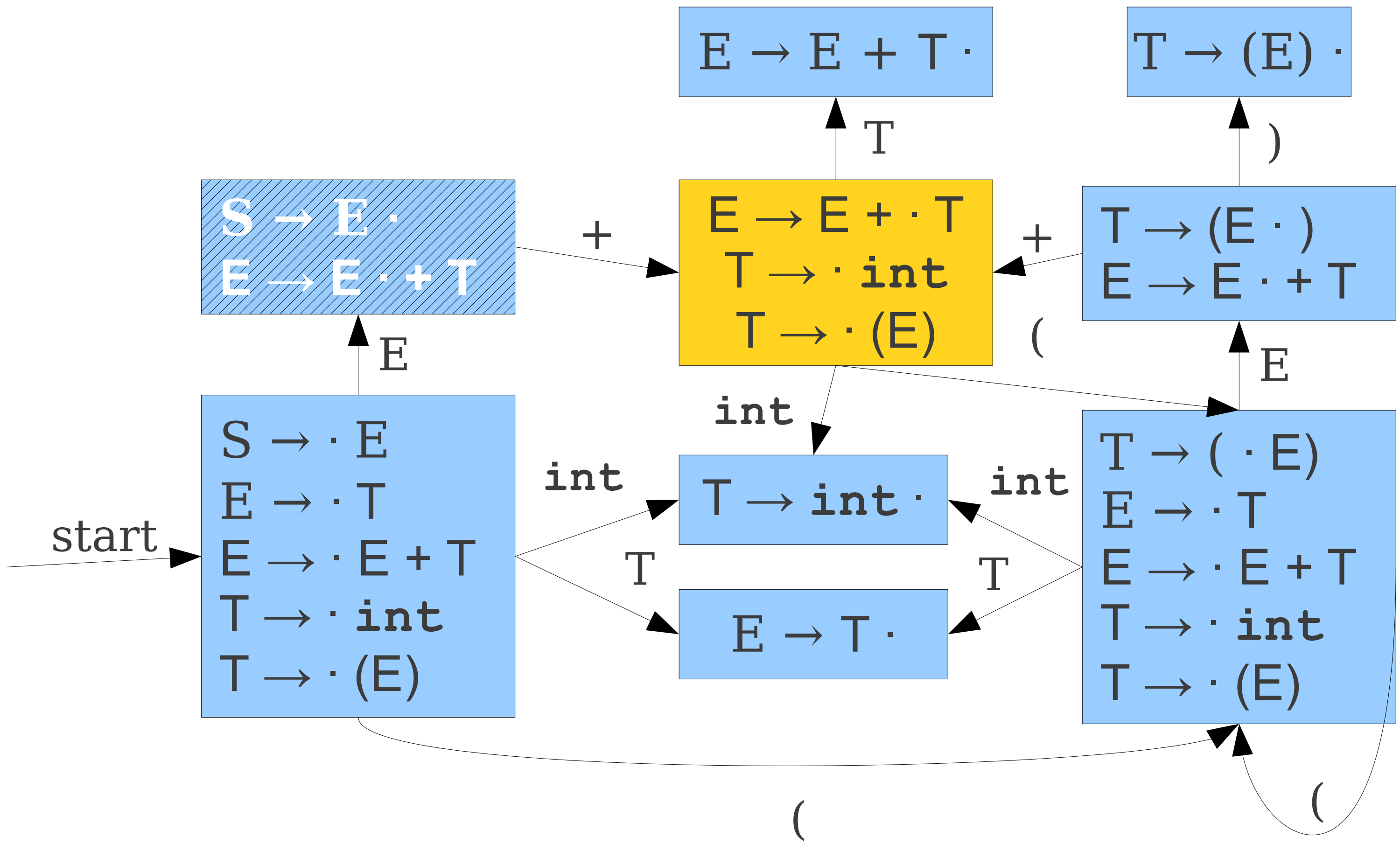



| E | + | | ( | int | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → · (E) | $ |

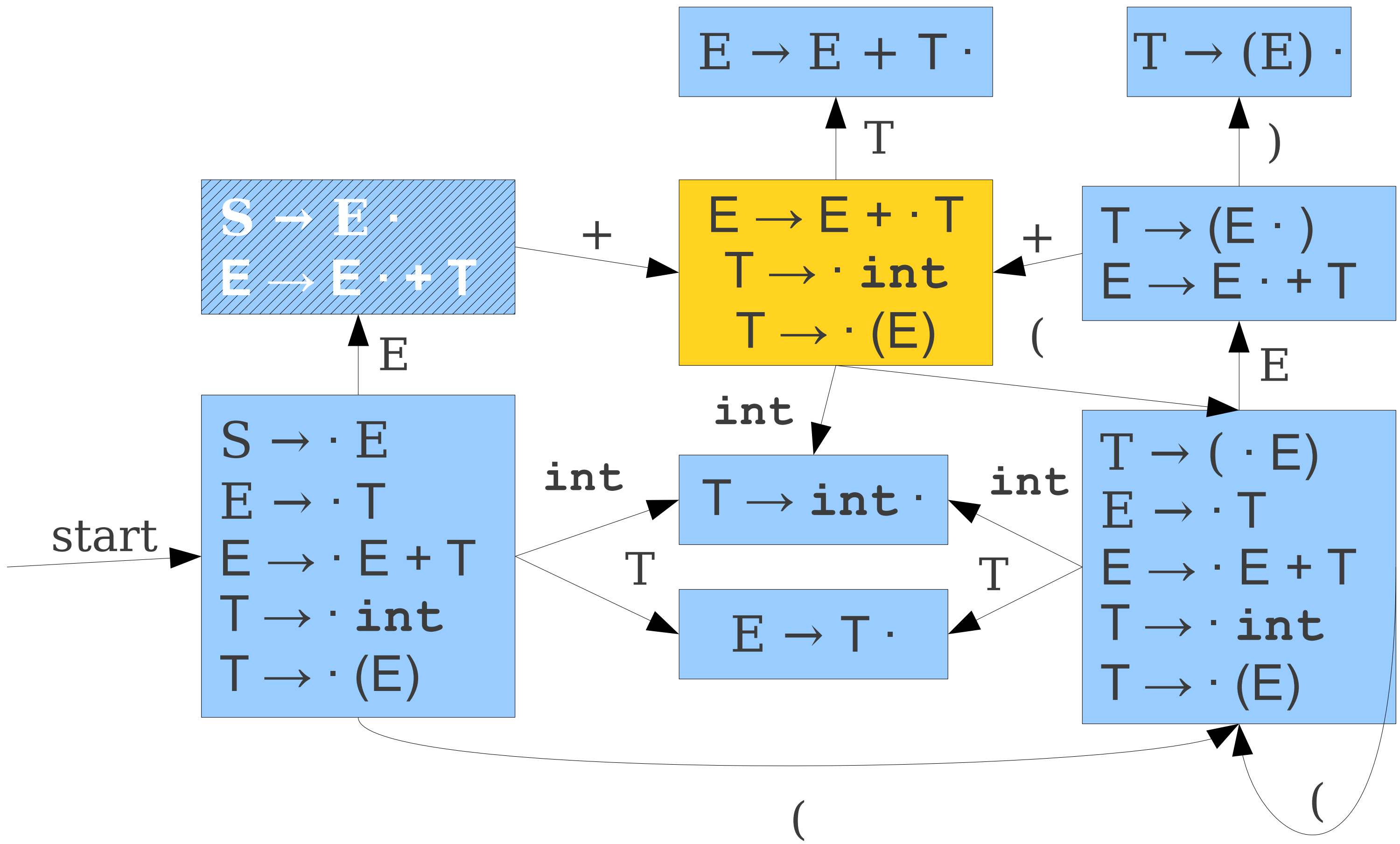



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| E | + | | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → · (E) | $ |

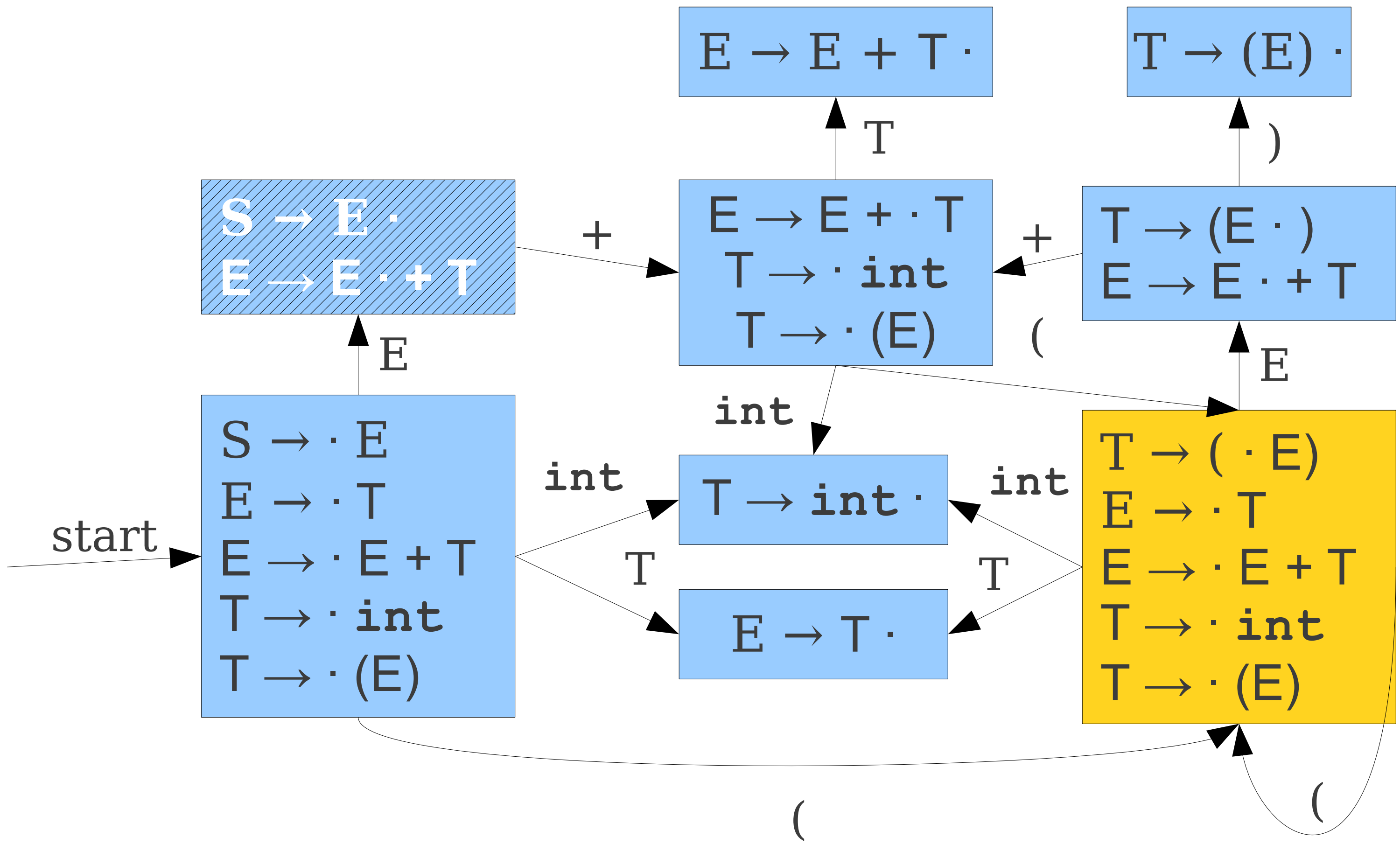



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| E | + | ( | \| | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |

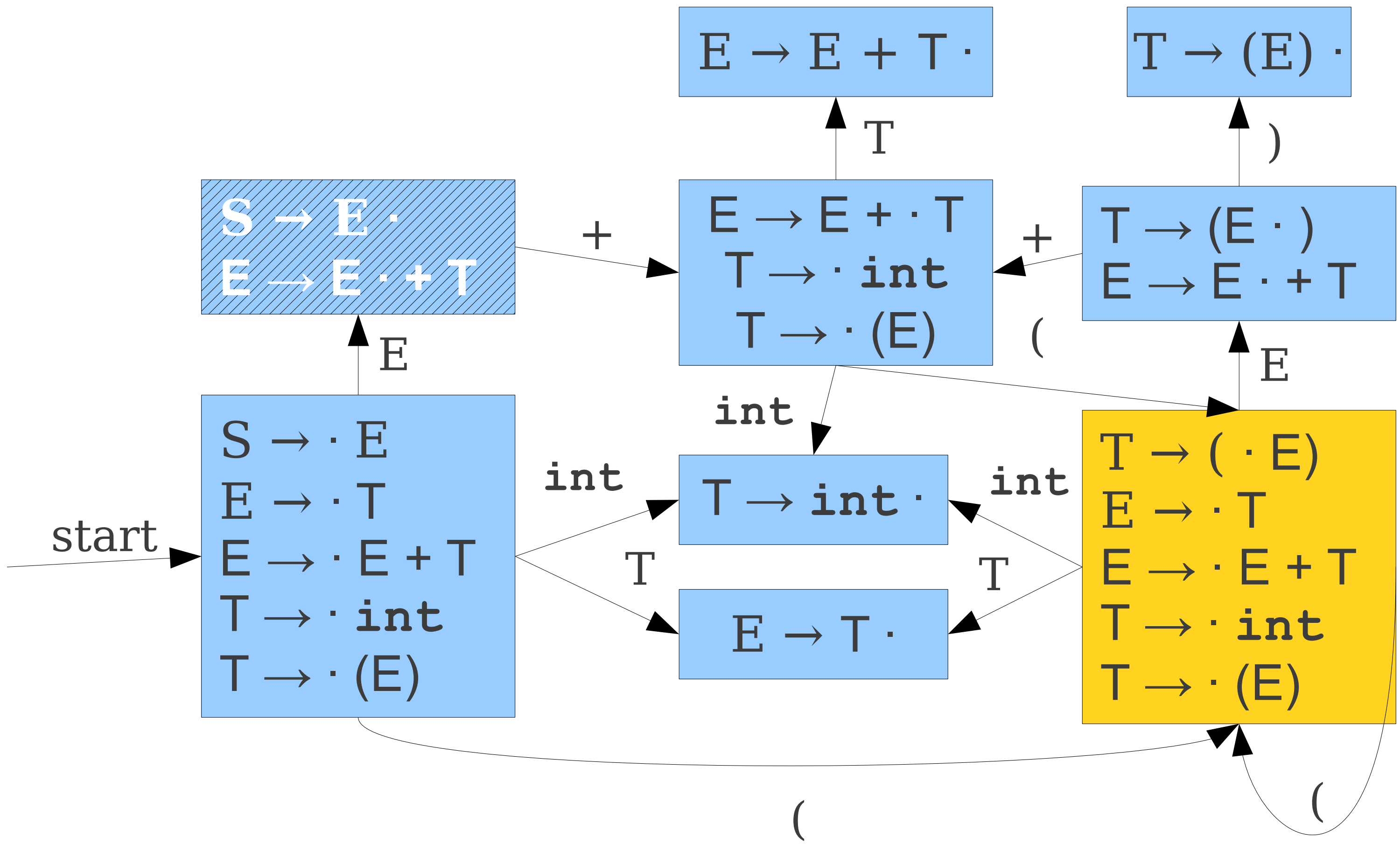



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| E | + | ( | \| | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

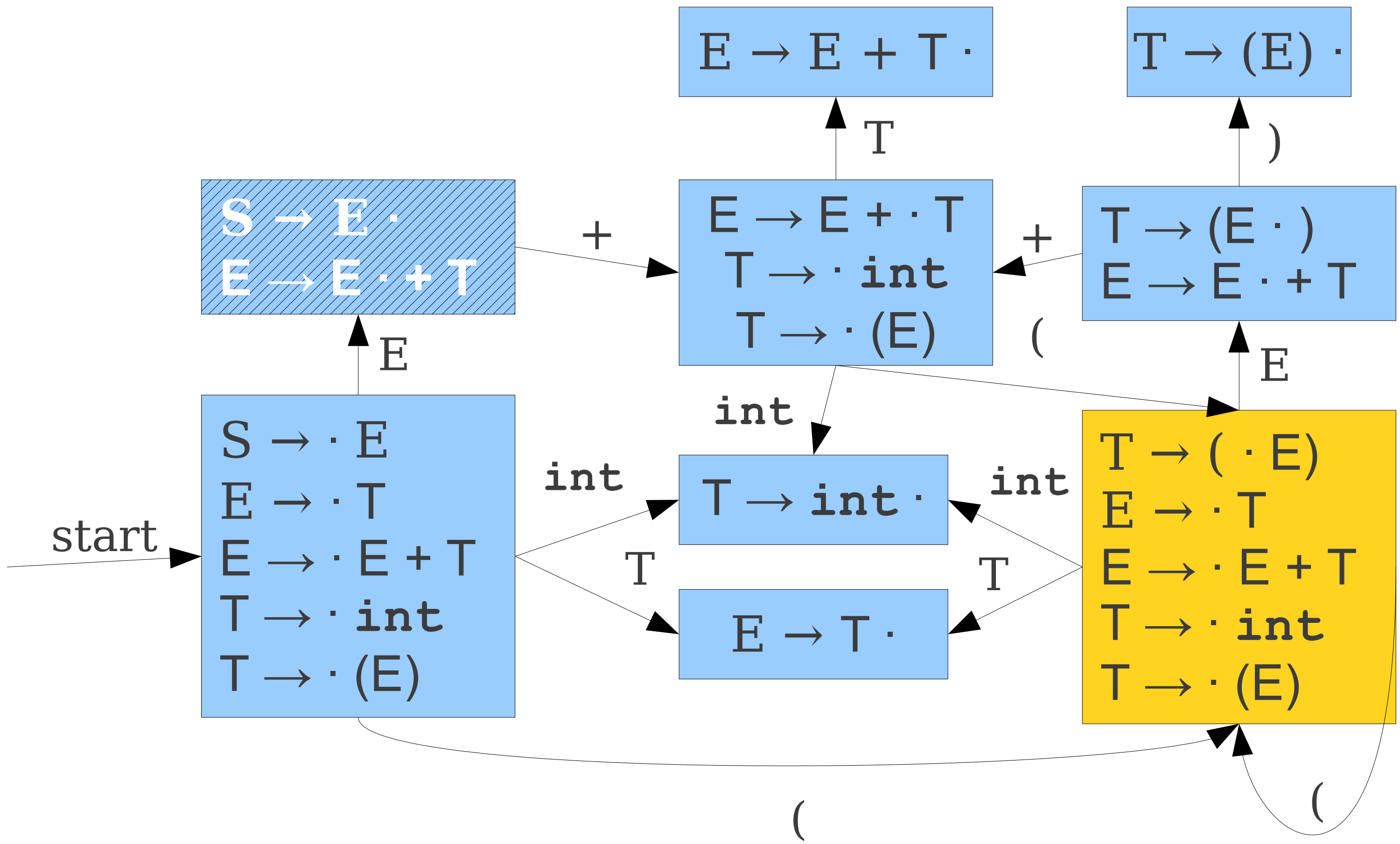
| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |
| E → · E + T | ) |




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| E | + | ( | | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

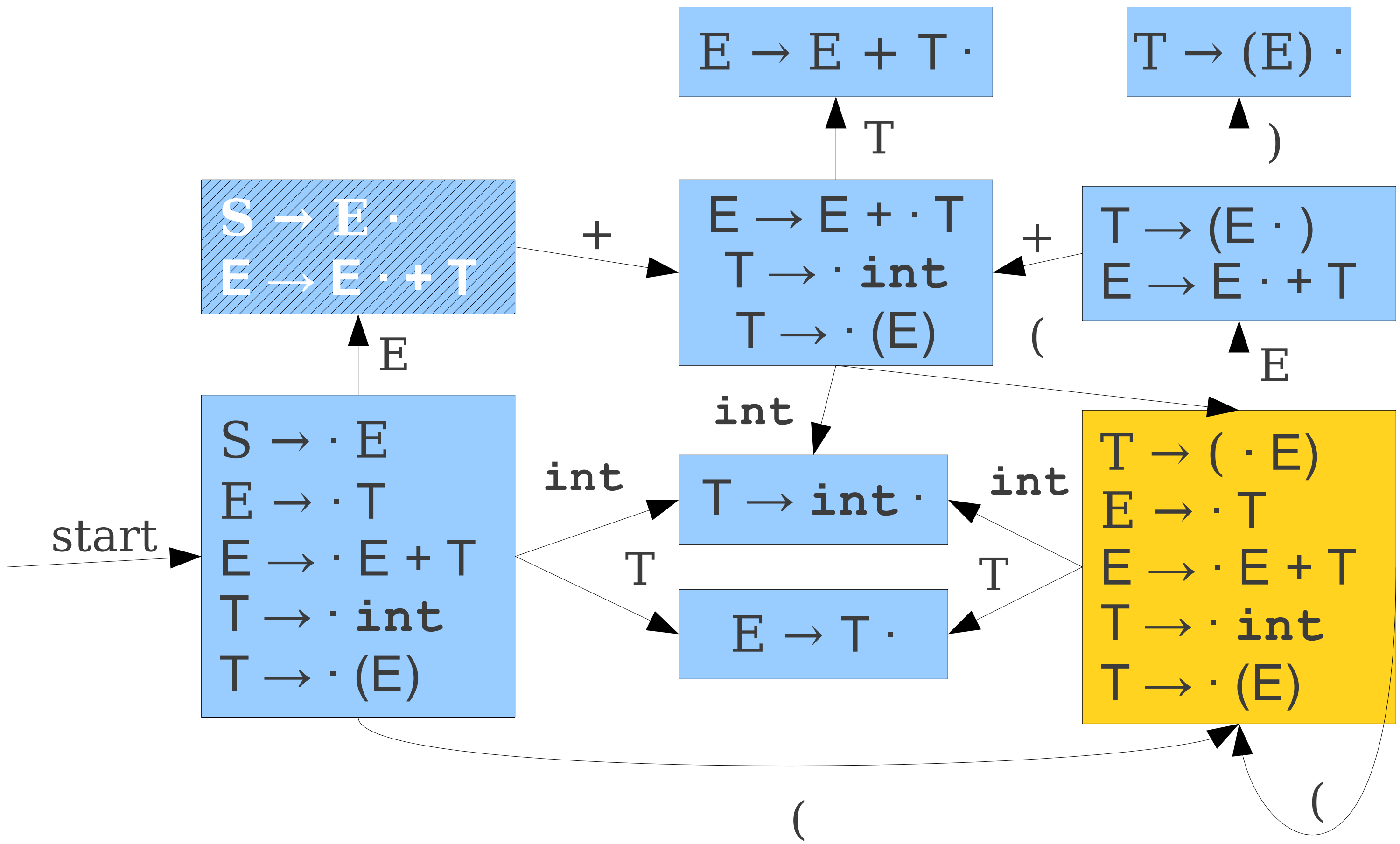



| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |
| E → · E + T | ) |
| E → · T | + |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| E | + | ( | int | + | int | + | int | ) | $ |

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**

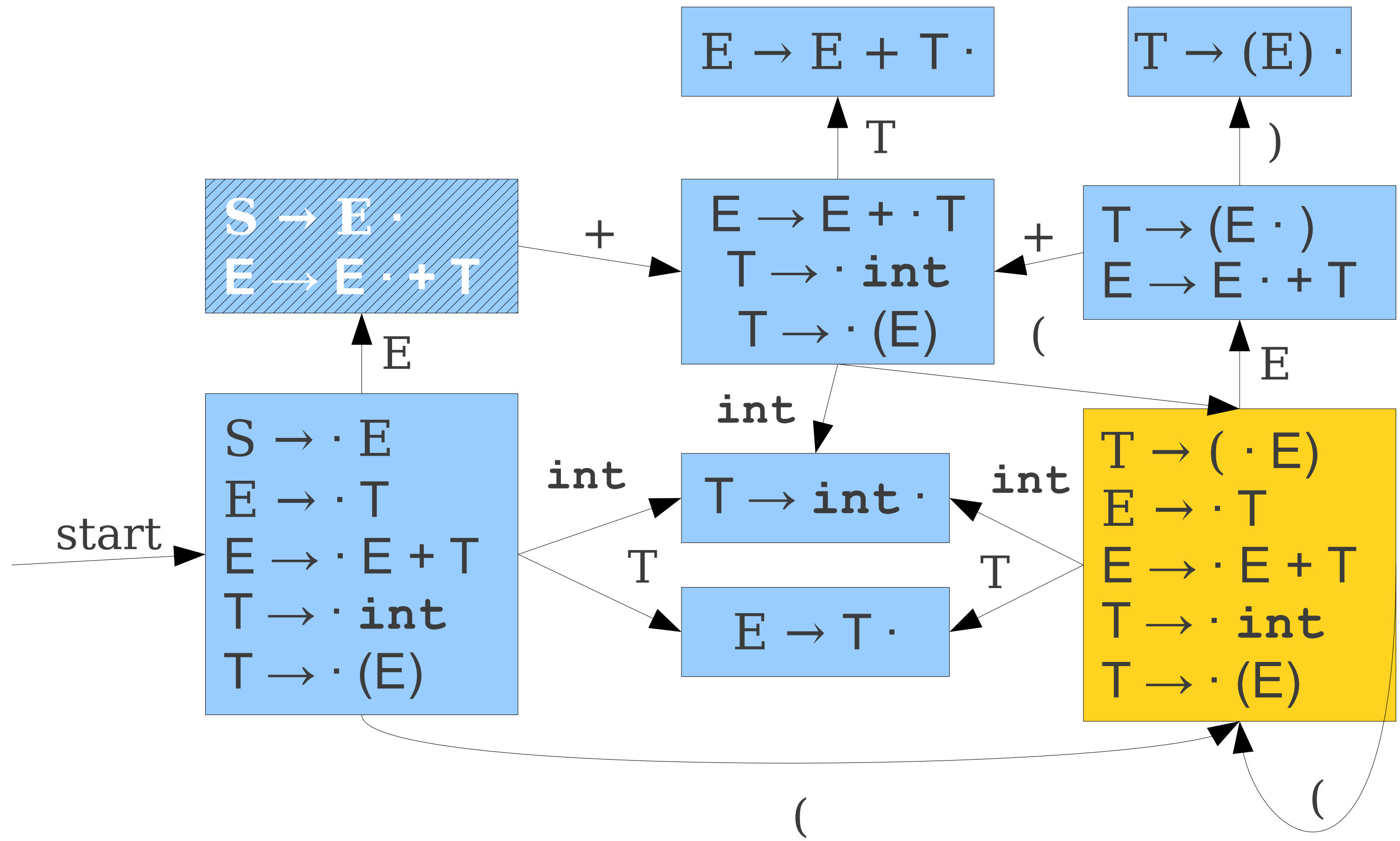



| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |
| E → · E + T | ) |
| E → · T | + |
| T → · int | + |

| E | + | ( | \| | int | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S** → **E**
**E** → **T**
**E** → **E + T**
**T** → **int**
**T** → **(E)**

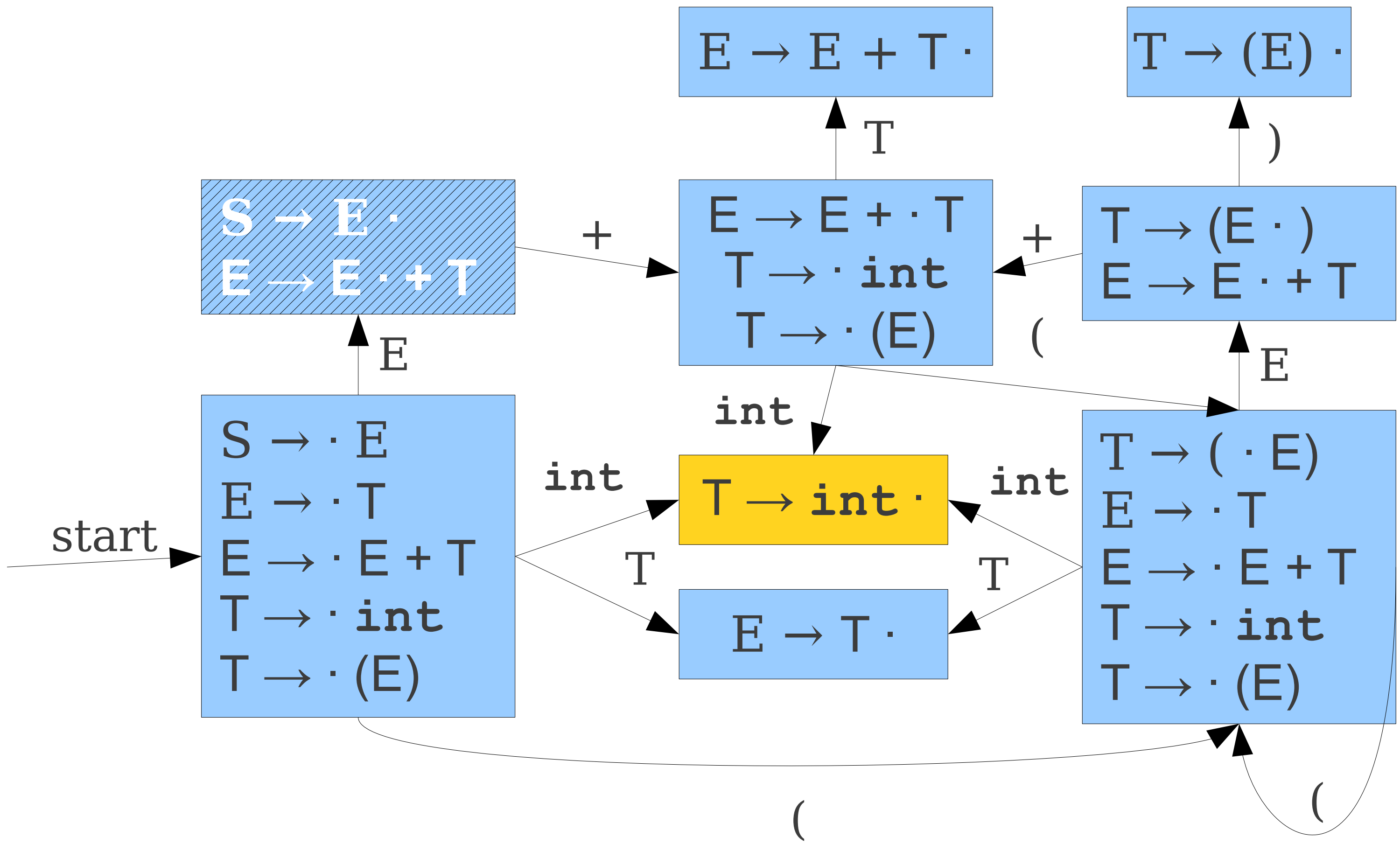



| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |
| E → · E + T | ) |
| E → · T | + |
| T → · int | + |

| E | + | ( | int | \| | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|---|

# LR(1) Parsing: The Intuition

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**




| | |
|---|---|
| S → · E | $ |
| E → E + · T | $ |
| T → ( · E) | $ |
| E → · E + T | ) |
| E → · T | + |
| T → int · | + |

| E | + | ( | int | \| | + | int | + | int | ) | $ |
|---|---|---|---|---|---|---|---|---|---|---|

# The Intuition behind LR(1)

- Guess which series of productions we are reversing.
- Use this information to maintain information about what lookahead to expect.
- When deciding whether to shift or reduce, use lookahead to disambiguate.

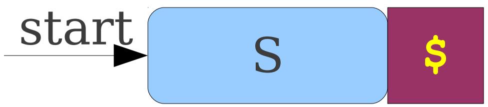
# Tracking Lookaheads

- How do we know what lookahead to expect at each state?
- Observation:
  - There are only finitely many productions we can be in at any point.
  - There are only finitely many positions we can be in each production.
  - **There are only finitely many lookahead sets** at each point.
- Construct an automaton to track lookaheads!

# Constructing LR(1) Automata

**S** → **E**
**E** → **T**
**E** → **E** `+` **T**
**T** → `int`
**T** → `(`**E**`)`

# Constructing LR(1) Automata




**S** → **E**

**E** → **T**

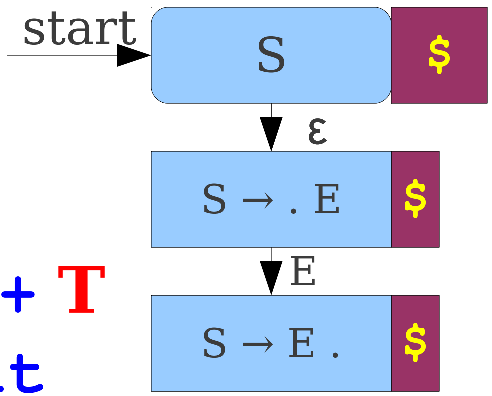
**E** → **E** **+** **T**

**T** → **int**

**T** → **(E)**

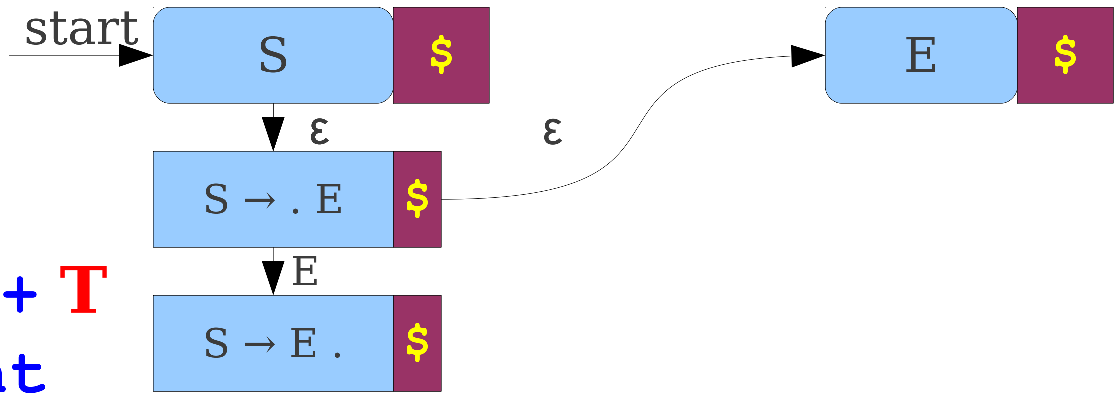
# Constructing LR(1) Automata

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



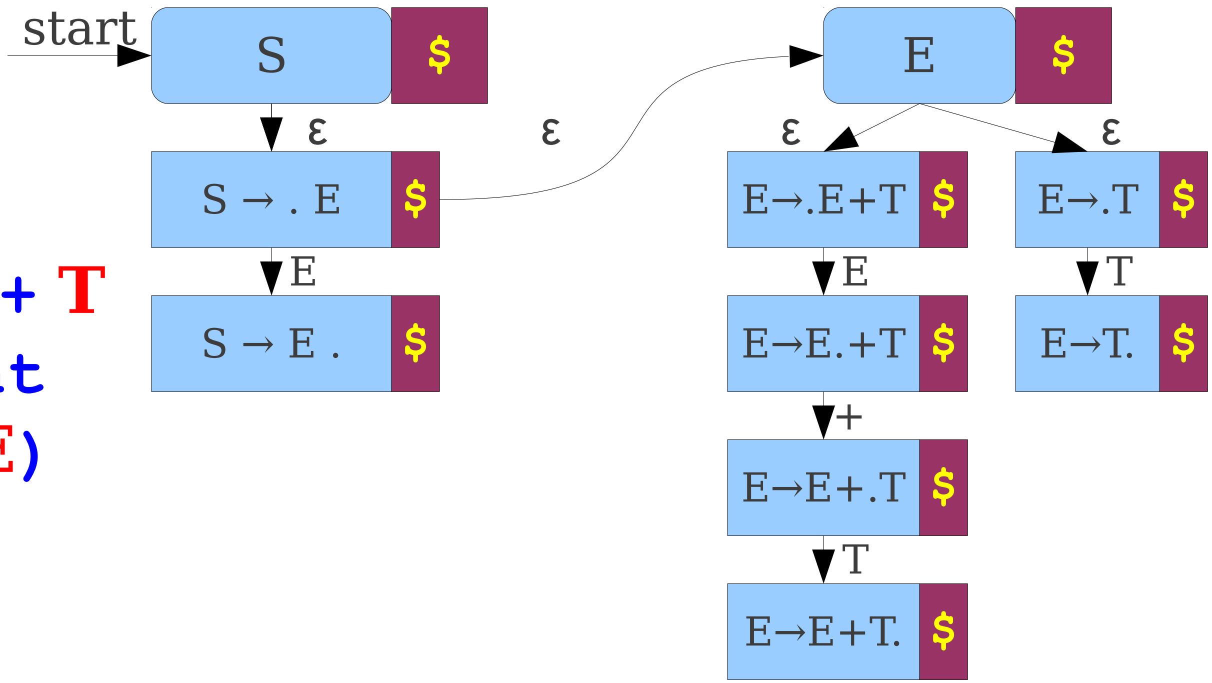
# Constructing LR(1) Automata

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



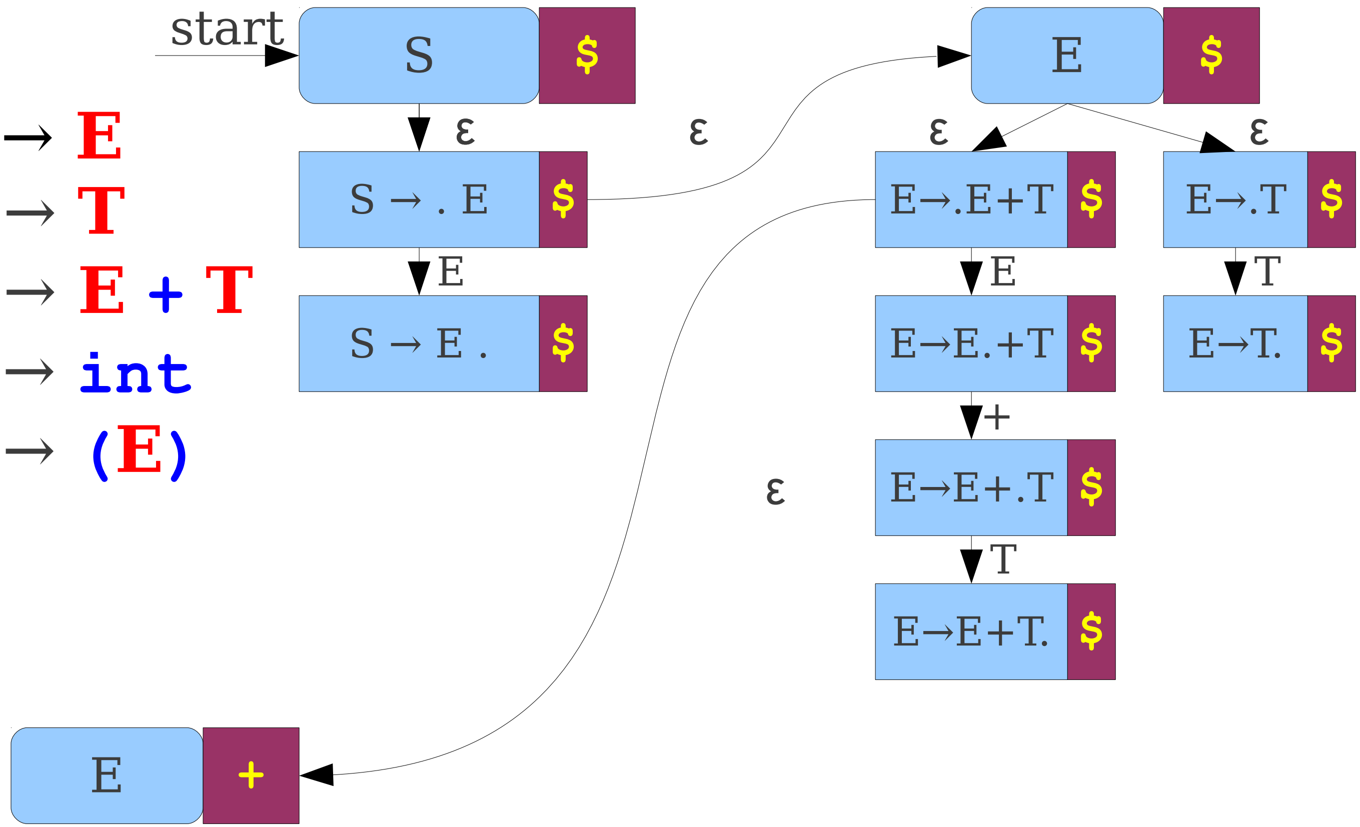
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



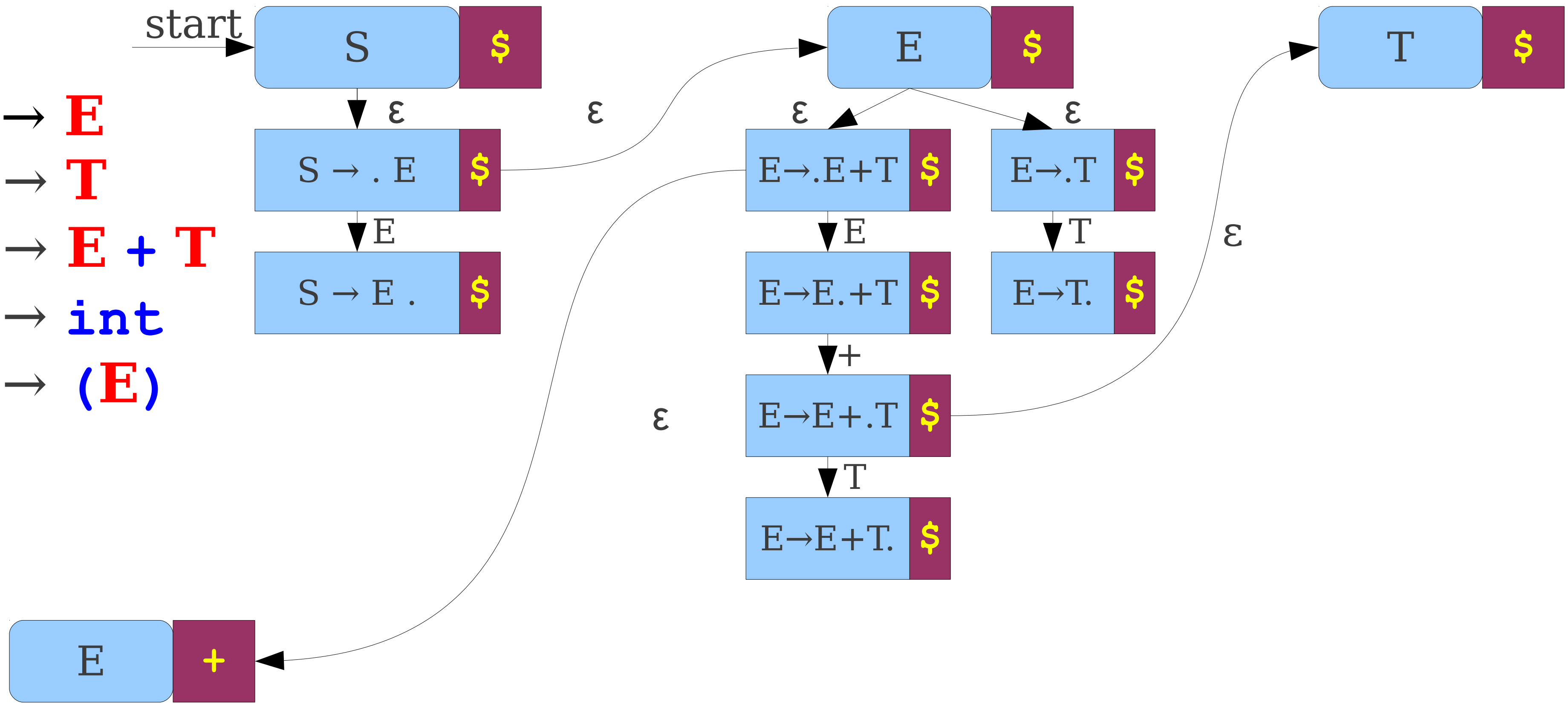
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → `int`**
**T → (E)**



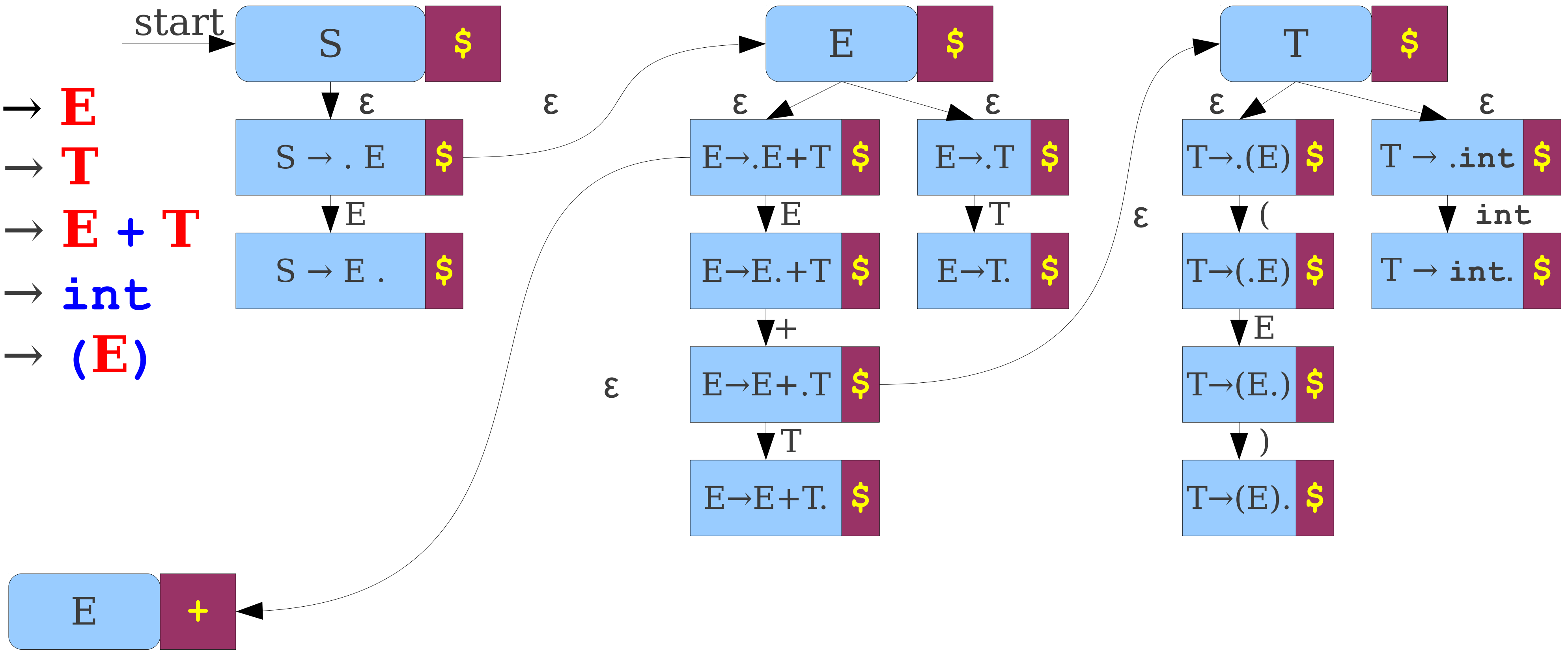
# Constructing LR(1) Automata

**S** → **E**
**E** → **T**
**E** → **E** **+** **T**
**T** → **int**
**T** → **(E)**



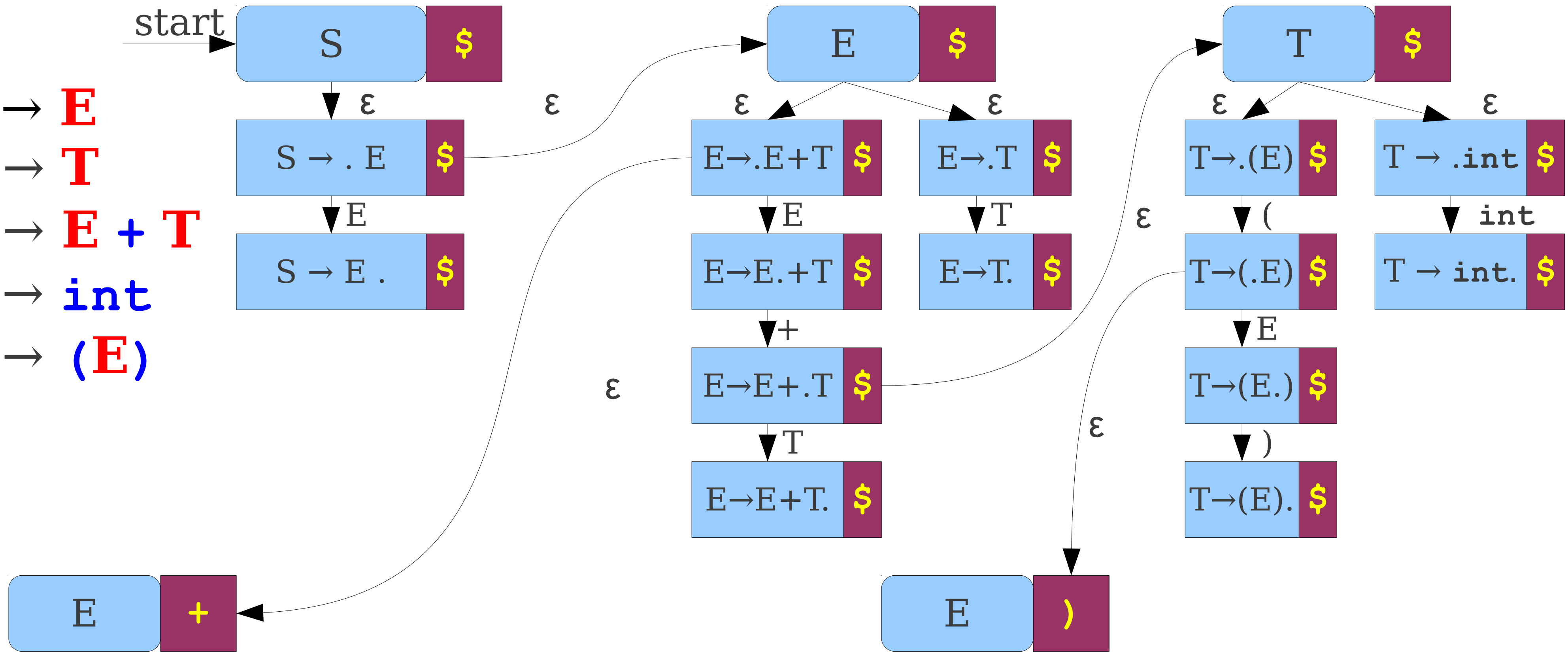
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



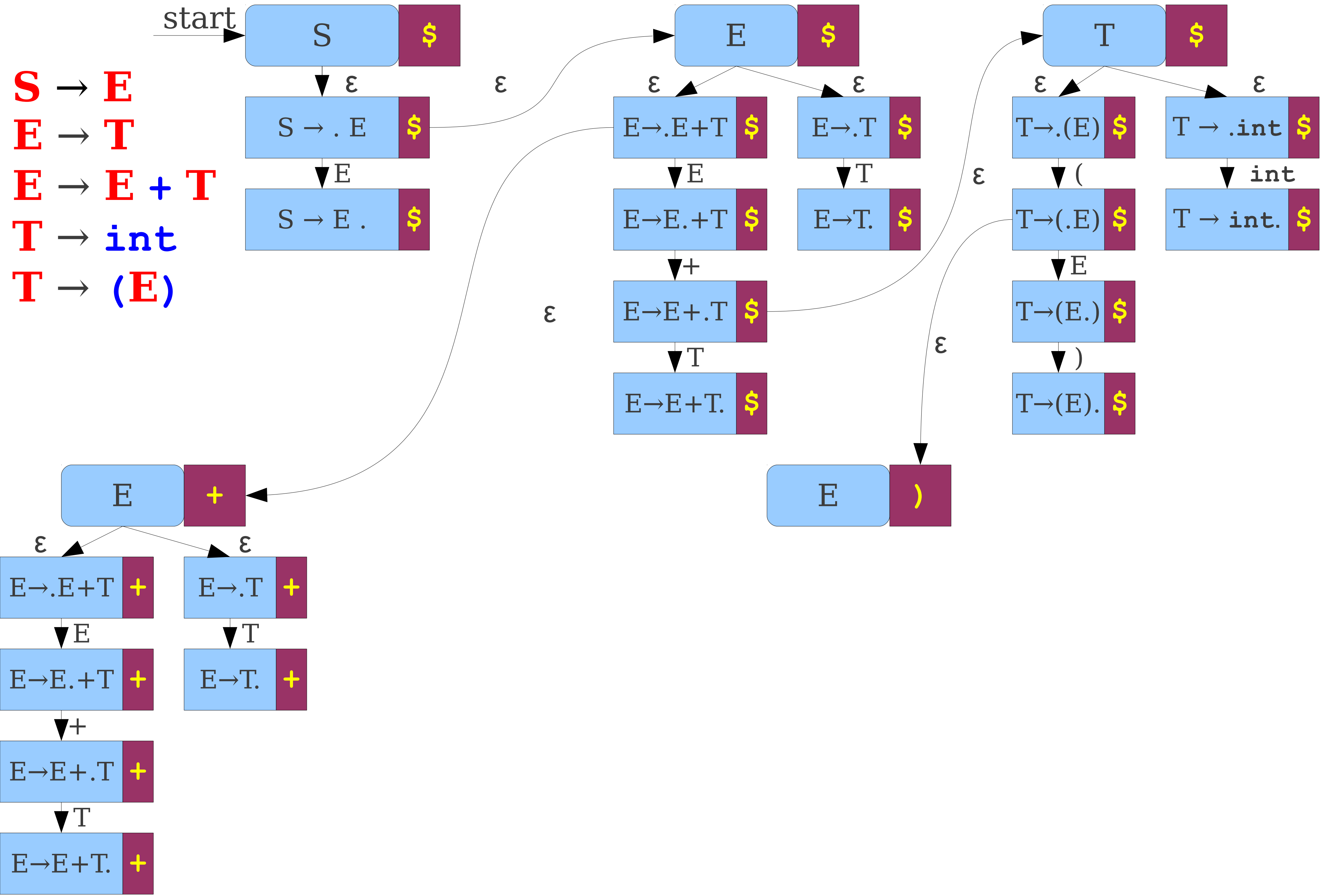
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



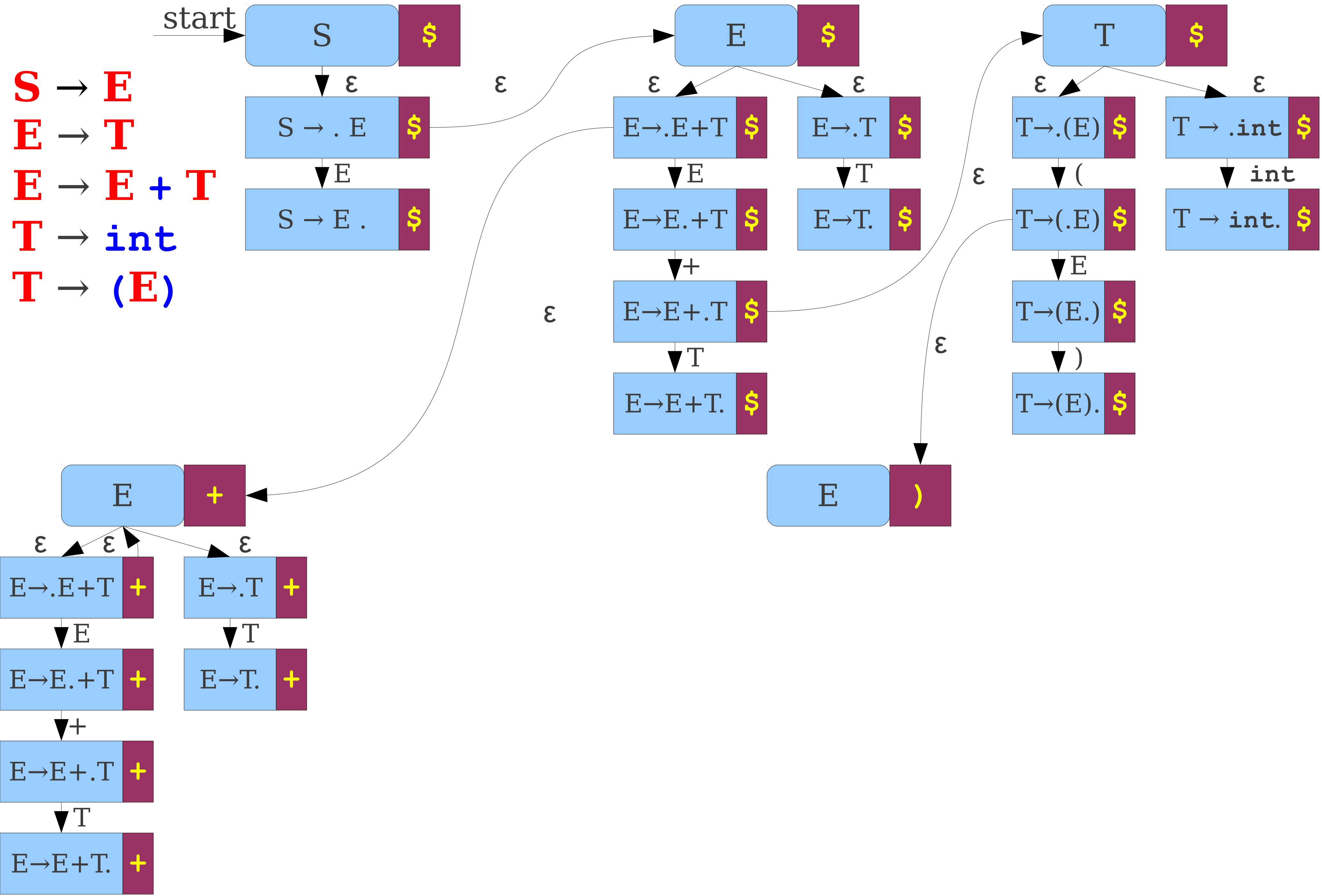
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

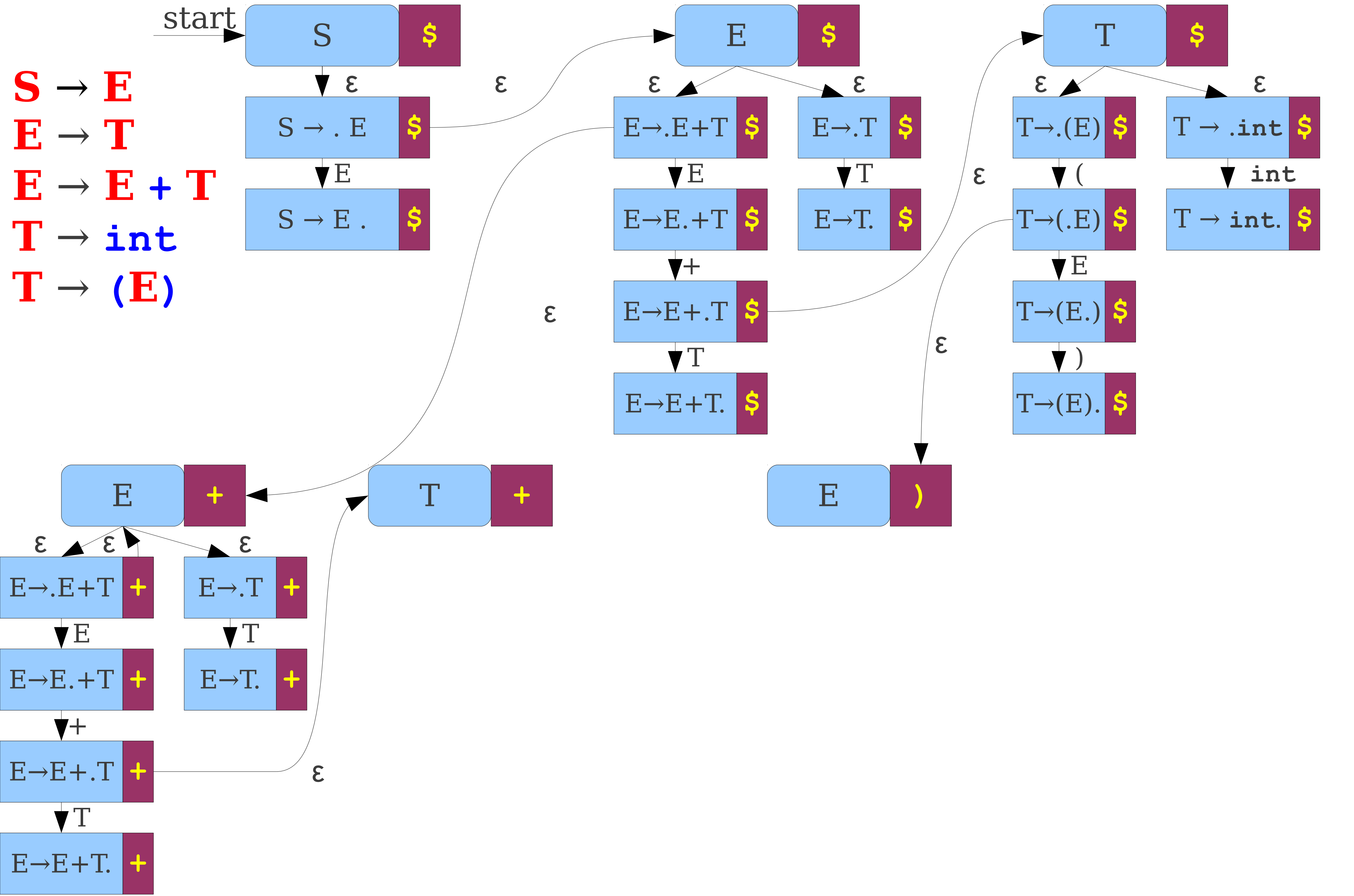


# Constructing LR(1) Automata

S → E
E → T
E → E + T
T → int
T → (E)

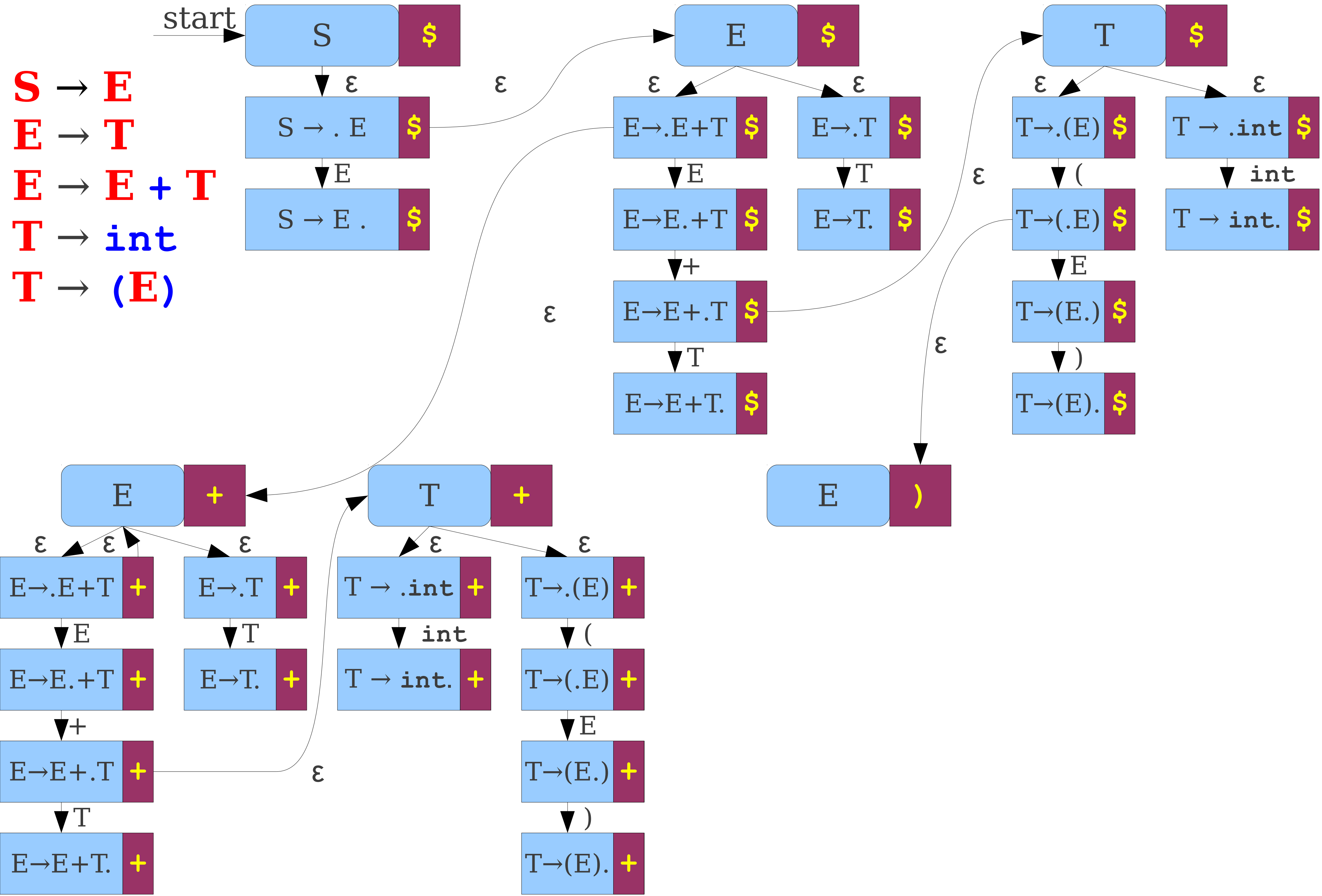


# Constructing LR(1) Automata



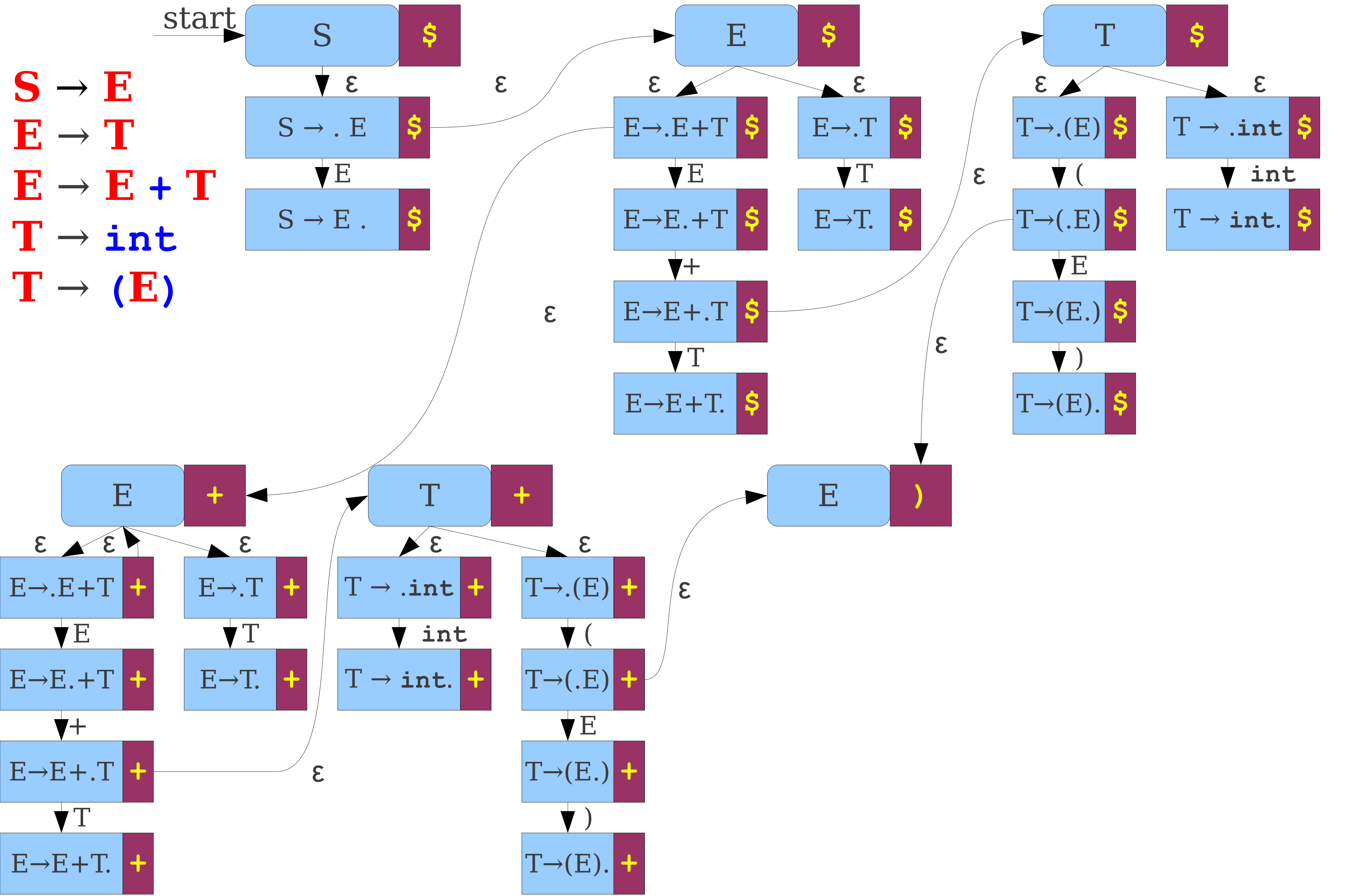
# Constructing LR(1) Automata

S → E
E → T
E → E + T
T → int
T → (E)



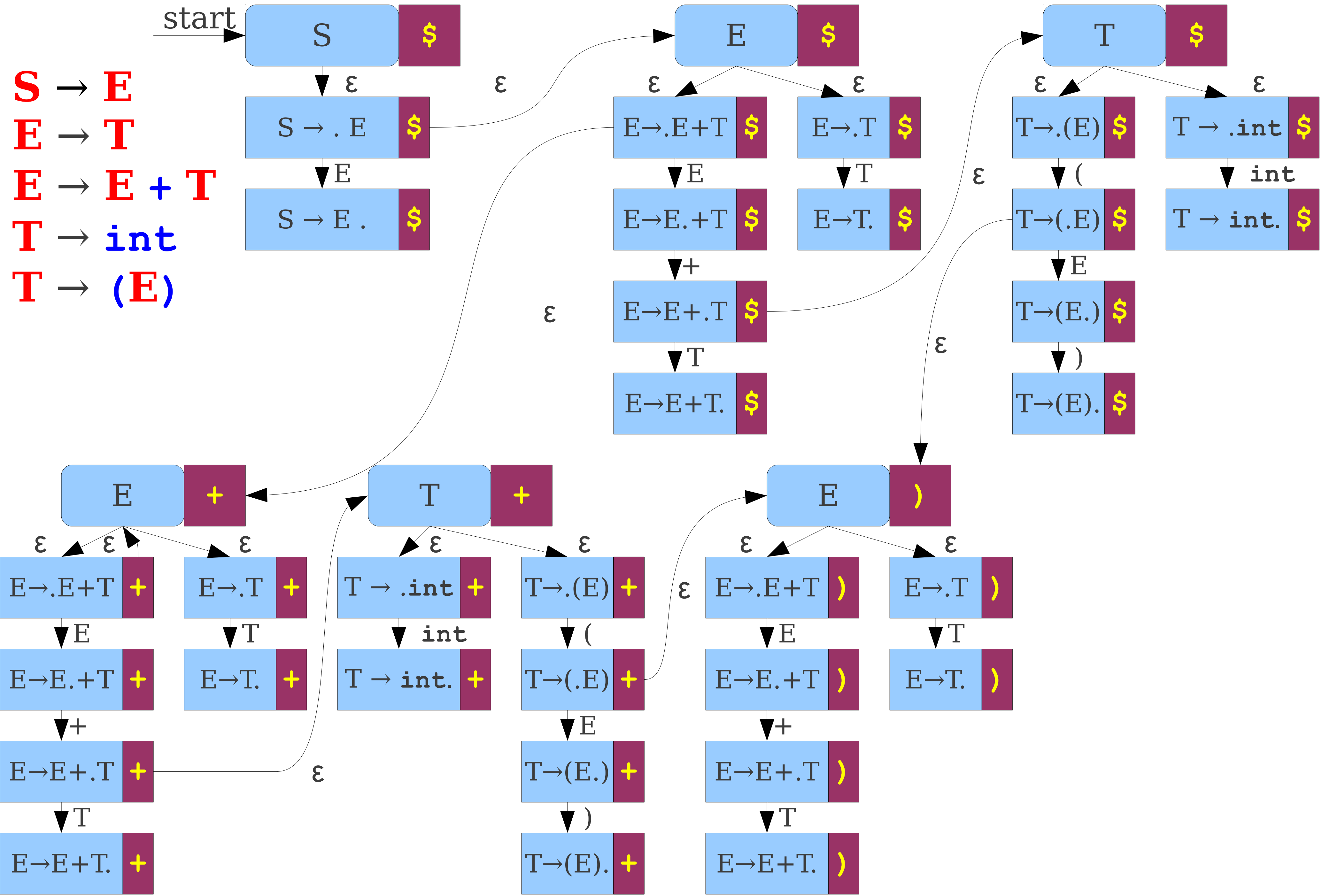
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



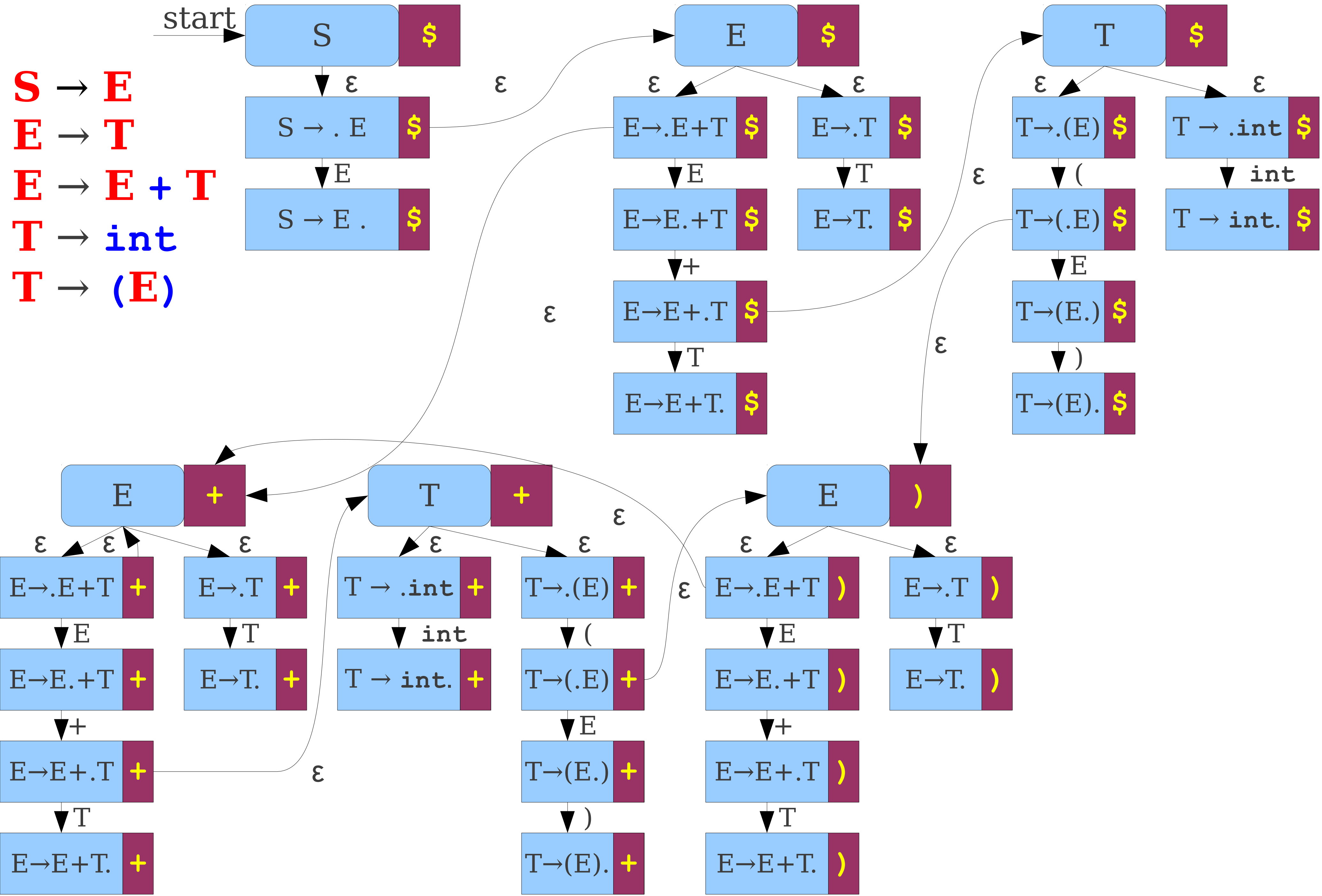
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



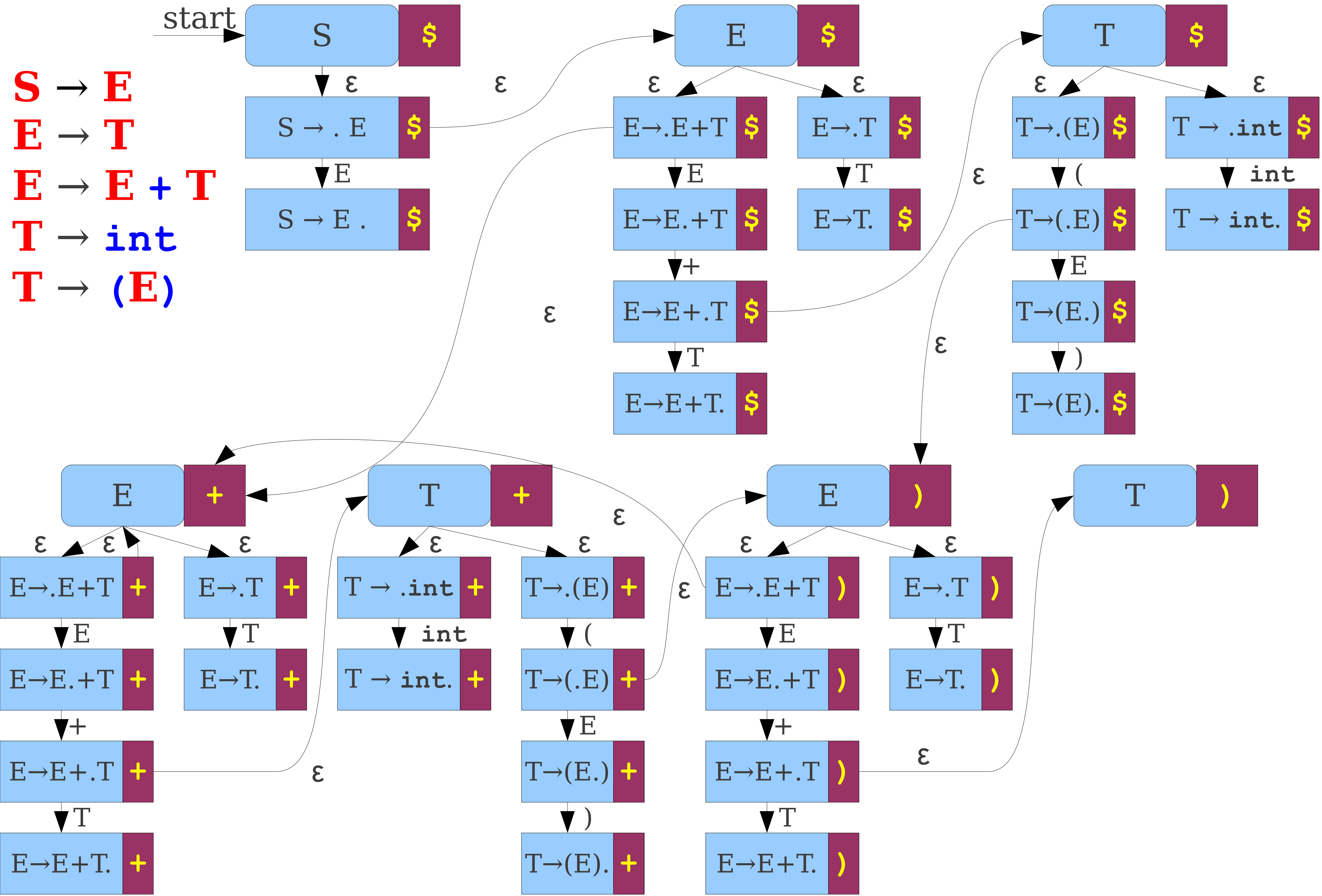
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



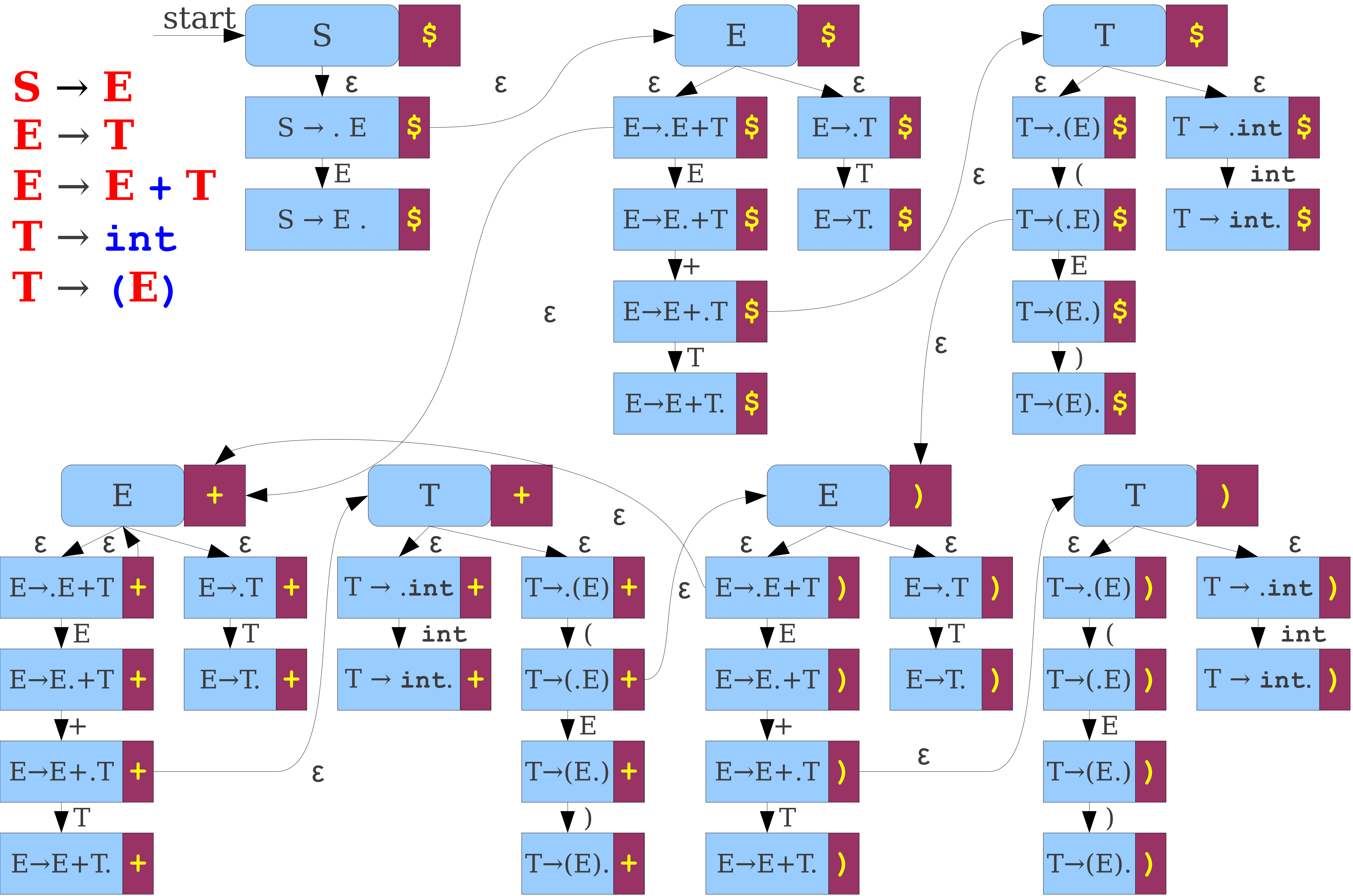
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



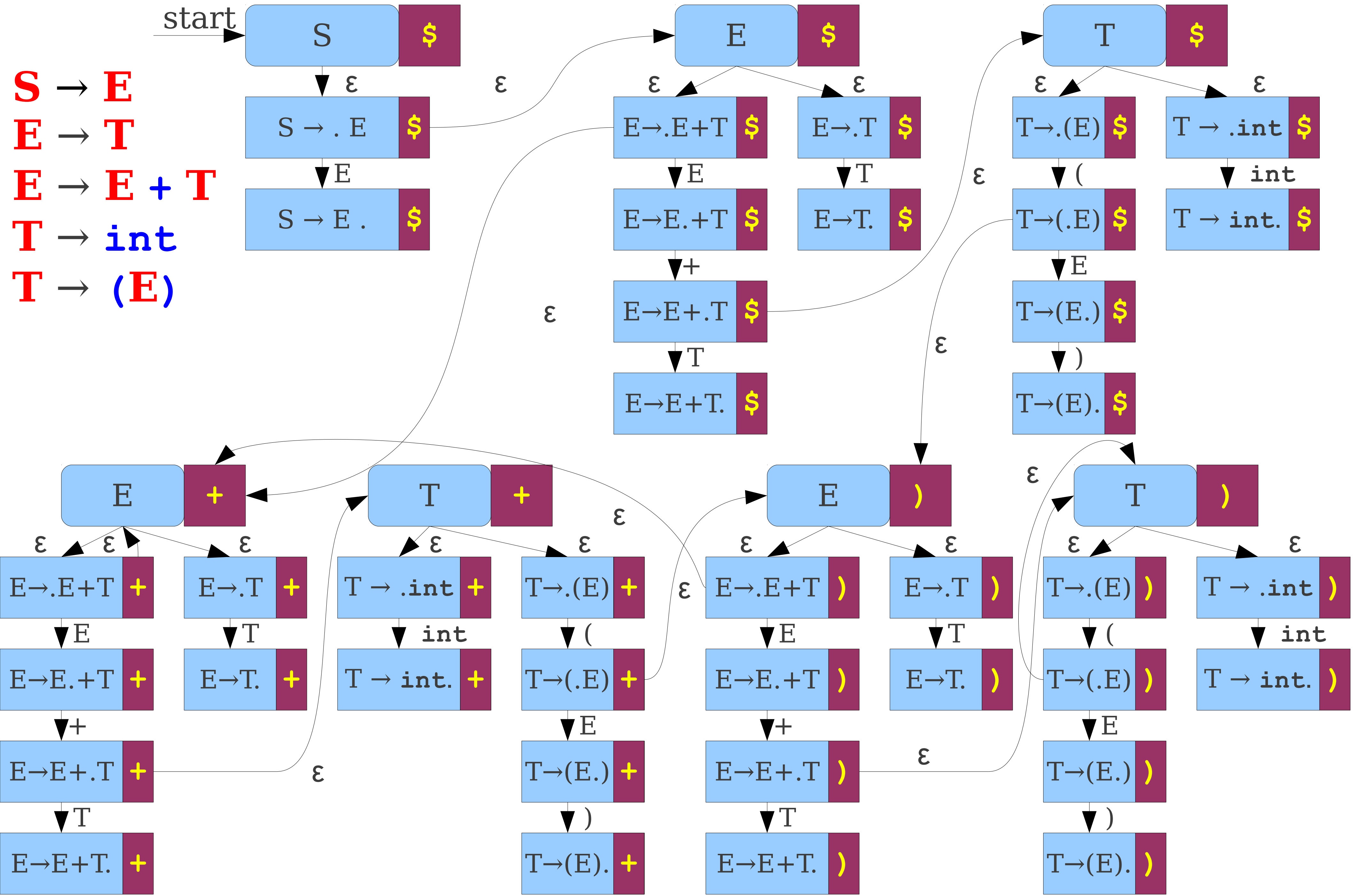
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



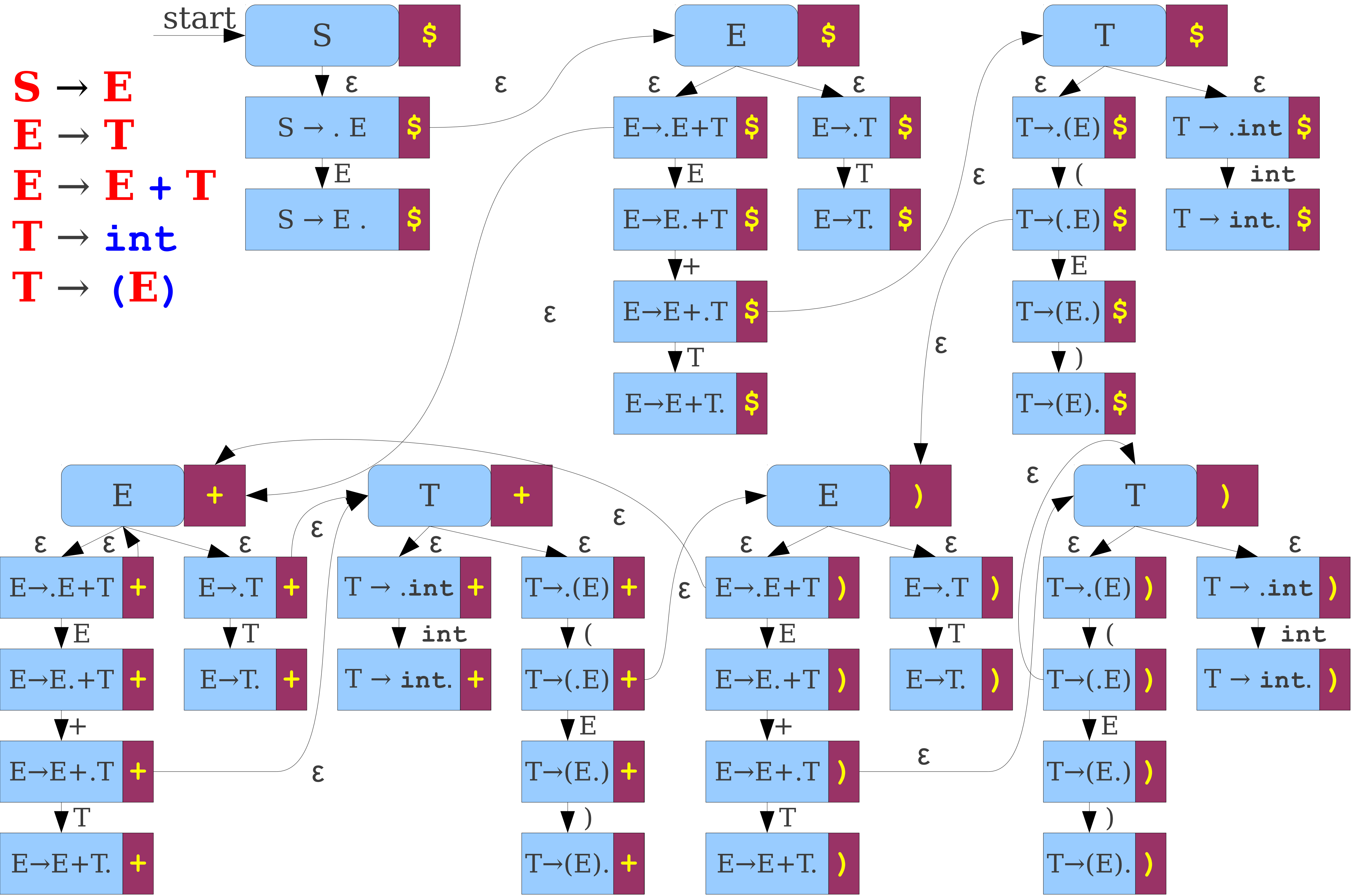
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



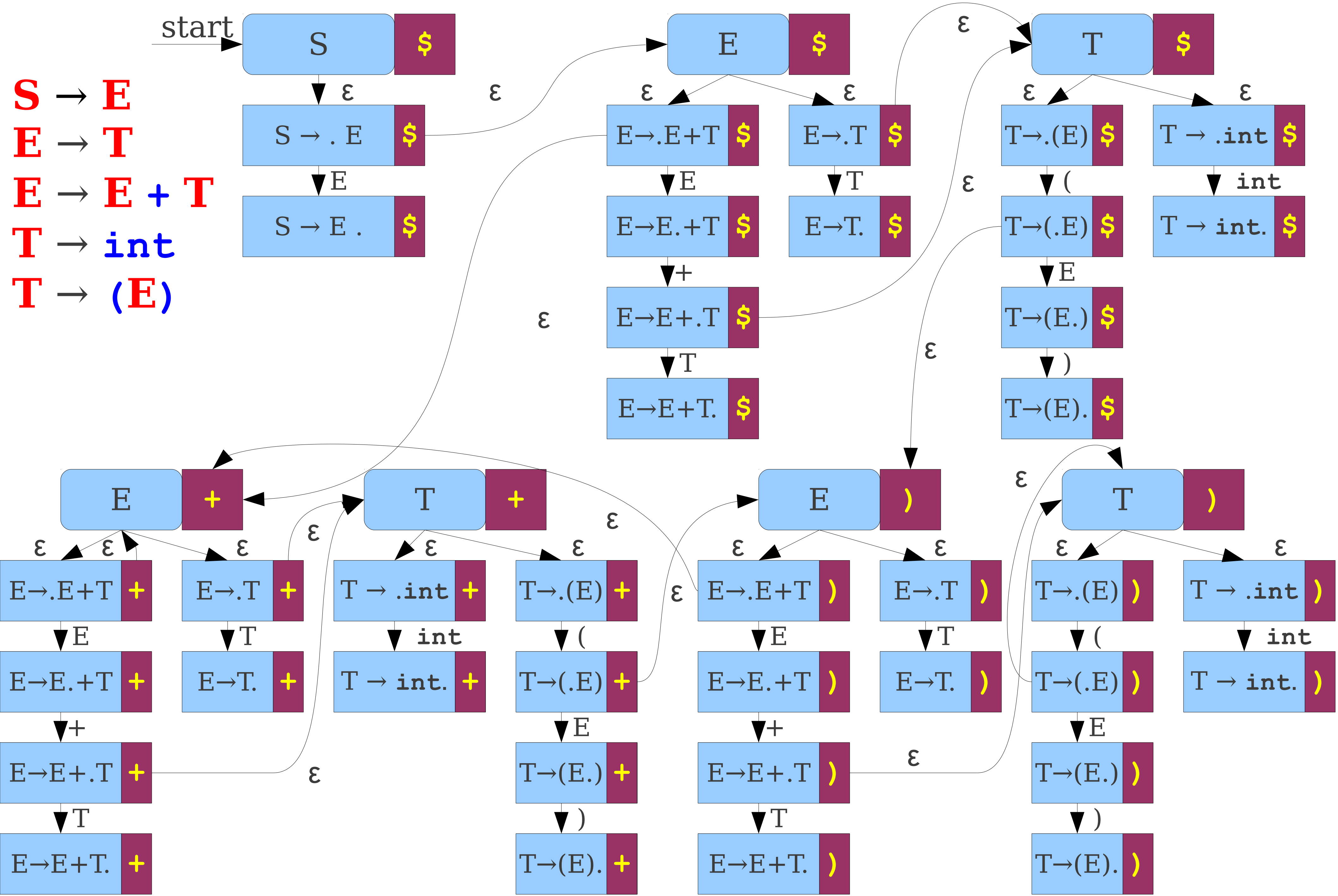
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



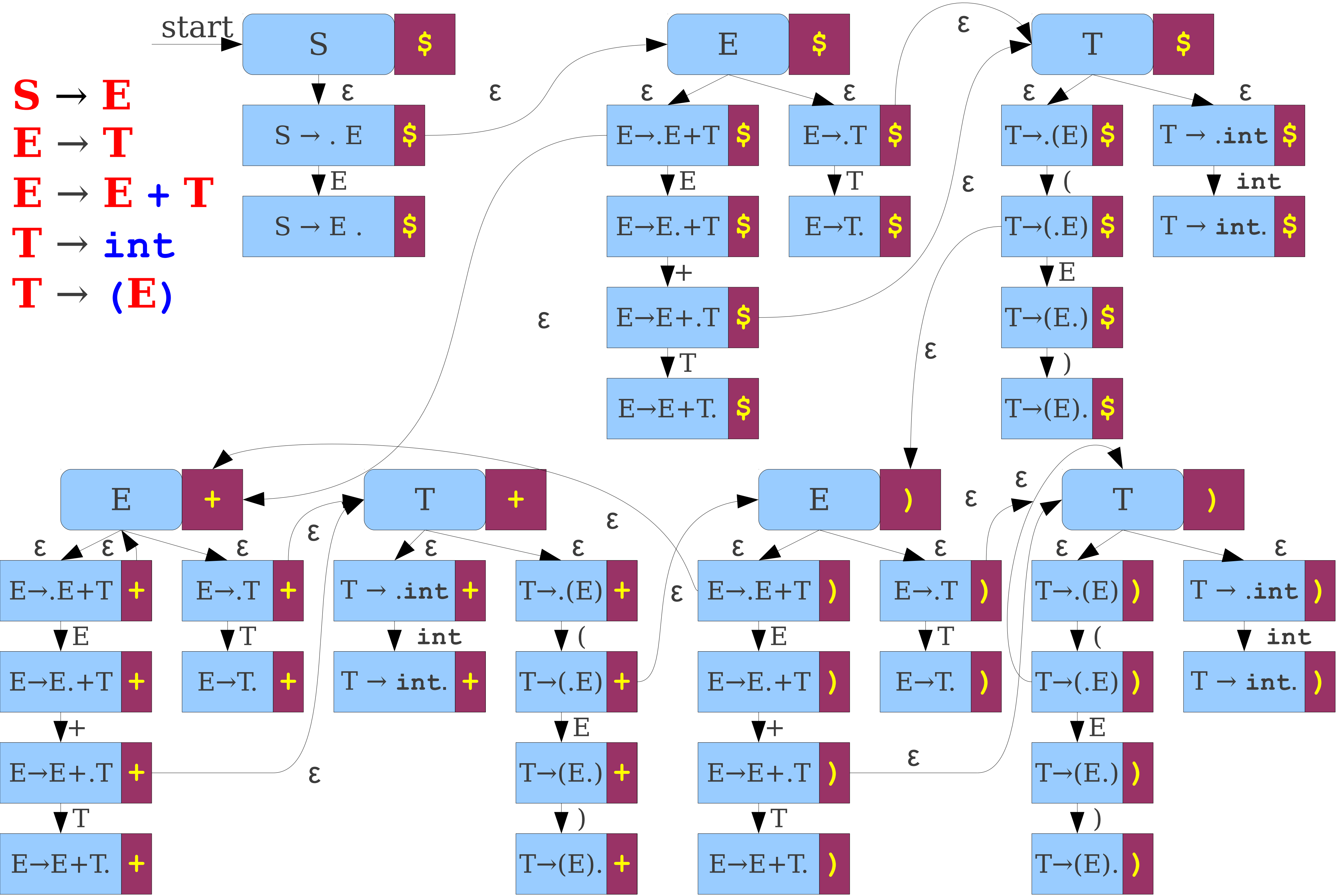
# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**



# Constructing LR(1) Automata

**S → E**
**E → T**
**E → E + T**
**T → int**
**T → (E)**

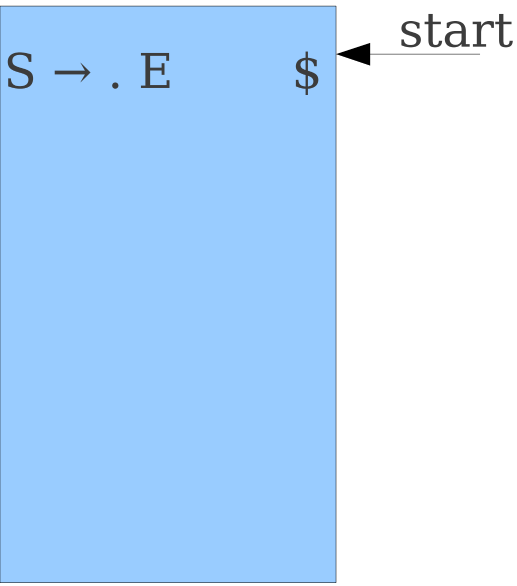


# Constructing LR(1) Automata

- Begin with a state **S** [**$**].
- For each state **A** [**t**], for each production **A** → ***γ***:
  - Construct states **A** → ***α·ω*** [**t**] for all possible ways of splitting ***γ*** = ***αω***.
  - Add an ε-transition from **A** [**t**] to each of these states.
  - Add transitions on ***x*** between **A** → ***α*** · ***xω*** [**t**] and **A** → ***αx*** · ***ω*** [**t**]
- For each state **A** → ***α*** · **B*ω*** [**t**], add an ε-transition from **A** → ***α*** · **B*ω*** [**t**] to **B** [**r**] for each terminal **r** ∈ FIRST*(***ω*t**).

# Deterministic LR(1) Automata

# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

start

S → . E $
E → . T $
E → . E + T $

# Deterministic LR(1) Automata

start

| | |
|---|---|
| S → . E | $ |
| E → . T | $ |
| E → . E + T | $ |
| E → . T | + |
| E → . E + T | + |

# Deterministic LR(1) Automata

start

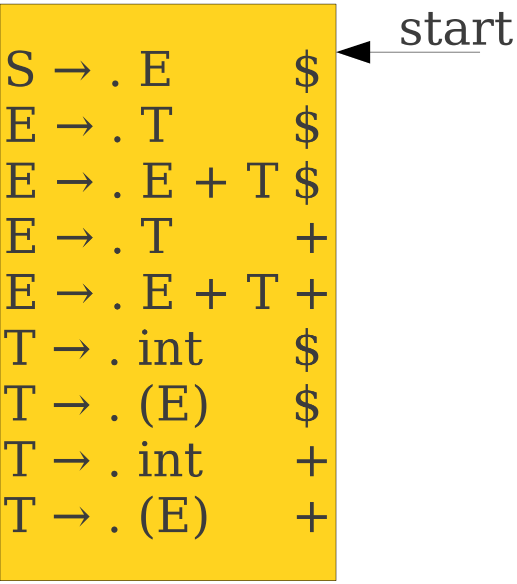
| | |
|---|---|
| S → . E | $ |
| E → . T | $ |
| E → . E + T | $ |
| E → . T | + |
| E → . E + T | + |
| T → . int | $ |
| T → . (E) | $ |

# Deterministic LR(1) Automata

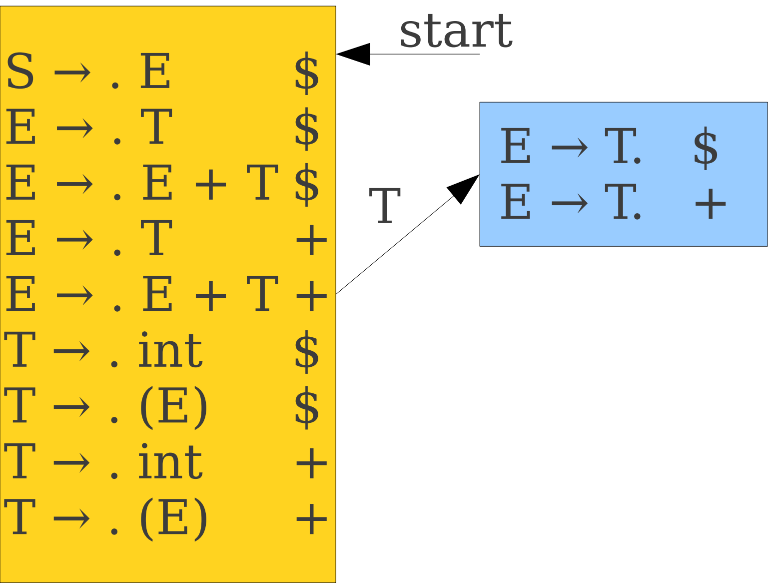
start

| | |
|---|---|
| S → . E | $ |
| E → . T | $ |
| E → . E + T | $ |
| E → . T | + |
| E → . E + T | + |
| T → . int | $ |
| T → . (E) | $ |
| T → . int | + |
| T → . (E) | + |

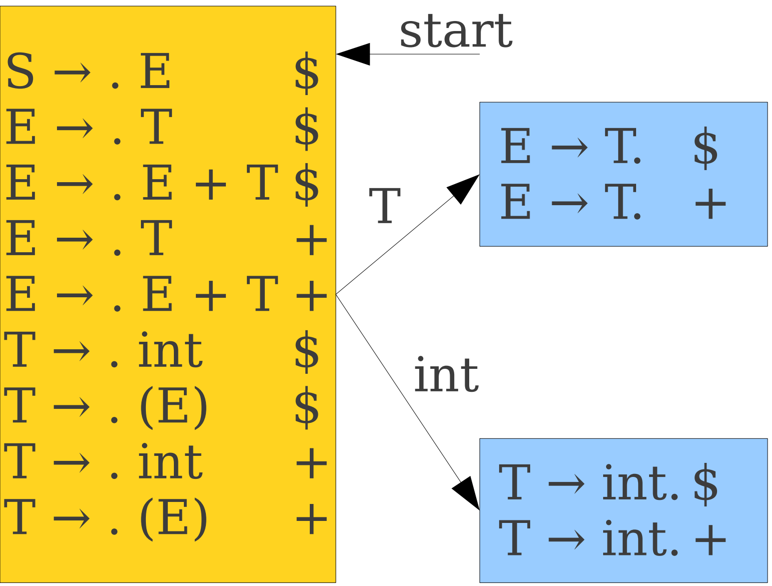
# Deterministic LR(1) Automata



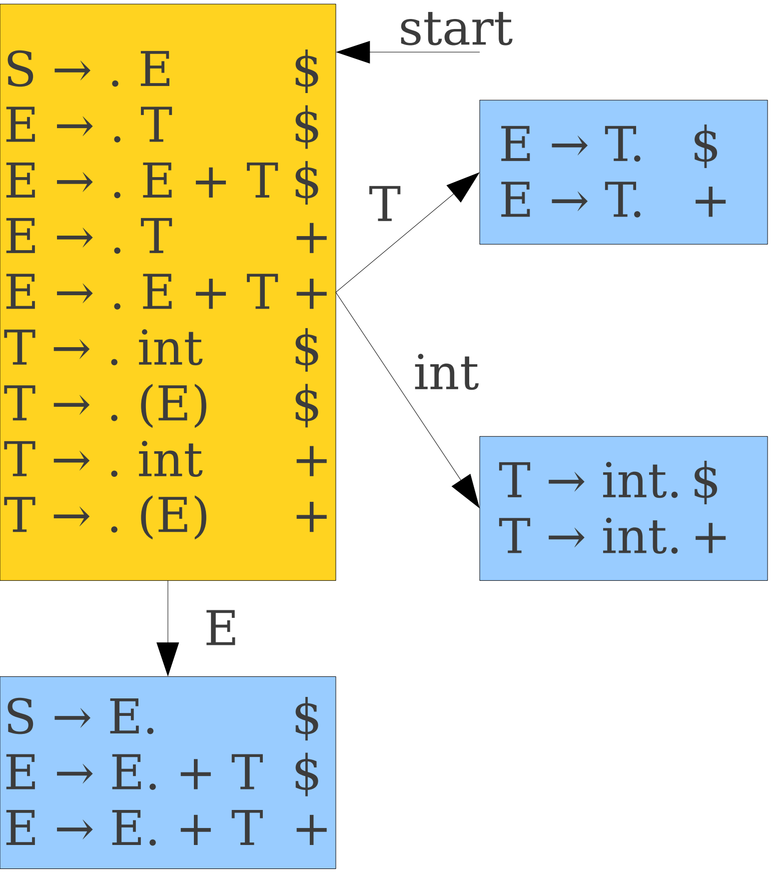
# Deterministic LR(1) Automata



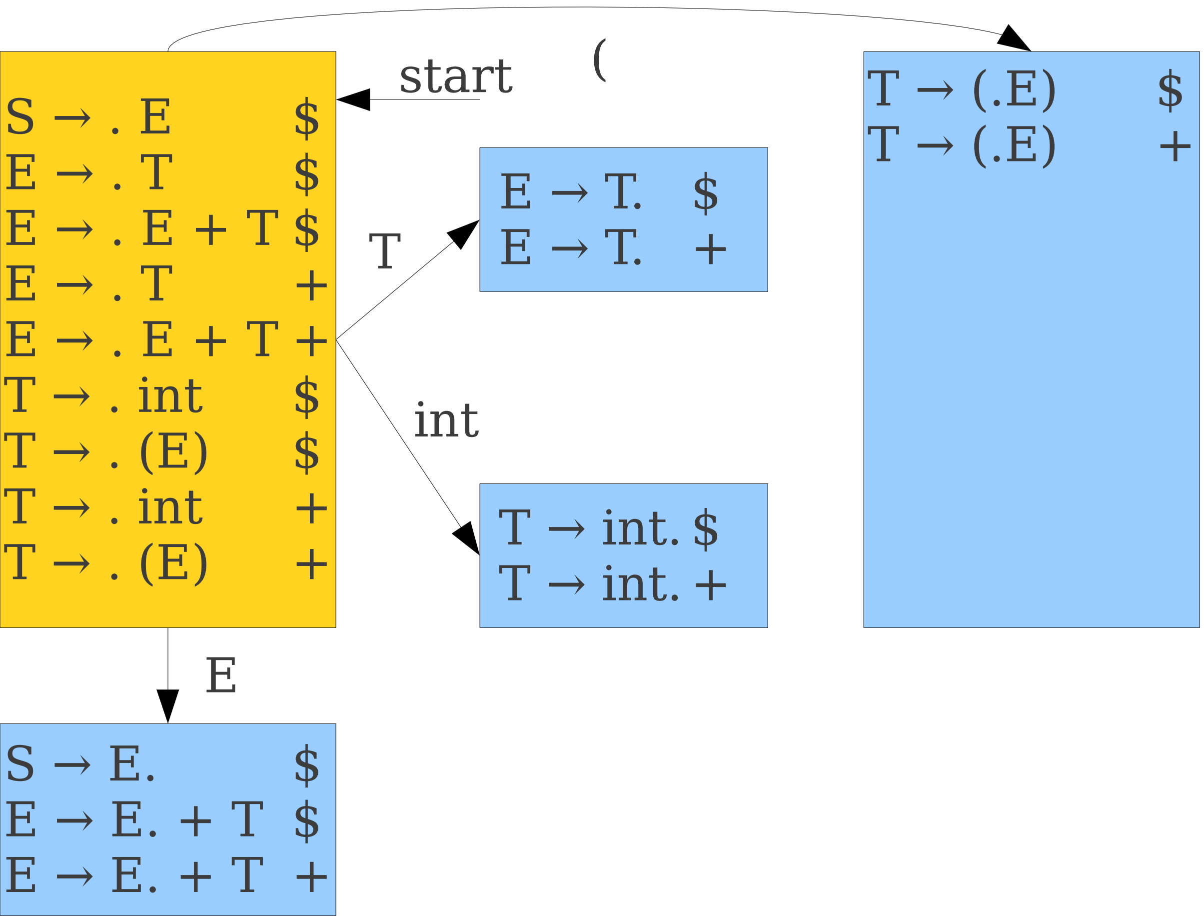
# Deterministic LR(1) Automata



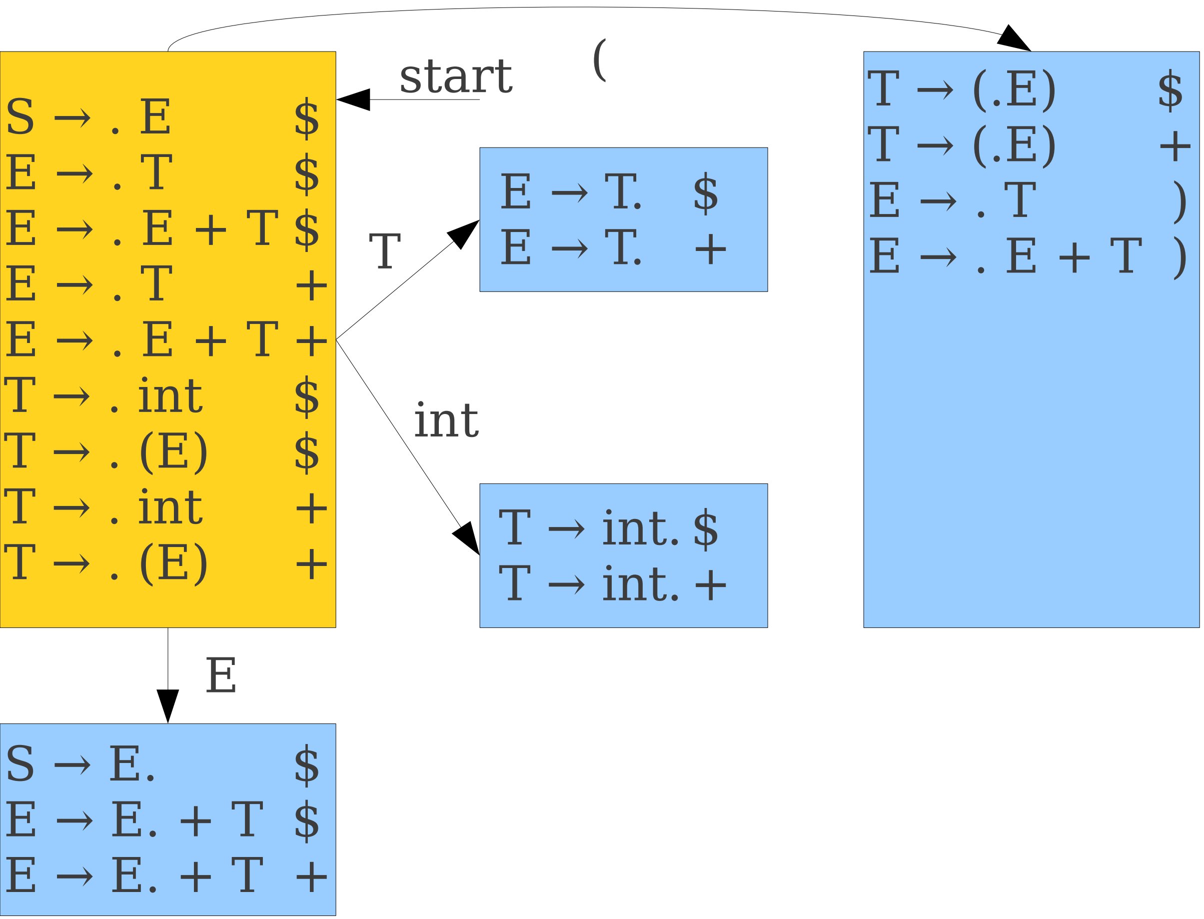
# Deterministic LR(1) Automata



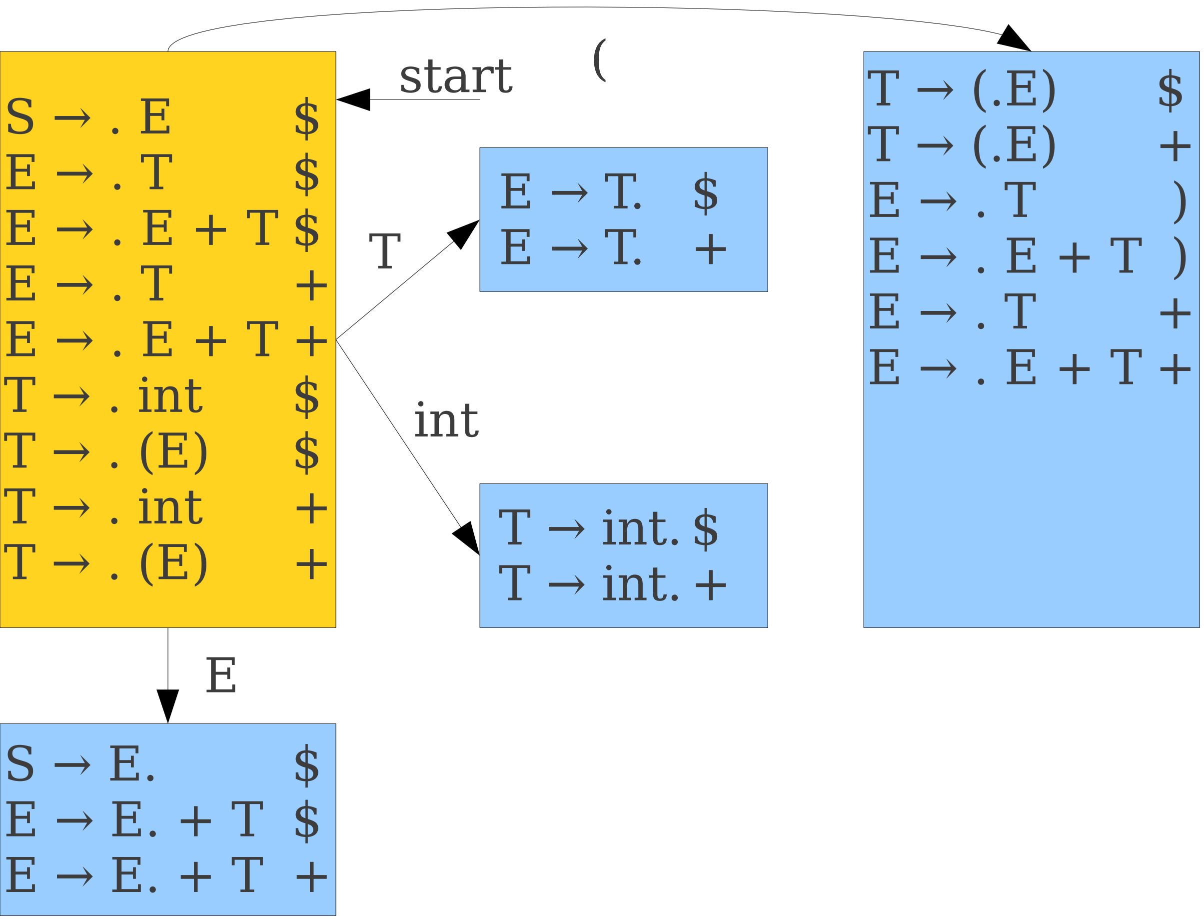
# Deterministic LR(1) Automata



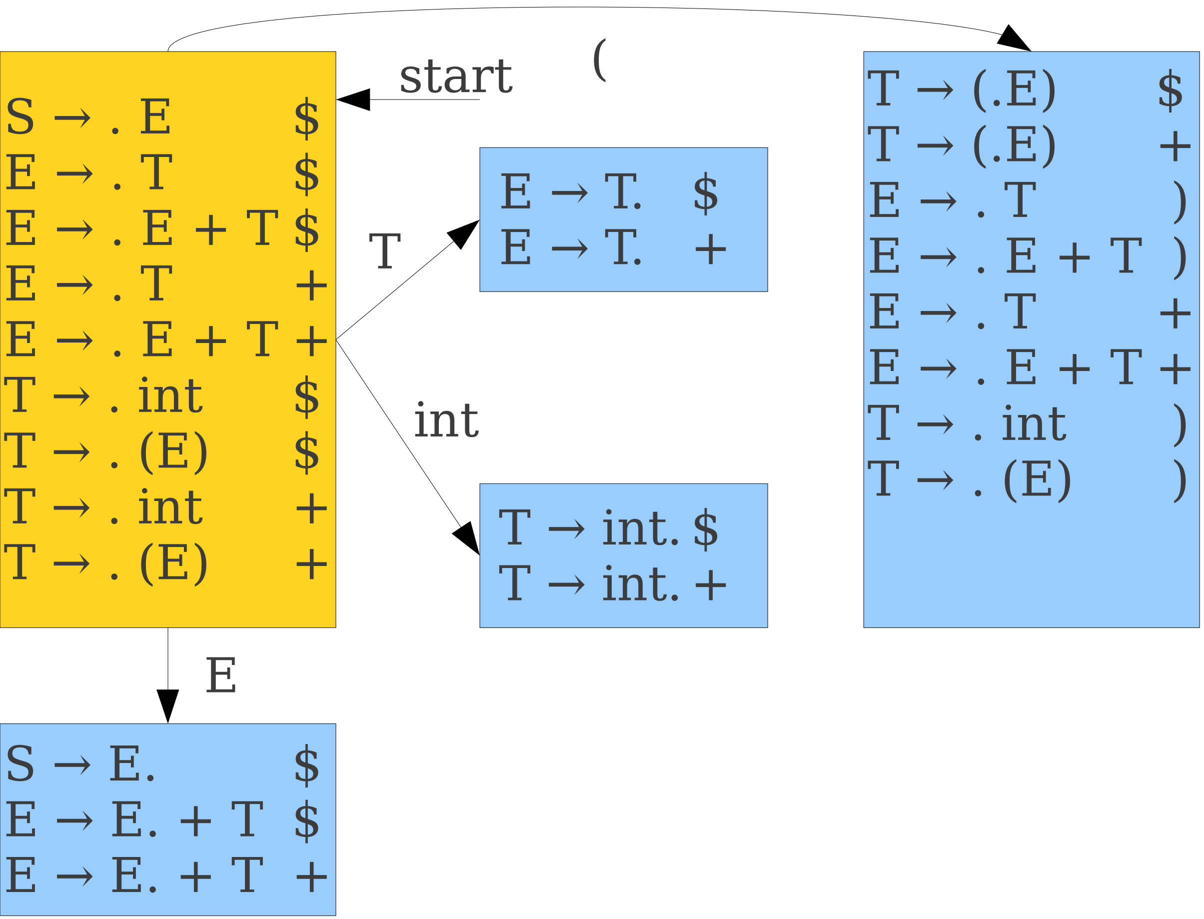
# Deterministic LR(1) Automata



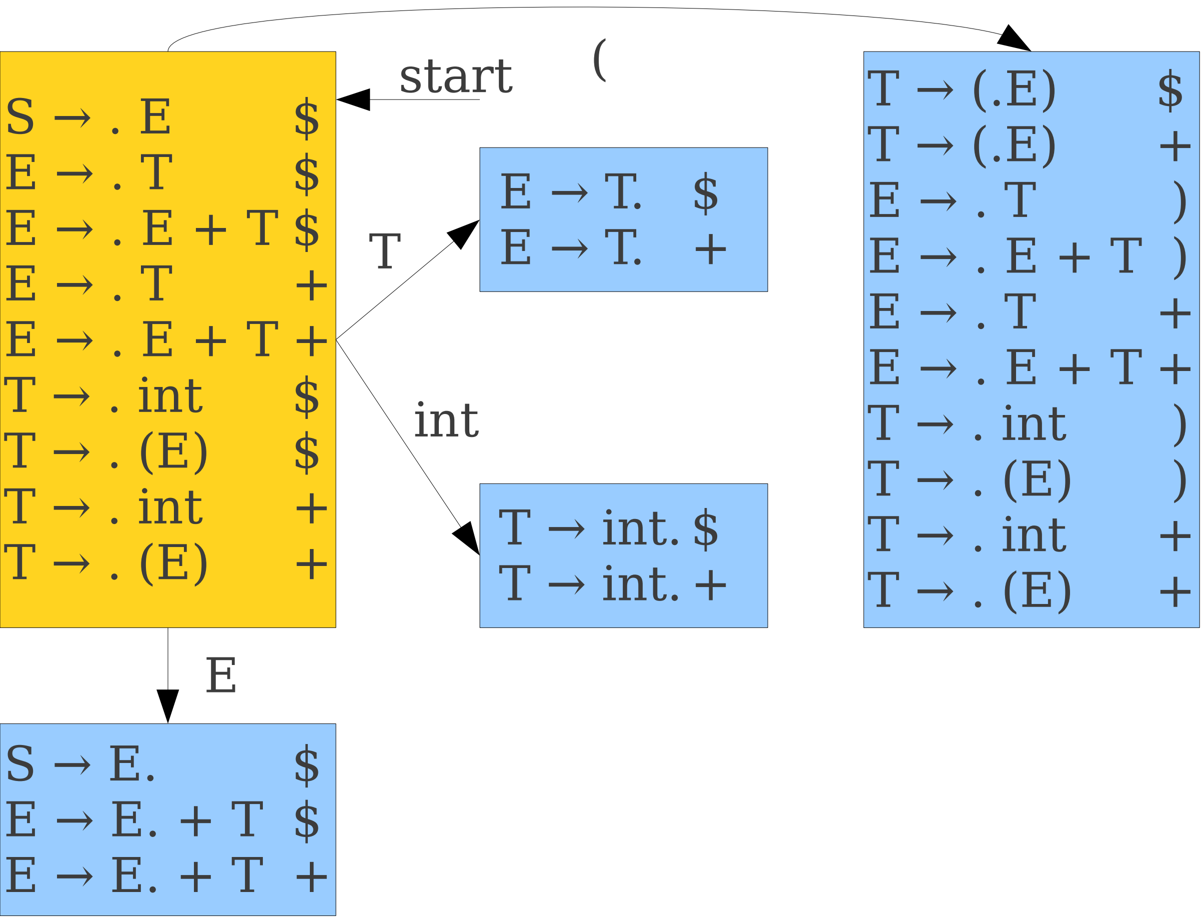
# Deterministic LR(1) Automata



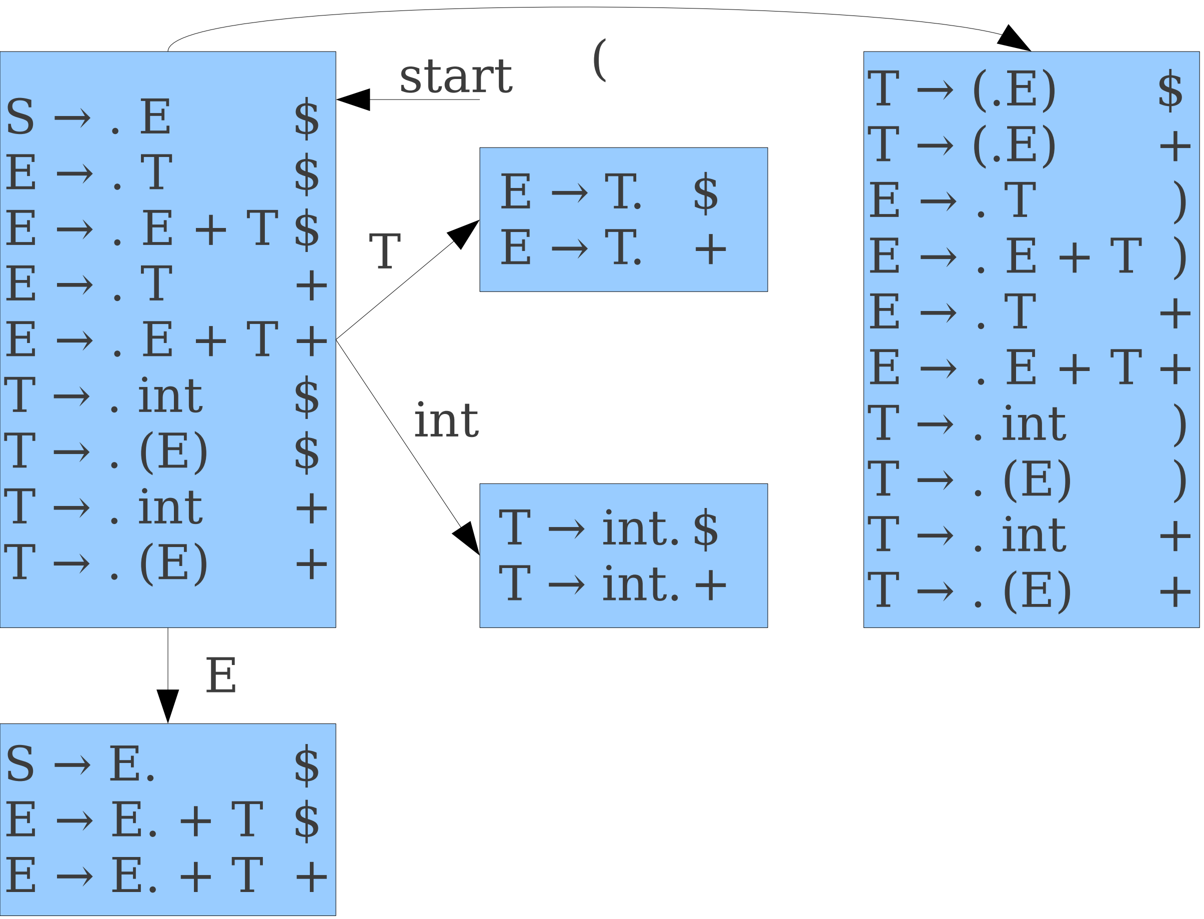
# Deterministic LR(1) Automata



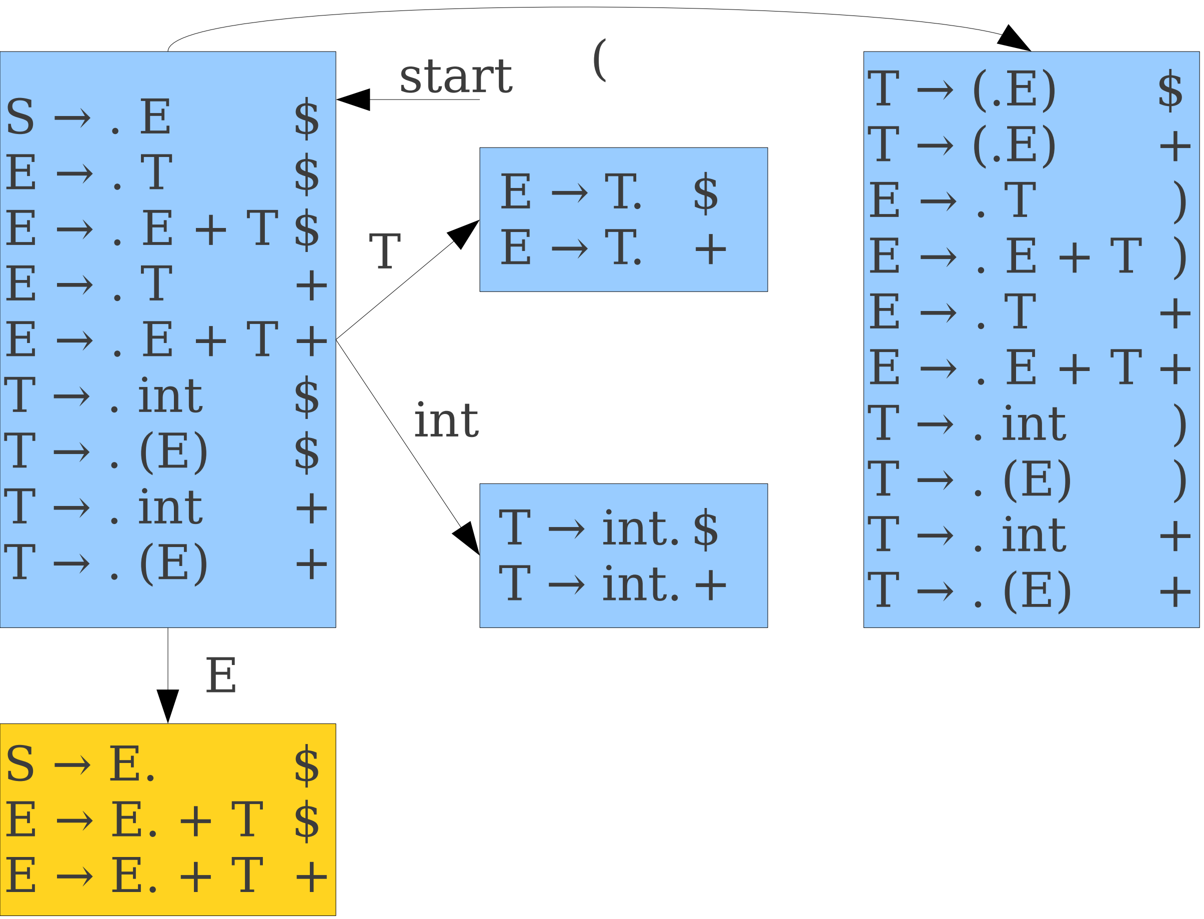
# Deterministic LR(1) Automata



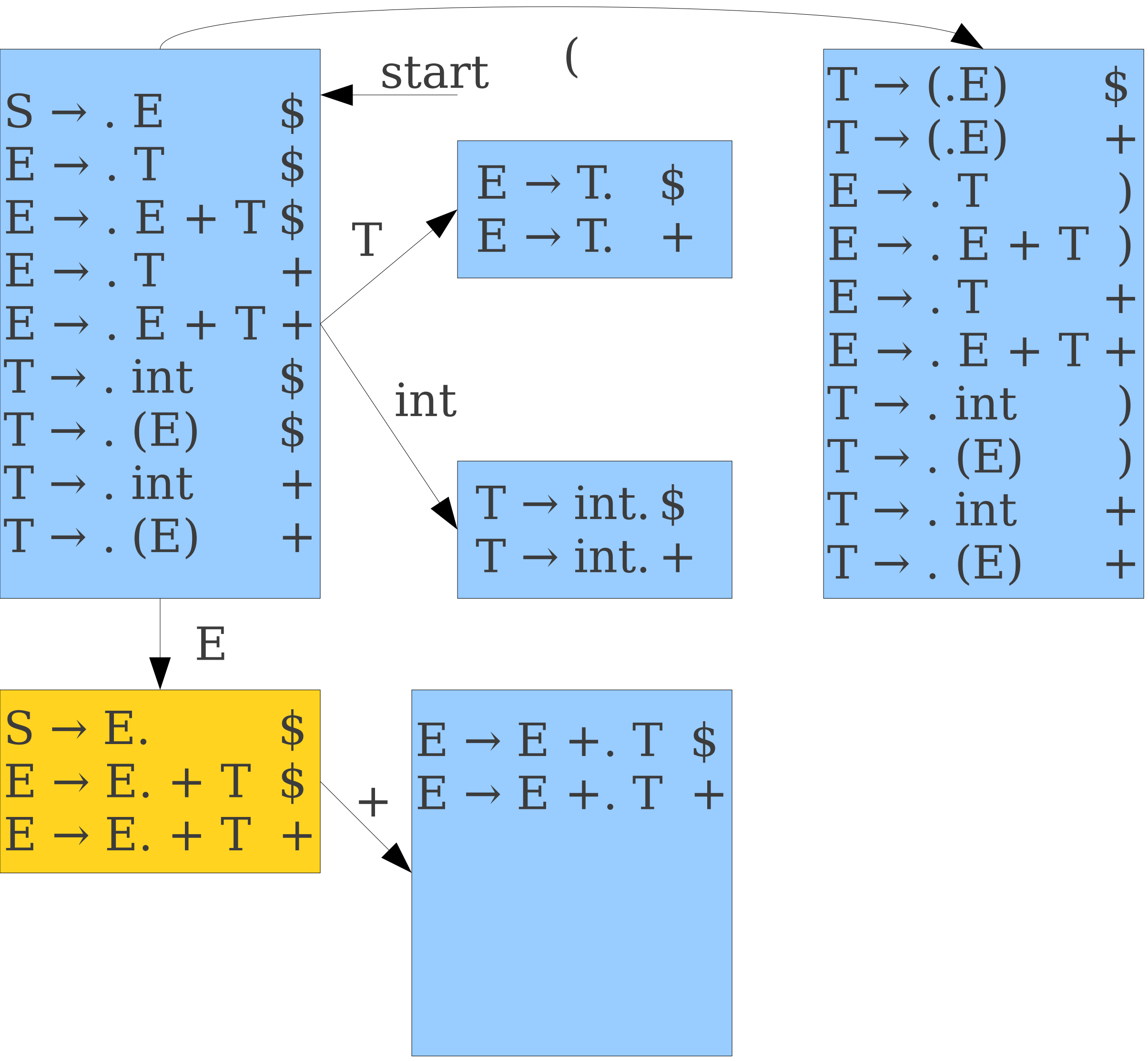
# Deterministic LR(1) Automata



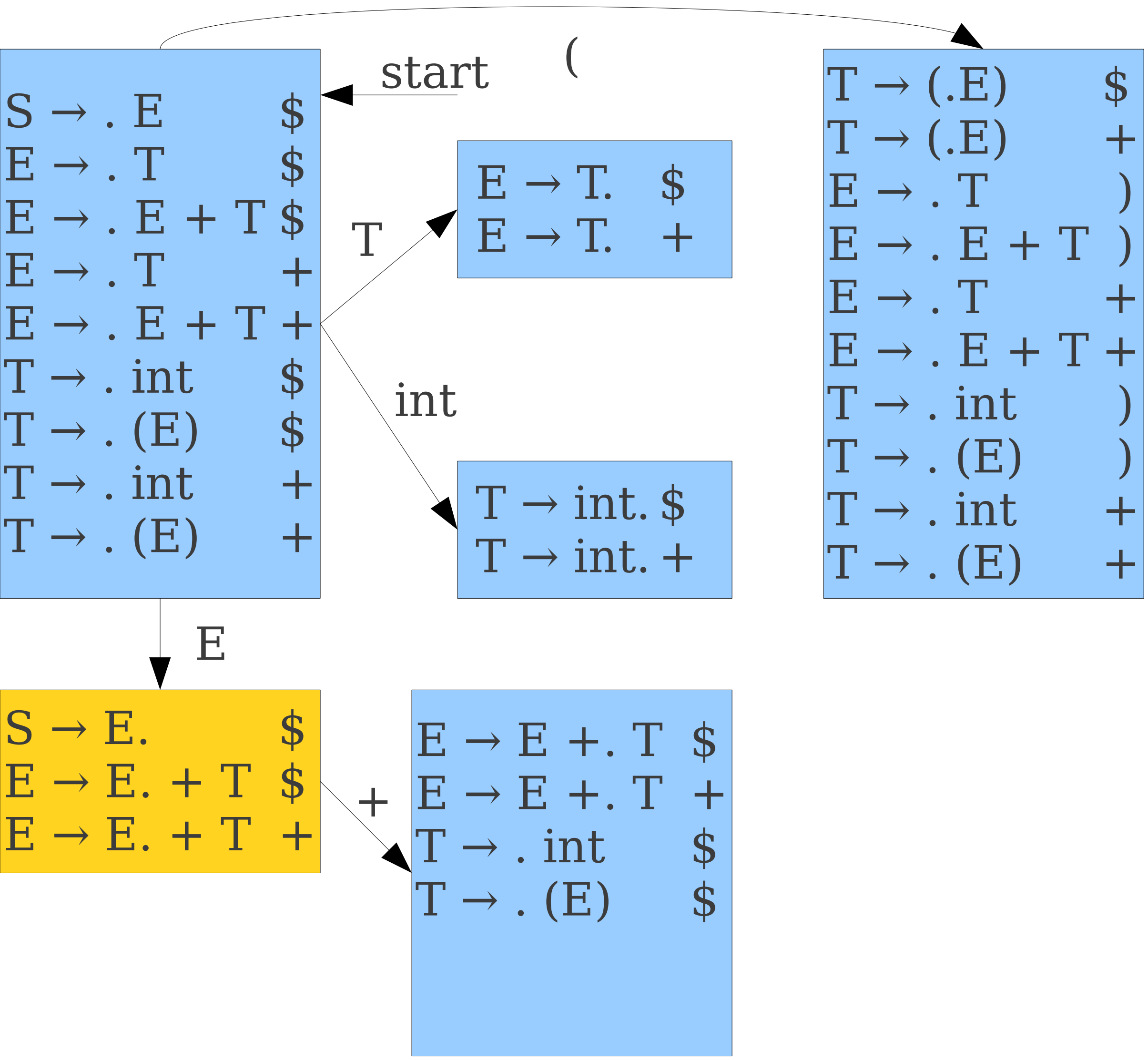
# Deterministic LR(1) Automata



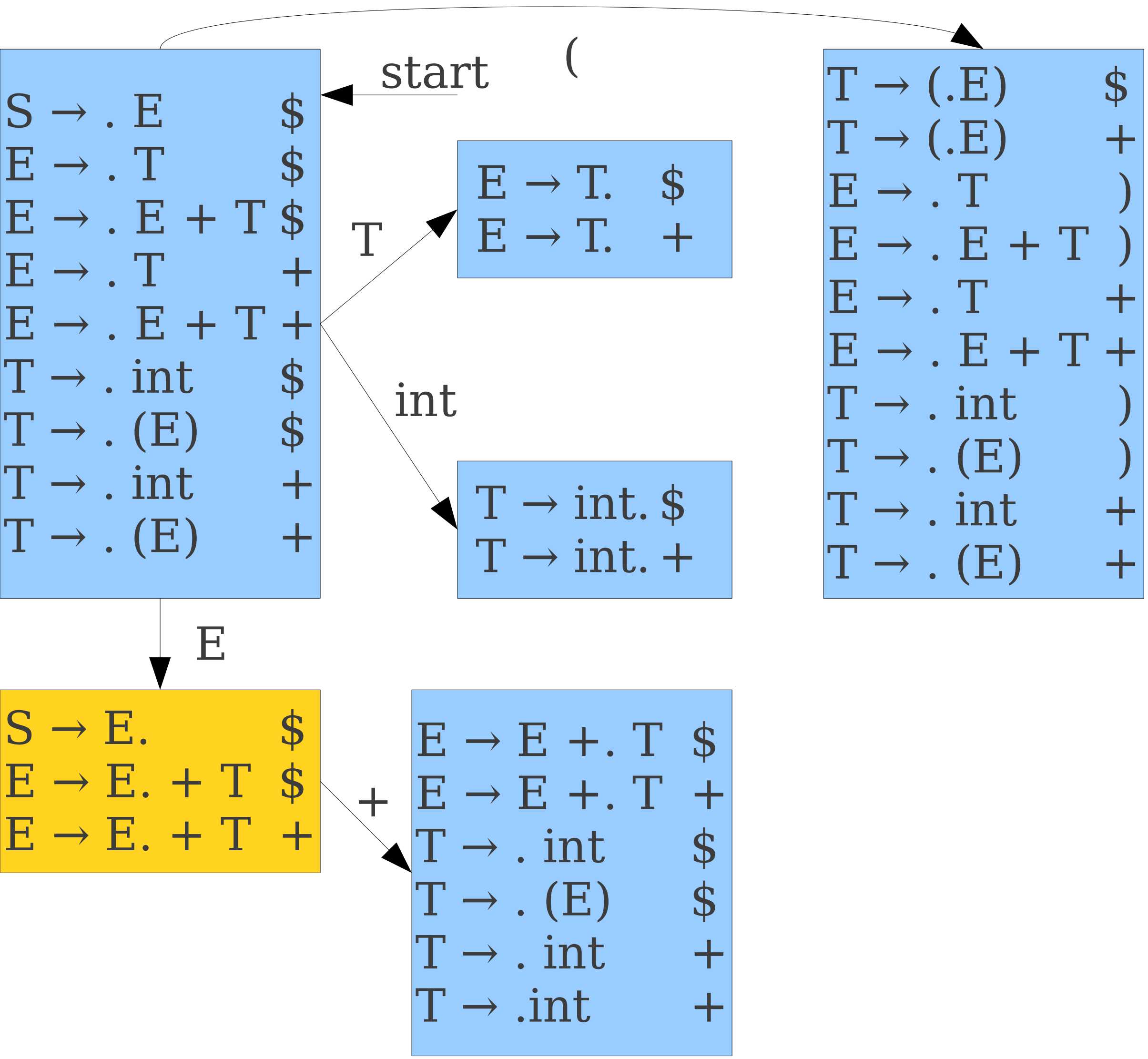
# Deterministic LR(1) Automata



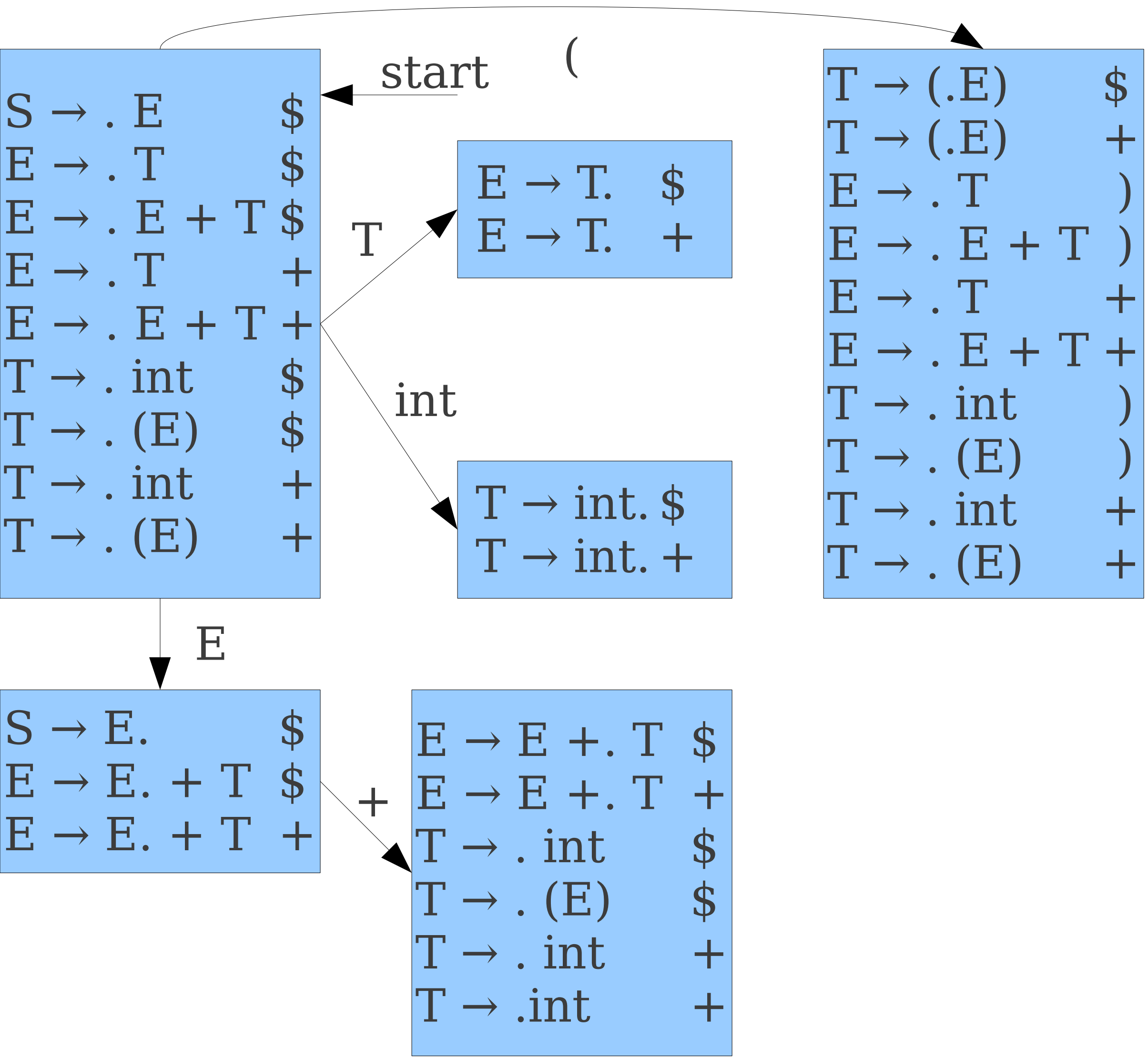
# Deterministic LR(1) Automata



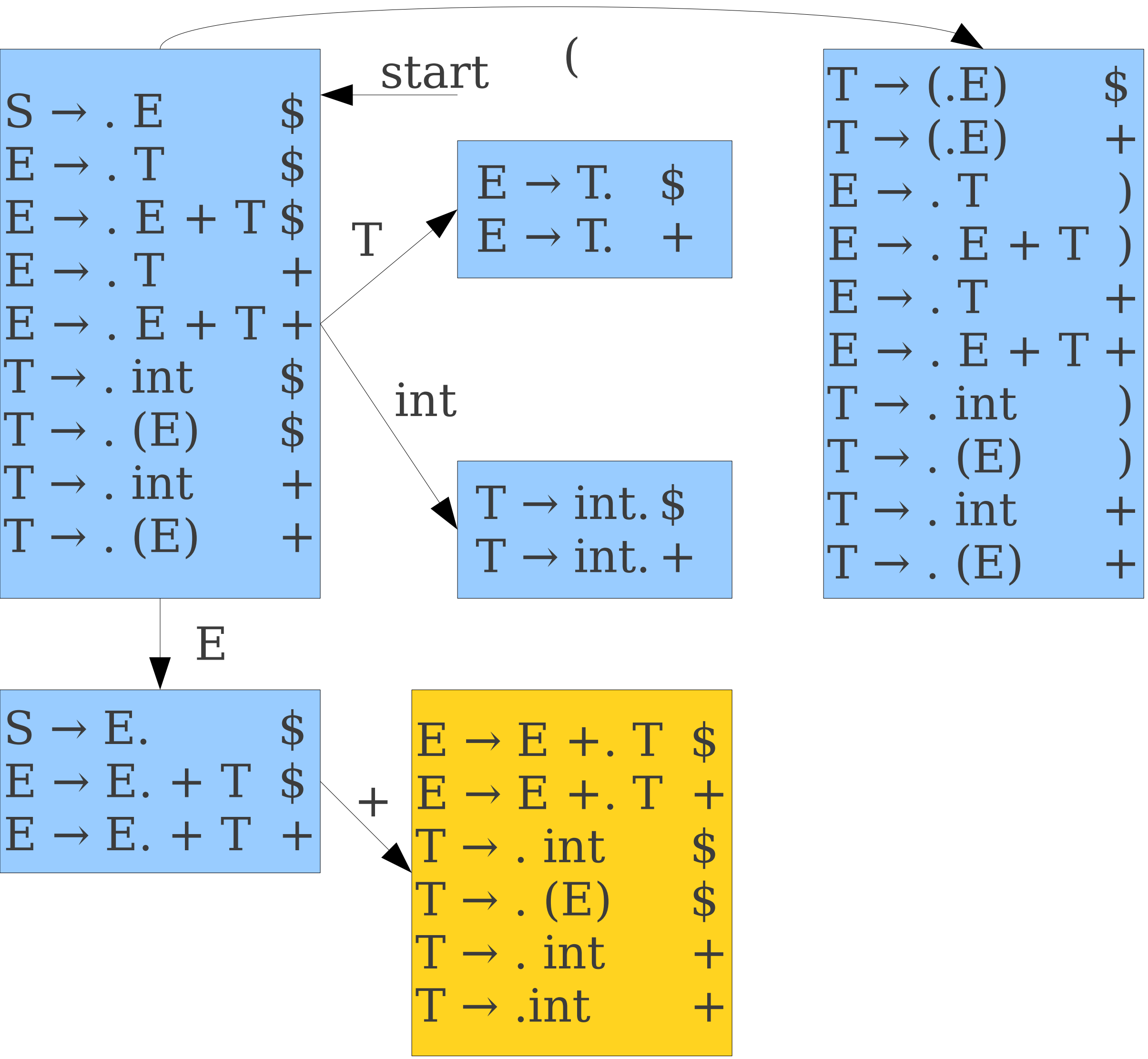
# Deterministic LR(1) Automata



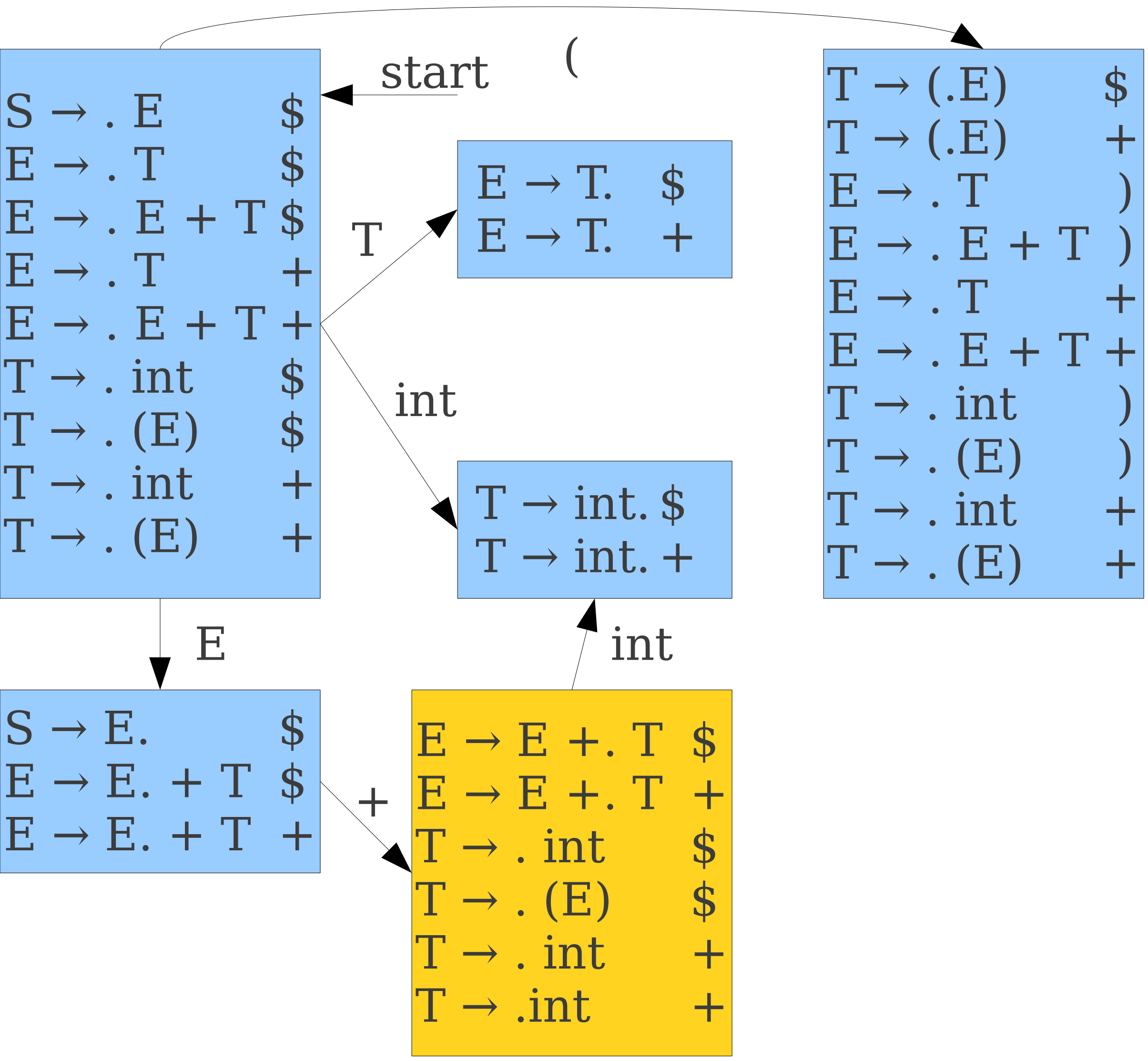
# Deterministic LR(1) Automata



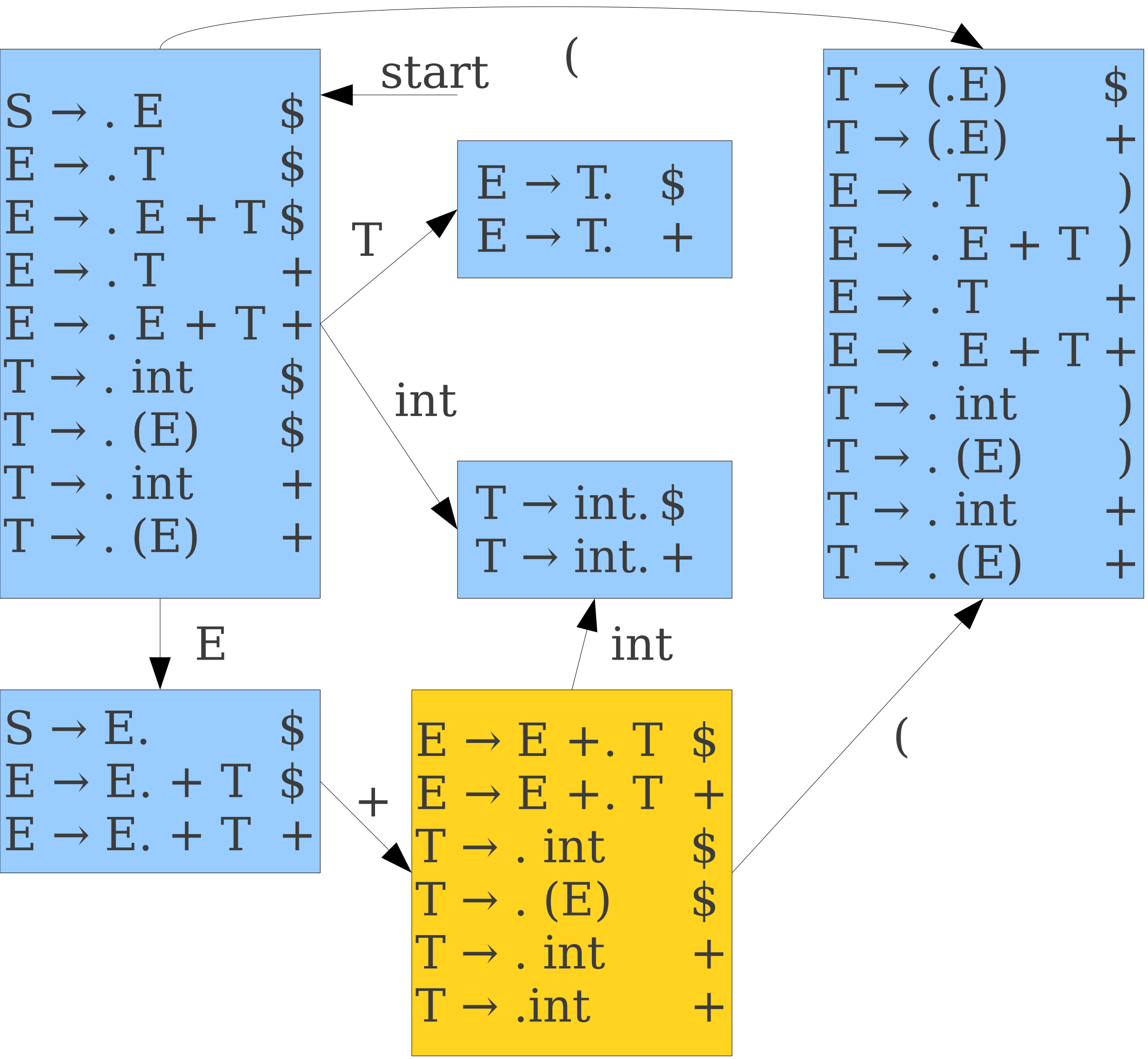
# Deterministic LR(1) Automata



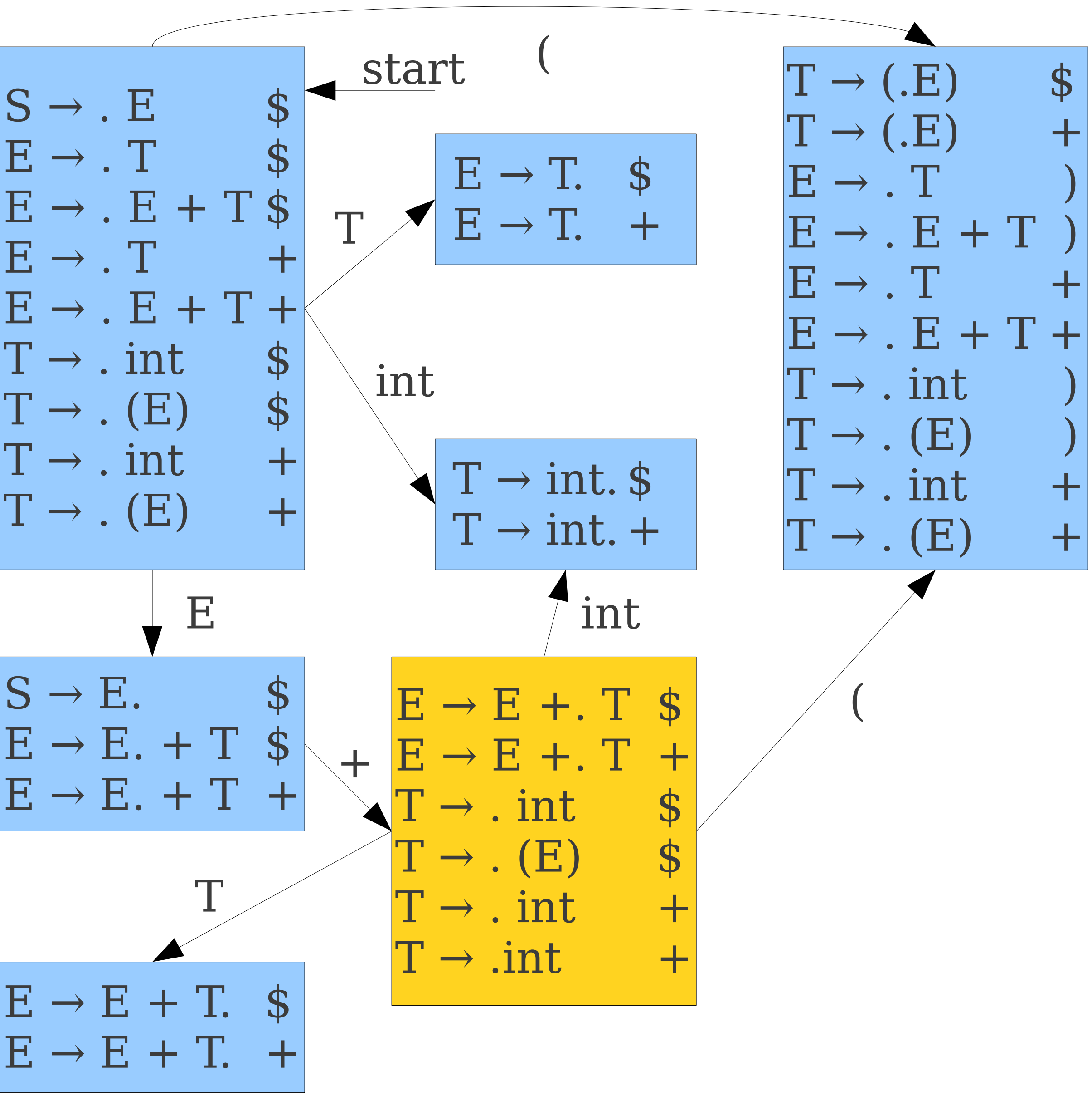
# Deterministic LR(1) Automata



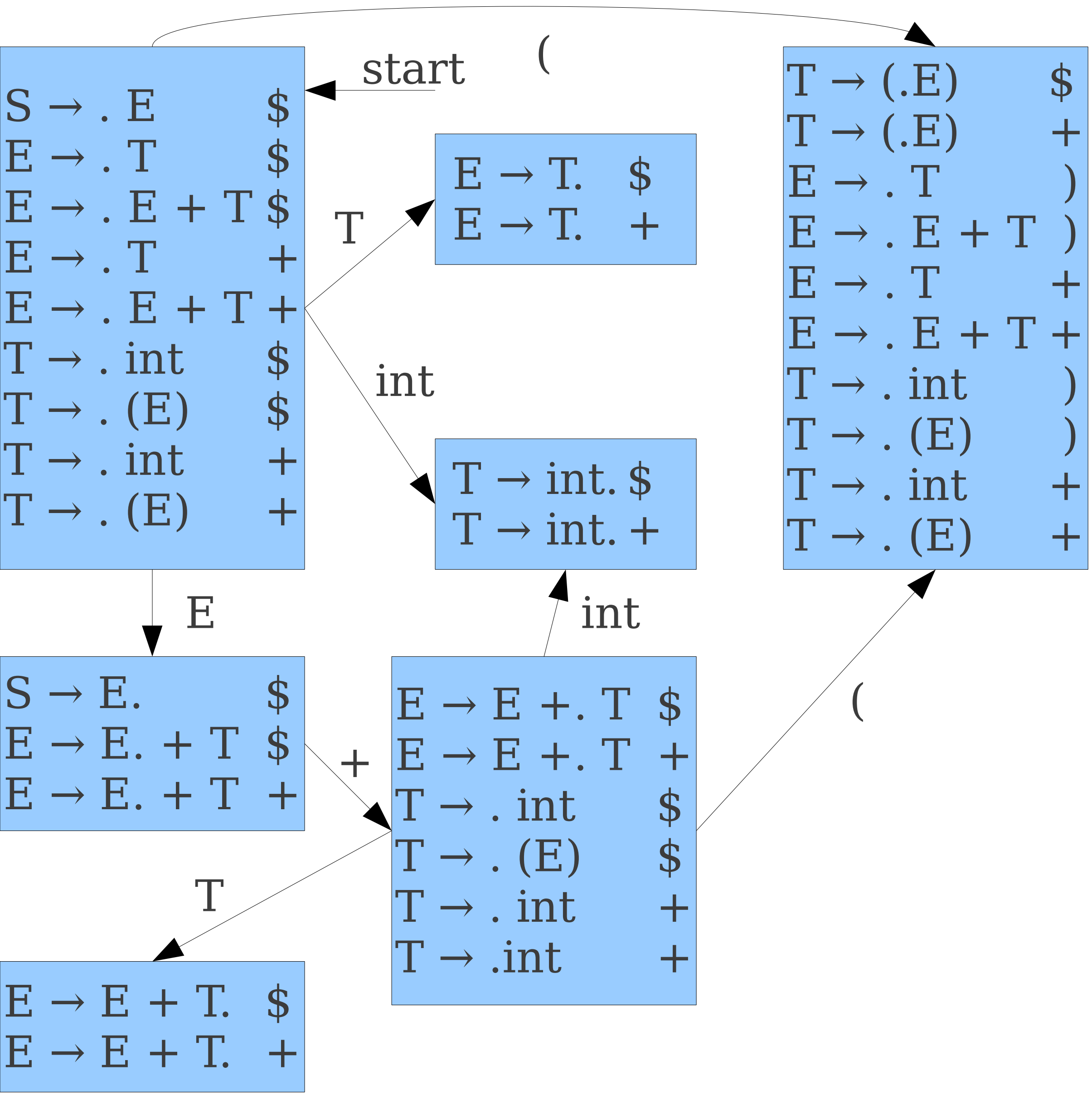
# Deterministic LR(1) Automata



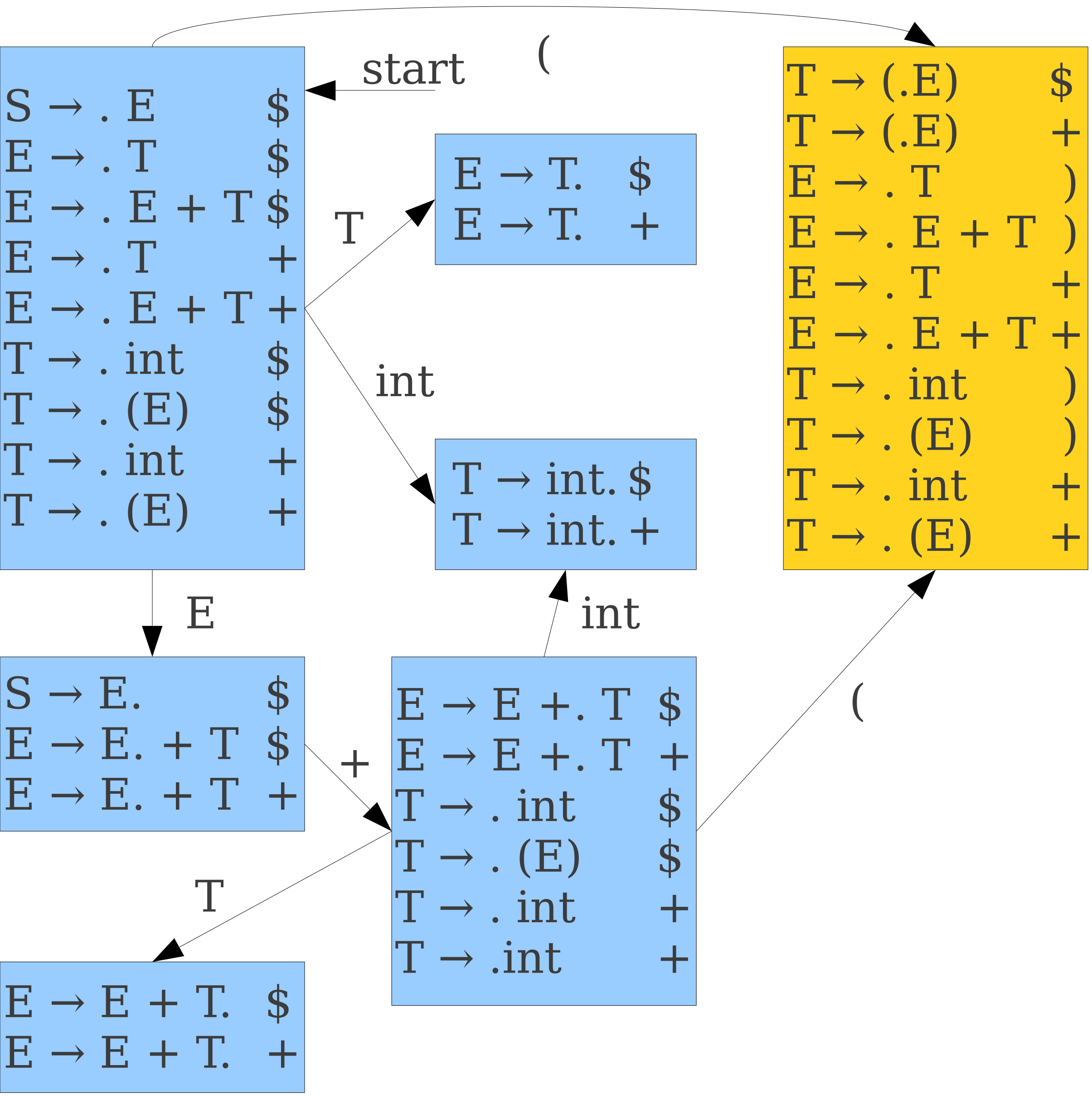
# Deterministic LR(1) Automata



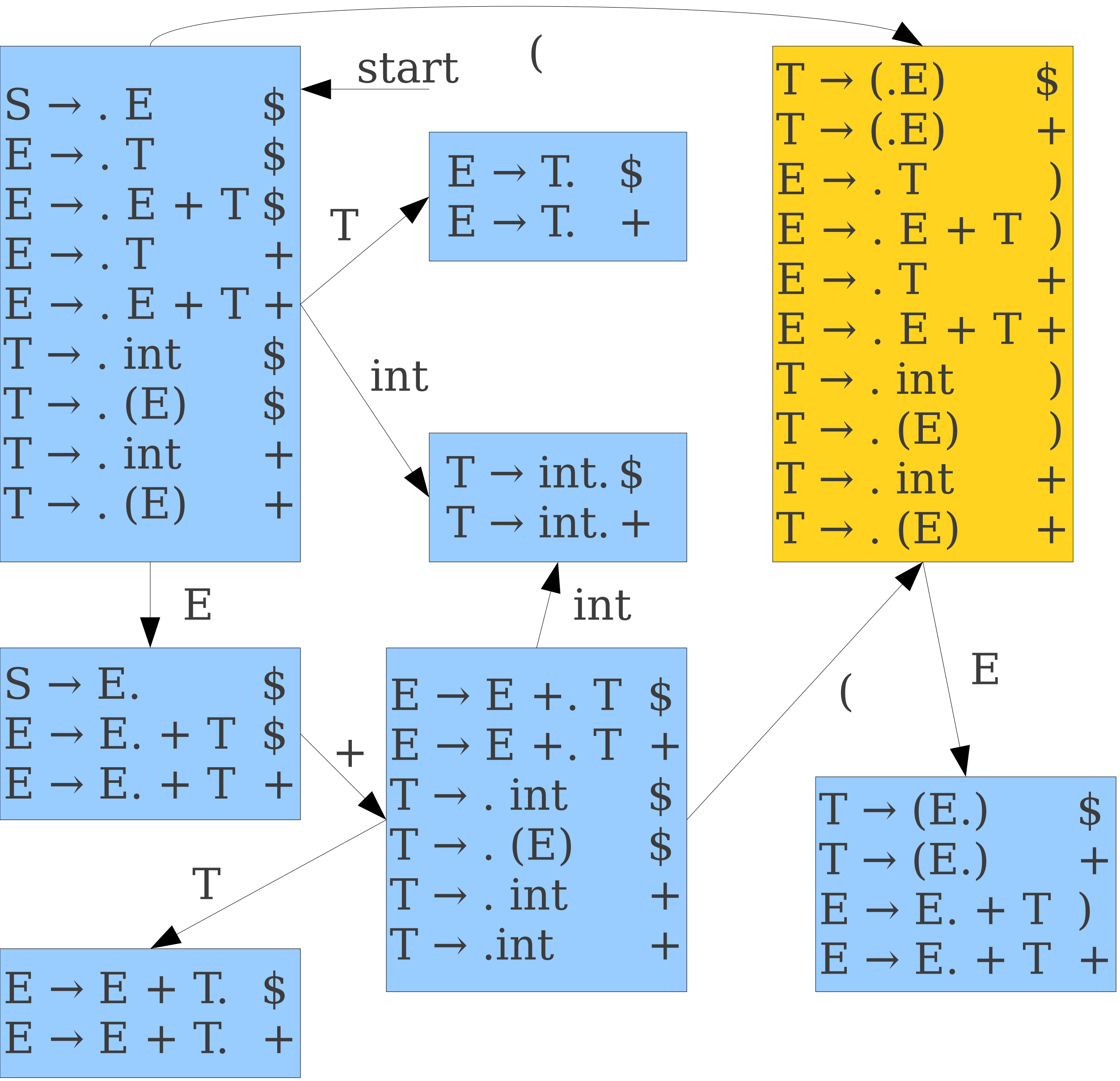
# Deterministic LR(1) Automata



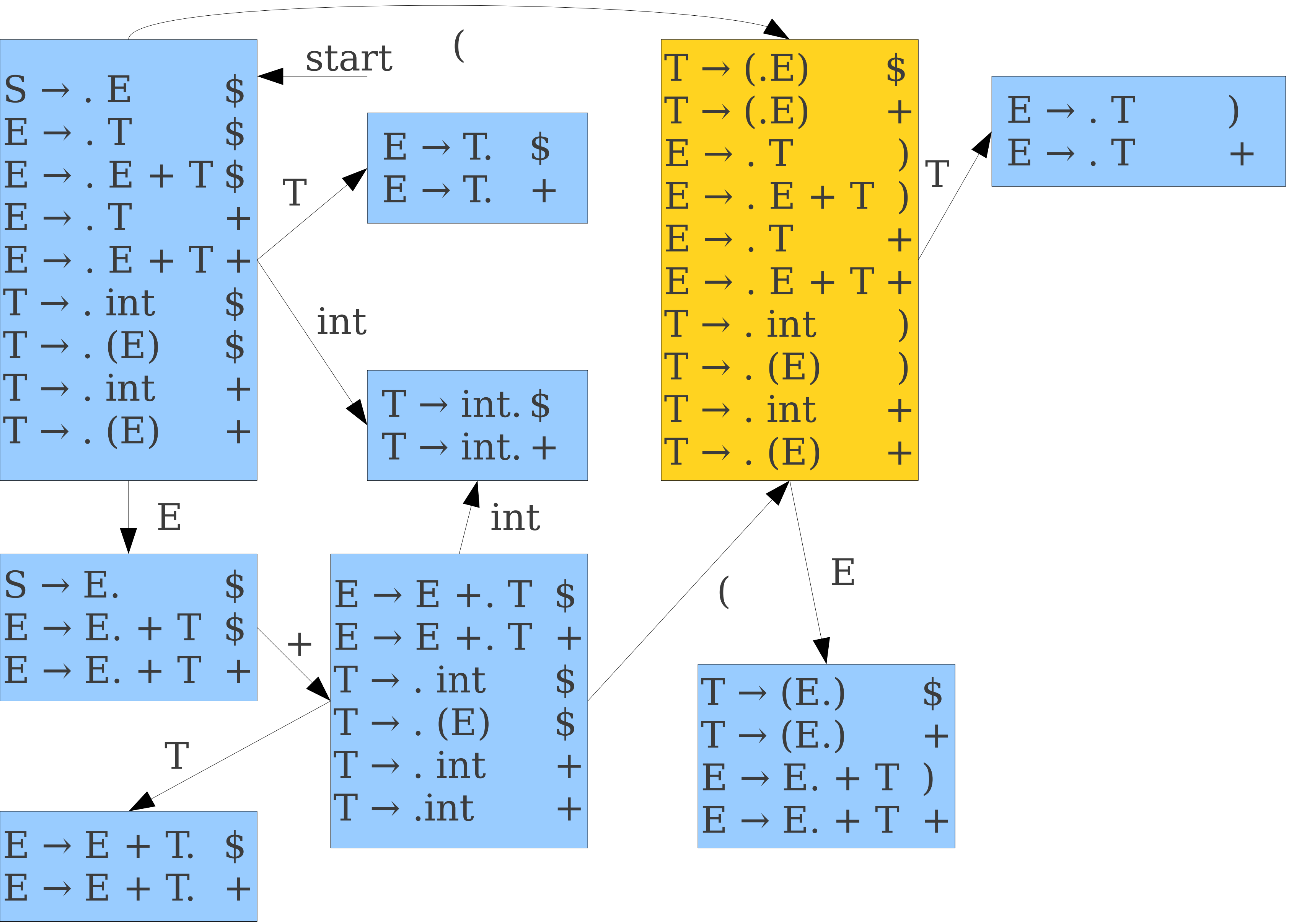
# Deterministic LR(1) Automata



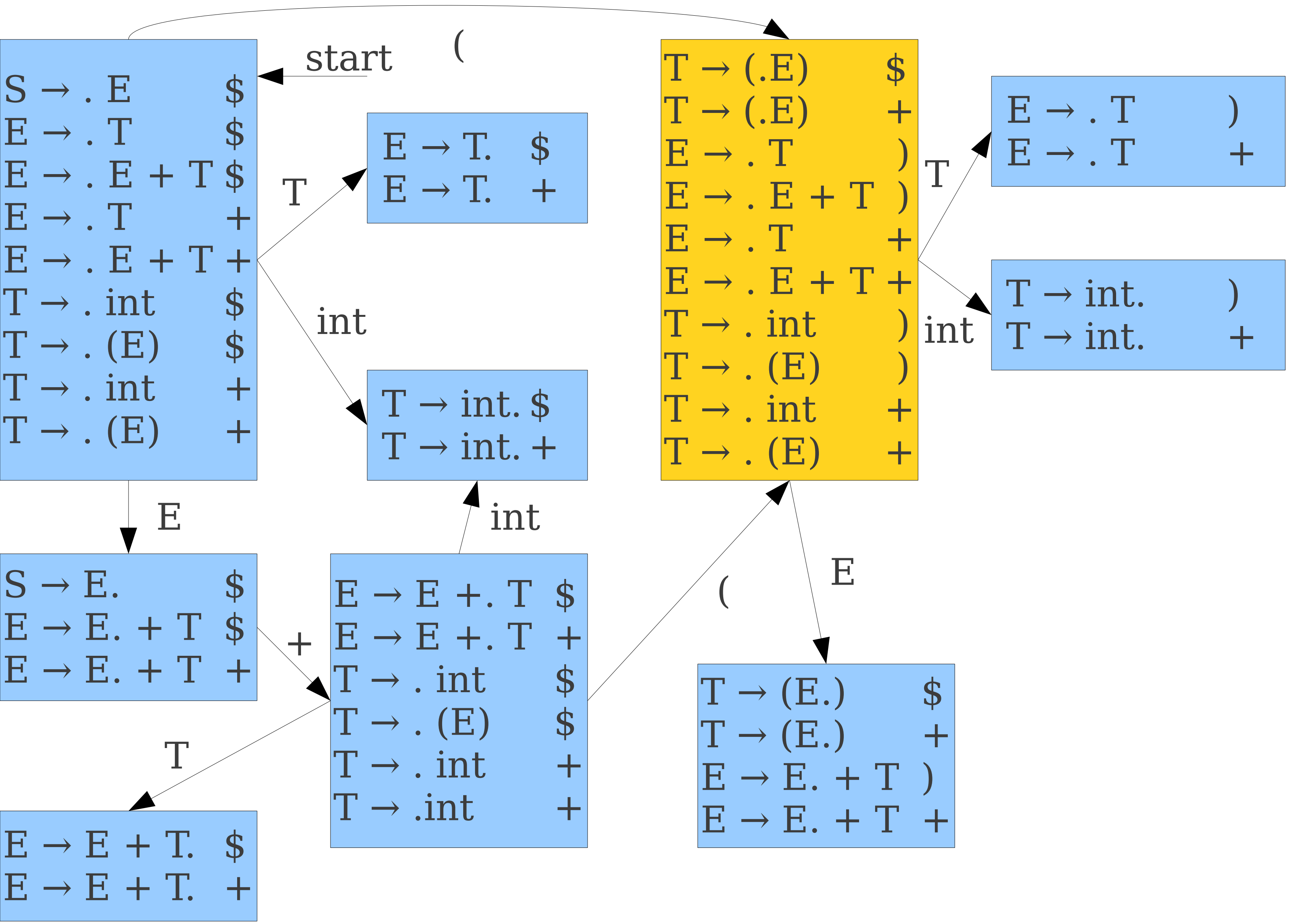
# Deterministic LR(1) Automata



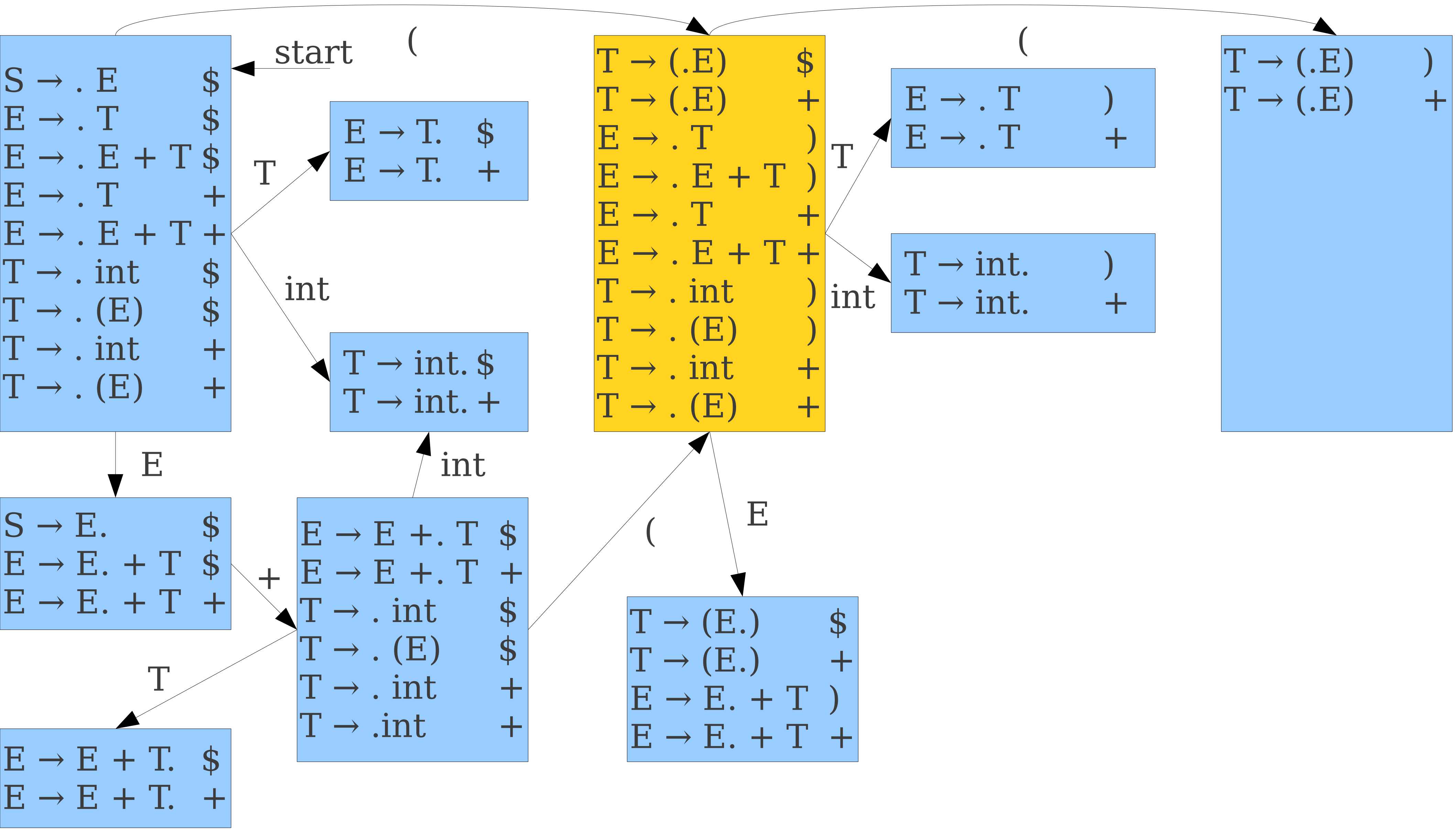
# Deterministic LR(1) Automata



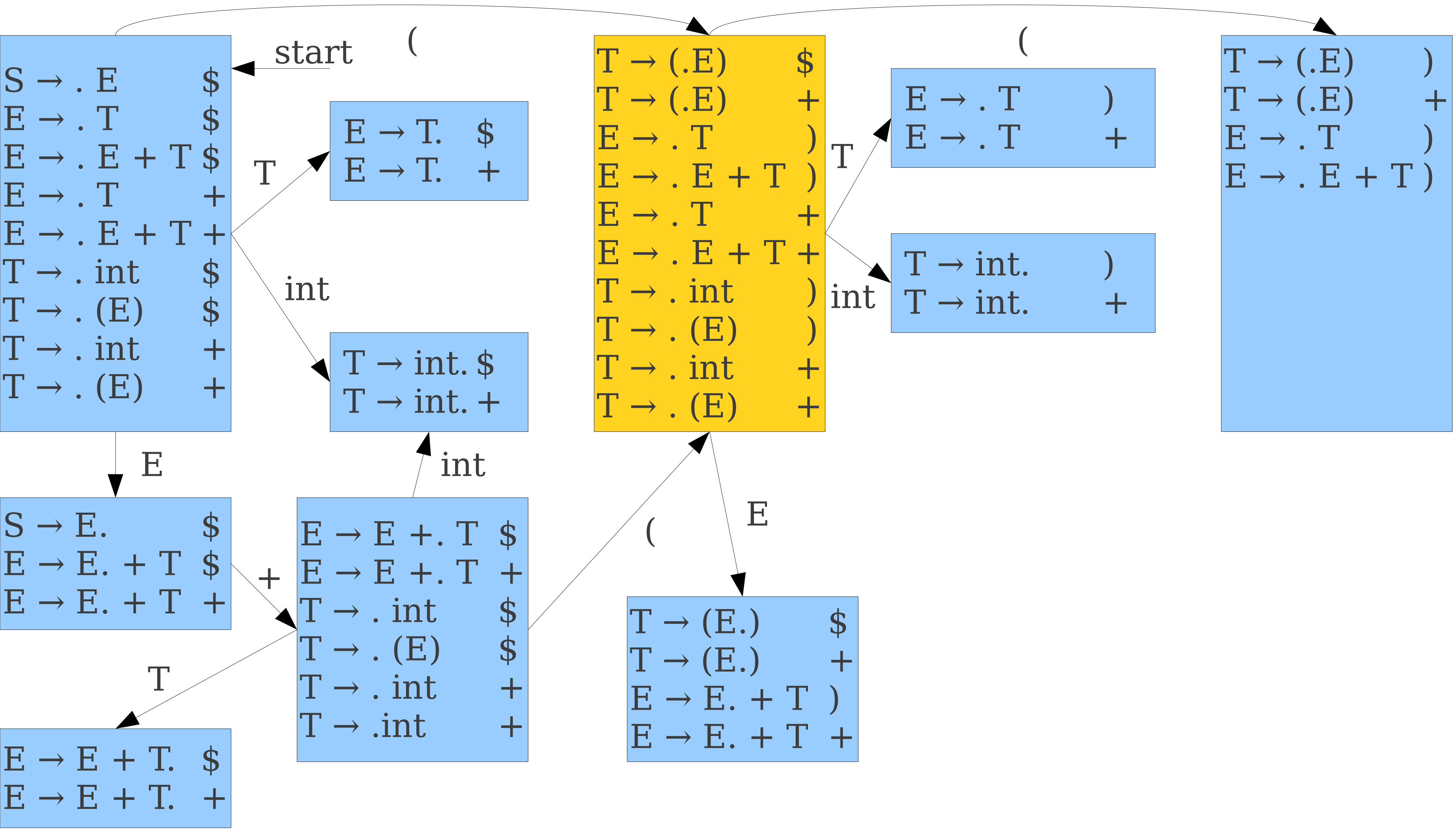
# Deterministic LR(1) Automata



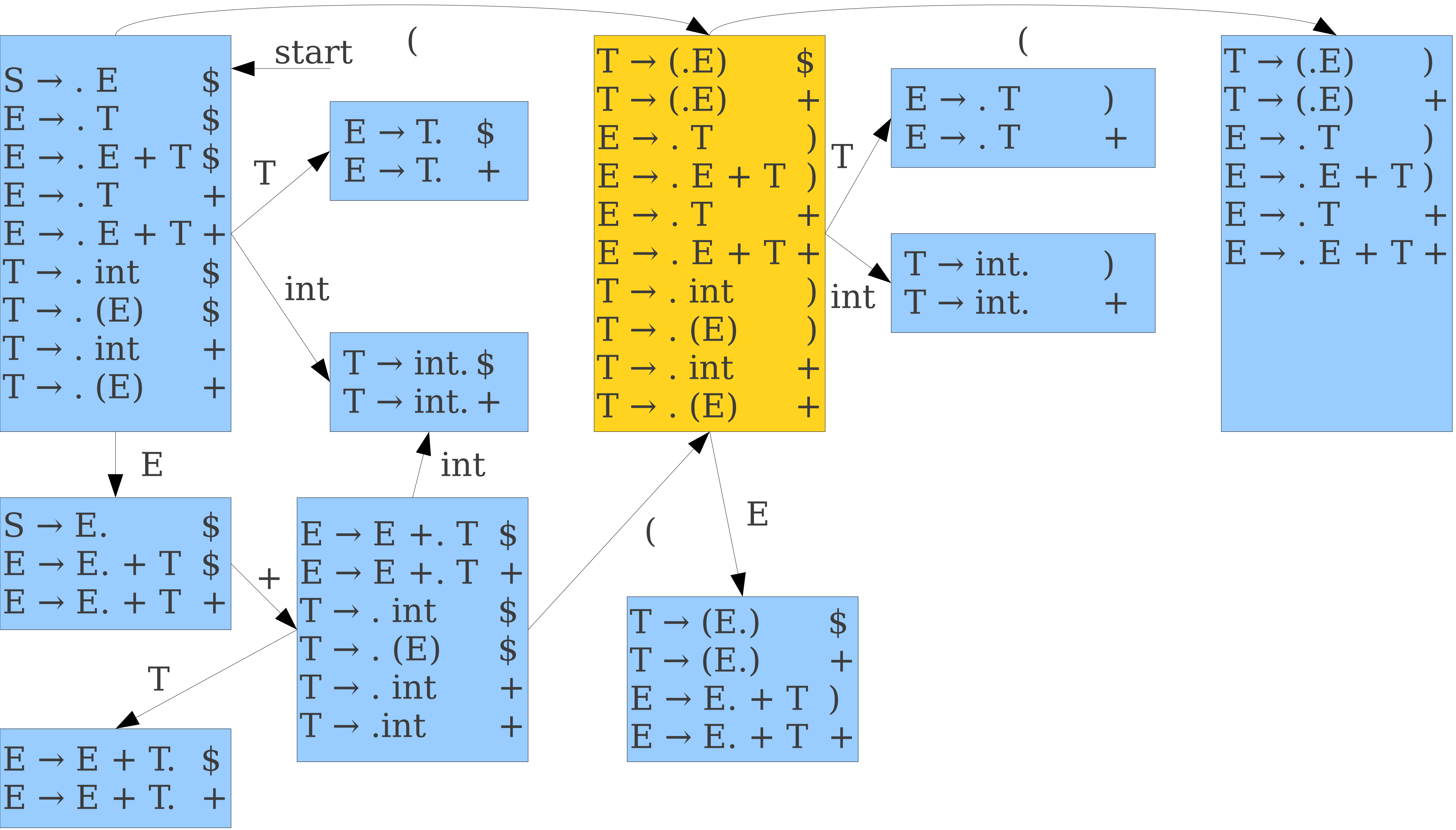
# Deterministic LR(1) Automata



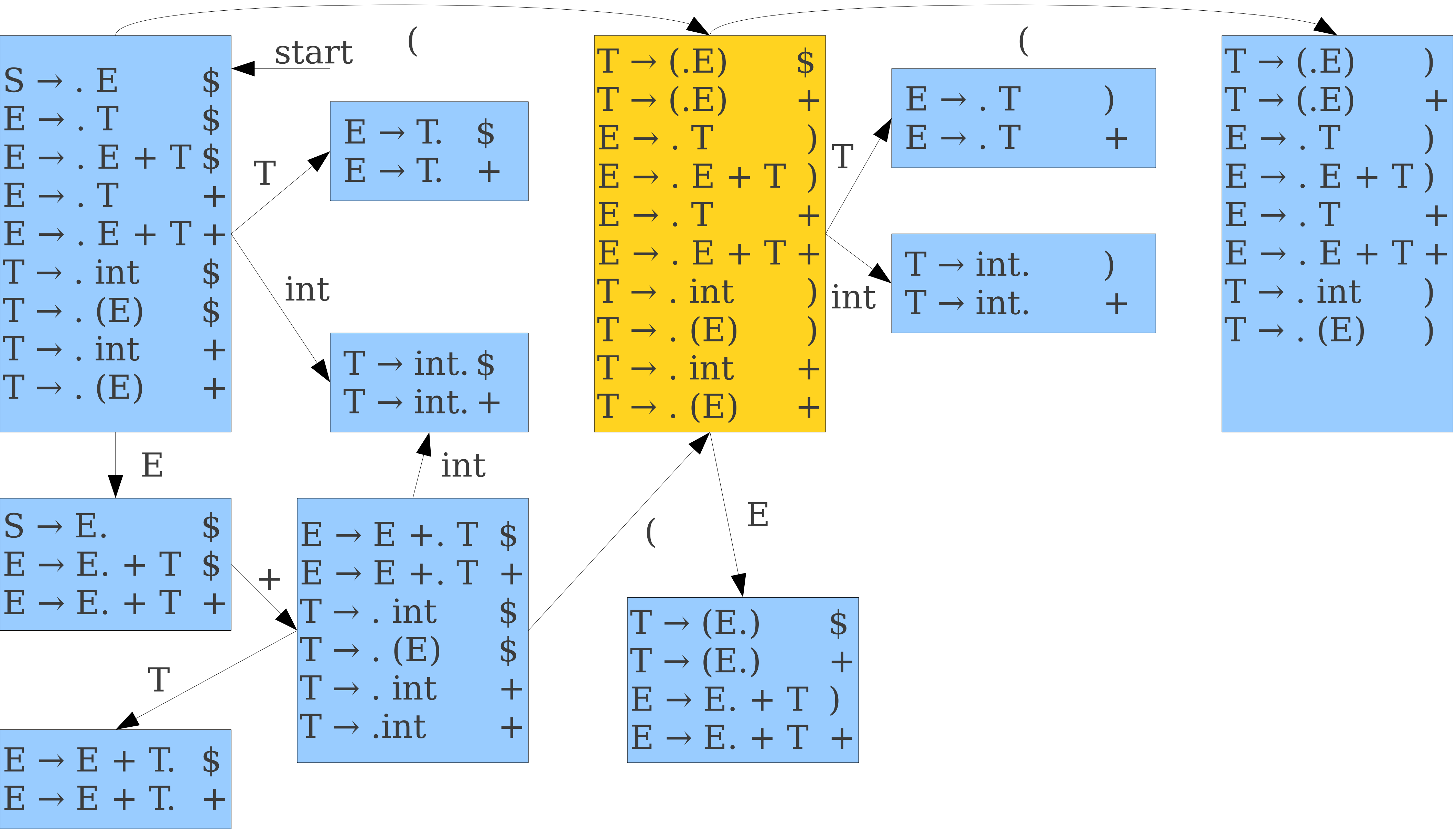
# Deterministic LR(1) Automata



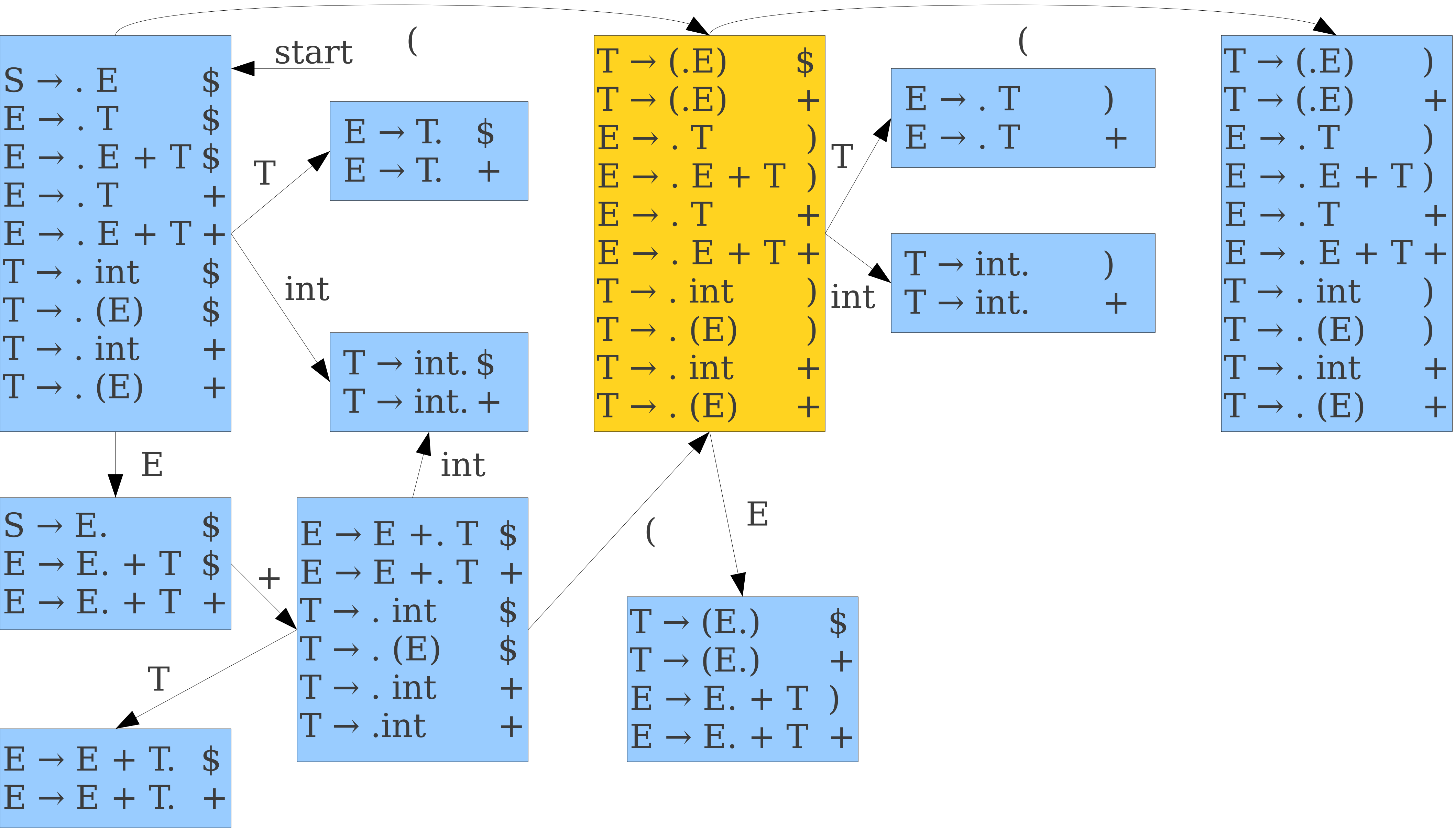
# Deterministic LR(1) Automata



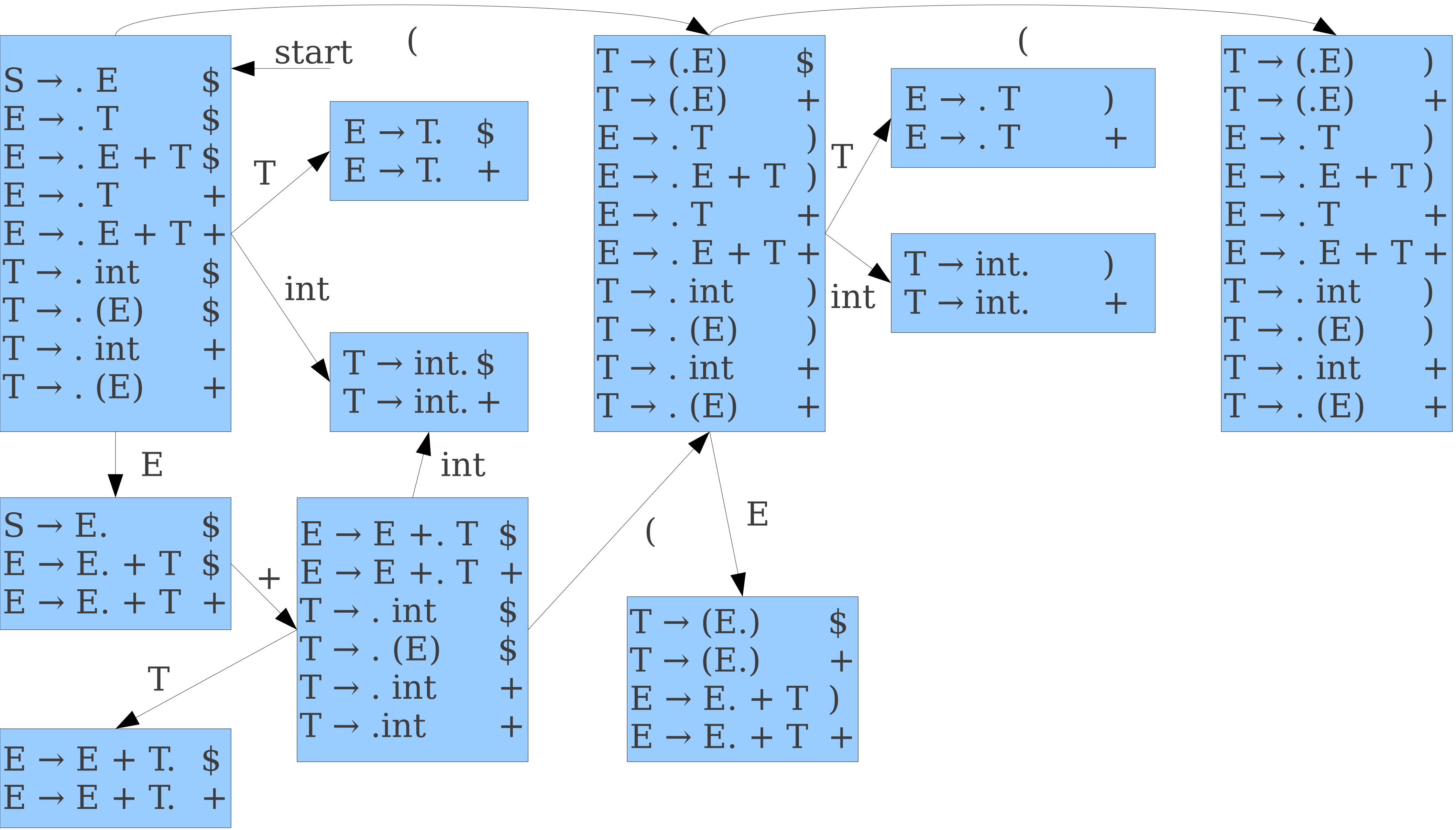
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



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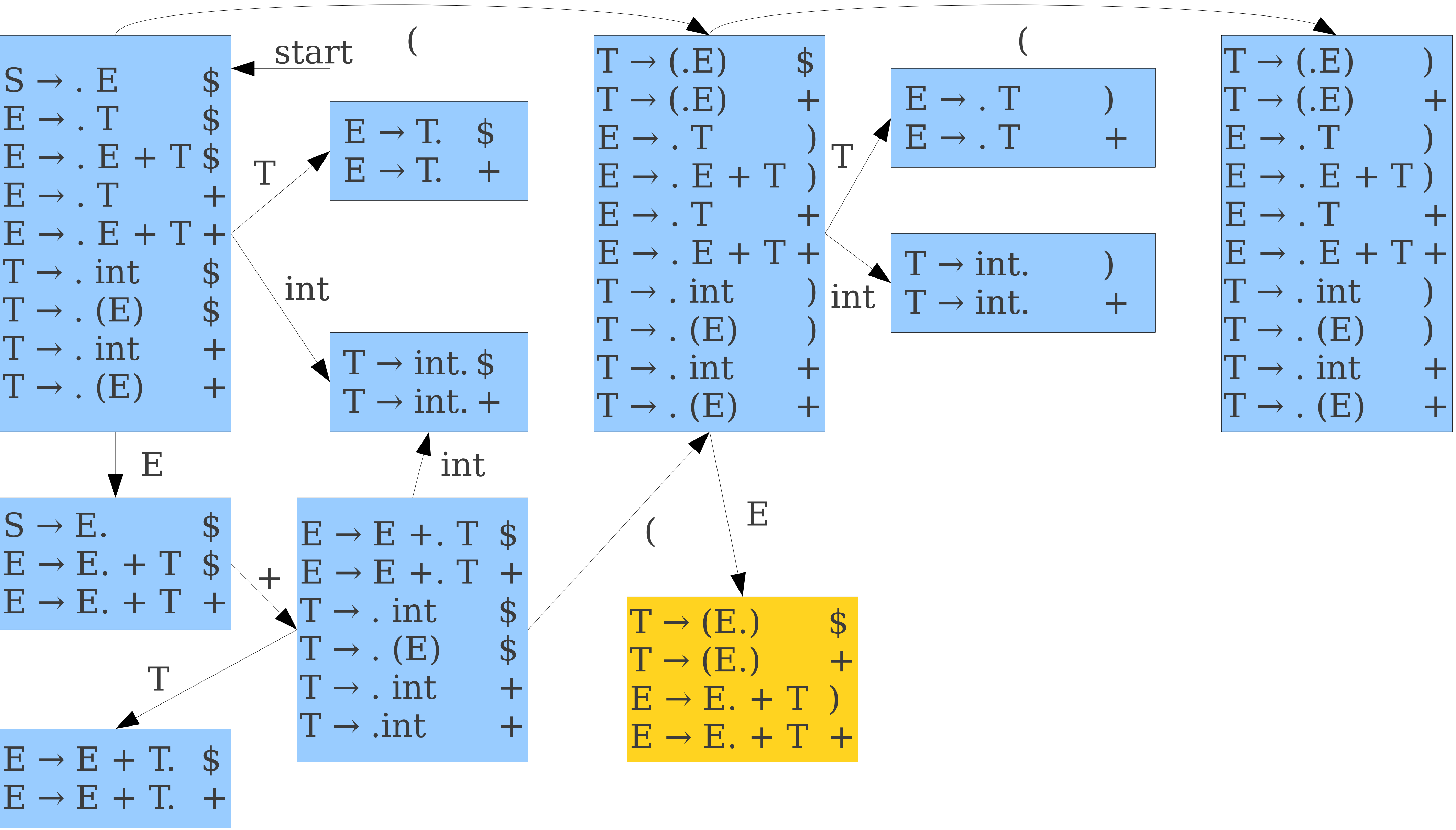
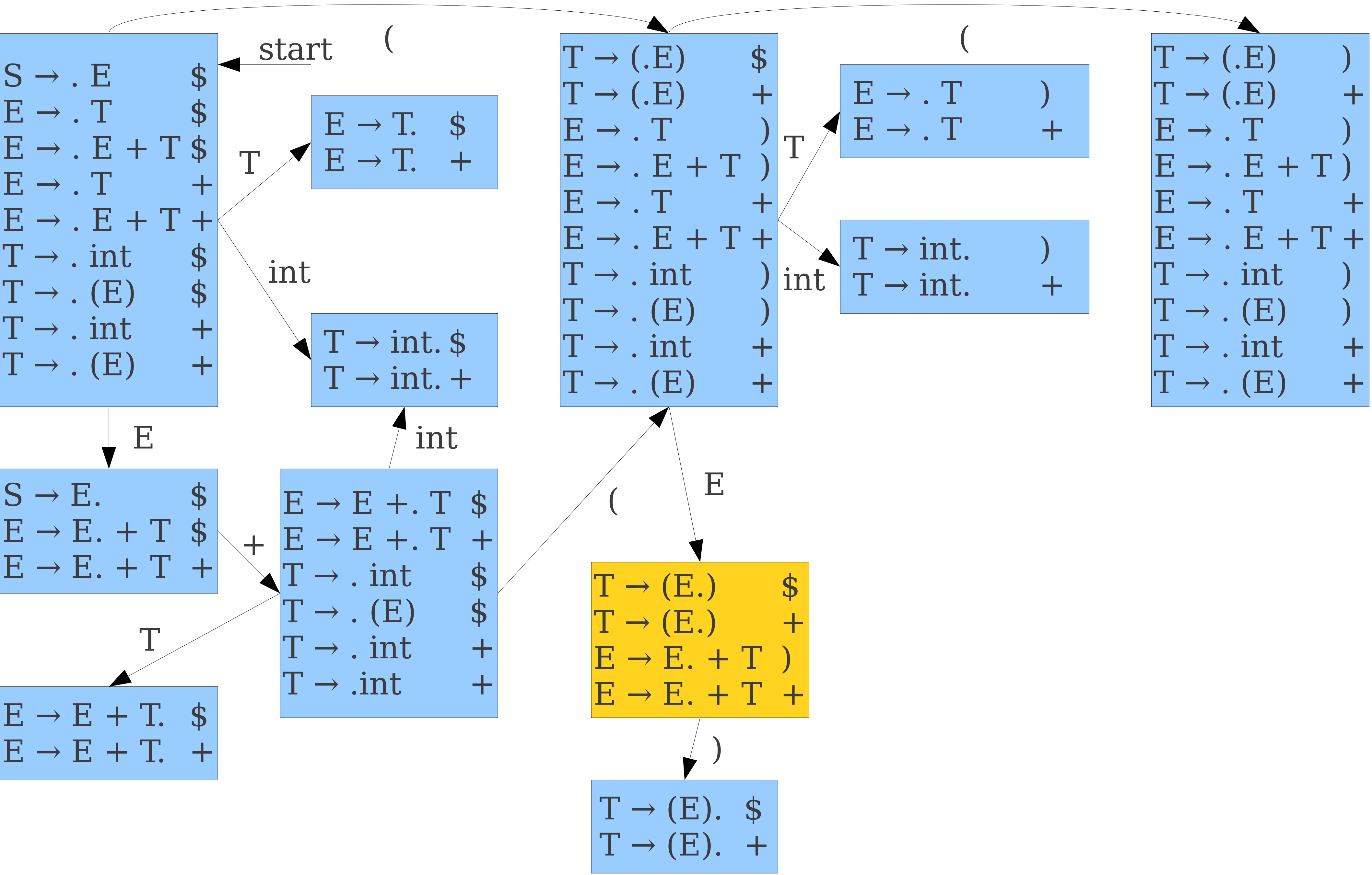
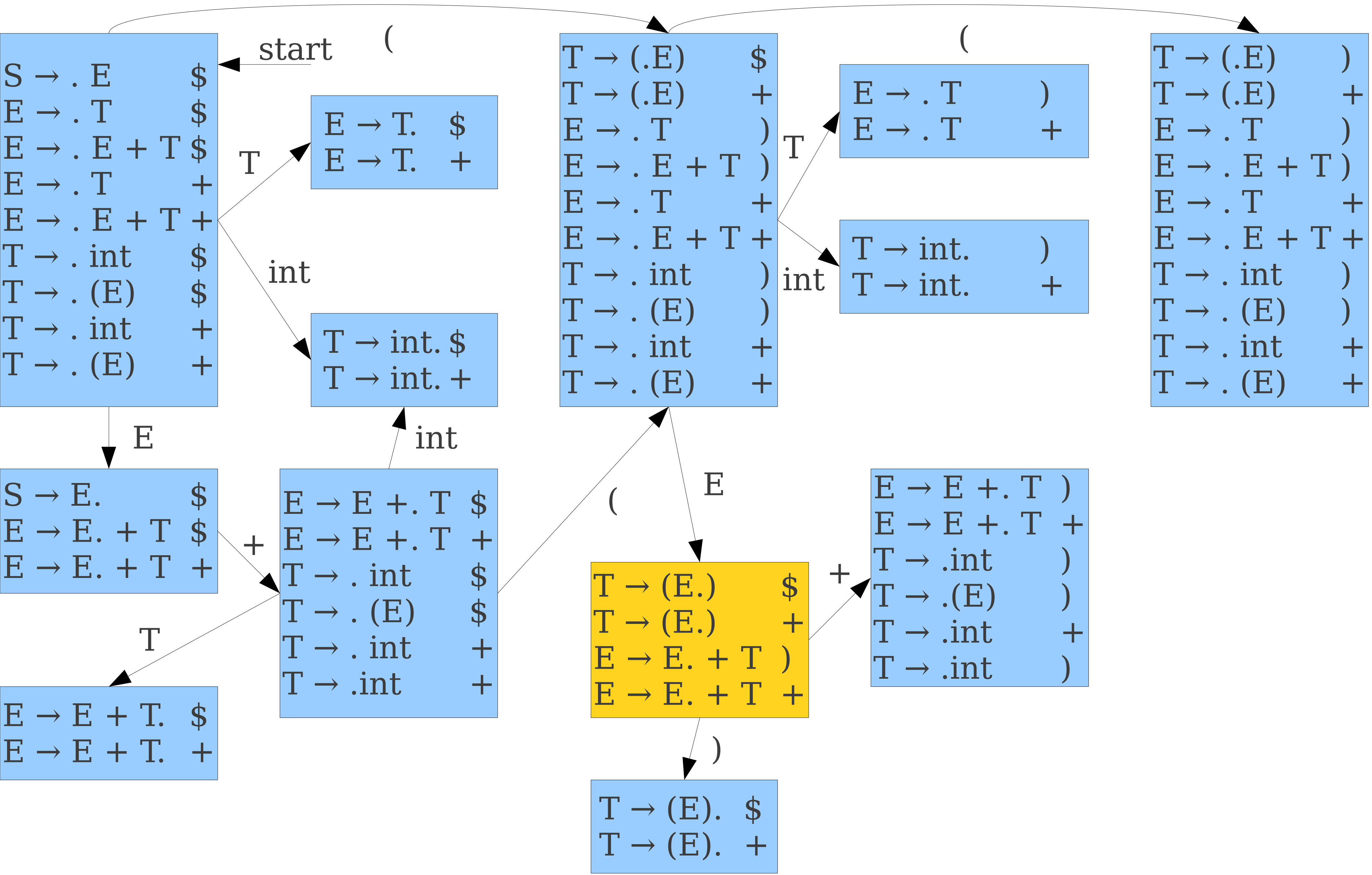


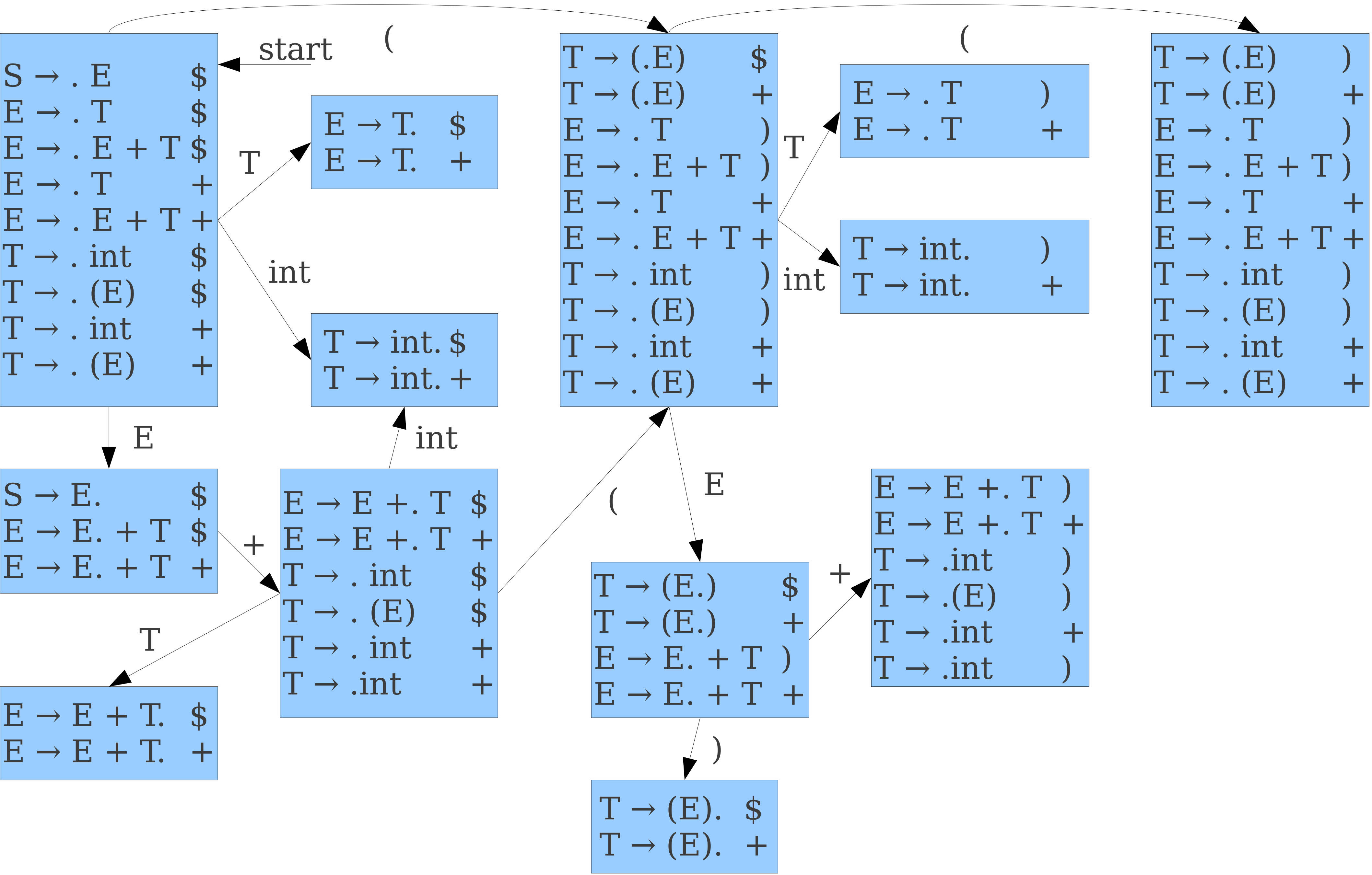
# Deterministic LR(1) Automata



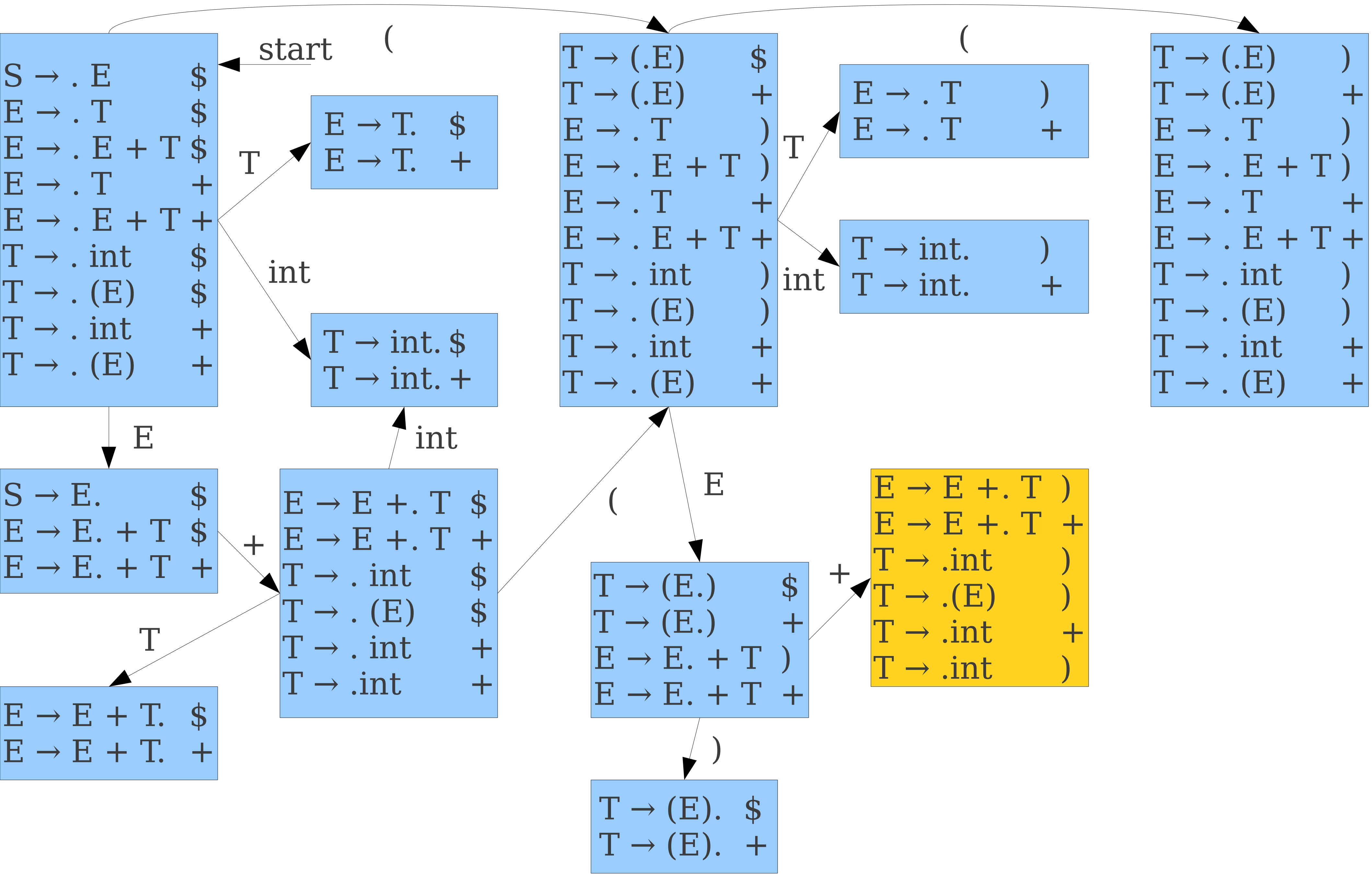
# Deterministic LR(1) Automata



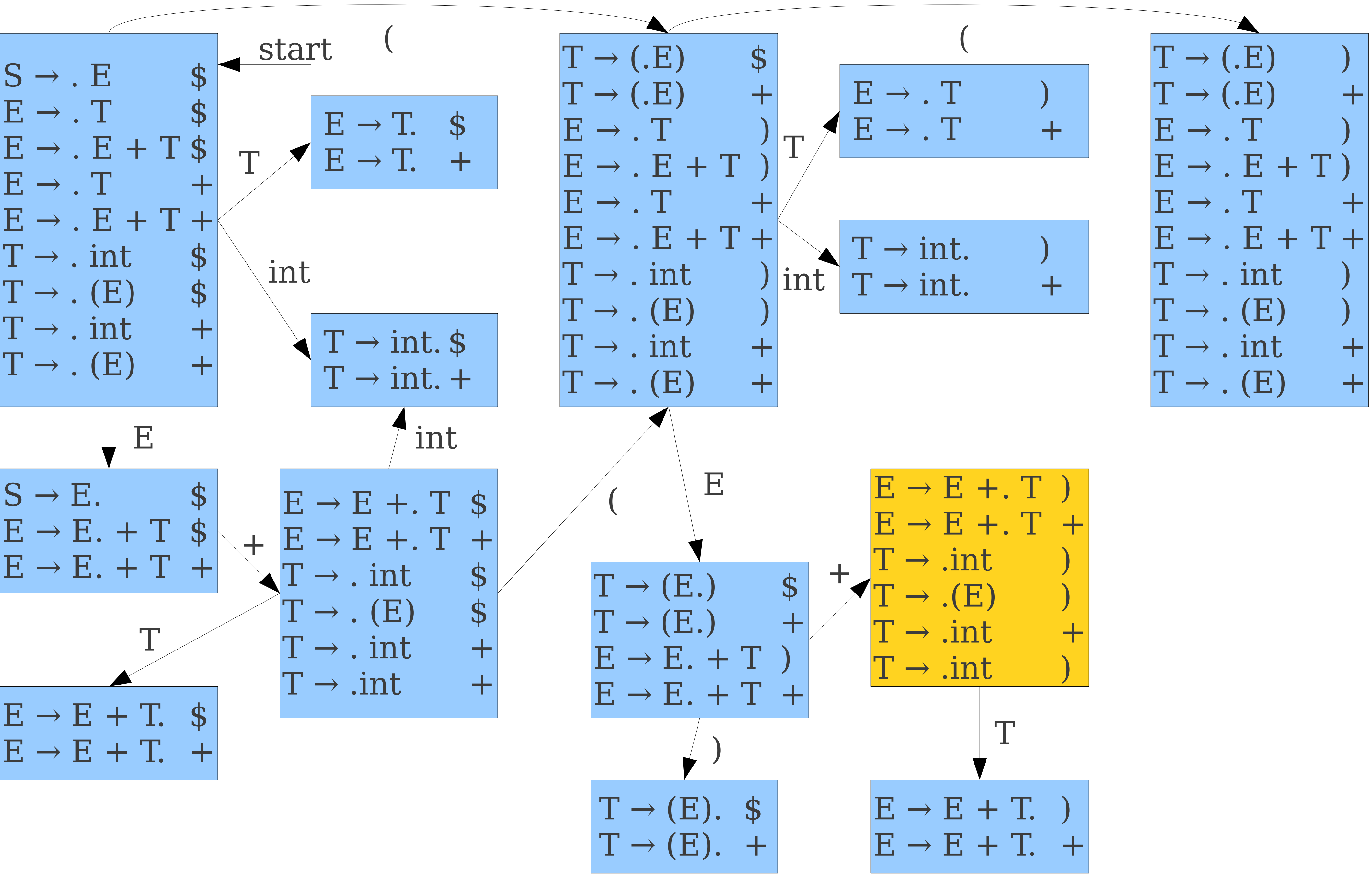
# Deterministic LR(1) Automata



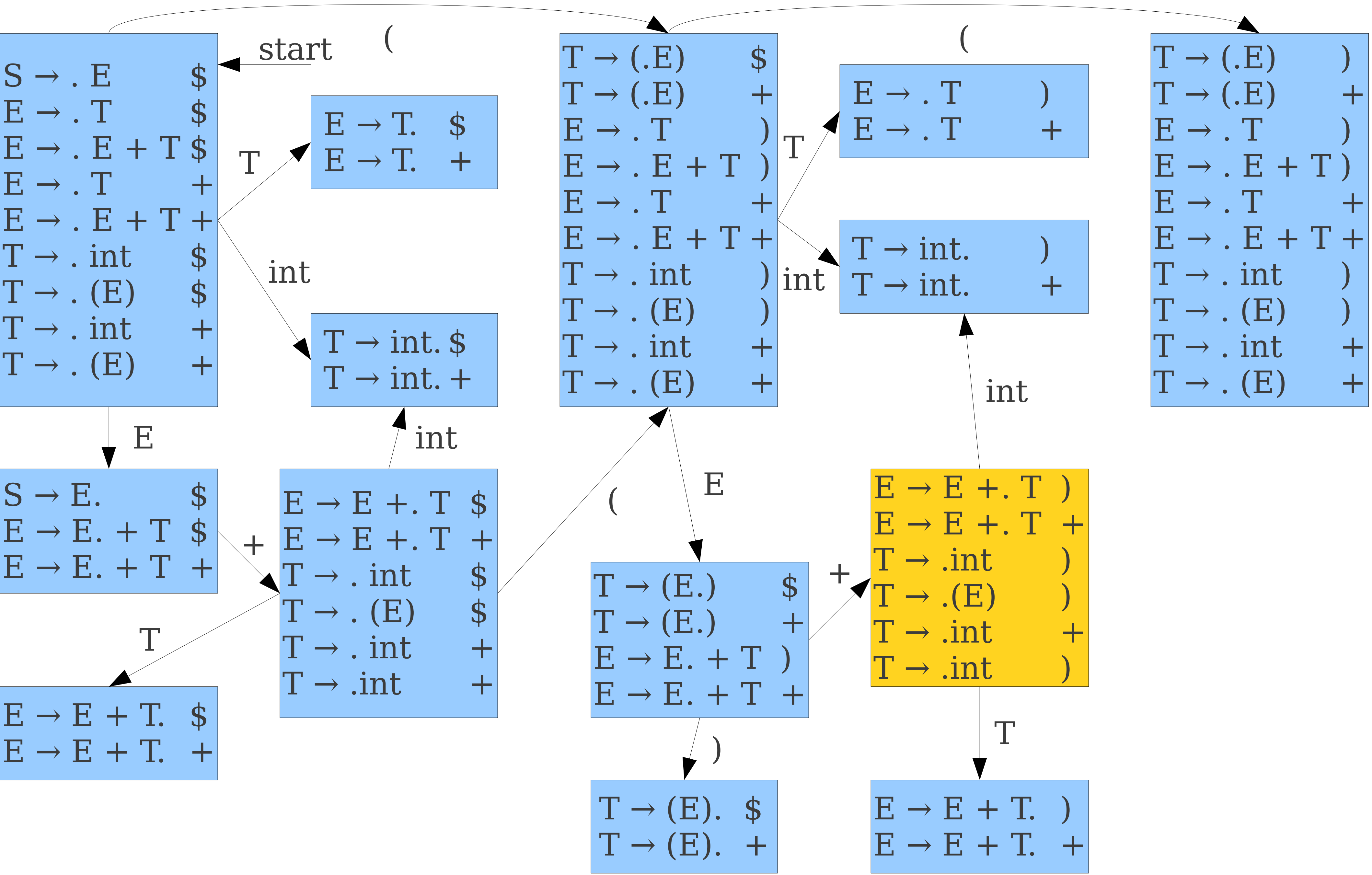
# Deterministic LR(1) Automata



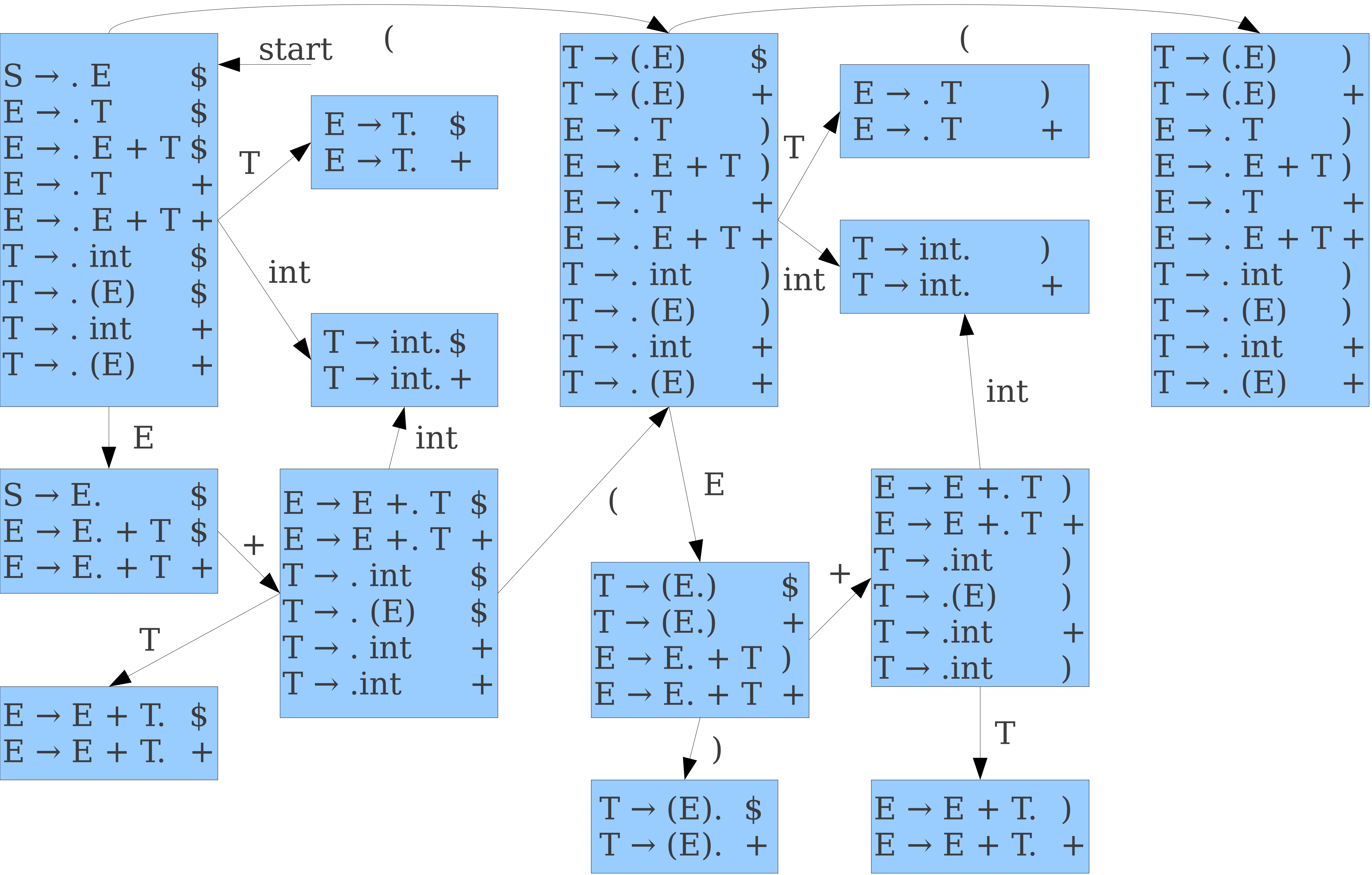
# Deterministic LR(1) Automata



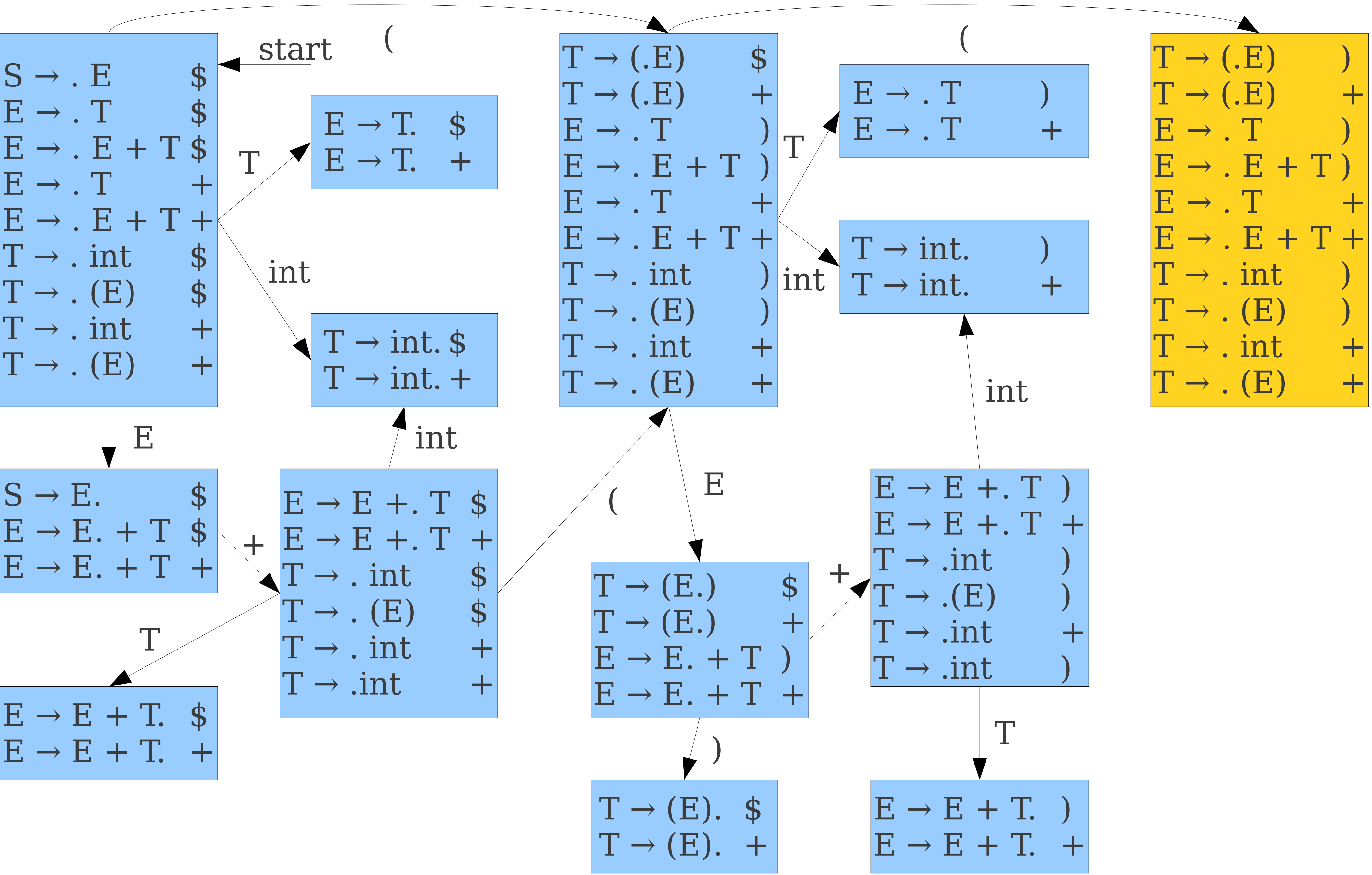
# Deterministic LR(1) Automata



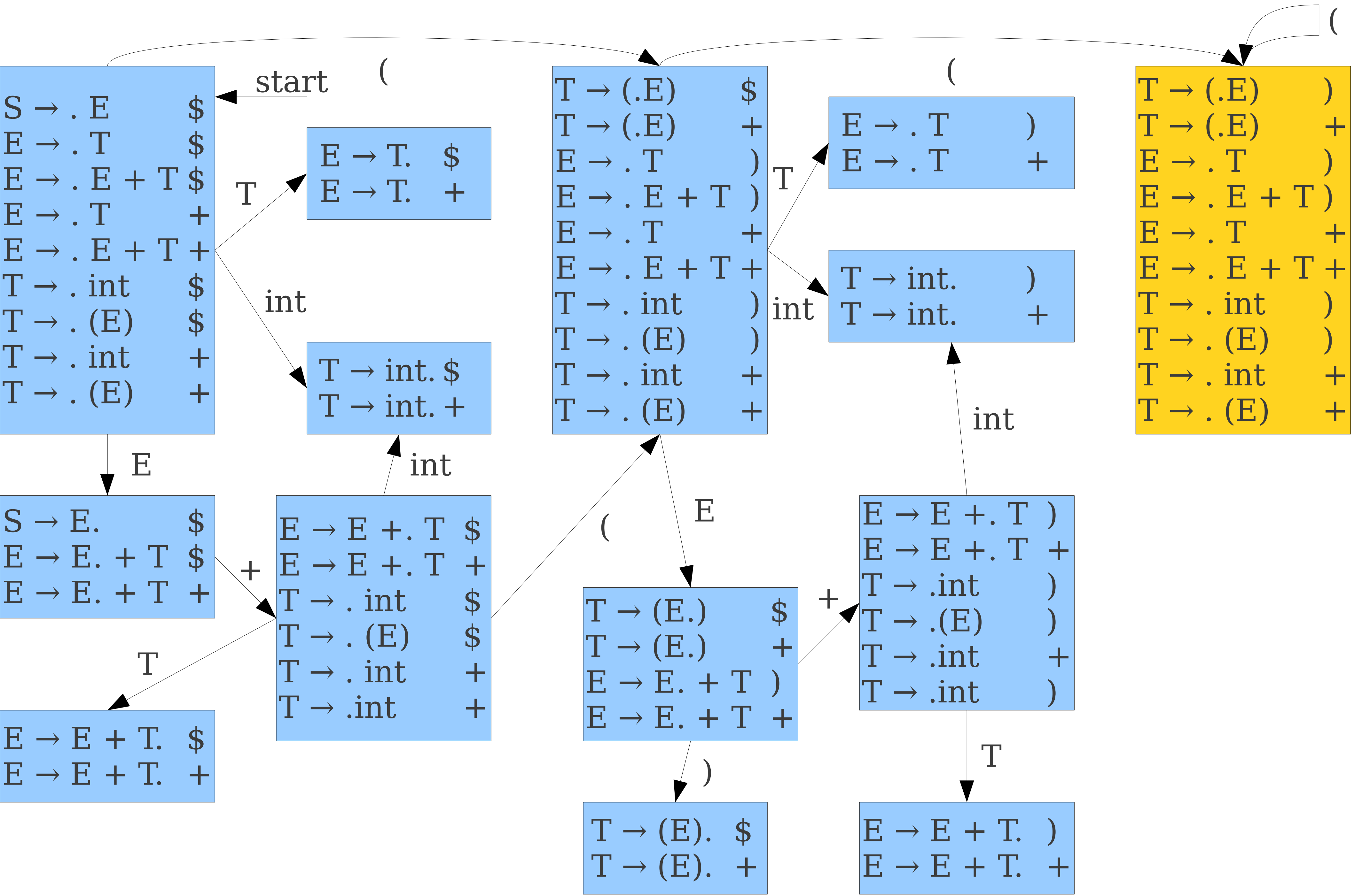
# Deterministic LR(1) Automata



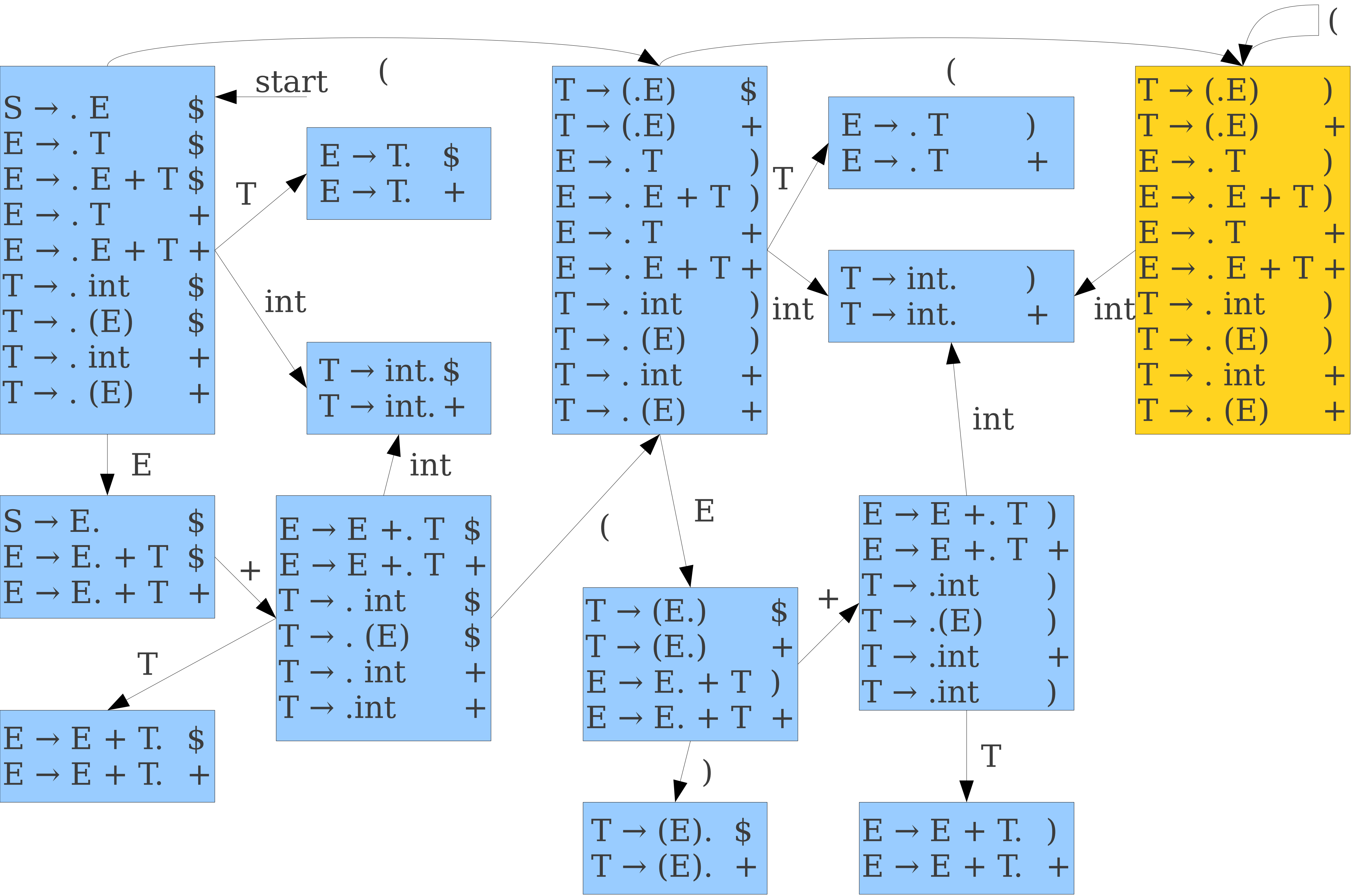
# Deterministic LR(1) Automata



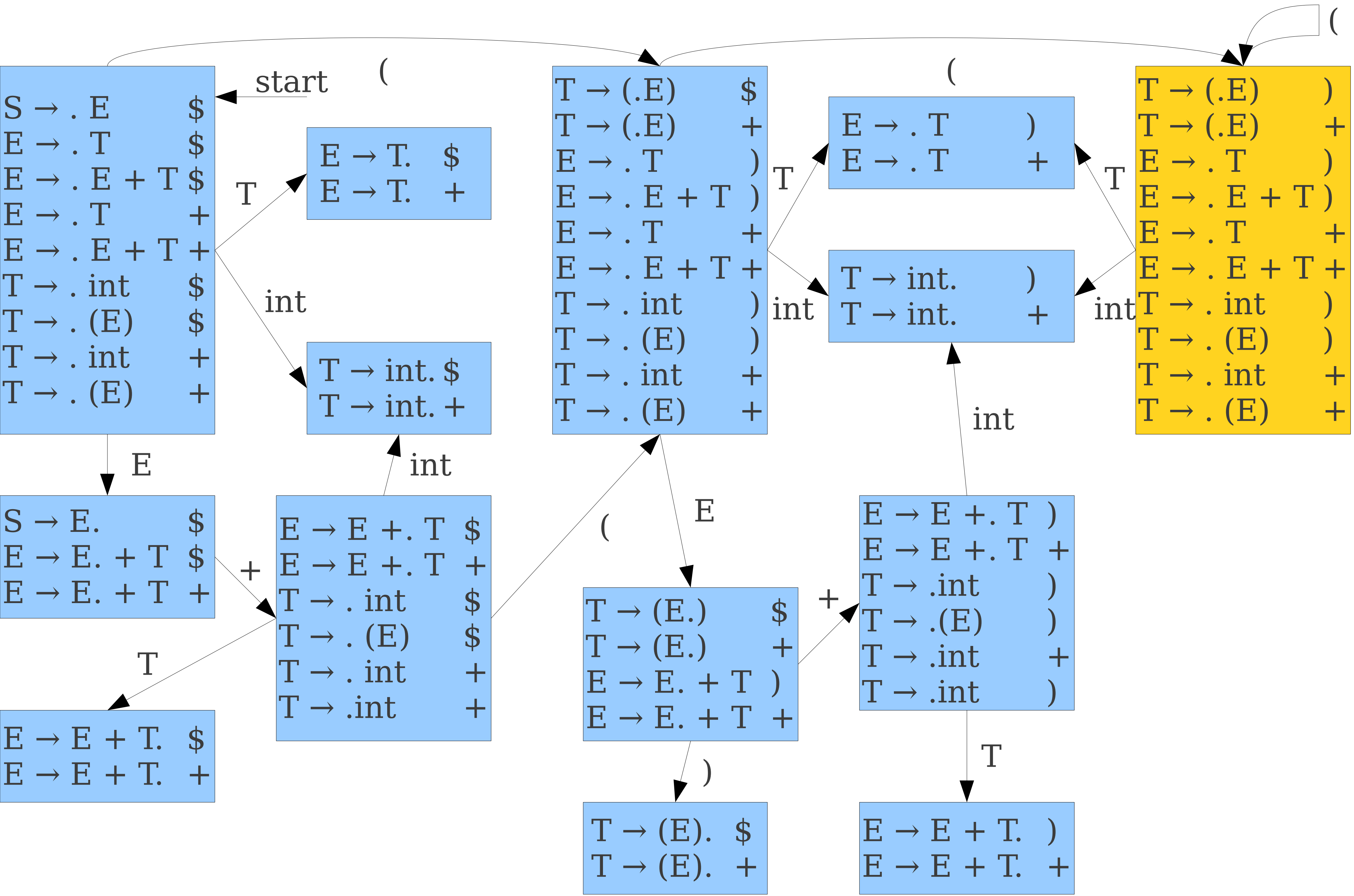
# Deterministic LR(1) Automata



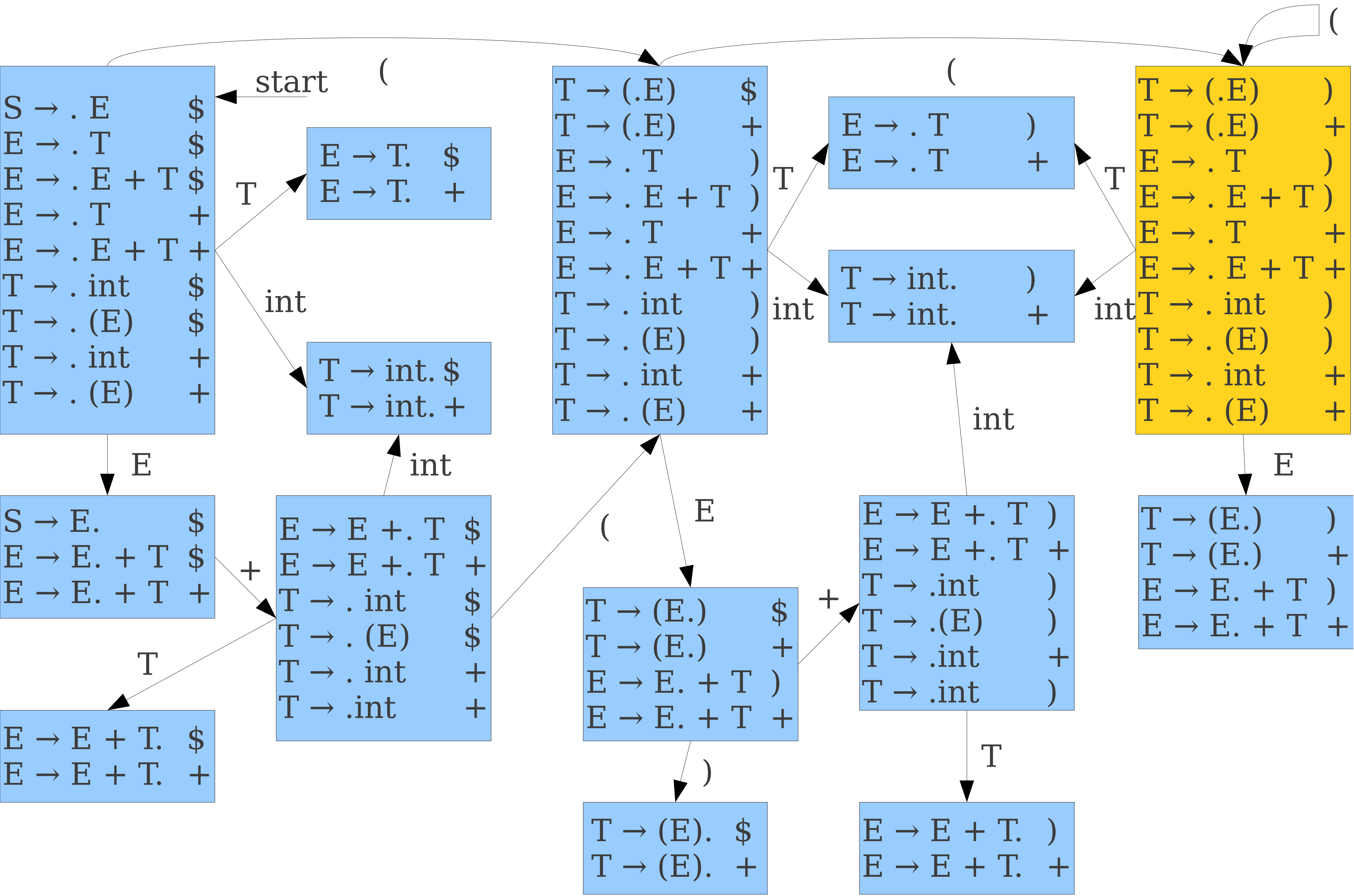
# Deterministic LR(1) Automata



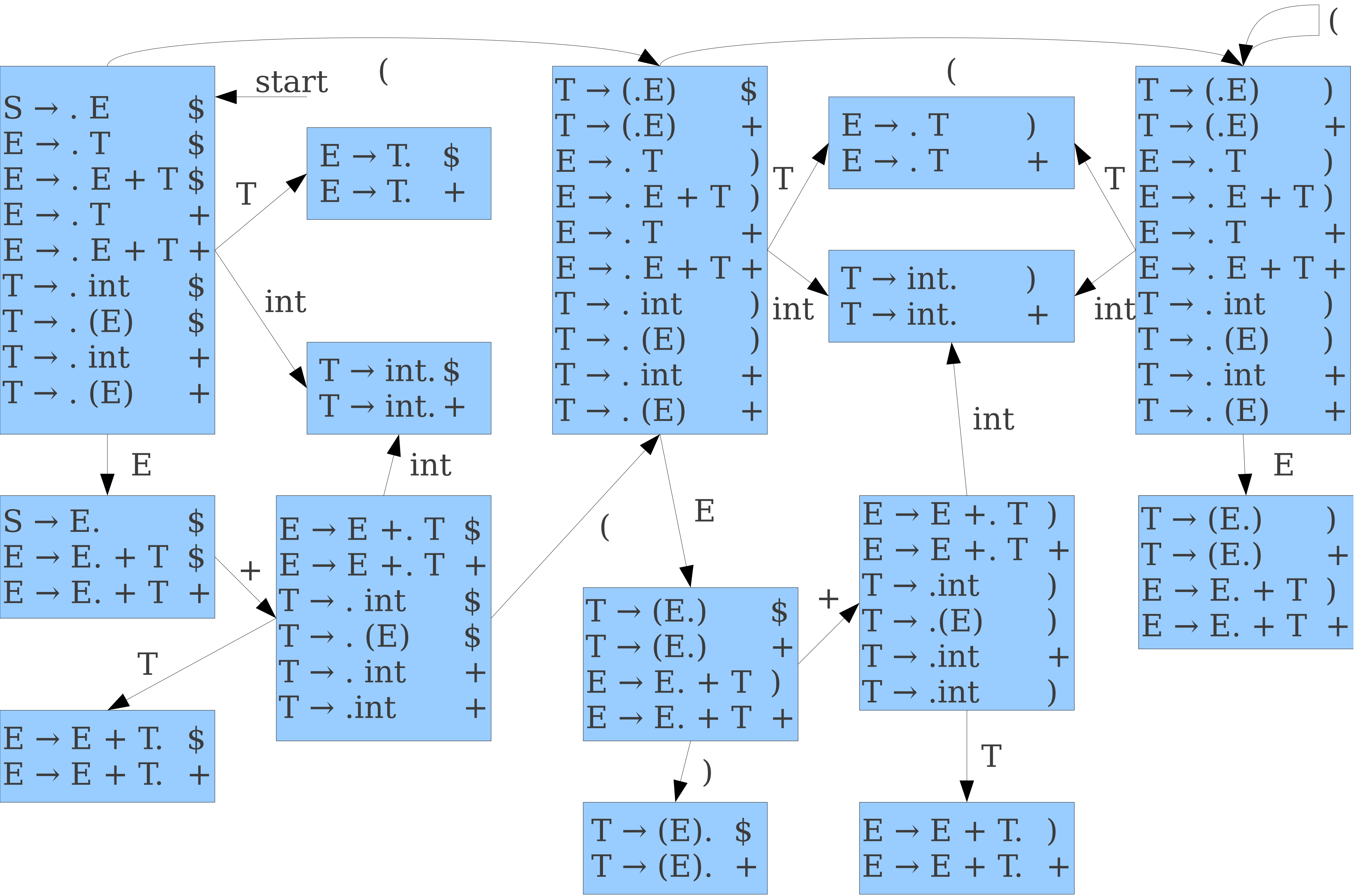
# Deterministic LR(1) Automata



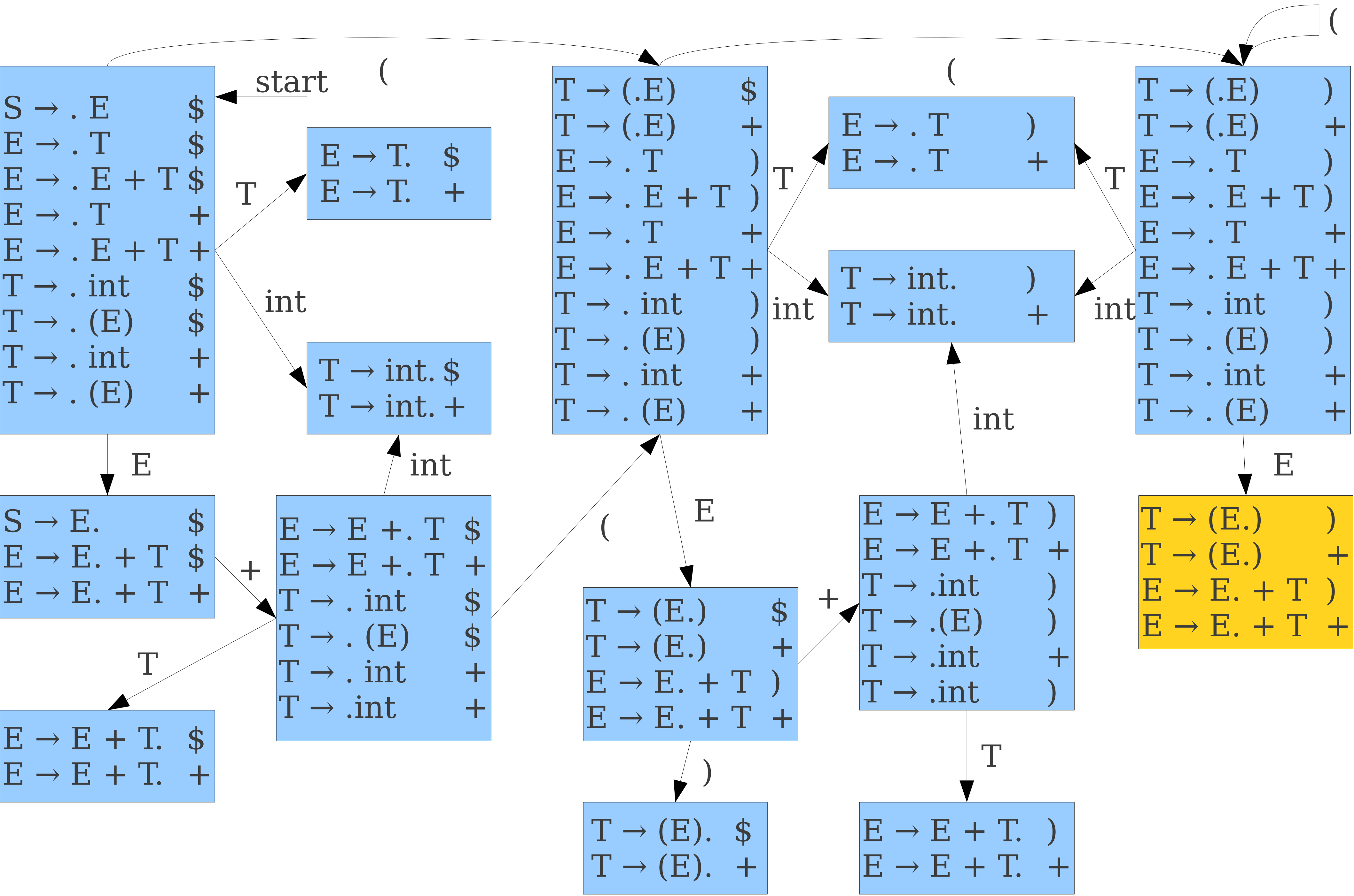
# Deterministic LR(1) Automata



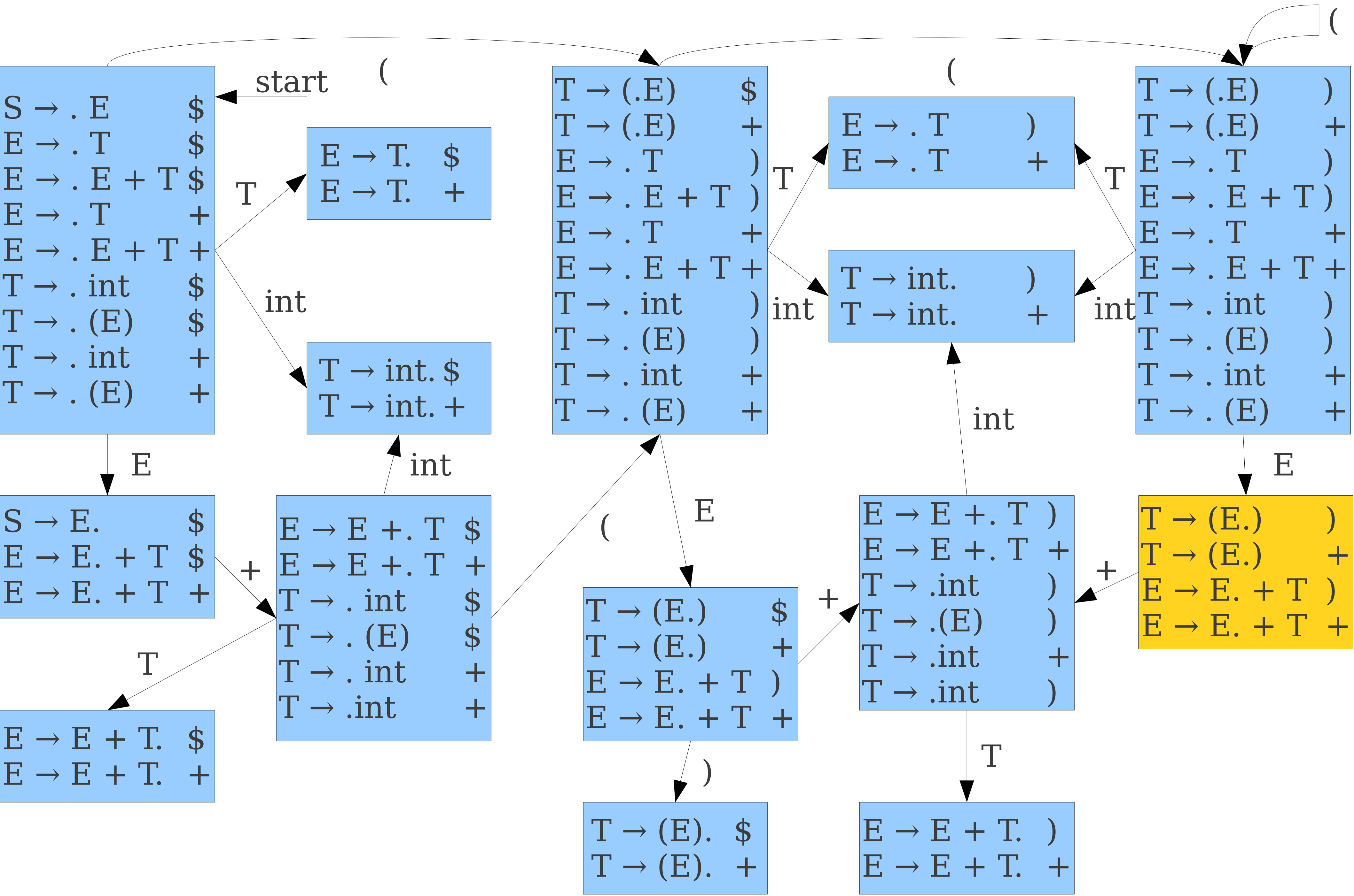
# Deterministic LR(1) Automata



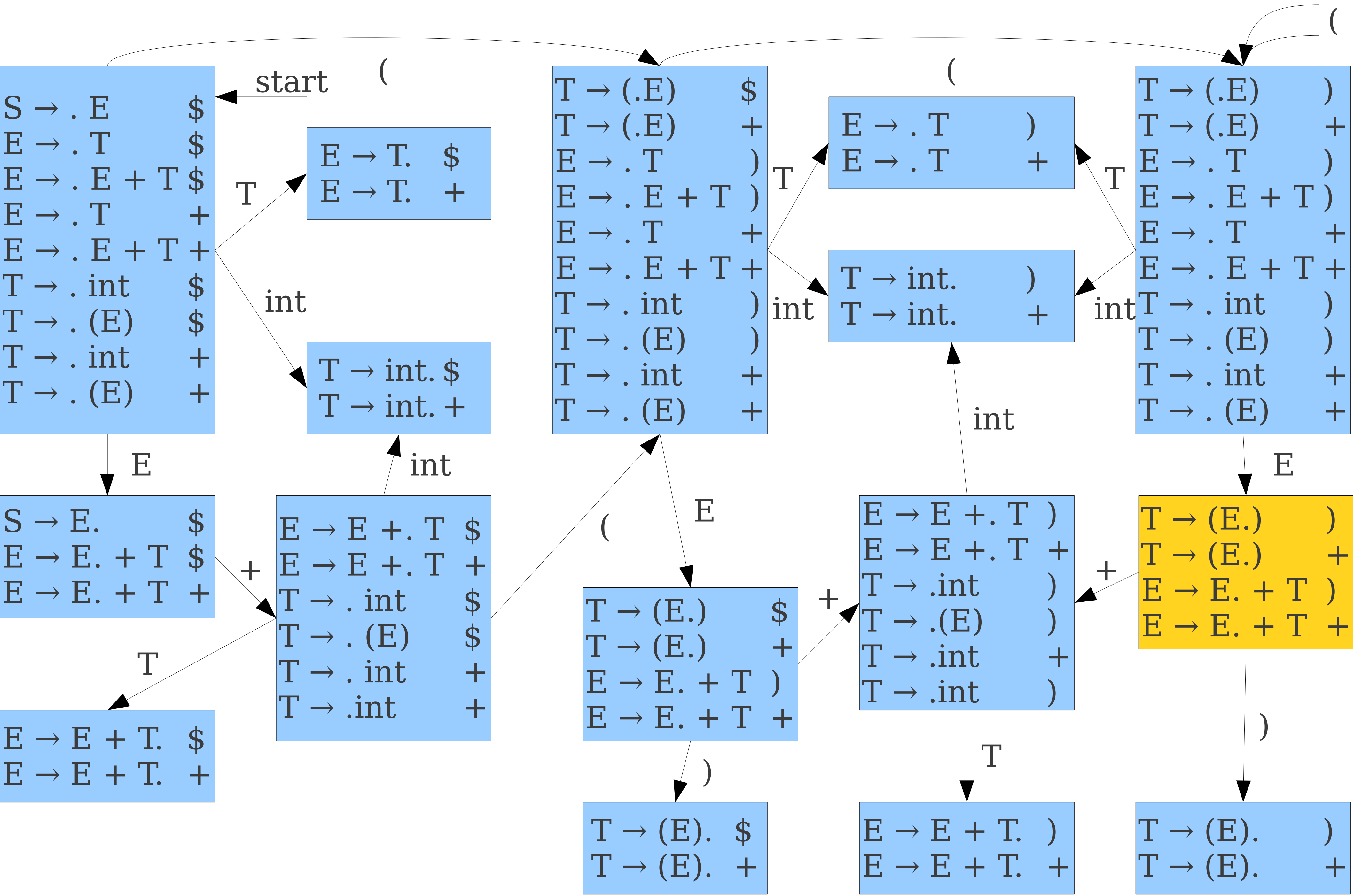
# Deterministic LR(1) Automata



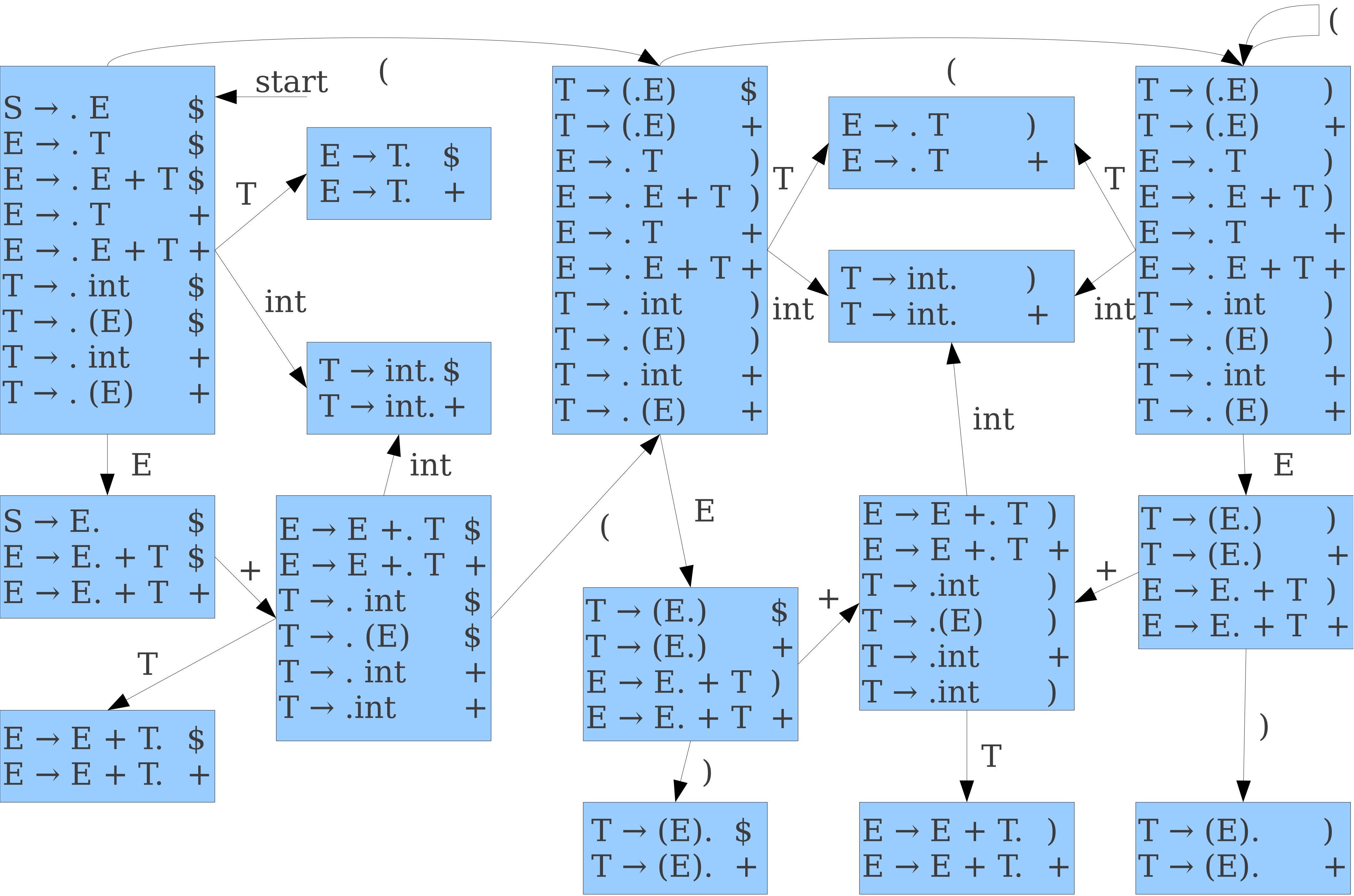
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Constructing LR(1) Automata II

- Begin in a state containing **S** → · **E** [**`$`**], where **S** is the start symbol.
- Compute the **closure** of the state:
  - If **A** → ***α*** · **B*ω*** [**`t`**] is in the state, add **B** → · ***γ*** [**`t`**] to the state for each production **B** → ***γ*** and for each terminal **`t`** ∈ FIRST*(***ω*****`t`**)
- Repeat until no new states are added:
  - If a state contains a production **A** → ***α*** · ***xω*** [**`t`**], add a transition on ***x*** from that state to the state containing the closure of **A** → ***αx*** · ***ω*** [**`t`**].

# Structure of LR(1) Automata

- Every LR(1) automaton simulates two processes simultaneously:
  - An **LR(0) automaton** for finding handles.
  - A **lookahead tracker** for determining what the lookahead is.
- Removing the lookaheads from an LR(1) automaton results in a (much larger) LR(0) automaton for the same grammar.

# Representing LR(1) Automata

- As with LR(0), use **action** and **goto** tables.
- **goto** table defined as before; encodes transition table as map from (state, token) to states.
- **action** table maps pairs (state, lookahead) to actions.
- Commonly combined into a single **action/goto** table.

S → E (1)
E → T (2)
E → E + T (3)
T → int (4)
T → (E) (5)

| | int | ( | ) | + | $ | T | E |
|---|---|---|---|---|---|---|---|
| 1 | s5 | | | | | s4 | s2 |
| 2 | | | | s6 | ACCEPT | | |
| 3 | | | | r3 | r3 | | |
| 4 | | | | r2 | r2 | | |
| 5 | | | | r5 | r5 | | |
| 6 | s5 | s7 | | | | s3 | |
| 7 | s10 | s14 | | | | s10 | s8 |
| 8 | | | s9 | s12 | | | |
| 9 | | | | r5 | r5 | | |
| 10 | | | r2 | r2 | | | |
| 11 | | | r4 | r4 | | | |
| 12 | s11 | | | | | s13 | |
| 13 | | | r3 | r3 | | | |
| 14 | s11 | | s14 | | | s10 | s15 |
| 15 | | | s16 | s12 | | | |
| 16 | | | r5 | r5 | | | |

# The LR(1) Parsing Algorithm

- Begin with an empty stack and the input set to ***ω*$**, where ***ω*** is the string to parse. Set **state** to the initial state.
- Repeat the following:
  - Let the next symbol of input be **t**.
  - If **action[state, t]** is **shift**, then shift the input and set **state = goto[state, t]**.
  - If **action[state, t]** is **reduce A → *ω***:
    - Pop |***ω***| symbols off the stack; replace them with **A**.
    - Let the state atop the stack be **top-state**.
    - Set **state = goto[top-state**, **A]**
  - If **action[state, t]** is **accept**, then the parse is done.
  - If **action[state, t]** is **error**, report an error.

# Constructing LR(1) Parse Tables

- For each state $X$:
  - If there is a production **A** → $\boldsymbol{\omega}$ · [t], set **action[**$X$**, t]** = reduce **A** → $\boldsymbol{\omega}$.
  - If there is the special production **S** → **E** · [$], where **S** is the start symbol, set **action[**$X$**, t] = accept**.
  - If there is a transition out of s on symbol t, set **action[**$X$**, t] = shift**.
- Set all other actions to **error**.
- If any table entry contains two or more actions, the grammar is not LR(1).

# Next Time

- **SLR(1) Parsing**
  - A smaller, simpler, and weaker variant of LR(1).
- **LALR(1) Parsing**
  - An excellent tradeoff between SLR(1) and LR(1).
- **Parsing Ambiguous Grammars**
  - Manually tweaking LR parsers.
- **Error Recovery**
  - Report all the errors!